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ON FACTORIZATION, INTERCONNECTION AND REDUCTION OF COLLOCATED PORT-HAMILTONIAN SYSTEMS

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Abstract: Based on a geometric interpretation of nonlinear balanced reduction some implications of this approach are analyzed in the case of collocated port-Hamiltonian systems which have a certain balance in its structure. Furthermore, additional examples of reduction for this class of systems are presented.

Keywords: Nonlinear systems, Hamiltonian systems, model approximation, model reduction.

1. INTRODUCTION

Given a minimal nonlinear system, the problem of *equilibrated*[1] reduction seeks to perform a reduction of the dimension of the state-space based on a criterion which characterizes a submanifold over which the reduced system can be constructed. Following the ideas of the dissipativity approach of balanced reduction (Lopezlena *et al.*, 2003), it can be said that a particular realization of a dynamic system is equilibrated when in such realization the square root of the required supply function and the corresponding square root of the available storage function have the same value in each point of the state space. In order to find an equilibrated representation, a relation between both functions can be defined in a certain form, for instance, an induced norm relating both functions within a certain submanifold.

The main idea of this paper is to apply several concepts of such nonlinear equilibrated reduction procedure to reduce port-Hamiltonian systems (PHS).

[1] The term "balanced" applies in this context, but some authors reserve such term to the balancing method of (Moore, 1981) only. The concept of equilibrated reduction includes the balanced reduction as a particular case.

This paper is organized as follows. In Section 2, using Legendre Transforms to factorize PHS, it is shown that a natural representation of Hamiltonian systems is the collocated form introduced in (Lopezlena *et al.*, 2003) which is just a particular structure of the Port-Hamiltonian paradigm. In Section 3 it is shown that the interconnection of two collocated-PHS (CPHS) yields newly a CPHS with Hamiltonian equal to the sum of the Hamiltonians of the component subsystems. The possibility of interconnection of this structures brings about the inverse operation of reducing the dimension of the state space according to its input-output or port relations. Therefore in Section 4 a factorization is useful to provide a structure preserving reduction method.

2. HAMILTONIAN SYSTEMS AND FACTORIZATION

A class of dissipative systems are port-Hamiltonian systems with dissipation (PHSD) (van der Schaft, 2000). In this section we present two representations of PHSD and their relations as an antecedent of the collocated representation of PHS presented in Sec. 3.

A PHS is characterized by a Dirac structure, an energy (Hamiltonian) function and a dissipative structure. The Dirac structure defines the power conserving interconnections in the system. More formally, given two spaces $\mathcal{F}$, $\mathcal{F}^*$, the Dirac structure $\mathcal{D}$ is a subspace $\mathcal{D} \subset \mathcal{F} \times \mathcal{F}^*$ such that $\mathcal{D} = \mathcal{D}^{\perp}$ for the symmetric bilinear form

$$\langle (f_1, f_1^*), (f_2, f_2^*) \rangle_{\mathcal{F}\times\mathcal{F}^*} = \langle f_1^* | f_2 \rangle + \langle f_2^* | f_1 \rangle \quad (1)$$

where $(f, f^*) \in \mathcal{F} \times \mathcal{F}^*$. If we consider the particular spaces defined as

$$\mathcal{F} = \{ f \mid f = -\dot{x},\ x \in \mathcal{X},\ f \in T_x\mathcal{X} \} \quad (2)$$

$$\mathcal{F}^* = \{ f^* \mid f^* = \frac{\partial H}{\partial x}(x),\ x \in \mathcal{X},\ f^* \in T_x^*\mathcal{X} \} \quad (3)$$

where H is a function (the Hamiltonian), then with structure matrix $J(x)$ ($J(x) = -J^T(x)$) such that $\mathcal{D} = \{ (f, f^*) \in T\mathcal{X} \oplus T^*\mathcal{X} \mid f(x) = J(x) f^*(x),\ x \in \mathcal{X} \}$, the triad $(\mathcal{X}, \mathcal{D}, H)$ is a conservative PHS (van der Schaft, 1998). Dissipation can be added with a symmetric structure R by adding a feedback loop interconnection, (van der Schaft, 2000).

In the behavioral approach, a system is conceived as an exclusion law which discards any outcome outside a subset of time-trajectories called the behavior of the dynamical system. Furthermore, there in no particular distinction in the set of dynamic variables between state variables, input and output variables. For our purposes, consider a Hamiltonian system $(\hat{\mathcal{X}}, \hat{\mathcal{D}}, \hat{H})$ where $\hat{\mathcal{D}} \subset \hat{\mathcal{F}} \times \hat{\mathcal{F}}^*$, $\hat{H}(x, w) \in C^{\infty}$ and the extended state space $(x, w) \in \hat{\mathcal{X}}$ includes those variables w associated to inputs and outputs. Such system can be represented by

$$\begin{bmatrix} \dot{x}(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} M_1^1(x) & M_1^2(x) \\ M_2^1(x) & M_2^2(x) \end{bmatrix} \begin{bmatrix} \partial_x \hat{H} \\ \partial_w \hat{H} \end{bmatrix} \quad (4)$$

where the inclusion of dissipation is provided by defining a symmetric dissipation structure $R(x) = R^T(x) > 0$ to the system resulting in a matrix $M(x) = J(x) - R(x)$ with structure matrix $J(x) = -J(x)^T$. Alternatively the same system can be represented with a Hamiltonian function $H(x) \in C^{\infty}$, $x \in \mathcal{X}$ in the form

$$\begin{bmatrix} \dot{x}(t) \\ -y(t) \end{bmatrix} = \begin{bmatrix} M_1^1(x) & M_1^2(x) \\ M_2^1(x) & M_2^2(x) \end{bmatrix} \begin{bmatrix} \partial_x H \\ u(t) \end{bmatrix} \quad (5)$$

The triad $(\mathcal{X}, \mathcal{D}, H)$, $\mathcal{D} \subset \mathcal{F} \times \mathcal{F}^*$, defines a port -Hamiltonian system. Legendre transforms are used very frequently to transform functions in a certain vector space into functions of its dual space. For a PHS $(\mathcal{X}, \mathcal{D}, H)$ and a state space locally partitioned as $x = (x_1, x_2)$ the (Legendre) transformation $\mathcal{L} : \mathcal{X} \to \mathcal{X}'$ defined by $\mathcal{L} : (x_1, x_2) \to (x_1, z_2)$ with $z_2 = \partial H(x_1, x_2)/\partial x_2$ provides a new Hamiltonian defined by $H'(x_1, z_2) = H(x_1, x_2) - z_2^T x_2$ that provides a factorization of the state space $\mathcal{X}$ such that it becomes useful for reduction. In (Lopezlena *et al.*, 2003) a partial Legendre transform was used for reduction purposes. In the following result we use such transform in order to express equivalence conditions to express such Hamiltonian system in the common representation of a PHS with explicit collocated inputs and outputs.

Proposition 2.1. The PHS (4) can be represented equivalently by the PHS (5) by defining inputs and outputs as

$$\begin{cases} u(t) = \partial \hat{H}(x, w)/\partial w \\ y(t) = -\dot{w}(t) \end{cases} \quad (6)$$

furthermore for small variations of u, the Legendre transform

$$\hat{H}(x,z) = H(x,w) - y^T \frac{\partial \hat{H}}{\partial w}, \tag{7}$$

can be used for state transformation from (4) to (5).

Proof. Due to (6) the Hamiltonian functions of systems (4) and (5) are related by $\hat{H}(x,z) = H(x,w) - \int \frac{\partial H}{\partial w}\dot{w}dt$. Consider the Hamiltonian function defined by (7) such that $u = \partial\hat{H}(x,w)/\partial w$. Then system (4) can be transformed to

$$\begin{bmatrix} \dot{x}(t) \\ d_t\left(-\partial_u \hat{H}\right) \end{bmatrix} = \begin{bmatrix} M_1^1(x) & M_1^2(x) \\ M_2^1(x) & M_2^2(x) \end{bmatrix} \begin{bmatrix} \partial_x \hat{H} \\ u \end{bmatrix}$$

which is equivalent to (5). ■

The Legendre transform used provides a partition of the state space allowing for a factorization of the system. In our particular case the system (4) is factorized in the system (5) and the following (sources) PHS

$$\begin{bmatrix} \dot{w}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} \partial_w H \\ \partial_z H \end{bmatrix}$$

with Hamiltonian function $H(w) = \int \left(\frac{\partial H}{\partial z}\right) \dot{z}\; dt$.

Proposition 2.2. The resulting structure $\hat{\mathcal{D}}(x)$ of the PHS defined in the extended manifolds $\hat{\mathcal{F}}$ and $\hat{\mathcal{F}}^*$ is power-conserving.

Proof. Using the bilinear form (1) for the spaces $\hat{\mathcal{F}}$ and $\hat{\mathcal{F}}^*$ constructed over (3) yields

$$\langle (\hat{f}, \hat{f}^*), (\hat{f}, \hat{f}^*) \rangle_{\hat{\mathcal{F}} \times \hat{\mathcal{F}}^*} = 2\dot{x}\frac{\partial H}{\partial x} - 2y^T u = 0.$$

■

Proposition 2.3. The energy balance of the PHS in Eq. (5) is given by

$$\frac{dH}{dt} = y^T u - \begin{bmatrix} \frac{\partial H}{\partial x} \\ u \end{bmatrix}^T \begin{bmatrix} R_1^1 & R_1^2 \\ R_2^1 & R_2^2 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x} \\ u \end{bmatrix}. \tag{8}$$

Proof. Since

$$\frac{dH}{dt} = \frac{\partial^T H}{\partial x}\dot{x} = \frac{\partial^T H}{\partial x}\left[M_1^1(x)\frac{\partial H(x)}{\partial x} + M_1^2(x)u(t)\right]$$

direct substitution of $\frac{\partial^T H}{\partial x}J_1^2$ from y^T from (5), since $J_2^1 = -J_1^2$, and $u^T J_2^2 u = 0$ the result (8) follows. ■

3. SERIES INTERCONNECTION OF COLLOCATED PHS

By *collocated PHS* (CPHS) we refer to a structured representation of PHS where all possible inputs and outputs are paired at the ports and included in one composite (structured) matrix, see also (Lopezlena *et al.*, 2003; Lopezlena and Scherpen, 2004). In particular, the class of CPHS in the following form are explicitly written as

$$\begin{bmatrix} \dot{x} \\ -y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} M_1^1 & M_1^2 & M_1^3 \\ M_2^1 & M_2^2 & M_2^3 \\ M_3^1 & M_3^2 & M_3^3 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x} \\ u_1 \\ u_2 \end{bmatrix} \tag{9}$$

with $H(x)$ the Hamiltonian function and $M(x) = J(x) - R(x)$ where $J(x) = -J^T(x)$, $R(x) = R^T(x)$ and as usual, the system is dissipative for a supply rate $r = y_1^T u_1 - y_2^T u_2$.

The class of CPHS has a particular advantage in terms of series interconnection. Given two systems of this class connected by the ports that satisfy some compatibility relations, the resulting series interconnected system belongs to the collocated class. More formally, consider two CPHS. The first one from Eq. (9) with Hamiltonian $H(x)$ and with the purpose of series interconnection consider a second system with Hamiltonian function $H(w)$ in the form

$$\begin{bmatrix} \dot{w} \\ -z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} N_1^1 & N_1^2 & N_1^3 \\ N_2^1 & N_2^2 & N_2^3 \\ N_3^1 & N_3^2 & N_3^3 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial w} \\ v_1 \\ v_2 \end{bmatrix} \tag{10}$$

with $H(w)$ the Hamiltonian function and $N(w) = J_N(w) - R_N(w)$, and dissipative for a supply rate $r = z_1^T v_1 - z_2^T v_2$.

Proposition 3.1. Assume that the interconnection is compatible, *i.e.* $y_2 = v_1$ and $u_2 = z_1$, then the series interconnected system with inputs u_1 and v_2 and outputs $-y_1$ and z_2 is again a CPHS with Hamiltonian $H(x,w) = H(x) + H(w)$ and is expressed in Eq.

$$\begin{pmatrix} \dot{x} \\ \dot{w} \\ -y_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} M_1^1 - M_1^3\psi N_2^2 M_3^1 & -M_1^3\psi N_2^1 & M_1^2 - M_1^3\psi N_2^2 M_3^2 & -M_1^3\psi N_2^3 \\ -N_1^2\Omega M_3^1 & N_1^1 - N_1^2\Omega M_3^3 N_2^1 & -N_1^2\Omega M_3^2 & N_1^3 - N_1^2\Omega M_3^3 N_2^3 \\ M_2^1 - M_2^3\psi N_2^2 M_3^1 & -M_2^3\psi N_2^1 & M_2^2 - M_2^3\psi N_2^2 M_3^2 & -M_2^3\psi N_2^3 \\ N_3^2\Omega M_3^1 & N_3^1 - N_3^2\Omega M_3^3 N_2^1 & N_3^2\Omega M_3^2 & N_3^3 - N_3^2\Omega M_3^3 N_2^3 \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial x} \\ \frac{\partial H}{\partial w} \\ u_1 \\ v_2 \end{pmatrix} \tag{11}$$

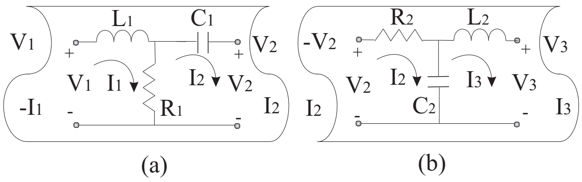

Fig. 1. Interconnection of elemental RLC circuits

(11). whenever $\Omega = (I + M_3^3 N_2^2)^{-1}$ and $\psi = (I + N_2^2 M_3^3)^{-1}$ exist.

Proof. Using the compatibility relations $y_2 = v_1$ and $u_2 = z_1$, the series interconnection of system (9) and (10), results straightforwardly in system (11). ■

Example 3.1. Consider the electrical circuits presented in figure 1. The circuit in figure 1 (a) has a CPHS given by

$$\begin{bmatrix} \dot{q}_1 \\ \dot{\lambda}_1 \\ -I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{-1}{R_1} & 1 & 0 & \frac{-1}{R_1} \\ -1 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 \\ \frac{-1}{R_1} & 1 & 0 & \frac{-1}{R_1} \end{bmatrix} \begin{bmatrix} \frac{\partial H_1}{\partial q_1} \\ \frac{\partial H_1}{\partial \lambda_1} \\ V_1 \\ V_2 \end{bmatrix}$$

for a Hamiltonian given by $H_1 = \frac{1}{2C_1}q_1^2 + \frac{1}{2L_1}\lambda_1^2$. The CPHS of circuit in figure 1 (b) is

$$\begin{bmatrix} \dot{q}_2 \\ \dot{\lambda}_2 \\ -V_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ -1 & 0 & -R_2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_2}{\partial q_2} \\ \frac{\partial H_2}{\partial \lambda_2} \\ I_2 \\ V_3 \end{bmatrix}$$

with a Hamiltonian given by $H_2 = \frac{1}{2C_2}q_2^2 + \frac{1}{2L_2}\lambda_2^2$. In both representations it can be clearly distinguished that the interconnection matrices M and N can be factorized into a skew-symmetric part accounting for the energy conserving interconnections and a symmetric part for the dissipation. The compatibility conditions $y_1 = v_1$ and $u_2 = z_2$ are satisfied allowing then the series interconnection of this systems. The resulting interconnected system has the form

$$\begin{bmatrix} \dot{q}_1 \\ \dot{\lambda}_1 \\ \dot{q}_2 \\ \dot{\lambda}_2 \\ -I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} \frac{-1}{R_{12}} & \frac{R_1}{R_{12}} & \frac{-1}{R_{12}} & 0 & 0 & 0 \\ \frac{-R_1}{R_{12}} & \frac{-R_1 R_2}{R_{12}} & \frac{-R_1}{R_{12}} & 0 & 1 & 0 \\ \frac{-1}{R_{12}} & \frac{R_1}{R_{12}} & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q_1} \\ \frac{\partial H}{\partial \lambda_1} \\ \frac{\partial H}{\partial q_2} \\ \frac{\partial H}{\partial \lambda_2} \\ V_1 \\ V_3 \end{bmatrix}$$

where we denoted $R_{12} = R_1 + R_2$ with Hamiltonian $H = \frac{1}{2C_1}q_1^2 + \frac{1}{2L_1}\lambda_1^2 + \frac{1}{2C_1}q_1^2 + \frac{1}{2L_1}\lambda_1^2$, which is a collocated port-Hamiltonian system and whose matrix again can be factorized into a skew-symmetric and symmetric part.

In (Lopezlena *et al.*, 2003) a dissipativity approach of balanced reduction was presented for nonlinear systems using storage functions. In the case of collocated port-Hamiltonian systems there is a very natural form to define such storage functions in terms of port variables. Define the storage functions through the ports

$$\begin{aligned} S_r^\star(x_0, r) &= S_r(x_0, y_1^T u_1) \\ &= \inf_{\substack{u(\cdot)\in\mathcal{U} \\ x_0=x,\ T\geq 0}} \int_{-T}^{0} y_1^T u_1 dt \end{aligned} \tag{12}$$

$$\begin{aligned} S_a^\star(x_0, r) &= S_a(x_0, -y_2^T u_2) \\ &= -\inf_{\substack{u(\cdot)\in\mathcal{U} \\ x_0=x,\ T\geq 0}} \int_{0}^{T} y_2^T u_2 dt \end{aligned} \tag{13}$$

which can be recognized as the *physical energy* supplied to the system, $\langle u_1, y_1 \rangle_{\mathcal{L}_2}$, by its input two-port, and the physical energy deliverable by the system, $\langle -u_2, y_2 \rangle_{\mathcal{L}_2}$, through its output two-port.

Proposition 3.2. Given the collocated PHS (9), for vectors partitioned as $u = (u_1^T, u_2^T)^T$ and $y = (-y_1^T, y_2^T)^T$, where the input energy is associated to $\langle y_1, u_1\rangle_{\mathcal{L}_2}$ and the output energy to $\langle y_2, -u_2\rangle_{\mathcal{L}_2}$, then $S_r(x)$ and $S_a(x)$ can be written as

$$S_r(x) = H(x) + D(x,t)|_{-T}^{0} \quad (14)$$
$$S_a(x) = H(x) - D(x,t)|_{0}^{T} \quad (15)$$

where $H(x)$ is the Hamiltonian of the system and

$$D(x,t) = \int \begin{bmatrix} \frac{\partial H}{\partial x} \\ u_1 \\ u_2 \end{bmatrix}^T \begin{bmatrix} R_1^1 & R_1^2 & R_1^3 \\ R_2^1 & R_2^2 & R_2^3 \\ R_3^1 & R_3^2 & R_3^3 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x} \\ u_1 \\ u_2 \end{bmatrix} dt.$$

Proof. Initially assume that system (9) for $u_2 = 0$ is excited by the input u_1^* which satisfies the variational problem defining S_r in order to reach the initial state x_0. Furthermore, departing from x_0 with $u_1 = 0$ assume that the systems is excited with the input u_2^* such that the variational problem defining S_a is satisfied. This sequence of operations can be described by the decomposition of the PHS as two separate systems. For $u_2 = 0$

$$\begin{bmatrix} \dot{x} \\ -y_1 \end{bmatrix} = \begin{bmatrix} M_1^1(x) & M_1^2(x) \\ M_2^1(x) & M_2^2(x) \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x} \\ u_1 \end{bmatrix} \quad (16)$$

and for $u_1 = 0$

$$\begin{bmatrix} \dot{x} \\ y_2 \end{bmatrix} = \begin{bmatrix} M_1^1(x) & M_1^3(x) \\ M_3^1(x) & M_3^3(x) \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x} \\ u_2 \end{bmatrix} \quad (17)$$

By the definition of the storage functions as $S_r^\star(x_0, r) = S_r(x_0, y_1^T u_1)$ and $S_a^\star(x_0, r) = S_a(x_0, -y_2^T u_2)$. The use of Eq. (8) for the inputs and outputs defined previously yield Eqs. (14) and (15) respectively. ■

4. EQUILIBRATED REDUCTION AND CPHS

In this subsection the reduction of collocated PHS is discussed under the light of the equilibrated reduction procedure presented in (Lopezlena *et al.*, 2003; Lopezlena, 2004) . The set of points reachable from x_0 at time T, for each $T \geq 0$ and each $x_0 \in \mathcal{X}$ is denoted by $\mathcal{A}(x_0, T)$. Since the storage functions (14) and (15) essentially are composed of the Hamiltonian and a dissipation function, only differing by a sign, it can be asserted that both are supported by the same subset of $\mathcal{A}(x_0, T) \subset \mathcal{X}$. This type of systems can be considered equilibrated in their controllablity and observability properties, as already anticipated in (van der Schaft, 1982).

The next step on the reduction procedure is to find a partition of the state space $\mathcal{X} = \mathcal{X}_a \oplus \mathcal{X}_b$ such that the highest concentration of energy remains in a certain submanifold $\mathcal{X}_a$. For such partition of the state space the procedure of equilibrated reduction on manifolds (Lopezlena, 2004) can be applied.

As it could be seen in this section, the fact that port-Hamiltonian systems have a certain structural balanced form in terms of its storage functions and in terms of its controllability and observability properties, shows that the importance of the nonlinear balanced reduction methods for this class of systems lies more on the correct partition of the state space in submanifolds with a certain stored energy in the Hamiltonians associated to such partition. Unfortunately we do not have at hand general procedures to decide on such factorization. In the following result we provide a factorization of the state space variables once such partition has been decided in a certain form and a structure preserving reduction method follows.

Proposition 4.1. Consider the PHS given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ -y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} M_1^1 & M_1^2 & M_1^3 & M_1^4 \\ M_2^1 & M_2^2 & M_2^3 & M_2^4 \\ M_3^1 & M_3^2 & M_3^3 & M_3^4 \\ M_4^1 & M_4^2 & M_4^3 & M_4^4 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x_1} \\ \frac{\partial H}{\partial x_2} \\ u_1 \\ u_2 \end{bmatrix}$$

with a Hamiltonian given by $H(x_1, x_2)$, where $M_i^j(x) = J_i^j(x) - R_i^j(x)$ and assume that $\partial^2 H/\partial x_2^2$ has full rank and $\det(M_2^2(x)) \neq 0$. Assume that the variations of the state vector x_2 can be neglected, then the state trajectories of the system lie in the submanifold defined as

$$N = \left\{ (x_1, x_2, u_1, u_2) \quad | \quad M_2^1(x)\frac{\partial H}{\partial x_1}(x) + M_2^2(x)\frac{\partial H}{\partial x_2}(x) + M_2^3(x)u_1 + M_2^4(x)u_2 = 0 \right\},$$

and the dynamics of the system can be represented in a reduced form as follows

$$\begin{bmatrix} \dot{x}_1 \\ -y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} W_1^1 & W_1^3 & W_1^4 \\ W_3^1 & W_3^3 & W_3^4 \\ W_4^1 & W_4^3 & W_4^4 \end{bmatrix} \begin{bmatrix} \frac{\partial H^\star}{\partial x_1} \\ u_1 \\ u_2 \end{bmatrix}$$

where $W_i^j = M_i^j - M_i^2 \Xi M_2^j$, for a Hamiltonian given by $H^\star = H(x_1, x_2) - \frac{\partial H}{\partial x_2} x_2$, where $\Xi = [M_2^2]^{-1}$.

Proof. It is a slight variation of (van der Schaft, 2002; Lopezlena *et al.*, 2003) which now considers collocated PHS and therefore is omitted. ■

Example 4.1. (Generalized electromechanical machine). Define the vector of fluxes as $\Phi = (\phi_d^r, \phi_q^r, \phi_d^s, \phi_q^s)^T$ and the rotational moment as $h = J\dot{\theta}$. Define $\Delta = L_s L_r - M^2$. The Hamiltonian for this system is given by $H(\Phi, h) = \frac{1}{2J} h^2 + \frac{1}{2} \Phi^T \Gamma(\theta) \Phi$, where L and $L^{-1}(\theta) = \Gamma(\theta)$, are such that

$$L(\theta) = \begin{bmatrix} L_r I_2 & M e^{-\mathcal{J}\theta} \\ M e^{\mathcal{J}\theta} & L_s I_2 \end{bmatrix}; \; e^{\mathcal{J}\theta} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\Gamma(\theta) = \frac{1}{\Delta} \begin{bmatrix} L_s I_2 & -M e^{-\mathcal{J}\theta} \\ -M e^{\mathcal{J}\theta} & L_r I_2 \end{bmatrix}.$$

The CPHS representation is given by

$$\begin{bmatrix} \dot{h} \\ \dot{\theta} \\ \dot{\phi}^r \\ \dot{\phi}^s \\ -I^r \\ -I^s \\ \omega \end{bmatrix} = \begin{bmatrix} -B & -1 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -R^r & 0 & I & 0 & 0 \\ 0 & 0 & 0 & -R^s & 0 & I & 0 \\ 0 & 0 & -I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial_h H \\ \partial_\theta H \\ \partial_{\phi^r} H \\ \partial_{\phi^s} H \\ V^r \\ V^s \\ \tau_L \end{bmatrix}$$

Several degrees of reduction can be of interest in these type of electromechanical machines (Richards and Tan, 1981). We consider two cases:
(a) Whole electrical transient dynamics discarded. Using the reduction procedure presented yields

$$\begin{bmatrix} \dot{h} \\ \dot{\theta} \\ -I^r \\ -I^s \\ \omega \end{bmatrix} = \begin{bmatrix} -B & -1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -R_r^{-1} I & 0 & 0 \\ 0 & 0 & 0 & -R_s^{-1} I & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial_h H \\ \partial_\theta H \\ V^r \\ V^s \\ \tau_L \end{bmatrix}$$

with a Hamiltonian given by $H(h, \theta) = \frac{1}{2J} h^2 - \frac{1}{2} \Phi^T \Gamma(\theta) \Phi$ where $\Phi = \Phi_0$ is constant in a certain operating point.
(b) Just stator transient dynamics discarded. This amounts to ignore the energy stored in stator flux-linkages ϕ_d^s and ϕ_q^s but including the electrical transients in the rotor windings, resulting in

$$\begin{bmatrix} \dot{h} \\ \dot{\theta} \\ \dot{\phi}^r \\ -I^r \\ -I^s \\ \omega \end{bmatrix} = \begin{bmatrix} -B & -1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -R^r & I & 0 & 0 \\ 0 & 0 & -I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -R_d^{-1} I & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial_h H \\ \partial_\theta H \\ \partial_{\phi^r} H \\ V^r \\ V^s \\ \tau_L \end{bmatrix}$$

where $H(h, \theta) = \frac{1}{2J} h^2 - \frac{1}{2} \Phi^T \Gamma(\theta) \Phi$.

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