

SECRETS OF THEIR SUCCESS: A MULTIPLE CASE STUDY OF
MATHEMATICALLY PROFICIENT HOMESCHOOL GRADUATES

by

Betty Roberts Cardinale

Liberty University

A Dissertation Presented in Partial Fulfillment

Of the Requirements for the Degree

Doctor of Education

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ABSTRACT

The purpose of this multiple case study was to explain how mathematically proficient homeschool graduates acquired their proficiency. Bronfenbrenner's (1979, 1999) ecological and Process-Person-Context-Time (PPCT) models were used as theoretical frameworks to analyze the development of the graduates in the homeschool environment. The National Research Council's (NRC) Five Strands of Mathematical Proficiency (Kilpatrick, Swafford, & Findell, 2001) was used as a conceptual framework. I collected four types of data: (a) surveys; (b) National Assessment of Educational Progress; (c) Mathematical Self-Efficacy Scale scores; and (d) interviews. These data items were analyzed with explanation building, a special case of pattern-matching logic. Many themes emerged from this research: strong nuclear families, direct teaching, self-study or self-directed education, mastery learning, purposive conversations, and challenging curriculum (Saxon). As they matured, these graduates built upon the foundation set by their parent educators and took ownership of their studies.

Descriptors: homeschooling; bio-ecological theory; PPCT model; mathematical proficiency; case study; explanation building; MSES; SAT; NAEP; Saxon curriculum

Dedication

As I complete this section, I am reminded of the words of Elizabeth in Luke 1:25, “The LORD has done this for me.” I would not be at this place without His grace, mercy, and guidance. He deserves all the praise.

I dedicate this work to the homeschooling families who blazed the trail and those who will continue. Also, I want to honor the homeschooling families and their graduates who made this research possible. Without their input, this project would not have been accomplished.

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List of Abbreviations

Academic Engaged Time (AET)

Bob Jones University Press (BJUP)

California Learning Assessment System (CLAS)

General Education Development (GED)

Grade Point Average (GPA)

Home School Legal Defense Association (HSLDA)

Iowa Test of Basic Skills (ITBS)

Mathematics Self-Efficacy Survey (MSES)

National Assessment of Educational Progress (NAEP)

National Assessment Governing Board (NAGB)

National Council of the Teacher of Mathematics (NCTM)

National Mathematics Advisory Panel (NMAP)

National Research Council (NRC)

Process, Person, Context, Time (PPCT)

Science, Technology, Engineering, Mathematics (STEM)

Tampere University of Applied Science (TAMK)

University Model School (UMS)

University of Chicago School Mathematics Project (USCMP)

CHAPTER ONE: INTRODUCTION

This dissertation utilized qualitative analysis to examine the acquired mathematical proficiency of homeschool graduates. The first chapter of the dissertation presents the background of the study, identifies the call to research for this topic and the corresponding gap in the literature, classifies the problem that the study addresses, clarifies the researcher's relationship to the problem, and describes the study's significance. The chapter concludes with the study's definitions, delimitations, a brief research plan, and the justification for this research.

Background

Ray (2011) approximated the current number of homeschoolers at 2.04 million. Compared to the Census Bureau's latest figure of 54.117 million United States school-aged children, Ray (2011) estimated that 3.8% of that population is homeschooled (also referred to as home educated). The growth of homeschooling, in the past several years has been dramatic. The US Department of Education's National Center for Education Statistics (NCES) found that this population has increased from 850,000 in 1999 (its first year of recording that information), to 1.5 million in the spring of 2007 (NCES, 2008), a 76.5 % increase over 8 years. This amount exceeded the individual school-age populations (5-17 years old) of all but five states—California, Florida, Illinois, New York, and Texas (Annie E. Casey Foundation, 2013), which justifies investigation.

The consensus of homeschooling literature suggested that homeschooling provides students with a comparable education to conventional schooling. In a seminal study by Rudner (1999) and a follow-up study by Ray (2010), the academic achievement (measured in both mathematics and English) of homeschoolers was perceived to be as

good or better on standardized tests than non-homeschoolers. Additional studies indicated that these alternatively educated youth were as well prepared for college, if not better prepared, than their conventionally schooled peers (Bolle, Wessel, & Mulvihill, 2007; Goodman, 2008; Jones & Gloeckner, 2004; Ray, 2000, 2010).

Findings from several quantitative studies suggests that homeschool learners perform at least as well as or better than their conventionally schooled counterparts. Ray (2010) stated, “to date, no research has found homeschool students to be doing worse, on average, than their counterparts in state-run schools” (p. 11). K-12 homeschool students showed high academic achievement on standardized tests and exhibited collegiate preparedness (Bagwell, 2010; Basham, Merrifield, & Hepburn, 2007; Clemente, 2006; Jones & Gloeckner, 2004; Klicka, 2004; Ray, 2000, 2004, 2010; Rudner, 1999; Van Pelt, Allison, & Allison, 2009). A limited amount of research dealt with special education homeschooling and those findings indicated that students with special needs benefit from school at home (Duffy, 2002; Duval, Delquadri, & Ward, 2004; Ensign, 2000; Ray, 2002). In their research, Duval et al. (2004) found that the homeschool students with special needs engaged in more Academic Engaged Time (AET) and less competitive behavior than their non-homeschooled peers. According to the overwhelming majority of homeschooling socialization literature, self-concept, community involvement, work ethic, and social group behavior of home-educated students (both college and K-12) appeared to surpass that of their conventionally schooled peers (Basham et al., 2007, Bolle et al., 2007; Byers, 2005; Goodman, 2008; Medlin, 2000; Ray, 2004; Shyers, 1992; Thomas, 1998; 2002). Results indicate that homeschooling benefited these students.

Available homeschooling research suggests that homeschooled children perform as well as or better than conventionally educated youth in mathematics and English

(Clemente, 2006; Ray, 2000; 2010; Rudner, 1999). Notably, the difference was more pronounced in verbal standardized college entrance tests than in mathematics ones. Both the 2000 SAT Verbal and Math averages of homeschoolers were above the national average, but the 63 point Verbal score difference was much greater than the 18 point Math score difference (HSLDA, 2001). Bagwell (2010) found that homeschoolers performed significantly better in reading and writing parts of the standardized college COMPASS tests, but not in the algebra subtests.

Current research suggests that at least some homeschool graduates feel unprepared for college mathematics. Jones (2010) found that 25% of surveyed homeschool graduates—as compared to 24% of public school graduates and 20% of private school graduates—wished that they had been better prepared for math in college. In the qualitative phase of his research, Bagwell (2010) investigated the feelings of selected students concerning their mathematical preparedness and discovered that all of them, both homeschooled and public-schooled, expressed a lack of confidence in their mathematical ability. Bagwell (2010) contended that these results would trigger future dialogue concerning mathematical preparation in homeschools.

Some researchers describe, in-depth, the types of instruction that home educators provide. Homeschooling instructors utilize flexible schedules and structures, adapting to the learner (Meighan, 1995; Patterson et al., 2007; Williams, 1991). From survey data, Williams (1991) established a recurring pattern in homeschooling instructors, flexible approaches with a moderate to high level of autonomy, encouraging intrinsic motivation. These respondents believe learning is a process, not a goal. Informal learning took place in homeschooling families (Meighan, 1999; Thomas 1998). Home educators selected curriculum that corresponded to the individual needs of the student and the instructor,

changing texts and methods as needed (Clements, 2002; Hanna, 2011; Meighan, 1995, 1999; Patterson et al., 2007). They valued rigor and repetition (Clements, 2002) and believed that immediate feedback was critical for promoting interest and engagement and utilized various assessments to evaluate their students' retention and curriculum effectiveness (Banner, 2007). Also, they taught for complete mastery while acknowledging learning styles. Martin-Chang, Gould, and Meuse (2011) suggested that more structured homeschooling instruction improved standardized test scores among younger children. Shepherd (2010) described the homeschooling instructional pedagogy he found as traditional – rote learning, extensive memorization, and worksheet practice. Hanna (2011) showed increased usage of computer technology and networking of home educators over a 10-year period. Banner (2007) suggested that future research needs to discover and report educational best practices in alternative educational systems, asserting, “Rigorous studies, both qualitative and quantitative, should be conducted to seek out and reveal the best practices in assessment, curriculum design, curriculum evaluation, and teaching techniques of educators in domains outside of mainstream higher education” (Banner, 2007, p. 66). Shepherd (2010) recommended further in-depth study of homeschool instruction, stating

The curricular plan and instructional approaches are the areas most in need of greater, more formal study on the part of educators. . . . What is lacking is greater insight into how they accomplish as successfully as they do. . . . What is the secret of their effectiveness? (p. 103)

Limited current research exists concerning mathematical curricula or instruction in homeschooling families. Consistent parental involvement in correcting work and keeping current with seminars correlated with mathematical achievement on standardized

tests (Richman, Girtten, & Snyder, 1992). Peterson (2011) found that out of 57 homeschool respondents, the most common curricula chosen were Saxon and Math-U-See (tied at 11 votes each). Interestingly, this selection shows an equal split between secular (Saxon) and religious (Math-U-See) programs. Ortiz (2000) reported mathematically stimulating environments, rich in conversation, puzzles, and brainteasers. These learning environments were child-led, interest-driven, conversation-abundant, and full of mutual trust and respect, with no central source of knowledge (e. g., teacher dominated). Studying Science, Technology, Engineering, and Mathematics (STEM) education in homeschooling families, Bachman (2011) investigated their mathematical practices. Confirming Thomas's (1998) findings, Bachman (2011) found the extensive use of practical mathematics applications, such as cooking and shopping. Additionally, Bachman (2011) reported the eclectic utilization of puzzles, games, toys, Lego blocks, manipulative objects, online helps (KhanAcademy.org and other mathematics teacher websites), packaged curriculums, and their own uniquely designed curricula.

There has been a call for research to explore the homeschooler educators' instructional practices and curriculum (Cizek & Ray, 1995; Sutton & Galloway, 2000). Cizek and Ray (1995) surveyed home education researchers. The respondents called for further research, about the instruction home educators provide and how it could be linked to the conventional classrooms. They wanted details about,

what home educators actually do . . . the teaching strategies used by home educators, the quality and effectiveness of home instruction, the role that each of the parents actually plays in home education, or the effects of long term home schooling (including psychological effects) on both students and parents.

Additionally, several respondents indicated that few attempts have been made to

establish linkages between home education research and educational research generally. For example, it would appear that research findings on effective practices in home education would be germane to current debates on general educational reform. . . . The home education research agendas identified in this study may provide a critical testing ground for enlightened practice in other settings if further research shows that what works in home schools may be efficaciously imported into other educational environments, especially in the traditional educational setting. (Cizek & Ray, 1995, pp. 7-8).

Sutton and Galloway (2000) called for future research focusing on the impact of high school curriculum on the college success of students from homeschool compared with non-homeschooled ones. They acknowledged the challenges of conducting quantitative research concerning curriculum,

Future research should also focus on the impact that high school curriculum may have on college success among students coming out of home schools, private schools, and public schools. In a rare 10-year longitudinal study, Adelman (cited in Bracey, 1999) discovered that curriculum intensity and quality was the best predictor of college success, not the more traditional predictors of senior class rank, grade point average, aptitude test, or even socioeconomic status. Controlling for variance in curriculum used by students coming from home school settings, as opposed to more standardized curricula found in public and private school settings, may be problematic for investigators, however (Sutton & Galloway, 2000, pp 144-145).

There has been a call for research to further investigate the homeschool mathematics-learning environment. Ortiz (2000) recommended that further study

concentrate on why homeschoolers succeed on standardized achievement testing, proposing, “New research, instead of focusing on validating homeschooling learning environments as successful learning environments, could do a huge service by concentrating on being able to explain in more detail why homeschoolers are successful on these standardized achievement test” (p. 143). Likewise, in her conclusion and discussion of homeschool research involving comparison of college SAT scores, Clemente (2006) posited, “Have home educators latched onto something we should be paying attention to, if so, then what?” (p. 47). Based on his quantitative and qualitative research concerning college students, Bagwell (2010) described a need for future research in the area of mathematical instruction and homeschooling. Bachman (2011) challenged homeschool and mathematics researchers to investigate the mathematical learning activity of homeschoolers, writing,

It is worth looking closer and more specifically at homeschool mathematics learning activity. Is “everyday” or life embedded mathematics activity incidental mathematics learning or more intentional for homeschooling families? It seems possible “everyday” mathematics could be different for home-educating families than for schooling families. How are academic and lifelong mathematics learning goals playing out (p. 405)?

Additionally, she made a general charge to researchers and educators, to go beyond the simplistic view of what is learned, and “to include why, how, where, when, with what tools, and with whom do learners learn, a systems view emerges that reveals the rich diversity and potential of family-led learning” (Bachman, 2011, p. 407).

Very little recent research exists that gives the perspectives of the homeschool student. Van Schalkwyk and Boucher (2011) interviewed younger homeschool students

about their homeschool experience, utilizing Bronfenbrenner's (1999) bio-ecological model as her theoretical framework. She found inconsistency between the parent's perception of the homeschool environment and the student's. Ortiz (2000) conducted structured interviews with the students concerning their views on mathematical topics: math anxiety, feelings about math, and advantages and disadvantages of learning math in a homeschool setting. The parents and the children appeared to share similar positive attitudes toward the homeschool situation. Goodman (2008) surveyed and interviewed homeschool graduates about their homeschool learning environments, finding three dominant patterns—personal, familial, and autonomous—that derived from the personalities, interests, and family responsibilities of the students and parents.

There has been a call for further investigation for the perspective of the homeschool student and a mathematical proficient student. Research on the perspective of the homeschool student is important and needed (Goodman, 2008; Green & Hoover-Dempsey, 2007; Shepherd, 2010). Perceptions of the mathematically proficient student would be beneficial to research literature (Schoenfeld, 2007). No current in-depth case study research exists that reports how mathematically proficient students (homeschooled or non-homeschooled) acquired their proficiency.

There is a gap in the literature for what works in homeschool mathematical instruction, as well as the perspective of the homeschool student. Although research seems to indicate that homeschoolers excel when compared with non-homeschoolers, some research suggests that the spread in homeschool mathematical achievement is markedly lower than that of language skills. Mathematical proficiency best practice is necessary research, since mathematically proficient citizens are critical to the United

States (National Mathematics Advisory Panel [NMAP], 2008). Furthermore, there is need for investigation from the perspective of the mathematically proficient student.

Situation to Self

In conducting credible research, the researcher must make the paradigm and philosophical assumptions explicit. Researchers need to articulate the paradigm—post-positivism, constructivism, advocacy/participatory, or pragmatism—that they employ (Creswell, 2007). Within each paradigm, five philosophical assumptions of qualitative research exist. They are ontology, epistemology, axiology, rhetorical, and methodology. In this case study, I mainly used the social constructivism/interpretivism paradigm to interpret the meaning that the homeschool graduates give to their mathematical proficiency. Since I also wanted to provide potential best practices in homeschool mathematics, I also considered the data analysis from a pragmatic viewpoint.

I have taught in public education and been a home educator. Before having children, I taught mathematics and computer programming for 10 years in a New Jersey public high school; at the same time, I conducted teacher in-service on the use of the computer in the classroom. After the birth of my first child, I worked part-time in an adult school helping General Education Development (GED) students pass the mathematics portion of the GED. After the birth of my second child, my family moved to South Carolina. When my first child turned four years old, our family began to homeschool. We have continued to this day, 19 years later. As a math teacher and a home educator, I desire to find what works for mathematical instruction, to offer help to those struggling with math. I interviewed homeschool graduates who scored 600 or above on the SAT college entrance mathematics subtests, to share the “secrets of their success.”

Definitions

The homeschool environment includes: the mathematical experiences of the graduate (e.g., curriculum, instructional practices), the physical locations where the mathematical experiences took place (e.g., home, math class at a co-op, online class, tutor, math club, etc.), the relationships of the student (with parents and siblings and other teachers), and the attitudes of the student and educators toward homeschooling and math, during the graduate's period of homeschooling.

Mathematical proficiency was determined as the ability an individual possesses to comprehend math concepts, be fluent in math operations, represent math problems, justify strategies used in solving math problems, and see mathematics as useful and worthwhile (Kilpatrick, Swafford, & Findell, 2001). This definition was based on the National Research Council's (NRC) Five Strands of Mathematical Proficiency involving: conceptual understanding; procedural fluency; strategic competence; adaptive reasoning; and productive disposition. In this research, mathematical proficiency was measured by a modified form of the National Assessment of Educational Progress [NAEP] Grade 12 mathematics test (Appendix A) for the first four components of the NRC model and the revised Mathematics Self-Efficacy Survey [MSES] (Betz & Hackett, 1993) and interview responses, for the productive disposition..

Problem Statement

Homeschool students outperformed non-homeschool students on standardized tests and college GPAs, both in reading levels and mathematics (Bagwell, 2010; Clemente, 2006; Cogan, 2010; Jones & Gloeckner, 2004; Ray, 2000; 2004; 2010; Rudner, 1999). Clemente (2006) found that homeschoolers scored significantly better than their non-homeschooled peers, on the composite SAT score. Yet, some research

indicated that the spread of mathematical achievement on college entrance exams was not as great as the reading levels (Bagwell, 2010; HSLDA, 2001; Jones & Gloeckner, 2004). Recent research suggested that some homeschooling graduates wished they had been better prepared mathematically (Bagwell, 2010; Jones, 2010). These findings imply that homeschool graduates who scored significantly higher on standardized mathematical achievement tests were a select group. They may be considered mathematically proficient. There is a need for research on the mathematically proficient student's perspective of their proficiency and how they acquired it (Schoenfeld, 2007).

There is a dearth of research available on the perspectives of mathematically proficient homeschooled students. Limited recent research exists that gives the perspectives of the homeschool student (Goodman, 2008; Ortiz, 2000; van Schalkwyk and Bouwer, 2011). Research from the homeschool student's perspective is needed (Green & Hoover-Dempsey, 2007; Shepherd, 2010). No research has been found that studies mathematically proficient homeschoolers from their perspective.

Qualitative inquiry, specifically case study research, allows a researcher to explore the how or why of a situation (Stake, 2006; Yin, 1994; 2009). Qualitative research is needed to understand and explain how these students acquired their mathematical proficiency. A multiple case study of some mathematically proficient homeschool graduates would serve to inform homeschooling families what environments and experiences supported the acquisition of mathematical proficiency in these participants, and if they could be replicated, as well as add to the literature concerning best practices of homeschool mathematical instruction.

Purpose Statement

The purpose of this multiple case study is to understand and explain how some mathematically proficient homeschool graduates acquired their mathematical proficiency. Despite criticism over its predictive validity (FairTest, 2001), the SAT math score positively correlated with college freshmen (.52) and high school (.48) mathematics course grades (Mattern, Patterson, & Kobrin, 2012). College and university admissions have long relied upon this reliable tool for predicting student success and performance (Long, 2011). Regarding the 2012 SAT test taking population, approximately 25% scored a 600 or above on the math section, well above the average of 510 (The College Board, 2012). More years of homeschooling showed more significant differences in achievement tests (Ray 2004, 2010; Rudner 1999; Van Pelt, Allison, & Allison, 2009). Ray (2004) surveyed families who had homeschooled for seven years or more because they exhibited the long-term effects of homeschooling. Based on these factors, the phenomenon examined in this study is the acquisition of mathematical proficiency in home school graduates who scored 600 or above on the SAT math subtest of the college entrance exam and had been homeschooled for seven or more years.

Research Questions

The purpose of this multiple case study is to understand and explain how mathematically proficient homeschool graduates acquired this proficiency. This collective case study focused on the central question: How does the homeschool environment, including attitudes (of both graduate and educators), instructional practices, relationships, and physical surroundings (may include more places than the home) contribute to the mathematical proficiency of homeschool graduates?

The following five subquestions guided this study:

- (a) How do mathematically proficient *homeschool graduates* describe their mathematical homeschool environment?
- (b) How do mathematically proficient *homeschool graduates* describe their mathematical self-efficacy?
- (c) How do *home educators* of mathematically proficient homeschoolers describe their mathematical self-efficacy?
- (d) How do *home educators* of mathematically proficient homeschoolers describe their homeschool graduate's homeschool mathematical environment?
- (e) Are there any apparent patterns or relationships between the descriptions of the mathematically proficient *homeschool graduates* and their *home educators* concerning the homeschool environment and their mathematical self-efficacy?

Significance of the Study

In an era of high stakes testing, strategies and methods that enhance mathematical learning and subsequent proficiency are greatly valued (Ball, 2003; Kilpatrick, et al., 2001; National Council of the Teachers of Mathematics [NCTM], 2000; Schoenfeld, 2007). There is need for further in-depth inquiry into the homeschooling learning environment (Banner, 2007; Shepherd, 2010) and instructional practices of homeschoolers (Cizek & Ray, 1995; Sutton & Galloway, 2000), integrating this inquiry with general educational research (Cizek & Ray, 1995), especially as it relates to the mathematical competence of homeschooling students (Bachman, 2011; Bagwell, 2010; Ortiz, 2000). This in-depth inquiry must proceed beyond quantitative research to qualitative research (van Schalkwyk & Bouwer, 2011) to better understand and explain how they acquired their mathematical ability. The perspective of the homeschool student

is underrepresented (Green & Hoover-Dempsey, 2007; Shepherd, 2010), as well as the mathematically proficient student (Schoenfeld, 2007).

Case study research focuses on the how of a phenomenon (Stake, 2006; Yin, 2009). Morgan (1991) conducted case studies of 15 Virginia families that evaluated their coverage of seven areas of socialization listed in the Virginia Middle School guidelines and found that the families covered the seven areas, some with specialized curriculum and some without. Morgan (1991) asserted that the findings of her research would provide the home school community with information that would help them “to evaluate their own methods and improve the quality of their instruction” (p. 5).

In this multiple case study, the NRC’s Five Strands of Mathematical Proficiency (Kilpatrick et al., 2001) served as the guiding model for mathematical proficiency. The meaning that mathematically proficient homeschool graduates gave to their homeschool mathematical experiences, their attitudes towards mathematics, and their mathematical self-efficacy was examined. Did the homeschooling families interviewed facilitate the five strands of mathematical proficiency set forth by the NRC’s 2001 report? As with the Morgan (1991) study, home educators would be able to utilize the results of this study to assess their own practices and improve the quality of their mathematical instruction. Results of this study would also provide information to public and private school educators seeking to provide services to homeschooling families, as well as the parents who seek to enrich the mathematical learning of their conventionally educated students.

Delimitations

Placing boundaries on a case study keeps the research from becoming too broad or having too many objectives (Stake, 1995). Binding a case in terms of time and activity kept the research reasonable in scope. Purposely selected participants bounded the case

to keep the research manageable and align it with the goals of the study. In this study, participants were selected on the basis of their math subtest SAT scores (600 and above) and the number of years homeschooled (seven or more).

To improve internal validity, Yin (1994, 2009) advocates the use of several cases. Two to three cases can be used for literal replication, and four to six additional cases for theoretical replication. In this study 16 mathematically proficient homeschool graduates were selected, to provide a rich amount of data, looking for commonalities.

Research Plan

A qualitative approach provides robust information about a phenomenon (Ary, Jacobs, Razavieh, & Sorensen, 2006) and case study focuses on small samples, for rich description of variables and environments (Bodgan & Bilkan, 2007). Multiple case studies provide information from individual cases and cross-case analysis (Stake, 2006; Yin, 1994, 2009), facilitating literal or theoretical replications (Yin, 1994, 2009). This multiple case study focused on how mathematically proficient homeschool graduates acquired their proficiency.

Yin (1994) advocates the use of pilot studies, “assisting an investigator to develop relevant lines of questions—possibly even providing some conceptual clarification for the research design as well” (p. 74). Initially, I conduct a pilot study with one graduate that refined the survey and interview process. The homeschool graduate filled out the Mathematics Self-Efficacy Scale [MSES] (Betz & Hackett, 1993), took a modified National Assessment of Educational Progress (NAEP) test (Appendix A), and answered interview questions (Appendix B), based on current literature, at their home. The home educators filled out a survey about the homeschool environment (Appendix C) based on

current literature, provided SAT scores and a final transcript, and also completed the MSES (Betz & Hackett, 1993).

Stake (2006) and Yin (1994, 2009) advocate the use of purposeful sampling to bind the study. After this pilot study, 16 other homeschooling participants were recruited using purposeful sampling. Four gate-keeping organizations sent out an email describing this case study research to their homeschooling constituents, with my contact information. One gatekeeper provided two qualifying participants. Through my personal contacts, I gathered 14 graduates. Altogether, 16 graduates, who scored 600 or above on the SAT math subtest and had been homeschooled for seven or more years, participated. Each graduate was considered as an individual case. Both parents of each student also participated for the purpose of better understanding the graduate.

Explanation pattern logic design was used to examine the data, using within case and cross case analysis (Yin, 1994, 2009). NAEP and the MSES (Betz & Hackett, 1993) scores were used to evaluate the mathematical proficiency of the homeschool patterned after the NRC's five strands of mathematical proficiency (Kilpatrick et al., 2001). Bronfenbrenner's (1979, 1999) ecological and PPCT models, were used to analyze the homeschool environment, as explained through the voice of the participants and the meaning the homeschool graduates give to their homeschool mathematical experiences.

Summary

There is a need for case study research examining how some mathematically proficient homeschool graduates acquired their proficiency. Given the tremendous growth of homeschooling in the United States over the past forty years, this alternative to conventional schooling has not been a passing fad, as many educational reform movements have been. Homeschooling research has consistently demonstrated that

homeschooling students have performed as well or better than non-homeschooling students. Most of this research has been quantitative and researchers have called for a more in-depth look at this success and what home educators are doing to promote it (Bachman, 2011; Clemente, 2006; Cizek & Ray, 1995; Ortiz, 2000; Shepherd, 2010). There is a critical need of best practices of instruction (Banner, 2007), especially in mathematics (NMAP, 2008). Furthermore, the perspective of the homeschool student is limited in research (Ortiz, 2000; van Schalkwyk, 2011) and the perspective of the mathematically proficient student was underrepresented in the literature (Schoenfeld, 2007). This research seeks to fill the void in those areas. Case studies can be used to develop possible explanations for a phenomenon (Gall, Gall & Borg, 2003). The purpose of this multiple case study is to explain how mathematically proficient homeschool graduates became proficient, from their perspective.

CHAPTER TWO: LITERATURE REVIEW

Introduction

Case study research needs a general strategy to assist in data analysis (Yin, 2009). A review of the homeschooling literature sets the stage for explaining the environment of the homeschool and the potential acquisition of mathematical proficiency in some homeschool graduates. The use of theoretical and conceptual frameworks provides researchers a basis for explanation building. These two frameworks provide a basis for analyzing the acquired mathematical proficiencies of homeschool graduates. The theoretical serves as a lens for the homeschool environment and the conceptual model sets the standard for assessing mathematical proficiency.

Theoretical Framework

By definition, the organic environment of homeschooling is the home. The location of this schooling seems inherently fitting for positive human development and ultimately has educational value. A theoretical model of the effects of the structural environment is attributed to the developmental psychologist, Urie Bronfenbrenner. This model seems to fit the naturalistic setting of home education.

Bronfenbrenner (1999) proposed that human development takes place in proximal processes,

Especially in its early phases, and to a great extent throughout the life course, human development takes place through processes of progressively more complex reciprocal interaction between an active, evolving biopsychological human organism and the persons, objects, and symbols in its immediate external environment. To be effective, the interaction must occur on a fairly regular basis

over extended periods of time. (p.5)

Bronfenbrenner (1999, 2000) used recurring parent-child and child-child activities as examples of these types of interactions. Bronfenbrenner (1979) identified the powerful relationships of primary dyads, in which two people formed an influential bond that exists even when they are not in the same setting. Bronfenbrenner (1979) asserted that a “child is more likely to acquire skills, knowledge, and values from a person with whom a primary dyad has been established than from one who exists for that child only when both are actually present in the same setting” (p. 58). Home educators have had the opportunity to form primary dyads with their students, setting up the optimal environment for their children’s human development.

In related research, older children living with both biological parents, on the average, earned the highest grade point average (GPA) than those students in single parent, mother homes or single-parent mother and stepfather homes (Bronfenbrenner, 2000). This finding appeared in homeschooling research. Planty et al. (2009), Ray (2010), and Rudner (1999) found that married parents led most homeschooling families in their sample populations (89.4%, 97.9%, and 97.2%, respectively) and that variable correlated significantly with high-standardized test scores (Ray, 2010; Rudner, 1999). Importance of the home environment may be the key to the apparent success of homeschoolers. Results from available homeschooling literature indicate that achievement and socialization of homeschool students matched or exceeded that of conventionally schooled students (Blok, 2004; Klicka, 2004; Morgan, 1991; Ray, 2000, 2004, 2010; Rudner, 1999; Shepherd, 2010; Shyers, 1992).

Along with the idea of proximal practices, Bronfenbrenner (1999, 2000) purported a second, corresponding proposition. The person’s characteristics, the

environment (immediate and distant), the person's emerging qualities, and the social continuities of historical change over time shaped human development. He labeled the most familiar, common environment, a microsystem, and called increasingly distant environments, mesosystems, exosystems, and macrosystems, the basis for the original ecological theory of human development (Bronfenbrenner, 1979). Later, he added a time element, the chronosystem (Bronfenbrenner, 1994). Homeschooling takes place in the microsystem (home) of the educator and student, with extensions to the mesosystems where extra-curricular activities do occur.

The microsystem involved social roles, interaction with physical things, interpersonal relationships, and the pattern of activities experienced by the developing individual's immediate surroundings (Bronfenbrenner, 1994). Family, school, peer group, and workplace were examples of microsystems. Proximal processes, increasingly more complex recurring types of interaction with the immediate environment (people or objects), activated and sustained the person's development. In the home, proximal processes were witnessed in "parent-child and child-child activities, group or solitary play, reading, learning new skills, studying, athletic activities, and performing complex tasks" (Bronfenbrenner, 1994, p. 38).

Homeschooling involves all these activities. Since homeschooling depends heavily on the family and the home environment, that setting is a significant microsystem of the homeschooled child. Relevant research on the microsystems has centered on the family, not with other important developmental situations like classrooms and schools (Bronfenbrenner, 1994). Studying the homeschooler in the context where family life and schooling is an overlapping microsystem would provide developmental information.

Bronfenbrenner (1994) classified the mesosystem as “the linkages and processes taking place between two or more settings containing the developing person” (p. 40). For example, these are the relations between the classroom and home, or the workplace and school. It is a system of microsystems. Within the homeschool community, the mesosystem involves the interaction of the homeschooled child’s home environment with the other supplemental settings such as community service, athletics, music lessons, co-operative classes, and homeschooling social events. The homeschooler is actively involved within each these settings, unlike the exosystem.

An exosystem involves the connection between two or more environments, where at least one of the settings does not contain the developing person (Bronfenbrenner, 1994). The events of that setting (or settings) directly influence the environment containing the developing individual. Specifically, three exosystems that affect child development are the parent’s workplace, the family’s social networks, and neighborhood and community happenings. Within the homeschool community, most families have one full-time working parent (Planty et al., 2009; Ray, 2010; Rudner, 1999), have regular contact with other homeschooling families (Basham et al., 2007), and participate frequently in extracurricular and community activities (Basham et al., 2007; Ray, 2004). An interrelated collection of micro, meso, and exosystems are considered a macrosystem.

Bronfenbrenner (1994) envisioned a macrosystem as “a societal blueprint for a particular culture or subculture” (p. 40). It consists of an embedded pattern of the micro, meso, and exosystems defined by the belief systems, bodies of knowledge, material resources, customs, life-styles, opportunity structures, hazards, and life course options. Identification of particular social and psychological features at the macrosystem level shed light on the microsystem. The society that surrounds an individual—their

community, state, country, and prevailing belief systems of a government—is considered a macrosystem. Possible homeschooling macrosystems include a local homeschooling support group (also possibly a mesosystem), a homeschool accountability organization, Home School Legal Defense Association (HSLDA), and homeschoolers within a geographic location. Beyond the physical locations, relationships, and experiences of the four systems, the chronosystem, evolved in Bronfenbrenner's (1994) theory.

This fifth system, the chronosystem, was comprised of a third dimension incorporating time into the ecological model. The chronosystem involved the chronological growth in the developing human, but the changes that took place in the environment of the individual (e.g., family structure, family income, location of residence, historical events as they directly or indirectly impacted the person, such as wars, changes in government, natural disasters) were included in this system as well. Longitudinal research would be necessary to adequately incorporate this element.

Bronfenbrenner (1999) updated his earlier theory with a four-part structure labeled PPCT (process, person, context, time). This more developed model, labeled the bio-ecological model, incorporated PPCT as variables. Tudge, Mokrova, Hatfield and Karnik (2009) purported that proper implementation of Bronfenbrenner's (1999) mature theory need to focus on proximal processes and the PPCT variables. Proximal processes linked person and context. The process was that which could explain the connection between the context, or some aspect of the person (e.g., being homeschooled), and an outcome of interest (e.g., acquisition of mathematical proficiency).

Recent research has utilized Bronfenbrenner ecological and PPCT models in educational settings (Esmonde, 2009; Porter, Cartwright, & Snelgar, 2006; Seginer, 2006, van Schalkwyk & Bouwer, 2011). Using the ecology model, Esmonde (2009) analyzed

equity in mathematical cooperative learning. Van Schalkwyk and Bouwer (2011) conducted case studies with homeschoolers using the ecological model. Porter et al., (2006) examined college students in statistics classes with the PPCT model. Seginer (2006) used the ecological model in a comprehensive literature review of parental involvement, with recommendations for the use of the PPCT model with children.

Esmonde (2009) used ecological theory to study equity in mathematical cooperative learning. The equity was studied in light of social ecology: (a) the activity; (b) the resources; (c) the patterned interactions between people; and (d) the emergent goals. The four focal points of the study included: (a) the learning through participation; (b) the learning in relation to the social ecology; (c) the learning through the process of identity development; and (d) the learning through communication about mathematical content. The ecological approach emphasized the cultural relationships of the student to the home, community, and school practices and how it affected the cooperative groups. Esmonde (2009) illustrated how the social ecology of the classroom supported interactions that resulted in learning outcomes.

Utilizing the bio-ecological theory with homeschooling families, Van Schalkwyk and Bouwer (2011) investigated the homeschool learner's participation and perceptions with respect to their homeschooling environment. Several interviews were conducted with the homeschooling parents and their child. The microsystem of the child appeared to be driven by the mother's discourses and goals, a macro system influence. The mesosystem, consisting of the home and the homeschool microsystems, seemed to be isolated from the much of the outside world, with the exception of a very restricted involvement with their church circle. The father's demanding work schedule, along with a desire to be with other children—exosystems to the child—produced stress in the

homeschool student, who missed time with her father and wanted to spend more time with peers. Results showed a radical disharmony existed at the chronosystem level, the perceived academic and socialization needs of the adolescent were not being met.

Porter et al. (2006) used the PPCT model to investigate and create effective strategies for teaching statistics to their diversified student body. The PPCT framework situated the students at the center of a set of ecological subsystems. The researchers examined students in Westminster College statistics classes in terms of their previous experience, expectations, anxiety levels, academic integration and social integration. The microsystem included the activities, roles, interpersonal relations experienced by the developing person. The mesosystem involved the interactions between the microsystems of the student and the exosystem, which is comprised of indirect effects of the microsystems of other individuals. The macrosystem was the distance environmental influences, such as government policy, that invaded the student's life. These systems pointed to the complex interactions the students experienced. Porter et al. (2006) targeted the microsystems of the students to discover what would support them in their statistics classes through interventions reducing potentially competing mesosystem activity.

In a literature review of Israeli parental involvement, Seginer (2006) used the ecological model terminology to identify and describe each of the four systems. Within the microsystem, Seginer (2006) studied the home-based parental involvement (motivation and support of school learning), the education-relevant family environment, the family structure (single or two parent families) and family size, and the physical aspects of the home learning environment. Mesosystem study focused on the school-based parental involvement, parent-teacher interaction, and the factors affecting school-based involvement. The parents' social networks, workplaces, neighborhoods, and the

legislation and policymaking contributed to the exosystem study. The macrosystem was classified as the immigrant and the minority groups. The parental involvement in immigrant or minority groups, as well as their belief systems, was studied. Seginer (2006) recommended that future research use Bronfenbrenner's (1999) PPCT model as a framework to study the individual child within the parent involvement research.

Seginer (2006) proposed that three important, but under-investigated, issues be studied with the Process-Person-Context-Time model. These issues related to interpersonal relationships (proximal processes), emotional characteristics (person), and social/historical across-time events (time). With respect to parental involvement research, they situate three questions: (a) the effect of child and parent characteristics (person variables) and parent-child, parent-parent, and parent-system (process variables) as they relate to parental involvement and the child (person variables) and the educational results; (b) the effect of person and process variables on the link between parental involvement and educational results; and (c) do these connections differ by social and historical events (time variable) and ethnic groups (context variables).

The PPCT model provides an effective framework to analyze the homeschool environment. Mathematics curriculum, instruction and experiences (the parent-child interactions) comprise the process component. The person variables involve the characteristics, perceptions and feelings of the homeschoolers, educator and child, toward mathematics. Environments in which mathematics were encountered make up the Context factor. The time spent on mathematical activities and the changes that took place over the years describe the time element.

Bronfenbrenner's (1979, 1999, 2000) ecological and PPCT models provide a framework for researchers to analyze environments of developing individuals. These

models give representations to varying systems that the individual participates in and is influenced by. Viewing an environment (complete with physical items, experiences, relationships, and interactions) through an ecological lens provides a concrete way to explain how these environments affect a developing individual. Since the home environment is fundamental to the practice of homeschooling, Bronfenbrenner's (1979, 1999, 2000) ecological and PPCT models are appropriate for its evaluation. Focusing on the proximal processes, the connection between the context of the homeschool graduate (being homeschooled) and the outcome of interest (mathematical proficiency) necessitates a model that can examine many contributing factors, simultaneously. The ecological model can explain the interrelated variables of the homeschool: the relationships of its members, the curriculum and experiences of the homeschooled individual, the motivations of the parent educator, and the attitudes of both the developing homeschooling student and the home educator. The PPCT model relates how these specific variables affect the developing individuals' environment. It provides a platform for dissecting and connecting the varying complexities of the homeschool environment and relating that to mathematical proficiency in homeschool students.

Conceptual Framework

Mathematical proficiency was examined through the lens of the Five Strands of Mathematical Proficiency (Kilpatrick et al., 2001). The NRC commissioned the 18-month study that produced this model. This model has been utilized in K-12 and college research settings (Ball, 2003; Kinnari, 2010; Schoenfeld, 2007; Suh, 2010).

Five Strands of Mathematical Proficiency

For the purposes of this study, a five-strand conceptual framework defined mathematical proficiency. The five components include: (a) conceptual understanding;

(b) procedural fluency; (c) strategic competence; (d) adaptive reasoning; and (e) productive disposition (Kilpatrick et al., 2001). Ball (2003) recognized that these five strands are appropriate to use as a foundational framework to conduct mathematics study with students. Kilpatrick et al. (2001) claim that the “best source of information about student performance in the United States is . . . the National Assessment of Educational Progress (NAEP), a regular assessment of students’ knowledge and skills in the school subjects” (p. 136). Three of the five strands are directly evaluated by the NAEP tests, one is indirectly tested, and the last is addressed by a separate NAEP student questionnaire. Schoenfeld (2007) referred to the NAEP tests as “the nation’s report card” (p. 3) and stated it “provides some ‘benchmarking’ of American student performance” (p. 280). NAEP results show groups of students as Below Basic, Basic, Proficient, and Advanced (NAEP website). A shortened 12th grade mathematics sample test from NAEP’s website served as a benchmark to assess homeschooled graduates, in this study.

The theoretical framework acts as a funnel (Ary et al., 2006) for the homeschool environment: attitudes, instructional practices, relationships, and physical surroundings. Bronfenbrenner’s (1984, 1999, 2000) ecological and PPCT models served as the theoretical framework. NRC’s five strands of mathematical proficiency (Kilpatrick et al., 2001) served as the conceptual framework. These two frameworks provided the basis to understand and explain how the homeschool graduate makes meaning of their mathematical proficiency and their homeschool mathematics environment.

Review of the Literature

To examine acquired mathematical proficiency in homeschooling graduates requires knowledge of contributing factors. First, an examination of mathematical proficiency and its assessment, as well as an understanding of mathematical self-efficacy

is necessary. Then, homeschooling origins, demographics, motivations, instructional methods, curriculum, academic achievements, and research complexities were discussed.

Describing Mathematical Proficiency

Mathematics proficiency is critical to American independence and leadership in mathematics (National Mathematics Advisory Panel [NMAP], 2008). It is also critical to individual citizens because it provides them college and career opportunities and increases income potential. Currently, quantitative comparisons with other countries demonstrate our mediocre level of mathematical achievement. On our national report card, the National Assessment of Educational Progress (NAEP), in Grade 8, 32% of students scored at proficient levels, and in Grade 12, 23% were ranked proficient. This finding correlates with the growing demand for remedial mathematics education in colleges and universities throughout America (Hall & Ponton, 2005; NMAP, 2008) and the lack of preparedness of freshman for college mathematics (Corbishley & Truxaw, 2010; Kinnari, 2010). To stay globally competitive, mathematics education must explore methods that graduate mathematically proficient students.

Mathematical proficiency is a complicated concept involving precision, stages, and problem solving. Milgram (2007) suggested, “the best we can do is to discuss the most important characteristics of mathematics” (p. 33). These characteristics included precise definitions of terms, operations, and properties, as well as posing and solving well-defined problems in a single situation. Individuals need to perfect skills at the different stages of arithmetic (Arithmetic) and abstraction (Algebra). Milgram (2007) purported that students use both verbal and nonverbal methods in their brains to solve problems. Skill and automaticity with numbers developed the verbal mechanisms which facilitates the nonverbal problem solving ability. Milgram (2007) stated “students must

practice with numbers. They must add them until basic addition is automatic. The same for subtraction and multiplication. They must practice until these operations are automatic . . . to facilitate the nonverbal processes of problem solving” (p.48). In addition to competent number skills, abstraction is needed. The study of algebra provides practice with problems and symbolic manipulation, increasing abstract reasoning. This thinking facilitates abstraction, exploring and working through a situation so that the nonverbal mechanisms in the brain can determine the major focus of the problem (Milgram, 2007). Problem solving entailed “solving a problem where the answer is not immediate and requires a novel answer from the student” (Milgram, 2007, p. 33).

Schoenfeld (2007) stated that mathematically proficient people have mathematical knowledge, as well as the capacity to use that knowledge in appropriate circumstances. He purported, “Good problem solvers are flexible and resourceful” (Schoenfeld, 2007, p. 59). They stuck with the problem until it was solved, drawing upon a diverse array of methods. If one strategy didn’t work, others were employed until a justifiable solution obtained. The mathematically proficient persisted when others would quit. Mathematics was not just viewed as a noun (a structured body of knowledge) but also a verb, the solving problems and proving theorems by mathematicians. Schoenfeld (2007) recommended research that uses extensive interviews with both the students and the teachers to understand their levels of mathematical proficiency.

The NRC commission concluded that, “mathematical proficiency cannot be characterized as simply present or absent” (Kilpatrick et al., 2001, p. 135). Each significant concept had the potential to be understood at many levels and in varying ways. Time was needed to attain proficiency. Prolonged periods of time devoted to

solving problems, reasoning, expanding understanding, reflecting, practicing, and forming connections with old and new mathematical knowledge was required.

Assessing Mathematical Proficiency

Mathematical assessments are diverse, depending on the needs of the stakeholders (Antigue, 2007). These evaluations embody deeply held mathematical beliefs. Schoenfeld (2007) stated, “What You Test is What You Get WYTIWYG” (p. 12). Mathematicians want to judge students’ understanding of mathematics’ central ideas. Mathematics educators believe a broad spectrum of content and processes, as put forth by the National Council of Teachers of Mathematics (NCTM) and the NRC, are primary. Parents desire to know their children’s standing with others and the areas they need to improve. Professional development and administrators assess what teachers need to do better to understand student knowledge and needs. Teachers need items and procedures that help them diagnose and remediate students (Antigue, 2007; Schoenfeld, 2007). The government and policy makers want a general statistical snapshot of its students’ progress (or lack of it) based on representative samples. Students need to realize what they do and don’t know (Schoenfeld, 2007). Finally, publishers and policy makers need to address rigorous constraints: reliability, construct validity, predictive validity, and liability. Schoenfeld (2007) warned that one size assessment does not fit all; most likely does not fit any group. Despite its limitations, “assessment is necessary to the life of educational systems, to the point that any form of knowledge which is not assessed or cannot be assessed lacks legitimacy” (Antigue, 2007, p. 298).

NCTM (2000) addressed the improvement of students’ mathematical thinking. Universally enriching mathematical understanding will advance overall learner achievement. Based on best practices research, NCTM (2000) purported an ambitious

vision for mathematics education, “Achieving it [vision] requires solid mathematics curricula, competent and knowledgeable teachers who can integrate instruction with assessment, education policies that enhance and support learning, classrooms with ready access to technology, and a commitment to both equity and excellence” (NCTM, 2000, p. 3). Six general principles guided the construction of thriving classroom environments: equity, curriculum, teaching, learning, assessment, and technology. All students regardless of ethnic background or ability deserved equity; this involved accommodating differences to help everyone learn mathematics.

NCTM (2000) standards involved content and process. The content standards (i.e., number and operations, algebra, geometry, measurement, and data analysis and probability) intersected with the process standards (i.e., problem solving, reasoning and proof, communication, connections, and representation) throughout the prekindergarten to twelfth grades. Intensity of these mathematical strands varied in individual grades. However, mathematical maturity developed by annually building upon these topics. Effectual instruction enabled all students to experience the interconnectivity of the topics intertwined in all grades, culminating in a well-rounded comprehension of the mathematical topics by the end of the high school years. In the ninth through twelfth grades, high school students needed to focus on the more sophisticated topics of functions and relations, invariance, and transformation; judging the reasonableness of their answers and other materials (e.g., graphs, statistics) was a critical life skill (NCTM, 2000). A high degree of quantitative literacy and mathematical knowledge equipped students “for citizenship, work, and further study” (NCTM, 2000, p. 289). This proficiency provided students “with the disposition, knowledge, and strategies to deal with the new challenges they will encounter” (NCTM, 2000, p. 334).

Schoenfeld (2007) discussed four aspects in assessing mathematical proficiency. The first was mathematical knowledge. This knowledge was not merely stored but connected and structured for appropriate retrieval and application, learning with understanding versus rote memorization (Kilpatrick et al., 2001). Secondly, the individual must have had the ability to employ problem-solving strategies (Schoenfeld, 2007). Thirdly, the individual must have been able to link what they know to the problem being solved. Fourthly, the mathematical beliefs and temperaments of character, tenacity, and persistence were critical to mathematical proficiency.

Moving beyond the straightforward, multiple-choice format, New York City and California instituted extended multiple choice and open-ended questions in their assessments. In New York City, students were required to solve problems and provide explanations for their answers (Taleporos, 2007). The California Learning Assessment System (CLAS) incorporated extended multiple-choice and constructed response type problems, with traditional questions, to assess the more demanding curricular aims that the traditional items could not appropriately appraise (Stage, 2007). Unfortunately, CLAS was abandoned due to political and procedural issues.

In 2001, NRC study proposed a five-strand model for promoting and evaluating mathematical proficiency (Kilpatrick et al., 2001). These components included procedural proficiency, but went beyond that limited definition to include rich conceptual understanding, problem solving of novel situations, explaining and justifying their procedures, and positive attitudes towards mathematics and their mathematical ability. This model has been utilized across many grade levels, from elementary to college (Kinnari, 2010; Sabey, 2009; Suh, 2007). Yet, this research does not include homeschooling students.

Kilpatrick et al. (2001) presented a framework for assessing mathematical proficiency. The five interdependent strands intertwined like a rope to illustrate a strong relationship: (a) conceptual understanding – understanding concepts, operations, and relations; (b) procedural fluency – using appropriate procedures with accuracy and flexibility; (c) strategic competence – precisely and symbolically representing and solving problems; (d) adaptive reasoning – thinking logically, reflectively and justifying the solution process; and (e) productive disposition – perceiving mathematics as a useful tool, together with a self-confidence in their mathematical ability. The core issue was one of addressing all the strands for completeness and balance, because of the interconnectedness and interdependency of the five components, (Ball, 2003).

Students with conceptual understanding possess an integrated grasp of mathematical ideas (Kilpatrick et al, 2001). Knowing more than isolated facts and procedures, they appreciate why a mathematical concept is useful. They organize their mathematical knowledge, enabling them to learn new concepts by connecting them to those already learned. Students with conceptual understanding are able to “represent mathematical situations in different ways and knowing how different representations can be useful for different purposes” (Kilpatrick et al., 2001, p. 119).

Procedural fluency involves both the skillful knowledge of procedures and the how and when to use them (Kilpatrick et al, 2001). This fluency is demonstrated by using appropriate procedures accurately, efficiently, and flexibly. Estimation is also necessary. These skills need to be perfected with well-timed practice to facilitate the other strands.

Strategic competence refers to the ability to understand a problem, represent it, and then solve it (Kilpatrick et al., 2001). This is similar to problem solving in the research literature. Mathematically proficient students construct a mental image of the

problem's essential components and then generate a representation ignoring extraneous elements. Equations or other representations are used to work through to the solution.

Adaptive reasoning involves thinking logically about the relationships between concepts and situations (Kilpatrick et al., 2001). This reasoning stems from careful considerations of alternative solution methods. Mathematically proficient students repeatedly justify their answers in a deductive manner. Justifying and explaining their processes facilitates clear reasoning and improves conceptual understanding.

The tendency to view mathematics as useful and worthwhile is referred to as productive disposition (Kilpatrick et al., 2001). Students with high productive disposition believe that mathematics is understandable and not arbitrary. They perceive that they are effective learners and doers of mathematics. With a high motivational level, they demonstrate persistence in their approach to all types of problems.

The NRC committee formulated this five-part mathematical proficiency model based on grades K–8 mathematical research (Kilpatrick et al., 2001). Although the 2001 NRC report focused on number, the authors maintained that the five strands apply well to other areas of mathematics, such as geometry, measurement, probability, and statistics. Conceptual understanding applied to a broad range of mathematical ideas. Procedural fluency applied to skill in strategies involving shapes, space, probabilities, and data description. Strategic competence incorporated the ability to analyze, represent, and solve problems in number, algebra, geometry, measurement, probability, and statistics. Adaptive reasoning, thinking logically about mathematical relationships, and productive disposition, seeing mathematics as worthwhile and useful, applied to every mathematics domain, not just number. Kilpatrick et al. (2001) believed, “the strands of proficiency are interwoven across domains of mathematics in such a way that conceptual understanding

in one domain, say geometry, support conceptual understanding in another, say number” (p. 142). Researchers have used the five strands framework in higher education and teacher development (Kinnari, 2010; Sabey, 2009).

Kinnari (2010) studied the mathematical proficiency of first year engineering students at Tampere University of Applied Sciences (TAMK). TAMK had designed a diagnostic procedure based on the NRC’s five strands of mathematical proficiency framework. The students took a 20 multiple question online survey based on three of the strands: (a) conceptual understanding; (b) procedural fluency; and (c) strategic competence. Kinnari (2010) found that the students scored from 8.1 to 18.3 points (out of 40), with a mean score of 13.8, representing a significant lack of mathematical literacy.

The National Assessment of Education Progress (NAEP) is considered to be the Nation’s Report Card (NMAP, 2008; Schoenfeld, 2007). Initiated in 1973, this assessment was designed to measure U.S. student achievement (National Assessment Governing Board [NAGB], 2008). Twelve different areas in the language arts, arts, science, social science, and mathematics subject, are evaluated.

The mathematical section of the NAEP has a consistent design that measures achievement for long-term trends, evaluated in the fourth, eighth, and 12th grade years (NAGB, 2008). It measures what students know and can do. Three achievement levels are reported: (a) Basic – partial mastery of grade level knowledge and skills; (b) Proficient – competency in grade level subject matter knowledge, real world application of that knowledge, and appropriate grade level analytic skills; and (c) Advanced – superior performance in grade level knowledge, real world application, and analytic skills (NAGB, 2008). Mathematical knowledge is evaluated with respect to mathematical complexity and five content areas: (a) number and operations; (b) measurement; (c)

geometry; (d) data analysis, statistics, and probability; and (e) algebra, mirroring the content areas named by the NCTM (2000). Both multiple choice and open-ended items are included in the NAEP (NAGB, 2008). At the 12th grade level, the NAEP combines the measurement and geometry areas into one content strand. The balance of content for Grade 12 test items includes: number and operations – 10%; measurement and geometry – 30%; data analysis, statistics, and probability – 25%; and algebra – 35%.

NAEP assessments have been continually evaluated and updated by a comprehensive national process involving mathematicians, educators, and the general public since they were first introduced (National Center for Education Statistics [NCES], 2009). The NRC 2001 report linked the NAEP framework with three of the NRC's five strands of mathematical proficiency components – conceptual understanding, procedural knowledge, and problem solving (Kilpatrick et al., 2001). The NCES website provided 2009 NAEP test items through the NAEP Questions tool which required student explanations and justifications as part of the answers (NCES, 2009). This added feature provided for the fourth strand of the NRC mathematical proficiency model – adaptive reasoning (Kilpatrick et al., 2001). The National Mathematics Advisory Board (2008) referred to the statistics generated by the NAEP as consistent with the research showing the growing need for remedial mathematics education in two and four year colleges. Using the NRC's five strands of mathematical proficiency framework, Sabey (2009) selected certain geometry NAEP test items to evaluate the mathematical proficiency of pre-service teachers enrolled in a middle school mathematics methods education course.

Sabey (2009) studied 15 secondary pre-service mathematics teachers using the NRC's five-strand framework and NAEP geometry items. The NAEP assessment divided the subjects into three distinct groupings. Three groups of participants emerged: (a) three

members in the low proficiency collection; (b) nine members in the middle proficiency cluster; and (c) three members in the high proficiency set. Interviewed participants in the low set showed lack of proficiency in all five strands, while interviewed participants in the high set demonstrated proficiency in all five strands. An interviewed middle group participant shared more closely the proficiency of the high group subjects.

NAEP assessments corresponded with the NRC five-strand framework (Kilpatrick et al., 2001). NAEP items focused on the mathematical abilities of conceptual understanding, procedural knowledge and problem solving (strategic competence), and includes other measurement of reasoning (adaptive reasoning), connections and communication (Kilpatrick et al., 2001; NAGB, 2000). Although the NAEP assessments were not designed to correlate with the five-strand model, they provided useful information about the strands (Kilpatrick et al., 2001).

NAEP provided a longitudinal picture of student achievement. Procedural fluency has steadily increased over the years from 1973–1999. The same assessments also suggested a weak conceptual understanding about number system properties. NAEP results indicated a deficit in problem solving. Some NAEP items required logical thinking in novel types of situations (adaptive reasoning) and student performance on this component (both in simple number facts and more complex ideas) was lower than the corresponding procedural fluency level. NAEP also administered a student survey that measures students' general and educational backgrounds, somewhat analogous to productive disposition. The 2009 NAEP assessment contained four types of questions: (a) multiple-choice; (b) short constructed-response; (c) extended constructed-response; and (d) scoring constructed response (NAGB, 2009).

The NAEP website has available problems to download according to level of difficulty. There are three sample tests (one each for Grade 4, Grade 7, and Grade 12) with 10 items that evaluates a test-taker with labels of Below Basic, Basic, Proficient, and Advanced. Since the NAEP assessments are closely aligned with the NRC five-strand framework (Kilpatrick et al., 2001), the Grade 12 sample test and corresponding student survey is an appropriate benchmark device to quickly and informally ascertain whether a high school senior is potentially mathematically proficient. Further one-on-one questioning can expound on that judgment by having students explain their procedures and answer the most profound assessment “What do you think” (Stage, 2007, p 361).

Existing mathematical proficiency research illustrates the need for purposeful and innovative teaching that facilitates mathematical learning beyond computational and procedural fluency. NCTM (2000) has set 10 content and process standards to guide mathematical instruction. Kilpatrick et al., (2001) has provided a five-strand model by which educators can formulate mathematical curriculum and instruction and then evaluate the student proficiency. NAEP assessments are aligned with the NCTM standards and the NRC model.

Self-Efficacy

Bandura (1994) defined self-efficacy as individuals’ beliefs about their own capabilities to generate performance outcomes, which influence significant events in their lives. Persons with a strong sense of self-efficacy viewed difficult tasks as challenges rather than threats and persisted in their efforts despite setbacks; they thought, motivated, felt, and behaved differently than those with low self-efficacy. Individuals with a robust sense of efficacy visualized success scenarios and employed good analytic thinking. They motivated themselves with higher goals and were willing to put greater effort into

tasks, for longer lengths of time despite failures. The stronger the sense of self-efficacy, the more control a person exerted over stressors and resists anxiety and avoidance behavior. Strong self-efficacy influenced the choices people made, developing the competencies, interests, and social networks that established their life direction.

Bandura (1994) described four sources of self-efficacy: (a) mastery experiences; (b) vicarious social models; (c) social persuasion; and (d) reduction of negative stress reactions. Mastery experiences were the most influential factor, developing from meaningful successes despite adversity. Homeschoolers provided mastery experiences in their instruction (Banner, 2007; Clements, 2002; Meighan, 1992, 1995, 1999, 2010; Peterson, 2011; Ray, 2000, 2002). Observing similar people modeling successful skills and strategies for handling difficulties strengthens self-efficacy (Bandura, 1994). Home educators have fought to obtain and retain the right to homeschool legally in the United States (Carper, 2000; Lyman, 1998; Tyler & Carper, 2000). Homeschooling parents/guardians made a great emotional, time, and physical commitment, as well as resource allocation, to educate their children at home (Hanna, 2011; Meighan, 1999, 2010; Ray, 2000, 2002; Thomas, 1998). The parents influenced their children in their homeschooling dedication. Successful efficacy builders verbally persuade others in their capabilities and also structure situations that bring meaningful success, avoiding placing people prematurely in situations where they will regularly fail (Bandura, 1994). Homeschoolers removed their children from school because they were not in situations that encouraged the students to succeed (Arora, 2006; Collum, 2005; Duffy, 2002; Duvall, Delaquadri, & Ward, 2004; Ensign, 2000; Green & Hoover-Dempsey, 2007; McDowall et al., 2000; Morton, 2010; Planty et al, 2009; Thomas, 1998; Wichers, 2001). Individuals with strong self-efficacy viewed stress as motivating rather debilitating

(Bandura, 1994). Their environment may shape people. Reducing negative stress factors and channeling emotional patterns helped individuals view stress as motivating rather than debilitating. Parents chose to homeschool their children to protect them from unhealthy stress in negative school environments (Arora, 2006; Collum, 2005; Duvall et al., 2004; McDowell et al, 2000; Morton, 2010; Planty et al., 2009; Thomas, 1998).

Mathematical self-efficacy. Applying self-efficacy to mathematical achievement, research suggested that higher levels of mathematical self-efficacy predicted higher levels of academic performance (Clutts, 2010; Pajares & Miller, 1994; Watts, 2011). Gender also affected mathematical self-efficacy. Pajares and Miller (1994) found that with equivalent prior experience, men scored higher on both mathematical self-efficacy and performance than women. Yet, among those professionals with high mathematical self-efficacy, successful men and women in Science, Technology, Engineering, Mathematics (STEM) vocations attributed their strong self-efficacy to different factors (Zeldin, Britner, & Pajares, 2008). The women participants claimed that role model vicarious experiences and social persuasion contributed to their success, while the interviewed men believed that mastery experiences were foundational to their accomplishments. Mathematical self-efficacy may have had different contributing sources dependent on gender. Interestingly, Ray (2010) and Rudner (1999) found that gender was not a contributing factor in the standardized academic achievement of homeschooling students.

Among the instruments used to measure mathematical self-efficacy, the Mathematical Self-Efficacy Scale (MSES) has been normed, standardized, and used extensively in research (Betz & Hackett, 1993; Ciechalski, 1993; Clutts, 2010; Hall & Ponton, 2005; Smith, 1993; Watts, 2009). The MSES (Betz & Hackett, 1993), published

by Mindgarden, Inc. is used to measure an individual's confidence in their ability to execute daily math tasks and solve mathematics problems, and persistence in the process of those tasks. The original MSES instrument had three subscales: (a) mathematics problems (18 items); (b) mathematics tasks (30 items); and (c) mathematics related courses (27 items) – with a total of 75-items (Ciechalski, 1993; Smith, 1993). The 1993 version of the MSES (Betz & Hackett, 1993) was a shorter, more refined version with 34 items and two subscales: everyday math tasks, and math courses. Ciechalski (1993) claimed that the MSES was easy to use, with detailed instructions and only took 15 minutes to administer. The 1983 MSES had high reliability coefficients: .92 for Math Courses; .96 for Math Problems; .92 for Math Tasks; and .96 for Total scale. The coefficient alpha was reported at .92, with the 2-week interval test-retest reliability reported at .94. The 1993 instrument retained the Math Tasks and the Math Courses subscales, without updated validity coefficients reported. The MSES was used to compare the mathematical self-efficacy and mathematical performance of college students (Clutts, 2010; Hall & Ponton, 2005; Watts, 2009). This theory-based instrument made a significant contribution to mathematical self-efficacy research (Smith, 1993). Besides Peterson (2011), I have not found any homeschooling research that utilized the MSES or any other mathematical self-efficacy instruments.

Homeschooling

The home was the base camp of operation for homeschooling. Ray (2000) stated that homeschooling is parent-controlled or directed, but also may be student-directed, taking place during conventional school hours on conventional school days. The children were between 5–17 years old, whose parents report them as being schooled at home where enrollment at a public or private school does not exceed 25 hours per week (Planty

et al., 2009). West (2009) classified homeschooling as “teaching one’s children at home rather than entrusting their education to either a public or private school” (p. 7).

History of home education. Initially a private matter and the social standard, home education was the norm in American colonial days (Carper, 2000; Glenn, 2005; Hill, 2000; Nemer, 2002; Wilhelm & Firmin, 2009). Wilhelm and Furman (2009) stated, “Home school education has a rich history. It is embedded in America’s most early form of education practice, with character education being a central component” (p. 303). The term ‘education’ was not mentioned in the Constitution (Basham, 2001; Holt, 2011). In *Plyler v. Doe* (1982), the majority opinion stated, “Public education is not a ‘right’ granted to individuals by the Constitution (Brennan, 2011, p. 183).

Later, as public or “common” schools became popular, they still clung to the earlier spiritual values embodied in homeschooling (Carper, 2000). Many rural one-room schools formed by communities were often religiously affiliated, church-led, and private (Hill, 2000; Wilhelm & Firmin, 2009). The public school movement gained much momentum throughout mid to late 1800s, deviating from its spiritual roots to embrace humanism and Darwinism (Carper, 2000; Nemer, 2002). Institutionalized schooling progress culminated in 1917–1918 with Massachusetts compulsory education laws, rendering most homeschooling illegal (Glenn, 2005; Nemer, 2002).

The groundwork for a home education resurgence originated from Supreme Court decisions outlawing Bible reading and prayer in public schools. In the 1960s, these landmark rulings sparked a Christian school movement, which eventually contributed to a homeschooling revival (Carper, 2000; Wilhelm & Firmin, 2009). A Christian missionary, Dr. Raymond Moore gathered a large following, including conservative, religious-right families (Lyman, 1998). Starting in the 1970s, Dr. Moore endorsed an ideological

Christian home education movement, which offered a common message. Lyman (1998) wrote, “The Moore’s advocate a firm but gentle approach to home education that balances study, chores, and work outside the home in an atmosphere geared toward a child’s particular developmental needs” (para. 19).

Concurrently, John Holt advocated academic freedom in the 1970s. Disheartened by the inflexibility of the public school system, he promoted a liberal, leftist counter-cultural educational agenda; a homeschooling exodus from governmental schools followed (Basham, 2001; Lyman, 1998). These pedagogues subscribed to Holt’s progressive education ideals. Holt (2011) believed in the learner’s right to control their own learning: “Young people should have the right to control and direct their own learning, that is, to decide what they want to learn, and when, where, how, how much, how fast and with what help they want to learn it” (p. 25).

Public appearances and court testimonies of Holt and Moore reignited home education. This reemergence started slowly in the 1970s and gained momentum in the 1980s and 1990s (Carper, 2000; Lyman, 1998). Initially, the liberal left group dominated. In the middle 1980s, the religious right became the majority (Lyman, 1998).

Much legal wrangling occurred from the late 1980s–1990s, culminating in more legislation that favored home-based education (Carper, 2000). Home School Legal Defense Association (HSLDA) and Association for Christian Schools International were instrumental in the battle to legalize home education (Wilhelm & Firmin, 2009). In 1993, homeschooling became legally established in all 50 states (Wilhelm & Firmin, 2009).

Home education has experienced significant growth since 1970. Homeschooling increased from about 15,000 participants in 1970, to 50,000 in 1985 (Wilhelm & Firmin, 2009). In 1999, National Center for Educational Statistics (NCES) report estimated the

number of homeschooled children at 850,000 (Planty et al., 2009). Noteworthy increases showed in subsequent NCES reports, 1.1 million in 2003, and 1.5 million in 2007 (Planty et al., 2009). Home-educated learners comprised approximately 2.9% of America's school-age population, a 74% relative increase over the 8-year period from 1999–2007 (Planty et al., 2009). Currently, the number of homeschoolers was estimated at 2.04 million members (Ray, 2011). Ray (2011) consulted past research from U.S. Department of Education, U.S. Census Bureau, data from 13 state departments of education, surveys of home-education groups, and information from five nationwide, private companies that serve home educators. Given the U.S. Census Bureau estimate of the current school-age population, 54.117 million, Ray (2011) estimated that 3.8% of those children participate in home-based education. The current home education movement transformed from the “extreme to the mainstream” (Basham, 2001, p. 3).

Demographics. Researchers attempted to categorize the different types of homeschoolers. Planty et al. (2009) found that the overwhelming majority of homeschooling families contained two parents (89%); half of those families had one parent in the work force and one-third send two parents into the labor force. Caucasian homeschoolers comprised three-fourths of the population; home-educated African American and Hispanic made up 4% and 10%, respectively (Planty et al., 2009). One-third of the home educated households earned \$75,000 or more (Planty et al., 2009). The highest percentage of homeschooling parents (36%) reported having some college or a technical background (Planty et al., 2009). Most homeschool households (60%) contained three or more children (Planty et al., 2009). Rudner (1999) and Ray (2010) reported similar male and female statistics: 50.4% female, 49.6% male; 50.3% male, 49.7% female.

The homeschooling population encompassed many diverse members with a wide-range of political, ideological, religious, and educational convictions (Apple, 2007). This non-homogenous movement cut across racial and class lines. The largest subgroup of home educators consisted of conservative, evangelical Christians. Inherently, these diverse families shared a common mistrust of the standardized schooling system.

Motivations for homeschooling educators. Researchers reported diverse reasons for homeschooling. Stevens (2001) and Apple (2007) called homeschooling a social movement. Labeled ideologues, the largest subgroup of home educators claimed to be fundamental, evangelical Christians who wanted to promote their values and family unity (Apple, 2007; Basham, 2001; Hanna, 2011; Lyman, 1998; Ray, 2010; Rudner, 1999). They worked collectively to preserve their identity and beliefs. Pedagogues generally adhered to John Holt's theories of academic freedom and resistance to lack of individuality in conventional schooling (Basham, 2001; Lyman, 1998).

Hanna (2011) found homeschoolers' motivation stayed consistent over a 10-year period. A longitudinal study was conducted from 1998-2008 and studied 250 homeschooling families in urban, rural, and suburban families in Pennsylvania. Over the years, these homeschoolers consistently (98.7%) identified themselves as ideologues (46.8%), pedagogues (24.6%), a combination of both (26.4%), or other (2.2%).

Academic and social concerns prompted participation in homeschooling (Planty et al., 2009; Ray, 2010; Rudner, 1999; Thomas, 1998). McDowell et al., (2000) found that differing cultures cite varying reasons to homeschool; specifically, African American home educators wished to protect their students from the racism that pervaded public schools. In a survey study of a homeschool charter school, Collum (2005) found that the participants loosely fit into four categories: critical of public schools; attracted to charter

schools; ideological reasons; and family and student needs, showing a more heterogeneous and complex set of motivations. Home educators of gifted learners and children with special needs believed that unique needs were not met in the traditional schools (Arora, 2006; Bannier, 2007; Duffy, 2002; Duval et al., 2004; Ensign, 2000; Planty et al., 2009; Thomas, 1998). Home-based education provided the opportunity for these special children to have the focused attention of an educator (their parent/guardian) committed to their well-being (Ray, 2000). Based on interviews and observations, Morton (2010) classified her participants in terms of the choice to homeschool: (a) natural – rejecting false and restrictive structures of conventional schooling; (b) social – wanting to promote their moral and social behaviors; and (c) last resort – providing an escape route and period of recovery for their children damaged by the schools. There was a common thread of perceiving their children as individuals and the parents as experts on them, as well as dissatisfaction with the traditional schools. Parents' choices to homeschool are becoming more complex (Goodman, 2008).

Spiegler (2010) conducted a meta-analysis of 12 homeschool motivation studies. Parents believed that home education enabled four major areas better than conventional schooling. These areas involved: (a) curriculum, academics, and educational approaches; (b) values, moral instruction, and socialization; (c) safety and well-being of the student; and (d) family unity. Motives differed with the methodology used and were shaped by the perception of environmental benefits.

Homeschooling instruction. Homeschooling instruction was not a replication of conventional classrooms (Meighan, 1999; Patterson et al., 2007; Thomas, 1998). Thomas (1998) observed that each of the 100 families in his study personalized their own homeschool, ranging from unstructured curriculum to those with set curriculums and

lessons. A large majority of the homeschoolers used some combination of more structured curriculum and less organized activities. There was intensive and concentrated instruction, due to individualized attention. Meighan (1999) labeled homeschooling as a “Do It Yourself” educational method. Meighan (1995, 1999) observed 11 behaviors which he classified as reasons for homeschoolers’ success: (a) non-hostile environments; (b) more efficient time usage; (c) purposive conversation, as opposed to crowd instruction; (d) a catalogue curriculum approach; (e) access of information-rich society at will; (f) “plan, do, review” type of learning, managed by the learner; (g) social skill development in the community (the real world); (h) learning style adaptation; (i) parents/guardians acting as learning agents; (j) first-hand experience; and (k) natural learning approaches. Home educators “either intuitively understand or figured out that the traditional school structures were not effective in the home school” (Patterson et al., 2007, p. 82). Although not specifically named, it would be reasonable to conclude that these descriptions applied to mathematics instruction.

Collum (2005) found strong consistency among significant determinants of student achievement. Parent education and political affiliation significantly affected reading, writing, and math scores. Parents with higher levels of education and conservative politics had children who scored better on standardized tests. Results suggested that teaching styles may have been influenced by these factors.

Richman et al., (1992) explored what mathematical instruction worked well in home schooling. Home educator survey responses were correlated with their homeschooled students’ CTBS/4 standardized achievement test scores. Results suggested that the more parents corrected student work and attended math seminars and/or presentations, the better the homeschoolers scored on the tests. Non-significant

variables included the number of hours spent weekly in formal lessons, parental education level, and parents' income. Intentional parental instructional involvement seemed to be a major factor in student achievement.

Cai, Reeve, and Robinson (2002) linked more controlling teaching styles to motivations for homeschooling. Results showed that religiously oriented home educators reported more controlling instructional techniques than public school teachers or other homeschoolers. The authors believed that this would cause a decrease in motivation for the homeschool students, because they would be less autonomous. These opinions were not confirmed by Bannier's (2007) findings that homeschoolers, regardless of religious motivation, encouraged appropriate pacing and motivating behavior in their children.

Bannier (2007) interviewed 11 homeschooling parents. Consistent with the homeschooling literature, several of the parents reported homeschooling primarily for religious reasons and others homeschooled primarily for the opportunity to better educate and motivate their children, according to their learning styles. These parents shared their instructional practice of mastery learning at an appropriate pace for their child. This individualized approach included taking extra time with topics the student needed to review and moving quickly when the student completely grasped the content. These home educators conducted frequent and varied evaluations to assess their students' retention and the effectiveness of the curriculum and methods utilized. They believed that immediate feedback promoted the continued interest and engagement of their children. Further research on homeschooling instruction, with respect to mastery and learning styles, was needed to discover and report on educational best practices.

Ortiz (2000) found a child-led non-structured mathematical learning environment in homeschooling families. He conducted cases studies of two families with children

from early elementary to middle school age. A series of separate interviews were conducted with the parents and the students. Analysis of these interviews showed a child-led environment with little structured mathematical instruction, yet an environment rich in practical mathematical application. Children choose to do what interested them. The environment was social with many conversations on many topics, including math. A high degree of mutual trust and respect existed in both homeschools. No central source of knowledge (like a teacher in a conventional classroom) was found in either homeschool. These homeschools utilized puzzles, brainteasers, and practical mathematics (e.g., cooking, shopping), rather than a formal mathematics curriculum.

Goodman (2008) surveyed and interviewed 16 college students who had been homeschooled. She investigated their pre-college activity to discover the characteristics of their learning, including the types of schooling, the curricula utilized, and classroom experiences. During their homeschooling experience, most participants (14 out of 16) had experienced co-op classes or tutoring in selected subjects (e.g., mathematics, foreign language, science labs). Two students had attended community college classes while homeschooling. Although each participant had an individualized experience, three overarching patterns of homeschool environments emerged: personal, familial, and autonomous. The personal environment was described as one that showed competitiveness with the traditional school environment; the parent served as a teacher or facilitator. Formal grades and report cards were given to measure achievement; generally, these students experienced high participation rates in co-ops or other classes outside the home. Familial homeschoolers' activities revolved around family cooperation and needs. Less formal feedback and accountability was found in these family-oriented homeschoolers. Parents were leaders. Students, who were given more freedom and choices, were

autonomous in their learning; their home educators acted as guides. Goodman (2008) found that many of the homeschoolers fit into more than one group, requiring subgroups of personal-autonomous and familial autonomous, because many of the homeschool students were permitted more autonomy during their high school years. Specific disciplines, such as mathematics, were not discussed in detail.

In a multiple case study, Bachman (2011) investigated the STEM instruction of eight homeschooling families. These findings indicated that younger children were taught mathematics in an experiential method, with practical applications and games. Older children were taught algebra and geometry with curriculum materials. Overall,

Home-education mathematics learning activity seems to evolve with child age and experience. In this study parents more often provided younger children with mathematics embedded in life experiences and games, and as the children became older, they turned to curriculum for algebra and geometry (Bachman, 2011, p. 363).

Conventional school advocates also studied the instructional practices of homeschooling educators (Patterson et al., 2007). In an effort to better understand the motivations of homeschoolers and how to encourage them to enroll their children in the public schools, a team of three college professors and four public school administrators conducted a case study of rural Kansas homeschooling families. Through interviews and observations these researchers found home educators to be motivated and committed, wanting the best for their students. They networked with other homeschool families to capitalize on strengths. These educators attended conferences and researched curricula and instructional techniques. Their child-centered pedagogy focused on curriculum and resources that matched individual learning styles, with flexible structures and schedules.

Most of the 10 families incorporated year-round schooling for maximum retention.

In another case study, Shepherd (2010) interviewed and observed three Indiana homeschooling families. These homeschoolers tended to use traditional methods of instruction and curriculum, with one family utilizing the classical (Trivium instruction – grammar, logic, and rhetoric stages) method of instruction. Shepherd (2010) reported,

There was no overwhelmingly unique instructional approach that dominated the teaching or learning in the homeschool studies. In fact, it appeared that the opposite was the case. Many public schools would reject rote learning, extensive memorization, and worksheet practice as outdated instructional techniques or at least as unreflective of current research-based best educational practice. Yet, these apparently were the primary methods used by the interviewed homeschooling mothers (p. 100).

Reading and completing assignments took most of the student's time. Results showed that all students tested at or above average on the Iowa Test of Basic Skills (ITBS) subtests and composite scores, including their mathematical scores. Despite the absence of current best practices teaching, these students performed well on standardized testing.

The longitudinal study by Hanna (2011) focused on the methods, materials and curriculum of homeschooling education. Concerning homeschool instructional practices, a dramatic increase in the diversity of methods and resources occurred in the 10-year time frame. Overall, these 250 Pennsylvania families became less dependent on district texts and other resources. Two of the most significant factors studied were the increase in the use of computer technology and the networking of the parent educators. Specifically, the use of computers and Internet facilitated the purchasing of homeschooling resources, communication with other homeschooling families, access to legal advice, research on

curricula, sources for instructional materials, and online courses for homeschool students. Home educators connected with other homeschooling families to share teaching expertise, materials, and private school facilities. Additionally, local religious groups and churches provided religiously oriented resources to homeschoolers. In 10 years, these parents also attained higher levels of education, many earning college degrees. These home educators matured in instructional practices and their own learning. They had kept up with innovation and instructional practices, and

upgraded and diversified their choices of pedagogy and their modalities for delivering instruction . . . choosing very specific methods and carefully selected materials for their children's instruction and most are conducting their business on a much larger stage than in the mid-1990s (Hanna, 2011, p. 19).

Research demonstrated the varied types of instruction that home educators utilize. Styles ranged from traditional, structured, and controlling (Cai et al., 2002; Martin-Chang et al., 2001; Richman et al., 1992; Shepherd, 2010) to flexible and child centered (Meighan, 1995, 1999; Ortiz, 2000; Thomas, 1998). Homeschooling parents selected curriculum based on learning styles, continually performed assessment of the curriculum and the children's learning, and sought out training and advice, through attending conferences, online helps, and networking with other homeschoolers (Bachman, 2011; Bannier, 2007; Hanna, 2011; Patterson et al., 2007; Richman et al., 1992). These motivated parents were committed to doing the best for their children (Bannier, 2007; Patterson et al., 2007), staying current with innovation and instructional practices and creatively integrating mathematics into their daily lives (Bachman, 2011; Ortiz, 2000).

Homeschool curriculum. Many different mathematic programs are available for the homeschooling market. They have different emphases. They may refer to spiritual

beliefs (labeled religious) or refrain from any mention of religious activity (labeled secular). Certain ones utilize manipulatives, while others are taught more abstractly. Various texts extensively employ real-life problems and stress problem solving. Selected programs are video-driven, others require the students read and learn from the textbook. Some curricula stress review. The following curriculum comprises those utilized by the 16 study participants. They include: Saxon, ABeka, Chalkdust, Teaching Textbooks, Math-U-See, University of Chicago School Mathematics Project (USCMP), and Bob Jones University Press (BJU).

A secular program, Saxon is used broadly in public, private, and home schools. More traditional and scripted in design, Saxon has emerged as one of two top programs in an experiment conducted for the U.S. Department of Research (Viadero, 2009). Concepts are presented in smaller increments, and then extensively reviewed throughout the textbook. Within each lesson, a few problems with the new concept are given, followed by several review exercises incorporating previously learned information. The focus of this program involves mastery and cumulative review (Saxon Math, 2012).

Promoting Christian values, ABeka provides traditional mathematics curriculum. Published by Pensacola Christian College, these texts are designed to teach students to believe in absolutes, that mathematics laws are based on God's creation. Biblical references are found throughout the lessons. The content stresses basic life skills. (ABeka Book, 2013). Besides textbooks, ABeka offers an optional DVD program in which instructors from their private school, ABeka Academy, provide direct teaching.

Houghton Mifflin Company publishes the Chalkdust program. This program consists of DVDs with direct teaching. The student then completes exercises from the accompanying textbook. A solutions manual with step-by-step explanations is provided

to check the answers. Reviews and tests are included. Online and telephone help are provided (Chalk Dust Company, 2011). Cathy Duffy recommends these independent study math courses for homeschoolers, because they enhance conceptual understanding with real world applications and word problems (Duffy, 2012).

A homeschool dad and mathematics teacher, Steve Demme, developed Math-U-See for homeschooling children. To meet the needs of individual students, this curriculum utilizes manipulatives, an instruction DVD, and consumable textbooks. Students learn abstract concepts concretely. Throughout this K–12, skill-based, multi-sensory program, students apply math concepts to everyday life. Mastery learning is emphasized (Math-U-See, 2013).

Teaching Textbooks is a computer-based program with accompanying texts. Designed for independent learners, step-by-step instructions are taught for every problem in each course, on the CD-ROM. This course combines review with new material in each homework assignment (Teaching Textbooks, 2012).

Founded to raise the level of mathematics instruction in elementary and secondary schools, UCSMP highlights real world applications. As the largest existing university-based mathematics project, this secular, classroom-based program emphasizes reading, problem solving, real-life applications, and technology usage. This rigorous curriculum stresses more complex understanding of mathematics (Usiskin, 2012).

BJU publishes traditional curriculum with a Christian emphasis. These textbooks incorporate manipulatives in the younger grades. Attention is given to the basic skills and review concepts systematically. Christian content is integrated to cultivate a Christian worldview (BJU Press, n.d.).

Academic proficiency of home educated students. Research demonstrated that homeschool learners perform as well as their conventionally schooled counterparts, if not better. Ray (2011) purported, “To date, no research has found homeschool students to be doing worse, on average, than their counterparts in state-run schools” (p.11). Studies affirmed the academic achievement and college preparedness of K-12 home-based learners, as well as positive benefits for special education homeschooling students (Bagwell, 2010; Basham et al., 2007; Bolle et al., 2007; Clemente, 2006; Duffy, 2002; Duval et al., 2004; Ensign, 2000; Goodman, 2008; Jones & Gloeckner, 2004; Klicka, 2004; Ray, 2000, 2004, 2010; Rudner, 1999; Van Pelt et al., 2009).

K-12. Regarding elementary and secondary learners, research supported the assertion that homeschoolers perform as well or better than their traditionally schooled peers (Martin-Chang et al., 2011; Ray 2010; Rudner, 1999). Martin-Chang et al., (2011) compared homeschooling and non-homeschooling youth in academic achievement. As a whole, the homeschoolers performed better than the non-homeschoolers, but not with statistical significance. The homeschoolers were then identified by structure. Unstructured or “unschooling” homeschools did not have set curriculum or a set time for schoolwork, while the structured homeschoolers had varying levels of structured school time and curriculum usage. The structured homeschoolers scored significantly higher than the non-homeschooled, who scored higher than the unstructured homeschoolers. Results suggested, “structured homeschooling may advance the development of academic skills (as measured by standardized tests) beyond what is experienced by attending traditional public school” (p. 201).

Rudner’s (1999) seminal study compared 20,760 homeschoolers’ standardized test scores with the normed public school sample. Findings showed that the home

educated, on average, performed above the means of the public educated in every subtest at every grade (typically in the 70th to 80th percentile). Students who had been exclusively homeschooled had higher achievement scores, than those who had attended public or private schools. Homeschooler achievement was not significantly related to gender, full-service curriculum, or parent teacher certification.

Following up Rudner (1999), Ray (2010) compared 11,739 homeschoolers' scores with a normed public school sample. This nationwide cross-sectional descriptive study found that homeschoolers' standardized test scores substantially exceeded the public schooled students' averages (homeschool subtests national percentile ranged from 84 to 89 as compared to public school national percentile of 50 in all subtests). No significant difference existed between achievement and several characteristics (i.e., number of years homeschooled, degree of state regulation, enrollment in a full-service curriculum). Ray (2010) stated, "that the results of this present study are consistent with preceding studies of the academic achievement of homeschool students" (p. 25).

College preparedness. Research showed that the home educated are well prepared for college level work (Bagwell, 2010; Bolle et al., 2007; Byers, 2005; Clemente, 2006; Cogan, 2010; Galloway, 1995; Jones & Gloeckner, 2004). They performed as well or better than their peers in standardized testing, GPAs, and retention.

Galloway (1995) compared freshman performance of three distinct subgroups, 60 homeschooled, 60 public schooled, and 60 private schooled. The ACT norm-referenced score and criterion-referenced English course scores provided the needed assessment to evaluate each of the groups. The English scores reflected cognitive and writing skills necessary for success, typical of practices necessary for many college classes.

Homeschool graduates averaged superior scores on all but one of the evaluated items,

scoring significantly higher on the ACT English subtest. Home education graduates performed as well as the other freshman, if not better; they exhibited comparable academic preparedness for college English work.

In Colorado public colleges and universities, Jones & Gloeckner (2004) compared 108 freshmen – 55 home school graduates and 53 randomly chosen traditionally schooled students with respect to ACT scores, GPA, credits earned freshman year, and retention. Homeschool graduates averaged higher on all four dependent variables, without statistical significance. Notably, ACT composite scores of the home school and the conventional student populations matched national averages (home schooled 22.8, identical to 2000 ACT national homeschool average; conventional schooled 21.3, close to the 2000 ACT national traditional school average of 21). These outcomes suggested that homeschoolers' collegiate readiness paralleled that of non-homeschoolers.

Clemente (2006) conducted a causal comparative study investigating college aptitude and achievement of home educated and conventionally schooled freshmen. Representing 23 states and seven Christian colleges, the 2959 participants included 1792 public schooled, 945 private schooled, and 222 home schooled, with respective mean scores of 1429.60, 1497.61, and 1811.91. The homeschoolers' higher mean SAT rank significantly differed from the public and private schooled groups, unlike the non-statistical difference between the public and private schooled. Results suggested home educators provided a suitable option to conventional schooling models.

At York Technical Community College in South Carolina, Bagwell (2010) compared 273 homeschool graduates with 273 matched peers from traditional schools. Homeschoolers outperformed traditional students in seven out of eight measures. Statistically significant differences were found in the writing and reading sections on the

COMPASS test (a nationally-normed college placement test), the mean first semester GPA, the mean math GPA, the mean science GPA, the mean English composition GPA, and the mean overall GPA. No significant difference existed between the home-educated graduates and the traditionally schooled graduates on the Compass math section. Bagwell (2010) suggested that further research address the mathematical shortfall.

Complexities of research involving home education. The intricacies of educational research necessitate the use of different inquiry methods. Furman and Shields (2005) alleged that educational research must be handled distinctively. Research issues have deeply moral roots and the selection of research questions, the way research is conducted, and the discussions of interpretations must be sensitive to those moral principles. Each research strategy must be tailored to the specific context in which it is studied. The organic context of home-based education calls for specialized inquiry.

The natural environment of home schooling presents another obstacle (Thomas, 1998, 2002). Since these students learn at home, gaining access may be cumbersome due to the impracticality of visiting each homeschool, and the reticence of many parents about being “spied upon” (Thomas, 1998). Each setting varies from the next, ranging from flexible, accommodating family needs and student selections, to very structured schedules (Goodman, 2008; Ortiz, 2000; Thomas, 1998). Normally, educators from public/private schools do not recognize this distinction. Adaptable to different contexts, case study research is an appropriate method for homeschooling study.

Studies involving a large homeschool population involve unique challenges. Homeschoolers exist out of the mainstream, most reporting only to accountability or support groups within their state, while some remain “underground”—not officially recorded. No centralized agency takes in information. Instruction and curricula have

differed (Slatter, 2009). Not every homeschooler could be part of a study because not all homeschoolers take standardized achievement tests. Only a sample, not necessarily representative, could be selected.

However, standardized tests provide a means of comparison with public, private, and homeschooled students. Universal tests like the ITBS and TAP supply a means of comparison with elementary and middle grades (Ray, 2000, 2010; Rudner, 1999). The SAT and ACT tests evaluate college readiness. These assessments offer a starting place.

Summary

Homeschooling research to date has demonstrated that homeschooling students perform at least as well or better than their conventionally schooled peers in academic achievement. Mathematics achievement is included in those results, although not as dramatic as reading achievement. Limited research addresses homeschool instruction and very little deal directly with mathematics, one being 20 years old. There is a dearth of intensive study dealing with mathematics proficiency in homeschool students. No studies deal with the homeschool graduate's perception of their mathematics instruction and how they attained such proficiency. A study of mathematically proficient homeschoolers would provide valuable insight to the homeschool and conventional education communities. Interpretive case study is needed to evaluate how mathematically proficient homeschool graduates may have benefited from the homeschool environment in obtaining their proficiency. This study examined the perceptions of homeschool graduates, who performed well on the college SAT mathematics subtest and the adapted National Assessment of Education Proficiency Grade 12 mathematics subtest.

CHAPTER THREE: METHODOLOGY

Introduction

The purpose of this multiple case study was to understand and explain how mathematically proficient homeschool graduates acquired mathematical proficiency. This collective case study research focused on the central question: How does the homeschool environment, including attitudes (of both graduates and educators), instructional practices, relationships, and physical surroundings (may include more places than the home), contribute to the mathematical proficiency of homeschool graduates?

The following five subquestions guided this study:

Research subquestion 1: How do mathematically proficient *homeschool graduates* describe their mathematical homeschool environment?

Research subquestion 2: How do mathematically proficient *homeschool graduates* describe their mathematical self-efficacy?

Research subquestion 3: How do *home educators* of mathematically proficient homeschoolers describe their mathematical self-efficacy?

Research subquestion 4: How do *home educators* of mathematically proficient homeschoolers describe their homeschool graduate's mathematical homeschool environment?

Research subquestion 5: Are there any apparent patterns or relationships between the descriptions of the mathematically proficient *homeschool graduates* and their *home educators* concerning their homeschool environment and their mathematical self-efficacy?

Research Design

Qualitative research provides an in-depth, robust look at a phenomenon to give clarity and describe the problem (Ary et al., 2006). Specifically, case study research focuses on a small sample to richly identify the environment and contributing variables (Bogdan & Biklen, 2007). Yin (2009) posits that case studies are the preferred form of qualitative study when the focus of the study is on the “how” or “why” of a current phenomenon, occurring in a naturalistic setting unhindered by the researcher. They are exploratory in nature. Multiple case studies provide more information, both from the individual cases and cross-case analysis (Stake, 2006; Yin, 1994; 2009). Results generated from the individual case studies can be compared with one another to replicate the findings, a literal replication, or confirm a negative occurrence by means of a rival theory, a theoretical replication (Yin, 1994; 2009). Within this multiple case study, I seek to explain how purposefully selected homeschool graduates acquired mathematical proficiency, based on their perceptions of their mathematical experiences within their homeschool and their mathematical self-efficacy.

Case study may involve a holistic or embedded approach and one or more sample cases (Yin, 1994). The holistic case study focuses on one unit for analysis, while the embedded design involves multiple components. Stake (2006) labels the focus of the study, such as the unit of analysis, to be a quintain, much like a target on a dartboard. When the purpose of a case study goes beyond the case itself—to explain the quintain—the case study is instrumental. In this instrumental, holistic study, the mathematical proficiency of the homeschool graduates was the unit for analysis, or quintain.

Furthermore, case study may involve one single case or multiple cases (Stake, 1995, 2006; Yin, 1994; 2009). Multi-case design provides more in-depth, rich

information by comparing single case study results (Bogdan & Biklen, 2007; Yin, 1994, 2009). They have the advantage of showing literal replication when similar results are found across the individual cases (Yin, 1994, 2009). Collective case study focuses on one targeted issue as it relates to multiple cases (Stake, 1995). In this collective, multiple-case study, comparing the common themes in the acquisition of mathematical proficiency provided insight as to how the homeschooling ecological environment, relationships, experiences, and mathematical self-efficacy, contributed to that proficiency.

Replication logic is critical to the selection of the sample case study participants. Yin (1994) purports that, “Each case must be carefully selected so that it either (a) predicts similar results (a literal replication) or (b) produces contrasting results but for predictable reasons (a theoretical replication)” (p. 46). Developing a rich, theoretical framework includes stating the conditions that an event is expected to occur (a literal replication) and also the conditions when the event does not happen (a theoretical application). The within case analysis is the literal replication analysis and the across-case evaluation is the theoretical replication (Yin, 1994). Selecting mathematically proficient graduates is important; describing what works for one student may be applied to others.

Researcher’s Role

In this case study, I took on the role of an interpreter. I mainly used the social constructivism/interpretivism paradigm because I sought to understand and explain mathematical proficiency acquisition through the reality that the participants give to that phenomenon. Also, more pragmatically, I was concerned with patterns that emerged from the data, to provide potential best practices in homeschool mathematics education.

My ontological assumption was that the homeschool graduates construct a social and individual reality as they explain the meaning they give to the acquisition of their mathematical proficiency with respect to their homeschool environment. Using transactional epistemology, I viewed the graduates as both the key to understanding the phenomenon and the phenomenon entwined together. Methodologically, I interpreted their explanations of how they acquired mathematical proficiency within their homeschooling context and how that related to their mathematical self-efficacy scores and their NAEP item answers. My rhetorical assumption was that I reported their “voice” in the data analysis; the participants were essential co-researchers. The axiological assumption I brought to the study was the great value I gave to homeschooling education.

The experience that I brought to this research included working as a mathematics educator in a New Jersey public high school for 10 years and homeschooling my three children for over 18 years. In both these capacities I have come in contact with many students and parents who claimed a lack of math confidence. They labeled math their least favorite subject and experienced much frustration with applying mathematics to their everyday lives. They scored disproportionately better on reading tests, than on mathematical tests. I also knew homeschool students who scored well on standardized tests and stated that mathematics was one of their best subjects. The question arose whether mathematical ability and perception came from the home environment, an inborn talent, activities outside of the home, or a combination of these factors.

I was the human instrument in this study, acting as a participant-observer. My experience with mathematics instruction and homeschooling allowed me to look at homeschooling mathematical instruction both from a best practices teaching and realistic

home education perspective. I acknowledged my personal bias with the advantages of homeschooling and my professional bias as to the importance of mathematics instruction. Homeschool graduates were part of a special group that I related with; their home educator parents were my peers. I acknowledged their participation in this research as co-researchers. I gave a voice to the participating graduates by using rich text and quotes, carefully analyzing the transcript data, analyzing the mathematical self-efficacy survey results, conducting member checks, and employing other mathematical and homeschooling experts to review my analysis. This type of analysis helped counteract any personal and professional bias. Additionally, the literature review process served to focus my thoughts onto this research topic and hone my skills as a human instrument.

Participants

A study, reasonable in scope, must be bounded (Stake, 1995, 2006; Yin, 1994, 2009). Careful selection of participants kept the research managed and focused. In this research homeschool graduates were initially recruited on the basis of SAT scores and the length of their homeschool study. These initial volunteers were screened with the NAEP Grade 12 Mathematics Proficiency Test; those scoring 80% or above were selected to participate in the study. One participant scored a 70%, but was included as an outlier, because of her 'A' in her college mathematics course Ray (2004, 2010) and Rudner (1999) found that the number of year's homeschooling was a significant contributing variable in homeschoolers' standardized test scores. Ray (2004) chose participants who had been homeschooled for seven years or more. Those who score above average on the mathematical sections of the SAT college entrance exams have the potential to be mathematically proficient; they are desirable candidates for college admissions' counselors. The average homeschooled student's score on the mathematical subtest of

the 1999 SAT was 532 (HSLDA, 2001). In the 2011-2012 SAT math subtest, about 25% of the test takers scored 600 or above; only 7% scored 700 or above (The College Board, 2012). Students scoring 80% or better on the NAEP Mathematical subtest are labeled as proficient (NAEP website). For the purposes of this study, homeschool graduates who scored 600 or above on the math subtest of the SAT, received a proficient score on the NAEP Grade 12 Mathematical subtest (with one exception), and had been homeschooled for seven or more years were eligible.

The number of individual cases varies with the focus of the research. Shepherd (2010) and Clements (2002) interviewed three homeschooling families to identify what each family used for instructional purposes. Morgan (1991) interviewed 15 families to find if their homeschooling programs were addressing the Virginia middle school socialization guidelines. Concerning mathematical learning environments and including the perspectives of the homeschooled students, Ortiz (2000) interviewed and observed two homeschooling families, while Bachman (2011) observed eight homeschools. For the purpose of understanding the homeschool learning environments and the ways of knowing of homeschool graduates, Goodman studied the perspective of 16 formerly homeschooled students. Thomas (1998) observed 100 home education settings to find commonalities in home education and compare those similarities with theories of informal learning. Yin (1994) recommends two to three cases for a literal replication and four to six additional cases for a theoretical replication. Through multiple case study research, I sought to understand and explain how 16 mathematically proficient homeschool graduates acquired their proficiency. Both the graduates and their home educators were involved. Pseudonyms were selected to protect their confidentiality.

Initially, I planned to seek the assistance of four institutions as gatekeepers. A North Carolina homeschooling business that offers high school laboratory classes emailed its homeschooling customers about this research and gave them my contact information. A South Carolina homeschooling accreditation association and a South Carolina legislative support group sent out an email to members with information about this study and invited them to contact me by email. Two Virginia online academies, affiliated with an accredited college/university, emailed their customers about this study and invited them to contact me by email. Through a personal friend I invited a homeschooling family to participate. That mother invited several other families, illustrating the snowballing sampling technique recommended by Bogdan and Biklen (2007). The 16 participants came from personal invitations, snowballing, and one gatekeeper, the South Carolina accreditation association. None of the other gatekeepers provided any participating families.

The 16 participants came from nine families in South Carolina and one in Virginia. Seven graduates, from four families, resided in an urban area (JONES, KING, RICE, SIMS). The other nine, from six families, lived in suburban areas (COX, FORD, HALL, LEE, TATE, WEBB). Each graduate was considered a single case (Table 1).

Table 1

Participants' Demographics and Characteristics

Name	Gender	Family	Age	SAT Math Score	Grades Homeschooled
Ned	M	COX	19	740	K–12 (13 years)
Cali	F	FORD	18	660	2–12 (11 years)
Sue	F	FORD	22	620	3–12 (10 years)
Ela	F	HALL	19	620	K–12 (13 years)
Ola	F	HALL	18	660	K–12 (13 years)
Tad	M	JONES	21	780	K–12 (13 years)
Rob	M	JONES	20	750	K–12 (13 years)
Wes	M	KING	20	700	K–12 (13 years)
Mae	F	KING	21	640	K–12 (13 years)
Ali	F	LEE	23	660	K–12 (13 years)
Hali	F	RICE	19	620	K–12 (13 years)
Ben	M	SIMS	20	610	K–12 (13 years)
Que	F	SIMS	21	600	K–12 (13 years)
Kim	F	TATE	18	730	1–12 (12 years)
Lea	F	WEBB	23	610	K–12 (13 years)
Don	M	WEBB	20	710	K–12 (13 years)

Setting

There were twelve settings for this study. To utilize Bronfenbrenner's (1979, 1999) ecological and PPCT framework, it was advantageous to interview the homeschool graduates in their homeschool environment, if possible. For the participants'

convenience, the locations varied. The interview sites included: five graduate homes (with seven graduates), two rental properties (with two participants), three college rooms (with five participants), a restaurant (with two graduates), and the researcher's home (one participant). The interviews took place in South Carolina and Virginia.

In these two states, the homeschooling population percentages align somewhat with the state school age population percentages for the nation. Using available data from research, 1% of homeschoolers have resided in South Carolina, and 1% in Virginia. In 2009, 1% of the nation's school aged children lived in South and 1% lived in Virginia (see Table 2). These percentages were identical, justifying the use of these states as a reasonable geographic location for participating homeschooling families.

Table 2

National Percentages of Homeschool Students (HS) in South Carolina, and Virginia as Compared to Percentage of School Aged Students (SA) in SC and VA

State	% HS	#HS in State/Total HS	(Year)	% SA	#SA State/Total SA (2009)
South Carolina	1%	8,494/0.85 million	(2000)	1%	0.8 million/53 million
Virginia	1%	24,682/2.04 million	(2010)	2%	1.3 million/53 million

Note. These percentages were calculated based on data obtained from NHES (2008); Ray (2010); and Ziese (2011).

Data Collection Procedures

Initially, IRB approval was obtained (see Appendix A). Then a pilot study was conducted. Since Yin (1994) advocated the use of pilot studies, I incorporated this strategy within my research. This practice clarified the scope of the research and provided reliability to the researcher-made instruments, the interview and survey questions. I selected one homeschooling graduate that fit my delimiters of a 600 or better SAT mathematics subtest score and had been homeschooled for at least 7 years, for the pilot study. I analyzed the data from the NAEP mathematical proficiency test, survey, interview, math self-efficacy survey, and document data with Bronfenbrenner's (1979, 1999) ecological and PPCT theoretical models and the NRC's Five Strands of Mathematical Proficiency (Kilpatrick et al., 2001). Then, I refined my interview and survey instruments and develop a preliminary set of data codes to help with the data analysis during the main research. Goodman (2008) used the results of her pilot study to generate data codes for her data analysis. She refined those codes as she conducted the main study. Following the revision of my researcher-made instruments, I recruited homeschool families through one South Carolina gatekeeper and personal contacts

located in two southeastern states. The remaining graduates were interviewed based on the lessons learned from the pilot. I was more careful to stay on topic, using the interview questions to focus my attention, rather than allow for other “off-topic” discussion. The process in taking the survey for the home educators was also slightly refined through the pilot study process. I offered the educators the option of mailing or emailing the survey. Most of the surveys were mailed with return self-addressed stamped envelopes. Additionally, I mailed all of the MSES (Betz & Hackett, 1993) forms with self-addressed stamped envelopes. The pilot study helped structure the organization of the survey, MSES, NAEP, and SAT documents.

The gatekeepers were educational organizations located within South Carolina and Virginia. One of the South Carolina organizations was a state licensed homeschool accountability group that services over 50 homeschooling families. The other South Carolina group acted as a legislative watchdog and helped sponsor homeschool conventions in South Carolina. Two of the organizations were online schools affiliated with accredited institutions of higher learning based in Virginia. After IRB approval (Appendix D) and completion of the pilot study, the gatekeepers sent out an email to its constituents with the intent of this study, the delimiters of the study, and an invitation for qualifying families to volunteer for the study, with my email contact information (Appendix E). Only one of these gatekeeper provided participants. The remaining participants came from personal contacts and snowballing.

After identifying potential volunteers, I sent out informed consent forms via email attachments (Appendix F, G). When I received the signed consent forms, data collection began. Multiple sources of data, data triangulation, is needed to provide dependability and credibility to a qualitative study (Ary et al., 2006). In this multiple case study, five

types of data were collected: (a) a mathematical proficiency test consisting of NAEP standardized items given to the homeschool graduates; (b) documents verifying the SAT math subtest scores and final mathematics course grades (c) mathematical self-efficacy scores of both the graduates and their home educators; (d) semi-structured interviews of the homeschool graduates; and (e) surveys of the home educators.

Standardized Instruments

I utilized two standardized instruments in the data collection phase. After the potential participants contacted me, I emailed (or mailed) a mathematical proficiency evaluation to the homeschool graduate. The mathematical proficiency evaluation (Appendix A) was a modified form of the standardized and normed NAEP Grade 12 Mathematics Assessment (NCES, 2009). I gave each graduate a \$25 gift card of his or her choice for participating. Additionally, from those who completed member checks, I randomly selected a graduate for a \$50 iTunes gift card. Graduates who scored 80% or better on the NAEP and their parents were mailed the Mathematical Self-Efficacy Scale (Betz & Hackett, 1993). I scheduled an interview with the participants after I received their permission to do so.

Mathematical proficiency questionnaire. I emailed or mailed a modified version of the NAEP Grade 12 Mathematical Proficiency Test to the participating homeschool graduates. This 10-item instrument was made up of questions covering all four of the Grade 12 content areas. This test was used to evaluate four of the five strands of the NRC's (2001) mathematical proficiency: conceptual understanding, procedural fluency, adaptive reasoning, and strategic competence.

For the purpose of this study, the sample Grade 12 NAEP Mathematical Proficiency Test was downloaded from the NCES website (NCES, 2009) and saved into a

WORD document. This online instrument consisted of 10 items selected from previous assessments, released for general publication. I modified the NAEP evaluation with directions added to the beginning of the document and the phrase “SHOW YOUR WORK AND EXPLAIN YOUR METHOD” added after each of the 10 questions (Appendix A). According to the NAEP website, these 10 items were broken down into three levels of complexity (3 easy, 6 medium, 1 hard) and overlapped in five content areas (4 number properties and operations, 3 measurement, 2 geometry, 4 data analysis and probability, 4 algebra). I sent this evaluation to the homeschool graduates via an email attachment. They returned the completed test via email or snail mail.

Mathematical Self-Efficacy Scale. The Mathematics Self-Efficacy Scale [MSES] (Betz & Hackett, 1993) published by Mindgarden, Inc. was utilized for this study (Appendix H). The MSES has been normed, standardized, and used extensively in research (Betz & Hackett, 1993; Ciechalski, 1993; Clutts, 2010; Hall & Ponton, 2005; Smith, 1993; Watts, 2009). The 1983 MSES had high reliability coefficients: .92 for math courses; .96 for math problems; .92 for math tasks; and .96 for total scale. The Coefficient alpha was reported at .92, with the 2-week interval test-retest reliability reported at .94. The 1993 instrument retained the Math Tasks and the Math Courses subscales, without updated validity coefficients reported (Betz & Hackett, 1993; Ciechalski, 1993; Smith, 1993). I purchased a one year license to administer the instrument to up to 100 participants, which more than covered the scope of this study. The MSES consisted of 34 items and two sub scales. Part One included 18 items measuring confidence with everyday math tasks. Part Two lists 16 classes. All items were rated on a Likert scale of 0 to 9, ranging from no confidence at all to complete confidence. Both the homeschool graduates and the home educators completed this scale.

Interviews/Surveys

The interview and survey instruments were grounded in the literature and with help from experts in the educational field. The semi-structured interview and survey questions were very similar, both measuring the constructs of the homeschool environment. The interview was aimed at the perspective of the homeschool graduate, while the survey gathered corresponding information from the parent to correlate or contradict the graduate's interview responses. Initially, the items were generated from homeschooling research literature. These items were evaluated and revised with the help of six educational experts. Three had conducted homeschool research: one had completed many homeschool studies and headed a homeschool research organization; another worked as a dean in a community college; the third was an educational psychology professor. Another reviewer was a professor that works extensively with doctoral research candidates. The final two evaluators worked in a public school setting; one administrated federal programs and the other was a middle school team leader.

Interview. An interview is “one of the most important sources of case study information” (Yin, 2009, p. 106). McCracken (1988) recommended one long interview driven by research. A review of the literature helps define the problems to be studied and construct the interview questions. Interviewers must synthesize the literature findings with their personal experience and the focus of the study. The interview questions need to begin with biographical questions, followed by a group of question areas specific to the topic. Effective interviewers set the tone of a session by eagerly listening to the answers and presenting oneself as curious, but not inquisitive.

The purpose of the interview is to understand and explain how the participants make meaning out of their experiences (Maxwell, 2005). Good interviewers need to put

themselves in their interviewee's place. In this study, the phenomenon investigated with the interview was the meaning that homeschool graduates gave to their mathematical proficiency acquisition. Since I have homeschooled for over 18 years and know many homeschooled students, I understood some of their experiences and the environment they came from. Furthermore, my 10-year experience as a high school mathematics teacher provided me the subject area background for evaluating mathematical proficiency.

After the receipt and evaluation of the NAEP mathematical proficiency test and MSES, I contacted each of the proficient homeschool graduates to arrange a face-to-face semi-structured interview (see Appendix B) in the home base of the graduate. With the consent of the participants, all the interviews were audio-taped. The purpose of the interview was to understand and explain how the homeschool graduates became mathematically proficient. Interviewing some of the former homeschoolers in their homeschool environment helped me understand the dynamics of that environment and place the interviewee in that context. Out of necessity, some interviews took place in other homes, dormitories, a restaurant, and college classrooms. Interview questions involved the participants remembering and describing their perceptions of significant feelings and events that gave meaning to their homeschool experience as it related to their acquisition of mathematical proficiency. As co-researchers they contribute their voice to this case study research.

All 20-interview questions were grounded in research (see Table 2). They correlated with the first 20 items in the survey. The data from these items was used to compare the meaning the homeschool graduates' gave to their homeschool environment with the home educator perceptions. The questions asked for descriptions about the family, homeschooling motivation, graduate's coursework, graduate's experiences with

the homeschooling environment, preparation for college, mathematics, and any changes that the graduate would make to the homeschooling experience.

Question 1 asked demographic questions to determine if the family was representative of what was known about the homeschooling population. Planty et al. (2009) found racial demographics of 76.8% Caucasian, 4% African American, 9.8% Hispanic, 9.3% other races. Ray (2010) and Rudner (1999) studied homeschooling populations with 91.7% Caucasian, 1.2% African American, and 2.2% Hispanic populations. Ray (2010) and Rudner (1999) did not find gender or race to be significant variables with respect to standardized test scores. Researchers have found most homeschooling families (61% or greater) have three or more children (Planty et al., 2009; Ray, 2010; Rudner, 1999; Van Pelt et al., 2009), three times the nationwide average of 19.8% (Ray, 2010).

Questions 2 and 3 addressed the reasons that families started and continued to homeschool. The most popular reason was religion and morals, followed by academic concerns (Barwegen, Falciani, Putnam, Reamer, & Stair, 2004; Collum, 2005; Green & Hoover-Dempsey, 2007; Ice & Hoover-Dempsey, 2011; Ray 2000; 2004; 2010; Van Pelt et al., 2009), although recent research has showed these motivations becoming more complex (Collum, 2005; Goodman, 2008). Homeschooling motivation affected homeschool-learning environments (Cai et al., 2002; Goodman, 2008).

Question 4 dealt with the number of years that the graduate had been homeschooled. Ray (2000, 2010) and Rudner (1999) found this variable to be significant to standardized test scores.

Questions 5 through 7 were included based on input from the expert review. Two reviewers specifically asked that the interview include more open questions, to express

opinions about their overall experience and favorite and least favorite subjects. Giving voice to homeschool students required them to express their opinions about their experiences (L. van Schalkwyk, personal communication, February 9, 2012).

Questions 8 through 10 considered the mathematics classes the student had taken and whether they felt their mathematical experience prepared them adequately for their future pursuits. Question 8 was asked to determine whether the graduate had taken algebra, geometry, and any other upper level mathematics classes. Abstraction reasoning was critical for mathematical proficiency and the study of algebra provided practice with problems and symbolic manipulation, increasing reasoning skills (Milgram, 2007). Students who took Algebra II were found to double their odds at completing a bachelor's degree by eight years after high school (Trusty & Niles, 2003). Algebra courses were considered a gatekeeper for college retention and success (Hall & Ponton, 2005; Milgram, 2007; NMAP, 2008). Homeschoolers who had exposure to geometric concepts scored better on standardized tests, than homeschoolers who had not been exposure to geometric concepts (Richman et al., 1992). Questions 9 and 10 asked whether the graduate considered their mathematics experience was adequate or inadequate and if they felt prepared for their college mathematics course (if one or more was taken). All four homeschool participants in Bagwell's (2010) study and 25% of the homeschool sample in Jones' (2010) research felt a lack of confidence in their mathematical background, wishing they had been better prepared.

Clemente (2006) found that homeschooled students significantly outperformed non-homeschoolers on the college SAT test. Ortiz (2000) asked why homeschoolers performed better on these tests. Question 11 was added to find out why the graduates believed they performed above average (top 25%) on the mathematics SAT subtest. With

regard to mathematical proficiency assessment, Stage posited that “What do you think” (p. 361) is the most profound question to be asked.

Questions 12 and 13 asked for a description of the mathematics curricula and instruction that occurred in their homeschool. Clements (2002) identified two major types of homeschooling teaching, direct instruction and self-study, and seven types of homeschooling curricula: (a) textbook-based; (b) literature-based; (c) computer-based; (d) video/satellite; and (e) unschooling. Peterson (2011) found that parents preferred pre-made structured curriculum, religious or secular. Martin-Chang et al. (2011) found that elementary students from structured homeschool environments significantly outperformed non-homeschooled students and homeschool students from unstructured homeschool environments. Richman et al. (1992) found that most participants liked Saxon math books for their cyclical content structure and homeschool students whose parents routinely corrected their answers scored better on standardized measures. Shepherd (2010) reported that the participants preferred a more traditional approach to mathematics with much drill and practice activities. Bannier (2007) found that home educators taught until mastery, pacing the work according to student needs. Cai et al. (2002) found controlling teacher behaviors in religiously motivated home educators. Goodman (2008) found that high school students take many classes outside of the home environment; this could conceivably include mathematics courses. NCTM Principles and Standards (NCTM, 2000) guided public school teachers. The six guiding principles of equity, curriculum, teaching, learning, assessment, and technology were accomplished through the five content standards—number and operations, algebra, geometry, measurement, data analysis and probability—and the five process standards: problem solving, reasoning and proof, communication, connections; and representation. It would

be beneficial to both the homeschooling and the public school community to identify whether these principles and standards are implemented in the homeschooling mathematical curricula and instruction.

Questions 14 and 15 asked about the relational parts of the homeschool environment and the changes that took place over the years of homeschooling. Question 14 involved the interactions that occur during mathematical instruction and experiences. Question 15 inquired about any changes or adaptations the family had made in the mathematical instruction. This related to Bronfenbrenner's (1979, 1999) ecological and PPCT models and how relationships and environment worked together in the proximal processes of an individual's human development. Von Schalkwyk and Bower (2011) used Bronfenbrenner's bio-ecological framework to analyze the homeschool environments comparing and contrasting the parents and children's perspectives.

Comparing the mathematics homeschool experiences with those in other subjects, Question 16 was added during the expert review. A leading home education researcher suggested that the graduates reflect on the similarities and differences of their math studies and other courses. (B. D. Ray, personal communication February 9, 2012).

Questions 17 and 18 involved the hindsight perceptions of the homeschool graduate. Their perspective was critical to the bio-ecological model (L. von Schalkwyk, personal communication, February 9, 2010). Future research needed to address what the students have to say about their mathematical experiences and their proficiency (Schoenfeld, 2007) and "what homeschool students think themselves about the educational choice made by their parents" (Shepherd, 2010, p.107). Question 18 asked what the former homeschoolers would have done differently.

Questions 19 and 20 were included to discern the mathematical attitudes of the homeschool graduates and their educators. These questions complemented component five of the NRC's Five Strands of Mathematical Proficiency, productive disposition (Kilpatrick et al., 2001). The graduate responses of these questions were compared with the survey answers of the home educators and the MSES (Betz & Hackett, 1993) results.

Table 3

Semi-Structured Interview Questions and Corresponding Research Citation

Questions	Corresponding Research
1) What is your age, birthplace, birth order? Describe your siblings (age, male/female, race, type of schooling).	1) McCracken (1988); Planty et al. (2009); Ray (2000, 2010); Rudner (1999).
2) What was/were the initial reason(s) your family started to homeschool?	2) Barwegan et al. (2004); Cai et al. (2002); Collom (2005); Goodman (2008); Green & Hoover-Dempsey (2007); Ice & Hoover-Dempsey (2011); Ray (2000, 2004, 2010); Van Pelt, Allison, & Allison (2009).
3) What is/are the reason(s) your family continues to homeschool?	3) Barwegan et al. (2004); Cai et al. (2002); Collom (2005); Green & Hoover-Dempsey (2007); Ice & Hoover-Dempsey (2011); Ray (2000, 2004, 2010); Van Pelt, Allison, & Allison (2009).
4) How long has your family been homeschooling?	4) Ray (2000, 2010); Rudner (1999).
5) Tell me more about your experience with homeschooling.	5) L. van Schalkwyk, personal communication, February 9, 2012.
6) Describe your favorite subjects during your homeschool years.	6) L. van Schalkwyk, personal communication, January 18, 2012.
7) Describe your least favorite subjects during your homeschool years.	7) L. van Schalkwyk, personal communication, January 18, 2012.
8) Describe what math classes you have taken. (looking for algebra, “gateway to college”)	8) Milgram (2007); NMAB (2008); Schoenfeld (2007); Stage (2007).
9) Describe your math experience during your homeschool years (adequate/inadequate?).	9) Bagwell (2010); Jones (2010).
10) How well were you prepared for college mathematics courses?	10) Bagwell (2010); Jones (2010).

Questions	Corresponding Research
11) Why do you think you did so well on the SAT math subtest?	11) Ortiz (2000).
12) Describe the mathematics curricula you used in homeschooling.	12) Clements (2002); NCTM (2000); Peterson (2011); Richman et al., (1992).
13) Describe what your family's mathematics instruction was like (e.g. style of instruction – structured or non-structured, direct teaching, reworked incorrect problems until mastery; group work, self-study, online course, combination of activities, co-op classes, other).	13) Bannier (2007); Cai, Reeve, & Robinson (2002); Clements (2002); Goodman (2008); NCTM (2000); Peterson (2011); Richman et al., (1992); Shepherd (2010).
14) Describe the personal interactions in your homeschool during the time that mathematical instruction took place (e.g. some or limited – self-study; parent student interaction – conversations, lessons—stressful or enjoyable; working with siblings—stressful or enjoyable).	14) Bronfenbrenner (1979, 1999); Ortiz (2000); van Schalkwyk & Bouwer (2011).
15) Describe changes (if any) that have taken place in your mathematical instruction and experiences over the years (e.g., curriculum, DVD classes, online classes, classes outside of home).	15) Bannier (2007); Bronfenbrenner (1979, 1999); Hanna (2011); Richman et al., (1992).
16) How was your homeschool experience in math the same or different from your homeschool experience in other subjects?	16) (B. D. Ray, personal communication February 9, 2012).
17) How do you view your overall homeschool math experience?	17) Ortiz (2000); Schoenfeld (2007); Shepherd (2010).
18) How would you change your homeschool math experience?	18) Shepherd (2010).
19) Do you view mathematics as useful, sensible, or worthwhile?	19) Kilpatrick et al. (2001).
20) What are your parents' attitudes about and experiences with math?	20) Betz & Hackett (1993).

Home educator survey. The survey I designed for this study consisted of 23 items (see Appendix C). As in the interview, the survey was grounded in literature (see Table 4). This survey contained questions about the family's demographics, how many years the graduate was homeschooled, the motivations for homeschooling, the structure of the homeschool, the mathematical curricula utilized by the graduate, the interactions that occurred during mathematical instruction, and experiences and the support systems utilized by the home educators. The first 20 items of the survey asked the same questions as the interview, from the perspective of the home educator. This information correlated the information given by the homeschool graduate. Question 21 dealt with the homeschooling support the home educator received. Research has demonstrated that homeschool educators attend conferences, workshops, network with other homeschoolers, form co-ops to provide instruction, search online for curriculum, and seek out training for their homeschool instruction (Banner, 2007; Hanna, 2011; Richman et al., 1992). Question 22 asked for other potential homeschool volunteers, snowball sampling recommended by Ary et al. (2006). The final item was for additional comments.

Table 4

Home Educator Survey: Open-Ended Questions and Corresponding Research Citations

Questions	Corresponding Research
1) Tell me about your children. The graduate being interviewed: age, birthplace, birth order. Describe your other children (age, male/female, race, type of schooling).	1) McCracken (1988); Planty et al. (2009); Ray (2000, 2010); Rudner (1999).
2) What was/were the initial reason(s) your family started to homeschool?	2) Barwegan et al. (2004); Collom (2005); Goodman (2008); Green & Hoover-Dempsey (2007); Ice & Hoover-Dempsey (2011); Ray (2000, 2004, 2010); Van Pelt, Allison, & Allison (2009).
3) What is/are the reason(s) your family continues to homeschool?	3) Barwegan et al. (2004); Collom (2005); Green & Hoover-Dempsey (2007); Ice & Hoover-Dempsey (2011); Ray (2000, 2004, 2010); Van Pelt, Allison, & Allison (2009).
4) How long has your family been homeschooling?	4) Ray (2000, 2010); Rudner (1999).
5) Tell me more about your home school graduate's experience with homeschooling.	5) L. van Schalkwyk, personal communication, February 9, 2012.
6) Describe your homeschool graduate's favorite subjects during your homeschool years.	6) L. van Schalkwyk, personal communication, January 18, 2012.
7) Describe your homeschool graduate's least favorite subjects during homeschooling.	7) L. van Schalkwyk, personal communication, January 18, 2012.
8) Describe what math classes your homeschool graduate took during homeschooling (looking for algebra, "gateway to college").	8) Milgram (2007); NMAB (2008); Schoenfeld (2007); Stage (2007).
9) Describe your homeschool graduate's experience with math during homeschooling (adequate/inadequate?).	9) Bagwell (2010); Jones (2010).
10) How well was your homeschooler prepared for college mathematics courses?	10) Bagwell (2010); Jones (2010).

Questions	Corresponding Research
11) Why you think your homeschool graduate did so well on the SAT math subtest (scoring in the top 25%)?	11) Ortiz (2000).
12) Describe the mathematics curricula you used with your homeschool graduate.	12) Clements (2002); NCTM (2000); Peterson (2011); Richman et al. (1992).
13) Describe what your homeschool graduate's mathematics instruction and experiences were like (e.g., style of instruction – structured or non-structured, direct teaching, reworked incorrect problems until mastery; group work, self-study, online course, combination of activities, coop classes, other).	13) Bannier (2007); Cai, Reeve, & Robinson (2002); Clements (2002); Martin-Chang, Gould, & Meuse (2011); NCTM (2000); Peterson (2011); Richman et al. (1992); Shepherd (2010).
14) Describe the personal interactions in your homeschool during the time that mathematical instruction took place for the homeschool graduate (e.g. some or limited – self-study; parent/student interaction conversations, lessons – stressful or enjoyable; working with siblings – stressful or enjoyable).	14) Bronfenbrenner (1979, 1999); Ortiz (2000); van Schalkwyk & Bouwer (2011).
15) Describe changes (if any) that have taken place in your homeschool graduate's mathematical instruction and experiences over the years (e.g., curriculum, DVD classes, online classes, classes outside of home).	15) Bannier (2007); Bronfenbrenner (1979, 1999); Hanna (2011); Richman et al. (1992).
16) How was your homeschool graduate's experience with math during homeschooling the same or different from their homeschool experience in other subjects?	16) B. D. Ray personal communication, February 9, 2012.
17) How does your homeschool graduate view his/her overall homeschool math experience (advantages/disadvantages)?	17) Ortiz (2000); Schoenfeld (2007); Shepherd (2010).
18) What would you have changed about your homeschool graduate's math experience?	18) Shepherd (2010).

Questions	Corresponding Research
19) Does your homeschool graduate view mathematics as useful, sensible, or worthwhile?	19) Kilpatrick et al. (2001).
20) What are your attitudes about and experiences with math?	20) To correlate with the MSES (Betz & Hackett, 1993).
21) Describe any homeschooling support that you have received (e.g., conferences, math workshops, support groups, online resources and coops).	21) Bronfenbrenner (1979, 1999); Hanna (2011); Richman et al. (1992).
22) If you know of other homeschooling families that might participate in this study, please ask them to contact me: Betty.Cardinale.Research@gmail.com .	22) Ary, Jacobs, Razavieh & Sorensen (2006).

Document Collection

Bogdan and Boyton (2007) recommend the use of document analysis to give another perspective on the phenomenon. These documents can be official in nature, giving objective information about standardized test scores and course grades. I asked to view a copy (either the actual report or an online copy) of their SAT mathematics subtest scores and a copy of their high school transcript, showing final grades of completed mathematics courses. Due to the self-reporting nature of the SAT scores and the mathematically self-efficacy questionnaire, validity of the given information may be questioned. These documents provided verification of the students' SAT mathematics subtest score and mathematical course information on the MSES. I set up a special email address for participant contact (Appendix G).

Data Analysis Procedures

Data analysis is the most challenging aspect of the multiple case study (Stake, 2006). The potential difficulties can be ameliorated through the use of a general strategy for analyzing the data, such as relying on theoretical propositions, developing a case description, using both qualitative and quantitative data, and examining rival explanations, or descriptive frameworks (Yin, 2009). Relying on theoretical propositions is the most preferred of the general strategies. These propositions guide the data collection process. This case study used Bronfenbrenner's (1979, 1999) four-system ecological model and PPCT model to understand the effects of the homeschool environment (relationships, materials, processes, and time) upon the acquired mathematical proficiency of the homeschool graduate, and the NRC's Five Strands of Mathematical Proficiency conceptual model to evaluate the graduate's level of proficiency. Additionally, quantitative data was collected. The mathematical self-efficacy scales of the graduates and the home educators were examined to see if any patterns arose from the collective case study. A document analysis of the SAT scores was used to verify the self-reported scores and the high school transcripts were analyzed to verify the interview data about the math courses taken. Finally, the graduate NAEP ratings of mathematical proficiency were compared with their SAT math subtest scores and their mathematics self-efficacy scores. Primarily, this multiple case study employed the theoretical propositions strategy, but also utilized the quantitative and qualitative strategy.

For use with each general strategy, Yin (2009) specified five types of data analysis for case studies. These types included: (a) pattern matching; (b) explanation building; (c) time-series analysis; (d) logic models; and (e) cross-case synthesis. I used explanation building, which was a special case of pattern matching. Yin (2009) stated

that pattern matching-logic is “one of the most desirable techniques” (p. 136) in case study analysis. The goal of explanation building was to analyze the data through a pre-built explanation about the case. This case study used the theoretical and conceptual frameworks of Bronfenbrenner’s (1979, 1999) four ecological systems, PPCT, and the NRC’s Five Strands of Mathematical Proficiency Model (Kilpatrick et al., 2001).

As data was collected, I utilized several techniques for maximum reliability. The five types of information, home educator survey, mathematics self-efficacy scale, homeschool graduate interview, official documents (SAT math subtest scores and final high school transcripts) and memoing, provided data triangulation. Data triangulation was necessary for validity and reliability in case study research (Yin, 2009).

Data analysis triangulation was conducted through transcription, memoing, member checks, expert review, and audit trail. I transcribed the audiotapes, analyzed the results and submitted the analyses to the participants for member checks. I used the software Express Scribe to facilitate transcription. Bogdan and Biklen (2007) recommended writing observer comments after reviewing the transcripts and interviewer notes. They recommended that the researcher write memos about what they are learning (Bogdan & Biklen, 2007). I memoed after many of the interviews (Appendix I) and kept an audit trail of all documents. Guided by Goodman (2008), I assigned data codes to the responses from the interviews and surveys. Goodman (2008) open coded data from the pilot study interview and created a set of initial codes (e.g., SL – self as learner; MT – mother as teacher), utilizing these with the formal data analysis. Similarly, with the data from the pilot study, I created an initial set of codes (e.g., SS – self-study; ME – math experiences) and refined them throughout the participant analysis. These codes helped

identify patterns within and across cases. Each of the 16 individual cases had a story to tell (Yin, 2009), explaining their acquired mathematical proficiency, in their own voice.

As each interview was completed, I analyzed that case (i.e., within case analysis) with the data collected: (a) interview, (b) survey, (c) mathematical self-efficacy scale scores, (d) SAT and high school transcripts, and (e) memos made after the graduate interviews. The specific purpose was to build an explanation as I analyzed the data through the theoretical and conceptual frameworks. This pattern was repeated for each within case analysis. Data that fit with the explanation was literally replicated (Yin, 2009). Data that contradicted the explanation was considered a rival theory. Subsequent data that supported the rival explanations was considered theoretical replications. The general explanation was altered slightly as the research continued, revising the original statement or proposition (Yin, 2009). Tables 3, 4, and 5 summarize the theoretical and conceptual frameworks used in the individual and cross-case analyses.

Summary

Yin's (2009) explanation building theory was utilized to analyze the data collected. Bronfenner's (1979, 1999) theoretical framework served as a filter for the information collected about the homeschool graduate's environment, involving attitudes, instructional practices, relationships, and physical surroundings. The data analyzed included the interview transcriptions, the survey responses, the MSES scores, the SAT and transcript documents, and the researcher memos. Literal replication of the data provided common themes organized by pre-determined data codes. First, 16 within case studies were completed, followed by a between case study.

Table 5

Bronfenbrenner's Four Ecological Systems (Bronfenbrenner, 1979)

System Name	System Description
Microsystem	The interactions between the physical things and the relationships (primary dyads, other dyads and triads) experienced by the developing individual in a regular, familiar environment.
Mesosystem	The collection of microsystems that the developing individual experiences on a regular basis.
Exosystem	The microsystems of others members of the dyads that the developing person does not belong to, but affects the developing individual.
Macrosystem	The overarching geographic or cultural group that the developing individual is a member – it may also include the societal belief system and structure that the developing individual lives within.

Table 6

Bronfenbrenner's PPCT Model (Bronfenbrenner, 1999) Applied to Homeschooling

Variable Types	Homeschool Applications
Process variables	Mathematical curriculum, instruction, experiences.
Person variables	Perceptions, feelings, choices of homeschool graduate and home educators.
Context	The environment in which the experiences and instruction took place.
Time	The evolving process as evaluated over time – how long did it take, how did changes take place, why did the changes take place.

Table 7

Five Strands of Mathematical Proficiency Model (Kilpatrick et al., 2001, p. 115)

Component	Description
Conceptual Understanding	Comprehension of mathematical concepts, operations, and relations.
Procedural Fluency	Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
Strategic Competence	Ability to formulate, represent, and solve mathematical problems.
Adaptive Reasoning	Capacity for logical thought, reflection, explanation, and justification.
Productive Disposition	Ability to see mathematics as sensible, useful, and worthwhile.

Trustworthiness

Also labeled rigor (Lincoln & Guba, 1986), trustworthiness involves confirmability, credibility, dependability, and transferability. In qualitative research, confirmability is analogous to objectivity and credibility matches with internal validity, of quantitative research. Dependability (qualitative) matches with reliability (quantitative). Transferability in qualitative research is analogous to the external validity of quantitative (Ary et al., 2006; Lincoln & Guba, 1986).

Confirmability deals with objectivity and neutrality in research (Ary et al., 2006). An audit trail provides the method for other researchers to be able to replicate the same results from a given set of data. Throughout this multi case analysis, peer review of documentation was enforced.

Credibility relates to the integrity of the research (Ary et al., 2006). It establishes the truthfulness or accuracy of the findings (Ary et al., 2006; Lincoln & Guba, 1986).

Method and data triangulation enhance credibility. Member checks and participant feedback add to the credibility of the results. Rich description boosts credibility. Also, Lincoln and Guba (1986) advocate prolonged engagement, persistent observation, peer debriefing, and negative case analysis to increase credibility. I employed five types of data collection: survey, NAEP assessment, MSES results, in-depth interview (of student), and document analysis (SAT scores and high school transcripts) and several types of analysis: transcription and thematic coding, memoing, audit trail, expert review, and ongoing member checks. These types of analysis add to the rigor of the study.

Dependability in qualitative research refers to the consistency of the results, “the extent to which variation can be tracked or explained” (Ary et al., 2006, p. 509). Use of the audit trail establishes dependability (Ary et al., 2006; Lincoln & Guba, 1986). An effective audit trail includes thorough record keeping: raw data; careful notes describing choices made about the research and its procedures; detailed records of the research activities; and rich journal entries of hypotheses formation (Ary et al., 2006). The expert review of my dissertation committee members, all who have conducted homeschooling research, to corroborate coding techniques and findings, data triangulation, and methods triangulation enhance the dependability of my research.

Transferability deals with the generalization of the findings to other contexts. Thick and rich descriptions, such as the use of meaningful quotes, cross-case comparisons, limitations discussion, detailed account of methods, and reflective statements are effective means of establishing transferability (Ary et al., 2006). Such descriptive narrative is “developed about the context so that judgments about the degree of fit or similarity may be made by others who may wish to apply all or part of the

findings elsewhere” (Lincoln & Guba, 1986, p. 77). An audit trail and updated reflective journal provided rich descriptions and quotes and gave “voice” to the participants.

More recently, Lincoln and Guba (1986) have proposed a synthesis of rigor and reliability, labeled authenticity. This paradigm included five criteria: (a) fairness; (b) ontological authentication; (c) educative authentication; (d) catalytic authentication; and (e) tactical authenticity. Lincoln and Guba (1986) defined fairness as, “a balanced view that presents all constructions and the values that undergird them” (p. 78). Evaluators and participants have been classified as equal research partners. Fairness involved transparency of the inquiry process fully informed consent as a continual process, and constant implementation of the member check practice. Ontological authentication included raising consciousness of some phenomenon previously not (or little) appreciated. Educative authentication represented the gatekeeper’s involvement in the inquiry process, enhancing their awareness of the population they serve. Catalytic authentication attempted to give “voice” and empowerment to the participant group, and tactical authenticity evaluated the extent of the given “voice” and empowerment.

Given my experience as a home educator, I viewed other homeschoolers, including participants of this study, as equal partners in this research and I relied upon ongoing member checks, provided to the participants after the analysis of their individual case to enhance fairness. I believe it was critical to raise consciousness about homeschooling, given the constant challenges by legislators, public educators, and those critical of home education, addressing ontological authentication. Attending to educational authentication, I kept the gatekeepers involved in this research process, being careful to use pseudo-names of the participants. My desire in this research was to give homeschoolers a “voice” and empower them to confidently continue the form of

education they have chosen for their family, dealing with catalytic authentication. Potentially raising tactical authenticity, I also wished to make known productive mathematical strategies available for other homeschooling families so that they could evaluate and improve their own programs.

Acknowledgement of researcher bias is necessary for trustworthiness (Maxwell, 2005). Personal, professional, and practical interests of the researcher need to be identified to keep integrity within the project design and analysis. Honest evaluation of the situation to self can reduce bias, protecting against invalidation of the study. I continually acknowledged my personal and professional bias through journaling and discussions with my expert reviewers.

Ethical Issues

In keeping with ethical research principles, I sought to protect all participants from any potential harm that might come to them through my research. The identities of participants and gatekeepers were protected. I devised a way to select their pseudonyms and they were allowed input with their member checks. None sought to change those pseudonyms. The gate-keeping institutions were identified by geographic location and general description pseudonyms such as VAC (Virginia online institution affiliated with an accredited college) and NCS (North Carolina science business). Confidentiality is the top priority to the American Psychological Association (APA, 2010). In case study research, the APA recommends preparing the descriptive case material and handing it over to the subjects of the study for their written approval. I followed this advice by obtaining written consent prior to the study and showing the data analysis of the interviews for member checks. Also, I made available the conclusions to the participating families for their information. One family sat down with me after their

interviews and I debriefed them on the progress of this study.

Addressing security and confidentiality concerns, I stored all raw data in locked or secured cabinets and password protected computer files. Video recordings and transcriptions were stored separately in secured storage areas. All backup copies of the information were stored separately in a locked cabinet. The expert reviewer was the only person with access to the raw data with pseudonyms inserted. All research raw data will be destroyed five years after project completion.

Integrity remained my goal. I sought to conduct authentic homeschooling research that accurately portrayed what mathematical methods homeschoolers utilized. I acknowledged my positive bias towards homeschooling, but I presented the results in an accurate way, even if the results did not positively represent homeschooling practices. Researchers must take care to carefully collect the data, accurately represent the “voice” of the participants, and truthfully display the results, whether or not the findings coincide with their beliefs. I intended to be an exemplary Christian example through the entire research process, keeping in mind “Work hard so that you can present yourself to God and receive his approval; Be a good worker one who does not need to be ashamed and who correctly explains the word of truth” (2 Timothy 2:15, NLT).

CHAPTER FOUR: FINDINGS

The purpose of this study was to understand and explain how mathematically proficient homeschool graduate acquired this proficiency. This multiple case study focused on the central research question of how does the homeschool environment, including attitudes (of both graduate and educators), instructional practices, relationships, and physical surroundings (may include more places than the home) contribute to the mathematical proficiency of the homeschool graduates. Five sub questions guided this study: (1) how do the homeschool graduates describe their mathematical homeschool environment, (2) how do the homeschool graduates describe their mathematical self-efficacy, (3) how do their home educators describe their mathematical self-efficacy, (4) how do their home educators describe the graduate's homeschool environment, and (5) are there any apparent patterns or relationships between the descriptions of the graduates and their home educators concerning the homeschool environment and their mathematical self-efficacy?

This chapter contains 16 within case studies, followed by a between case study. Both the within and between cases are analyzed through the filters of Bronfenbrenner's (1979, 1999) ecological theoretical framework and the NRC's (2001) Five Strands of Mathematical Proficiency conceptual framework. For the between case study, I used explanation building (Yin, 2009), to highlight literal replications of the within cases. This was done to understand and explain the graduates' acquisition of mathematical proficiency.

Participant Portraits

Each of the 16 graduate participants represented an individual within case study, organized by family names. Family demographics and parental information preceded the graduate presentation. Interview, educator survey, MSES scores, NAEP assessment, SAT score report, and high school transcript data was triangulated, synthesized, and presented as a participant portrait. Appendices J–Y contain additional details.

Cox Family (Ned)

The Cox family resided and homeschooled in a South Carolina suburb (66,000), within 25 miles of a large metropolitan area (750,000). This family consisted of two Caucasian parents and four biological children. Their homeschool program continued for 23 years. Three of the four completed their K-12 education at home, while one went to public high school, 10-12. The youngest, Ned graduated his K-12 homeschool in 2011.

Ned's parents devoted time and resources to his education. Their homeschooling motivations included faith and individualized, flexible schooling. Ned saw it as family unity and his mother, "wanted to be sure to spend more time with us." The mother previously worked as a RN, before she became the primary teacher. Both she and Ned shared that she found math difficult. Confirming that, her MSES scores showed a wide spread, a 7.4 on the everyday math tasks and a 4.5 on math courses sections. The total score (6.1) ranked in the female 60 percentile (Table 13). This indicated confidence with practical math, but not with upper level math. The father, a retired radiologist, helped with harder math courses, because he liked math. Predictably, his total MSES score (8.1) ranked in the male 90 percentile. They outsourced Ned's math from eighth grade on.

Ned. In the den of his parents' home, this astute, sincere, 19-year old young man willingly answered all questions in his 28-minute interview. He believed that

homeschooling provided a “perfect fit for his personality” and his mother verified that. His homeschooling math experience afforded him “the best plan for myself” and his mother called it “adequate.” Aligning with his beliefs, he scored in the top 7% (The College Board, 2012) with his SAT math score (740) and in the male 95 percentile (Table 10) with his MSES score (8.8). Also he fulfilled all the mathematical proficiency components with his: NAEP score (90%), explanations and justifications for the NAEP items, MSES score, and certainty that mathematics was useful, sensible, and worthwhile. His high school transcript verified excellent grades, Algebra 1 through Calculus 2.

Despite his stellar performance, math frustrated Ned throughout his homeschool years. He affirmed his flexible homeschool math experience as, “it fit my personality with how I get frustrated and I would still be able to come back and not be penalized . . . eventually . . . learn what I needed to learn.” Ned mistakenly thought that math should come easily and he revealed, “I didn’t understand a lot of the things initially because you have work at it. When I was younger, I didn’t entirely grasp that concept. . . . I thought that it was supposed to kind of come to you.” He struggled with his math work, even to the end of high school, describing it as, “To the very end of the last couple of math classes. . . . Math was always a source of frustration for me.” Although the mother did not mention Ned’s frustration, she expressed that she would have “gotten more outside help,” if she was to make any changes; this may be an indication that Ned struggled with math at home. His chemistry major helped him to see the study of math was worthwhile, stating, “I found out that there’s a lot of math in chemistry. . . . So that’s another reason I had to start liking math. . . . But . . . did not get too fond of it.”

Ned’s homeschool instruction included: direct teaching from his mother, a consistent schedule, personal attention from his parents, Saxon curriculum, progression to

self-study, and outside classes. Both Ned and his mother reported direct teaching in the early years, followed by a progression to self-study using Saxon. He described, “As I got older, it became more and more that I had to go to it and make sure the check marks were all in the assignment book and actually taking the test problems . . . and do them seriously.” Saxon was tedious but provided needed repetition; Ned related, “I had Saxon math for a long time. . . . Sometimes I wanted to burn my Saxon book. . . . It’s very tedious but I definitely needed some sort of review here and there.” From eighth to tenth grades he attended a university model school, which met three times weekly for homeschool students. Ned enjoyed and learned from these classes, reflecting, “After we heard so many different things so many times from her it really did start to stick . . . she had a passion for math . . . and it made it that much better for us.” He liked the self-study at home and the university model, “both of them pretty equally.” Both served as building blocks for the next level of math; he confirmed, “I wouldn’t be able to understand any of that [algebra] if I hadn’t learned from my parents the more basic things earlier on with Saxon. . . .The algebra I learned from [university model] . . . has stuck with me.” His math classes at the community college had a downside including, “We wasted massive amounts of the day because one person . . . didn’t understand it yet . . .we didn’t get through as much in calculus because there . . . were more people holding it up.”

Ned experienced good relationships with his family. He viewed his parents and siblings as resources, reflecting, “I definitely needed their structure on me to make sure I was getting through enough of it in a timely manner.” He appreciated his parents’ persistence, describing, “My parents were on me everyday to make sure I was doing [math]. . . .The personal attention from my parents . . . really helped me understand.” Their patience paid off; he admitted, “Sometimes I would storm off . . . eventually I

would sit back down. . . .They were like, are you ready to learn this now?” His mother provided, “a tender touch”, while his dad showed, “tough love.”

Ford Family (Cali and Sue)

The Ford family resided in a South Carolina suburb (66,000), within 25 miles of a large metropolitan area (750,000). This family consisted of two Caucasian parents and two biological children. These two sisters completed most of their K-12 education at home, the younger Cali, 2-12, and the older, Sue, 3-12. Both participated in this study.

Cali and Sue’s parents assumed different roles. They wanted to provide a good Christian education for their children. Cali thought, “My parents believed that the education was not as good in the public system as could be given at home” and Sue echoed that view, “the quality of education and my mother wanted to be able to tailor my education to my needs.” The mother instilled a good work ethic; Cali remembered, “Must work before you play.” Formerly employed as an industrial engineer, the mother acted as the primary teacher. She liked math and considered a strong math foundation very important; it was useful for science. Part of her MSES scores reflected that, 7.9 on everyday math tasks and 6.4 on math courses, indicating confidence with practical math and less certainty on college level. The mother’s total score (7.2) ranked above the female 90 percentile (Table 10). Cali reported that her mother, “still can do her basic math . . . geometry she was very good at it and . . . algebra she’s good at.” Sue included, “My mom was always very adamant about everything in my homeschooling. That everything was good and necessary and my dad supported her.” To support the homeschooling, Cali’s father worked as a controller, dealing with company finances. Unsurprisingly, his total MSES score (8.4) placed him near the male 95 percentile.

Cali. Bright and precocious, 19-year old Cali willingly answered all questions in her 38-minute interview. This took place in my home, near to her parents', for convenience and to visit with my daughter afterwards. She positively described her homeschool math experience as, "It was um, very good. . . . I'm really glad I was homeschooled in math." Cali felt well prepared for her Biology major (and possibly Chemistry minor) at college, asserting, "I've taken Chemistry at [university] and have been A-Ok with all the math in there. . . . I was still so prepared for the chem . . . basic math principles." She called herself, "a little math and science buff." Aligning with her perceptions, Cali scored in the top 13% on the SAT math (The College Board, 2012) with a 660, and her MSES score (7.6) ranked in the female 90 percentile (Table 10). She satisfied all requirements for mathematical proficiency: NAEP (80%), explanations and justifications for the NAEP items, MSES score, and belief that mathematics was useful, sensible, and worthwhile. Her high school transcript verified that she completed Algebra 1 through College Trigonometry, earning very good grades, 90 and above.

Although Cali enjoyed math, she became frustrated with it at times. It stressed her because, "I would work through them [problems] over and over again and not get it. . . . I would stress out if something took me too long." Yet, she believed it to be for the best, stating, "It was quite stressful for me going through the answers not understanding and having to go over them and go over them until I got it . . . but it was good because that's the way you learn." Aligning with that, her mother reported, "We spent much time on basics, much repetition of math facts—made sure she got foundation—worked hard on math daily. We repeated if she didn't understand." This affected the mother daughter relationship. The mother alluded to this and Cali revealed, "Maybe I didn't always enjoy

being at home all the time cause sometimes it's a little trying on relationships, especially with my mother." Yet, Cali reflected, "I will be eternally grateful."

Cali's instruction was structured and consistent. She and her mother discussed one-on-one instruction in the early years. Cali described the rigor of mastery learning, "So, we would work on them until you got it right until you understood. . . . Everything was checked that day and redone that day. It was very fresh." She progressed to independent study with algebra and pre-calculus and relied on her mother for geometry. She remembered, "For algebra . . . I didn't really need my mother's help. . . . With geometry she helped me a lot. . . pre-calculus I took using a Chalkdust video. . . . I did that myself, almost, she corrected my work." Cali used a variety of curriculum, but only remembered the Chalkdust. Her mother reported others, one being Saxon. Cali's homeschool math was completed at home, until her senior year, when she attended a community college for two math classes. She reflected, "It says something that I could go to a tech college and not learn anything [new] . . . I may have learned a little bit but everything else was review. . . . I'm really glad I did that and then I went off to college."

Despite her overall positive attitude, two things concerned her: her SAT math score and not taking Calculus. Her 660 math score disappointed her, stating, "I am dissatisfied with my SAT scores in math. I feel like I should have done better, because I'm not necessarily a great test taker." That seemed incredible; in this study 660 was one of the highest female SAT scores (Table 1). Also, she shared, "I wish that I had taken Calculus. That was one thing I really wish I had done . . . so that I would feel better about going into college taking a college level Calculus." However, Cali expressed confidence in her abilities by stating, "I feel like I know how to work hard and how to renew self; I guess that's probably self-study. . . . If I don't understand it, I can learn it."

Sue. Amidst a middle class neighborhood a few miles from her parents' house, 22-year old Sue ate the Chinese food I brought to her rented house. She willingly shared her homeschool experiences in a 26-minute interview. Affirming the flexibility of homeschooling, she said, "I rode horses professionally, while in high school that afforded me a flexible schedule so that I could complete my schooling and pursue my interests." Sue described her homeschool math experience as, "It was a positive experience. . . . It prepared me well for college. I felt capable and proficient in all of my college math classes. That I had the tools I needed to succeed . . . the basics of algebra and geometry." Verified by her high school transcript, her SAT math score (620) and her good grades in high school Algebra 1 through Calculus and college Calculus 1 and 2, aligned with her perceptions. Also, she fulfilled all five strands of mathematical proficiency: NAEP (90%), explanations and justifications of NAEP items, MSES score (7.6) ranking of 90 female percentile (Table 10), and belief that math was useful, sensible, and worthwhile.

Despite her good performance, Sue experienced frustration with math. She expressed her stress as, "Math is probably more structured . . . [took] an hour to an hour and a half daily. . . . I remember not enjoying my math. I remember sometimes the lessons taking a long time." Yet upon reflection, she acknowledged, "I didn't enjoy that, but I'm sure that it was good for me. So, I don't have any regrets. . . . I don't know that I would change anything." She specifically remembered struggling with her community college math classes after a break in her mathematical studies. Sue related, "I had possibly a year, year and a half gap . . . to when I took trigonometry . . . I had forgotten a lot of my math. . . . I did work hard and ended up . . . with an A." The mother reported that this situation was so stressful that she did not push Cali into calculus, as she had Sue.

At university she recalled, “I had been out of math for about a year and a half and it was just an adjustment period. . . . I did make an A in the course. I took Calculus 1 and 2.”

Sue homeschool instruction included: direct teaching, progression to more autonomy, mastery learning, use of Saxon curriculum, and community college classes. She related that her mother, “would sit down with me for my lessons and help teach them to me.” Her learning became more self-directed, remembering, “As I got older, and more independent, I taught most of my lessons.” The mother reported reworking of incorrect problems and Sue remembered, “Incorrect problems were reworked. Potentially, if I was only working even numbers and I had a trouble spot where I consistently missed a problem of the same type, I could also go back and work the odd numbers.” About curriculum, Sue simply stated, “We used Saxon math.” Her mother described others, as well as Saxon. Both Sue and her mother referred to the community college classes.

This confident young woman graduated from university a semester early and then started working full-time in marketing. Sue felt homeschooling was, “a really good fit for my family.” Notably, she runs a business on the side, buying, training, and selling horses.

Hall Family (Ela and Ola)

The Hall family resided and homeschooled in a South Carolina suburb (11,000), within 10 miles of a large metropolitan area (750,000). The family consists of two Caucasian parents and four biological children, who have been homeschooling for 15 years. The oldest females, Ela and then, Ola, completed their K-12 education at home; two younger children continue to homeschool. Ela and Ola participated in this study.

Both parents greatly contributed to Ela and Ola’s mathematical program. According to the mother, they home-schooled to “include spiritual training/guidance along w/ academics.” Ela believed, “Primarily for religious reasons” and Ola confirmed,

“They felt that God was calling them to . . . being home educators. . . . They believed that that was the best way for us to receive a good education.” They desired to instill a good work ethic. The mother wrote, “School was their job, Work, then play.” Ela Before homeschooling, their mother worked as a certified elementary education teacher. According to Ela, her mother had trouble with math until high school. She informed, “she feels like if she had had personal one-on-one attention and a better teacher earlier, she would have started enjoying math.” Providing her children with a good math background may have been another incentive to homeschool. Her MSES scores reflected a strong confidence in practical math (8.4 on Part I), but much less confidence with math courses (6.5 on Part II). Her total score (7.2) placed her between the female 80 and 90 percentiles (Table 10). The mother, Ela, and Ola highly respected the father’s mathematical ability. He worked in banking and had earned degrees in math and economics. Ela affirmed, “My dad always loved math. He did extremely well in school”, Ola asserted, “My dad, definitely, math, science genius”, and the mother credited him and the Saxon curriculum for the girls’ mathematical proficiency. He tutored his daughters in the harder math classes. Unsurprisingly, his total MSES score (8.6) ranked above the male 95 percentile.

Ela. Confident and articulate, 19-year old Ela willingly participated in a 41-minute interview, held in her parents’ upper middle class house. She believed that homeschooling cultivated learning, sharing, “My experience has been that it fostered a love of learning because I was able to focus on things I really loved. . . . I love to learn. I love to study.” Her mother verified Ela’s responses describing her as, “glad to be homeschooled . . . self-motivated and taught . . . good problem solver.” Ela felt very prepared for college, affirming, “For the most part not only have I been equal to the level academically with other people I come into contact with in college, in some ways I was

even more prepared academically just as far as learning on my own.” She has taken four semesters of calculus for her chemistry major and has maintained a 4.0 GPA at college, recently installed into an academic society. Her high school transcript verified her SAT math score (620) and her Algebra 1 – Calculus math grades (all As). She overwhelmingly satisfied the five components of mathematical proficiency with her: NAEP (100%) score, detailed explanations and justifications of NAEP items, MSES score (8.7) ranking above both the male and female 95 percentile (Table 10), and her firm belief that mathematics was useful, sensible, and worthwhile. Amazingly, Ela had not always thought this way.

Ela struggled with mathematics until middle school, at which time her attitudes drastically changed. She discovered a love for chemistry, was influenced by her father, and became serious about her faith. Ela disclosed, “Around seventh or eighth grade time when I decided I loved chemistry and then in the ninth grade . . . I started to see the usefulness of math.” Her father’s tutoring and conversations with her affected her mindset; she revealed, “My dad starting helping with math in the evenings . . . in seventh and eighth grade . . . having my dad tutor me through high school in the evenings . . . changed my attitude. . . . Obviously, this was very important.” They would discuss mathematical applications; she reminisced, “math is going to explain not only physics and chemistry but . . . explain every biological process quantitatively.” Ela considered her faith integral to her studies, declaring,

School is part of my faith. . . . Eighth grade . . . I became very serious about my faith and took it on as my own. . . . It was kind of a turning point and I decided I really needed to take my studies seriously.

Ela’s math instruction consisted of: direct teaching, self-study, consistent curriculum, mastery learning, tutoring, and dedication. Her mother read the lessons to her

during the early years, using ABeka curriculum. Her parents purposively transitioned her to Saxon curriculum, where she became more independent. Ela reflected, “In the late elementary and middle school years . . . I’d go from being taught by my mom to being self-taught. . . . The biggest part of that was my parents’ desire for me to become self-motivated and self-taught.” The mother confirmed, “structured curriculum; self-study; tutoring.” They used Saxon from the fourth to twelfth grades. Ela related, “In the fourth grade, we switched to Saxon because my mom feels like the repetition and the constant review of older problems is really helpful. . . . That has been really helpful for me.” Her parents stressed mastery; Ela confirmed, “When I got things wrong . . . we’d go back and rework the ones we got wrong until they were right. That was useful . . . In Algebra 2, my dad would start checking the lessons.” She spent much time, stating, “Ninth, tenth grade . . . two to three hours . . . pre-calculus and especially calculus . . . three to four hours.”

Ela completed all her homeschool math at home, from elementary, directly taught with her mother, to middle and high school, guided by her father. She tutored her younger siblings as well. Their location and mindset contributed to this situation. The mother shared that the nearest community college was too far away to consider that option. Ela liked the independence and ability to focus in on her studies. Yet, despite the advantages of independence and her father’s expert help, she noted one disadvantage as, “Disadvantages, I guess that would be the lack of group work. . . . I think it would have been a good thing to be able to build those group study skills.”

Ola. Following her sister, 18-year old Ola interviewed for 32 minutes. This pleasant young woman eagerly answered all questions. She freely shared her faith, disclosing, “Having that grounding in the Bible with my parents integrating that as well, I think that that was really good for me; it helped me to grow in grace as well as in

knowledge. . . . I am a Christian.” Ola treasured her homeschool experience, voicing, “I love it! I would not have survived without homeschooling. . . . Knowing that my teacher loves me and cares about me and wants me to succeed has been really great for me.” She appreciated the flexibility of homeschooling, “with scheduling, really great . . . to go on a field trip or visit family, but take school with you. . . .In high school . . . I would do math at night . . . when my brain sort of starts functioning.” She especially liked math, sharing, “I love math. . . . I loved doing it because it came easily and so it was kind of like solving puzzles.” Her mother verified Ola’s responses describing her as, “thrilled to be homeschooled . . . self-motivated and taught, very successful.”

Ola felt well prepared for her upcoming college mathematics, informing, “I’m going to be very well prepared, especially as a music major . . . math is the foundation of music. . . .That’s just going to really help me in theory classes. . . . My major requires only Math 101.” Confirming Ola’s confidence, her high school transcript verified a good SAT math score (660), in the top 13% of test takers (The College Board, 2012) and excellent grades in Algebra 1 through Calculus, earning all As. Ola overwhelmingly fulfilled the components of mathematical proficiency with her: NAEP (100%) score, explanations and justifications for the NAEP items, MSES score (8.1) in the female 95 percentile (Table 10), and her strong conviction that mathematics was useful, sensible, and worthwhile.

Ola’s math instructional experiences included: direct teaching, progression to self-study, ABeka and Saxon curriculum, mastery learning, and tutoring from her father. She explained the mother’s role and the progression to self-study, “In the younger years, she would read the lesson to me. Um, probably did that up to . . . sixth grade . . . but after that . . . I started doing it by myself.” She enjoyed her textbooks, describing, “When we

started out we used the ABeka curriculum. . . . It was really good. . . . Fourth grade we moved into Saxon math, which I absolutely love, lots of repetition . . . through my senior year.” Both parents helped her with mastery learning, she recalled, “My mother would correct the lesson. . . . If I couldn’t solve them . . . I would either go to my mom . . . or if she didn’t know how to do it, I would go to my dad.” The mother verified the dad’s tutoring. She also went to Ela for help in calculus, when she was home from college. Occasionally, Ola helped by tutoring her younger brother and sister.

Jones Family (Tad and Rob)

The Jones family resided and home-schooled in a South Carolina city (130,000). This family consists of one Caucasian and one Indian (Asian) parent, with six biological children. Their homeschool program has continued for 20 years, graduating the three oldest; the three youngest still homeschool. Brothers Tad and Rob participated.

The mother and father took different roles in their homeschool. The mother related their primary motivation as, “to be able to teach them from the Bible and to protect them from ungodly influences.” Tad confirmed, “They wanted us to be taught through like a religious system of education” and Rob reiterated, “Probably religious reasons, ah, to keep us insulated from other kids who might lead us astray morally.” The mother directed the homeschool activity, especially the math part. She loved math and had taught middle school, but did not finish her certification program. Both Tad and Rob valued their mother’s expertise with math. Rob described, “That’s one advantage that we had having a mother who has a Masters in math. . . . She wanted us to love it and care about it too, which we did in time” and Tad echoed, “She did her Bachelor’s and Master’s in mathematics.” Tad thanked his mother afterwards; she reported, “At the end

of it all he said: Thanks for teaching me Mom; it was fun.” Her total MSES score (8.7) exceeded both the male and female 95 percentiles (Table 10), reflecting their remarks.

Their father worked to support the homeschooling. Tad explained, “My dad like was always working to support the family because homeschooling is not necessarily cheap . . . more expensive than public school . . . curriculum and books . . . transportation.” Rob described him, “He runs the ministry for . . . students . . . buys apartment buildings . . . rents them to university students. That’s a whole lot of finance and business. . . . Problem solving . . . does embody it a lot in his work.” The mother explained, “My spouse does not like math courses but is a business/banking genius.” His MSES scores mirrored those comments, with a 9.0 on the everyday math tasks and a 6.5 on the math courses. His total score (7.8) ranked between the male 80 and 90 percentiles.

Tad. In his parents’ upstairs apartment on a busy city street, 21-year old Tad freely participated in the 48-minute interview. This prodigious young man provided rich dialogue about his homeschool experience. Tad described homeschooling in economic terms, asserting, “To a certain degree, [its] an ideal kind of education. Like it’s a free market solution . . . personally tailored to every single one of your kids.” He appreciated its flexibility, describing, “I think it was really good. . . . I could do what I wanted. . . . I play a lot of guitar. . . . And allowed me to be like very self-driven. And I was self-motivated because of it.” He felt well prepared for college mathematics, elaborating, “I think kids who are homeschooled tend to do better. . . . I think I was very well prepared.” He described it as, “It was kind of like a push and a pull. Like, I pulled myself because I liked it and I was kind of pushed into it by my parents.” Confirming his perceptions, his high school transcript verified his stellar SAT math score (780), scoring in the top 2% (The College Board, 2012) and his excellent grades in Algebra 1 through Calculus 2,

earning all As. He took his calculus classes at a university honors college as a high school student. His male 95 percentile (Table 10) ranked MSES score (8.7) further corroborated his statements. Tad prodigiously established mathematical proficiency with his: NAEP (100%) score, unique problem solving strategies and justifications for all NAEP items, MSES score, and his robust view of math's usefulness, sensibility, and worth.

Tad's accomplishments extended beyond mathematics. During homeschool, he stayed active describing, "It was broad . . . like a wide range of things. I had a music teacher; I swam for a swim team. . . . I ran for a cross country team . . . played soccer for a recreational team." He greatly enjoyed, "Writing became one of my favorite things . . . I did homeschool debate. Um, so I did that for four years, I think. And that was something I really, really, really enjoyed." Tad also pursued music, relating, "I competed in guitar and performed a lot in guitar." He continued with that, double majoring in Biological Sciences and Classical Guitar in a prestigious university in California. Now he has completed his first semester at a well-known medical school in Maryland. Yet, his only objection with homeschooling involved socialization. He clarified, "The social aspect of it was like kind of one of the more difficult things. And transitioning to university was also kind of complex, given my homeschool background. . . . It wasn't like the easiest thing at first." Yet, he reflected, "Now looking back at it, I'd say like I've learned, yeah, I know what it means to learn and be happy." His mother labeled Tad as, "He was a breeze. He was so advanced and I could relax at his testing."

Tad's thorough homeschool instruction included: direct teaching, rigorous curriculum, learning in a group, mastery learning, self-study, and university classes. Through most of his homeschool math, his mother faithfully taught Tad, his sister, and sometimes other homeschoolers with a rigorous math curriculum. He detailed,

Usually me and my older sister. . . . Just my mom sitting down with us, we'd read the chapter together. And then, we tackled some of the problems . . . once some friends came over . . . same style, just extra people. . . . The University of Chicago. . . . the mathematical curriculum that they publish. . . . It's a large, large textbook. Um, tons and tons of like detail put into everything. . . . It wasn't a homeschool only curriculum. But the way it was delivered was very homeschool. . . . One teacher to two students, so, lots of personal attention, lots of getting pushed, um and a lot of one-on-one instruction.

The mother reported the curriculum as, "University of Chicago Student Math Project (USCMP). It is real situations in the world—applied math/scientific." She showed me one of the course textbooks; it was thick, colorful, and full of pictures with real-life applications, just as Tad and she had described. According to the mother, Tad progressed to self-study in pre-calculus; then he attended the university honors calculus classes. His mother stressed mastery learning; he related, "If we did not get good scores on the A sheet . . . we would retake it if we didn't get everything right." The mother added, "If he got something wrong we went over it." The book had to be finished, taking the summers. Tad explained, "Didn't get that many breaks over the summer. We kind of had this structure idea that we need to finish one textbook per year. But it never really like exactly worked out to just fit into the school year." His rigorous training allowed him to excel.

Rob. Interviewed for 41 minutes in his university dormitory lobby, 20-year old Rob thoughtfully answered all questions. As the interview progressed, he became more comfortable and was very informative. He shared that he had also done research and was familiar with the IRB process and gathering participants; making him willing to help other researchers. Rob described his homeschooling as, "unique . . . at home all day long .

. . with mom teaching them . . . until middle school . . . we really started taking classes with other teachers . . . like a mini private school environment.” His mother added, “He was very creative. . . . Homeschooling was the best thing. . . . At school he would have been constrained to a desk for hours and would not have had the opportunity to do the things he liked.” Rob felt well prepared for his college math courses, explaining, “I knew that I was prepared for college math because I had already taken college math. I took Calc 2 my first semester here. It was easy.” Aligning with his perception, his high school transcript verified his top 3% (The College Board, 2012) SAT math score (750) and excellent math grades, Algebra 1 through Calculus. His mother called him, “a great student.” Rob achieved mathematical proficiency, proved by his: NAEP score (100%), explaining and justifying his problem solving strategies for the NAEP items, MSES total score (8.6) ranking in the male 95 percentile (Table 10), and his love of mathematics. He double majored in Computer Science and Political Science.

As with Tad, the mother provided a rigorous, thorough math program. Rob used the same UCSMP texts as his siblings. However, he had one-on-one instruction, describing, “Most of the time spent on math work, was with my mom, one-on-one . . . at our school table. . . . She would basically just read the textbook; I would listen. Um, and once that was done, there was always exercises in the book.” Although the time fluctuated, every textbook must be completed, he remembered,

There’d be weeks where it was somewhat consistent. There were also times . . . we would not do math for several days. . . . Then we would have to overload the next week. . . . But it all got done. . . . And then we would have to do it during the summer . . . always had to finish the book; we weren’t done until we were done.

Rob provided more detail about the mastery learning, clarifying, “With incorrect problems, my mom would make us like do homework and homework until we didn’t make any mistakes.” This practice included tests; he explained,

If I made a couple of mistakes on the test. . . . She would make me take the next test and if I didn’t get that perfect she’d make me take the next test and the next test. So, that was kind of the rework until mastery. So, no tolerance for mistakes.

Rob did not mention self-study, but his mother reported, “The last two books . . . he did on his own and came to me if he didn’t get something.” Rob enjoyed his calculus class the best. His mother reported, “He took calculus as dual enrollment at [university] and probably enjoyed that the most with the challenge and the use of the computer.” Not nervous socially, but academically, Rob reminisced,

I was a little bit worried if I was going to be prepared for it. But then I quickly found out that . . . it was not hard to get good grades in that class. I was actually really good at it. I really enjoyed it . . . We learned about Maple, which is a math software. . . . I didn’t really think about the difference in social context. . . . In high school, we took a lot of classes with homeschoolers and other teachers. . . . I was actually surprised that even in the honors college, there were so many students that would never show up to class and would just talk to each other during class. . . . I hadn’t experienced that in the homeschool world.

Insightfully, Rob reflected on the tension in his math experience. He considered it stressful, explaining, “For the most part, probably stressful. . . . I enjoyed the material. But during the lessons, it was usually stressful. And the interaction was limited to . . . my mom and I . . . and it was stressful”, which he attributed to his teenage rebellion phase as,

There were definitely times when having to be taught math by your mother was not always the most enjoyable thing, just cause it was based on her schedule. . . . It was just a lot of my youthful rebellion against my mother's desire for me to learn.

His mother wrote that, "It may not have been super enjoyable . . . but he was very grateful . . . at the end of the summer and right after that took the [university] math placement test; he was so ready and aced it." Despite the tension, Rob acknowledged,

Another time I definitely would have said that I would have like to have a class, um, just that it wasn't one-on-one. But in retrospect that was probably more beneficial for me. . . . I might not have liked it but I don't think I would, like now if I could change it, I don't think I would. . . . I think it was great.

At the end, Rob pondered, "The past couple of years I've wondered how all of us are good at math. . . . It's weird that all of us are pretty apt with math and problem solving." He then speculated, "I wonder if it has to do with math specifically with the way my mom taught it or was it just our general academic experience [homeschooling] that brought up every single subject." This summed my motive for this inquiry.

King Family (Wes and Mae)

The King family resided and homeschooled in a South Carolina city (130,000). This family consisted of two Caucasian parents and three biological children. All three completed their entire K–12 education at home. Their homeschool program continued for 16 years. Wes, the youngest, and Mae, the middle child, participated in this study, interviewing at a Dunkin Donuts restaurant, a few miles from their parents' residence.

Both parents participated with Wes and Mae's math instruction. Their reason for homeschooling included, "poor schools in area and God plus character as part of their

education.” Mae echoed, “My family lived in a school district that wasn’t the greatest. . . . They also wanted to be able . . . to teach us from a Christian perspective” and Wes verified, “We were kind of were in a bad school district.” Both parents thought math proficiency important. Wes stated, “Both my parent think it’s important to be good at math” and Mae added, “They are both very supportive of being proficient at it [math].” The mother acted as the primary teacher and coordinated the homeschool. She used, “basic math as a swim coach.” Wes and Mae confirmed that their mother helped with math until the harder classes. Her MSES everyday math tasks score (7.6) reflected confidence with practical math applications and her total (7.1) placed her between the 80 and 90 female percentiles (Table 10), displaying a fairly high math self-efficacy.

The father helped with the harder math classes. According to the mother, “Husband uses math as engineer and enjoys it.” Wes informed, “My dad loves math. We’ve had competitions against each other. . . . My dad does math a lot with his job. He’s an engineer” and Mae reported, “My dad’s an engineer and knows math and is able to use it. . . . The upper level math, my dad helped with.” Predictably, his total MSES score (8.7) exceeded the male 95 percentile (Table 10).

Wes. The youngest, 20-year old Wes seemed reticent at first, but became more comfortable, during the 25-minute interview. Except for a couple of clarifications on curriculum, his mother sat quietly nearby and filled out the educator surveys. Wes answered all the questions and offered extra explanations when prompted. He enjoyed math, stating, “Math, I just find fun. Like the survey [MSES], I thought it was fun. . . . Sometimes I do math problems in my head, just for fun, to pass the time.” He positively viewed his homeschool math experience as, “I would say overall is good. I think I did well. I’ve done well in my college math courses and on the SAT.” His high school

transcript verified good grades (90s) in Algebra 1 through Pre-Calculus and his top 7% (The College Board, 2012) SAT math score (700).

However, his MSES scores reflected a large spread, 7.3 on everyday math tasks and 6.1 on math courses. This showed confidence with practical math but insecurity in math-related college courses. The low part II score affected his total score (6.7), placing him in the male 60 percentile, well below the scores of both his parents and his sister. This did not match with his other performance markers. At the time of the interview, he was not planning to return to college that semester and when I sent out the within case studies for member checks, the mother informed me that Wes was working with a Christian landscaping company. This may be a factor in the low part II score. However, Wes satisfied all five strands of mathematical proficiency with his: NAEP score (100%*, Appendix Q), very clear and precise explanations and justifications for NAEP items, MSES score (60 percentile, above 50%), and value attributed to mathematics.

Wes perceived one disadvantage. He wanted to be around peers, noting, “It did get lonely without having other people around. I didn’t like that, especially around the end of my homeschool years.” Wes wanted to attend public school, but his parents kept him home. He explained, “I asked my parents if I could go to public school in tenth grade to play soccer for high school. But they wouldn’t let me cause they thought I might be badly influenced.” His mother reported that Wes required boundaries, writing, “Wes had the most challenges of our children, because he probably needed more structure and accountability.” She acknowledged his personality as, “HE IS SOCIAL, so probably missed interaction.” Yet, Wes affirmed his homeschool as, “It was overall very positive.”

Wes’ mother tailored his homeschool instruction. She taught him with manipulatives in the early years. He recounted, “One of the earlier things I remembered

was . . . multiplication problems; I wasn't getting them . . . we had . . . math wrap ups. And it was fun. . . . I'm a hands-on learner, so it helped me." The mother verified the use of, "a lot of manipulatives like math wrap ups", along with, " ABeka for younger grades . . . Saxon for some middle grades; Chalkdust videos for high school, ABeka for Consumer Math." Wes remembered self-study with Saxon and self-directed learning with Chalkdust, explaining, "It's a lot of teaching yourself. . . . I'm either doing the book on my own or I watched videos a lot. . . . Saxon, it was just the book and I kind of went through the book. He liked Chalkdust videos, describing, "The teacher I took like from a video course. He was really good. . . . I did like getting taught from a teacher. And I could rewind if I didn't get it." As he progressed to self-directed learning, he checked his own answers, remembering, "To check my answers, I usually do it on my own . . . figure out why I got one wrong. And if I couldn't figure it out I would . . . get my dad to help." Wes saw working at his own pace as a benefit, he related, "I could do it at my own pace. Um, not have to rush through things and get behind." He enjoyed taking a class with his sister, informing, "One year my sister and I took the same math class and she was helping me if I didn't get it . . . It was good because we're both really competitive." He concluded with advice for other students, asserting, "Definitely take consumer math in high school. Even though it's easy, it will help you the most with practical life situations." His homeschool provided him with a good math foundation.

Mae. Arriving later, after Wes had started, 21-year Mae listened with interest for a few minutes. Wes and their mother left right after he finished. She willingly answered all the questions, during her 34-minute interview. Homeschooling suited her schedule and she described, "Homeschooling was good for me. . . . It worked well. I feel like I learned what I needed to and I enjoyed the flexibility of being able to start whatever time I

wanted.” She felt well prepared mathematically for her nursing major, asserting, “I’m a nursing major. The math I had in high school was very adequate. . . . I was very well prepared. Like it was kind of easy at times. I made an A in it.” Her mother described her good performance, as, “Great grades. . . . She took the time to do well, was thorough with her homework . . . self-motivated.” Her high school transcript verified her good SAT math score (640) and very good grades in Algebra 1 through Geometry and Consumer Math, validating her statements. Also, her total MSES score (7.4) placed her near the female 90 percentile (Table 10). Mae fulfilled all five components of mathematical proficiency with her: NAEP score (90%), explanations and justifications for all NAEP items, MSES score, and her belief that mathematics was useful, sensible, and worthwhile.

Despite her mathematical proficiency, Mae disliked math. She revealed, “I didn’t like math; I didn’t like all the work and having to do the calculations.” She recalled stressful situations as, “Sometimes math was kind of stressful. I remember crying, sitting there with my younger brother and my dad when he was trying to explain things.” All Mae’s math instruction took place in the home.

Mae detailed her math instruction. In the elementary years, she liked the ABeka curriculum, recalling, “I started in elementary school with . . . ABeka. . . . I liked ABeka. . . . They had nice colorful pictures. And it was fairly easy. . . . My mom probably worked with me more, then.” Next, she used Saxon, recalling, “For middle school it was Saxon math. . . . The textbook was kind of boring, black and white. . . . I taught myself. I think I would read the textbook and then do the problems.” In high school, she used Chalkdust, describing, “Then, high school, we used Chalkdust and the videos were good. . . . If didn’t understand it I could just rewind it. You can’t do that in a regular class. You can’t rewind the teacher.” She liked working at home, asserting,

Math was always at home. . . . Overall, I think it was good. I think an advantage was that I was able to work at my own pace. . . . I didn't have to sit in a classroom, where other people are trying to grasp it and I have to then spend time on them. If I understood it and can just go ahead and I don't have to wait.

From middle school on, she mostly graded her own work and she saw that as an advantage, emphasizing, "I think probably having to correct my own work, probably is an advantage. Um, because it made me really have to think through, 'ok, how do I get this right.'" Her good homeschool experience prompted her closing remarks, "I am very supportive of homeschooling. I think it worked very well for me and my siblings."

Lee Family (Ali)

The Lee family resided and homeschooled in a South Carolina suburb (66,000), within 25 miles of a large metropolitan area (750,000). This family consisted of two Caucasian parents and five biological children. All five completed their K-12 education at home. Their homeschool program continued for 20 years. Ali, the second oldest participated in this survey, interviewed in her current residence, near a city (123,000).

Both parents contributed to the homeschool math program. Their primary motivation included "religious reasons." Ali established this, declaring, "It was straight up God's leading." Her mother acted as the primary educator. She established a good work ethic through consistent routine; Ali remembered, "No schoolwork in your pajamas. You had to be dressed for the day. . . . Had Bible time with Dad before he went. . . . Then, story time with Mom . . . chores and then . . . our homework." Specifically, Ali admired her math instruction, asserting, "She was sort of the resident expert. . . . Three other girls from the homeschool group, their parents had Mom teach them math, too. . . . She had this fantastic way of being able to explain things." The mother reported that she

was, “always the ‘go to’ mom in the support groups as not many were good at math. I love it up to Alg II.” Ali appreciated the practical applications her mother stressed, affirming, “Homeschooling for us wasn’t ever *you* sit down and *you* do your work. Its, *let’s* go to the store and *do* math . . . bake cookies; *let’s do* chemistry. . . . She was always much more practical.” Remarkably, this skillful educator had one year of college; yet, Ali highly respected her abilities, explaining, “She never graduated from college. She has no degrees, but she’s been brilliant . . . to meet us where we were and teach us these things and get us excited.” Her MSES results matched this description, everyday math tasks (7.1), math courses (6.3), and total (6.7), ranking between the female 70 and 80 percentiles (Table 10). Ali considered her father the “ultimate say in all things math.”

First the father worked as a carpenter, then an engineer. Ali and her mother reported that he was the final authority with harder math problems. Ali described, “Dad worked as a carpenter and that clearly has a lot of math . . . [he became] an engineer. . . . He was always very, very clear with math and science.” The mother wrote, “If I ever had a problem figuring anything out, [father] was able to do it.” He talked “engineerese” at the dinner table and Ali enjoyed their conversations, elaborating, “My dad always told me that math is the language of science and he was absolutely right and it always made sense to me.” His MSES scores matched these statements, everyday math tasks (8.8), math courses (8.3), and the total (8.6), exceeding the male 95 percentile (Table 10).

Ali. Bubbly and expressive, 23-year old Ali willingly participated in the 88-minute interview. She exuded a passion for math and science,

Pretty much all math; I was an absolute huge fan. . . . Chemistry is my other absolute favorite. . . . It was the concept of being able to take this and this and put it together and get something completely new and different and possibly

dangerous. So just that math has this magical quality to it that I absolutely loved. . . . I liked math because it made me feel smart.

Math connected with her faith; she elaborated,

Math is the same everywhere and it's really just a beautiful picture of God at work and nature. . . . There is a Bible verse in *Psalms* that talks about thinking God's thoughts after Him and that is what math and science is to me . . . this system of letters and numbers . . . it's just as much part of His creation as, you know, you or me or an artist's painting. Like I mean it's all equally beautiful.

Ali's mother described her mathematical talent as, "She understood numbers well and loved playing with them. . . . She'd get out our white board and ask for a height and a diameter so she could figure out the volume of whatever shape she chose." Both Ali and her mother acknowledged her preparation for her chemistry major. Ali stated, "I always felt really confident and able" and her mother echoed, "Extremely well [prepared]."

Matching her father's very high MSES score (8.6), she ranked in the male and female 95 percentile (Table 10). Verified by her high school transcript, Ali's top 13% (The College Board, 2012) SAT math score (660) and excellent math grades for Algebra 1 through Calculus 2 confirmed these perceptions. She overwhelmingly met the criteria for mathematical proficiency with her: NAEP score (100%), clear explanations and justifications for the NAEP items, MSES results, and positive mathematical attitude.

Ali valued the close family relationships fostered by homeschooling. She described, "We were a really close family and mostly because we were homeschooling. And we were always together. . . . I had so many siblings, I was never bored . . . because there was always someone to play with." She tutored her siblings, explaining, "When I was older, I would kind of take over going over math problems . . . if Mom was busy. . . ."

I was the mini math tutor.” Together, Ali and her older sister completed most math courses; she reported, “I took it with my sister” and the mother explained, “We would watch the [math] video together, [Ali], [sister], and I.”

Although Ali’s math instruction mostly took place at home, it included two community college classes. Her mother taught Ali and her older sister with Math-U-See. Praising that curriculum, Ali asserted, “We used Math-U-See. . . .They were always really good with making it very visual, tactile. . . .The guy who did the video, he was brilliant cause he was able to make interesting.” They watched the video, the mother went over the concepts on a white board, the girls completed their exercises, and they corrected the work. Incorrect problems were reworked until mastery; Ali described, “After we went through all 20 problems. . . .Which one did you get wrong? . . . Sometimes it would have been botched completely and we would go back and do the problem again until we got it right.” With trigonometry, Ali ended up teaching herself. The mother reported that she was, “rusty on trig . . . and half way through the book, she [Ali] just took it over and it was self-directed with questions to [father].” Enjoying that, Ali revealed, “Self-study was quote ‘sort of my thing’. . . .By the time I got to Trig like I was old enough that I could read and understand the text. . . . Teaching myself, that was probably my favorite.” She valued her community college calculus experience, advocating,

Calculus was interesting because it was one of my first experiences in a typical classroom setting . . . [it] went really well. I made As. . . . I highly recommend um, homeschooling families who, if their children are planning on going to a university, to get them into local tech school classes before they graduate; because college is a culture shock.

Ali attributed her math and science propensity to homeschooling. She purported,

I honestly don't think I would have liked math and science if I wasn't homeschooled. . . . I wasn't lectured constantly and then given homework and then be held back . . . cause you get so bored in those situations and then you end up hating those classes.

She appreciated its flexibility, maintaining, "It let us all work at the speed that we could remain interested in and we could be challenged in, without being overwhelmed . . . the freedom to . . . be in trigonometry in the tenth grade." Subsequently, Ali experienced much success at university and in the workplace, as a quality control chemist, with her double major in chemistry and English. She credited her parents, asserting, "It's absolutely because way back when, Mom and Dad decided to homeschool us."

Rice Family (Hali)

The Rice family resided and homeschooled in a South Carolina urban area (130,000). This family consisted of two Caucasian parents and six biological children. Four completed their entire K-12 education at home, while the oldest went to public school for a few years. Their homeschool program has continued for 25 years. The youngest still is homeschooled. The fifth child, Hali participated in this study.

Her parents home-schooled to "provide a quality Christian education that glorified God." Hali confirmed, "My parents are very passionate about education. . . . And so for them it worked in incorporating their secular desire for just good education with their desire to raise their children in their faith . . . Christian faith." Both Hali's parents possessed science backgrounds, understanding the importance of mathematics. Hali explained, "My father has an excellent background in math. He studied chemistry in undergrad and then computer engineering in graduate. My mother was a biology/pre-med in her undergraduate degree." Her mother acted as the primary educator and her father

engaged Hali in financial conversations. Hali reminisced, “I remember, from an early age my father giving me financial lessons that were mathematics-based. . . . And we had a lot of conversations like that. Even today, we still do.” Predictably, both the mother’s total MSES score (7.9) and the father’s (8.5), ranked in the 95 percentile (Table 10). This closely aligned with their college backgrounds and enthusiasm for math education.

Hali. Confident and relaxed, 19-year old Hali interviewed for 50 minutes in her honors college dorm room. Hali labeled her math education as, “Very adequate. I think it was wonderful.” Her homeschooling experience prepared Hali for college mathematics. The mother wrote, “[Hali] was well prepared for mathematics and was able to take a college level course while a senior in high school” and Hali confirmed, “For my major [Nursing] there’s only one math course required . . . which I took in high school. . . . But for chemistry . . . or other courses that require math . . . I feel very adequately prepared.” Verifying their perceptions, her high school transcript verified her SAT math score (620) and excellent grades in Algebra 1 through statistics. Hali’s MSES score (7.9) matched her mother’s, also ranking in the female 95 percentile (Table 10). She met the requirements for mathematical proficiency with her: NAEP score (90%), explanations and justifications for the NAEP items, MSES score, and opinion of, “I consider mathematics to be of an extremely useful nature.”

Hali’s instruction progressed from direct teaching to independence, using Saxon curriculum and eventually taking classes outside the home. She described the early years, “When I was a little girl and going through my math course, my mother would go through it with me every day.” Then, Hali recalled, “My mother was big on independence and so, she would usually step back and let me finish the lesson. . . . By the time I was getting in pre-algebra and Algebra 1, um, it was very independent.” Hali’s mother firmly

believed, “Mastery is essential. If the student did not ‘get’ material we honed in on this area” and Hali reiterated, “The rule from all my math history, was that if you missed more than three problems in the lesson, then you had to correct all of the problems.” The mother preferred Saxon curriculum and specified, “Their integrated approach to the material affords the student the repetition necessary for mastery.” Neither was impressed with the academic quality of the homeschool pre-calculus at the resource center. Hali expressed, “It was taught by a teacher not suited to teach the course” and her mother wrote, “It was not well instructed.” Hali described her Statistics as, “The community college structure of the class was memorize the formula, come into class, regurgitate it for the test.” Although she liked the socialization of these classes, especially the pre-calculus, the math part was inadequate. She pondered, “I did so much better in my enjoyment of math when I was taking it with peers. . . .The one I enjoyed more, but the other [self-study] I actually think I learned more from.”

Before college, Hali detested mathematics, but that attitude changed. She revealed, “My consistently least favorite subject was always math. . . . It was not that I had any major issue that was identifiable or any handicap or thinking block. But I was completely unmotivated” and her mother described, “[Hali] demonstrated the least zeal for pure mathematics.” Hali spent hours daily. She detailed, “I spent a lot of time on math. . . . I would get up at five . . . start on my math and . . . around eight o’clock . . . finished my math lesson for the day and it was grueling.” This stressed the mother-daughter relationship. Ali admitted, “Math was especially an area where we didn’t get along.” Her mother also addressed this, reporting, “This [math] was NOT this student’s favorite subject! Stress was common.” Hali’s attitude transformed; she reflected,

I have a much greater appreciation for math as a subject, than I used to. And I in a lot of ways, long for those years back that I can do them over again. . . . [I] am appreciating more what my parents, mother especially, were doing for me.

Towards the end of the interview, Hali made a thought-provoking statement. She asserted, “I wish that girls would go farther with math than I think that they do. . . . I just think we have some sort of weird gender misconception that [only] boys are good at math and I don’t see any real reason for that.” I’ve been pondering that idea ever since.

Sims Family (Ben and Que)

The Sims family resided in a South Carolina city (130,000). The family consisted of two Caucasian parents and four biological children. Two completed their K-12 education at home, while two still are homeschooled. Their homeschool program has continued for 15 years. Ben and Que interviewed in a classroom on their college campus.

Both parents wanted to be very involved in their children’s education. They home-schooled because it, “Best meets the intellectual and spiritual needs of each child.” Ben viewed their motivation as, “They thought I was going to be bullied. . . . I think Mom and Dad they really wanted to have a . . . big part of our learning career” and Que believed, “My mom wasn’t ready for me to leave home, yet. . . . To foster a close-knit family atmosphere . . . my parents really wanted to be able to shape and form our foundational beliefs and worldviews.” Ben labeled his father the “pusher” and his mother as the “enforcer.” The mother acted as the primary teacher, especially with math. She shared, “I was more hands on with this subject. . . . I like math and see it as ‘puzzle working.’” Ben confirmed, “Mom she’s done a lot of math. . . . Mom was great at math” and Que added, “She is a physical therapist . . . her strengths are in the sciences and in

math.” Predictably, their mother’s MSES score (7.9) ranked in the female 95 percentile (Table 10).

Ben and Que admired their father’s practical ability with math in building and fixing things. Ben asserted, “He’s very, very intelligent, like being able to go out and work on a car. . . . He can fix anything. . . . How to fix things, do things with my hands, I learned that from Dad” and Que informed, “My dad, he, I mean he uses math when he builds things. He is very handy and he builds shelves. . . . He uses basic math with proficiency.” The mother reported that the father took math in college. His MSES total score (7.6) ranked above the male 80 percentile (Table 10). Based on the complements by Ben and Que, his everyday math tasks score (7.4) seemed lower than expected.

Ben. In the quiet college classroom, 20-year old Ben freely participated in the 78-minute interview. This earnest young man seemed a study in contrasts. He disliked math, although he claimed that math came easily to him. He stated, “Surprisingly, I never really liked math. . . . I was good, better at the math stuff and I could learn it. . . . I can memorize math stuff and then apply it.” Appreciating homeschooling, he affirmed, “My experience with homeschooling, I just have to say that it was, I mean, it was fantastic”; but he didn’t want to be stereotyped, asserting, “I’m not a homeschooler, I was homeschooled. . . . I’m a normal person.” Community college calculus came easily and he didn’t apply himself, yet he struggled with college calculus, explaining, “I took calculus at a tech school. . . . I really didn’t apply myself in that class . . . and made As because I could memorize. . . . When the [college] class started . . . I had some really bad tests.” He recovered by the end of the semester, describing, “I did fantastic on my final exam.” His SAT score (600) disappointed him; he admitted, “I didn’t do nearly as well as I wanted. . . . The SAT is not my test.” The MSES scores indicated he felt more

comfortable with everyday math tasks (8.0) than the math courses (6.5). His total score (7.3) ranked between the male 70 and 80 percentiles (Table 10). However, Ben qualified as mathematically proficient with his: NAEP score (90%), clear explanations and justifications for NAEP items, MSES score (above 50%), and a strong belief that mathematics was useful, sensible, and worthwhile.

As the interview progressed, Ben seemed to put his ideas about homeschooling into perspective. Labeling himself as a trouble child, he admitted, “I’m amazed that Mom has put up with me that long. . . . I was kind of a rough child. . . . I hated doing work.” His mother wrote, “He was lazy about doing daily work.” In middle school, he went to his father’s office to finish his school. Ben reflected,

It was the best thing for me and also at least shaping who I am. . . . In today’s society where like the schooling system where everything is taken and put down to the lowest common denominator. . . . So that no one feels bad about themselves. I think that homeschooling was definitely the way to go for me, cause I would have been that kid that would be like, pretend that I’m stupid . . . just glided through school. . . .When I was younger, I wouldn’t have cared; sure, I’ll drink.

Grateful for his parents’ persistence and leadership, Ben admitted,

My mom and dad were really strict with me for a while because I kind of was a trouble child. . . . I am really realizing now is that they were giving me a good strong foundation. . . . I didn’t want to succeed, that was something I had to learn.

Ben felt his Saxon curriculum and self-study were most beneficial. He used different curricula, but he believed Saxon worked best for him. He explained,

The courses that I took in the beginning, which was Saxon, they gave me the foundation that I needed . . . I would look at the problems. And I would remember how it worked. . . . Saxon was hard . . . but you learn a lot.” Commending self-study, he asserted, “ [Self-study] worked well for me. . . . Being able to do things on your own, really prepares you well for real life. . . . It teaches you to be able to assess things and learn things on your own.”

Que. Immediately following her brother’s dialogue, 21-yr old Que interviewed for 52 minutes. Poised and mature, she freely provided thoughtful answers. Que greatly appreciated her homeschooling, comparing it to a collegiate environment. She informed, I really, really loved the experience. It gave me opportunities to grow. . . .The homeschool experience I had was much more like a college setting and a college course load . . . I was able to go to a class once or twice or sometimes three times a week . . . but spend the rest of my days really in independent study and research in reading. . . . It really allowed me to flourish.

Que felt well prepared because she learned how to learn, explaining, “Dorothy Sayers talks about students not just learning facts, but learning how to learn. And I did really learn how to learn as a homeschooler. And because of that I was able to do well on the math course.” That math course referred to her college statistics. She described, “My first test I actually got an 82 . . . I was very upset. . . . After that test, I studied a good deal for that course. . . . I got an 99 on the final.” Her mother verified that class, “only statistics and made an A.” Her high school transcript verified her SAT math score (600) and her very good grades in Algebra 1 through pre-calculus. Her MSES total score (7.3) ranked between the female 80 and 90 percentiles, demonstrating a fairly confident attitude towards everyday math tasks and math courses. Que satisfied all five strand of

mathematical proficiency with her: NAEP score (80%*, Appendix V), clear explanations and justifications for NAEP items, MSES score, and adequate view of mathematics.

The NAEP and MSES scores hinted at her struggles with math. Que revealed her frustration with mathematical study. She disclosed, “It’s ironic because I really loved every academic subject, apart from math. . . . I just have never really been able to wrap my mind around it easily at all. . . . I worked hard to get As.” Her mother verified this struggle, reporting, “Reluctant, had trouble grasping concepts. . . . [Que] had to have more individual attention working problems.” Needed to talk through problems.”

Describing the ensuing mother/daughter tension, Que acknowledged, “Most of the hostile harsh interaction . . . almost all of that was over math. . . . I would get frustrated with her, often, if I couldn’t understand a concept. . . . As I got older, I realized that’s wrong.” She also recognized a sense of accomplishment, informing, “If there was a concept that I was struggling with, when you know the light dawned and I understood it and I could do the problems, that was satisfying . . . a sense of fulfillment and satisfaction.” Que readily admitted that mathematics was valuable, asserting, “I know it reflects the order of God’s creation and it is valuable as a discipline. . . . It is valuable, even if it’s not your skill set.”

Que maintained that her homeschool math program was the best option for her.

All Que’s math was completed at home. It fit her learning style and she declared,

Because I was homeschooled, I had more time, probably spent more time on the math problems than I would otherwise in a classroom setting. . . . I learn better by reading something and really thinking about it, on my own. . . . So I think that had I been in school, and only had limited amount of time in the evenings to do homework. . . . I probably would have gotten lost easily. . . . My experience was the best way for me to learn.

Her mother confirmed, “Home study was best for [Que].” Que also pointed to the lack of quality math classes, stating, “There weren’t a lot of great math opportunities.” Similar to other homeschoolers, Que related her mother’s direct teaching, “In elementary school my mom would go through the lessons with us on a daily basis and then we would work the problems. She’d kind of leave us to do the problems and then we would talk about it” and her progression to self-study, “By . . . middle school/high school I would grade the lesson. . . . I spent a lot of time in independent study. . . . If I . . . didn’t understand the concepts, my mom would explain them to me.” Both reported primarily using Saxon, except for one year. Que explained, “We used Saxon from the time I was very young. . . . The only different curriculum we used . . . was Chalkdust for geometry” and the mother supplemented, “Saxon mostly: Algebra I (Saxon), Algebra II (Saxon), Geometry (Chalkdust), Advanced Mathematics (Saxon).”

Que valued homeschooling. She desired to continue it with her family disclosing, “If I do marry and have kids, and if I homeschool that’s what I’d like to do . . . have my mom have a [math] class with my kids.”

Tate Family (Kim)

The Tate family resided and home-schooled in a South Carolina suburb, within 2 miles of a large metropolitan area (750,000). This family consisted of two Caucasian parents and two biological children. The oldest, Kim, completed almost all of her education at home, 1–12. They continued to homeschool after 12 years, the younger brother still at home. Kim participated in this study.

Her parents incorporated many resources to offer Kim a quality Christian education. According to her mother, they homeschooled because it provided a better atmosphere, academic freedom, and ability to use a Christian curriculum.” Kim

described, Mom found out about homeschooling and the benefits of it . . . a better option than the public school that we were zoned for . . . a better option than the private schools cause they are really expensive.” Kim’s mother viewed math as a, “gateway to better jobs.” The mother experienced a poor math instruction, not mastering basic math principles until later; she wanted a better math foundation for her children.

Both parents worked at math-related vocations. The mother earned her degree in pharmacy and continued to work part-time as a pharmacist, while directing her children’s homeschool. Kim reported that her mother taught math for a Classical Conversations® community, explaining, “She is more science-focused and there’s a lot of math in that, obviously. And now, as a homeschool mom she’s done a lot in learning math and teaches a math class now for students in high school.” Corroborating that, the mother’s total MSES score (8.1) exceeded the female 95 percentile (Table 10). She reported that the father worked as a financial planner, with a business major and a master’s degree. Kim echoed, “My dad is actually a financial planner. So, he uses math a lot.” His overall MSES score (6.8) ranked slightly above the male 60 percentile. Based on his vocation, those results seemed lower than expected. His everyday math tasks (7.4) demonstrated better than average practical math confidence, but his math courses (6.1) showed insecurity with college level math coursework.

Kim. Kim’s 43-minute interview took place at her parents’ kitchen table. At first, the 18-year old seemed reserved. Sitting at the table, her mother worked on her laptop, while Kim answered questions. As the dialogue continued, Kim relaxed and eagerly participated. She felt very positive towards homeschooling, relating, “It’s been the better option for me. . . . This was the way that I learned to learn the most and there’s the most flexibility and most opportunities. . . . I’m really glad that I did it.” That good attitude

included mathematics, sharing, “I also enjoyed my math class. I always was blessed with good teachers for math.” Although math was stressful at times, she experienced a sense of accomplishment when mastering a hard concept. She disclosed,

I would say the stress came in when there was something that I really got stuck on and I didn’t understand. . . . I wouldn’t say that math was easy for me but it um was possible. . . . I worked hard enough at it like I was able to get things and move forward. So that was always really rewarding for me when I would accomplish things that were challenging.

Because she completed calculus as a high school senior, she felt very prepared for her university calculus class, describing, “This semester I took calculus again. . . . I was able to do things faster and get through it a lot more smoothly than the first time. . . . That was a good preparation.” Her mother agreed, reporting her as, “Well prepared, made an A in college level calculus.”

Kim’s performance corroborated this information. Her high school transcript and College Board SAT report verified her good grades in Algebra 1 through Calculus, as well as her top 4% (The College Board, 2012) SAT math score (730). Her MSES results (8.3) exceeded the female 95 percentile (Table 10). Although adequate, her NAEP score (80%) seemed lower than expected, based on the other data. Nonetheless, Kim demonstrated mathematical proficiency with her: NAEP score, clear explanations and justifications for NAEP items, MSES score, and high opinion of mathematics.

Kim’s math program included both teaching from her mother and tutoring from outside sources. In the elementary years, her mother used direct instruction for math. Kim remembered, “With elementary school, um, it was really just my mom teaching me, mainly for math.” She also recounted practical math conversations as, “My mom was

always looking to find teachable moments, especially with math. . . . If we were going to buy something . . . like, it's not really going to be that, it's going to be more with tax."

Managing the homeschool math program from middle school to high school, she enrolled Kim in a good math program, at a nearby Classical Conversations® community. She still kept track of the tests and grades. Kim explained, "The people that I was being taught by weren't technically my teacher, my mom was my teacher; I was accountable to her . . . taking the tests, I would give them to her and she would grade them." She spoke highly of these classes.

Kim enrolled in a resource center type environment with other homeschoolers, throughout middle and high school. She attended a Classical Conversations® community for many of her classes. An affiliate of the national Classical Conversations® organization, this group was independently organized. Kim explained,

In middle school and throughout high school, it was sort of set up like the co-ops. . . .They were mainly in like classes around like 10 students or less. . . . I would go like once a week. And um, the teacher would give us a seminar on like four lessons or something like that and then by the next week we were expected to do those lessons. . . .That always worked really well, especially for Saxon.

The mother clarified that Classical Conversations® was not the same as a co-op because the parents paid qualified math tutors and did not share teaching responsibilities. Kim prospered in this situation, asserting, "It helped me become more independent . . . helped me learn how to learn. . . . It was also a good way to get to know kids who were sort of similar to me in that they were homeschoolers." Kim believed the repetitive Saxon curriculum, which she used from elementary to high school, greatly helped her retention,

claiming, “It was definitely helpful to get through the problems that were harder and to see them over and over again so that I could apply um, what I was learning.”

Homeschooling allowed Kim extra time to enjoy her education. She appreciated the flexibility it afforded, the opportunities to do many things, and the organizational skills she acquired. She shared,

I had more flexibility with my time than my friends who were not homeschooled. And so that was nice. I had the opportunity to make the most of that or abuse it and I’m not going to lie; I did both sometimes. Ultimately I think that was very beneficial and it helped me with time management in college. . . . I was able to do other extracurricular activities that I may not have been able to do if I had been super busy with being in school eight hours a day and then coming home and killing myself with tons of homework. . . . It just opened up the door for me to um, enjoy my high school experience and my middle school experience.

Webb Family (Lea and Don)

The Webb family resided and homeschooled in an upper middle class suburb (15,000) residents, within 16 miles of a major city (210,000). This family consisted of two Caucasian parents and four biological children. All four completed their entire K–12 education at home. Their homeschool program continued for 20 years. The two youngest, Lea and Don participated in this study, being interviewed after dinner in their kitchen. Recently, their grandmother came to live at their house. Before the interviews, this proud grandmother showed me Lea’s certificate of a prestigious award earned when she graduated a university nursing Bachelor’s program with a 4.0 GPA. She also brought out a letter that verified Don’s 4.0 GPA of his sophomore year at university.

These parents home-schooled as a response to God's calling. The father reported this as, "To instill God-fearing/respecting values in our children and teach them well. Help them love & bring glory to the almighty God." Lea added, "They felt like God was calling them to do it . . . to nurture relationships in the family and them to be a big part of our lives, counselors, and then we could gain wisdom" and Don confirmed, "They wanted to have times with us, and as well as building in godly values into our lives."

Profoundly impacting their children, both parents participated in homeschool math instruction. The mother used direct teaching for Lea and Don, in the early years. Lea informed, "My mom, um, was proficient at math and helped us early on." The father acted as the primary teacher with the harder math classes. Lea related, "My dad is good in math and was the main one who helped us especially later on." Describing their math-related vocations, Lea explained, "My dad is a chemical engineer with a Bachelor's degree and my mom is/was a registered nurse with a Bachelor's degree." Both graduates appreciated their mother and her contribution to their lives. She modeled a work ethic for Lea and encouraged Don in his abilities. Lea recounted, "I feel like my parents, especially my mom, like she really put into us a good work ethic." and Don related, "Mom was a nurse, so she's smart. . . . She'd say thing like oh you do great at math. She'd recognize it and encourage me in it." The father used math principles in his work, stating, "I enjoyed math and so I described how I used the concepts in my work as a an engineer." Lea described his unique ritual when a math book was completed, informing, "He would be so excited that when we finished the math book, he would do a headstand in the kitchen on the hard floor." Sharing his father's motivation for being more thorough with his math work, Don recounted,

Dad was pushing me a lot when I was in high school. I think he was trying to get me to use more paper and be thorough. What he kept on telling me was [Don], as you're doing your math right now I wouldn't want to drive on a bridge that you built. I want to help you so that you do things right, so that I would drive on a bridge that you built.

The parents' MSES results correlated with the interview and survey data. The mother's MSES results demonstrated high confidence in practical math applications, with her everyday math tasks score (8.0) and insecurity with her math courses score (5.3). Her total score (6.7) ranked between the female 70 and 80 percentiles. Likewise, the interview and survey statements about the father and his MSES results (8.1), ranking at the male 90 percentile, confirmed the father's high confidence with mathematical applications.

Lea. After a full day of work, 23-year old L seemed tired. While her parents, grandmother, and brother quietly moved throughout the house, L freely answered all questions during the 58-minute interview. This busy and altruistic young woman worked as a nurse and volunteered at a clinic. She seemed well adjusted and content.

Lea appreciated her math homeschool program and felt well prepared for her university experience. Valuing its flexibility, she related, "It was good, in that I could fly through the stuff that was easy for me . . . I could take a little more time for the things that were hard. . . . I could work at my own pace." Yet, the downside involved working in the summer, because, "We always had to do math in the summer because it was more self-directed . . . the thing you can do tomorrow . . . the flexible subject. . . . In some ways it was the subject that never ended." She described her math preparation as, I took a statistics class. . . . I felt very prepared . . . nursing math . . . it was easy." At first, she

enjoyed math, stating, “I actually like math. Well, I liked algebra because . . . some tests and problems were easy for me.” Later advanced math posed a problem in that,

I took advanced math with Saxon . . . I didn’t really like it that much. Um, because I had a hard time understanding all of it . . . I realized . . . that I had just been memorizing how to do a problem and not the logic behind it.

Her performance showed a mixed picture. Despite the advanced math hurdle, she passed a CLEP test for college math. A College Board SAT report verified her SAT math score (610) and her high school transcript confirmed very good grades (all As) in Algebra 1 through advanced math. Her MSES everyday math tasks score (6.3) indicated a limited confidence with practical math, while the math courses score (7.4) showed more confidence with math-related college courses. Her overall results (6.8) ranked at the female 80 percentile (Table 10). She seemed not to relate practical math with her outstanding performance in nursing school and her work at the hospital. Her NAEP score (100%) indicated a thorough and careful attention to detail and basic math principles. The MSES results seemed low compared with her good math performance. Lea satisfied all mathematical proficiency requirements with her: NAEP score, clear explanations and justifications for NAEP items, MSES score, and belief that mathematics was useful, sensible, and worthwhile.

Lea positively described her math homeschool program. After direct teaching with her mother, she progressed to “a lot of self-directed work.” She elaborated,

I did a lot of figuring out of how to find resources, which I really thing helped prepare me to go to university, because I wasn’t dependent on the professor to tell me exactly what I needed to do . . . not having it all spelled out for me.

Although Lea completed all her K-12 math at home, she took other subject classes outside the home. She described her socialization as, “I um, was obviously at home and was except for doing classes with other homeschoolers, which was pretty frequent . . . I did, was able to socialize with other homeschoolers and I also did soccer.” Enjoyable to her, she related, “I liked the social aspect of that because I was with my friends.”

Don. Even after a day of university classes, 20-year old Don seemed focused during the 45-minute interview. While his family quietly moved throughout the house, Don freely answered all questions, some with a dry wit. He majored in chemical engineering and minored in math. At dinner, he eagerly discussed his summer job at an engineering firm. Don’s father described him as, “He has a natural aptitude and interest in math and science.” This sharp young man appeared confident in his engineering abilities.

Don portrayed a good homeschool program. He liked the flexibility of his homeschool, describing, “Into middle school, it kinda got more where I could do my own stuff and I’d try to do it as quickly as possible so I could get it done before lunch.” After the early years of direct teaching, he progressed to self-study with his math, relating, “Having to teach myself was actually a good thing in a way, even if I didn’t teach myself as well as maybe I should have. But you learn how to learn and that’s more worthwhile than learning knowledge.” Overall, the Saxon curriculum worked well for Don because, “You really had to read that stuff. . . . It was all in there. . . . It did a good job at mixing up the problems. . . . You had to know your stuff. . . . You couldn’t forget it.”

Eventually, Don heeded that advice in his community college calculus classes, revealing, “When I’m actually getting graded for something outside of the house, then it got to be a bigger deal to me . . . and I ended up being more thorough.” He summarized his homeschool experience as, “I’m happy with it.”

Don's performance corroborated the interview and survey data. A College Board SAT report confirmed his top 6% (The College Board, 2012) SAT math score (710) and his high school transcript verified his B grades in Algebra through Advanced Math and his As in community college Calculus. His MSES score (8.3) ranked between the male 90 and 95 percentiles (Table 10). The NAEP score (80%) seemed low compared with the other data; perhaps he rushed through the assessment. Nonetheless, Don met all the mathematical proficiency requirements with his: NAEP score, explanations and justifications for NAEP items, MSES score, and firm belief in the useful, sensible, and worthwhile nature of mathematics.

Between Case Analysis

Multiple case studies are constructed upon the careful analysis of individual cases. This multiple case study used explanation building, which is a special type of pattern making (Yin, 2009). The iterative process of explaining individual cases and then conducting a cross case analysis strengthens the study. Good explanatory case study reflects a theoretically significant proposition. Following the completion of the 16 within case study analyses, a between or cross case study was conducted.

In the same format of the within case analyses, the between case analysis used the central research question and five subresearch questions based on Bronfenbrenner's (1979,1999) ecological framework using the PPCT variables and the NRC's five components of mathematical proficiency (Kilpatrick, et al., 2001).

Writing the multiple case study report in a question/answer format allows the reader to analyze the information and create their own cross case comparisons (Yin, 2009). Contributing to the consistency of the report, the between case analysis was completed in the same format as the within case analysis. First, the participating families'

demographics are described. Then, the combined graduate mathematical proficiency results are presented. Each of the five sub-questions of this study is rewritten, followed by the collective findings. Lastly, the central question is restated and addressed.

Demographics

The 16 participants came from ten homeschooling families, located in two different states. Most (14) resided in South Carolina. Two lived in Virginia. Two graduates were an Indian (Asian) and Caucasian, mix; all others were Caucasian. They all lived in two parent families with biological children, with the number of children per family ranging from two to six; in measures of central tendency, the mean, median, and mode of the number of children per family were four. Most (13) graduates home-schooled for 13 years (K–12), Three were educated at home for at least 10 years.

Mathematical Proficiency of Graduates

All 16 graduates were evaluated with the conceptual framework of NRC's (2001) model of mathematical proficiency (Kilpatrick, et al., 2001). The five components included: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. As in the within case analysis, the modified NAEP assessment (Appendix A) was used to evaluate the first four items of the NRC model.

The interview and the MSES confirmed production disposition.

Graduates demonstrated strategic competence, adaptive reasoning, and productive disposition. Their explanations of work in solving the NAEP items displayed strategic competence. Justifications for their solution methods confirmed their adaptive reasoning. The interviews and the MSES scores verified their productive disposition. Based on the interviews, each of the graduates believed mathematics to be useful, sensible, and

worthwhile. All above average, their MSES scores ranged from 60 percentile (one graduate) to the 95 percentile (seven graduates) (Table 11).

Fifteen of the graduates scored at least 80% on the NAEP. They showed conceptual understanding and procedural fluency by selecting the correct answers. However, one scored 70% and was labeled as an outlier. Upon inspection of the explanations and justifications, I deduced that there might have been a procedural error on one item and a conceptual error on another item. Additionally, this participant was inducted into a national honor society admitting only students with a perfect 4.0 cumulative average. This reasoning allowed me to classify this graduate as mathematically proficient, thereby all graduates were mathematically proficient.

SAT and NAEP Gender Comparisons

In this study, the male SAT and NAEP scores exceeded the female ones (Table 8). Five of the six male graduates scored 700 or higher on the SAT math section. Nine of the ten females scored between 600 and 660, with one outlier scoring 730. The male SAT average of 715 topped the female average of 642 (Table 9). Similarly, the male NAEP mean (93.33) surpassed the female mean (89.00).

Table 8

Graduate SAT Math and NAEP Scores

Graduate	Gender	SAT Math Score	NAEP Score
Ned	M	740	90
Tad	M	780	100
Rob	M	750	100
Wes	M	700	100
Ben	M	610	90
Don	M	710	80
Cali	F	660	80
Sue	F	620	80
Ela	F	620	100
Ola	F	660	100
Mae	F	640	90
Hali	F	620	90
Ali	F	660	100
Que	F	600	70
Kim	F	730	80
Lea	F	610	100

Table 9

Averages of the SAT and NAEP by Gender

Gender	SAT Math	NAEP
F	642	89.00
M	715	93.33

Research Subquestion One

How do mathematically proficient homeschool graduates describe their mathematical homeschool environment?

Attitudes. All graduates reported a positive attitude towards their homeschooling math experience. Fifteen described it very positively with comments such as: “It was perfect for me” (Ned), “No regrets” (Sue), “Overall, very positive” (Wes), “I’m very supportive of homeschooling” (Mae), and “It really turned out well for us” (Ola). Several reported that they were grateful for their homeschooling math experience and Cali stated, “I’m eternally grateful for being homeschooled” (Cali). Four liked that they were good at math. Ali revealed, “I liked math because it made me feel smart.”

Homeschooling fostered individuality and success. Graduates appreciated the flexibility to complete their math work on their schedule and it allowed them to focus on their interests. All 16 felt that their homeschool math experience prepared them for college math. They felt capable, proficient, and well prepared, with a strong background or foundation in mathematics. Several believed they had “learned how to learn.” Ben related that to life skills stating, “I think that being able to do things on your own really prepares you well for real life. Cause in real life you don’t have someone standing behind you with a whip, going, do this, do this”. Several felt that their home training was

superior to outside math classes. However, Hali explained her neutral positive attitude due to the stressful nature of studying mathematics

Mathematical study caused frustration because of its hard work and commitment. Most graduates (12) revealed that they felt stress during their mathematical experience. Some experienced tension with their mothers during instruction, as well as with the subject itself. Math work involved focused effort and time, causing frustration. Several reported spending at least two hours daily. A few remembered struggling in elementary school and in learning a new concept. Some even referred to it as the subject that “never ended”, because it continued year round. Yet, some participants recalled a sense of accomplishment when grasping a difficult concept.

Ela described mastering a mathematics concept as “enjoyable and rewarding”. Rob explained that he “loved the rewarding experience of transitioning from not-understanding to understanding.” Kim informed, “that was really rewarding for me when I would accomplish things that were challenging” and Ali exclaimed, “You get that rush when you understand like that light bulb moment. It’s just a fantastic feeling.” Uniformly, all respondents believed the knowledge of mathematics to be valuable.

All 16 participants believed mathematics to be useful, sensible, and worthwhile. They reported that it was useful to their study of chemistry and everyday life skills of budgeting, shopping, computing gas mileage, and taxes. Math applied to music, engineering, and calculating drug doses. It was fundamental to wise decision-making. Ned labeled math as the “backbone” of the world. Math made sense to them; it was understandable, usually with one answer. Noting that math was worthwhile, many indicated the necessity of basic mathematical fundamentals. Ben reported that he used math principles a lot in his summer construction job and when he rebuilt a car.

Additionally, Ali, Hali, and Que acknowledged the beauty of math and how it represented God's creation. Hali asserted that girls needed to study more math declaring, "I wish that girls would go farther with math than I think that they do. . . . I just think we have some sort of weird gender misconception that boys are good at math and I don't see any real reason for that". All acknowledged the value of mathematics, but they expressed mixed feelings about their natural ability.

Slightly more than half the students thought they possessed natural math ability, while the others admitted that they struggled with math. More than half the participants thought math came easily to them and most of these really enjoyed their math studies. Others believed it took them longer to understand concepts and had to work harder. Nine participants reported that math was one of their favorite subjects. Conversely, seven graduates revealed that math, at some point, to be one of their least favorite subjects. As these seven reflected back, they all reconsidered their former attitudes about math and now appreciated their hard work and acquired mathematical proficiency. They recognized their parents' involvement.

Parent commitment. Parents profoundly impacted their students. All interviewees acknowledged their parents commitment to their math education. Uniformly, they believed that their parents strongly supported mathematics education. Sue explained, "My mom was always very adamant about everything in my homeschooling. That everything was good and necessary. Ah, and my dad supported her" and Wes informed, "Both my parents think it's important to be good at math." In retrospect, all acknowledged the value of their hard work and appreciated the work ethic modeled by their mothers and fathers. Some commented, "Must work before you play" (Cali), "Especially my mom, like she really put into us a good work ethic" (Lea), and

“All time she put into it, which resulted in time that we had to put into it, um meant we were doing it a lot” (Rob). A few believed their father pushed them to achieve, while their mothers motivated and encouraged them to succeed. Don’s father motivated his son with a bridge analogy stating, “[Don], as you’re doing your math right now I wouldn’t want to drive on a bridge that you built. I want to help you so that you do things right, so that I would drive on a bridge that you built.” That same educator would stand on his head when his children would finish a math book (Lea). The participants also revealed their parents’ reasons for homeschooling.

Motivation. Homeschool families were strongly motivated by academics, faith, family unity, flexibility, and values. According to the graduates, most parents initially homeschooled for religious reasons, followed by academic concerns. The parents wanted Christian principles integrated within the curriculum. Desiring to have a big impact on their children’s education, they wanted to provide a better education than the area public schools. Some wanted to promote family unity. As they continued, these educators experienced success. Their children were learning, and it fostered good behavior. Homeschooling provided a safe environment and individual needs were met. They had the freedom to instill spiritual values. Ultimately, homeschooling proved to be flexible, allowing for varied opportunities.

Instructional practices. Mothers greatly influenced their children. Eight reported their mother as the main math go-to person for the duration of their homeschool. The other half of the participants informed that their mother acted as the main math facilitator in the early elementary and some middle school years, while their father then assumed that responsibility in later middle school and high school. In the early years, the mother provided direct teaching to the students, in terms of reading the textbook aloud to them

and working problems with them. Past the elementary years, many of the participants were involved with self-study.

Self-study emerged as a major theme. Most of the students reported that they progressed to self-study with their mathematics study sometime in middle school to early high school. This involved them independently reading the textbook lesson and working on the exercises. Eventually, ten corrected their own problems and the mothers of the rest checked the work throughout their home-based math courses. All participants reported that at least one of their parents was available for questions.

Many curricula were discussed, the most common being Saxon. Thirteen of the graduates used Saxon curriculum. Several greatly appreciated Saxon's repetition and thoroughness, while others disliked those textbooks. A few students used Chalkdust curriculum, reporting that they liked the video instruction. In the early elementary years, four used ABeka curriculum and three others used Bob Jones; all seven of these reported that they liked the colorful pictures of those curriculums as compared to the black and white pages of Saxon. A pair of brothers used The University of Chicago Math Project, a rigorous public school program, that their mother directly taught them. Ben and Kim used Teaching Textbooks for a short time and both reported that they thought it was not the best program for them; they liked Saxon better. Finally, Ali learned from Math-U-See and loved its practical applications and the entertaining video instructor. All 16 participants reported mastery learning.

All the home educators required mastery learning. Whether or not the graduate corrected the work, all missed problems were to be reworked until completely correct. Rob and Tad's mother carried this philosophy over to the tests. In the UCSMP curriculum, alternate tests were provided. If their test answers contained mistakes, they

were redone. Rob commented, “No tolerance for mistakes.” Ben and Hali’s parents would assign half the problems (either even or odd) and if the student missed too many they would have to do all the problems the next day, as well as completely redo the incorrect work. Additionally, some graduates reported that conversations took place, reinforcing math principles.

Math conversations took various forms. Many reported such interactions that displayed mathematics in use. Hali fondly remembered family conversations, mostly led by the father, about finances. Kim described these as “teachable moments”, when her mother would use shopping, budgeting, and taxes to illustrate mathematical application to life. Ali enjoyed shopping trips with her mother where she would be challenged to keep a running total of the items and then calculate the sales tax. Her engineering father often discussed his work at the dinner table. Likewise, Don and Lea recalled dinner conversations with their engineering father. Regardless of whether or not the graduates took part in these types of interactions, they all finished challenging mathematics courses.

All 16 completed Algebra 1 and 2, and geometry. All but Mae went onto pre-calculus or trigonometry. Several finished Calculus 1, while three accomplished Calculus 2. Mae and Wes took consumer math together, while Hali, Rob, and Tad completed statistics courses. Many noted homeschooling’s flexible nature.

Homeschooling was adaptable to graduate needs. Some noted that they were able to work at their own pace. As the “flexible subject”, they postponed their math work for other classes; completing the math lessons on the weekends or even in the summer. Others liked the option to finish their work early and participate in extracurricular activities. Ola worked best at night, stating, “And when I got into the upper grades in high school, um, I would do math at night because I’m a night person. And I would do

the next day's math lesson at night and it would just go a lot better." Homeschooling facilitated an individualized, needs-based education. Most graduates took math classes from resource centers, community colleges, and other school type environments.

The participants discussed their math instruction outside the home. Kim and Ned's parents monitored this work, even though other educators taught these classes. In eighth through tenth grade, Ned enrolled in a University Model school, which met three days a week. He gave high praise to this instruction, saying he learned a lot from this experience. However, his community college experience frustrated him, because a lot of time was wasted on answering people's questions. Kim enjoyed her resource center classes in middle and high school, learning and interacting with peers. Ben and Hali did not learn math from their resource center class or their community college classes, but they enjoyed the socialization. Cali felt that her community college math classes were easy, only learning a couple of concepts and skills. Other graduates reported really enjoying the learning environment of their community college or university courses and appreciating the ability to take such classes before homeschool graduation.

Relationships. All graduates grew up in a two-parent home. The mother acted as the primary instructor in the elementary years. The primary dyad (Bronfenbrenner, 1979) would have been the mother and the child during those years. As the graduates progressed in their math studies, either the mother or father acted as the math resource; Ali reported both parents participated until trigonometry when her father took over. Some participants reported stress in these instruction times with their mother and sometimes with their fathers. Some females reported that these interactions with their fathers were positive and uplifting and other graduates were grateful for their mothers' encouragement and care. Ben appreciated his father's efforts to make math practical to him in the

workshop. Ned appreciated the “personal attention from my parents.” Several participants worked with their siblings on mathematics.

Sibling interaction took place in the form of taking math classes with brothers or sisters and in math tutoring. Four completed math classes with siblings, sharing curriculum and the mother’s direct teaching. Five tutored their younger family members. Older brothers and sister tutored three participants. The time spent working with parents and siblings promoted family unity.

Many graduates commented about the closeness of their families. Ali stated, “We were a really close family and mostly because we were homeschooling.” Participants talked about family unity as a motivation for homeschooling. Ned reported, “The interactions with my parents and my siblings were just really good.” Notably, several participants brought up the issue of socialization.

Several participants addressed socialization. Cali reported that she was not “scarred socially,” while her sister, Sue believed homeschooling taught her to relate to people of all ages, adults in particular. Que appreciated the opportunity to interact with various types of people with differing belief systems. In retrospect, Ben was glad not to have been badly influenced by public school peers, appreciating the positive influence of good homeschool friends. He needed time to mature, admitting, “I never wanted to go to public school. I was like, ‘I don’t want to have to deal with all that social, like the social craziness.’ . . . Now, I’m really glad I didn’t have to deal with this [partying] when I was younger.” Don and Lea enjoyed their many outside classes and activities, where they interacted with both homeschooling and non-homeschooling peers. Ben, Hali, Kim, and Ned enjoyed outside math classes with other homeschooling students for peer interaction, yet only Kim and Ned believed that the learning environment was adequate. Rob

discussed the social aspect of attending a university class in high school. He didn't give it much thought because he was used to taking outside classes with 25 to 30 other homeschoolers. He didn't find it much different taking classes in the university honors college, except that those students talked when the professor talked and skip class. Ali highly recommended attending community college to prepare homeschoolers for the "culture shock" of college life. Four graduates felt that homeschooling limited some socialization. Wes wanted to attend public high school to be with friends and play sports, while Tad felt that his adjustment to university life took longer due to his homeschooling. Hali and Que believed that some of the stress of the math instruction would have been mitigated with a very small co-op situation with two knowledgeable parents teaching the math class. Mostly positive, the graduates acknowledged socialization.

Physical surroundings. Homeschool mathematics instruction took place in the home, at resource centers or educational facilities, and at community colleges or universities. During the elementary years, all homeschool math instruction took place in the home. Starting in the middle school years and continuing through high school, Kim enrolled in math classes at a resource center servicing homeschoolers; the classes met once or twice, weekly. Ned took math courses at a University Model school, three times a week, for three years. Both Kim and Ned completed their math exercises at home; their classes consisted of a lecture time to introduce the material. Ben and Hali took one high school class at a resource center and then one community college class. In her junior and senior years, Sue one student took her math at a local community college. Five others enrolled in community college during their senior year. Six completed all their math instruction at their home. In this research, most math instruction took place in the home.

Research Subquestion Two

How do mathematically proficient homeschool graduates describe their mathematical self-efficacy?

The Mathematical Self Efficacy Survey [MSES] (Betz & Hackett, 1993) was used to determine the mathematical self-efficacy of the 16 participants. Betz and Hackett (1993) conducted research at two universities to determine male and female percentiles. Male and female respondents were ranked differently (Table 10).

In this research, all graduates scored above the 60 percentile ranking (Table 11). Notably, all but two female scores clustered around the 90 and 95 percentiles; one ranked mid-way in the 80 and 90 percentile range and another near the 80 percentile cutoff. Five of the six males ranked at or above the 90 percentile, with one outlier at 60. Especially for the female participants, this demonstrated a very high math self-efficacy.

The averages of the graduates are shown in Table 12. The average total male score (8.07) exceeds the average total female score (7.83) by 24-hundredths (3%). Although the average total score of the males exceeds than that of the females, the average female percentile rank (95) surpasses the male one (90).

Table 10

Percentile Equivalents for Math Self-Efficacy Scores (Betz & Hackett, 1993)

Percentile	Female Total Score	Male Total Score
95	7.9	8.5
90	7.5	8.1
80	6.9	7.5
70	6.5	7.1
60	6.1	6.7
50	5.8	6.4
40	5.5	6.1
30	5.1	5.7
20	4.7	5.3
10	4.1	4.7
05	3.7	4.3

Table 11

MSES Scores of 16 Graduates with Percentiles Gender Based (Betz & Hackett, 1993)

Family	Name	Gender	Part I	Part II	Total	Percentile
COX	Ned	M	8.9	8.8	8.8	95
FORD	Cali	F	7.8	7.4	7.6	90
	Sue	F	7.5	7.6	7.6	90
HALL	Ela	F	8.6	8.9	8.7	95
	Ola	F	8.4	7.6	8.1	95
JONES	Tad	M	8.7	8.8	8.7	95
	Rob	M	8.2	9.0	8.6	95
KING	Wes	M	7.3	6.1	6.7	60
	Mae	F	7.4	7.4	7.4	90
LEE	Ali	F	8.7	8.4	8.6	95
RICE	Hali	F	7.7	8.1	7.9	95
SIMS	Ben	M	8.0	6.5	7.3	70–80
	Que	F	7.2	7.4	7.3	80–90
TATE	Kim	F	8.6	7.9	8.3	95
WEBB	Lea	F	6.3	7.4	6.8	80
	Don	M	8.1	8.6	8.3	90–95

Table 12

Averages of the MSES (Betz & Hackett, 1993) Graduate Scores

Type	Part I	Part II	Total	Percentile
All	7.96	7.87	7.92	-----
Females	7.82	7.81	7.83	~ 95 (little lower)
Males	8.2	7.97	8.07	~ 90 (little lower)

Research Subquestion Three

How do home educators of mathematically proficient homeschoolers describe their mathematical self-efficacy?

The MSES (Betz & Hackett, 1993) was used to determine the mathematical self-efficacy of the 20-homeschool educators involved in this study. The mothers and the fathers' scores were ranked differently into percentiles, according to gender (Table 10). All the mothers and fathers ranked at or above the 60 percentile (Table 13). The females ranged from a low of 60 percentile (one mother) to a high of 95 percentile (four mothers). The males also ranged from a low of 60 percentile (one father) to a high of 95 percentile (five fathers). Seven of the ten fathers ranked at or above the 90 percentile and seven of the ten mothers ranked above the 80 percentile. Although the average total male score (8.12) exceeded the average total female score (7.36) by 76 hundredths (10%), both groups ranked closest to the 90 percentile (Table 14).

Table 13

MSES Scores of Home Educators with Percentiles Gender Based (Betz & Hackett, 1993)

Family	Parent	Part I	Part II	Total	Percentile
COX	Father	8.0	8.2	8.1	90
	Mother	7.4	4.5	6.1	60
FORD	Father	8.3	8.4	8.4	95
	Mother	7.9	6.4	7.2	80–90
HALL	Father	9.0	8.3	8.6	95
	Mother	8.4	6.5	7.2	80–90
JONES	Father	9.0	6.5	7.8	80–90
	Mother	8.4	8.9	8.7	95
KING	Father	9.0	8.4	8.7	95
	Mother	7.6	6.4	7.1	80–90
LEE	Father	8.8	8.3	8.6	95
	Mother	7.1	6.3	6.7	70–80
RICE	Father	8.4	8.6	8.5	95
	Mother	8.3	6.3	7.9	95
SIMS	Father	7.4	7.8	7.6	80
	Mother	8.1	7.8	7.9	95
TATE	Father	7.4	6.1	6.8	60
	Mother	8.2	7.9	8.1	95
WEBB	Father	8.6	7.6	8.1	90
	Mother	8.0	5.3	6.7	70–80

Table 14

Averages of the MSES (Betz & Hackett, 1993) Home Educator Scores

Type	Part I	Part II	Total	Percentile
All	8.17	7.29	7.74	-----
Females	7.94	6.76	7.36	~ 90 (little lower)
Males	8.39	7.82	8.12	~ 90 (little higher)

Research Subquestion Four

How do home educators of mathematically proficient homeschoolers describe their homeschool graduate's homeschool mathematical environment?

Attitudes. The educators' Christian faith influenced their decision to homeschool. They cited religious reasons or faith, the freedom to teach Christian values within their children's education. They also reported academic motivations.

These educators took charge of their children's academics. Another motivation for starting and continuing homeschooling, they believed this form of education superior to the public school because of the better home environment that met individual needs, allowing for curriculum choice, outside the influence of poor school districts. Mrs. Hall wrote, "With a daughter beginning her third year of college, with great success, we have gained confidence in homeschooling. We have Results."

All educators reported an overall positive homeschool experience. The homeschooled graduates liked being at home. Their students thought their experience positive or great. A few highly praised their graduates, while two mothers acknowledged homeschooling struggles that ultimately worked out for the best. Also, they provided insight into their graduates' favorite and least favorite subjects.

The homeschool educators confirmed the variety of their students' interests and disinclinations. Several graduates favored the sciences and mathematics. Some preferred reading. Hali developed a passion for languages, Sue, for horseback riding, Tad for guitar performance, and Wes for physical education. Rob and Tad really enjoyed debate competitions for many years. Some students disliked mathematics, or biology. Other graduates disliked English and writing.

All but Mrs. Sims believed their graduates obtained an adequate education, prepared for mathematics in college. The graduates' mathematical experiences were adequate or very successful/advanced; they were well prepared for math in college or fairly well prepared. Mrs. Sims thought Ben did not diligently prepare and that Que struggled with math concepts.

According to the educators, hard work, natural ability, and a good mathematical foundation were the keys to their graduates' good SAT Math Subtest scores. They reported that their graduates studied SAT problems and took practice tests. Some believed natural ability helped their students. A good mathematical foundation in the basics and math fundamentals was "critical."

Educators of 15 graduates reported that their students considered mathematics to be useful, sensible, and worthwhile. Mrs. Ford thought that math did not have much practical application to Sue. Mrs. Tate believed math to be a "gateway to better jobs," and Wes's mother believed he thought math fun. Notably, every family had at least one parent confident with math.

At least one parent in every family liked math and felt comfortable with its application. Five mothers reported enjoying math and using it in a job, while four were

comfortable with math up to a certain level. According to the surveys, several fathers enjoyed mathematics and used math extensively.

Instructional practices. Educators indicated the use of direct and structured teaching, self-study or self-directed learning, and mastery education. According to the surveys, 11 of the graduates received direct teaching from the mother, eight only in elementary school, and three up to high school. Some defined instruction as structured, especially in the younger years of the graduates. Reaching a level of maturity, eleven of the homeschooled students self-taught or self-directed their math courses. Four educators defined mastery learning as the reworking of incorrect problems until understanding. Mrs. Rice summed it up with, “Mastery is essential. If the student did not ‘get’ material we honed in on this area.” Choice of curriculum facilitated these practices.

Most homeschool educators favored Saxon math curriculum. Twelve graduates used Saxon curriculum for most or part of their homeschool careers. Five used Chalkdust video curriculum for selected courses, with good results. For the younger years, K–3rd, seven participants used either ABeka workbooks or BJU textbooks. After using some basic math workbooks, Rob and Tad transitioned into the challenging University of Chicago School Mathematics Project classes. Ben and Kim utilized Teaching Textbooks for one course and Ali used Math-U-See exclusively for math courses completed at home. Graduates took challenging high school math classes.

All students finished at least Algebra I and II, and geometry. All but one completed pre-calculus and nine took calculus. Additionally, three had a statistics course. Mae and Wes worked on consumer math. Although graduates took most of these courses at home, outside resource centers and community were utilized.

Many graduates attended math classes outside the home. Although five students stayed home for all their K–12 math classes, four experienced courses at resource centers with other homeschoolers and nine enrolled at the local community college or university. Hali’s mother expressed her pleasure with the community college experience stating, “We used our local college to teach upper level math courses with great success.” When the students took math at home, at least one of the parents helped them.

Corroborating graduate information, one or both parents acted as a math resource for their children. According to the educator surveys, the mother served as the primary math “go-to” in the elementary years, as well as later on. At some point, with the more advanced math, the father assumed the “go-to” role with eight students.

Relationships. In this research, all participating families consisted of two parents with two to six biological children, an average of four children per family. Fourteen of the sixteen participants were Caucasian, while two were Asian Indian and Caucasian mix. All but three participants were homeschooled from K–12th grades. These educators sought to positively influence their children with Christian values and a good educational environment.

As did their students, the educators described the personal interactions during math instruction. One on one instruction took place with the mother, during the earlier years. Some of the fathers tutored their children. Math-related conversations occurred with the mother and the father. Some described these interactions as enjoyable, while others alluded to its stressful nature. As the students matured, the personal interaction declined due to self-study. In some families, siblings worked together.

The parents confirmed that siblings interacted with the graduates as classmates, tutors, or the ones being tutored. According to the surveys, four participants took classes with their siblings. Three tutored their siblings, (e.g., Ola received help from Ela).

Physical surroundings. As mentioned previously, the home, resource centers, and community colleges or a university served as the physical locations for the graduates' math courses. All but five participants enrolled in mathematics courses away from home. Four attended classes in nearby resource centers and a total of nine students completed community college or university courses, close to their home.

Research Subquestion Five

Are there any apparent patterns or relationships between the descriptions of the mathematically proficient homeschool graduates and their home educators concerning the homeschool environment and their math self-efficacy?

The data from the graduate interviews, educator surveys, and the MSES scores were evaluated for reoccurring relationships. Interview and survey information closely aligned, providing several main topics. Although this group of participants ranked well above the MSES norm, no apparent patterns due to gender or interest in mathematics emerged from the comparison of parent and graduate MSES scores.

When comparing the interview and the survey data, common themes emerged. A few slight inconsistencies occurred. Overall, both data sets corroborated each other. For the purposes of comparison, Appendix Z contains a chart with the overall themes addressed by the graduates and the educators.

Differences involved the labeling of a course and some of the least favorite subjects. A few educators and their graduates used the math course names trigonometry and pre-calculus interchangeably. Trigonometry is a branch of mathematics usually

incorporated in a pre-calculus class. This switching of names is understandable. Five educators' responses about the least favorite subjects did not match up with their graduates' information; however, none of these involved mathematics. Even with these dissimilarities, no inconsistencies affected this research.

Interview and survey data supplemented each other. Graduates described parts of their instruction that they vividly remembered, while forgetting curriculum names. The educator filled in the missing curriculum names. Likewise, the educator provided very brief explanations of the graduates' view of their math experience or their preparedness for mathematics in college, while most students elaborated on these. Likewise the educators' wrote brief summaries concerning self-study or self-directed education, while most graduates described this in great detail. All the students seemed very sincere and willing to share their stories. The educator survey data balanced the interview evidence.

Homeschool environment. Several themes emerged in all participating families. Participants described their family demographics. Faith played an important in the decision to homeschool. Parents reported college education. Direct teaching took place. Graduates completed challenging math classes. All students believed their math education to be adequate, preparing them well for mathematics in college and all but one educator, agreed. All participants and most of the educators described mastery learning. All students believed mathematics to be useful, sensible, and worthwhile. The interview and survey data corroborated the findings.

Family demographics. Since the majority of graduates were obtained by snowball sampling, the family demographics were skewed towards the researcher's range of contacts. All families consisted of two parents with two to six biological children. Two families had two children, one family had three children, four families had four children,

one family had five children, and two families had six children. Nine of the ten participating families were Caucasian and one family had an Asian Indian parent and a Caucasian parent. Nine of the ten families lived in South Carolina, five in the suburbs and four in an urban area, clustering around two cities. The Virginia family resided in the suburbs, near a major city. These families had homeschooled twelve or more years, some over 20 years. All but three participants were homeschooled K–12.

Faith. As with the family demographics, the faith-base of these graduates was skewed due to the range of the researcher’s contacts. All families reported faith-based motivations for homeschooling. They referred to their Christian faith in the survey, desiring to impart Christian values within their children’s education. Also, some of the graduates referred to faith as a motivation for homeschooling and a few shared their own Christian faith, Kim sharing her interest in theology.

Parent education. All parents reported attending college. All but Mrs. Lee completed a Bachelor’s degree and several had graduate degrees. Four earned specialty degrees (an educational doctorate, a pharmacist, a physical therapist, and a radiologist). Sixteen of the twenty parents had science or math-related majors and/or vocations (Table 15). Mrs. Hall parent had elementary teaching certification and experience and Mrs. Jones had public school teaching experience, but had not completed teacher certification.

Direct teaching. Graduates and educators described direct teaching. During the elementary years, mothers taught their students directly one on one, using worksheets or basic math curricula or in small group situations. This individualized teaching continued until middle school, Tad and Rob until high school. Some fathers tutored.

Mathematics courses. Students completed challenging courses. All completed Algebra I and II, as well as geometry. All except Mae took pre-calculus and ten finished

calculus. Ali, Don, Ned, and Tad took Calculus 2. Many felt their mathematics preparation helped them on the SAT math subtest, dual or concurrent enrollment math classes, and with mathematics skills needed at college. All participants performed well.

Well prepared. All graduates and most of their educators believed them to be equipped for mathematics in college. Fourteen felt well prepared, while Ben and Wes thought they were fairly well prepared. Most reported doing well in their college math courses, or classes that required an understanding of math principles. Several reported that they did very well in university or college math classes. Ben and Sue reported that they were rusty on skills but since they had the mathematical background, they were able to catch up and perform well in their math classes.

Comparing the MSES (Betz & Hackett, 1993) Part II (Mathematical Classes) results (Tables 11 and 13), the level of mathematics classes, the perceptions of being well prepared for college mathematics, and the SAT math subtest scores, did not produce any clearly defined patterns. Four of the six males scored from 8.6 to 9. Notably, each of these four also scored very well on the SAT (710, 740, 750, and 780, respectively) and reported that they performed well in dual enrollment calculus. Ned and Tad also completed Calculus 2. The other two scored 6.1 and 6.5 (Wes and Ben, respectively). Wes's SAT (700) did not seem to match with this score; neither did he take Calculus during his homeschool years. Ben's SAT score (610) was lower, yet he took dual enrollment calculus and earned an A. The mathematical courses taken, the SAT scores, and the MSES Part II results lined up for four of the six male participants. For these four, the MSES Part II results indicated they were very confident in their ability to perform well in math-related college courses. Ben and Wes did not exhibit that confidence.

The female MSES Part II scores clustered closer together than the males, from 7.4 to 8.9. Hali, Ali and Ela scored above 8: (8.1, 8.4, and 8.9, respectively). These were very high MSES scores for females. Yet, their SAT scores did not seem to match up (620, 660, and 620, respectively), as did most of the males in this study. The female with the highest SAT (730), Kim scored a 7.9 on the MSES Part II. This was still a high score for women, but lower than the top three. Kim had just finished her first semester at university and perhaps was not as confident as Hali, Ali, and Ela who were further along in their studies. Overall, the ten females scored well on the MSES Part II, showing confidence in their ability to perform well in math-related college courses. They had indicated in the interviews that they had performed well in college, aligning somewhat with the MSES Part II results.

Mastery learning. The entire student group described mastery learning. Several educators also discussed this philosophy. Mr. Webb used a bridge analogy to emphasize accuracy. Graduates were expected to rework incorrect problems until consistently correct. Some worked extra problems and Rob and Tad even retook tests.

Productive disposition. All graduates agreed that mathematics was useful, sensible, and worthwhile. Math applied to budgeting, shopping, and taxes, as well as the sciences, nursing, and engineering. All but Mrs. Ford reported that their student believed mathematics to have application to their lives. Ali, Hali, and Mrs. Rice related math to God's creation.

Other topics emerged from the interview and surveys. Saxon was the most popular curriculum. Participants described self-study or self-directed education. Several parents and students showed interest in math and science-related fields. Similarities existed in the amount of math classes completed inside and outside the home. Several

graduates and a few educators acknowledged the stressful nature of mathematical study, while a few graduates and educators described a sense of accomplishment. Several participants and some educators mentioned flexibility. Some graduates worked with siblings. Many participants and their educators mentioned socialization.

Curriculum. Of the six curricula mentioned, Saxon dominated. Thirteen graduates used Saxon textbooks. Seven students and four educators commended the Saxon curriculum for its repetition and incremental approach. Mrs. Rice used Saxon for her six children and appreciated its adaptability for independent learning. Some students acknowledged ABeka and BJU curriculum for its attention to basic foundational math principles in the elementary years. A few educators and graduates liked Chalkdust video teaching segments. Ben and Kim used Teaching Textbooks, but were not satisfied. Rob and Tad used UCSMP, appreciating its rigor and application to real life. Ali loved Math-U-See and in part, credited it for her proficiency.

Self-directed learning. Fifteen graduates and their educators discussed self-study or self-directed learning. As they matured in their mathematical studies, the students read the textbook by themselves, completed the exercises independently, and then either corrected their work or had their parent (usually the mother) check the answers. For some, this started in middle school, while others started in high school. While the graduates self-directed their math studies, one of the parents still tutored when necessary. Graduates and educators alike praised this type of learning because it taught them how to learn and prepared them for college.

Interest in math and science related fields. At least one parent of each participating family possessed a math or science-related college degree used math-related skills at work (Table 15). Eight parents majored in science or engineering, and two

majored in Math for their undergraduate degrees. Two earned their Bachelors in Nursing and worked as RN's. Three went onto earn Masters degrees in Computer Engineering, Economics, and Mathematics. Four completed specialty degrees (Educational Doctorate, Pharmacist, Physical Therapist, and Radiologist). Several used mathematics principles in their jobs (bank teller, coordinating rental properties, engineering, financial controller, financial planner, pharmacist, and sales). The graduates also tended to major in these disciplines (Table 15). Nine majored in the sciences, three in nursing, one in marketing, with one minoring in mathematics. Two graduates finished college and worked, one as a nurse and the other as a chemist.

Table 15

The College Majors and Vocations of Graduates and Their Parents

Graduate	College Major	Father Degrees/Vocations	Mother Degrees/Vocations
Ned	Chemistry	Radiologist	Nursing/RN
Cali	Biology	Business/Controller	Industrial Engineering
Sue	Marketing	Business/Controller	Industrial Engineering
Ela	Chemistry	Math/Economics/Mortgages	Elementary Education
Ola	Music	Math/Economics/Mortgages	Elementary Education
Tad	Pre-Med	Ed.D/Rental properties	Math undergraduate/graduate
Rob	Computer Science	Ed.D/Rental properties	Math undergraduate/graduate
Wes	-----	Engineering	Journalism
Mae	Nursing	Engineering	Journalism
Ali	Chemistry	Chemical Engineering	Bank Teller
Hali	Nursing	Chemistry/Computer Eng.	Pre-Med
Ben	Microbiology	Media Arts	Physical Therapy
Que	English Spanish	Media Arts	Physical Therapy
Kim	Civil Engineering	Business/Financial Planner	Pharmacist
Lea	Nursing	Chemical Engineering	Nursing/RN
Don	Chemical Engineering/ Math minor	Chemical Engineering	Nursing/RN

Math classes completed at home. Fourteen graduates completed all or a majority of their math classes at home. Starting in middle school, only Kim and Ned participants took math classes at a separate location with other homeschoolers. Six finished all their homeschool math classes at home. The others completed their math classes at home until sometime in their high school career.

Math classes completed outside of home. Ten graduates completed some math coursework outside of their home. Four took classes in an educational setting that exclusively involved other homeschoolers. Kim and Ned praised these classes. Seven took community classes, while Rob and Tad enrolled in a local university, during their later high school years. Graduates and educators appreciated these college courses. Four students discussed the wait time in those classes, due to individuals who slowed up the class pace with their questions.

Stress in mathematical study. Both educators and graduates reported stressful situations with math. They explained tension between mother and the student over math instruction and Mae reported stress during tutoring from her father. Several wanted to take mathematics classes with other homeschoolers to help alleviate the stress. Ela struggled with math in her elementary years, Don and Lea struggled with Advanced Math, and Hali, Ned, and Que were stressed by math throughout their homeschooling.

Sense of accomplishment. Several graduates reported a sense of accomplishment with their homeschool mathematical study. During college they realized their homeschool math courses had well prepared them for challenging college situations. Ali said, “It made me feel smart.” Ali, Ola, and Wes maintained that math was, “fun.”

Flexibility. Graduates and educators appreciated the flexibility that homeschooling provided. It served as a reason to start and continue homeschooling.

Flexible scheduling accommodated special interests, like academic subjects, extracurricular activities, music lessons, community service, and sports. Graduates and educators acknowledged the ability to control the pace of the math lessons to meet individual needs. Parents had the flexibility to spend more time with their students in what Kim called “teachable moments.” Mothers imparted practical math applications in shopping, baking, and taxes, while fathers participated in conversations about their work, building things, and finances.

Working with siblings. Most of the graduates worked with their siblings. Ten worked with them in mathematical study. Two took math classes with a sister or brother. Six tutored younger siblings. Siblings tutored three graduates.

Socialization. Participants addressed some aspect of socialization in their homeschooling math classes. Most math classes took place in the home apart from the graduates’ peers. As mentioned previously, all graduates completed their elementary school math at home one on one with their mother or one on two with a sibling. In middle school Kim and Ned took math classes with other homeschoolers and excelled academically as well as enjoyed the socialization with peers). In high school, Ben and Hali took a pre-calculus class at a resource center; although they enjoyed the social aspect of the class, they both admitted not learning anything new. Graduates reported taking classes with homeschoolers outside the home for subjects other than math. Cali stated she was not “scarred socially.” Sue related that she enjoyed being around adults. Que appreciated the ability to interact with people of all ages and backgrounds. Ben prized the great people he met during homeschooling. Nine graduates enjoyed their community college/university math classes from a social perspective; Ali reported that it helped her make the transition to university. Notably, Hali, Que, and Wes wished that they could

have taken math classes with at least a few other. Tad noted that homeschool educators must be intentional about providing social experiences for their children; it was hard to adjust to university life his freshman year. Upon reflection, none of the graduates expressed resentment about their homeschool socialization experience.

MSES. As a group, these graduates and their parents scored well above average on their math self-efficacy surveys (Table 16). They scored at or above 6.1 (one mother). Thirty out of the thirty-six participants scored at or above 7.3. Twenty-two scored 8.1 or higher. When translating these numbers to the percentile equivalents based on gender (Table 10), these numbers show a high math self-efficacy.

All student and parent participants ranked at 60 percentile or better (Table 18). One mother, one father, and one male graduate placed in the 60 percentile. One male graduate and two mothers fell in the 70 to 80 percentile range. Two graduates, two fathers and three mothers ranked somewhere in the low to middle 80 percentile range. Notably, the other 23 participants, including 12 graduates and 11 parents, placed at or above the 90 percentile. Most of the students (75%) and over 50% of the parents ranked in the top 10-percentile, indicating a very high math self-efficacy for this population.

However, comparing the student total scores with their parents' does not reveal a pattern, either for gender or mathematical interest. Each graduate's total score correlates more closely with either one of the two parents (Table 16). Notably, not one set of parents has identical or very close MSES scores. All but the Rice and Sims families have a spread of approximately one point. Nine graduates' total scores align more closely with their mothers, four of which match very closely or exactly. Seven student participants' total score line up more with their fathers and Ali's and Ela's of these very closely or exactly match. These did not all align according to mathematical interest. Wes

scored well below both his parents. No predictable pattern appears with gender and mathematical interest.

Slightly more female graduate scores align with their mothers, while the male graduate scores are equally split among the parents. Six female scores are closer to their mothers, with Hali and Lea closely or exactly matching their mothers' score. The remaining four females are closer to their fathers and Ali and Ela closely or exactly match. Rob, Tad, and Wes line up with their mothers. Rob and Tad closely or exactly match. Ben, Don, and Ned come closer to their fathers' scores. No strong relationship emerges from this data.

The correlations in Table 17 corroborate these comparisons. The mostly weak correlations (-0.13, 0.07, 0.09, 0.10, and 0.12) demonstrate that no predictable pattern exists in these participants concerning graduates MSES scores with the scores of their mothers or fathers. The strongest correlation coefficient occurs between the six male participants and their fathers (-.47), a stronger negative relationship. However, with such a small sample size, no pattern can be determined from this larger negative correlation.

Table 16

Graduates' MSES Total Score Side by Side With Parents

Graduate	Gender	Graduate MSES	Father MSES	Mother MSES
Ned	M	8.8	8.1	6.1
Cali	F	7.6	8.4	7.2
Sue	F	7.6	8.4	7.2
Ela	F	8.7	8.6	7.2
Ola	F	8.1	8.6	7.2
Tad	M	8.7	7.8	8.7
Rob	M	8.6	7.8	8.7
Wes	M	6.7	8.7	7.1
Mae	F	7.4	8.7	7.1
Ali	F	8.6	8.6	6.7
Hali	F	7.9	8.5	7.9
Ben	M	7.3	7.6	7.9
Que	F	7.3	7.6	7.9
Kim	F	8.3	6.8	8.1
Lea	F	6.8	8.1	6.7
Don	M	8.3	8.1	6.7

Table 17

Correlations of Total MSES Score of Graduates with Parents

Type	With Father	With Mother
All Graduates	-0.13	0.12
Female Graduates	0.07	0.10
Male Graduates	-0.47	0.09

Table 18

Graduates' percentiles (Betz & Hackett, 1993) Side by Side With Parents

Graduate	Gender	Graduate %	Father %	Mother %
Ned	M	95	90	60
Cali	F	90	95	80-90
Sue	F	90	95	80-90
Ela	F	95	95	80-90
Ola	F	95	95	80-90
Tad	M	95	80-90	95
Rob	M	95	80-90	95
Wes	M	60	95	80-90
Mae	F	90	95	80-90
Ali	F	95	95	70-80
Hali	F	95	95	95
Ben	M	70-80	80	95
Que	F	80-90	80	95
Kim	F	95	60	95
Lea	F	80	90	70-80
Don	M	90-95	90	70-80

The Central Question

How does the homeschool environment, including attitudes (of both graduate and educators), instructional practices, relationships, and physical surroundings (may include more places than the home) contribute to the mathematical proficiency of homeschool graduates?

Attitudes. All homeschool graduates reported a positive attitude towards their homeschooling experience. They expressed their faith as a primary motivation for homeschooling, as well as academics and protection. Parents valued higher education and modeled that by earning college degrees. Most parents (80%) showed interest in mathematics with their college majors and choice of vocation. Likewise, most graduates (75%) chose math or science-related majors. As a group, especially the fathers, the parents exhibited a high math self-efficacy. Not surprisingly, the graduate MSES scores and interview results demonstrated productive disposition, confidence in their mathematical ability and belief that mathematics was useful, sensible, and worthwhile. Many participants reported stress with mathematical studies, working with their mother and with the subject matter. Yet, there was a sense of accomplishment when understanding a challenging concept and graduates were prepared for college mathematics. Most believed that they were well prepared; all but one educator agreed. Educators and students appreciated the flexibility of homeschooling in scheduling, selection of curriculum and courses, and pacing; this facilitated family interaction, extracurricular activities, the pursuit of special interests, as well as successful completion of their mathematical studies.

Instructional practices. Five major themes emerged in this category: direct teaching, mastery learning, self-directed learning, Saxon curriculum, and challenging

math courses. Direct teaching dominated the elementary years. Mothers taught with basic mathematics curricula, reading the text and completing the exercises with their students. Some families continued with this format until middle school, and even into high school. Several fathers helped tutor their children, especially with upper level math courses. Emphasized in all homeschools, mastery learning took the form of consistent monitoring and reworking of incorrect problems. Under parental supervision, students progressed to self-study or self-directed learning at some point in their homeschool career. A majority of the graduates used Saxon curriculum. Most of the students and some educators appreciated the rigor, repetition, and incremental approach. Other curricula positively mentioned included: ABeka and BJU (for elementary), Chalkdust, UCSMP, and Math-U-See. All the graduates completed the traditional high school math classes, Algebra I and Algebra II, and geometry. Except for one, these students finished pre-calculus and several went onto to calculus.

Relationships. Mostly familial interactions occurred in homeschooling, but other topics emerged. Family demographics coincided. All families consisted of two parents and two to six biological children, with an average amount of four children. Nine of the ten families were Caucasian and one family had a Caucasian mother and an Indian Asian father. All families had homeschooled for twelve or more years and most graduates were homeschooled from K-12. Several graduates described the closeness of their families. Some worked with their siblings: taking mathematics courses together, tutoring in math, or receiving help with their math lessons. Each graduate discussed socialization. They took classes with other homeschoolers, some in math. Other wished they had taken math with other students for the social aspect. Only one reported a somewhat difficult

transition to university social life. None of the graduates expressed discontent regarding their homeschool socialization experience.

Physical surroundings. The graduates lived in three main geographic areas. Six of the families resided in the suburbs, five in South Carolina near a major metropolitan area and one in Virginia. The other four families lived in a major South Carolina city, an urban area. Most math classes took place in the home, especially in the elementary and middle school years. Math classes outside the home were located at community colleges or a university or an educational facility with other homeschoolers.

CHAPTER FIVE: DISCUSSION

The purpose of this multiple case study was to understand and explain how 16 mathematically proficient homeschool graduates acquired their proficiency. All participants scored 600 or above on the math subtest of the SAT and were homeschooled a minimum of seven years. A modified sample NAEP Grade 12 math assessment (Appendix A), supplied further evidence. Their educators filled out a survey that mirrored the interview questions. Both parents and the graduates completed the Math Self Efficacy Survey [MSES] (Betz & Hackett, 1993). Bronfenbrenner's (1979, 1999) bio-ecological theory served as the theoretical basis and the NRC's Five Strands of Mathematical Proficiency (Kilpatrick et al., 2001) provided the conceptual framework.

Bronfenbrenner's (1979, 1999) theory framed the central research question and the sub research questions. How did the homeschool environment, comprised of family member attitudes, instructional practices, relationships, and physical surrounding contribute to the mathematical proficiency of the homeschool graduates? The five supporting questions involved the graduate and educator descriptions of the homeschool environment, the MSES scores of the graduates and their parents, and the surfacing patterns between the graduate and educator information.

The Five Strands of Mathematical Proficiency (Kilpatrick et al., 2001) verified the graduates' mathematical proficiency. These strands included: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The modified NAEP assessment, the MSES scores, and the interview data supplied the data used to evaluate each graduate. Additionally, their SAT math subtest scores provided further insight into their mathematical competence.

Each graduate demonstrated mathematical proficiency. Most completely fulfilled all five components of mathematical proficiency. Those who lacked in one strand exhibited proficiency in scholastic profile, SAT math subtest scores, and college performance, thereby, qualifying them for this study.

This group of students constituted a select segment of the homeschool population. Two gatekeepers reported that such a selection of homeschoolers with higher SAT math subtest scores was difficult to find. Their stories offer much concerning math preparation.

Summary of the Findings

Gender

Notably, gender issues surfaced. Despite one female graduate's comment about debunking the gender myth of boys being more mathematically inclined than girls, those stereotypes seemed to be fulfilled, in part, by certain comparisons. The average male SAT math subtest scores surpassed the female mean. Likewise, the average male NAEP assessment exceeded the female one. Although the male average total MSES score slightly topped the mean female score, the average female percentile ranking was five points higher than the average male one. However, the small sample size of six males and ten females, does not allow any type of generalization to the homeschooling population.

Homeschool Environment

Bronfenbrenner PPCT variables (1999) served to formulate measurable categories within the homeschool environment. Attitudes represented the person variable. The academic activity, or instructional practices covered the process variable. The context variable, encompassed the homeschool environment, created by the attitudes, instructional practices, relationships, as well as physical surroundings. The graduates' development from their elementary years to the present time addressed the time element.

The systems: micro, meso, exo, macro, and chrono, (Bronfenbrenner, 1979, 1994) enabled the identification of different elements of the homeschool and provided organization to the PPCT variables data collection.

Comparing the within case analyses of the 16 proficient participants led to the discovery of several emergent concepts. Several topics surfaced from all or most of the graduates. These themes arose in the context of the attitudes, instructional practices, relationships, and physical surroundings of the homeschool environment.

Graduates and their parents portrayed similar attitudes. All educators referenced their Christian faith in the survey (Appendix Z). Several graduates confirmed those values. The transmission of Christian values was the main homeschooling motivation, followed closely by academics and safety. The majority of parents and graduates showed interest in STEM college majors and vocation. All but one parent completed at least one college degree. A majority of the parents (80%) majored in a math-related subject or currently worked in a vocation requiring math-related skills. Similarly, 75% of the graduates had math or science-related majors. All participants' MSES scores ranked at the 60 percentile or above. Most clustered around the 90 percentile and several exceeded the 95 percentile. This demonstrated productive disposition, the fifth strand of the NRC's mathematical proficiency model. Many graduates and educators reported stress in studying mathematics. The graduates felt prepared for college mathematics; only one educator did not agree. Participants noted the flexibility of homeschooling in: scheduling, course and curriculum selection, and pacing. They felt it contributed to successful academics, family unity, and extracurricular interests.

The participants lived in two parent families with all biological children. One family had Asian (India) and Caucasian children and the rest consisted of all Caucasian

members. The amount of children per family ranged from two to six, with a mean, median and mode of four children per family. They homeschooled for at least 12 years and many families had finished homeschooling, having graduated most of their children from that program. Participants mentioned close-knit nuclear families. Some took math classes with their siblings and some tutoring occurred between siblings. Each of the graduates referred to socialization issues, but none expressed discontent regarding their homeschool socialization experience.

The graduates and their families resided throughout two states. Most lived in suburban areas, while the rest came from urban locations. More than half of the South Carolina participants were located in suburbs near a major metropolitan area. The other South Carolina participants made their home in an urban area. The majority of math classes took place in the participant's homes, especially from the early to the middle years (K-8). Many graduates attended classes outside the home: in community colleges or a university, or in a classroom setting with other homeschoolers.

Correlating Findings with Homeschool Research

The findings of this study confirmed and extended existing homeschool research. Certain family demographics were repeated. Mathematical instruction was examined further. Graduates were given a voice. The acquisition of mathematical proficiency was addressed in detail. The discussion is organized according to attitudes, instructional practices and physical locations.

Attitudes

Graduates and educators shared positive attitudes concerning the graduates' homeschool experience. Most responses aligned very closely. Ortiz (2000) found that the

homeschool students and their educators shared similar positive viewpoints towards the homeschool experience. However, Van Schalkwyk and Bower (2011) found dissimilar views between younger students and their parents about the homeschool environment.

Positive attitude. Homeschooling agreed with these graduates. They thought it a good fit. None of the graduates reported any significant socialization problems adapting to university; one did allude to some difficulties, but he adapted in due time. Above average MSES scores (the majority in the top 90 percentile) demonstrated high math self-confidence. This corroborates homeschooling literature, which reports that homeschooling students appear to surpass the academic learning, social group behavior, and self-concept of their non-homeschooling peers (Basham et al., 2007; Bolle et al., 2007; Byers, 2005; Goodman, 2008; Medlin, 2000; Ray, 2004; Shyers, 1992; Thomas, 1998, 2002). The majority of participants related they were well prepared for college mathematics, correlating with research indicating that homeschool graduates were well prepared for college level study (Bagwell, 2010; Bolle et al., 2007; Byers, 2005; Clemente, 2006; Cogan, 2010; Galloway, 1995; Jones & Gloeckner, 2004). Yet, their high math self-efficacy seemed to contrast with Bagwell (2010) and Jones (2010), who reported homeschool students insecure of their mathematical preparation.

Homeschooling motivation. In a meta-analysis of homeschooling motivation, Spiegler (2010) found four major areas: academics, morals and socialization, safety, and family unity. This study's findings support those categories, substituting faith for morals.

Faith. In this population, their Christian faith provided the biggest motivation for homeschooling. Homeschooling literature reports the largest subgroup of homeschooling parents considers themselves fundamental, evangelical Christians; they want to impart

their values and family unity to their children (Apple, 2007; Basham, 2001; Hanna, 2011; Lyman, 1998; Ray, 2010; Rudner, 1999).

Academics. Within this study population, academic concerns and flexibility offered secondary reasons for homeschooling. They wanted to select appropriate curriculum reflective of their values and that best met their students' needs. These ideals align with homeschool literature (Basham, 2001; Lyman, 1998; Spiegler, 2010).

Protection. Graduates talked about their parents protecting them from bad influences and keeping them safe from bullying, racial aggression, and gang activity. Graduates appreciated their parents' persistence and care for them. Specifically, one was eternally grateful; two others were very thankful that their teacher was someone who loved them. According to Ray (2000), homeschooling afforded children the focused attention of a parent educator, committed to their welfare.

Instructional Practices

The instructional practices revealed in this research align with a plethora of homeschooling literature. Parent educators attended homeschool conferences and support group meetings, as well as carefully selected curricula based on learning styles. Graduates benefitted from direct teaching, carefully selected curricula, mastery learning, outside classes, flexibility, and meaningful conversations.

Direct teaching. All graduates received direct teaching from their mothers in the elementary years. They read the text with the students and helped them complete the exercises. They utilized basic mathematics workbooks and/or worksheets. Martin-Chang et al. (2011) found that more structured teaching correlated with higher standardized test scores among younger children.

Curriculum. Saxon was the most popular curriculum choice. Also, used in conventional schooling, Saxon emerged as one of two top programs in a study conducted for the U.S. Department of Research (Viadero, 2009). The homeschool graduates and educators liked Saxon's more traditional approach, with the no frills black and white pages and repetition, as well as its rigor. Homeschooling parents used a more traditional approach of drill and practice (Shepherd, 2010) and valued rigor and repetition (Clements, 2002). In her survey, Peterson (2011) found Saxon and Math-U-See to be the most popular curricula. Math-U-See was highly praised by one graduate and her educator. Only a few graduates mentioned manipulatives, for basic math concepts.

The committed educators in this studied matched with those identified in homeschooling literature. Home educators researched options. They carefully chose curricula based on individual needs and recommendations gathered from homeschooling conferences and networking with other homeschoolers (Bachman, 2011; Bannier, 2007; Hanna, 2011; Patterson et al., 2007; Richman et al., 1992). Patterson et al. (2007) found committed parent educators, wanting the best for their children, practicing child-centered instruction with curriculum that matched individual learning styles, with flexible structures and schedules, many incorporating year-round schooling. In this study, some graduates reported regularly completing math in the summer.

Mastery learning. Graduates consistently reported mastery learning, which they defined as regular correcting of work (mostly by parents) and reworking of problems. Mastery learning is found throughout homeschooling literature (Bannier, 2007; Clements, 2002; Meighan, 1992, 1995, 2010; Peterson, 2011, Ray, 200, 2002). Results suggested that the more parents corrected student work and attended math workshops, the better

their students scored on standardized testing (Richman et al., 1992), while immediate feedback remained critical to maintaining interest (Banner, 2007).

Self-directed learning. The graduates participated in self-study or self-directed learning. Ray (2000) reported that homeschooling may be student-directed, as well as parent directed. These homeschools fit that pattern; mothers directed the graduates, at least in the early years and facilitated their outside class schedules and then the students progressed to self-study or self-directed learning. They took ownership of their studies. Holt (2011) promoted self-direction of learners. Meighan (1995, 1999) labeled “plan, do review” type of learning managed by learner as a successful educational behavior. In homeschooling students, Williams (1991) found recurring patterns of flexible structures, which promoted a moderate to high level of student autonomy, and Goodman (2008) found them to be autonomous in their learning, with their parents as guides.

Outside classes. All but five graduates participated in outside math classes. Two graduates attended these classes starting in middle school, with other homeschoolers. Others waited until high school for community college or university courses. Goodman (2008) found that 12 out of 16 college students, previously homeschooled, attended co-op classes or tutoring in core classes.

Flexibility. These participants acclaimed the flexible aspect of homeschooling. Their schedules were adjusted to allow for different interests, in courses and extracurricular activities. Some reported that math was completed on weekends and summers to accommodate other opportunities. Homeschooling educators incorporated flexible scheduling that adapted to learner’s needs and interests (Meighan, 1995; Patterson et al., 2007; Williams, 1991).

Conversations. Graduates reported conversations that stressed mathematical application to life skills. Fathers discussed their work at the dinner table. One father talked about finances with his children. Others related how useful mathematics was as a language, describing and quantifying scientific principles. Mothers took children shopping and practiced math skills. One graduate labeled these conversations as “teachable moments.” Ortiz (2000) described conversations that applied mathematics to everyday life. Meighan (1995, 1999) identified purposive conversation and parents/guardians as learning agents as two of eleven homeschooling behaviors, which promoted successful education. Bachmann’s (2011) homeschool STEM research, identified practical application of mathematic principles in shopping and cooking. In this research, most parent educators were not certified teachers. One possessed elementary certification, with teaching experience. Another taught middle school math, but had not completed the final phase of certification. Homeschool achievement was not significantly related to parent teacher certification (Rudner, 1999).

Dedication of these homeschool educators in direct teaching, providing them with rigorous and meaningful curriculum and outside classes as needed, individualizing their education, moving them into self-study as they matured, aligned with homeschool research (Hanna, 2011; Meighan, 1999, 2010; Ray, 2000, 2002; Thomas, 1998).

Relationships

Demographics. The ten two-parent families involved in this research matched other homeschooling families highlighted in homeschool literature. Two parent homes constituted 89% of the homeschooling population (Planty et al., 2009). In this study 90% of the homeschooling families were Caucasian, compared with a national average of 75%

(Planty et al., 2009). The average amount of four children per household matched research findings (Van Pelt et al., 2009).

Gender

Comparing the average SAT math subscores, MSES total scores, and NAEP assessments according to gender, shows that the averages of the six male graduates exceeds the ten female scores. This seems to contradict Ray (2010) and Rudner (1999) in that gender did not contribute to standardized academic achievement among homeschoolers. However, Ray (2010) and Rudner (1999) were evaluating elementary and middle school students, not university students

Connecting Findings with Non-Homeschool Research

Mathematical proficiency maximizes college and career opportunities and increases income potential (MNAP, 2008). One graduate's mother believed that homeschooling facilitated acceptance into better colleges and more scholarships. An educator perceived mathematical proficiency as a "gateway to better jobs."

Milgram (2007) suggested that students needed to perfect mathematical skills at arithmetic and abstraction (algebra) levels to acquire mathematical proficiency. All the graduates in this study completed algebra, as well as more advanced math courses.

NCTM (2000) recommended equity, curriculum, teaching, learning assessment and technology, for the development of thriving classroom environments. All of these homeschools exhibited multiple examples of the first five principles. Technology existed in the families with the videos and was discussed in one interview where the graduate explained that his father was ahead of the technology curve with cell phones, computers, and even providing a domain for personal website development.

NCTM (2000) standards involved content and process. Content (arithmetic, algebra, geometry, and probability and statistics) intersects with process (problems solving, reasoning and proof, communication, connections, and representations). Both content and process were exhibited in the Saxon curriculum. Among the graduates, there was mention of their attainment of problem solving and reasoning skills.

Schoenfeld (2007) purported four aspects of mathematical proficiency: mathematical knowledge, problem solving strategies, ability to connect knowledge to the problem being solved, and tenacity. The 16 graduates demonstrated these components in their NAEP assessments, MSES scores, SAT math subtest scores, and interviews.

Researchers in elementary and college settings (Kinnari, 2010; Savey, 2009; Suh, 2007) have used the Five Strands of Mathematical Proficiency (Kilpatrick et al., 2001). This study uniquely utilizes this model in homeschool inquiry.

Bandura (1994) described four sources of self-efficacy: mastery experiences, vicarious social models, social persuasion, and reduction of negative stress reactions. These homeschool environments provided all four to the graduates. Graduates practiced mastery learning, which led to mastery experiences. Most parents (80%) modeled the importance of STEM education in their majors and employment. Parent educators demonstrated commitment and persistence in homeschooling, contributing to social persuasion. Flexible pacing, one on one tutoring, as well as the absence of bullying, gang problems, reduced potential stress reactions. Those who successfully build self-efficacy in others, do so by encouraging individuals in their competencies, carefully constructing situations where success is attainable, and not place them in a position prematurely, where they will fail (Bandura, 1994). These graduates received encouragement from their mothers and fathers. They progressed to self-study as they matured.

Bronfenbrenner (2000) reported that family involvement stimulated children's growth processes and cited research that showed children in two parents families scored better academically. These families remained intricately involved throughout homeschooling. The graduates' mathematical proficiency validated their efforts.

The average MSES of the male graduates slightly exceeded the female ones, as in the Pajares and Miller (1994) and Betz and Hackett (1993) studies. However when the average scores were adjusted for gender (Betz & Hackett, 1993), the female graduates ranked in the 95 percentile, exceeding the male's 90 percentile.

Implications of this Study

This multi case study has answered several calls to research and addressed a dearth in the literature concerning mathematical proficiency in high school graduates, specifically homeschooling ones. Identified mathematically proficient students were interviewed as recommended by Schoenfeld (2007). This study investigated an underrepresented population in the homeschooling curriculum as suggested by Green and Hoover-Dempsey (2007), Shepherd (2010), and Van Schalkwyk and Bouwer (2011). Reported in this study were potentially effective homeschool instructional methods including: mathematical learning activity, mastery learning, curriculum choice and implementation, diagnostic and remediation procedures, teaching techniques, impact of high school curriculum on college success, and best practices in alternative education. These findings addressed recommendations from Antique (2007), Bachman (2011), Bagwell (2010), Bannier (2007), Clemente (2006), Cizek and Ray (1995), Ortiz (2000), Schoenfeld (2007), Shepherd (2010), and Sutton & Galloway (2000). As advocated by Cizek and Ray (1995), this inquiry has linked homeschooling and general education

research in the use of Bronfenbrenner's (1979, 1999) bio-ecological theory and the Five Strands of Mathematical Proficiency (Kilpatrick et al., 2001).

Uniquely, this collective study gives a voice to homeschooling graduates proficient in mathematics. This information would be useful to homeschooling parent educators, non-homeschooling parents, as well as conventional (public and private) teachers and administrators. The findings of this research provide practical applications.

Homeschooling Parent Educators

Homeschooling parent educators seek to improve their instruction through networking and what works for other homeschools. This document provides information from ten families and their homeschool environments. NCTM (2000) notes that parents desire to know their children's standing with others and the areas in which they need to improve. Homeschooling educators are in a unique position to understand and help their children obtain the skills needed to become mathematically proficient. General recommendations are provided in bullet format: attitudes, instructional practices, relationships, and physical surroundings.

Attitudes.

- Model positive attitudes towards mathematics and math-related vocations.
- Put stress of working with child in perspective; it is part of the process and will ultimately reap rewards.

Instructional practices.

- Provide direct teaching to elementary and middle school children.
- Research and select rigorous curriculum with repetition (like Saxon).
- Incorporate mastery learning.

- Promote student's ownership of their education by purposely training the student to progress from direct teaching to self-directed learning.
- Research and enroll the student in outside classes, such as classes with other homeschoolers and community college courses (when ready).
- Be flexible in scheduling to accommodate student interests and pacing.
- Engage in purposive conversations which demonstrate the practical applications of mathematics, especially in the younger years.
- Take challenging math courses: Algebra I and II, geometry, pre-calculus and calculus.

Relationships.

- Have siblings work together, either taking classes together, tutoring, or in being tutored.
- Encourage the father's participation in math tutoring or in modeling of math-related skills.
- Promote family unity.

Physical locations.

- Conduct the mathematics classes in the home until sometime in middle or high school.
- Consider outside classes in an educational facility with other homeschoolers and/or community college courses to prepare the student for university type classroom situations.

Parents of Conventionally Schooled Children

Non-homeschooling parents of school age children would also benefit from this research. Certain behaviors may reinforce the conventional schooling. Certain attitudes and instructional practices from the homeschool educator list would prove helpful.

Attitudes.

- Model positive attitudes towards mathematics and math-related vocations.
- Put stress of working with child in perspective; it is part of the process and will ultimately reap rewards.

Instructional practices.

- Become involved early with the student's math work; help them with homework and consistently check their answers.
- Promote student's ownership of their education by purposively training the student to progress from direct teaching to self-directed learning.
- Engage in purposive conversations which demonstrate the practical applications of mathematics, especially in the younger years.
- Have both parents involved with their mathematical study.

Conventional School Educators

Public and private school teachers and administrators would utilize these findings to relate to homeschooling families. Some private schools partner with homeschool educators to offer classes and extracurricular programs. Some states allow homeschool students to participate in public school programs. Knowledge of homeschooling practices would be beneficial to those conventional educators seeking to attract homeschooling students through online education, part-time on campus academic programs, and

extracurricular activities. A resource center structure would appeal to many homeschooling families including:

- Flexible type scheduling with classes one to three days per week.
- Rigorous curriculum that incorporates repetition (like Saxon).
- Mastery learning.
- Allowing students to work at their own pace.

Trustworthiness

Trustworthiness, or rigor (Lincoln & Guba, 1986) comprises confirmability, credibility, dependability, transferability, and authenticity. Confirmability establishes the impartiality of the researcher. Credibility establishes the truthfulness of the findings. Member checks and participant feedback enhance credibility. Dependability adds to the consistency of the results. The expert review by six educational researchers increased the dependability of interview and survey questions. Peer review done by the dissertation committee verified the dependability of the findings. Transferability involves the generalization of the results to other contexts. The use of thick, rich descriptions in meaningful quotes contribute to transferability. Authenticity includes: transparency of the interview process, raising consciousness about a lesser-known phenomenon, giving voice to the participants, and evaluating the impact of the voice. This research employed authenticity with homeschool graduates as co-researchers.

Demonstrating confirmability, all data was organized and processed in the same manner. Every piece of data was carefully coordinated according to the graduates and the families. Each interview was conducted in the same manner. They were transcribed verbatim and checked over twice. Direct quotes from the interviews and surveys were inputted into identical charts for each participant. NAEP explanations and justifications

were also recorded in chart format for each graduate. All the within case studies took place in the same manner, all completed before the between case analysis.

Providing credibility, copies of their participant portraits and within case analyses were emailed to all graduates, encouraging their input and asking for corrections. Thirteen of the sixteen participants provided member checks, correcting minor errors, as needed. One clarified a statement she made, and another updated her employment status.

Ensuring dependability of the researcher-made interview and survey questions, each of the items was reviewed by educational researchers. This expert review was conducted by six professionals with terminal degrees, all with first hand knowledge of homeschool and/or qualitative research. Several conducted research cited in this study.

Also guaranteeing dependability of the findings, peer review was conducted on the within case and between case analyses by members of the dissertation committee. They are familiar with homeschooling research, two of which are cited in this study.

Confirming transferability of the results, participant portraits consisted of interview summaries with thick, rich description. Meaningful quotes illustrated various themes. These quotes served as the basis for the within case and between case analyses.

Guaranteeing authenticity, the homeschool graduates became co-researchers for this research. Transparency existed in the interview process. This research gave voice to the participants, as they shared their perceptions of their mathematical proficiency. This brings to light a topic that is underrepresented in the literature. Several pragmatic recommendations emerged from this cross case analysis.

Limitations

The small skewed sample size, the geographic locations, and the self-report nature of interviews and surveys are limitations to this multi case study. Due to the nature of

qualitative research, a small sample size is necessary. In this study, 16 homeschool graduates and 10 families were studied. Qualitative research requires intensive study of a small sample to provide rich text for evaluation. Yet, findings from a small sample do not allow for generalization to a larger population. The results of this inquiry may not represent the homeschooling population at large. Also, the location and settings of these homeschool programs is a limiting factor.

These 16 graduates, from 10 families, resided in urban and suburban areas. Seven of the students, from four families, lived in a major city in South Carolina. In a suburb of South Carolina, five families made their home near a major metropolitan area. The two participants from the Virginia family also lived in a suburb.

Also, the participants belonged to a select group of homeschooling students. Not all homeschool graduates score at or above 600 on the SAT math section. Considering the snowball sampling and my range of contacts, not all homeschool families are two-parent homes, with mostly Caucasian members, or have a faith-based motivation. Most of the participants (10 out of 16) were female, not applying to the approximately even amount of male and female homeschool students (Planty et al., 2009; Ray, 2010). Thereby, these findings may not generalize to all homeschooling families.

Additionally, the self-report nature of the interview and the survey are additional limitations to the findings of this study. Perceptions may be biased and not accurate. Memories of past events may be incorrect.

Recommendations for Future Research

To generalize results, further research with mathematically proficient homeschool graduates needs to be conducted with a larger sample size and a broader geographic area. This inquiry was limited to families in South Carolina locations and one Virginia site.

Locating more participants throughout the United States and other countries (in rural, suburban and urban areas) would provide more transferability.

Additionally, further longitudinal research with this group of graduates would provide additional information. Will they continue with STEM majors and vocations? Will they continue to hold positive views about homeschooling? Will they homeschool their children?

Qualitative work may uncover areas for future quantitative inquiry. Without any constraints on the SAT math subtest, a larger sample may be surveyed to compare MSES scores, SAT math scores, parental education, and vocations to see if there exists any strong correlation among these factors.

More research needs to give voice to homeschool graduates and their mathematical experience. Their reflection provides insight to learning. Many commented that they learned how to learn and that proved invaluable to their collegiate studies. Further insight is needed into how proficient students take ownership of their learning.

One of the female participants brought up a significant gender issue, stating, I wish that girls would go farther with math than I think that they do. . . . I just think we have some sort of weird gender misconception that boys are good at math and I don't see any real reason for that.

Further research into homeschooling and gender issues is needed. Does homeschooling promote this idea that males perform better at math, or mitigate it, allowing women to excel in math-related subjects?

Conclusion

These mathematically proficient homeschoolers had much to offer. Giving voice to their perceptions provided insight into effective educational environments. In unique

and creative ways, their parent educators created productive atmospheres, modeled positive attitudes towards math, and motivated their students to mastery. Many themes emerged from this research: strong nuclear families, direct teaching, self-study or self-directed education, mastery learning, purposive conversations, and challenging curriculum. As they matured, these graduates built upon the foundation set by their parent educators and took ownership of their studies.

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Appendix A: Modified NAEP Grade 12 Mathematics Proficiency Evaluation

Record time started:

Record time finished:

DIRECTIONS:

Circle the correct answer for each problem.

Near to the problem statement and answers, show your work for solving the problem. You may write anywhere on the problem.

Also, explain why you chose this method.

**TEMPERATURES ON OCTOBER 1ST
FOR FIVE CITIES (in °F)**

	High	Low
City A	72	50
City B	90	75
City C	83	72
City D	50	37
City E	92	72

1. The table above shows the high and low temperatures on October 1st for five cities. Which city had the greatest temperature range?
- A. City A
 - B. City B
 - C. City C
 - D. City D
 - E. City E

SHOW YOUR WORK AND EXPLAIN YOUR METHOD:

2. John is going to cover an attic floor with insulation. The floor measures 25 feet by 35 feet. If one roll of insulation will cover 64 square feet, how many rolls of insulation does John need?
- A. 1
 - B. 2
 - C. 8
 - D. 14
 - E. 110

SHOW YOUR WORK AND EXPLAIN YOUR METHOD:

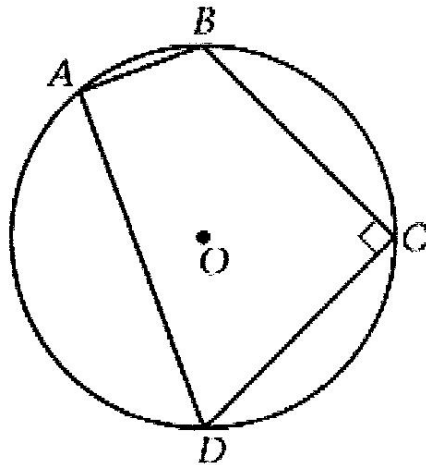
3. The first four terms in a sequence are shown below.

40, 8, 24, 16, ...

Each term after the first two terms is found by taking one-half the sum of the two preceding terms.
Which term is the first odd number in this sequence?

- A. The 5th term
- B. The 6th term
- C. The 7th term
- D. The 8th term
- E. The 9th term

SHOW YOUR WORK AND EXPLAIN YOUR METHOD:



4. Quadrilateral ABCD is inscribed in circle O, and $\angle C$ is a right angle, as shown above. Segment AB is not parallel to segment DC. Which of the following statements must be true?

- A. $\angle A \cong \angle B$
- B. $\angle B \cong \angle D$
- C. $\angle B$ is a right angle.
- D. \overline{AC} is a diameter of circle O.
- E. \overline{BD} is a diameter of circle O.

SHOW YOUR WORK AND EXPLAIN YOUR METHOD:

5. The manager of a company has to order new engines for its delivery trucks after the trucks have been driven 150,000 miles. One of the delivery trucks currently has 119,866 miles on it. This truck has the same delivery route each week and is driven an average of 40,000 miles each year. At this rate, the manager should expect this truck to reach 150,000 miles in approximately how many months?
- Less than 4 months
 - Between 4 and 6 months
 - Between 6 and 8 months
 - Between 8 and 10 months
 - More than 10 months

SHOW YOUR WORK AND EXPLAIN YOUR METHOD:

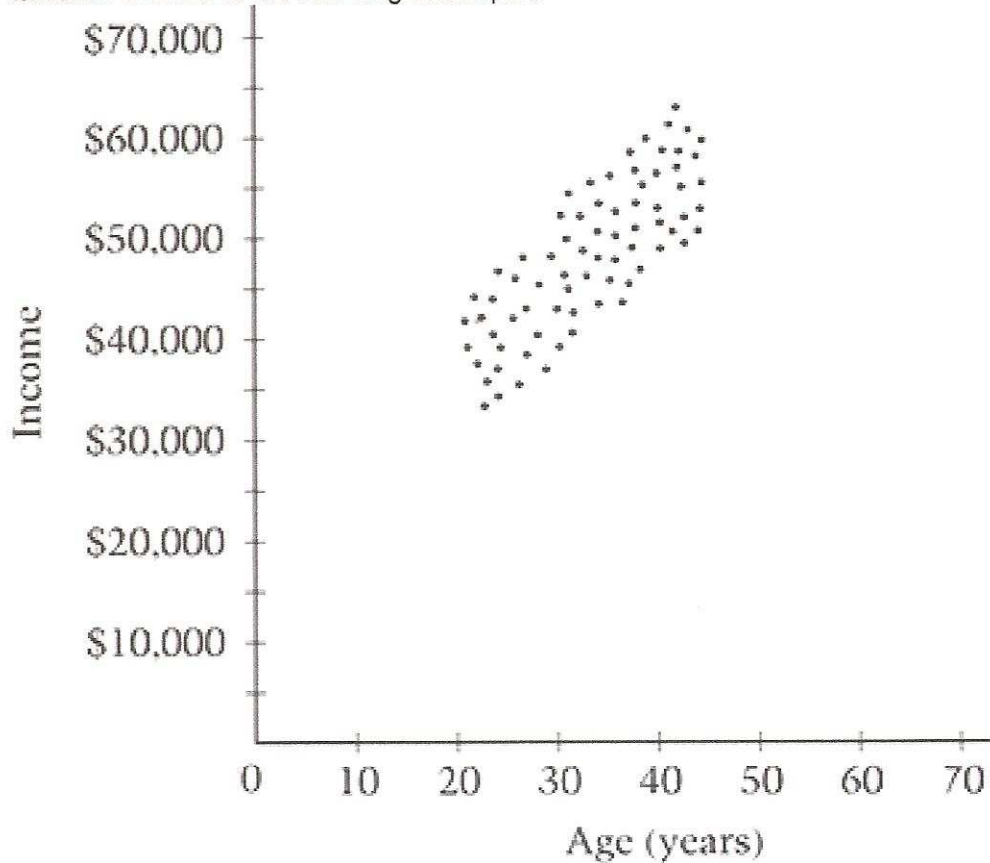
6. Which of the following expressions is NOT equivalent to $(a + b)(x + y)$?
- $(a + b)x + (a + b)y$
 - $a(x + y) + b(x + y)$
 - $(b + a)(y + x)$
 - $ax + by$
 - $ax + bx + ay + by$

SHOW YOUR WORK AND EXPLAIN YOUR METHOD:

7. The principal of a high school would like to determine why there has been a large decline during the year in the number of students who buy food in the school's cafeteria. To do this, 25 students from the school will be surveyed. Which method would be the most appropriate for selecting the 25 students to participate in the survey?
- Randomly select 25 students from the senior class.
 - Randomly select 25 students from those taking physics.
 - Randomly select 25 students from a list of all students at the school.
 - Randomly select 25 students from a list of students who eat in the cafeteria.
 - Give the survey to the first 25 students to arrive at school in the morning.

SHOW YOUR WORK AND EXPLAIN YOUR METHOD:

Question 8 refers to the following scatterplot:



A random sample of graduates from a particular college program reported their ages and incomes in response to a survey. Each point on the scatterplot above represents the age and income of a different graduate.

8. Of the following equations, which best fits the data above?

- A. $y = -1,000x + 15,000$
- B. $y = 1,000x$
- C. $y = 1,000x + 15,000$
- D. $y = 10,000x$
- E. $y = 10,000x + 15,000$

SHOW YOUR WORK AND EXPLAIN YOUR METHOD:

9. Angie has a bag containing n apples. She gives 4 to her brother and keeps 5 for herself. She then divides the remaining apples equally among 3 friends. Which of the following expressions represents the number of apples each friend receives?

- A. $\frac{n}{3} - 4 - 5$
 B. $\frac{3}{4+5-n}$
 C. $\frac{3}{n-4} - 5$
 D. $\frac{3}{n-5} - 4$
 E. $\frac{n-4-5}{3}$

SHOW YOUR WORK AND EXPLAIN YOUR METHOD:

10. Carlene told Kyle that a rectangular room measured 16 feet by 12 feet, to the nearest foot. This means that the length could measure between 15.5 feet and 16.5 feet and the width could measure between 11.5 feet and 12.5 feet. Kyle performed the following calculations.

Dimensions (feet)	Area (square feet)
15 by 11	165
15.5 by 11.5	178.25
16 by 12	192
16.5 by 12.5	206.25
17 by 13	221

Of the following intervals, which is the smallest interval that contains all possible values of the area of the room?

- A. Between 191.5 and 192.5 square feet
 B. Between 191 and 193 square feet
 C. Between 179 and 206 square feet
 D. Between 178 and 207 square feet
 E. Between 165 and 221 square feet

SHOW YOUR WORK AND EXPLAIN YOUR METHOD:

Appendix B: Semi-structured Homeschool Graduate Interview

- 1) What is your: Age ____ Birthplace _____ Birth order ____?
Describe your siblings:
Child 1 Age____, Male/Female, Race _____ (African American, Asian, Latino, White, Other), Type of schooling (home, public, private)
Child 2 Age____, Male/Female, Race _____ (African American, Asian, Latino, White, Other), Type of schooling (home, public, private)
Child 3 Age____, Male/Female, Race _____ (African American, Asian, Latino, White, Other), Type of schooling (home, public, private)
Child 4 Age____, Male/Female, Race _____ (African American, Asian, Latino, White, Other), Type of schooling (home, public, private)
Child 5 Age____, Male/Female, Race _____ (African American, Asian, Latino, White, Other), Type of schooling (home, public, private)
- 2) What was/were the initial reason(s) your family started to homeschool?
- 3) What is/are the reason(s) your family continues to homeschool?
- 4) How long has your family been homeschooling? 1-3 years, 4-6 years, 7-10 years, 11-14 years, 15 or more years
- 5) Tell me more about your experience with homeschooling.
- 6) Describe your favorite subjects during your homeschool years.
- 7) Describe your least favorite subjects during your homeschool years.
- 8) Describe what math classes you took during your homeschool years (e.g., Algebra, Geometry, Pre-Calculus, Calculus, Probability and Statistics).
- 9) Describe your experience with math in your homeschooling years (adequate/inadequate?).
- 10) How well were you prepared for college mathematics courses (if you have taken any)?
- 11) Why you think you did so well on the SAT math subtest (Scoring in the top 25%)?
- 12) Describe the mathematics curricula you used in homeschooling (e.g., Saxon, teacher-made).

- 13) Describe what your mathematics instruction and experiences were like (e.g. style of instruction – structured or non-structured, direct teaching, reworked incorrect problems until mastery; group work, self-study, online course, combination of activities, co-op classes, other).
- 14) Describe the personal interactions in your homeschool during the time that mathematical instruction took place (e.g., some or limited– self study; parent/student interaction – conversations, lessons - stressful or enjoyable; working with siblings – stressful or enjoyable).
- 15) Describe changes (if any) that took place in your mathematical instruction over the years of homeschooling (e.g., curriculum, DVD classes, online classes, classes outside of home).
- 16) How was your homeschool experience in math the same or different from your homeschool experience in other subjects?
- 17) How do you view your overall homeschool math experience (advantages/disadvantages)?
- 18) What would you have changed about your homeschool math experience?
- 19) Do you view mathematics as useful, sensible, or worthwhile?
- 20) What were your parents' attitudes about and experiences with math?

Appendix C: Homeschool Educator Survey

- 1) Tell me about your children: (Please fill in/circle the response)
The homeschool graduate being interviewed: Age _ Birth place ___ Birth order ___
Describe your other children:
Child 1 Age____, Male/Female, Race _____ (African American, Asian, Latino, White, Other), Schooling (home, public, private)
Child 2 Age____, Male/Female, Race _____ (African American, Asian, Latino, White, Other), Schooling (home, public, private)
Child 3 Age____, Male/Female, Race _____ (African American, Asian, Latino, White, Other), Schooling (home, public, private)
Child 4 Age____, Male/Female, Race _____ (African American, Asian, Latino, White, Other), Schooling (home, public, private)
Child 5 Age____, Male/Female, Race _____ (African American, Asian, Latino, White, Other), Schooling (home, public, private)
- 2) What was/were the initial reason(s) your family started to homeschool?
- 3) What is/are the reason(s) your family continues to homeschool?
- 4) How long has your family been homeschooling? (Please circle the response)
1-3 years, 4-6 years, 7-10 years, 11-14 years, 15 or more years
- 5) Tell me more about your homeschool graduate's experience with homeschooling.
- 6) Describe your homeschool graduate's favorite subjects during homeschooling.
- 7) Describe your homeschool graduate's least favorite subjects during homeschooling.
- 8) Describe what math classes your homeschool graduate took during homeschooling (e.g., Algebra, Geometry, Pre-Calculus, Calculus, Probability and Statistics).
- 9) Describe your homeschool graduate's experience with math during homeschooling (adequate/inadequate?).
- 10) How well was your homeschool graduate prepared for college mathematics courses (if any were taken)?
- 11) Why you think your homeschool graduate did so well on the SAT math subtest (top 25%)?
- 12) Describe the mathematics curricula you used with your homeschool graduate during homeschooling.

- 13) Describe what your homeschool graduate's mathematics instruction and experiences were like (e.g., style of instruction – structured or non-structured, direct teaching, reworked incorrect problems until mastery; group work, self-study, online course, combination of activities, co-op classes, other).
- 14) Describe the personal interactions in your homeschool during the time that mathematical instruction took place for the homeschool graduate (e.g., some or limited – self study; parent/student interaction – conversations, lessons - stressful or enjoyable; working with siblings – stressful or enjoyable).
- 15) Describe changes (if any) that took place in your homeschool graduate's mathematical instruction, over the years of homeschooling (e.g., curriculum, DVD classes, online classes, classes outside of home).
- 16) How was your homeschool graduate's experience with math during homeschooling the same or different from their homeschool experience in other subjects?
- 17) How does your homeschool graduate view his/her overall homeschool math experience (advantages/disadvantages)?
- 18) What would you have changed about your homeschool graduate's homeschool math experience?
- 19) Does your homeschool graduate view mathematics as useful, sensible, or worthwhile?
- 20) What are your attitudes about and experiences with math?
- 21) Describe any homeschooling support that you have received (e.g., homeschool conferences, math workshops, support groups, coops).
- 22) If you know of other homeschooling families that might participate in this study, please ask them to contact me at Betty.Cardinale.Research@gmail.com.
- 23) Other comments _____

Appendix D: IRB Approval Letter



The Graduate School at Liberty University

June 4, 2012

Betty Cardinale □ IRB Approval 1353.060412: Secrets of Their Success: Multiple Case Studies of Mathematically Proficient Home School Graduates

Dear Betty,

We are pleased to inform you that your above study has been approved by the Liberty IRB. This approval is extended to you for one year. If data collection proceeds past one year, or if you make changes in the methodology as it pertains to human subjects, you must submit an appropriate update form to the IRB. The forms for these cases were attached to your approval email.

Thank you for your cooperation with the IRB and we wish you well with your research project.

Sincerely,

Fernando Garzon, Psy.D.

Professor, IRB Chair

Counseling (434) 592-4054

Liberty University | Training Champions for Christ since 1971

Appendix E: Email of Intent to Potential Participants

My name is Betty Cardinale (Betty.Cardinale.Research@gmail.com) and I am conducting research for my dissertation at Liberty University, Lynchburg. I have homeschooled for 18 years. My oldest has graduated college, my second is a college junior, and my youngest is a high school senior.

The importance of homeschooling research cannot be understated.

Homeschooling research played a vital part in the recent California court case (*Rachel L.*). An appeals court denied the legal status of more than 150,000 homeschoolers based on one case of alleged neglect involving a homeschool family. Several months later, this same appeals court **overturned its own decision based on expert testimony that utilized homeschooling research.**

My case study research involves the mathematical proficiency of homeschool graduates and how homeschooling has contributed to that proficiency. *(name of gatekeeper)* has kindly agreed to send this email to you. I do not know the email addresses of the families this form has been emailed to, in order to protect your privacy. *(name of gatekeeper)* will not be involved in the data collection or analysis, nor will they have access to any data collected from this study.

You have been sent this email in hopes that **you and your homeschool graduate will choose to volunteer in this study and help further homeschool research.**

I am looking for homeschool graduates (ages **18-23**) who were homeschooled for **seven or more years** (these do not have to be continuous, but they do have to end in graduation [e.g. 1st-4th, 10-12th]) and who scored **600 or above on the mathematics section of the college SAT**. In this study I am asking for the **homeschool graduate** to participate in the following manner:

- 1) By completing a mathematical proficiency 10 question evaluation. This will be emailed or mailed.
- 2) By completing a mathematics self-efficacy questionnaire. This will be mailed.
- 3) By participating in an at-home interview concerning their mathematical proficiency.

In this study I am asking for the **home educator(s)** to participate in the following manner:

- 1) By completing a survey about your homeschool. This will be emailed.
- 2) By completing a mathematical self-efficacy questionnaire. This will be mailed.
- 3) By allowing me to view a copy of the homeschool graduate's mathematics score on the SAT college entrance exam and a high school transcript with the final grades of the homeschool graduate's mathematical courses.

Note: Any data collected will be kept confidential and only pseudo-names will be used.

If you are willing to participate in this research please email:

Betty.Cardinale.Research@gmail.com

Thank you,

Betty Cardinale

Appendix F: Consent Form for Homeschool Educators

Secrets of Their Success: Case Studies of Mathematically Proficient Homeschool Graduates

Betty R. Cardinale
Liberty University
Graduate School of Education

You are invited to be in a research study that seeks to explain how some homeschool graduates (18-23) became mathematically proficient. You and your graduate were selected as possible participants because your student graduated from your homeschool, scored 600 or better on the mathematics section of the college SAT entrance test, and was homeschooled for seven or more years. I ask that you read this form and ask any questions you may have before agreeing to be in the study.

This study is being conducted by:

Betty R. Cardinale, Liberty University Graduate School of Education

Background Information

The purpose of this study is to explain how the homeschool experience contributed to the mathematical proficiency of some homeschool graduates. Mathematics curriculum and instruction is based on research that demonstrates “what works” to benefit students. This study will investigate and explain the methods, experiences, perceptions, and curriculum that mathematically proficient homeschool graduates report contributed to their ability and achievement.

Procedures:

If you agree to be in this study, I would ask you to do the following things:

Complete a mathematical self-efficacy survey, (the survey will take approximately 10 minutes to complete), complete a survey about your graduate’s homeschool and mathematical experiences (this survey will take approximately 20-30 minutes to complete), and provide documentation showing your graduate’s college SAT mathematical section score and final high school transcript.

Risks and Benefits of being in the Study

No study is without risk. However, the risks are minimal and no more than the participant would encounter in everyday life. As a result of this study, awareness of uncomfortable and unpleasant thoughts associated with the experience may increase. The study may involve additional risks to the participants, which are currently unforeseeable.

The benefits to participation are not material but philanthropic, knowing that you added to the growing amount of homeschool research that seeks to explain the inner workings of successful homeschooling (in this study - mathematically proficient graduates) to the general public and other homeschoolers.

Compensation:

A \$50 gift card will be given to a randomly selected graduate participant who returns the mathematical assessment and the mathematical self-efficacy survey within two weeks after they receive those items. Additionally, all homeschool graduate participants will receive a \$25 gift card after they complete the interview and their parent completes the surveys.

Confidentiality:

The records of this study will be kept private. In any sort of report I might publish, I will not include any information that will make it possible to identify you or your family. Research records will be stored securely and only I will have access to the records. All data will be stored in my home computer (password protected) and separate locked containers in my home.

Voluntary Nature of the Study:

Participation in this study is voluntary. Your decision whether or not to participate will not affect your current or future relations with Liberty University. If you decide to participate, you are free to not answer any question or withdraw at any time without affecting those relationships.

Contacts and Questions:

The researcher conducting this study is: Betty R Cardinale. You may ask any questions you have by contacting me at (803) 517-0182 or Betty.Cardinale.Research@gmail.com. Also, you may contact my research chairman, Dr. Kathi Morgan at kcmorgan@liberty.edu or (434) 58202469. If you have questions later, **you are encouraged** to contact Dr. Morgan or myself by phone or email address given above.

If you have any questions or concerns regarding this study and would like to talk to someone other than me or my chairman, **you are encouraged** to contact the Institutional Review Board, Dr. Fernando Garzon, Chair, 1971 University Blvd, Suite 1582, Lynchburg, VA 24502 or email at fgarzon@liberty.edu.

You will be given a copy of this information to keep for your records.

Statement of Consent:

I have read and understood the above information. I have asked questions and have received answers. I consent to participate in the study.

Signature of homeschool educator: _____ Date: _____

Signature of Investigator: _____ Date: _____

Appendix G: Consent Form for Homeschool Graduates

Secrets of Their Success: Case Studies of Mathematically Proficient Homeschool Graduates

Betty R. Cardinale
Liberty University
Graduate School of Education

You are invited to be in a research study that seeks to explain how some homeschool graduates became mathematically proficient. You were selected as a possible participant because you graduated from a homeschool, scored 600 or better on the mathematics section of the college SAT entrance test, and were homeschooled for seven or more years. I ask that you read this form and ask any questions you may have before agreeing to be in the study.

This study is being conducted by:

Betty R. Cardinale, Liberty University Graduate School of Education

Background Information

The purpose of this study is to explain how the homeschool experience contributes to the mathematical proficiency of some homeschool graduates. Mathematics curriculum and instruction are based on research that demonstrates “what works” to benefit students. This study will investigate and explain the methods, experiences, perceptions, and curriculum that some mathematically proficient homeschool graduates report contributed to their ability and achievement.

Procedures:

If you agree to be in this study, I would ask you to do the following things:

Complete a 10-question assessment of 12th grade basic mathematics topics (these questions will take approximately 20-30 minutes to complete), **complete a mathematical self-efficacy survey** (the survey will take approximately 10-15 minutes to complete), and **participate in a ½ hour interview regarding your homeschool mathematics experience.**

Risks and Benefits of being in the Study

No study is without risk. However, the risks are minimal and no more than the participant would encounter in everyday life. As a result of this study, awareness of uncomfortable and unpleasant thoughts associated with the experience may increase. The study may involve additional risks to the participants, which are currently unforeseeable.

The benefits to participation in this study are not material, but philanthropic, knowing that you added to the growing amount of homeschool research that seeks to explain the inner workings of successful homeschooling (in this study - mathematically proficient graduates) to the general public and other homeschoolers.

Compensation:

A \$50 gift card will be given to a randomly selected homeschool graduate participant who returns the mathematical assessment and the mathematical self-efficacy survey within two weeks after receipt of those items. Additionally, all homeschool graduate participants will receive a \$25 gift card after they complete the interview and their parent completes the survey.

Confidentiality:

The records of this study will be kept private. In any sort of report I might publish, I will not include any information that will make it possible to identify you or your family. Research records will be stored securely and only I will have access to the records. All data will be stored in home computer (password protected) and separate locked containers in my home.

Voluntary Nature of the Study:

Participation in this study is voluntary. Your decision whether or not to participate will not affect your current or future relations with Liberty University. If you decide to participate, you are free to not answer any question or withdraw at any time with out affecting those relationships.

Contacts and Questions:

The researcher conducting this study is: Betty R Cardinale. You may ask me any questions you have now by contacting me at Betty.Cardinale.Research@gmail.com or (803) 517-0182. Also, you may contact my research chairman, Dr Kathi Morgan at kcmorgan@Liberty.edu or (434) 582-2469. If you have questions later, **you are encouraged** to contact Dr. Morgan or myself with the contact information, listed above.

If you have any questions or concerns regarding this study and would like to talk to someone other than me, **you are encouraged** to contact the Institutional Review Board, Dr. Fernando Garzon, Chair, 1971 University Blvd, Suite 1582, Lynchburg, VA 24502 or email at fgarzon@liberty.edu.

You will be given a copy of this information to keep for your records.

Statement of Consent:

I give my permission to be video recorded and photographed for the purposes of this study.

I have read and understood the above information. I have asked questions and have received answers. I consent to participate in the study.

Signature: _____ Date: _____

Signature of Investigator: _____ Date: _____

Appendix H: Mathematics Self-Efficacy Survey

(Betz & Hackett, 1989) (see Clutts, 2010, Appendix A)

No Confidence at all Very little Confidence Some Confidence 0 1 2 3 4 5
Much Confidence Complete Confidence 6 7 8 9

Part I: Everyday Math Tasks

How much confidence do you have that you could successfully:

1. Add two large numbers (e.g., $5379 + 62543$ in your head.
2. Determine the amount of sales tax on a clothing purchase.
3. Figure out how much material to buy in order make curtains.
4. Determine how much interest you will end up paying on a \$675 loan over 2 years at $14\frac{3}{4}\%$ interest.
5. Multiply and divide using a calculator.
6. Compute your car's gas mileage.
7. Calculate recipe quantities for a dinner for 3 when the original recipe is for 12 people.
8. Balance your checkbook without a mistake.
9. Understand how much interest you will earn on your savings account in 6 months, and how that interest is computed.
10. Figure out how long it will take to travel from Columbus to Chicago driving at 55 mph.
11. Set up a monthly budget for yourself taking into account how much money you earn, bills to pay, personal expenses, etc.
12. Compute your income taxes for the year.
13. Understand a graph accompanying an article on business profits.
14. Figure out how much you would save if there is a 15% mar-down on an item you wish to buy.
15. Estimate your grocery bill in your head as you pick up items.
16. Figure out which of 2 summer jobs is the better offer: one with a higher salary but no benefits; the other with a lower salary but with room, board, and travel expenses included.
17. Figure out the tip on your part of a dinner bill total split 8 ways.
18. Figure out how much lumber you need to buy in order to build a set of bookshelves.

Part II: Math Courses

Please rate the following college courses according to how much confidence you have that you could complete the course with a final grade of —A| or —B|. Circle your answer according to the 10-point scale below: [note: same scale as Part I]

19. Basic College Math
20. Economics
21. Statistics
22. Physiology
23. Calculus
24. Business Administration
25. Algebra II
26. Philosophy
27. Geometry
28. Computer Science
29. Accounting
30. Zoology
31. Algebra I
32. Trigonometry
33. Advanced Calculus
34. Biochemistry

Appendix I: Memo Samples

8/7/12: The Lord led me to pray with each participant before the interview. As the Lord leads, I would like to do that before every interview! 11/7/12 note – prayed with E, O, T, R, L & D, H, A in Charleston, B & Q; Not with W & M - in Dunkin Don)

E was very open and helpful. It surprised me that she classified herself as not having a good attitude towards math until she saw its usefulness in 7th-8th grades. Then she described herself as very focused. That is my impression of her academic endeavors, now.

O surprised me with her very positive attitude about math. It was not what I expected, with her music major (vocal) aspirations. Refreshing and wonderful.

It seems that Saxon is regimented and repetitive and what has helped these girls (as well as all my other participants). These girls took this curriculum to the full extent, where as the other students so far used it with other choices – especially later.

Beautiful home. They seemed to stay at home and focus on their schoolwork. I didn't get any other picture but studies and music. Were non-music extracurricular not that important? Their home was so far away from things – I wonder was that why they stayed home so much. (11/7/12 addition – I saw the mother last Wed – a week ago – and she mentioned that traveling to York Tech was a logistical issue: 40 min drive each way only 1 car available and they did well on their own, was it really necessary?) They were both strong about their Christian faith.

Their dad seemed very involved with the math for both the girls, especially in high school classes, although LB talked about time with Dad in 7th and 8th grades.

9/18/12: Concerning A, Math was “in-house” (at least through early high school, Algebra 2 or Trig/Pre-Calc) unlike other subjects such as History/Humanities; English; science labs; foreign languages

All participants, so far, participated in church, Christian faith is a core element, either student gave testimony a/o a reason for homeschooling involved the excising of their Christian faith in some manner, for protection, ability to include God in their studies
Self-study for all participants for at least part of their math experience

Dual/concurrent enrollment, most took tech/university classes in math(not M, W, Q)

10/17/12: As I attend the AX induction ceremony, I am amazed that 2 of my participants were inducted. That means they have a 4.0 GPA. I would say they are successful! There are other homeschool students in this group. The words that come to me are: Hard working; consistent, conscientious; all are females (hmm)

Appendix J: Ned

Ned interview. Interviewed in the den of his parents' upper middle class home 19-year old Ned willingly participated. He seemed very comfortable, being open and honest without any sign of nervousness or hesitation. The interview took place after Saturday lunch. Amidst a lot of family activity, the mother had all the documents ready for me as I entered the kitchen. While his mother and brother noiselessly moved throughout the house, Ned answered all questions and offered extra explanation when prompted. The interview lasted 28 minutes.

According to Ned, his family started to homeschool because his mother wanted family unity. Ned believed, "I think my mom wanted to be sure to spend more time with us." Positive outcomes stimulated further homeschooling. Ned perceived, "But we'd actually come to like it so much from the flexibility and for like family time and, self-motivation in learning that was so appealing. So these things kind of got us into and kept us going in homeschooling." His mother acted as the primary teacher with some help from the father, especially in the high school level mathematics courses.

Overall, Ned believed that his overall homeschool math experience, positive and well prepared him for college academics. He indicated,

I thought it was perfect for me . . . it fit my personality with how I get frustrated sometimes and I would still be able to come back and not be penalized . . . Eventually I was able to learn what I needed to learn what I needed to learn and that has proven valuable for I knew the math I needed to know for my classes.

Ned appreciated the mathematics help from his parents and siblings, sharing,

I had so many different resources between my parents and my siblings that it was always explained to me sooner or later. Sometimes, I would storm off . . . maybe later that night but eventually I would sit back down and get the problem done . . . They were like are you ready to learn this now?

Ned learned at his own pace, completing all his subjects quickly. He appreciated that flexibility when compared to his community college experience, recollecting,

That really frustrated me because . . . I would be able to get through school or a school day real quick because I understood things . . . There weren't other people to ask questions and wait for that . . . There were times at [community college] where I know we wasted massive amounts of the day because one person out of the 40 had a question because they didn't understand it just yet.

He affirmed, "I think I got the best plan for myself. I've been able to do all the math I needed to do for my first year of college; so I think I was prepared." Given the option to attend public school, he chose to stick with homeschooling. His excellent SAT math score aligned with his affirmative beliefs.

Ned believed his preparation enabled him to score well on the SAT math subtest (top 7% – 740 Math). Ned's transcript and SAT score record verified his score of 740. He completed a SAT prep class the summer before his first SAT test, in 10th grade. After that he used a SAT prep book. He reasoned,

I think a large part of it is preparation. My family, at least knew that it was important and they kind of stressed to me that you do actually need these practice test . . . it became more and more of my day as it got closer to the time to actually take the test . . . then I started doing multiple sections in a day of just practicing. Sometimes it was all math in a day, sometimes mixing it up with all different sections. But um, just ramping it up closer to the time. And in the end I probably did eight practice tests before the actual final time I took the SAT.

Interestingly, mathematics was not N's favorite subject during his school years.

Ned enjoyed biology and chemistry, not mathematics or writing. Writing was his least favorite, then math, declaring, "Math has never been one of my favorite subjects."

At times, Ned felt frustrated with his math work, revealing, "Stressful/enjoyable reminds of those days that I wouldn't catch onto something immediately . . . sometimes I would storm off with dividing or fractions." Math problems exasperated him, revealing, "I didn't understand a lot of the things initially because you have to work at it and when I was younger, I didn't entirely grasp that concept just yet. I thought that it was supposed to kind of come to you." He also expressed irritation with his math textbook, stating, "Sometimes I wanted to burn my Saxon book. But I think that might have happened with any math book." At the time, the repetition of Saxon irritated him, recalling, "That [repetition] frustrated me. But I understand that repetition, you need to understand it [math] . . . It's very tedious but I definitely need some sort of review here and there."

Although Ned currently appreciated math, it had distressed him. He learned what he needed to know for his chemistry major, but math frustrated Ned throughout his school career. Ned now appreciates math for its application to chemistry, saying "I found out that there's a lot of math in chemistry, anyway, so that's another reason I had to start liking math." Yet, he fell short of saying he now liked math, commenting, "We all knew that we had to do it [math] and appreciated it for what it was. But at the same time did not get too fond of it or friendly with it." Notably, his family helped him be successful.

Ned appreciated his parents and siblings' role with his homeschool math. He described his parents' persistence, stating, "My parents were on me everyday to make sure I was doing it [math]. . . The personal attention from my parents . . . really helped me understand things." He considered his parents and siblings to be resources, recalling, "I had so many different resources between my parents and my siblings that it was always explained to me sooner or later." He attributed his good foundation to them, maintaining, "I wouldn't be able to understand any of that

[algebra at the University Model School] if I hadn't learned from my parents the more basic things earlier on with Saxon." His mother taught him directly in the early years.

Ned's mother worked with him in his younger years. He remembered,

My mom would have assignments laid out for each day of the week . . . and then we would read these pages in a book and do these problems and when I was younger she would do it with usually making sure I'm actually keeping up with it.

As he grew older, Ned became more independent. He recalled,

But as I got older, it became more and more that I had to go to it and make sure the check marks were all in the assignment book and actually taking the test problems and everything and do them seriously.

Ned started at home with Saxon in the third grade, switched to a University Model School in the eighth grade, and then attended a local community college for his last two years of high school. He remembered,

I had Saxon Math for a long time. . . . Early on, I just had a textbook and my parents and my siblings asked questions. Um, I definitely needed their structure on me to make sure I was getting through enough of it in a timely manner.

Later, his mother enrolled him in University Model School classes. At this alternative school, Ned attended these classes, three times weekly, with other homeschooling students. He completed assignments at home. The class size varied from six to sixteen, with the same teacher for all three years. Ned really enjoyed the classes, reminiscing,

After we heard so many different things so many times from her it really did start to stick. And also you could tell she really did care about it. And she had a passion for math . . . and it made it that much better for us.

Ned felt the University Model School provided him with an excellent foundation in algebra and enjoyed the group work, explaining, "The algebra I learned at [University Model School] from [teacher] is what has stuck with me." Ned's transcript verified that he completed the high school math courses of Algebra 1 and 2, Geometry, Trigonometry, Calculus 1 and Calculus 2. Ned took Trigonometry, Calculus 1 and Calculus 2 at community college. Ned expressed irritation at the wait time, explaining,

There were times at [community college] where I know that we wasted massive amounts of the day because one person out of 40 had a questions because they didn't understand it just yet. . . . We didn't get through as much in Calculus because there was a bigger class and there were more people holding it up.

Ned affirmed small class sizes, stating, "So I almost feel that it is more effective to have a smaller class size where you do have some group work." Ned went from one-to-one study at home, to the University Model School, and then finally to the community college.

Ned detailed the changes in his mathematical experiences. He recounted,

I started out with the direct teaching from my parents until um about seventh grade. And then I did the University Model in eighth grade. In eleventh and twelfth grades I took Calculus 1 and Calculus 2 at [community college], oh and Trigonometry. . . . And as I got older it was more doing things with a group, too. Um, it was more of a self-motivated study. But then also I had ah to study in my own time and about once a week, Mom, a little accountability to keep me on task.

When asked about his preference, Ned was equally divided among his home instruction and the University Model, explaining, "I like both of them pretty equally." He felt he had an excellent home foundation, stating, "It fit my personality with how I get frustrated sometimes and I would still be able to come back and not be penalized . . . eventually I was able to learn what I needed to know." Ned believed the alternative University Model School experience provided a community for math, relating, "The community for math was helpful for me because not only would I sometimes get things explained to me different ways by other students, but also explain it to other students . . . it was actually real good."

During the interview, he established that mathematics was useful, sensible, and worthwhile. Emphatically he asserted, "Absolutely; mathematics is everywhere, all the time, when you are in a store, buying things. Whenever I get my tank filled up, I calculate my mileage to see how I am doing with my driving. Um, and literally it describes pretty much everything in the world. It's the backbone behind it."

Ned described his parents' attitudes and experiences with math. His dad enjoyed math. His mother taught him math in the earlier years, but as it became more involved, she handed it over to the father. Ned observed, "My dad really enjoyed math and we all knew we had to do it and appreciate for what it was. . . . So that's why whenever I was really, really having problems with it my mom would just kind of pass me onto him. She was also great though especially earlier she was with a lot too when I needed that more kind of tender touch from my mom, opposed to when I needed that tough love from my dad – that's fine but you still have to know this [math]."

Currently, Ned majors in biology at a large university over 300 miles from his home. He received a large scholarship to attend that university and performs very well.

Ned home educator survey. The data from the survey established the interview information as well as described three additional reasons for homeschooling. The mother filled in the survey prior to the student interview. The timing of the survey and the student's recent arrival home did not allow for much, if any, discussion of the survey questions. There was no direct contradiction between the interview and the survey data. Some survey answers clarified and supplemented interview information.

The survey data confirmed much of the graduate's interview. Identical items included the age of the student and the sibling information. All mathematics classes were mentioned (Pre-calculus was used for Trigonometry). The mother described the homeschooler's experience with homeschooling as "great", his experience with math in homeschooling as "adequate", and his preparation for college mathematics as "fine".

Ned's mother described reasons for starting to homeschool. These included individual learning styles, flexibility, and faith. These reasons stayed constant for the 23 years that the family continued to homeschool, until Ned, the youngest, graduated.

The mother described the math curricula, instruction, and experiences. She used self-produced worksheets and Saxon curriculum. Her homeschooling program included all the methods listed on the survey: structured, non-structured, direct teaching, reworked incorrect problems until mastery, group work, self-study, online course, combination of activities, and co-op classes. Ned completed daily math lessons. As Ned matured, there was more self-study. He liked mastering concepts on his own. This served him well at university. Later, Ned attended technical college classes.

Although Ned's mother viewed mathematics as useful, sensible, and worthwhile, mathematics was difficult for her. For support, she attended homeschool conferences, math workshops, support groups, and co-ops. When Ned had difficult math problems, he would have to wait for his dad or older siblings.

Both Ned's mother and father retired from the medical profession. Ned's mother was a RN in labor and delivery. N's father had worked as a radiologist.

NAEP assessment. Ned answered nine of the NAEP assessment questions correctly. He described his work in solving the problems for nine of the questions and justified his methods for three of the items. Fifty-five percent of the 12th grade students taking the NAEP test also answered incorrectly the same problem that Ned missed.

MSES scores. Ned scored 8.8 overall, an 8.9 on Part I and an 8.8 on Part II, ranking above the male 95 percentile (Table 10). His mother total score was 6.1, (7.4 on Part I and 4.5 on Part II), matching the female 60 percentile. The father scored 8.1 overall, (8.0 on Part I and an 8.2 on Part II), ranking near the male 90 percentile.

Appendix K: Cali

Cali interview. Interviewed in the kitchen of my home, located in the same town, 19-year old Cali willingly participated. Cali had been in my home many times visiting my daughter. In this relaxed setting, she seemed very comfortable. In the 38-minute interview, Cali answered all questions and offered extra explanations.

Cali's family started to homeschool because her parents believed that homeschooling would provide a better education. She explained, "I think my parents believed that the education was not as good in the public system as could be given at home." Success inspired them to continue to homeschool. She considered, "I was getting a good education, that they could probably keep track of my progress."

Cali's overall homeschool math experience was very helpful. It well prepared her for her college experiences, both at the community college and university levels. She explained, "It was um, very good. I'm really glad I was homeschooled. I'm really glad I was homeschooled in math." Cali described her university experience, relating,

I've taken Chemistry at [university] and have been AOK with all the math in there . . . It really is a good thing to know that you are prepared for what is coming because college is the next step. And going into college I was still so prepared for at least the Chem, you know like Chemistry basic math principles."

Her consistent and commendable SAT scores lined up with these beliefs.

Although Cali scored well on the SAT math section (660), she expressed disappointment. Her high school transcript and SAT score report verified her SAT scores (660, 650, 660). She wished to score higher, stating, "I am dissatisfied with my SAT scores in math. I feel like I should have done better, because I'm not necessarily a great test taker. I took several SAT prep classes and I took the PSAT multiple times. Um, and I guess that helped a lot."

Her scores stayed consistent. Her favorite subjects included Math and Science.

Cali liked Math and Science, but disliked English and History. She explained, "I have always been a little Math and Science buff. Ah, Math was always hard, but I understood it. But, Science was the one I really enjoyed." About the other subjects, Cali described her feelings, "And English was ah I was ok at it; I was good at it; but I didn't like it and I still don't like it and I hate writing papers. . . . History, just hated it." Despite Cali's propensity towards Math, she experienced frustration and stress with it.

Cali revealed that Math frustrated her out at times. She did not always grasp the concepts quickly, stating, "I would have a list of things in the curriculum and have a piece of notebook paper, then I would rework through them. Sometimes that was frustrating. I remember being frustrated going through them over and over again and not getting it. . . . I would stress out if something took me too long."

However, she believed that the frustration and stress was for the best, admitting,

Probably a good thing. So, we would work on them until you got it right until you understood. . . . It was quite stressful for me going through the answers not understanding and having to go over them and go over them until I got it. That was probably more stress for me but it was good because that's the way you learn."

Despite the stress of math studies, Cali appreciated her mother's helpful role. Cali's mother devoted time to math. She taught Cali, one on one. Cali described,

I did have one on one instruction with my mom. . . . I would miss half the problems and go back through and she'd be there one on one to instruct me and that's where it counted. . . . Once we worked through this problem together and she watched me [to see] where I would make my mistakes."

Mostly, this instruction involved Geometry and classes prior to Algebra. Cali clarified, "I guess for Algebra. . . . I didn't really need my mother's help. . . . With Geometry she helped me a lot. . . . She really helped me with the proofs." Although Cali's mother did not teach her Pre-Calculus, she researched the problems. Cali stated,

My mother actually researched other people who had the same problems and found like here are some problems that you probably should do. And then you don't have to do the rest of them. . . . She did lots of research. So she would be like we're not going to do this chapter, cause it's not relevant to what you need."

Her mom checked her work daily, requiring Cali to rework problems. Cali remembered,

Everything was checked that day and redone. . . . It was very fresh. . . . After I had done everything go back and check everything and then after everything was done, I was allowed to go. Must work before you play."

At times, homeschooling strained the mother-daughter relationship; Cali remembered,

"Maybe I didn't always enjoy being at home all the time cause sometimes it's a little trying on relationship, especially with my mother. . . . I will be eternally grateful."

Cali most vividly remembered her pre-calculus course.

Cali's recollection of pre-calculus clarified her mathematical learning environment. She described,, "Pre-calculus I took using a Chalkdust video. So, there was another teacher . . . my mother, I did that myself, almost, if she checked my work. But even when she checked my work, if something was like in a fraction and it should have been in a decimal or it doesn't matter she would circle it; that's not really wrong, different format. So, I pretty much taught myself using that curriculum. I taught myself pre-calc. . . . The videos could be 45 minutes long, which is substantial. So, I think I began figuring out that I could skip the videos and taught myself from the book. If I recall correctly, I think I remember doing that secretly, just skipping the videos and trying to do the problems myself."

She attributed her success, in part, to this course, stating, “I’m really glad I took Chalkdust Pre-Calculus. I really, that, that is what helped me. . . . I learned more about Trigonometry than I did from . . . a tech level trig class.” Although challenged, Cali learned a lot through all the effort. She reflected,

It was a very hard course, and it took a lot. . . . I remember one day I worked from 8 am, took an hour for lunch ‘til 3:30 or 4, consistent, not all Math of course, but like Math was what caused it to be so long . . . but definitely, [I] learned a lot.

Pre-calculus prepared Cali for her technical (community) college math classes.

Cali took College Algebra and College Trigonometry. She claimed,

It says something that I could go to a tech college and not learn anything. . . . I may have had one week at the end of each semester where I may have learned a little bit. But everything else was review.

However, Cali appreciated her community college math classes, explaining, “I went to [community college] for my two classes, which was very beneficial. I’m really glad I did that and then I went off to college.” She learned better in a classroom setting, explaining, “But I do learn better in a classroom listening and taking notes, than reading from a book. . . . Basically, that is the way I learn I do learn in the classroom.” However, homeschooling prepared for her college experiences, stating, “But the good news is that I can learn from a book if I want to and that is something that I definitely got from being at home.” Her transcript verified that Cali completed Algebra 1 and 2, Geometry, Pre-Calculus, College Algebra, and College Trigonometry, earning grades of 90-96.

Cali expressed that mathematics was useful, sensible, and worthwhile. She explained, “I’ll be going into Biology. I was considering majoring in Chemistry and it’s all over the place.” Math made sense to her; she claimed, “I have a math head; so it all makes sense to me.” Cali believed math to be worthwhile, commenting, “Very much, it’s everywhere. . . . There’s lots of good jobs out there that have math involved in them. So, definitely worthwhile.” Her parents shared her attitudes.

Both her parents held positive views towards math. Her dad used math-related principles, keeping track of company expenditures. She elaborated, “My dad does like financial stuff for a company. . . . He has to deal with very large numbers and balance out on all the little tiny things.” Her mother possessed basic mathematical skills and Geometry knowledge. Cali speculated, “She still can do her basic math. . . . Geometry, she was very good at it and you know Algebra she’s good at.”

Currently, Cali majors in Biology (and possibly minoring in Chemistry) at a prominent state university over 100 miles from her home. Receiving a substantial scholarship, she continues to earn very good grades. After this interview, Cali emailed her mother, thanking her for training Cali to finish schoolwork, “before she played.”

Cali home educator survey. The survey confirmed the interview information, as well as provided additional details concerning the curriculum utilized and the reasons for homeschooling. The primary teacher, the mother filled in the survey prior to the graduate interview, not discussing the questions with Cali. No direct contradiction existed between the interview and the survey data. Some answers supplemented interview information.

Survey responses corroborated interview data. Identical items included the student’s age and sibling information. The educator listed all home mathematics classes. She described Cali’s experience as: “the homeschooler liked to be home”, the experience with math in homeschooling as “adequate”, and Cali’s preparation for college mathematics courses as “still has not taken Calculus at [university].”

Cali’s mother described two main reasons for starting to homeschool. Christian education was important and she added, “public school was inadequate for educating our children.” She continued to homeschool because homeschooling was the best education and that she could impart “effective Christian values. Impress them on your children.” Cali’s mother clarified some instructional aspects.

The educator detailed their progression of mathematics instruction, curriculum, and experiences. She described it as: “structured when young [with] direct teaching” and “older, self-directed DVDs”. Over the years, she required daily math time and stressed the basics, explaining, “We spent much time on basics, much repetition of math facts—made sure she got foundation—worked hard on math daily. We repeated if she didn’t understand.” Pleased with all their curricula, she detailed, “Saxon, Singapore Math, Jacobs Geometry, Chalkdust—PreCalc, all were excellent in explaining and developing foundation.” She discussed changes in curriculum, writing, “We changed curriculum when something better was available.” Important to her, mastery and consistency were stressed. She reported, “Always reworked problems that got wrong – everyday. When younger, if we finished a book, we continued on to the next. We did not stop because we were at the end of the book.” Personal interactions decreased as the homeschooler became more advanced; Cali’s mother remembered, “Personal interaction became less as she advanced. In elementary years we worked closely one on one. As she matured, she became more independent – less interaction. No interaction with sibling.”

Cali’s parents exhibited positive mathematical attitudes. Her mother viewed mathematics useful for science. She liked math, working as an industrial engineer. Cali’s father worked as the controller in a defense company.

NAEP assessment. Cali answered eight of the NAEP assessment questions correctly. She described her work for all ten questions and justified her methods for all ten items. Comparing her answers with the 12th grade sample taking the assessment, forty percent of them missed the first one and 74% missed the second.

MSES scores. Cali scored 7.6 overall, a 7.8 on Part I and a 7.4 on Part II, ranking above the female 90 percentile (Table 10). Her mother’s total score was 7.2, a 7.9 on Part I and a 6.4 on Part II, ranking above the female 80 percentile. The father scored 8.4 overall, an 8.3 on Part I and an 8.4 on Part II, near the male 95 percentile.

Appendix L: Sue

Sue interview. Sue, 22 years old, was interviewed for 26 minutes in the kitchen of her rented house, located in a middle class subdivision. She seemed very comfortable, being open and honest without any sign of nervousness or hesitation. The interview took place after dinner. Sue answered all questions and offered extra explanation when prompted. Academics provided the main reason to homeschool. According to Sue,

I felt like I did not receive the attention that I needed to um, I guess get the best out of my schooling. . . . That was the main one that the quality of the education and my mom wanted to be able to tailor my education to my needs and my level.

Her family continued to homeschool because of its flexibility. She perceived,

It was a really good fit for my family. . . . In high school I could have gone to public or private school if I had had the choice; but I rode horses professionally, while in high school that afforded me a flexible schedule so that I could complete my schooling and pursue my interests and my hobbies.

Sue described her math instruction. In the younger years, her mother provided direct teaching. Sue recalled, “She, when I was younger, would sit down with me for my lessons and help teach them to me. I always had a list and a schedule of things I needed to complete throughout the day.” Sue became more autonomous; she related, “As I got older and more independent, I taught most of my lessons. . . . At that point [seventh or eighth grade] I was probably more independent and taught the Math on my own from a textbook.” Mastery learning was the norm. Sue emphasized, “Incorrect problems were reworked. Potentially, if I was only working even numbers and I had a trouble spot where I consistently missed a problem of the same type, I could also back and work the odd numbers.” Her parents helped her when needed. She explained, “If I had any questions or any struggles, uh, my mom was certainly there to help. . . . If there were some trouble spots, my mom or my dad would step in for some direct teaching.” Sue used Saxon, stating, “We used Saxon Math.” Sue’s homeschool math prepared her for college.

Sue’s homeschool math experience benefitted her college career. She expounded,

It was a positive experience. I think it prepared me well for college. Ah, I felt capable and proficient in all of my college math cases. That I had the tools I needed to succeed . . . the basics of Algebra and Geometry.

These basics carried her through her high school and college math experiences. During high school, she enrolled in community college math courses and was successful despite a gap in her math classes. She recounted,

I had possibly a year, year and a half gap from the time I finished my last Algebra class to when I took Trigonometry at [community college] and that was a difficult class for me because I had forgotten a lot of my math and my Algebra basics. Um, however, I did work hard and ended up . . . with an A and doing well.

This situation was repeated at the university level, but again Sue prevailed. She disclosed,

The same with the first one. I had been out of Math for about a year and a half and it was just an adjustment period to get used to it. I did make an A in the course. I took a Calculus 1 and 2 as well as a Business Statistics class at [university].

Her high school transcript verified her high school math classes of Algebra 1 and 2, Geometry, College Trigonometry, and College Calculus, as well as her SAT scores.

Practice and well-rounded education contributed to her good SAT math score. Her score (620) ranked in the top 25% (College Board Website). Sue thought the SAT math simple; she felt well prepared, indicating,

I thought that the SAT math was pretty simple. Um, nothing was too complicated and I do think that I had a well-rounded math education. . . . My mom did have me work through the Princeton Review practice.

She took the SAT in the seventh grade and then later on, explaining, “I took the SAT three times. Once in seventh grade with the Duke Tips program and once sophomore and possibly junior year, scoring highest my sophomore year, probably when Math was the most fresh.” Her verbal score (660) confirmed that she performed well in other subjects.

Sue equally enjoyed Math, Science and English elaborating, “I am pretty well rounded. I probably equally enjoyed, um, English as well as Math and Science. I was kind of equally interested in everything.” When asked about her least favorite subject, Sue commented, “Possibly History.” Although she enjoyed working on her subjects, the amount of time that math work required frustrated Sue.

More structured than other subjects, mathematics required a consistent time commitment. It was difficult at times, but ultimately good for her. She remembered,

Math is probably more structured . . . an hour to an hour and a half daily. . . . I would think that was consistent. . . . Sometimes I didn’t like it. . . . I remember not enjoying my math. I remember sometimes the lessons taking a long time. I didn’t enjoy that, but I’m sure that it was good for me. So, I don’t have any regrets. . . . I don’t know that I would change anything.

Sue expressed a positive attitude towards mathematics.

Sue agreed that Math was useful, sensible, and worthwhile. She explained,

It’s just good to know the basics. I do Math in my head. . . . I still have the math basics that I learned in high school and were reiterated um, in my college curriculum. Yes, and that’s what I use. . . . It’s always worthwhile to be more educated and to have more knowledge.”

Similarly, her parents valued mathematics.

Sue’s parents exhibited positive attitudes towards Math. Her dad worked in accounting. The driving force in their homeschool, the mother promoted learning. Sue remembered that, “My mom was always very adamant about

everything in my homeschooling. That everything was good and necessary.” She perceived that her dad agreed, recalling, “and my dad supported her.”

Sue works as a Director of Marketing. She graduated early and obtained a Bachelor’s degree in Marketing from a prominent public university, 125 miles from her home. Sue received substantial scholarship money and earned very good grades.

Sue home educator survey. Generally, the survey confirmed the interview information, and provided additional details concerning the reasons for homeschooling and the curriculum choices. The mother filled in the survey prior to the graduate interview. She indicated that there was not any discussion between them concerning the survey or the interview questions. Minor differences existed concerning favorite and least favorite courses and curriculum names. They supplemented interview information.

The survey data coincided with much of the graduate’s interview. Identical items included the age of the student and the sibling information. All the mathematics classes taken at home were listed. Her mother acknowledged Sue’s love for horses and described Sue’s homeschool experience as “[she] wanted to finish her school day early to take part in an activity that she loved, horseback riding.” As did Sue, her mother acknowledged the struggle that Sue had with Math at the community college, writing, “she feared the first math at [university], Calculus (2nd semester) because the 1st semester at [community college]. She did fine – A, I think.” Her mother confirmed the reasons for the good SAT math score, specifying, “We spent much time on basics, Much repetition of math facts. Made sure she got foundation. Worked hard on math daily. We repeated if she didn’t understand”. Instruction was tailored to Sue’s needs.

Sue’s family started to homeschool for two main reasons. A Christian education was important and the “public school was inadequate for educating our children.” She continued to homeschool because homeschooling was the best education and she could impart “effective Christian values. Impress them on your children.”

The educator detailed Sue’s math experience. She described it as, “structured when young [with] direct teaching . . . older, self-directed.” Satisfied with the curriculum they used, she listed, “Saxon, Singapore Math, Jacobs Geometry. All were excellent in explaining and developing foundation.” She discussed changes, writing, “We changed curriculum when something better was available. No DVDs but went from homeschool to [community college]”. She stressed mastery and consistency, reporting, “Always reworked problems that she got wrong—everyday. When younger, if we finished a book, we continued on to the next. We did not stop because we were at the end of the book.” She remembered, “Personal interaction became less as she advanced. In elementary years we worked closely one on one. As she matured, she became more independent—less interaction. No interaction with sibling.”

Sue’s parents held positive attitudes towards math. Her mother explained that she liked math and previously worked as an industrial engineer. Sue’s father worked in finance, the controller in a defense contracting company.

NAEP assessment. Sue answered eight of the NAEP assessment questions correctly. She briefly described her work for all ten questions and justified her methods for all ten items. Comparing her answers with the 12th grade sample taking the assessment, forty percent missed the first one and 38% missed the second.

MSES scores. Sue scored 7.6 overall, a 7.5 on Part I and a 7.7 on Part II, ranking above the female 90 percentile (Table 10). Her mother’s total score was 7.2, a 7.9 on Part I and a 6.4 on Part II, exceeding the female 80 percentile. Her father scored 8.4 overall, an 8.03 on Part I and an 8.4 on Part II, placing near the male 95 percentile.

Appendix M: Ela

Ela interview. Interviewed in the kitchen of her parents' home, located in an upper middle class subdivision, 19-year old Ela willingly participated. The 41 minute interview took place before lunch. While her mother and siblings quietly moved throughout the house, Ela provided detailed and complete answers to all questions.

Ela's parents started to homeschool for religious reasons, to impart Christian values. She explained, "Because primarily for religious reasons . . . they didn't want to send me away all day and they just felt that they could instill not only education information but also moral values that they wanted me to have." They persisted with homeschooling because they felt it to be successful, at least as good as conventional schooling. Ela related, "Since my parents have obviously had success in going from homeschooling to a college outside of homeschooling, we've found that educationally it prepares us just as well as a private or a public school would have." About the spiritual aspect, Ela added, "While there were some things I had been sheltered from and hadn't been exposed to in a public school, um, I was prepared and mature enough spiritually to face things that I came into contact with." She described their Christian values as,

We are Christians. . . . The foundation of our faith is that Jesus Christ is our Savior. We believe in the inerrancy of the Bible and um, we base everything that we do and everything that we learn in all areas of academia, on the Bible.

Faith fueled her academic motivation; Ela declared, "At this point, school is a part of my faith. It is my joy. I love to learn. I love to study. I love to help others learn and study."

Her attitude towards education transformed in middle school. She related,

Most of through elementary school, I was just your average student. . . . It wasn't until eighth grade, um, when I became very serious about my faith, and took it on as my own. Not just as something my parents held to. . . . It was kind of a turning point and I decided I really needed to take my studies seriously.

Homeschooling nurtured learning, with Chemistry and reading. She recalled,

My experience has been that it fostered um, a love of learning because I was able to focus on things I really loved. . . . I always loved reading. . . . In sixth grade there was a unit in my science book and it was Chemistry and I was just absolutely fascinated by the four models of the atoms. . . . When I got into ninth grade there was a module in my Apologia Biology book called the Chemistry of Life. And from that point on I knew I am going to major in Chemistry. . . . I love reading my science textbooks, so the two kind of fit into one.

Her love of Chemistry became the catalyst to a great interest in mathematics.

In her early years, Ela initially struggled with Math and disliked it, changing her mind only when she understood its importance in Chemistry. She revealed,

[I] really struggled with Math when I was younger . . . Then, kind of around seventh or eighth grade time when I decided I loved Chemistry and then in the ninth grade when I confirmed that, I started to see the usefulness of Math. I guess what had bogged me down in the earlier years was I didn't see how it applied to real life. . . . I didn't see the application to bigger picture life problems. Um, and when I started taking Chemistry and Physics in high school I realized just how useful and how essential it is and how central it is to everything.

She completed all of her math classes at home.

Ela's math instruction started with direct teaching from her mother and progressed to self-study with tutoring from her father. Ela recalled, "Elementary, direct teaching by my mom. She would read through the lesson, like I said. Very structured, daily lessons, tests after several lessons." Her mother ensured mastery learning by the reworking of problems until they were correct. Ela remembered,

When I got things wrong we wrote the problems that we got wrong at the top of the page and we'd go back and rework the ones we got wrong until they were right and sometimes that would take two or three times. That was useful.

As she matured, her parents trained her to become more independent.

Her transformation to independence started in seventh grade, with participation from both parents. Her mother still remained involved. Ela detailed,

That transition occurred somewhere in the late elementary and middle school years where I'd go from being taught by my mom to being self-taught. . . . I think the biggest part of that was my parents' desire for me to become self-motivated and self-taught. . . . Then, sixth through twelfth grade she just would have me read the lesson by myself, do all the practice problems and all the exercises by myself, and then she would check them for me.

Her father's tutoring helped change her attitude towards math. Ela related,

My dad starting helping with Math in the evenings a bit more, he would tutor me through some things I had trouble with, had trouble with finding the slope of a line for the longest time when it was introduced in seventh and eighth grade. . . . So that changed my attitude towards it. Obviously, this was very important and not just something I need to do. . . . That's how the tutoring kind of changed me.

Ela described her mathematics experience and the positive effect of the Saxon curriculum. Her textbooks stressed basic skills with repetition. During high school she mastered algebra through geometry, followed by pre-

calculus and calculus. ABeka was used for Kindergarten through third grade, followed by Saxon for the rest of her homeschool career. She remembered,

Kindergarten through third grade my mom used the ABeka textbooks . . . In fourth grade, we get switched to Saxon because my mom feels like the repetition and the constant review of older problems is really helpful. Um, and definitely for me, I learn by rereading things and the constant repetition.

Ela's dad became more involved during her Algebra 2 class. She recalled, "Beginning in Algebra 2, my dad would start checking the lessons." Ela devoted much time to her math study. She recollected, "Ninth, tenth grade I would say the Saxon math lessons that I did every day would take me two to three hours. . . . Pre-Calculus and especially Calculus those lessons would take me anywhere from three to four hours." Her high school transcript verified that Ela completed Algebra 1 and 2, Advanced Math (with Geometry and Trigonometry/Pre-Calculus), and Calculus, as well as confirm her SAT scores. Ela credited Saxon and her preparatory work for her SAT math score (620). She asserted,

My parents did provide me with the Princeton Review SAT study book. I didn't take any courses. . . . It's just from drilling foundational principles all through ah, my education. And I feel like Saxon math did a really good job of doing that constant repetition. Ah, that has been really helpful for me.

This mathematical background served her well in her college career.

Overall, homeschooling was very beneficial to Ela's college academic preparation. She noted two big advantages and one disadvantage, declaring,

The big advantage would be just that independence, that's sooo necessary in college. Um, and the other big advantage . . . was having a parent who was had been taught upper level math and was able to work through things with me and explain them in different ways than the textbooks. I feel like if I had been in public school, I may not have been able to get through Calculus, just because I needed so much help addressing things one-on-one. And once I understood those foundational principles, I was able to soar in college. . . .

Um, disadvantages, I guess, would be the lack of group work. . . . I think it would have been a good thing to be able to build those group study skills.

Despite the group work limitation, Ela felt very well prepared to learn at college, stating,

For the most part not only have I been equal to the level academically with other people I come into contact to/with in college. In some ways I was even more prepared academically just as far as learning on my own. I'm very much an independent learner. When professors ask you to read the textbook they are not going to lecture on everything, I am completely capable of doing that, where a lot of people have been spoon-fed all the information.

Algebra principles provided a good foundation for chemistry and physics courses. She affirmed, "I feel like that was excellent preparation for college, because in chemistry or physics you are not just referring back to calculus problems, you are referring back to algebra principles that you need to remember."

Emphatically, Ela answered that mathematics was useful, sensible, and worthwhile. Critical to biology, chemistry, and physics, she maintained,

It is so fundamental to everything, and I love to listen to my dad talk about it. How math pretty much will replace biology, chemistry, and physics because we're going to be able to explain everything with numbers. I already have a calculus professor at {college} who does biological research using math and can explain things quantitatively. Um, I did research with a professor two summers in a row at [college] and it's all on the computer. . . . So, math's behind it all.

Both her parents demonstrated positive attitudes about math. Ela recognized,

He and my mom have instilled a love of mathematics in all of us . . .

The one-on-one attention, I mean, obviously just showed me, it was obviously important not only to my parents but obviously something important overall that as a life skill that Math was very important and that my parents saw something fundamental to my education, and to my development as a person, as a whole.

Her mother's experience, somewhat mirrored her own, Ela related,

She is very much like me, probably in her younger years. Um, had a little bit trouble with math. . . .If she had had personal one-on-one attention and a better teacher earlier she would have started enjoying math. I feel like I can really relate to her in that. So, once she started getting it in high school, she loved it and wished she could have gone farther. She got her Bachelor's degree in Education.

Her father excelled with math; Ela expounded,

My dad always loved math. He did extremely well all through school: went to Naval Academy, worked on nuclear submarines. So, he's used Math extensively. . . . His undergraduate degree is in Mathematics. Um, and then he went on and actually saw applications of that in Economics. . . .He loves Business. He currently works at [large bank] and crunches numbers on the computer all day.

Ela admired her father's achievements and appreciated her mother's teaching ability.

Currently, Ela attends a private South Carolina college, about 130 miles from her home. She maintains a prominent academic scholarship. She has taken four calculus courses, earning all As. Ela has worked in research with a chemistry professor for the last two summers, working on computer models of molecules. She has substituted for one of her chemistry professors, while the professor had maternity leave. Recently she has been inducted into an academic society for her excellent grades.

Ela home educator survey. Ela's mother, the primary educator, filled out the survey prior to the interview. Similar in detail, the two forms of data differed structurally. Ela provided rich commentary, while the survey consisted of succinct phrases. No direct contradiction existed. Survey answers clarified and supplemented interview information.

The survey confirmed much of the interview. Identical items included the ages, the sibling information, the mathematical classes and curriculum, and the student's least favorite subject, French. Similar to Ela's responses, her mother explained the homeschooler's experience as "positive, experience; nurturing environment; glad to be homeschooled; was ready for college," her math experience as "self-motivated and taught. Very successful; tutored by Dad weekly; good grasp of concepts. Good problem solver," and her preparation for college mathematics courses as, "In her own words, 'Nothing I hadn't seen before. Saxon was an excellent preparative curriculum'".

This family started homeschooling for spiritual training and continued because of their success. Ela's mother wrote, "We wanted to include spiritual training/guidance along w/ academics. We also were concerned about the possibility of values contrary to our faith/values being taught." The parents gained confidence in their homeschooling and continued. The mother expounded, "With a daughter beginning her 3rd year of college, with great success, we have gained confidence in homeschooling. We have Results!"

The educator reported the math experiences. The curricula included, "K-3 ABeka, 4-12, Saxon". Instruction and experiences involved, "structured curriculum; self-study; tutoring". Personal interactions occurred, "(K-5) one on one, teacher taught (6-12) self-study/tutor interactions/conversations – mostly enjoyable, not stressful". She did not mention Ela's dislike of math during the elementary school years. Ela's mother wrote that there were "no changes" in Ela's instruction during the homeschooling years.

Ela's mother described her attitudes about mathematical instruction. She advised, "Make sure you build a firm foundation early on. Drill math facts. Repetition is essential—Saxon does this well. Be consistent. Best to stick with one curriculum to avoid gaps." This good foundation helped Ela score 620 the SAT math section. She wrote, "good foundation, consistent study in courses taken, studying the practice problems in the SAT book". This information aligned with her daughter's replies.

Her mother credited the father. She explained, "He was a math major, served on a submarine where he had to qualify as an engineer. I really credit him."

NAEP assessment. Ela answered all of the NAEP assessment questions correctly. She described her work and justified her methods for all items.

MSES scores. Ela scored 8.7 overall, an 8.6 on Part I and an 8.9 on Part II, ranking above the female and male 95 percentile (Table 10). Her mother's total score was 7.2, an 8.4 on Part I and a 6.5 on Part II, between the female 80-90 percentiles. The father scored 8.6 overall, a 9.0 on Part I and an 8.3 on Part II, surpassing the male 95 percentile.

Appendix N: Ola

Ola interview. Interviewed in the kitchen of her parents' home, located in an upper middle class subdivision, 18-year old Ola willingly participated. The 32-minute interview took place before lunch. While her mother and siblings quietly moved throughout the house, Ola answered all questions and offered extra explanation.

Ola's parents started homeschooling to respond to God's calling. She supposed,

I believe that the original reason really that they started to homeschool was they felt that God was calling them to do that to raise their children in the home, being home educators. And I think that they believed that that was the best way for us to receive a good education. . .academically as well as grounded spiritually.

Her faith grew out of this environment. She elaborated,

Your religion, whether you want it to or not, really affects the way that you think about everything. So um, having that grounding in the Bible with my parents integrating that as well. I think that that was really good for me and um, it just helped me to grow in grace as well as in knowledge. . . . I am a Christian.

They continued to homeschool to stay grounded in their faith. Ola perceived,

I think that one of the main reasons that we keep homeschooling is that it's just it helps us to remain grounded in our faith. . . . I think it has really turned out well for us having [Ela] go off to college and come back and tell us how you know, well she's doing and how she thinks homeschooling helped her with that [faith].

Ola treasured her homeschool experience. She valued its flexibility, exclaiming,

I love it! I would not have survived without homeschooling. Um, from a personal aspect, I'm not a morning person. So the flexibility there is great. Flexibility also with scheduling, really, really great, where you can have that flexibility to go on a field trip or visit family, but take school with you; it's not like you stop.

She appreciated working on her own schedule, revealing, "When I got into the upper grades in high school, um, I would do math at night because I'm a night person . . . that's when my brain sort of starts functioning." Her mother inspired Ola. She reminisced,

I have loved it, and having a parent who can empathize and just really cares about making sure you do well. And I'm sure there are teachers in the public and private institutions that do care; but knowing that my teacher loves me and cares about me and wants me to succeed has been really great for me.

An especially enjoyable part of her homeschooling revolved around math.

Excitedly, Ola related how math was fun, like a puzzle to her. Her mother kept them doing math during the summer months; she affirmed,

I love math. I really do. I, over the years, my mom would keep us doing math during the summer and there would be supplemental questions in the back of our books. And I would just, I loved doing it because it came easily and so it was kind of like solving puzzles. . . would do it for fun if I could.

All Ola's homeschool math classes took place at home. Her instruction started with direct teaching from her mother and progressed to self-study with tutoring. K-5, Ola's mother provided structured instruction, ensuring mastery. Ola remembered,

In the younger years, she would read the lesson to me. Um, probably did that up to fourth or fifth grade, maybe sixth grade, would read the lesson and I would do it. But after that, definitely after sixth grade, I started doing it by myself.

Describing her self-study and parents' assistance, she recalled,

I am self-taught, basically. Um, I would read the lessons and I would do the problems. My mom would correct the lesson and if I had problems I would either go back to them myself. If I couldn't solve them myself again, I would either go to my mom and she could figure it out. Or if she didn't know how to do it, I would go to my dad, which had to be scheduled 24 hours in advance.

Her dad would help at night, after work. She remembered, "Occasionally, at dinner we would bring up I need you to go over Math with me tonight and he'd do it." Early on, Ola sat at a table with her mom and later in various places. She reported, "starting out we would do it at our school table . . . middle school, high school, I started doing my school in my room or just anywhere throughout the house where I could find a comfortable place to sit." Additionally, Ola spent some time with her siblings doing math.

Occasionally, there was sibling interaction with Ola and math. Ela, her older sister helped her with calculus. She remembered, "[Ela] occasionally would help me with calculus problems if she was home from college and my dad's at work." At times, Ola would help her younger brother and sister. She remembered, "Um, and then I would help my siblings if my mom was out doing something and she just needed me to read the lesson or help them go over a problem". Ola spent different amounts of time with math.

Ola spent from 30 minutes to 2 hours on her math work. She recounted,

It may be used to take me an hour to do a math lesson, tops, maybe 30 minutes. And then, as I got into advanced math and calculus, it would take me anywhere from an hour and a half to two hours to do a math lesson.

Ola positively portrayed her mathematics courses and curriculum. She reported the classes, itemizing, "I took basic math until middle school, high school, I got into algebra. Um, I did like a pre-algebra, algebra 1/2, algebra, algebra 2. And then I got into advanced mathematics, and then pre-calc and calc". Her high school transcript verified

that she completed Algebra 1 and 2, Advanced Math (with Geometry and Trigonometry/Pre-Calculus), and Calculus. Her Saxon curriculum integrated geometry and trigonometry within the algebra and advanced math classes. She clarified, "The geometry was, as well as trigonometry was sort of um, combined with the advanced mathematics and algebra. So, it sort of brought along in those subjects. So, I did not take a specific geometry or specific trigonometry course. It was integrated." Ola enjoyed the repetition in her math textbooks, explaining,

I've always loved math. When we started out, we used the ABeka curriculum. . . . It was really good for us. Ah, to give you a foundation in math. There was lots of repetition, which as a musician for me, it's very important. That's how I learn . . . fourth and fifth grade we moved into Saxon math, which uh, I absolutely love, lots of repetition, there as well. No frills, just basic math due to the concepts laid out in the lesson and you can do the problems. Um, so we did Saxon and have done that through my senior year of high school. So, that's been great. . . . I really have loved what we've done with Saxon and it's worked well for me.

Ola credited her strong math background, along with some preparation, for her good SAT math score (660). Her high school transcript verified that score. She reported,

My approach to math is. . . if I can reason it out, I just feel confident with that. . . . I think it is because I had such a strong background with all that repetition throughout the years; [it] really grinds the concepts in your mind of how to solve those problems. Um, I did do an SAT study book. . . . I could see how the SAT is more of a 'how you do it' instead of 'what you know'.

Overall, homeschooling was beneficial to Ola's college academic preparation. She felt well prepared for her music major, asserting,

I think I'm going to be very well prepared, especially as a music major . . . math is really sort of the foundation of music in of itself. Um, you can't write music without mathematical structure although they try. Um, so, I mean, I think that that's just going to really help me in theory classes.

Ola was open to taking more math classes in college, declaring, "I can definitely see myself taking more math classes and possibly minoring in math or double majoring." She offered only one disadvantage, that of daily instruction. Ola contemplated,

I think that if, if at all the only con would be having that um, the instruction, having it explained to you being able to ask questions. You can't ask a book questions . . . the only con, instruction, daily instruction.

Still, Ola commended her homeschooling activity, asserting, "I've had a really good math experience. I think that the curriculum leaves um, really does prepare you well. I've heard [Ela] say, for the umpteenth time, how much Saxon prepared her for calculus." Pleased with Ola her homeschool experience, Ola would change, "Absolutely nothing".

Ola explained how mathematics was useful, sensible, and worthwhile, with budgeting, shopping, and music. She detailed,

Definitely. I mean in a store, that was one of the questions on the survey, being in a store. Being able to calculate in my head how much basically the discount would be, how much sales tax will be . . . doing your budgeting. . . . I remember watching, I think it was an adaptation of a Charles Dickens movie book. And this man says something about if your income is 20 and your expenditure is 20, that is not good. . . . It just helps you to make wise choices in your life. . . . Definitely worthwhile; knowledge is never wasted . . . leads to a just ah, better life.

Math correlated with music; she affirmed, "I think it's so useful as a musician. . . . A definite correlation and the more and more that I teach music, the more and more I see it and I get really excited." Her parents affirmed her view of math.

Math was important to her parents. A certified teacher, her mother worked in an elementary school. Ola shared, "My mom majored as a teacher; so, she had to have math for that I believe. And um, I mean she's really been able to help a lot." Her dad extensively used Math in his training and occupation. Ola reported,

My dad [is] definitely, [a] math, science genius. Um, definitely has been able to help me with that, because of that. And um, I think it really enabled him to do well in his schooling. Um, both at the Naval Academy and at Oxford and Duke. . . . He works with mortgages. . . . He's done risk management.

O highly recommended homeschooling. She advised,

I think everybody should homeschool. . . . When I have kids, I'm going to homeschool them. . . . It's such a wonderful way to learn. . . . Even if your kids aren't homeschooled technically they should be schooled in the home on other things and homeschooling really just sums all of that up.

Currently, Ola is a freshman at a private South Carolina university, 140 miles from her home. She has an academic scholarship and is doing very well.

Ola home educator survey. Ola's mother, the primary educator, filled out the survey prior to the interview. Ola provided detailed information, while the survey data consisted of succinct phrases. No direct contradiction existed. Survey answers clarified and supplemented interview information.

The survey confirmed the interview. Identical items included the ages, the sibling information, the mathematical classes and curriculum, and Ola's favorite subject, math. The graduate's experience with homeschooling was "positive, experience; nurturing environment; thrilled to be homeschooled; was ready for college; not hesitant socially," her experience with math in homeschooling as "self-motivated and taught. Very successful; Tutored by Dad weekly; good grasp of concepts. Good problem solver."

They started homeschooling for spiritual training and continued because of success. The mother wrote, "We wanted to include spiritual training/guidance along w/ academics. We also were concerned about the possibility of

values contrary to our faith/values being taught.” She stated, “With a daughter beginning her 3rd year of college, with great success, we have gained confidence in homeschooling. We have Results!”

The educator reported the math experiences. The curricula included, “K-3 ABeka, 4-12, Saxon.” Instruction and experiences involved, “structured curriculum; self-study; tutoring.” Personal interactions occurred, “(K-5) one on one, teacher taught (6-12) self-study/tutor interactions/conversations—mostly enjoyable, not stressful.”

Ola’s mother described her attitudes about mathematical instruction. She advised, “Make sure you build a firm foundation early on. Drill math facts. Repetition is essential – Saxon does this well. Be consistent. Best to stick w/ one curriculum to avoid gaps”. This good foundation helped Ola score 660 on the SAT math section. She wrote, “good foundation, Consistent study in courses taken, studying the practice problems in the SAT book”. This information lined up with Ola’s responses.

Her mother credited the father. She explained, “He was a math major, served on a submarine where he had to qualify as an engineer. I really credit him.”

NAEP assessment. Ola answered all of the NAEP assessment questions correctly. She described her work and justified her methods for all items.

MSES scores. Ola scored 8.1 overall, an 8.4 on Part I and a 7.6 on Part II, ranking above the female 95 percentile (Table 10). Her mother’s total score was 7.2, an 8.4 on Part I and a 6.5 on Part II, between the female 80-90 percentiles. The father scored 8.6 overall, a 9.0 on Part I and an 8.3 on Part II, exceeding the male 95 percentile.

Appendix O: Tad

Tad interview. Interviewed in the kitchen of his home, 21-year old Tad willingly participated. He appeared very open and honest, without any sign of nervousness or hesitation. The 48-minute interview took place mid-afternoon. While his mother worked in the kitchen, Tad answered all questions, offering extra explanations.

Religious and academic reasons motivated Tad's family. He explained,

Mostly, I think probably religious reasons. They wanted us to be taught through like a religious system of education, a Christian one as opposed to going to public school. . . . Part of that had to do with the fact around this area there are not great public schools. . . . But I think mainly it was the religious convictions.

Academic success encouraged them to continue. He specified,

I think the reason why they keep homeschooling everyone is because they've seen like there is a lot of success I think academically with it. Um, I think kids who are homeschooled tend to do better. I think that shows on the SAT scores and stuff.

T believed that his homeschool experience worked very well. In addition to good preparation for college academics, he enjoyed extracurricular activities; he explained,

I was very well prepared. . . . It was broad. . . . It was really good, . . . I could do what I wanted . . . a wide range of things. I had a music teacher; I swam for a swim team. . . . I ran for a like cross-country team; played soccer for a recreational team. So, kind of like a hodgepodge of things. . . . I did the homeschool debate. So, I did that for four years, I think; and that was something that I really, really enjoyed. . . . I was a musician. I competed in guitar.

He appreciated the flexibility of the homeschool schedule. Tad elaborated,

If I was in school, I would have all sorts of problems with attendance policy and stuff like that. But being homeschooled, I could you know, do what I wanted and . . . I was self-motivated because of it. I think it worked out well.

Tad talked about his favorite and least favorite classes. Specifically he enjoyed physiology and math, stating, "Physiology was like the only one [of the sciences] that I kind of liked. . . . I really like math, too." He also enjoyed writing and debate,

Writing became one of my favorite things. . . . [A] unique area of learning that I got to participate in, doing like debates and going around to lots of different tournaments and competing. And so that was another huge part of my life, in my last four years of high school. And so that was one of my favorite things.

Tad did not like French or a lot of the sciences; he revealed, "I never really liked French. French was kind of boring to me. . . . I didn't really like a lot of my sciences. I thought Physiology would probably be the sole exception under like the natural sciences."

Tad positively portrayed his homeschool math experience. He explained, "It was more than adequate. My mom was a math teacher . . . Your parents' favorite subjects are taught the most." He participated in many math competitions, describing, "So I was taught a lot of Math and I competed in several different math competitions in the south". He described it as a push from his mother and a pull from himself, relating,

It was kind of like a push and a pull. Like I pulled myself into because I liked it and I was kind of pushed into it by my parents because, I mean, my mom really liked math, too. But I was very eager to learn and something about the numbers and the math was just like all fits nicely and it's very understandable.

He related homeschooling to the economic principle of free markets, stating,

But I think homeschooling like is really, to a certain degree, is an ideal way kind of education. Like it's a free market solution to do whatever you want. So like it's personally tailored to every single one of your kids. . . . I wouldn't say that it's necessarily right for everybody; but, it can work really well in the appropriate environment . . . the mom's able to teach well. . . and can work well with the kids.

In the early years, Tad worked with basic math concepts. He remembered,

I mean as far back as my memory could go, I remember like sitting at that table and like adding penguins. It's like one of the first things I can remember. . . . I thought that was hilarious. . . . I was like 3 or 4. . . . [Later] I think we did workbooks, like very simple workbooks, up until I was probably like, maybe 9 or 10 . . . subtraction, division, and I even think we started algebra in some of those.

His mother chose to use a mainstream, public school curriculum. Tad explained,

We used like something that the University of Chicago Press put out. Um, so we used a different type of math curriculum, which is something more akin to what you see in the public schools . . . she didn't really like the homeschool curriculum, like Saxon or some of those math curriculums.

Designed by faculty and students, Tad detailed this curriculum as,

Each textbook is probably like about 15 different units and. . . . It breaks down the different concepts dedicated to each unit. . . . You go through the history; you learn all the theorems that have to do with it. Um, and then there is problem solving at the end of each section . . . and so we would take the test at the end of each one. . . . Some of them were more challenging than others . . . like random stuff that is kind of integral to like mathematics. . . . It covered all the basics. . . . It's a large, large textbook. Um, tons and tons of like detail put into everything.

T described his "very homeschool" math instruction. He recounted,

It wasn't a homeschool only curriculum. But the way it was delivered was very homeschool . . . my mom teaching us and giving us a lot of personal attention . . . a lot of time to work through the problems . . . one teacher to two student . . . lot of getting pushed, um and a lot of one-on-one instruction. . . . I think almost every day. . . . I got paired up with my older sister . . . I had to like match what she was doing . . . a little over 2 years older than me. . . . Just my mom sitting down with us, we'd read the chapter together. And then, we tackled some of the problems. . . .After we finished that my mom would be like ok do this, like matched lesson mastery sheet at the end of each section. . . . We spent at least an hour every day.

His mother stressed mastery learning. Tad reported,

If we did not get good scores on what they would call the A sheet . . . then she would make us do the B sheet or the B um, test. Which is very similar in style. So we would retake it if we didn't get everything right. So, we would go through the old problems, learn how to get them right, learn how to solve it, and then we would get another chance on the B test. And I think she would take the average.

The math classes continued year-round. Tad explained,

My math education between maybe like 9 or 10 until 15 when I went to college it was like continuous. So we really didn't get that many breaks over the summer. . .to finish one textbook per year. But it never really like exactly worked out to just fit into the school year. It was just kind of like continuously going through books.

His mother also taught other homeschoolers. Tad recalled,

I would say that there was probably like maybe two years where it wasn't just me and my sister. There was like once we had some friends come over and so we went through might have been trigonometry together with them. . . .It was the same style, just extra people. And there was another time where another girl joined me and my sister for like one of the more advanced, maybe Statistics.

This group activity would spark conversations, encouraging teamwork. Tad described,

We would always like talk about the questions and like we would usually try and follow the problems together. And there were times, where we would all get stuck on a problem. And so, we would be talking about how to figure it out. And sometimes, the answer keys don't show how they arrived at the problems. . . . And so there's a lot of like discussion; um, a lot of like, I think it was a lot probably like team-work in like trying how to figure out how we could solve this, um, how we could solve the different problems.

T completed rigorous mathematical courses. After the basic arithmetic principles, he took the standard high school math courses, as well as advanced ones at a local university while still in high school. He specified, "I did algebra. I did geometry. I did statistics. I did a pre-calc. . . . I did a trigonometry course. And then when I went to university, I took calculus 1 and 2." He described his calculus experience, relating,

I would say I had a lot of experience with it. And when I went to university down the street, I took classes in the Honors College, there. . . . I performed better than pretty much all the other kids in my classes. . . . I think I was well prepared.

T attributed his stellar SAT math score (780) to math competitions. He stated,

I think part of it was that I had done a lot of math competitions. . . .The AMC 8 is for like eighth grade and below. AMC 10 and the AMC 12 and I took all of those tests, as I did Math Counts. So, I was very well prepared for being like a good test taker when it came to Math and under time pressure and all that stuff.

His high school transcript verified that Tad completed Algebra 1 and 2, Geometry, Functions/Statistics/Trigonometry, Pre-Calculus, and Calculus, as well as his SAT scores.

Tad honestly admitted that homeschooling had potential drawbacks. He was not able to find many other participants for a competitive math team. Tad remembered,

We would have like team competitions sometimes. And like the homeschool community here it was really hard to put together a good team. And uh, we tried several years to put together a competitive team . . . [it] was like almost impossible to get together a few different kids who were like all really good at math, who could compete with some of the really good public schools.

Another concern involved socialization. Thoughtfully, Tad recognized,

I think maybe the biggest downfall is or falls is one of the more concerns for homeschooling is that you don't always get the same amount of social interaction. I think you can if you work at it; you can if you are intentional about getting kids out and getting them to interact with other kids; it's not that hard but, if you're not careful, that can be something that doesn't naturally happen. . . . Like that's one of the downfalls with social, with homeschooling. In like, even when I went to the university down the street for calculus, it took me a while to be able to like talk to the other kids. Kind of like making friends and stuff like that. . . .

Transitioning to university was also kind of complex, given my homeschool background. But, like I was able to adjust. . . .But, it wasn't like the easiest thing at first.

Mathematics was useful, sensible, and worthwhile to Tad. He detailed,

As a science student, like when I took physics I was supposed to have taken Vector Calculus already. . . . I had some experience with it in the past. And so they said, 'Alright, fine; we'll let you like get a chance to get into this physics class.' It was like an honors physics class with one of the best professors at my university. And uh, actually I did really well in the course. . . . I had the basic mathematical tools. . . . I found like math very useful in my physics and my overall science education . . . having a good mathematical understanding helps you so much in problem solving in any of those subjects.

Math made sense to him. Tad recounted, “It makes sense. . . . That’s one of the things I love the most about mathematics. It’s able to be grasped. . . . It’s just very nice and it all fits together.” Tad thought math worthwhile, stating,

They do studies on CEOs . . .one or two things they said that were the most important things for CEOs. . . .One was the basic mathematical proficiency. Like understanding um, mathematics is like sooo important for a broad range of things. Like just from living your life from managing your budget, you know. Figuring how far you can drive what gasMathematics is like something that uh, is really, really important, and very, very worthwhile for anybody who is going into higher education. . . . Pretty much every discipline in you know, in our world, can benefit from mathematics. . . . Having to read complicated music sometimes, you need to know Math in order to figure out complicated rhythms. And for artists, like knowing Geometry like is not going to hurt you.

Both parents contributed to the homeschool. His dad worked to provide for the family so they could keep homeschooling. Tad acknowledged his father’s indirect role as,

My dad like was always working to support the family because homeschooling is not necessarily cheap. . . . Like the amount of curriculum and books you need to buy. Um, and even just like anything, transportation, having to get to like all these different places; it’s not like everything in one building, like you would go to in public school. . . .It’s pretty expensive.

His mother acted as the primary teacher, especially focusing on math,

She’s taught us math and she did her Bachelor’s and Master’s in mathematics, Applied Mathematics. And she was a teacher, teaching assistant at [university] for a while. She was a teacher for middle school kids. . . . As soon as we started getting homeschooled, then she quit and just focused on teaching us. Um, but she didn’t teach us just math; she taught us French and how some English grammar and writing and stuff like that.

Tad double majored in Biological Sciences and Classical Guitar from a major university in California. Then, Tad earned acceptance in a well-known medical school in Maryland, over 500 miles from his home. According to his mother in a follow-up email, he was doing very well in medical school.

Tad home educator survey. Survey data mostly confirmed the interview information, and provided additional detail about the math instruction. The primary teacher, the mother filled in the survey prior to the student interview. Identical items included the age of the student, the sibling information, the length of homeschooling time and the student’s favorite subject, math. All mathematics classes were mentioned, clarifying that the trigonometry course included functions and statistics. The mother praised Tad, “He was a breeze. He was so advanced and I could relax at his testing.” She described his college preparation as, “He was excellently prepared.” She attributed his stellar SAT math score to his rigorous math courses and mastery learning, claiming, “He did so well because of the challenging math curriculum we used. I also made him do every problem and 100% of the textbooks. If he got something wrong we went over it.”

Similar to Tad, the mother reported that their Christianity and curriculum choice provided the main reasons for homeschooling. She wrote, “To be able to teach them from the Bible and to protect them from ungodly influences.” Likewise, they continued homeschooling, “also to select what curriculum we want to use.”

Tad’s mother described his mathematical experiences. She described the curriculum, “We used the University of Chicago Math all the way which is a lot of applied math—real situations in the world. It used to be in the public schools.” Writing about his math experience, she recounted, “He did his math at home. It was direct teaching until the last book, pre-calculus—he did self-study. He took two calculus courses as dual enrollment at the local university.” He enjoyed his homeschool math experience and was grateful. She shared, “For the most part with this student it was enjoyable and interesting. At the end of it all he said, “Thanks for teaching me Mom; it was fun.” Instruction continued through the summer months; she reported, “It went into the summer to finish.” She explained that she did not prepare for each math lesson, but sat down with them and went through the reading and problems with them as a tutor.

Tad’s parents used math. His father provided housing for college students and ran a ministry for international students, both involving financial matters. Tad’s mother was not certified as a math teacher, but had experience and a positive attitude towards Math. She wrote, “I love Math. I still teach my kids left at home. I was almost certified but didn’t finish the summer requirement. I did teach at middle school for one year.”

NAEP assessment. Tad answered all of the NAEP assessment questions correctly. He described his work and justified his methods for all ten questions.

MSES scores. Tad scored 8.7 overall, an 8.7 on Part I and an 8.8 on Part II, ranking above the male 95 percentile (Table 10). His mother’s total score was 8.7, an 8.4 on Part I and a 8.9 on Part II, above the male and female 95 percentiles. The father scored 7.8 overall, a 9.0 on Part I and a 6.5 on Part II, between the male 80-90 percentiles.

Appendix P: Rob

Rob interview. Interviewed in the lobby of his dormitory, 20-year-old Rob appeared relaxed and comfortable. He answered all questions and provided extra explanations. The interview took place after Saturday lunch. Quiet with good acoustics, this location provided an ideal setting for the interview. The interview lasted 41 minutes.

His family home-schooled for religious reasons. Rob shared,

I think that mostly probably religious reasons. Ah, to keep us insulated from other kids who might lead us astray morally. . . . They [his parents] are of the Christian faith. Um, I believe that they just wanted us to be in an environment where we weren't, I guess, constantly challenged while we were young.

The academic success encouraged them to continue. He explained,

I think that as time went on, their reasons for homeschooling probably um, morphed a little bit in that they, especially my mom, saw the benefits of homeschooling as far as academic excellence. Uh, but I think that as time went on, homeschoolers really could not only be on par with private and public schoolers, but even be more advanced than them.

Rob took many classes outside the home. He described,

I think our experience was pretty unique. . . . People who are outside of the homeschool world think homeschooling, a lot of time they just think that people, who um, are at home all day long, just sit with their mom teaching them. But, for us, it was like that for maybe the first uh, half of our homeschooling lives. But once it started getting into middle school and high school, um, we really started taking classes with other teachers, other groups, um, co-ops or resource centers, as they like to call them . . . like a mini private school environment.

Notably Rob completed most of his math classes at home, as well as some social studies.

His homeschool math instruction started with basic workbooks, progressed to UCSMP, and then finished with calculus at a local university. He recounted,

Starting out in our mathematical careers . . . they were just basic problem books, I guess that got progressively harder and harder. You learned new concepts. But you just basically did problems and problems. So it was kind of a lot of repetition.

Later, he used UCSMP textbooks, explaining,

The math was University of Chicago . . . very much like a college level type textbook. . . . It wasn't super interesting or anything or care to entertain you with the textbook. But it just cared to inform you . . . It did a good job of that.

The mother acted as the primary teacher in Rob's math instruction. Ensuring mastery, she checked work daily and required the reworking of incorrect problems; he described, "And when we were finished with the homework, we would show it to our mom. She would check it. And if it was wrong, questions, and she would correct them for us and make us do more, of course." When he switched to the UCSMP texts, his mother used direct teaching. He recollected,

Most of the time spent on math work, was uh, with my mom, one on one, um, I guess at our school table in our schoolroom. . . . She would basically just read the textbook; I would listen. Um, and once that was done, there was always exercises in the book. . . . The teaching was pretty structured. Um, we would always do the same format: she would go straight from the book. . . . As far as the homework, the quizzes and the test, I guess it was a little bit less structured cause it was more on my time, whenever I was at that point, to take the test, or take the quiz or do the lesson masters . . . With incorrect problems, um, I mean my mom um, would make us like do homework and homework until we didn't make mistakes. So if, I made a couple of mistakes on the test for the chapter, they actually had like back up tests, I guess for this curriculum. So she would make me take the next test and if I didn't get that one perfect, she'd make me take the next test and the next test. So, that was kind of the rework until mastery. So, no tolerance for mistakes.

Although Rob finished all his textbooks, the time spent on mathematics instruction varied; sometimes there would be lapses; but they always finished the book. He recollected,

There'd be like weeks where it was somewhat consistent, maybe uh, an hour a day or so. There were also times . . . we would not do math for several days at a time, maybe even a week. Then we would have to overload the next week and do 2 hours at a time or so. . . . But it got done. And then there were times like maybe a month would go by and we would kind of pause on math because we were busy with other things, like debate tournaments. And then we'd have to do it during the summer . . . always had to finish the book. We weren't done until we were done.

Although Rob now appreciated his mathematical studies, he experienced stress during them. He revealed, I think for the most part, probably stressful. Um, yeah, I think, like I enjoyed the material. But during the lessons, it was usually stressful. And the interaction was limited to . . . my mom and I. . . . It was stressful.

He attributed that tension to his teenage rebellion, disclosing,

As growing up teenagers, there were definitely times when having to be taught math by your mother was not always the most enjoyable thing, just cause it was based on her schedule. And she'd be like, alright we're doing math now. Um, so there I mean definitely um, was some tension, there. . . . I went through a teenager

phase; I think we all do. And so, it was just a lot of my youthful rebellion against my mother's desire for me to learn.

Yet, he affirmed his instruction, maintaining,

Definitely times that it wasn't as enjoyable but um, I don't know; somehow it ends up ok. . . . Another time I definitely would have said that I would have liked to have a class, um, just that it wasn't one-on-one. But in retrospect that was probably more beneficial for me. Um, so I don't know. I think what happened, happened, and it was probably for the best. . . . I might not have liked it but I don't think I would, like now if I could change it, I don't think I would."

Rob completed pre-algebra through calculus. He detailed,

I started taking pre-algebra . . . then I took ah, algebra, geometry, uh trigonometry, functions and statistics, um, and then I think after that I can't remember if I took a pre-calc course, but I took Calc 1.

His math classes contributed to his high SAT math scores (750), verified by his high school transcript. He asserted, I think that all of the math that I did take in high school and before then, um, just the whole way of thinking helped me be prepared for that [SAT]. Um, problem-solving techniques and seeing a problem and knowing what they're asking and thinking of how to solve it."

He admitted that he took practice tests, but maintained they only helped him get acclimated to the SAT test format. R acknowledged, "I did take some uh, of the practice tests. But that was just more to get, I guess, um, acclimated to the test itself." His outstanding SAT performance correlated with his good calculus 1 experience.

Rob really enjoyed the calculus class he took at a local university. He described the class, the adjustment, and his gratification in mastering the course. He reminisced,

Senior year is when I took the class at [local university], at the Honors College, Calc 1. And um, that was just a normal college class. . . . I think there was four days a week of class: three days of lectures; one day with a math software lab where we learned how to use Maple, um, which is a math software. . . . But, that was pretty standard, we did lectures; had suggested homework assignments, tests.

Although nervous academically, he was not nervous socially; he revealed,

For me it wasn't that much different because I had been in classes with 25, 30 people, um, all through high school. And so it was really, the only difference for me was that it was college-level; it wasn't high school level. I really loved it. Um, I was actually surprised that even in the honors college, there were so many students that would like never show up to class and would just talk to each other during class. And that was, I mean, that was I guess, new; I hadn't really experienced that in the homeschool world.

Rob believed he was well prepared for his college math courses. He affirmed,

I would say I was very well prepared for college math classes. . . . I took Calc 2 my first semester here. It was easy. And then I took Number Theory last year. And it was really hard but I really enjoyed it as well.

His high school transcript verified his math classes, with grades ranging from 95-99. To R, mathematics was useful, sensible, and worthwhile. It was valuable knowledge that he found challenging and rewarding, asserting,

Just like in day to day, uh, figuring out how to budget stuff. . . . The algorithms [in Computer Science] we have to design are all based on Logic. It's all problem solving, which is what we do all day long. . . . I think it's a necessary part of everyday like, even um, just being able to sustain yourself, economically. I think a lot of people do not have a solid understanding of um, math. And it just makes them confused when they have huge credit debts, overspend their budget and stuff like that. . . . And that rewarding experience of being able to transition from not understanding-to-understanding is a lot of the reason why I really love it.

His parents also thought math very important. His mother earned two mathematics degrees and taught math before homeschooling. He explained,

Math was what my mom did. . . . She used to teach math, um, for like school students I believe. But um, she definitely had a pretty strong basis in math, um, majored in math; got a Masters in Math. Um, so she was definitely qualified.

The father's work demonstrated the importance of math. Rob described,

He runs the ministry. . . . He also in order to get an income, buys apartment buildings downtown [city] and rents them out to university students. That's a whole lot of finance and business and I think he has like a really good understanding of the basic uh, math. . . . But as far as like problem solving skills, he definitely has a great understanding of that. Um, yeah, he definitely does well with that. . . . embody it a lot in his work.

R recognized and appreciated the importance of his mother's dedication: repetition, problem solving, and advanced classes. He detailed,

I think it was great. I don't really know how people not homeschool their child math, but, at least for me, my mom since she really truly loved math and cared about the subject. She wanted us to love it I guess and care about it too, um, which we did in time. And uh all the time she put into it, which resulted in time that we had to put into it, um meant that we were doing it a lot. So I think that a lot of repetition, um, in problem solving really helped us develop mathematical ways of thinking. Um, and so because of that focus I think that we might have had that advantage over students in school where math is just another subject, because um, my mom really took care to make sure that we knew not just the basic high school math, but also functions and statistics and pre-calculus and a bunch of other classes that people might not necessarily take. . . . But she made definitely sure that we all were always learning new things in math. Um, and we had that ability, that

flexibility um that we could take math earlier than, I guess, kids in school could. We could take high school math in like 7th, 8th grade and then we had more time to take harder classes later on.

Currently, Rob attends an academically acclaimed private South Carolina university with 2600 students, about 100 miles from his home. He double majors in Computer Science and Political Science, using the computer that he built this summer from scratch.

Rob home educator survey. Overall, the data from the survey confirmed the interview information. The primary teacher, the mother completed the survey after the interview took place. I do not believe there was any discussion between the mother and the student about these questions because the student was at college. No direct contradictions existed; some answers differed, both only in point of view and minor curricula descriptions.

Identical items that included the age of the student, the sibling information, the length of homeschooling time, and the student's favorite subject, math. All mathematics classes were mentioned. She described,

He was very creative when he was young. Homeschooling was the best thing for him. . . . If he was at school he would have been constrained to a desk for hours and would not have had the opportunity to do the things he liked. He invented a lot of other things as well (scientific) and soon became a genius at computers and filming. He was very advanced as well in his work and so made my life easy.

She labeled his experience with math in homeschooling, as "It was totally adequate. It was advanced." and his preparation for college mathematics courses as "He was excellently prepared, he either got one or no mistakes on the [local university] Math Placement Test." Confirming Rob's information about his preparation for the SAT math test, his mother credited the challenging curriculum for his excellent score, stating, "He did so well because of the challenging math curriculum we used. I also made him do every odd problem (even problems too if he got any odds wrong) and 100% of the textbook." She provided extra information about competitive testing, informing, "He actually got 1 mistake only on the math part of the SAT. He also participated in the AMC8, Mathcounts and AMC10 math competitions when younger." This educator provided more information about the reasons for homeschooling.

Like Rob, his mother revealed that the initial reason for homeschooling involved their faith and the continuing reason depended on academic concerns. She expounded, "To be able to teach about God (Jesus) from the Bible and to protect them from ungodly influences from other kids at school and false teachers." She explained that the freedom to choose rigorous curriculum inspired to continue, stating,

same as above [initial reasons] and also to select exactly curriculum we want to use and the great books we want to read. To study in detail what we want to like, geography or French. To use the most challenging math book we can find where it will be explained and do it right. To have the freedom to travel.

Rob's mother explained the math curricula, instruction, and experiences. She detailed the applications of the University of Chicago Math Project as, "It is real situations in the world – applied math/scientific. Every picture is a math problem. It was used in the public schools." Similar to Rob, the mother specified,

He did his math, at home at his desk. He spent an hour per section. It was direct teaching until the last two books, func/stats/trig and precal We didn't finish in the year but did all summer (it seemed like drudgery in the summer but ok.) He took calculus as dual enrollment at [local university] and probably enjoyed that the most with the challenge and the use of the computer.

Her answers on the survey added information about Rob's self-study, expounding, "which he did [the last two books] on his own and came to me if he didn't get something." This would indicate that Rob became more self-sufficient as he matured in his studies, eventually enrolling in a university class during high school. She portrayed a less stressful situation than Rob, but she did confirm much of Rob's interview information, disclosing,

It was enjoyable and interesting. I taught him at first then he couldn't wait for me and went ahead on his own but he came to me if he didn't understand something. I graded his work and we talked about what he got wrong (not much). He was a great student. It was not stressful. It may not have been super-enjoyable like playing xbox soccer but he did seem to like it and how it made sense. He was very grateful that when he finished his pre-calculus text (end of the summer) and right after that took the [university] Math Placement Test, he was ready and aced it.

The mother explained their mathematical attitudes. She loved math and had teaching experience. She related, "I love math and still teach my kids left at home. I was almost certified in SC but didn't finish the summer requirement. I did teach middle school for one year and Algebra at the {local university} one semester." She reported, "My spouse does not like math courses but is a business/banking genius which deals with money and interest." For encouragement, she attended homeschool conferences and support group meetings, clarifying, "I went to homeschool conference once a year but missed the last 3 years. I belong to a local support group and go to their mom's meetings when I can."

Rob's mother reflected on the past years of their homeschooling. She reported, "Thank you for asking these questions It made me reflect back on the memories of teaching [T] and [R] and evaluate what I am doing." Reaffirming her commitment, she mused, "It is not any easier to homeschool but I am still convinced that it is God's will for our family and the best for our children in terms of learning and being protected."

NAEP assessment. Rob answered all of the NAEP assessment questions correctly. He described his work for all ten questions and justified her methods for all ten items.

MSES scores. Rob scored 8.6 overall, an 8.2 on Part I and a 9.0 on Part II. His total score ranked above the male 95 percentile (Table 10). His mother's total score was 8.7, a 8.4 on Part I and a 8.9 on Part II, exceeding both the male and female 95 percentiles. The father scored 7.8 overall, a 9.0 on Part 1 and a 6.5 on Part II, between the male 80-90 percentiles.

Appendix Q: Wes

W interview. Despite a busy dinner crowd, 20-year old Wes and his mother seemed comfortable. The mother filled in the educator survey during the 25-minute interview. She sat quietly nearby and offered one comment when Wes had trouble remembering a name. Wes answered all questions and offered extra explanation

Wes's family home-schooled for academics and character development. His neighborhood fed into substandard schools that experienced racial tension. Wes explained,

I think the main reasons were that we kind of were in a bad school district, the schooling wasn't great; and also, we would have been, at the high school there was like five white people. We would have really been in the minority, there. And just the schooling wasn't great. And my parents didn't want me to be badly influenced, either, by public schooling.

They realized the benefit of homeschooling and continued. Wes recounted,

Homeschooling was working for us. We were learning a lot, and making good grades and still being relatively good kids. So, they thought it was working. . . . They thought it was better schooling to be like one on one with teachers.

He asked to enroll in public school in tenth grade; Wes remembered,

I asked my parents if I could go to public school in 10th grade to play soccer for high school. But, they wouldn't let me cause they thought I might be badly influenced. And so, I didn't like that but it was their choice I guess.

Despite his denied request, Wes positively viewed homeschooling. He was well prepared for college math. He related, I've taken trigonometry and statistics. . . . I was pretty well prepared because they weren't that difficult.

There was kind of like the stuff I'd already taken. But it had been like maybe a few years since trigonometry. So I had to relearn it. But it came back pretty easily. . . . I did well. I've done well in my college math courses and on the SAT. I think it was overall very positive.

He described his math experience as,

I think it was pretty adequate. . . . My experience with homeschool. Um, I guess it's a lot of teaching yourself. . . . I'm either doing the book on my own or I watched videos a lot. . . . It was more independent than other schools. And I can do it on my own time. I can get it done quicker than other schools.

W enjoyed physical education and math, while disliking most of the sciences. Speaking fondly of his favorites, he reported,

My favorite subjects were easily, PE and math, probably. . . . I like to exercise and math I just find it fun. Like that survey, I thought it was fun. . . . All sports I find fun. I like to get outside. And then, math, I like figuring out stuff in my head. Like, I'm a lifeguard right now and it can be really boring. Sometimes I do math problems in my head, just for fun, to past the time.

He disliked biology and chemistry vocabulary, but he liked physics more, revealing,

Stuff where I have to memorize a lot of terms. I don't see any use for it again, except just to pass the tests. So, like biology, maybe chemistry. Physics was better because it had math in it that I liked.

His natural math affinity and good testing taking skills helped him score 700 on SAT math. His high school transcript verified his score.

Wes attributed the good score to time management and finding most math problems to be easy. He disclosed, "I didn't waste too much time on the problems, when I was going through them. I think there was like 25 problems. Like the first 22 are easy and like the last 3 are super hard." Wes acknowledged he completed practice tests but he didn't need to take a prep class, stating, "I did a lot of practice tests. . . . I went to a prep class for English section for the English bit. But not the math, cause I was struggling more with English than the math."

Wes completed his homeschool math at home. He named his courses (verified by his high school transcript), "I took Pre-Algebra, Algebra, Algebra 2, I guess. Geometry, Pre- Calc. . . . I took Consumer Math. . . . I did it all at home". His His earliest memories involved math wrap-ups, stating,

One of the earlier things I remembered was like I think multiplication problems; I wasn't getting them. And then, we had something like string {Mother: math wrap-ups} math wrap-ups. And it was fun. You like connecting the strings to each one and I'm a hands-on learner so it helped me get my multiplication down.

Later he worked with Saxon, describing, "then like I think, Saxon, it was just the book and I kind of went through the book. And it would explain it and the lesson, like um, and that would work for me." Wes checked his work, explaining,

To check my answers, I usually do it on my own. I check the back of the book and see if I could figure out why I got one wrong. And if I couldn't figure out, I would go back to the beginning the lesson and look it over or get my dad to help.

He used Chalkdust video curriculum for the high school courses, explaining, "PreCal, Geometry, and I think Algebra and Algebra 2 I all took on video from Chalkdust. And so it was a really good course, taught it well." Wes liked the Chalkdust curriculum. He appreciated the teacher and the rewind option of the video, explaining,

The teacher I took like from a video course. He was really good. It was Chalkdust Math. . . . I did like getting taught by a teacher. And I could rewind if I didn't get it. That was another thing that helped cause sometimes like in a classroom I won't understand something like in college and I can't rewind it. . . . Um just the way he taught was simple. And seeing it on a board, seeing how he did it, it all made sense, step by step.

Despite his good experience with Chalkdust, Wes desired to take math classes with peers.

Wes described the advantages and a disadvantage with his homeschooling. He believed that self-pacing and his father's availability to help were beneficial, claiming, "I guess the advantages would be I could do it at my own pace. Um, not have to rush through things and get behind and um, I could ask, I could get my Dad's help." The one disadvantage involved not taking an outside class with math. He declared,

One disadvantage would be able to um, do it with others, do math with other kids, to study with them, um, to motivate me a bit more. . . . So, I could get to do math with other kids. It would have been more fun.

Notably, Wes took one math course with his sister and he enjoyed the competition.

Wes and Mae worked together. Their natures fostered success despite logistical issues. He explained, "One year my sister and I took the same, took the same math class and she was helping me if I didn't get it. . . . It was good because we're both really competitive. . . . The bad part was that we would try and watch the videos together sometimes and we were not always ready at the same time and we would get mad."

Wes believed mathematics to be useful, sensible, and worthwhile. He understood its practical nature, contributing, "I do view it as useful. . . . It is useful in calculating everyday stuff . . . like college tuition, going to a grocery store, and how much it is going to cost at checkout." Math made sense to him; he declared, "Almost all math makes sense to me, except logarithms never made sense to me." Wes noted that math knowledge makes you less dependent on calculators, claiming,

It's worthwhile; there's some stuff you probably will never use. But, the stuff you will use will help you in your life and you won't be helpless and have to use a calculator and there's none around or something.

Both his parents believed good mathematical skills to be important. He explained,

I guess both my parents think it's important to be good at math. . . . They both have their Bachelor's degrees; neither one was in Math. . . . My dad's is in engineering and my mom's in English, journalism. Um, neither one is a certified teacher. I guess my dad does math a lot with his job. He's an engineer for DHEC. Um, so he uses it a lot in his job. . . . My dad loves math. We've had competitions against each other in our heads to see who could figure out problems the fastest.

Wes recommended that all students take consumer math. He asserted,

Definitely take consumer math in high school. Even though it's easy, it will help you the most with practical life situations like buying a car, figuring how much a mortgage on a house is going to be . . . even if you have to cut out advanced calculus or something. Because unless you're going to major in that or take that in college, I think it would help you to take a business or consumer math.

Currently, Wes works for a Christian landscaping company.

Wes educator survey. The survey data confirmed the interview information, as well as provided more specifics about the math curriculum. The mother, who was the primary teacher, filled in the survey while the interview was taking place. No direct contradiction existed and some survey answers clarified and supplemented interview information. Identical items included the age of the student, the sibling information, and the one of the student's favorite subjects, physical education. All mathematics classes were mentioned. The mother described the homeschooler's experience with homeschooling as he, "needed more structure and accountability", his experience with math in homeschooling as, "Adequate—pretty good", and his preparation for college mathematics as, "Fairly well."

His mother described the two main reasons for starting to homeschool. These included the poor schools and Christian character development. She wrote the causes as "poor schools in area; wanted God + character"

The mother provided more detail about the math curricula, instruction, and experiences. She used a combination of manipulatives because Wes was a kinesthetic learner, "He was a kinesthetic learner so we used a lot of manipulatives. . . . and a lot of manipulatives like math wrap ups, Flash cards, JumpStart K, 1st, 2nd computer programs; GAME; Blocks unifix cubes." Curriculums utilized included ABeka, Saxon, and Chalkdust. She detailed, "ABeka for younger years. . . . [used] Saxon for some middle grades. Chalkdust videos for high school [and] ABeka for Consumer Math." The Chalkdust videos provided structure for Wes and allowed his learning to be home based and more self-directed. She explained, "DVD for high school, self-directed; very little help from parents, except for pre-calculus . . . math was always at home, DVDs were awesome; great teacher on DVD. . . . The DVDs provided structure, which was good for Wes." His mother believed Wes thought his instruction was good, but recognized his social nature, acknowledging, "I think he viewed it as a good experience but he is social so probably missed interaction." She confirmed that Wes thought math to be useful, sensible and worthwhile, as well as fun, writing, "Yes + fun, too!"

Both parents played a significant role in W's math education. The mother selected W's curriculum and helped him when needed. For support, she attended homeschool conferences, math workshops, support groups, and co-ops, writing, "All of the above were helpful to me as a homeschool mom." When Wes had difficult problems in more advanced math classes, he went to his dad, who the mother described as, "Husband uses math at work." Wes's mother used basic math in coaching; she described, "I use basic math as a swim coach." Neither one had teaching certification.

NAEP assessment. Wes answered all of the NAEP assessment questions correctly. He described his work and justified his methods for all questions. Although he marked one answer incorrectly, his work demonstrated his understanding and the item was marked correct.

MSES scores. Wes scored 6.7 overall, a 7.3 on Part I and a 6.1 on Part II. His total score fell at the male 60 percentile (Table 10). His mother total score was 7.1, a 7.6 on Part I and a 6.4 on Part II, ranking between the female 80-90 percentiles. The father scored 8.7 overall, a 9.0 on Part I and an 8.4 on Part II, exceeding the male 95 percentile.

Appendix R: Mae

Mae interview. Interviewed after her brother Wes, 21-year old Mae seemed comfortable with the process. Arriving during her brother's time, she listened in. Mae answered all questions and offered extra explanation. The interview lasted 34 minutes.

Mae's family home-schooled for academics and character development. The poor public school environment provided the motivation to start homeschooling. Mae clarified,

I think it had to do with that fact that my family lived in a, um, school district that wasn't the greatest and they knew that um, the schools that we would be going to was not um, not probably be a good environment. Her parents wanted to instill Christian principles; she described, "They also wanted to be able to um, kind of guide our schooling and be able to teach us from a Christian perspective and not have us learn about evolution. They wanted to be able to teach us Creation." Her family experienced success and continued. Mae reported,

The reasons I stated for um, why we started initially, probably were some of the reasons we continued. But, also, it seemed to be working for us. And I think we were um, doing well, learning and . . . they thought it was just a good fit for us.

Mae described her homeschool experience in a positive manner. She asserted,

Homeschooling was good for me. I never knew anything different. So, um, for me it worked well. I feel like I learned what I needed to . . . I think it worked very well for me and my siblings. . . . For me it was a very good option.

She appreciated the flexibility, expressing,

I enjoyed the flexibility of being able to start whatever time I wanted to in the morning and um, take a day off here if I needed to go do some kind of service project or go on a trip with the family. So, I also liked the fact that I could pick and choose my classes.

Mae enjoyed human anatomy, while disliking history and math. She explained, "Human Anatomy was probably my favorite or one of my favorites. I really enjoyed learning about the human body." Although she performed well in history and math, she disliked them, revealing, "I did well in both of them. So, it was not like I was failing. But I just – history, I felt like it was boring. And math, I didn't like all the work and having to do the calculations." Despite her distain, she performed well on the SAT Math.

Mae scored well on the Math SAT subtest (640) and earned good math grades, verified by her high school transcript. She credited the score to her high school math classes and practice. She stated,

The math classes that I took in high school prepared me well. And then the SAT prep that I did, you know. I don't know that I did a lot; but I did some practice tests, a lot of practice questions. So I think those helped me be prepared.

Her high school math classes provided adequate preparation for her nursing major in college. They provided a good foundation. She described,

I took um, algebra 1 and geometry and algebra 2 and . . . consumer math. . . . It was adequate for me and for the major I was going into in college; I'm a nursing major. The math I had in high school was very adequate and I think that the curriculum that we used taught me well. I got a good foundation. . . . I was very well prepared. Like it was kind of easy at times. I made an A in it.

All Mae's math courses took place in the home. She declared, "Math was always at home. . . . I never took any outside math classes." In the elementary years, her mother provided help with ABeka and other resources. Mae explained,

I started in elementary school with . . . ABeka . . . arithmetic . . . I liked ABeka when I did it in elementary school. They had nice colorful pictures. And it was fairly easy and understandable. . . . We may have had flash cards for multiplication or something. I just heard my mom mention the math wrap-ups, which I probably would not have thought of. But I do remember doing those. Um, like for multiplication it kind of helps you get those tables in your head.

She used Saxon in the middle school, remembering, "In the middle school I think we moved into Saxon. . . . Saxon, I wasn't so crazy about, the textbook was kind of boring, black and white." In high school, Mae used Chalkdust and ABeka. She reported,

I did Chalkdust for Algebra, Geometry, Algebra 2. And then, my Consumer Math was ABeka again. So, I really liked Chalkdust and we did the videos with that. Um, and the teacher who did the lectures and explained everything was very good at explaining things clearly and it was good to have a like a teacher actually planning and going through um, you know calculations and equations.

Mae remembered mostly working independently in mathematics. Her mother helped her more in elementary school and some in middle school. She recounted,

In elementary school with ABeka, um, my mom probably worked with me more, then. Kind of going through workbooks or stuff if I could kind of understand it. Um, but I think I did a lot of it on my own. Um, and also in middle school, she probably would go through concepts if I didn't understand them.

She became more self-directed in middle school, reporting,

Mostly it was just working on my own, self-study. . . . For middle school it was Saxon math. . . . I would read the textbook and then do the problems. I probably corrected them by myself. Um, and took it to my mom or dad if I had problems.”

In high school she relied on the Chalkdust videos, describing,

[In] high school I guess that was with Chalkdust and videos. So, I watched the videos in the living room and then I would do the work probably on my bed. Um, I’m not quite sure how much time I spent. Probably, an hour to 2 hours a day - roughly, maybe sometimes less. . . . A lot of it was just working on my own.

Mae corrected her exercises, informing,

I would correct them, look in the back of the book and get the answers. And then, yeah, if I had them wrong I would just try to figure out why and I think the back of the book showed kind of how to work the problems.

Um, and if I absolutely couldn’t figure it I would take it to my dad to figure it out and help me through.

M described the advantages of her homeschool math program. Like her brother, she affirmed the value of setting her own pace and rewinding the videos, maintaining,

Overall I think it was good. I think an advantage was that I was able to work at my own pace. Um, if you know, I didn’t understand something; I had the freedom to spend more time on it. It’s not like it was a set class structure. I think another advantage is with the videos, if I didn’t understand it I could just rewind it. You can’t do that in a regular class. You can’t rewind the teacher.

She viewed self-correcting as an advantage, claiming,

Having to correct my own work probably is an advantage. Um, because it made me really have to think through ‘ok how do I get this right?’ . . . I had to actually like, deal with my own mistakes. Um, disadvantages, I can’t really think of any.

Mae also saw the value of not waiting on others in a class setting, revealing,

I think another advantage is that I didn’t have to sit in a classroom, where other people are trying to grasp it and I have to then spend time on them. If I understood it and can just go ahead and I don’t have to wait on other people.

Mae and Wes worked together on at least one Chalkdust math class. Overall, this experience worked out well. Mae reported,

My younger brother and I, we, um, took some of the same; it may have been Algebra 1, Geometry, and Algebra 2 that we took it at the same time, because he was just a year behind me. . . . I think it was fine. . . . It was good that we could watch the video at the same time. Um, and also when we had problems my dad could sit down with both of us and go over it.

However, there existed some logistical stress. She disclosed, “Sometimes it was a stress because we only had one textbook; so, only one of us could do it at a time. Like do the problems or we’ve have to sit there beside each other and share the textbook.” She remembered some upsetting times with math, recalling, “Sometimes math was kind of stressful. I remember crying, sitting there with my younger brother and my dad when he was trying to explain things.”

Mae agreed that mathematics was useful, sensible, and worthwhile. She acknowledged its practicality, recognizing,

There’s a lot of basic concepts in math, that are useful for everyday things, like the things that were mentioned on the everyday math test like, calculating the tip, estimating your grocery bill. . . . Some like for calculating drug doses, very useful. Um, when you have to, you have to be able to know how to do that. Um, kind of like algebra, you are putting in numbers for different . . . a formula.

Math made sense and was worthwhile to Mae. She admitted,

There’s one right answer, which I like. So, yeah it makes sense . . . worthwhile to know how to do those things to get a firm foundation for the basics in math.

Both parents believed good mathematical skills to be important. She explained, “I think my parents both think that it’s useful. Um and sensible. . . . They both are very supportive of being proficient in that.” Being an engineer, her father seemed to place a higher value on its usefulness; she revealed,

I think my dad probably would say that math is more worthwhile and useful than my mom. . . . He’s an engineer and so he had to use all of that and know all of that. Where my mom was a journalism major, so math, so she didn’t need math as much. So, she’s probably wouldn’t view it as, as useful as my dad.

Both her parents participated with Mae’s math experience. She recounted, “In high school it was mostly my dad who would help me out when we had problems. Um, and then middle and elementary, my mom helped more. She would help me go over things.”

Currently, Mae pursues a Bachelor degree in Nursing at a Christian South Carolina university, 100 miles from her home. She is doing very well.

Mae home educator survey. The survey data confirmed the interview information. The primary teacher, Mae’s mother filled in the survey before the interview. This limited the possibility of discussion about the questions prior to the interview. No direct contradiction existed and some survey answers clarified and supplemented interview information. Identical items included the age of the student, the sibling information, and the student’s least favorite subject, math. All mathematics classes were mentioned. The mother described the homeschooler’s experience with homeschooling as “Good experience; self-pacing was helpful; great grades,” her experience with math in homeschooling as, “It was adequate; she took the time to do well; was thorough with her homework,” and her preparation for college mathematics courses as, “Very well.”

Mae's mother confirmed the reasons for starting to homeschool. These included the poor schools and Christian character development. She wrote the causes as "poor schools in area; wanted God + character as part of their schooling."

The survey information verified the curriculum used. The mother detailed, "ABeka for younger grades[;] Saxon for some middle grades; Chalkdust videos for high school [and] ABeka for Consumer Math." The Chalkdust videos provided independence and allowed M's learning to be home based and more self-directed. She explained, "DVD for high school – self directed; very little help from me or her dad." The mother thought that Mae viewed the math experience as positive, informing, "I think she would say it was positive." She expressed that Mae thought basic math skills to be useful, sensible and worthwhile, reporting, "Yes – basic math skills yes."

Both parents played a significant role in M's math education. The mother selected M's curriculum and helped her when needed. For support, she attended homeschool conferences, math workshops, support groups, and co-ops, writing, "All of the above apply to me." As needed, Mae would seek help from her father, described as, "My husband is gifted in math + helped her as needed." Neither one had teaching certification.

NAEP assessment. Mae answered nine of the NAEP questions correctly. She described her work and justified her methods for all ten items. Comparing her answers with the 12th grade sample taking the assessment, 38% of them missed the same item.

MSES scores. Mae scored 7.4 overall, a 7.4 on Part I and a 7.4 on Part II. Her total score ranked near the female 90 percentile (Table 10). Her mother total score was 7.1, a 7.6 on Part I and a 6.4 on Part II, between the female 80-90 percentiles. The father scored 8.7 overall, a 9.0 on Part I and an 8.4 on Part II, exceeding the male 95 percentile.

Appendix S: Ali

Ali interview. Interviewed in the kitchen of her residence, 23-year old Ali willingly participated. She seemed very comfortable, being open and honest without any sign of nervousness or hesitation. The interview took place after lunch. With no one else present, Ali answered all questions and offered extra explanation when prompted. The interview lasted 88 minutes.

Her parents started homeschooling as a response to God's calling. She explained,

They just really felt God leading them to homeschool. . . . It was back when homeschooling was still kind of really fuzzy and the legality of all of it was you know, still trying to be sorted out. For the longest time, my sister and I didn't have social security numbers because they wanted to homeschool us. But they didn't necessarily want to know the government to know that we were around, so that no one would come to the house. Um, so it was straight up God's leading.

Ali shared their faith, especially when it concerned mathematics, articulating,

Math is the same, everywhere and it's really just a beautiful picture of God at work and nature. And I think it's fantastic. There is a Bible verse in *Psalms* that talks about thinking God's thoughts after Him. And that what math and science is to me, because you're figuring out how the world works and how He know this, this system of letters and numbers and how it can always make sense and, and it's, it's just something God created and I love learning about it because it's just as much part of His creation as you know, you or me or an artist painting. Like I mean it's all equally beautiful. And I just absolutely love it."

Ali's family established themselves within a support group and was able to continue their individualized instruction. Ali remembered, "we found a support system that we needed with other third option groups to kind of help cover areas of schooling that mom wasn't necessarily, completely comfortable teaching, like high school English. So we ended up finding other teachers." The flexibility of homeschooling allowed them to be creative and work at their own pace; she clarified,

The longer we homeschooled and the longer Mom saw it working . . . she just knew that that's what we needed to keep doing. And the public school would end up squashing a lot of what of what was going on with our creativity. . . . So it let us all work at the speed that we could remain interested in and we could be challenged in, without being overwhelmed and being stuck to a set system.

Ali appreciated the family closeness and the training. She recalled,

We were a really close family and mostly because we were homeschooling. . . . I was never bored with, you know, I've done my homework now, and now there's nothing to do, because there was always someone to play with.

She also enjoyed reading, relaying, "I've always really loved reading. And so I would often grab a book and you know, sit in my room or kind of hide off somewhere"

Her parents instilled good habits and routines; she described,

No schoolwork in your pajamas. You had to be dressed for the day. We every morning we got up and had Bible time with Dad before he went to work. And then, we would have story time with Mom. . . . We would do our chores and then by 9 o'clock in the morning we would have to start with our homework. But we had to be dressed and up, and have been ready for the day.

Her love of mathematics and science sprung from this experience. She believed,

I think that if I had not been homeschooled, I probably would have hated Math. I really honestly believe that, because I would have been absolutely been bored to tears. Because I was always ahead; I ended up skipping a whole grade because, and a lot of it was because I was so ahead in math that you know I could work extra on the other subjects to kind of pull me along faster. . . . I honestly don't think I would have liked math and science if I wasn't homeschooled because I was allowed to run with it. . . . I wasn't lectured constantly and then give homework and then be held back and then, cause you get so bored in those situations and then you end up hating those classes.

She also worked with her siblings, acting as a tutor. She described, "When I was a little older, I would kind of take over going over math problems with my younger, if Mom was busy. . . . I was the math, mini math tutor of the house."

Math excited A. She described her mother's instruction, explaining,

She was sort of the resident expert. So, she taught me and my sister who were taking the same math because I could and it made life so much easier for mom. . . . Then, three other girls from the homeschool group, their parents had Mom teach them math, too, because she was really good at it. . . . Most of the lecturing happened in the living room, as we were on the couch and Mom had a white board that she would stand on a chair.

We'd be in the living room because a lot of the math lessons would start with a video. You have to go where the TV is.

Incorrect problems were done over until mastered. Ali recalled,

You would check each problem. . . . We would read our answer. Um, if it matched, then you know, that it worked out. . . . After we went through all 20 problems, we'd go back and say ok, which one did you get wrong? . . . And sometimes it would have been botched completely and we would go back and do the problem again until we got it right. . . . Or if we couldn't figure it out what was going on, I would go through

it with Mom and we would look at it together. In extreme cases we would call in Dad. . . the ultimate say in all things math.”

Her mother stressed practical math skills. Ali recalled,

She would take one or two of us with her to the store and we would practice estimating. And we would, you know, practice adding, you know, keeping track of the total cost of, of buying the groceries. . . . Then we hit the clothing department; we started getting sales. . . . Its let’s go to the store and do math. . . . I want to bake cookies; let’s do chemistry. . . . It was all the time.

A reported another benefit concerning neatness, explaining, “One of the other things that was great about math, it taught me the importance of being neat; because if you garble your numbers too much you’re going to get the complete wrong answer”.

A used Math-U-See curriculum. She recounted, “Algebra 1 and 2. Trig, Geometry.” She enthusiastically remembered,

We used Math-U-See, which was a fantastic curriculum . . . came with these brilliant blocks . . . kit with fractions . . . always really good with making it very visual, tactile. . . . The guy who did the video, he was brilliant, cause he was able to make it interesting and fun. . . . There was always a story that went with it. . . . He just does a really good job of tying to actual life as possible.

During tenth grade, she taught herself trigonometry; she explained,

I basically taught myself trig. I would read through the teacher book and then I would work the problems. And then, Mom would check it with me. And anything I got wrong, instead of her explaining to me why I got it wrong, I would just go look at it again. And if I couldn’t figure it out, my dad could, cause he’s an engineer. . . . I think when I was teaching myself that was probably my favorite. I’m very much a, I, I still don’t play well with others. Not that I can’t get along with people, but when it comes to school, I know really well what I can do and how I work. And so, I am most comfortable doing it by myself.

A completed Calculus I and II at community college. She relayed,

Calculus was interesting because it was one of my first experiences in a typical classroom setting. . . . Ali month into the class, I finally understood what was going on, and how it all worked together. . . . It still went really well. I made A’s. . . . I highly recommend um, homeschooling families who, if their children are planning on going to a university, to get them into local tech school classes before they graduate; because college is a culture shock. But then to add the culture shock of the academic system on top of that is just a lot. . . . That was really, really good especially for me to kind of be familiar with how that all worked.

Her high school transcript verified her SAT math score and her excellent grades in Algebra 1 through Calculus 2. Ali attributed her good SAT math score (660) to her math classes. She acknowledged, “Math has just always made sense to me. . . . At that point I had gone through calculus . . . I’ve always loved it I’ve always been good at it for some reason. . . . We had a couple of DVD programs that we used.”

Ali double majored in chemistry and English. She felt very prepared, explaining,

I took calculus and Diffy-Q in college. And then, you know I was a chemistry major and so there was, physical chemistry is lot of math. Physics itself is just applied math, so there was a ton of math in my college career. I always felt really confident and able to, because it was a classroom setting. . . . I wasn’t the one controlling the speed of the information going on, it was you know usually the group of slow, the slowest people you ever, you know what that means. The people that had a hard time getting it, you know you went by their pace. So that gave me more than enough time to be able to figure out . . . what’s going on.

Emphatically, Ali affirmed the value of mathematics. She elaborated,

Even if science is not your thing . . . you still have to have it to you know, when you’re shopping. . . . If you’re painting your house, you got to figure out how much paint you need to buy to paint this room that’s this big. Um, you know, gas mileage, how much are you going to be paying for gas? I mean it’s literally everywhere and it’s everything that we do and you got to know it. You don’t have to like it but you have to at least know how to use some of it.

She remembered enlightening conversations with her dad; she related,

My dad always told me that math is the language of science and he was absolutely right and it always made sense to me. That it, in this crazy insane world, different things and different people would and cultures and languages like all this madness, math is the same everywhere. . . . It’s just that order, its simplicity, and its following the rules you get the right answer, and there’s only the one answer. . . . If you follow the rules, you can find the right answer. And that order, that has the complexity and brilliance behind it has always spoken to me.

Worthwhile, math provided a language for science and a picture of God. Ali described,

It’s worthwhile because it is the basic language for science which gives us one picture of how the world works . . . a picture of God in a sense because there is a system and there is logic there and it is true everywhere.

Both parents valued math. Her parents provided her with a balance she shared, “Dad philosophized about Math until the cows came home. But Mom was like, ok, what is this good for? Tell me on a basic, fundamental level why do I need to know this?” Her father first worked as a carpenter, then as an engineer. Ali clarified,

Dad worked as a carpenter and that clearly has a lot of Math just right there. Um, so, so he’s always loved math and science. . . . Dad became a Chemical Engineer. . . . He was always very, very clear with math and

science and that was always very easy for him to work with. So, whenever we would have a problem, you know, in our curriculum or whatever, we would ask Dad. . . . Engineeersse was often the language of choice at dinner, because Dad would talk about his day.

Ali's mother inspired a love of learning. Ali explained,

She's not certified as a teacher; she never graduated from college. She has no degrees. But she's been brilliant with all of us. We were all different and she was able to meet us where we were and teach us these things and get us excited about this stuff. . . always had that passion that she was able to give to us.

Majoring in Chemistry and English, Ali graduated with Honors from a prominent South Carolina university. She now works as a chemist in quality control and research.

Ali home educator survey. Ali's mother filled out the home educator survey. The survey responses confirmed the graduate interview data. As the primary teacher, she filled in the survey prior to the student interview. I do not believe that there was any collaboration between the mother and the daughter. The survey answers clarified and supplemented interview information.

Identical items included the ages, sibling information, mathematical classes and curriculum, and the graduate's favorite subject, math. Similar to Ali's responses, her mother explained: the homeschooler's experience with homeschooling as "She loved it," her experience with math in homeschooling as, "Adequate, she loved playing with numbers;" and her preparation for college as, "Extremely well." She believed that Ali's good SAT math performance resulted from, "She understood numbers well and loved playing with them." The mother reported that they homeschooled for "religious reasons."

Her mother identified the math curricula and instruction. Substantiating Ali's responses, she named the curricula as, "Math-U-See." She described the instruction,

For the most part in high school we would watch the video together . . . occasionally we would have other students—then I would explain the newest concept. They would do the pages and we would check them together going over anything that was missed. We did this for algebra, algebra I & II and geometry."

Her mother explained the transition to self-study, reporting, "I was very rusty on trig so we used the same format, but we figured it out together. . . . Half-way through the book, she just took it over and it was self directed w/ questions addressed to [father]."

The mother enjoyed math, for the most part. She wrote,

I was always 'go-to' mom in the support groups as not many were good at math. I loved it up to alg II. I could get into trig if I had to. . . . If I ever had a problem figuring anything out, [Father] was able to do it, engineer.

NAEP assessment. Ali answered 10 of the NAEP assessment questions correctly. Efficiently, she described her work and justified her methods for all ten items.

MSES scores. Ali scored 8.6 overall, an 8.7 on Part I and an 8.4 on Part II. Her total score ranked above the female and male 95 percentiles (Table 10). Her mother's total score was 6.7, a 7.1 on Part I and a 6.3 on Part II, between the female 70-80 percentiles. Her father's score was an 8.6 overall, an 8.8 on Part I and an 8.3 on Part II, above the male 95 percentile.

Appendix T: Hali

Hali interview. Interviewed in her dorm room, 19-year old Hali seemed relaxed. The interview took place after Sunday lunch. Hali answered all questions and offered extra explanation when prompted. The interview lasted 50 minutes.

Her family home-schooled for faith and academic reasons. The public school environment prompted the parents to consider homeschooling. Hali clarified, “Homeschooling for them [aligned with] their faith based ideology for how they wanted to raise their children, especially in the environment that we lived in New Hampshire, at that time.” They wanted to direct their children’s education; Hali related, “My parents both are very passionate about education. . . . And so for them it worked in incorporating that secular desire for just good education with their desire to raise their children in their faith.” Hali defined her family’s faith as, “My family, my parents and my brothers and sisters and I are Christians. And we all are within a Presbyterian denomination, all of us. Um, we have more of a Reformed doctrine in our church practices.” Homeschooling aligned with their Christian principles and potential scholarships. Hali explained,

They still find that homeschooling aligns well with how they want to raise their children in terms of their children learning about their faith . . . being free to fully live out their faith, their Christian faith. . . .

Homeschooling provides my sister with the need-based kind of education that she needs to um, get a head start on getting scholarships and acceptances to good schools.

H represented both advantages and disadvantages. She identified, “Generally, my experience with homeschooling was positive. Um, but I consider myself to be more of a neutral positive.” She appreciated the flexibility and individualization, detailing,

There are many things that I really enjoyed about being homeschooled. I enjoyed the need-based educational approach because I was very proficient in some subjects and in other subjects I needed a lot more motivation. . . .

I really appreciated the opportunities to plan out my own high school uh, schedule of courses how I wanted to do things, especially in terms of, being a concurrent student at a community college and a university. And um, being able to plan out my AP courses . . . and really pursue the things I was interested in academically.

She exposed some disadvantages describing,

There was definitely deficiencies or disadvantages, just as I think there are within the educational system program. And um, some of that just involved not always, being a little slower to learn other teachers’ teaching styles. . . . Perhaps, too much focus on my education and I wish, that I think at times it was too much of an idol.

H felt passionate about languages, while disliking Math. She explained,

Definitely my favorite subjects during my homeschool years [were] foreign languages. . . . I kept adding on languages and keeping up with the previous ones. And I’d tutor Latin as well throughout high school to middle school students. . . . I loved everything about it, from learning different kinds of vocabulary to figuring out phonetics and sentence inflections, syntax. . . . I just found it all fascinating.

Although Hali earned A’s (verified by her high school transcript), she felt much frustration with math lessons. She revealed,

My consistently least favorite subject was always math . . . my absolute least favorite subject . . . would have been geometry. I really hated it. And it was not that I had any major issue that was identifiable or any handicap or thinking block. But I was completely unmotivated and that’s the only thing I can figure out about that. . . . My grades – I always got As in math. But in my mind, I hated math and I was really bad at it, because it was associated negatively with the repeated going over problems and getting into conflict with my mother.

Her dislike of mathematics caused conflict with her mother. Hali lamented,

The math academic work that I was undergoing was very stressful. And um, was really the crux of a lot of relationship issues I had with my mother, during those years. So, there was a lot of stress there. . . . The math was making our relationship less peaceful. . . . Actual math academic work was very stressful.”

Despite her stressful experience, she performed well on the math section of the SAT, verified by her high school transcript.

Although Hali felt her SAT score (620) unimpressive, she attributed her performance to her test preparation and her math foundation. She detailed,

I scored a 620 . . . in my mind that wasn’t not very high. . . . I certainly went through plenty of SAT math prep books. . . . I was very nervous about it. And those helped somewhat, but I think a lot of it was just the thoroughness of my math education up through Algebra II that was drilled into me. . . . I did practice tests, especially in math. . . . I definitely did focus a lot on that.

Hali progressed from one-on-one instruction to self-teaching. Initially, her mother taught her with Bob Jones curriculum and then she transitioned into Saxon textbooks. Her mother carefully trained Hali to become more independent. Hali specified,

We used Bob Jones University Press Mathematics books, which I liked because they were colored, from preschool. . . . [to] 3rd grade. . . . Then I used Saxon from 4th grade all the way through 9th grade, Algebra II. . . . When I was a little girl and going through my math course, my mother would go through it with me every day. She was the primary educator in our homeschooling family. And so, uh, she would be at my side and would teach me the basic concepts or whatever was the lesson. And then, she would usually, my mother was big on independence, and so, she would usually step back and let me finish the lesson. And then every day she would grade the lesson and go over it with me. So, as I got further into elementary school, the time that she spent explaining something lessened in the time that I spent going through the lesson on my own increased. And, then by the time I was getting into pre-algebra and algebra I, um, it was very independent . . . I was basically teaching the concepts to myself.

Hali devoted a huge amount of time to math work, up to three hours. She described, I spent a lot of time on Math. That was the one subject that took up more time than anything else, even the year that I was taking three languages and tutoring one. . . . I did have OCD at that time. . . . I would get up at 5 or so and I would just be alone at my desk and I would start on my math and by the time that we had breakfast, around 8 o'clock, I would probably have finished my math lesson for the day. And it was grueling. . . . It just wasn't learning the lesson, it was also correcting all of the errors from the previous day's lesson. . . . I redid all of the problems that I missed and then I would go through the lesson for the day and go through its concepts and the practice problems and then I would do the lesson. Starting with Geometry, Hali switched her learning environments. She remembered, Once I got into tenth grade and was taking Geometry, my math education started to go downhill. . . . That was the last math course I took at home . . . I didn't enjoy it and I didn't apply myself. And so I don't think I learned as much as I could have.

The next year she took an inadequate pre-calculus resource center class. She disclosed, Pre-calculus was the first math course that I took outside of the home and it was taught by a teacher who was not, uh, was not suited to teach the course. And it was much simpler than math I had done since middle school. And, so, I would say my pre-calculus, trigonometry, basics of probability and statistics, that kind of advanced math stuff was inadequate. . . . And I didn't have to study for that class. Other than doing the homework on my own and then being in class.

The next year, Hali enrolled in a local community college for statistics. She conceded, "By statistics, it was, the community college structure of the class was memorize the formula, come into class, regurgitate it for the test. So, I don't think that I actually really learned math past my year of geometry." Hali acknowledged that she enjoyed the classes outside the home, but did not learn as much as by self-study, admitting, I very much enjoyed it transferring the math authority figure in my life from my mother to another person. . . . Taking it with peers really just pushed me to want to do well. And I didn't have that problem with other courses, but I really, really enjoyed that peer group for math. . . . I do love the way that I was taught forced me to teach myself, because I really remembered it that way. But on the other hand, I did so much better in terms of my enjoyment of math when I was taking it with peers . . . so it's hard for me to decide which way worked better for me.

Because the one I enjoyed more, but the other one I actually think I learned more from. Hali felt well equipped her for college mathematics. She related, I've been adequately prepared. . . . For my major, there's only one math course required, which is a Statistics course, which I took in high school. . . . For . . . courses that require math . . . I feel very adequately prepared . . . goes back to the basics of . . . elementary school math principles, and discipline of how to approach problems. . . . Fundamentals I learned from algebra have stuck with me.

In college, Hali appreciated her homeschool math much more. She acknowledged, My perspective on math has changed, retroactively or retrospectively since high school. And I have a much greater appreciation for math as a subject, than I used to. And I in a lot of ways, long for those years back that I can do them over again, because I appreciate it as a subject now. So and appreciating more what my mother especially, both of my parents were doing for me with math education."

She realized the benefits of her thorough program, declaring, My math education I think was very adequate. I think it was wonderful. . . . It was very thorough. It was grueling to me because I didn't enjoy it. But it was very thorough and adequate, beyond measure, very extensive. Um, and I found that as I went into other subjects, such as chemistry in high school and chemistry in college, I really do have an excellent math foundation for understanding things. . . . That was the subject that I had to be diligent on, year after year after year, instead of just losing a year in math. Um, so I think math was being homeschooled was very advantageous to teaching me math. . . . I am very grateful that I was homeschooled for my math education because I wouldn't have had that ability to do that in another school environment. So I actually think that, even though math wasn't the subject that I excelled in, math like, homeschooling helped me to excel in the subjects that I worked, that I was really gifted at. But it was also really good for me in forcing me to keep up and be diligent in subjects that I would have totally dismissed in another schooling environment.

The only thing that Hali would have changed was to include another student, declaring, “I wish that I had taken math alongside a family friend or someone like that and shared the math teaching responsibilities with another homeschooling parent.”

Hali emphatically agreed that mathematics was useful, sensible, and worthwhile. Math enhanced your brain capability, she claimed, “I consider mathematics to be of an extremely useful nature. . . . The exercise that it gives to your mind to be able to solve it and the resourcefulness, the creativity, the logic that it takes is so healthy”. Describing math to be sensible, she explained,

I find math to be very sensible. I love it. . . . I wish that I had more of a broad-minded understanding of different disciplines within math. But what I do understand, I take pleasure in the fact that it is sensible.”

Interestingly, Hali believed math was worthwhile, especially for females, asserting,

I wish that girls would go farther with math than I think that they do. . . . I just think it’s very worthwhile and I don’t know. I just think we have some sort of weird gender misconception that boys are good at math and I don’t see any real reason for that.

Both Hali’s parents possessed science background. Hali explained,

My father has an excellent background in math. He studied chemistry in undergrad and then computer engineering for his graduate degree. My mother was a biology/pre-med in her undergraduate degree. So they both had this, they come from a science or math or applied math background.

Her father taught Hali about financial matters. She fondly remembered,

We really did have a lot of family conversation that was devoted in some way to math. I had brothers; my father is an engineer and is gifted in math and especially in economics and understanding financial matters. . . . Talking about compounded interest; and savings and mortgage rates and sales tax. . . . We had a lot of conversations like that. Even today, we still do.

Hali summed up, “My parents certainly are passionate about math education”.

Currently, Hali attends the Honors College within a large South Carolina public university, 130 miles from her home. She pursues a Bachelor of Science in Nursing.

Hali home educator survey. The survey data confirmed the interview information. The primary teacher, M’s mother filled in the survey after the interview. Due to the distance between them, there was little chance for collaboration. No direct contradiction existed and survey answers clarified interview information.

Identical items included the age of the student, the sibling information, and the student’s favorite and least favorite subjects, languages and math, respectively. All mathematics classes were mentioned. The mother described Hali’s experience as “Overall, I believe our student was thankful for home education and valued the effort and outcome. There were difficulties, some associated with being home schooled. However, difficulties are a part of the normal overall development of the student.”

Christian education motivated them to homeschool. She wrote, “We wanted to provide a quality Christian education that glorified God.” About the continuing reasons, her mother reported, “Our reasons remain the same, although we also see the benefit of filtering or administrating cultural influences.”

The mother’s answers aligned with Hali’s about the math experiences. She favored the Saxon curriculum, explaining, “My favorite curriculum vendor is Saxon Publishing. [Their] integrated approach to the material affords the student the repetition necessary for mastery.” She also acknowledged the graduate’s lack of motivation for math, admitting, “[Hail] demonstrated the least zeal for pure mathematics. Applied mathematics were of little appeal as well, but more acceptable to her as she saw the direct application.” Additionally, her mother recognized the desire for more peer interaction stating, “I would refine some of the presentation of these courses if done again. Perhaps I would provide a more social environment.” Hali’s math education prepared her for the college and SAT math, declaring, [Hali] was well prepared for mathematics. . . . Diligence, hard work and faithfulness! The tools of being a student and the application of these tools to the subject of mathematics afforded her the ability to do pretty well on the SAT. These same tools will enable her to be a good student in other academic arenas.

Her mother stressed mastery, asserting, “Mastery is essential. If the student did not ‘get’ material we honed in on this area. By the time she took a test, there was a fair certainty of mastery.” Praising Hali’s, her mother maintained,

This was NOT this student’s favorite subject! Stress was common as well as a general lack of zeal. Had a positive relationship with Christ and had a developing maturity that eased the process. I was proud of her.

The mother valued mathematics and acknowledged it took effort. She reported,

God is the master mathematician. To know this area, to value this area is to honor, to glorify to enjoy God. Although I am competent in math, I would not describe myself as a master of it. I too, have to work on learning math, re-learning math and applying math to opportunities I am afforded. I am not afraid to learn.

NAEP assessment. Hali answered nine of the NAEP assessment questions correctly. She described her work and justified her methods for all ten questions. Comparing her answers with the 12th grade sample taking the assessment, 72 percent also missed that item.

MSES scores. Hali scored 7.9 overall, Part I (7.7) and Part II (8.1). Her mother’s total score matched H’s, at 7.9, Part I (8.3) and Part II (7.6). The father scored 8.5 overall, Part I (8.4) and Part II (8.6). All three ranked in the 95 percentile (Table 10).

Appendix U: Ben

Ben interview. In the quiet classroom, 20-year old Ben eagerly participated. He answered all questions and offered extra explanation when prompted. The interview lasted 78 minutes.

His parents wanted to participate in their children's education and to protect them from bad influences. He explained, "It kind of started off as I think Mom and Dad they really wanted to have a big like a big part of our learning career. . . . they wanted to have a big part in our development." Until he was 10, Ben was small for his age and his parents worried about bullying. Ben related, "I know one of the things that Mom said was they were always afraid to send me to public school, cause they thought that I was going to be bullied." The schools were unsafe and not the right place for Ben. He informed,

Some of the schools that we were zoned for weren't necessarily the best schools in the area. Like especially the middle school. . . . during my high school years they started having a lot of gang issues. . . . I'm just kind of one of those people that who is out there and I would get in trouble a lot. So, it probably would have been bad for me to be in a situation like that.

Ben positively described his homeschool. He reported,

My experience with homeschooling, I just have to say that it was, I mean, it was fantastic. . . . I was like, 'I don't want to go to public school. I don't want to have to deal with all that social, like the social craziness'. Yet, he saw himself as a normal person, not a homeschooler stereotype, emphasizing,

I'm not a homeschooler, I was homeschooled. Because when people think homeschooler, they think calico dresses and socks, uh, jean shorts pulled up to your way past your navel . . . white T-shirt tucked in. . . . I'm a normal person.

He valued people that he interacted with, stating, "The people that I was around, it was really fantastic. . . . The people that I was around in homeschooling really influenced me." Ben enjoyed chemistry, history and English, but disliked math. He clarified,

Chemistry, of course. . . . My teacher for chemistry in eighth grade [Hali's mother] was absolutely phenomenal. . . . [she] taught me, her daughter [Hali], and my friend. . . . I enjoyed learning about history. . . . I enjoyed English. . . . Surprisingly, I never really liked math very much. . . . But it was just one of those things, it was kind of always there, 'Aw, I got to do math now'.

In retrospect, Ben realized his homeschool math benefited him greatly. His curriculum included basic math skills, Saxon, and Teaching Textbooks. He detailed, "We just did basic math. We did like times tables. . . . I did Saxon 4/5 . . . to Algebra I in eighth grade. . . . I took Algebra II and Geometry with Teaching Textbooks". His mother motivated him towards mastery and self-study. Ben remembered,

Mom was great at math. . . . Up to eighth, ninth grade year, she sat down and read the lesson with me. . . . If you get like this many right, she's like you can do half the problems. . . . If I got them wrong then the next day I would have to do all the problems, which I think back now is 25 problems. . . . It gave me incentive and that actually really, really worked for me. . . . cause I hated doing work.

Self-study worked well for Ben. He asserted,

And that worked well for me. . . . It was really self-driven because that was what I, I was good, better at the math stuff and I could learn it. And if I sit down with a math book. . . . I can memorize math stuff and then apply it. . . . I think that being able to do things on your own, really prepares you well for real life. Cause in real life you don't have someone standing behind you with a whip, going do this do this. . . . It teaches you to be able to assess things and learn things on your own.

Saxon provided a better foundation than Teaching Textbooks. He contended,

The courses that I took in the beginning, which was Saxon, they gave me the foundation that I needed. I took Saxon for Algebra 1, Teaching Textbooks for Algebra 2. I remember so much more Algebra 1 than Algebra 2. . . . Saxon . . . I would look at the problems. I'd say this is how it's solved. . . . And I would remember how it worked. . . . Saxon was hard . . . but you learn a lot. . . . Teaching Textbooks they did a good job . . . but for me it didn't work. . . . They tried to oversimplify it. Uh, and I would get bored . . . it would take them so long.

His high school transcript verified his math courses and his grades, 89 to 101. Ben took two classes outside of the home. He didn't learn much, asserting,

When I was at home, it was very adequate. When I left the home, it was inadequate. . . . Eleventh grade, I would basically say that that was the first year I stopped taking math because I took pre-calculus . . . it wasn't really pre-calculus . . . I did take it with [Hali] . . . I really honestly feel like I didn't take math that year. . . . The class was fun, but we just really didn't learn much. . . . Then, I took calculus at a tech school. . . . didn't apply myself in that class enough and I just went in and took the tests and made A's because I could memorize.

Ben felt he had a good math foundation, but expressed regret over his SAT math scores (610), which was verified by his high school transcript. He revealed, "I didn't do nearly as well as I wanted to. . . . The SAT was not my test. The ACT was my test". Still above average, he acknowledged, "It's because I knew math. . . . I would look at the problems and I'd be like, I've seen this before. . . . second nature type of thing".

Ben shared his initial struggles with college calculus. He reported,

The speed with which I jumped into calculus was difficult. . . . The wheels were rusty. And I did fantastic on my final exam. But before that. . . I had some really bad tests and I was just thinking I have this oh my gosh, I can't believe that this is what's happening. Like I'm trying to just scramble to catch up.

Reflecting on his homeschool experience, Ben believed it saved him from failure. He revealed, "I do think that it was the best thing for me and also at least shaping who I am today." Ben was a rough child, admitting, "I'm amazed that Mom has put up with us that long. I was kind of a rough child." Lacking motivation and discipline, he would have become the lowest common denominator in schools, disclosing,

In today's society where like the schooling system where everything is taken and put down to the lowest common denominator . . . so that no one feels bad about themselves. I think that homeschooling was definitely the way to go for me, cause I would have been that kid that would be like, pretend like I'm stupid . . . just glided through school. . . fairly recently did it really click, the true importance of having a good education and why I was doing it. Cause I realized that it's going to affect the rest of my life. And I want to be educated . . . I'm really glad I didn't have to deal with this when I was younger. . . . When I was younger, I wouldn't have cared; sure, I'll drink. But now . . . I kind of like have a check in my spirit.

Grateful for his parents' persistence and leadership, Ben admitted,

My mom and dad were really strict with me for a while because I kind of was a trouble child. . . I am really realizing now is that they were giving me a good strong foundation and they wanted me to understand why we believe this and why we believe that and why we did things this way and not that. So, I think that just it was definitely the best way for me. . . I didn't want to succeed, that was something I had to learn. . . . If you're pushed to your full potential; one day you'll realize why. . . . That was the most beneficial thing for me. And I think that it has really shaped who I am and I'm striving to be.

Ben's mother acted as the primary teacher and his father supported her. Ben labeled his Dad as the pusher and his mother as the enforcer, describing, "As for school, Dad's been like the, well Dad kind of pushes us, and Mom is the enforcer." His mother's family valued education most. Ben explained, "Academics has always been really important to her side of the family. Whereas, Dad, the way he grew up, academics wasn't just as important to them." Both parents obtained college degrees; his mother trained as a physical therapist and his father worked as a salesman. Ben detailed, "She's actually a physical therapist. . . . What he always wanted to do is what he is doing now, sales. . . . He's really good at interaction with people and getting things done." He spent time with B. Ben recounted,

I got a broken car and I rebuilt it. . . . I figured out what was wrong with it. And I put it back together again. . . . I took that project on as my own. And Dad kind of would come out and give me advice. . . . Every time we would go out to the shop . . . and we do stuff . . . learning how to fix things, do things with my hands.

Ben valued mathematics stating, "I think that math is useful, sensible, and worthwhile." He understood its practical nature, describing, "When I was younger I thought I never am going to use this. I worked construction this summer. . . . This assessment [NAEP], insulation thing I laughed cause . . . I knew the answer to this problem". B thought math sensible and essential, stating,

If you didn't have math, you wouldn't be able to function. . . . When I was building stuff, you use algebra a lot because like, you would have all these equations. . . . 'How much concrete are we going to need'?

Currently, Ben attends a South Carolina college, 100 miles from his home. He majors in Biochemistry.

Ben home educator survey. Ben's mother, the primary teacher, filled in the survey after the interview. I do not believe there was discussion between the graduate and the educator about the questions. No direct contradiction existed and some survey answers clarified interview information.

The survey data confirmed the graduate's interview. Identical items included the student's age, the sibling information, the mathematics classes, and the curriculum. The mother described the graduate's experience with homeschooling as, "positive," his experience with math in homeschooling as, "Grasps concepts quickly. Lazy about doing daily work," and his preparation for college mathematics courses as, "Had difficulty with calculus." Concerning math instruction, she answered, "All math until 11th grade done at home for [Ben]. Reworked incorrect problems."

The main reasons for homeschooling included academic and Christian character development. For the initial reason she wrote, "Not ready to have first child leave home for kindergarten." She reported the continuing reason as, "Best meets the intellectual and spiritual needs of each child." This matched with Ben's information.

The mother acted as the primary educator. She selected Ben's curriculum, assisting him when needed. She described her involvement as, "I was more hands-on with this subject." She liked math, reporting, "I like math and see it as 'puzzle working.'" She wrote that both she and her husband earned college degrees and completed math classes, answering, "We are both college graduates and took required math."

NAEP assessment. Ben answered nine of the NAEP assessment questions correctly. He described his work for all questions and justified his methods for eight items. Comparing his answers with the 12th grade sample taking the assessment, 78% of them missed the same item.

MSES scores. Ben scored 7.3 overall, an 8.0 on Part I and a 6.5 on Part II. His total score ranked between the male 70-80 percentiles (Table 10). His mother total score was 7.9, an 8.1 on Part I and a 7.8 on Part II, at the female 95 percentile. The father scored 7.6 overall, a 7.4 on Part I and a 7.8 on Part II, above the male 80 percentile.

Appendix V: Que

Que interview. In a quiet classroom, 21-year old Que appeared comfortable. She answered all questions. The interview lasted 52 minutes.

Que's family home-schooled for academic and spiritual reasons. Her mother thought it would be best to keep her home a little longer. Que narrated, "I was the first-born and my mom wasn't ready for me to leave home yet."

Family unity and their children's worldviews provided motivation to continue. Que explained,

We just continued. . . . To foster a close-knit family. . . . My parents really wanted to be able to shape and form our foundational beliefs and worldviews, in a way that they didn't feel they could do if we were spending eight hours of our day in a school setting under the teaching of someone who might or might not be a believer."

She defined a believer as, "When I say believer in context, a believer in the Lord Jesus Christ, someone who believes in the orthodox Christian faith."

Overall, Que loved her homeschooling years. She felt secure, clarifying, It's helpful for me to feel that my mom just sort of didn't want me to leave home yet and we just kept going. . . . They did want to provide that safe environment for us to grow up in especially when we were young and very impressionable.

Que appreciated the flexibility and exposure to different types of people; she affirmed,

I really, really loved the experience. It gave me some wonderful opportunities to grow in ways that I don't think I would have grown had I been surrounded by peers at a younger age. . . . gave me experience interacting with people who believe very different things from very different backgrounds."

Que likened her homeschooling to her college experience, asserting,

Because I wasn't spending hours a day in a classroom setting going at a pace set by, partly by the peers around me and by a teacher, I really was . . . having lots of time to simply read and to really explore intellectually and grow intellectually through reading, listening to lectures. . . . The homeschool experience I had was much more like a college setting and a college course load. . . I was able to go to a class once or twice or sometimes three times a week, usually once or twice. But, and then, spend the rest of my days really in independent study and research in reading, which was for me, it really allowed me to flourish.

Additionally, she took concurrent courses at a local university and classes at nearby resource centers. Que related, "My senior year I took concurrent classes, classes as a concurrent student at a local university. . . . Throughout high school I took classes at different resource centers." She enjoyed her classes, except for math. She admitted,

It's ironic, because I really loved every academic subject, apart from math, which is unfortunate. But I know it reflects the order of God's creation and it is valuable as a discipline. I just have never really, I, been able to wrap my mind around it easily at all. I forget it very quickly. . . . I'm just not very good at it."

She struggled with math related subjects, explaining,

Physics was challenging for me. And again using the math and the, you know the scientific concepts weren't something that came as easily. . . . math was kind of my subject that you know, I worked hard at it to get A's.

Yet, Que acknowledged a sense of accomplishment from mastering concepts, indicating,

It wasn't easy and it wasn't my favorite subject to study. But it was rewarding, once I learned the material. . . . When, you know the light dawned and I understood it and I could do the problems; that was satisfying. I would say, more of a sense of fulfillment and satisfaction."

Que praised a participating family, without knowing their involvement, saying,

[Jones] they are one family in the homeschool community, that we were in, just you know kind of our friend group, something that made them really stand out they got math. . . . who were four or three years ahead in their math curriculum. So that is something that really, they're very bright and you know [mother] was very well prepared to teach them high level math.

Que took challenging courses, using mostly Saxon curriculum. Her transcript verified these courses and her very good grades, 93-97. She specified,

Algebra 1, Geometry, Algebra 2. . . . In 11th grade I took, I was finishing up at home, with a trigonometry textbook. . . . We used Saxon from the time I was very young. . . . Chalkdust for Geometry.

Que progressed from direct teaching to self-study. She elaborated, "In elementary school my mom would go through the lessons with us on a daily basis and then we would work the problems. She'd kind of leave us to do the problems. And then we'd talk about it." As Que matured, she primarily taught herself, describing,

It was primarily self-instruction. . . . I had my math textbook; I would read the lesson. I would work the problems. . . . By the time I got into middle school/high school I would grade the lesson. . . . I did spend a lot of time in independent study . . . somewhere between an hour and 2 hours.

Her mother acted as a tutor and corrected Que's work. Que explained, If I was stuck and didn't understand the concepts my mom would explain them to me. . . . She would check the math. And if I missed a lot of problems, she would have me look at them and typically we would go over them together.

Math work was stressful at times, with her mother. Que admitted,

Most of the hostile harsh interaction. . . . I think almost all of that was over math. . . . But I would get frustrated with her, often, if I couldn't understand a concept. And so then, she would try to explain it to me

and I would get mad. You know, when, it wasn't her fault. . . . As I got older, you know I realized, that's wrong . . . that's not ok to be frustrated and turn angry when I'm having trouble with the concepts. . . . I do remember math lessons taking me a very long time.

A classroom setting would have not worked any better. She reasoned,

I don't really think that I would have done better in school. I think that because I was homeschooled I had more time, probably spent more time on the math problems than I would have otherwise uh, in a classroom setting. . . . I learn better by reading something and really thinking about it, on my own. . . . So I think that had I been in school, and only had limited amount of time in the evenings to do homework. . . . I probably would have gotten lost easily.

Her homeschool experience prepared her for the SAT and college math. She worked hard, describing, Math is a subject that, I, if I'm given a text, and given problems, I study it; I did the same thing with SAT and barely got a 600. But, you know, I worked to prepare. . . . I did a Kaplan course and took several practice tests. I used a Kaplan prep book and I had another prep book. I think maybe it was the Princeton Review. I worked through those. . . . So you know, because I don't tend to do well on standardized tests at all, I studied a good bit, and learning the math tricks.

Her high school transcript verified her SAT math score (600).

Que mastered her college statistics class, despite initial concern. She remembered,

My first test and I actually got an 82. . . . I never had gotten a B on a test before. I was very upset. . . . After that test, I studied a good deal for that course; I'm sure I went to the professor, probably spent an hour in his office each week if I didn't understand something. . . . By the final, I got a 99 on the final. So, you know, that was not because I'm so good at math; I'm definitely not. But I worked the problems and studied and um, the Lord was gracious in giving me stamina, when I didn't feel like continuing to study the subject.

She felt well prepared because she learned how to learn, affirming,

My homeschool experience prepared me well, in that I knew how to study. . . . I had the tools that you know, that Dorothy Sayers talks about students not just learning facts, but learning how to learn. And I did really learn how to learn as a homeschooler. And because of that I was able to do well on the math course.

Que agreed that mathematics was useful, sensible, and worthwhile. She recognized its value. She explained, "I do use math on a daily basis you know, not high level, complex math per se. But you know if you don't have confidence in using basic math, it is a handicap." As part of God's creation it made sense to her; she mused, "It reflects the order of God's creation . . . and it is valuable, even if it's not your skill set."

Both her parents modeled the importance of math. Her mother trained and worked as a physical therapist.

Que explained, "My mom you know went to medical, well you know to P.T. school and of course, took calculus. . . . She was always good at math." Her mother supervised Que's math, all done at home. Que conjectured, "I'm going to speculate why we also did math at home, I think it's because my mom. . . went to med school to be a P.T. . . Her strengths are in the sciences and in math."

Impressed by her father's practical application of math, Que relayed, "He is very handy. . . . He built an entertainment center. . . . He had to measure all the wood, cut it in all the right dimensions. . . . He uses basic math with proficiency."

Que wants to homeschool, stating, "If I do marry and have kids, and if I do homeschool that's what I'd like."

Que double-majors in English and Spanish at a small South Carolina college, located 100 miles from her home. Que belongs to Alpha Chi, an academic honor society.

Que home educator survey. Que's mother, the primary teacher, filled in the survey after the interview.

There was little chance of discussion between the graduate and the educator about the questions. No direct contradiction existed with interview data.

The survey data confirmed the graduate's interview. Identical items included the student's age, the sibling information, the mathematics classes, and the curriculum. The mother described the graduate's experience with homeschooling as, "positive", her experience with math in homeschooling as, "Reluctant had trouble grasping concepts. Needed to talk through problems", and her preparation for college mathematics courses as, "Only statistics and made an A." About instruction, she answered, "[Que] had to have more individual attention working problems."

The main reasons for homeschooling included academic and Christian character development. For the initial reason she wrote, "Not ready to have first child leave home for kindergarten." She reported the continuing reason as, "Best meets the intellectual and spiritual needs of each child." These matched with Que's answers.

Her mother played a significant role in Que's math education. She described her involvement as, "I was more hands-on with this subject. I like math and see it as 'puzzle working.'" Both she and her husband earned college degrees and completed math classes, answering, "We are both college graduates and took required math."

NAEP assessment. Que answered seven of the NAEP assessment questions correctly. She described her work for nine questions and justified her methods for all ten items. She missed items 8, 9, and 10, as did 55%, 42% and 72%, of the test sample. Upon closer examination of her work, one of the incorrect problems (#8) met the conceptual understanding standard, while another (#9) met the procedural fluency component. This would bring her to 80% in both categories, allowing her to satisfy those strands.

MSES scores. Que scored 7.3 overall, a 7.2 on Part I and a 7.4 on Part II. Her total score ranked between the female 80-90 percentiles (Table 10). Her mother's total score was 7.9, Part I (8.1) and Part II (7.8), matching the female 95 percentile. The father scored 7.6 overall, 7.4 on Part I and 7.8 on Part II, above the male 80 percentile.

Appendix W: Kim

Kim interview. Interviewed in her kitchen, with her mother sitting close by, 18-year old Kim seemed comfortable. The interview took place before Saturday lunch. While her parents and brother quietly moved throughout the house, Kim answered all questions and offered extra explanation when prompted. The interview lasted 43 minutes.

Her family learned of homeschooling and its benefits from family friends. Her mother saw the results and considered it a better option than the public schools they were zoned for and expensive private schools. Kim explained, Mom found out about homeschooling and the benefits of it. . . a better option than the public school that we were zoned for. . . We would have more opportunities to study things that were interesting and it just would be a better environment. . . a better option than the private schools nearby cause they are really expensive. Her family continued to homeschool based on the flexibility it afforded. Kim elaborated,

It's been the better option for me. Um, I think there's always been an opportunity for me, if I really wanted to go to private school or public school but I just found that this was the, the way that I learned to learn the most and there's the most flexibility and most opportunities and options of things that I wanted to study.

Kim positively described her homeschooling experience. She confirmed, "In hindsight now, I'm really glad that I did it [homeschooling] and I think it was a good decision for our family." Providing many learning experiences, she recalled,

When I was younger, we did a lot of curriculum specifically at home. . . . As I got older we started to do more co-ops and things where we would go to class once or twice a week and meet with other families. . . . I would be with students my age and um, with a different person teaching the class. . . . It helped me become more independent . . . and um helped me learn how to learn. . . . It was also a good way to get to know kids who were sort of similar to me in that they were homeschoolers. And so that was a good thing.

Kim enjoyed a theology class and her math, while disliking writing and biology. She recalled her theology class, "I really enjoyed taking theology last semester . . . it was discussion based and . . . hear different people's opinions." Kim discussed her math classes, "I also enjoyed my math class. I always was blessed with good teachers for math". Although she felt proficient at writing, she didn't enjoy it, revealing, "I feel like I have become a relatively proficient writer. I've never really enjoyed it and so that was always a struggle for me." Biology was difficult for her, but she liked it a little better, informing, "Biology was kind of tough too. But it was a little more interesting than writing so it was easier to pull through."

Kim discussed her math classes and instruction. She explained,

I did take a pre-calculus and for those courses geometry was sort of mixed in since we used Saxon. So geometry and trigonometry was in that. Then last year I took calculus. For most of it, we actually used Saxon because we found that that was just, I don't know, one of the better textbooks to use. At one point, we did use Teaching Textbooks, which was good. . . . Then last year for calculus . . . it was a college calculus book.

Kim recalled her early homeschool math instruction. She reported, "With elementary school, Um, it was really just my mom teaching me, mainly for math. And ah, so that we used Saxon and I don't remember that being much of a struggle." Kim also remembered conversations with her parents that she labeled as "teachable moments", recalling,

My mom was always looking to find teachable moments especially with math. And so, at the grocery store if we were going to buy something, you know, it was always a teachable moment. Like it's not really going to be that; it's going to be more with tax. Or um, with my first job, there was a lot to take into consideration with taxes and filing taxes. . . . To write a check, it was always a teachable moment.

Kim progressed to taking math classes with other homeschoolers. She detailed,

Starting middle school and through high school, it was sort of set up like the co-ops. . . . They were mainly in like classes around like 10 students or less. . . . I would go like once a week. And um, the teacher would give us a seminar on like four lessons or something like that and then by the next week we were expected to do those lessons. . . . That always worked really well, especially for Saxon, cause that's the way it was set up.

Her mother still supervised her assessment. Kim explained, "The people that I was being taught by weren't technically my teachers, my mom was my teacher; I was accountable to her . . . taking the test I would give them to her and she would grade them". Kim mastered the concepts with some help from Saxon DVDs and these tutors, revealing,

We always have the solutions manual so. . . . I was able to check it. . . . With Saxon they have the DVDs that help you like work through the different problems and so sometimes I would use that. But more often than not my tutors were always very accessible. So I could email them with any questions that I had.

The repetition from the Saxon curriculum greatly helped K. She claimed,

It was definitely helpful to get through the problems that were harder and to see them over and over again so that I could apply um, what I was learning. . . . It was helpful just that things come up again and again.

She spent differing amounts of time on her math, stating, "When I was a lot younger, I remember it being really easy so not much time commitment. But in high school, I say maybe like an hour or two at a time".

Kim completed calculus with the help of a well-qualified tutor. The tutor was Advanced Placement (AP) certified and selected a college level textbook. Kim described,

Last year I took calculus and it was technically AP Calculus cause my tutor was able to get AP level certified . . . My tutor thought it would be better. . . to get acquainted with a college textbook.

Her tutor met with her twice a week, Kim recalled, "When I took calculus . . . it was actually a one on one situation where my teacher taught me a section a week, and then um, another day of the week we would. . . trouble shoot. "

Kim spent more time on calculus than she had any previous math course, recounting, “With calculus, I probably put in more time than I had in the past. . . . Calculus was a big step from pre-calculus. So, it was a lot more challenging and I needed to put in a lot more work for that; so, maybe a couple hours at a time.”

Kim felt well prepared for her college calculus course. She elaborated,

This semester I took calculus again . . . I was able to do things faster and get through it a lot more smoothly than the first time and a lot more smoothly than I think I would have if I had just jumped ahead to the next one. So, um I think that was a very good preparation.

Kim experienced stress when she encountered a difficult concept. She admitted,

The stress came in when there was something that I really got stuck on. . . . I’m used to math coming relatively like smoothly to me. . . . When I took calculus, it was a big step up and it was a lot harder. So, that was a little bit more stressful. Um, but I was able to work through it and so it turned out, it turned out alright, but I think that’s when the stress kicked in when I really could not get something.

She felt a sense of accomplishment, when mastering difficult concepts, remembering,

I wouldn’t say that math was easy for me but it um was possible and it like, things made sense and um, if I worked hard enough at it like I was able to get things and move forward. So that was always really rewarding for me when I would accomplish things that were challenging.

In retrospect, she found math enjoyable. She affirmed,

Now I feel like it was more enjoyable. I don’t know if I would have told you that in the middle of it. But now I can see the benefits and I can see now how much I learned and how helpful it’s all been.

Discussing her stellar SAT math subtest results (730), (verified by a College Board SAT report), she asserted,

I got a 730 on my SAT and I think that a lot of that had to do with I actually worked with a lady who was in the SAT math tutor. . . . It was really helpful for me to work with her and understand the different questions on the SAT, cause we studied like exactly what the SAT is gonna ask.

Kim recalled working with her brother on math problems. She remembered, “Since we got older if we were at home alone together and he’s working on math and has a question. Then I do like to help him sometimes, if he needs it”.

Mathematics was useful, sensible, and worthwhile. Explaining its practical, sensible, and worthwhile nature, she stated,

Useful, even if you’re not going to be a civil engineer. Like I said with those teachable moments, like at the grocery store or with taxes. . . . And so, um, I think that we use it all the time and it makes things easier and quicker if you are proficient at it. . . . There is so much that you can do with math. There’s so many advances that can be made in society using like math concepts so I would say yes, definitely worthwhile.

Both her parents worked in math-related fields. Her mother was a pharmacist. Kim clarified, “My mom’s a pharmacist. And so she was a Bio-Chem major in college. Um, and so she is more science-focused and there’s a lot of math in that, obviously.” Her father worked as a financial planner. Kim explained, “My dad is actually a financial planner . . . more in the context of money and finances, obviously.

Currently, Kim majors in Civil Engineering in a large South Carolina university, 130 miles from her home.

Kim home educator survey. The educator information came in two different formats. The mother filled out the first page and answered the remaining items verbally. She provided insight into the reasons for homeschooling, established the interview report, and clarified the different math curricula.

The mother presented three main reasons for homeschooling. These included a better environment, academics, and Christian influence. She wrote, “Better atmosphere, academic freedom, and ability to use a Christian curriculum.” These reasons continued to motivate them throughout their homeschooling years.

She verified the graduate’s account. Identical information included the age of the student, the sibling information, the years homeschooling, and the graduate’s least favorite subjects. She confirmed Kim’s math classes and experience as, “It was varied and diverse, giving her a broad education”, her experience with math as “Adequate”, and her preparation for college mathematics courses as, “Well prepared, made an A in college level calculus.”

The educator supplemented the graduate information for the curricula for the early years and the brief usage of Teaching Textbooks. Before Saxon, they used a classic curriculum, Ray’s Arithmetic. These small hardback books (resembling McGuffey’s readers) provided practice with basic mathematical concepts. She reported that there was a lot of personal interaction among K, her brother and the mother throughout elementary school. K’s mother clarified the use of Teaching Textbooks for a brief time in middle school. She wanted to try that program; it sounded promising at a curriculum fair. Disappointed, the mother switched Kim back to Saxon after a few months. Kim continued to use Saxon until calculus, when the math tutor suggested a college level text. The mother managed Kim’s homeschooling program.

Mathematics knowledge was a “gateway” to better vocations. She selected Kim’s curriculum and provided resources. The mother talked about her horrible math experience and wanted a better math education for her children. She carefully researched options. As Kim progressed, she enrolled Kim in a tutoring program with other homeschooling students. This program differed from a co-op; the parents pay an outside tutor. She acquired a “fearless tutor” to teach Kim calculus. She worked as a pharmacist and the father had an MBA, working as a Financial Planner.

NAEP assessment. Kim answered eight of the NAEP assessment questions correctly. She described her work and justified her methods for all ten items. Comparing her answers with the 12th grade sample taking the assessment, 48% missed the first one and 55% missed the second.

MSSES scores. Kim scored 8.3 overall, an 8.6 on Part I and a 7.9 on Part II. Her total score exceeded the female 95 percentile (Table 10). Her mother’s total score was 8.1, an 8.2 on Part I and a 7.9 on Part II, also exceeding the female 95 percentile. The father scored 6.8 overall, 7.4 on Part I and 6.1 on Part II, above the male 60 percentile.

Appendix X: Lea

Lea interview. After a full day of work, 23-year old Lea seemed tired. While her parents, grandmother, and brother quietly moved throughout the house, Lea answered all questions and offered extra explanation when prompted. The interview lasted 58 minutes.

Her parents obeyed a calling from God to homeschool. They wanted to protect their children from worldly influence and promote family unity. She explained, “They felt like God was calling them to do it . . . to nurture relationships in the family and for them to be a big part of our lives . . . we could gain wisdom.”

They continued homeschooling, also for academic freedom. Lea clarified,

I was able to pick which classes I wanted to take, especially in high school. Um, and was able to self-direct a lot of my classes. . . . So, that was helpful, the reason they continued. Um, so, I think basically the initial reasons. . . . Senior year I took a class at [community college] and one at [university]. And so I was able to have the freedom to go to those. And I also did classes with other homeschoolers in the area . . . different opportunities like that, I wouldn’t been able to in public school.

Lea described her socialization in her homeschool experience, detailing,

I um, was obviously at home and was except for doing classes with other homeschoolers, which was pretty frequent. . . . I did, was able to socialize with other homeschoolers and I also did soccer and got socialization with that.

She developed independence during her education, asserting, “Some people envision homeschooling and you sitting in your room and your mother standing up on one side of the room and teaching you. But it was a lot of self-directed work.” Lea learned how to find resources, preparing her for university. She elaborated,

I did a lot of figuring out of how to find resources, which I really think helped prepare me to go to university, because I wasn’t dependent on the professor to tell me exactly what I needed to do or exactly explain everything to me. . . . I was used to finding the answers. Not having it all spelled out for me.

Lea enjoyed many different subjects. She described her humanities class, recalling,

There was a class that we took . . . The humanities, history, there was some art folded in. Um, I enjoyed that. . . . That was a class that a homeschool mom taught us. . . . I liked the social aspect of that because I was with my friends.

Lea liked some math classes, relating, “I actually liked math. Well, I liked algebra because . . . some tests and problems were easy for me.” Her favorite class included anatomy and physiology; she remembered, “My favorite, I think that actually even trumping that would be um, I took Anatomy and Physiology my senior year at [community college]. . . . But I really enjoyed that.” She enjoyed her Spanish courses, recounting, “I ended up minoring in Spanish. . . . I took 4 years of high school level Spanish. . . . I loved learning a language.” She didn’t like Advanced Math; she recalled,

I took Advanced Math with Saxon . . . I didn’t really like it that much. Um, because I had a hard time understanding all of it. . . . I realized when I got up to Advanced Math that I had just been memorizing how to do a problem Pre-calc things I struggled more. I also, um, it was harder for me to understand the reading part of Saxon.

Lea finally found a resource that helped her with pre-calculus, reporting, “My last year with Math we got this CD called the DIVE CD and that’s what helped me tremendously, because I am more of a visual person and so it explained on the automated chalkboard”.

Overall, Lea had a good homeschool math experience. She asserted, “I had a very good foundation.” Her family used Bob Jones and Saxon curricula. She informed, “We did Bob Jones through grade three. Then, switched to Saxon from then on”. Both parents participated. Lea recollected, “My dad is good in math and was the main one who helped us especially later on. My mom, um, was proficient at math and helped us early on”. Lea progressed to Saxon and self-study. She reported, “Saxon grade 4 and on, um, was harder, and took more effort to self-teach. . . . I would read the lesson and then, um, would do the problems”. Lea’s father acted as a tutor and test monitor. She related,

I corrected my work to my regular lessons and my dad would correct my tests. . . . I would do a lesson and correct myself . . . I would try to figure out what I did wrong and most of the time, I could figure it out. If I couldn’t . . . ask my dad.

Her high school transcript verified all As in Algebra 1 through Advanced Math. Math continued throughout the year, sometimes dragging on. However, it allowed for flexibility with her other studies. She recounted,

We always had to math in the summer because it was more self-directed and it was always the thing you can do it tomorrow; you can do it later. . . . It was the flexible subject. . . . It was a little of the dreaded subject because I had to do it in the summer. . . . In some ways it was the subject that never ended. . . . But then, you can’t do everything. So, I was able to focus more on the other subjects during the year, which was great, cause I learned a lot from them.

Finishing the book was important; her dad made a big deal of that. Lea relayed, “My dad is very excited when we finished a math book . . . He would be so excited that when we finished the math book that he would do a headstand in the kitchen on the hard floor.”

She appreciated the flexibility in pacing and one-on-one attention, reminiscing,

It was good, in that I could fly through the stuff that was easy for me . . . I could take a little more time for the things that were hard . . . I did get some one-on-one with my dad when I just couldn't understand. . . . I could work at my own pace.

She wished that she had found the DIVE disc earlier, remembering, "I wish I had the DIVE CD earlier because I'm a more visual learner; it is more visual".

Lea performed well in math testing and college. She scored 610 on the math SAT subtest, which was verified by her College Board SAT report, and passed a CLEP test for her college math. She remembered her SAT prep, stating, "Used a SAT study . . . maybe 10 hours studying. . . . I would go read the different teaching sections and then I would do practice tests. And also had a daily email with the question prepared." She fulfilled her college math requirement with the CLEP test explaining, "I took the CLEP test for college math. She felt well prepared for university, stating, "I took a statistics class. . . . I felt very prepared . . . nursing math . . . it was easy."

To Lea, mathematics was useful, sensible, and worthwhile. She used math in nursing, maintaining, "It definitely helps with calculating medications and um, I will be using it a lot in reading research articles for school". Math made sense; she stated, "Simple math, algebra does make sense to me." Math knowledge made her relatable to others; she specified, "I don't think everyone has to learn calculus or even pre-calc. But um, but in a good way, whether it's actually used or not to be relatable to people . . . so I'm at least relatable in math."

Both parents valued good mathematical skills. They obtained Bachelor's degrees in Chemical Engineering and Nursing. Lea detailed, "My dad is a chemical engineer with a Bachelor's degree and my mom is/was a registered nurse with a Bachelor's degree. Neither of them are a certified teacher." She saw a godly work ethic, explaining, "I feel like my parents, especially my mom, like she really put into us a good work ethic that she got from her parents. Um, of whatever you do, do it with all your heart unto the Lord".

Currently, Lea works as a registered nurse, in a hospital. She has passed her state boards with high honors. Volunteering at a non-profit clinic, she interprets for Hispanics, while providing nursing assistance. At her hospital, she has recently received an award for outstanding job performance. She has just started graduate school for nursing.

Lea home educator survey. Notably, Lea's father filled out the survey. He gave general information about the family's homeschool. Although he centered in on her younger brother, Don, much of the survey provided evidence to confirm Lea's answers. His responses also supplemented some math curriculum and instruction information.

The survey data confirmed much of Lea's interview. Identical items included the age of the student, the sibling information, and the curricula names, Bob Jones and Saxon. All mathematics classes were mentioned. The father reported, "Dad gave math/science assignments often reading textbooks & testing."

Christian education provided the motivation for homeschooling. He wrote, "To instill God-fearing/respecting values in our children and teach them well. Help them love & bring glory to the almighty God". They continued for the same reasons.

Her father provided detail about the math curricula, instruction, and experiences. He confirmed the curriculum, "Saxon Algebra, Algebra II, Advanced Math – proctored by Dad. . . . We used Bob Jones textbook curriculum thru 3rd grade (taught by [mom]) and then switched to Saxon (taught by [dad])." He described,

Early math lessons were done one-on-one. For all of Saxon [Lea] read the book + did the lessons. Early grades I corrected the homework & then [she] took the tests. I went over concepts [she] missed . . . Math was more self-study with one-to-one tutoring from me. Saxon had a DVD that DIVE produced to help explain the concepts—we didn't realize it was available 'til too late.

The mother taught Lea in the earlier years and the father tutored in the later. The father used math daily, writing, "I use algebra concepts every day." Yearly, they attended homeschool conferences. He reported, "We went to Virginia HEAV (homeschool state organization) conferences each year to learn about resources that were available."

NAEP assessment. Lea answered all of the NAEP assessment questions correctly. She described her work and justified her methods for all ten items.

MSES scores. Lea scored 6.8 overall, a 6.3 on Part I and a 7.4 on Part II, ranking at the female 80 percentile (Table 10). Her mother's total score was 6.7, an 8.0 on Part I and a 5.3 on Part II, placing between the female 70-80 percentiles. The father scored 8.1 overall, an 8.6 on Part I and a 7.6 on Part II, at the male 90 percentile.

Appendix Y: Don

Don interview. After a day of university classes, 20-year old Don seemed very comfortable. While his family quietly moved throughout the house, Don answered all questions and offered extra explanation when prompted. The interview lasted 45 minutes.

His parents wanted to instill godly values, promoting the fear of the Lord and encourage family unity. They wanted to spend time with their children. Don clarified,

They wanted to have time with us, as well as building in godly values into our lives, through just interacting with us as we were growing up and learning things and teaching us how to grow up in the fear of the Lord. This situation worked well for them and they continued. According to Don,

It seemed to go well and my mom could put up with us sufficiently enough. Um, to a certain extent they had us do classes outside for history or other things that weren't a specialty. But, Dad was good with math and science. . . . He would do that for us. Uh, so specialties and they just could keep up with it.

Don progressed from direct teaching to self-study. Early on he worked with his mother, explaining, "Generally Mom would teach in elementary school; Mom would teach a lot and then um, there would be kind of one-on-one for a certain amount". Gaining more independence in middle and high school, he remembered,

Into middle school, it kinda got more where I could do my own stuff . . . In high school it got, it probably got even more by myself. . . . I could always ask for help. But, normally it got more and more by myself.

Don completed his work quickly, revealing, "I'd try to do it as quickly as possible so I could get it done before lunch, was normally my goal . . . go outside and play. . . . I'd play a lot of soccer." Math was completed at home; he clarified,

I wouldn't correct my own work in other classes. . . . Took some literature, English classes. . . . Their redeeming quality was being with people.

Don and his sister took some math classes together; He enjoyed them recalling, "I generally followed along with [L], who is 2 ½ years older than me. So, it was going with her curriculum, though, starting with middle school, I think. And, I enjoyed it.

Don preferred math and science to writing. Describing his favorites, he responded, math and science, no doubt. Uh, probably science more interested. . . . I just get math. That's how my brain works. It comes very easily to me. Just deduction and I don't have to really memorize anything. I just, if I just see how it's worked once, then I can, then it's logical to me and I can work it again.

Yet, he revealed a couple of exceptions, disclosing, "I didn't enjoy biology and anatomy and physiology, because they were a lot more memorization than deduction. . . . Pre-calculus is the most dull math of all. It gets interesting again . . . at calculus."

He also took several high level science classes, detailing, "[In] high school I did chem, advanced chem, physics, advanced physics." Although he performed well with writing, Don didn't enjoy its elements, stating his least favorite subjects were, "I can write. I don't necessarily enjoy doing it. But, I can do it. It's not terrible. I didn't really have a problem with it. I just never liked diagramming sentences and I never was very good at spelling." Yet, Don enjoyed reading, revealing, "I'm a pretty good reader, I'd say. I can read quickly and still understand it. Um, so it went well."

Except for K-3 and his senior years, Don used Saxon math textbooks for his math classes. His high school transcript verified B's in Algebra 1 through Advanced Math and As in Calculus 1 and 2. He had trouble remembering some curriculum names, reporting,

I think it was ABeka grades 1-3. . . . A white book with a big letter 1 and a big letter 2 and a big letter 3 on the first, second and third grade books. . . . Saxon starting with 4th grade all the way through.

He remembered the benefits of persevering with Saxon, recounting,

Then I switched to Saxon 5/4 . . . 6/5, and I think there was a Pre-Algebra and then Algebra 1, Algebra 2, and then. . . . Pre-Calc, um, and that's when I hit the Advanced Math. . . . Saxon really taught it well but you really had to read that stuff a couple times. Didn't explain it in a very clear way but it was all in there. Um, so you could get it if you were determined to do it. . . . It did a good job at mixing up the problems. . . . You'd get problems that you'd been taught 40 lessons ago. And so you couldn't forget it and that was good

Teaching himself greatly benefited Don in his university math classes. He considered,

I'm a math minor. Um, I would say, having to teach myself was actually a good thing in a way. Even if I didn't teach myself as well as maybe I should have. But you learn how to learn and that's can be worthwhile more than learning knowledge. If you know how to learn you can push yourself to learn on your own.

Verified by a College Board SAT report, Don scored well on the SAT math subtest (710). He attributed that to his test taking skills, explaining, "I checked my answers quite a bit. I could do it quickly". He explained his minimal preparation for the SAT math subtest, admitting, "I got the SAT question a day and it was for 6 months before I took the SAT. That's all I did".

Senior year, he worked harder on his class at community college, revealing,

And then I did calculus at community college. . . . I just didn't really want to make mistakes, cause well when I'm actually getting graded for something outside of the house, then it got to be a bigger deal to me. Um, and so, I put more pressure on myself and I ended up being more thorough just cause I wanted to get it right, cause there's actually a grade that I could smell and taste and feel.

He relished being taught by an energetic and passionate teacher, explaining,

That was a luxury that they would actually explain everything to you. . . . I wasn't used to that. . . . Once somebody spelled it out for me, I didn't have any problems reworking or doing the math that way. Um, I had a really good teacher at the community college. He was very excited; he was very bubbly and excited about his math. It was kind of humorous as well. He just described it well.

Yet, it took longer to get through the material in a classroom; Don acknowledging, "I wouldn't have taken that long if it was one-on-one or just me reading the same material. Because you're teaching 30 people and you have to go a little bit slower."

Although Don enjoyed being taught, he ultimately acknowledged that he probably wouldn't change his self-study experience, recognizing,

I think it was great. Um, I think maybe, maybe it would have been good to do pre-calc with a teacher, but it all worked out. Uh, I think learning how to teach math to yourself I think that worked for me. . . . I'm happy with it.

Don agreed that mathematics was useful, sensible, and worthwhile. Math applied to his engineering major and the way he approached decisions; he discussed,

In engineering school, it's in everything. Math is in how I think through, like think through pros and cons of things, how I'm spending my money or whatever, just with math and I think I'm, I put a lot more thought into things that may not need extra thought, like maybe like thinking through what am I sacrificing with the value of my time if I'm doing this instead of this.

Mathematics made sense to him and Don elaborated, "That's why I enjoy it and it comes easily to me because I can follow it and I can reproduce it. But I don't like doing things that I'm just memorizing process". Everyone needed to take algebra, and he affirmed, "Through algebra, you use that a lot in life. . . . It's worth pushing through algebra".

His parents valued mathematics. They both earned a Bachelor's degree that involved some level of mathematical training. His father worked as a Chemical Engineer and his mother trained as a nurse. Don explained, "Dad is a Chemical Engineer. . . . He would say that math is critical. . . . Mom was a nurse, so she's smart. His father pushed him to excellence and his mother encouraged him in his ability. Don reflected,

Dad was pushing me a lot when I was in high school. I think he was trying to get me to use more paper and be thorough. What he kept on telling me was [Don], as you're doing your math right now I wouldn't want to drive on a bridge that you built. I want to help you so that you do things right, so that I would drive on a bridge that you built. . . . Mom didn't express things about pushing me in math as much. But she'd say things like oh you do great at math. She'd recognize it and encourage me in it, but it wasn't like she was necessarily pushing me. She would realize that I was doing well and would encourage that in me.

At a nearby Virginia university, Don majors in Chemical Engineering, with a Math minor. He has worked at an engineering firm this summer. Prior to his interview, his mother and grandmother told me Don earned a 4.0 last semester.

Don home educator survey. Don's father filled out the survey. It gave evidence to verify Don's answers. His responses also supplemented instruction information.

Identical items that were identical included the age of the student, the sibling information, and the Saxon curriculum. All mathematics classes were mentioned. He clarified the use of Bob Jones curriculum for K-3, which Don was not sure about. The father reported, "Dad gave math/science assignments often reading textbooks & testing."

They home-schooled for Christian training. The father wrote, "To instill God-fearing/respecting values in our children and teach them well. Help them love & bring glory to the almighty God." The family continued to homeschool for the same reasons.

Don's father provided detail about the math curricula, instruction, and experiences. He confirmed the curriculum, "Saxon Algebra, Algebra II, Advanced Math – proctored by Dad. . . . We used Bob Jones textbook curriculum thru 3rd grade (taught by [mom]) and then switched to Saxon (taught by [dad])." He described the instruction,

Early math lessons were done one-on-one. For all of Saxon [D] read the book + did the lessons. Early grades I corrected the homework & then he took the tests. I went over concepts he missed with him. . . . Math was more self-study with one-to-one tutoring from me (Dad).

Don was well prepared for college. The father wrote, "[Don] had an adequate math training from the Saxon curriculum & learned well by reading." According to his father, Don was, "very well prepared" for college and scored well on the SAT math subtest because, "he had a natural aptitude & interest in math & sciences."

Both parents participated in Don's education. The mother taught early on and the father tutored in the later. The father used math, daily, writing, "I use algebra concepts every day." Yearly, they attended the Virginia homeschool state organization conferences. The father informed, "We went to Virginia HEAV (homeschool state organization) conferences each year to learn about resources that were available."

NAEP assessment. Don answered eight of the NAEP assessment questions correctly. He described his work and justified his methods for all ten items. Comparing his incorrect answers with the 12th grade sample taking the assessment, 40% missed both.

MSES scores. Don scored 8.3 overall, 8.1 on Part I and 8.6 on Part II. His total score fell between the male 90-95 percentiles (Table 10). His mother total score was 6.7, 8.0 on Part I and 5.3 on Part II, between the female 70-80 percentiles. The father scored 8.1 overall, 8.6 on Part I and 7.6 on Part II, at the male 90 percentile.

Appendix Z: Graduate and Educator References to Major Between Case Themes

Graduate Interview References

THEMES in the Between Case Analysis	N e d	C a l i	S u e	E l a	O l a	T a d	R o b	W e s	M a e	A l i	H a l i	B e n	Q u e	K i m	L e a	D o n
Attitudes:																
Faith: Christian values, education as homeschooling motivation		X		X	X	X	X		X	X	X	X			X	X
STEM (1= 1 parent, 2= both parents, * = graduate)	2	2	2	2	2	2	2	1	1	1	2	1	1	2	2	2
Well prepared (+) Fairly well prepared (x)	+	+	+	+	+	+	+	X	+	+	+	X	+	+	+	+
Productive disposition (H – high, M – moderate, L – low)	H	M	M	H	H	H	H	L	M	H	H	L	M	H	M	H
Stress with math	x	x	x	x		x	x		x			x	x	x	x	
Sense of accomplishment				x						x		x	x	x		
Math ability (N – natural, H – have to work hard to understand)	H	N	H	H	N	N	N	N	H	N	H	N	H	H	N	N
Instructional Practices:																
Direct teaching	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Self-study or self-directed learning	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Mastery learning	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Saxon curriculum	x	x	x	x	x			x	x		x	x	x	x	x	x
Chalkdust curriculum		x						x	x							
Flexible	X		X	X	X	X	X	X	X	X	X		X	X	X	X
Relationships:																
Working with siblings				x	x	x		x	x	x					x	x
Father as tutor	x			x	x			x	x	x					x	x
Physical locations:																
All math at home				x	x			x	x				x		x	
Outside math classes with homeschoolers	x										x	x		x		
Community college or university dual enrollment classes	x	x	x			x	x			x	x	x				x

Educator Survey References

THEMES in the Between Case Analysis	N e d	C a l i	S u e	E l a	O l a	T a d	R o b	W e s	M a e	A l i	H a l i	B e n	Q u e	K i m	L e a	D o n
Attitudes:																
Faith: Christian values, education as homeschooling motivation	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
STEM education vocation (1= 1 parent, 2= both parents, * = graduate)	2	2	2	1	1	2	2	1	1	1	2			2	2	2
Well prepared (+) Adequate (x) Not well prepared (-)	+	+	+	+	+	+	+	x	+	+	+	-	x	+		+
Productive disposition	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x
Stress with math								x			x	x	x			
Sense of accomplishment				x	x	x	x	x		x	x			x		
Math ability (N – natural, H – have to work hard to understand)	x			N	N	N	N	N		N	H	N	H			N
Instructional Practices:																
Direct teaching	x	x	x	x	x	x	x			x					x	x
Self-study or self-directed learning (independence)	x	x	x	x	x	x	x			x	x				x	x
Mastery learning	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Saxon curriculum (Y – yes, N – no)	Y	Y	Y	Y	Y	N	N	Y	Y	N	Y	Y	Y	Y	Y	Y
Chalkdust curriculum		x	x					x	x				x			
Flexible	x									x				x		
Relationships:																
Working with siblings				x	x	x	x	x	x	x						
Father as tutor	x			x	x			x	x	x					x	x
Physical locations:																
All math at home				x	x			x	x							
Outside math classes with homeschoolers											x			x		
Community college or university dual enrollment classes			x			x	x			x	x	x				x