ELSEVIER

6 November 1997

PHYSICS LETTERS B

Physics Letters B 413 (1997) 70–78

Gauged supergravity from dimensional reduction

E. Bergshoeff [1], M. de Roo [2], E. Eyras [3]

Institute for Theoretical Physics, Nijenborgh 4, 9747 AG Groningen, The Netherlands

Received 11 August 1997
Editor: P.V. Landshoff

Abstract

We perform a generalised Scherk-Schwarz reduction of the effective action of the heterotic string on T^6 to obtain a massive $N=4$ supergravity theory in four dimensions. The local symmetry-group of the resulting $d=4$ theory includes a Heisenberg group, which is a subgroup of the global $O(6,6+n)$ obtained in the standard reduction. We show explicitly that the same theory can be obtained by gauging this Heisenberg group in $d=4, N=4$ supergravity. © 1997 Elsevier Science B.V.

1. Introduction

Dimensional reduction is a key to understanding the interplay between various dualities in string theory. Many results in the field of string dualities have been obtained using standard toroidal reductions. Recently there has been a renewed interest [1,2] in the generalisation of toroidal reduction introduced long ago by Scherk and Schwarz [3]. Its basic property is that it allows a certain dependence of the fields on the coordinates which are wrapped around the torus. The result is usually that parameters with the dimension of mass are introduced into the resulting theory.

A nontrivial feature of the Scherk-Schwarz reduction is that, although before reduction some of the fields depend on the torus coordinates, the reduced theory is independent of these coordinates. To achieve this one specifies the particular dependence of the fields on the torus coordinates by using a global symmetry of the theory. In the original work of Scherk and Schwarz, the global symmetry used for this purpose was a compact subgroup of the internal symmetry group of the theory. The Scherk-Schwarz mechanism in this form was applied to the six-index formulation of $d=10$ supergravity in [4] and to the effective action of the heterotic string on T^6 in [5], to obtain a gauged $d=4, N=4$ supergravity theory with a positive semidefinite potential; similar work on $d=11$ supergravity and M-theory was done in [6,7].

[1] E-mail: e.bergshoe@phys.rug.nl.
[2] E-mail: m.de.roo@phys.rug.nl.
[3] E-mail: e.a.eyras@phys.rug.nl.

PII S0370-2693(97)01111-8

Almost coincident with the work of Scherk and Schwarz, it was realized that supergravity theories have extra noncompact global symmetries [8]. It is natural to ask oneself whether one of these extra global symmetries can be used as well in the Scherk-Schwarz mechanism. Recently, it has been shown that indeed this can be done in the reduction of IIB supergravity to nine dimensions [1]. The motivation of [1] was to establish duality rules relating type IIB supergravity to the massive version of type IIA supergravity [9]. The relevant noncompact symmetry used was a particular $SL(2,R)$ symmetry involving the Ramond-Ramond scalar of IIB supergravity. Basically the RR scalar ℓ was replaced by $\ell' = \ell + my$, where y is the coordinate over which one reduces. The fact that this global shift of the field ℓ is a symmetry of the ten-dimensional action ensures that the linear y-dependence disappears in the reduction. The nine-dimensional theory then contains the massive parameter m.

The purpose of this letter is to show that the noncompact symmetry used in [1] can also be applied in a more general context involving more scalars. To be explicit, we will start with the standard toroidal compactification of the heterotic string effective action on T^5 [10]. This theory has a global $O(5,5+n)$ symmetry [4], which we use in a Scherk-Schwarz reduction to $d=4$ thereby giving a linear x^4-dependence to $n+8$ scalars. We find that the Scherk-Schwarz reduction induces a local nonabelian symmetry in the resulting $N=4, d=4$ supergravity theory. We establish that this generalised reduction is equivalent to the gauging of this nonabelian group in matter-coupled $N=4, d=4$ supergravity [11].

Let us first give an example of our use of the Scherk-Schwarz mechanism. Consider a complex scalar field $\hat\lambda$ coupled to gravity in d dimensions [5]:

$$\mathscr{L}_d = \sqrt{|\hat g|}\left\{\hat R + \frac{\partial\hat\lambda\partial\bar{\hat\lambda}}{|\hat\lambda - \bar{\hat\lambda}|^2}\right\}. \tag{1}$$

d-dimensional fields are indicated by hats. This lagrangian has a modular invariance $SL(2,\mathbb{R})/\mathbb{Z}_2$:

$$\hat\lambda \to \frac{c + d\hat\lambda}{a + b\hat\lambda}. \tag{2}$$

For $b=0$ and $a=d=1$, we have a 1-parameter subgroup isomorphic to $(\mathbb{R},+)$. The real components of $\hat\lambda$ transform as $\hat\lambda_1 \to \hat\lambda_1 + c$ and $\hat\lambda_2 \to \hat\lambda_2$. We use this subgroup in the generalised reduction. The spacetime coordinates are divided as $\hat x_{(D)} = (x_{(D-1)}, y)$ and the generalized reduction rules for the scalars become

$$\hat\lambda_1(\hat x) = \lambda_1(x) + my, \quad \hat\lambda_2(\hat x) = \lambda_2(x). \tag{3}$$

This results in the following lagrangian in $d-1$ dimensions:

$$\mathscr{L}_{d-1} = \sqrt{|g|}\left\{R + \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}e^{\sqrt{\frac{2(D-2)}{(D-3)}}\phi}F^2(A) + \frac{1}{2\lambda_2^2}\left[(\partial\lambda_2)^2 + (\mathscr{D}\lambda_1)^2\right] - \frac{1}{4\lambda_2^2}m^2 e^{-\sqrt{\frac{2(D-2)}{(D-3)}}\phi}\right\}. \tag{4}$$

The derivative $\mathscr{D}$ is defined as $\mathscr{D}_\mu\lambda_1 = \partial_\mu\lambda_1 - mA_\mu$. The modular symmetry is broken down to a one-parameter local subgroup, for which $\mathscr{D}$ is the covariant derivative [6]:

$$\Delta\lambda_1 = m\beta(x), \quad \Delta A_\mu = \partial_\mu\beta(x). \tag{5}$$

The gauge field A_μ has become massive, as can be seen by going to the gauge $\lambda_1 = 0$.

---

[4] For the heterotic string $n=16$. It is convenient to keep n arbitrary.

[5] This example is similar to the reduction of the type *IIB* string in [1].

[6] Throughout this paper we will use Δ to indicate finite transformation, defined for any field F by $\Delta F = F' - F$, where F' is the transformed field. We will use δ for infinitesimal transformations.

In this example it is straightforward to see that this result can also be obtained by first doing a standard reduction and by then gauging the appropriate one-dimensional subgroup of $SL(2,\mathbb{R})/\mathbb{Z}_2$ in $d-1$ dimensions. Note however that by doing this one does not recover the dilatonic potential. If this example is embedded in a supersymmetric theory, the potential can be recovered by requiring supersymmetry.

In the next section we will consider a similar, but more complicated example. A basic difference with the above example will be that, after reduction, three instead of one symmetries receive m-dependent modifications. Correspondingly, this example involves the gauging of a three-dimensional group, which turns out to be the Heisenberg group.

2. Generalized reduction

We carry out a standard reduction of the low energy effective action of the heterotic string on T^5 to obtain the five dimensional $N=4$ supergravity theory. The action for the bosonic fields is [10] [7]:

$$S=\int d^5x\sqrt{|\hat{g}|}\,e^{-2\hat{\phi}}\left\{\hat{R}-4(\partial\hat{\phi})^2-\tfrac{1}{8}\mathrm{tr}\left(\partial_{\hat{\mu}}\hat{\mathcal{M}}\partial^{\hat{\mu}}\hat{\mathcal{M}}^{-1}\right)+\tfrac{3}{4}\hat{H}^2_{\hat{\mu}\hat{\nu}\hat{\rho}}+\tfrac{1}{4}\hat{F}(\hat{\mathcal{A}})^i_{\hat{\mu}\hat{\nu}}\hat{\mathcal{M}}^{-1}_{ij}\hat{F}(\hat{\mathcal{A}})^{\hat{\mu}\hat{\nu}j}\right\}. \tag{6}$$

This action has $O(5,5+n)$ symmetry [10]. The scalars parametrize an $O(5,5+n)$ element $\hat{\mathcal{M}}^{-1}$:

$$\hat{\mathcal{M}}^{-1}=\begin{pmatrix} -\hat{\kappa}_1^2-\dfrac{\hat{\ell}^4}{4\hat{\kappa}_1^2}+\hat{\ell}^a\hat{M}^{-1}_{ab}\hat{\ell}^b & \dfrac{1}{2\hat{\kappa}_1^2}\hat{\ell}^2 & \hat{\ell}^a\hat{M}^{-1}_{ab}-\dfrac{\hat{\ell}^2}{2\hat{\kappa}_1^2}\hat{\ell}^aL_{ab} \\ \dfrac{1}{2\hat{\kappa}_1^2}\hat{\ell}^2 & -\dfrac{1}{\hat{\kappa}_1^2} & \dfrac{1}{\hat{\kappa}_1^2}\hat{\ell}^aL_{ab} \\ \hat{M}^{-1}_{ab}\hat{\ell}^b-\dfrac{\hat{\ell}^2}{2\hat{\kappa}_1^2}L_{ab}\hat{\ell}^b & \dfrac{1}{\hat{\kappa}_1^2}L_{ab}\hat{\ell}^b & \hat{M}^{-1}_{ab}-\dfrac{1}{\hat{\kappa}_1^2}L_{ac}\hat{\ell}^c\hat{\ell}^dL_{db} \end{pmatrix}. \tag{7}$$

Here $\hat{M}^{-1}$ is an element of $O(4,4+n)$, containing $4(4+n)$ independent scalars, and L is the invariant metric of $O(4,4+n)$. $\hat{\ell}^2$ equals $\hat{\ell}^aL_{ab}\hat{\ell}^b$. The indices have the range $i,j=1,\dots,10+n$, $a,b=1,\dots,8+n$.

For the Scherk-Schwarz reduction we use the subgroup of $O(5,5+n)$ under which the scalars $\hat{\ell}_a$ transform by constant shifts. These transformations take on the form:

$$\hat{\omega}(\Lambda)=\begin{pmatrix} 1 & 0 & 0 \\ -\dfrac{\Lambda^2}{2} & 1 & \Lambda^aL_{ab} \\ -\Lambda^a & 0 & \delta_{ab} \end{pmatrix}. \tag{8}$$

Under these transformations $\hat{\mathcal{M}}\rightarrow\hat{\omega}\hat{\mathcal{M}}\hat{\omega}^T$ [8], which implies that $\hat{\ell}^a\rightarrow\hat{\ell}^a+\Lambda^a$, while $\hat{M}$ and $\hat{\kappa}_1$ are invariant. The vector fields transform as $\hat{\mathcal{A}}_{\hat{\mu}}\rightarrow\hat{\omega}\hat{\mathcal{A}}_{\hat{\mu}}$. Writing out $\hat{\mathcal{A}}$ in terms of $n+8$ vectors $\hat{V}^a$ which transform as a

[7] The hats indicate five-dimensional fields and coordinates, the absence of a hat implies that the corresponding object is four-dimensional. We use the notation and conventions of [12].

[8] Recall that $\hat{\mathcal{M}}^{-1}=\mathcal{L}\hat{\mathcal{M}}\mathcal{L}$, where $\mathcal{L}$ is the $O(5,5+n)$ invariant metric.

vector under $O(4,4+n)$, and vectors A and B which are Kaluza-Klein and winding vectors obtained in the reduction from six to five dimensions, we get the following transformations under $\hat{\omega}$:

$$\hat{\ell}^a \to \hat{\ell}^a + \Lambda^a , \quad \hat{A}_{\hat{\mu}} \to \hat{A}_{\hat{\mu}} , \quad \hat{B}_{\hat{\mu}} \to \hat{B}_{\hat{\mu}} - \tfrac{1}{2}\Lambda^2 \hat{A}_{\hat{\mu}} + \Lambda^a L_{ab} \hat{V}^b_{\hat{\mu}} , \quad \hat{V}^a_{\hat{\mu}} \to \hat{V}^a_{\hat{\mu}} - \Lambda^a \hat{A}_{\hat{\mu}} . \tag{9}$$

In the implementation of the Scherk-Schwarz reduction to four dimensions we include in the relation between four- and five-dimensional fields a dependence on the fifth coordinate y, by choosing Λ y-dependent. The generalized reduction rules, expressing the 5-dimensional fields in terms of 4-dimensional ones, become

$$\begin{aligned}
&\hat{g}_{\mu\nu} = g_{\mu\nu} - (\kappa_2)^2 A^{(2)}_\mu A^{(2)}_\nu , \quad \hat{g}_{\mu y} = -(\kappa_2)^2 A^{(2)}_\mu , \quad \hat{g}_{yy} = -(\kappa_2)^2 , \quad \hat{\phi} = \phi + \tfrac{1}{2}\log \kappa_2 , \\
&\hat{\ell}^a = \ell^a + \Lambda^a(y) , \quad \hat{\kappa}_1 = \kappa_1 , \quad \hat{M} = M , \quad \hat{A}_\mu = A_\mu + a A^{(2)}_\mu , \quad \hat{A}_y = a , \\
&\hat{B}_\mu = B_\mu - \tfrac{1}{2}\Lambda^2(y) A_\mu + \Lambda^a(y) L_{ab} V^b_\mu + A^{(2)}_\mu \big(-\tfrac{1}{2}\Lambda^2(y) a + b + \Lambda^a(y) L_{ab} v^b \big) , \\
&\hat{B}_y = b - \tfrac{1}{2}\Lambda^2(y) a + \Lambda^a(y) L_{ab} v^b , \\
&\hat{V}^a_\mu = V^a_\mu - \Lambda^a(y) A_\mu + A^{(2)}_\mu \big(v^a - \Lambda^a(y) a \big) , \quad \hat{V}^a_y = v^a - \Lambda^a(y) a , \\
&\hat{B}_{y\mu} = B^{(2)}_\mu - \frac{a}{2} B_\mu - \frac{b}{2} A_\mu - \tfrac{1}{2} v^a L_{ab} V^b_\mu , \quad \hat{B}_{\mu\nu} = B_{\mu\nu} + A^{(2)}_{[\mu} B^{(2)}_{\nu]} - a A^{(2)}_{[\mu} B_{\nu]} - b A^{(2)}_{[\mu} A_{\nu]} - v^a L_{ab} A^{(2)}_{[\mu} V^b_{\nu]} .
\end{aligned} \tag{10}$$

In the reduction we will generate in the action terms proportional to $\partial_y \Lambda^a$, at most to the second power, so that the y-dependence disappears in four dimensions when $\Lambda^a = m^a y$. The scalar kinetic term reduces as follows:

$$\begin{aligned}
\tfrac{1}{8}\mathrm{tr}\big(\partial_{\hat{\mu}} \hat{\mathcal{M}} \partial^{\hat{\mu}} \hat{\mathcal{M}}^{-1} \big) &= \tfrac{1}{8}\mathrm{tr}\big(\partial_\mu M \partial^\mu M^{-1} \big) - (\partial \ln \kappa_1)^2 - \frac{1}{2\kappa_1^2 \kappa_2^2} m^a M^{-1}_{ab} m^b \\
&\quad + \frac{1}{2\kappa_1^2} \big(\partial_\mu \ell^a - m^a A^{(2)}_\mu \big) M^{-1}_{ab} \big(\partial^\mu \ell^b - m^b A^{(2)\mu} \big) .
\end{aligned} \tag{11}$$

As expected we find the local symmetry $\Delta \ell^a = m^a \beta(x)$, $\Delta A^{(2)}_\mu = \partial_\mu \beta(x)$, with $\beta(x)$ an arbitrary function of the four-dimensional coordinates. We also obtain a scalar potential that depends on the $O(4,4+n)$ scalars and, in the full action, on the dilaton.

For the vectors we can make the $O(6,6+n)$ structure explicit, by making m^a-dependent modifications: Define

$$\mathcal{F}_{\mu\nu}(\mathcal{A}^I) \equiv \begin{pmatrix} \mathcal{F}^{(2)}_{\mu\nu}(A) \\ \mathcal{F}^{(2)}_{\mu\nu}(B) \\ \mathcal{F}_{\mu\nu}(A) \\ \mathcal{F}_{\mu\nu}(B) \\ \mathcal{F}_{\mu\nu}(V^a) \end{pmatrix} = \begin{pmatrix} F^{(2)}_{\mu\nu}(A) \\ F^{(2)}_{\mu\nu}(B) + 2 m^a L_{ab} A_{[\mu} V^b_{\nu]} \\ F_{\mu\nu}(A) \\ F_{\mu\nu}(B) + 2 m^a L_{ab} V^b_{[\mu} A^{(2)}_{\nu]} \\ F_{\mu\nu}(V^a) + 2 m^a A^{(2)}_{[\mu} A_{\nu]} \end{pmatrix} \tag{12}$$

for $I = 1, \dots, 12+n$. All vector kinetic terms can now be gathered in the expression

$$-\tfrac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{N}^{-1} \mathcal{F}^{\mu\nu} , \tag{13}$$

where $\mathcal{N}^{-1}$ is an $O(6,6+n)$ element parametrizing the $6(6+n)$ scalars in four dimensions:

$$\mathcal{N}^{-1}=\begin{pmatrix} -\kappa_2^2-\dfrac{z^4}{4\kappa_2^2}+z^i\mathcal{M}_{ij}^{-1}z^j & \dfrac{1}{2\kappa_2^2}z^2 & z^k\mathcal{M}_{kj}^{-1}-\dfrac{z^2}{2\kappa_2^2}z^k\mathcal{L}_{kj} \\ \dfrac{z^2}{2k_2^2} & -\dfrac{1}{\kappa_2^2} & \dfrac{1}{\kappa_2^2}z^i\mathcal{L}_{ij} \\ \mathcal{M}_{ij}^{-1}z^j-\dfrac{z^2}{2\kappa_2^2}\mathcal{L}_{ik}z^k & \dfrac{1}{\kappa_2^2}\mathcal{L}_{ik}z^k & \mathcal{M}_{ij}^{-1}-\dfrac{1}{\kappa_2^2}\mathcal{L}_{ik}z^kz^l\mathcal{L}_{lj} \end{pmatrix}. \tag{14}$$

Here $\mathcal{M}^{-1}$ is the same matrix as (7), but now without hats. The $10+n$ scalars z^i correspond to a,b and v^a, z^2 is now $z^i\mathcal{L}_{ij}z^j$.

The 3-form strength tensor $H_{\mu\nu\rho}$ can be written as

$$H_{\mu\nu\rho}=\partial_{[\mu}B_{\nu\rho]}-m^aL_{ab}A^{(2)}_{[\mu}A_\nu V^b_{\rho]}+\tfrac{1}{2}\eta_{IJ}\mathcal{A}^I_{[\mu}\mathcal{F}_{\nu\rho]}(\mathcal{A}^J), \tag{15}$$

with η_{IJ} the invariant metric of $O(6,6+n)$. We will need its explicit form later on:

$$\eta_{IJ}=\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_1 & 0 \\ 0 & 0 & L_{ab} \end{pmatrix}. \tag{16}$$

The derivatives of the scalars z^i can be conveniently rewritten with m^a-dependent contributions:

$$\mathcal{D}_\mu z^i=\begin{pmatrix} \partial_\mu a \\ \partial_\mu b-m^aL_{ab}\left(V^b_\mu+v^bA^{(2)}_\mu\right) \\ \partial_\mu v^a+m^a\left(A_\mu+aA^{(2)}_\mu\right) \end{pmatrix}. \tag{17}$$

The complete action then takes on the form:

$$\begin{aligned} S_{4D,\text{massive}}=\int d^4x\sqrt{|g|}\,e^{-2\phi}\Big\{ & R-4(\partial\phi)^2-\tfrac{1}{8}\mathrm{tr}\left(\partial_\mu M\partial^\mu M^{-1}\right)+(\partial\ln\kappa_1)^2+(\partial\ln\kappa_2)^2 \\ & +\frac{1}{2\kappa_1^2\kappa_2^2}m^aM^{-1}_{ab}m^b-\frac{1}{2\kappa_1^2}(\partial^\mu\ell^a-m^aA^{(2)\mu})M^{-1}_{ab}\left(\partial_\mu\ell^b-m^bA^{(2)}_\mu\right)+\tfrac{3}{4}H^2 \\ & +\tfrac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{N}^{-1}\mathcal{F}^{\mu\nu}-\frac{1}{2\kappa_2^2}\mathcal{D}_\mu z\mathcal{M}^{-1}\mathcal{D}^\mu z\Big\}. \end{aligned} \tag{18}$$

3. Symmetries of the reduced action

The gauge transformations of the vector fields $A^{(2)}_\mu$, A_μ and V^a_μ, with parameters β,ξ and η^a, respectively, that leave the action (18) invariant obtain m^a-dependent modifications. To determine them, one should use the fact that the parameters $\hat{\xi},\hat{\alpha}$ and $\hat{\eta}^a$ corresponding to the gauge transformations of the vector fields $\hat{A}_\mu,\hat{B}_\mu$ and $\hat{V}^a_\mu$, respectively, are reduced as follows:

$$\hat{\xi}=\xi\,,\quad \hat{\alpha}=\alpha-\tfrac{1}{2}y^2m^aL_{ab}m^b\xi+ym^aL_{ab}\eta^b\,,\quad \hat{\eta}^a=\eta^a-ym^a\xi\,. \tag{19}$$

Using this, we find that the m^a-modifications in the transformations, which we call Δ_β, Δ_ξ and $\Delta\eta$, respectively, take on the form:

$$
\begin{aligned}
&\Delta_\beta A^{(2)}_\mu = \partial_\mu \beta\,, && \Delta_\beta B^{(2)}_\mu = 0\,,\\
&\Delta_\beta A_\mu = 0\,, && \Delta_\beta B_\mu = -\frac{m^2}{2}\beta^2 A_\mu + \beta m^a L_{ab} V^b_\mu\,,\\
&\Delta_\beta V^a_\mu = -m^a\beta A_\mu\,, && \Delta_\beta B_{\mu\nu} = -B^{(2)}_{[\mu}\partial_{\nu]}\beta\,,\\
&\Delta_\beta a = 0\,, && \Delta_\beta b = -\frac{m^2}{2}\beta^2 a + \beta m^a L_{ab} v^b\,,\\
&\Delta_\beta v^a = -m^a\beta a\,, && \Delta_\beta \ell^a = m^a\beta\,.\\
\\
&\Delta_\xi A^{(2)}_\mu = 0\,, && \Delta_\xi B^{(2)}_\mu = -\frac{m^2}{2}\xi^2 A^{(2)}_\mu - \xi m^a L_{ab} V^b_\mu\,,\\
&\Delta_\xi A_\mu = \partial_\mu \xi\,, && \Delta_\xi B_\mu = 0\,,\\
&\Delta_\xi V^a_\mu = m^a\xi A^{(2)}_\mu\,, && \Delta_\xi B_{\mu\nu} = -A_{[\mu}\partial_{\nu]}\alpha - B_{[\mu}\partial_{\nu]}\xi\,,\\
&\Delta_\xi a = 0\,, && \Delta_\xi b = 0\,,\\
&\Delta_\xi v^a = -m^a\xi\,, && \Delta_\xi \ell^a = 0\,.\\
\\
&\Delta_\eta A^{(2)}_\mu = 0\,, && \Delta_\eta B^{(2)}_\mu = m^a L_{ab}\eta^b A_\mu\,,\\
&\Delta_\eta A_\mu = 0\,, && \Delta_\eta B_\mu = -m^a L_{ab}\eta^b A^{(2)}_\mu\,,\\
&\Delta_\eta V^a_\mu = \partial_\mu \eta^a\,, && \Delta_\eta B_{\mu\nu} = -L_{ab} V^a_{[\mu}\partial_{\nu]}\eta^b\,,\\
&\Delta_\eta a = 0\,, && \Delta_\eta b = m^a L_{ab}\eta^b\,,\\
&\Delta_\eta v^a = 0\,, && \Delta_\eta \ell^a = 0\,.
\end{aligned}
\tag{20}
$$

Note that the Δ_β transformations are determined by performing a general coordinate transformation in five dimensions in the y-direction, with parameter $\beta(x)$. The remaining gauge transformations that do not obtain m^a-dependent modifications are given by:

$$
\Delta_\alpha B_\mu = \partial_\mu \alpha\,,\quad \Delta_\gamma B^{(2)}_\mu = \partial_\mu \gamma\,,\quad \Delta_{\alpha,\gamma,\Sigma} B_{\mu\nu} = \partial_{[\mu}\Sigma_{\nu]} - A_{[\mu}\partial_{\nu]}\alpha - A^{(2)}_{[\mu}\partial_{\nu]}\gamma\,. \tag{21}
$$

The generalised reduction has broken $O(6,6+n)$ invariance to an $O(4,4+n)$ global symmetry, which acts in an obvious way on the four-dimensional fields.

The local infinitesimal transformations δ_β, δ_η, δ_ξ, δ_γ and δ_α satisfy the following algebra:

$$
\begin{aligned}
&[\delta_\beta, \delta_\xi] = \delta_{\eta'}\,, \quad \eta'^a = m^a\beta\xi\,,\\
&[\delta_\eta, \delta_\xi] = \delta_{\gamma'}\,, \quad \gamma' = -\xi m^a L_{ab}\eta^b\,,\\
&[\delta_\beta, \delta_\eta] = \delta_{\alpha'}\,, \quad \alpha' = -\beta m^a L_{ab}\eta^b\,,\\
&[\delta_\alpha, \delta_{any}] = 0\,, \quad [\delta_\gamma, \delta_{any}] = 0\,.
\end{aligned}
\tag{22}
$$

We see that the symmetries with parameters β,ξ,η form an inhomogeneous Heisenberg [9] algebra. The corresponding global transformations have generators T_β, T_ξ and T_η^a. The algebra is

$$\left[T_\beta,T_\xi\right]=m^aL_{ab}T_\eta^b\,,\quad \left[T_\beta,T_\eta^a\right]=0\,,\quad \left[T_\xi,T_\eta^a\right]=0\,. \tag{23}$$

Note that the only nontrivial commutation relation in this algebra involves the combination $m^aL_{ab}T^b$. This can also be seen from the m^a-dependent terms in the transformations (20), which always contain the combinations $m^aL_{ab}V_\mu^{\ b}$ and $m^aL_{ab}\eta^b$. Therefore, effectively the nonabelian group which emerges from the Scherk-Schwarz reduction is three-dimensional.

4. Equivalence with a gauged symmetry

We will now show that (18) can also be obtained by starting from $m^a\equiv 0$ (the standard reduction) and by gauging an appropriate subgroup of $O(6,6+n)$. This will give all terms in (18), except the scalar potential, which will be obtained by supersymmetry.

Using indices $I,J=1,\dots,12+n$ as in (12), and by considering the algebra (22) we obtain structure constants f_{IJ}^K:

$$f_{3b}^2=m^aL_{ab}\,,\quad f_{b1}^4=m^aL_{ab}\,,\quad f_{13}^a=m^a\,. \tag{24}$$

Here the directions 1,2,3,4 and a correspond to the symmetries with parameters β,γ,ξ,α and η^a, respectively. Assuming local transformations of the gauge fields of the form $\delta\mathscr{A}_\mu^I=\partial_\mu\lambda^I+f_{JK}^I\lambda^J\mathscr{A}_\mu^K$ we recover the modified field-strengths (12). Since the transformations of the scalars under global $O(6,6+n)$ transformations are given, we are uniquely led to covariant scalar derivatives in (18).

By gauging the bosonic theory (18) for $m^a=0$ we do not recover the scalar potential in (18), since it is gauge invariant by itself. To obtain it we must use the fact that (18) is the bosonic part of a supersymmetric theory, namely some version of $N=4$ supergravity. We should therefore be able to obtain (18), now including the potential, from the results of [11].

In matter coupled $N=4$ supergravity the scalars parametrize an $O(6,6+n)/(O(6)\times O(6+n))\times SU(1,1)/U(1)$ coset. The $SU(1,1)/U(1)$ coset corresponds to the dilaton and to the dual of $B_{\mu\nu}$. The scalars of $O(6,6+n)/(O(6)\times O(6+n)$ can be expressed in terms of real fields Z_a^I, where $I=1,\dots,12+n$ and $a=1,\dots,6$, satisfying

$$Z_a^I\eta_{IJ}Z_b^J=-\delta_{ab}\,. \tag{25}$$

For our purposes it is convenient to introduce the combination [10] $Z^{IJ}\equiv Z_a^IZ_a^J$. The lagrangian of gauged $N=4$ supergravity can be expressed in terms of the Z^{IJ}. We give only the scalar kinetic terms and the potential, and to compare to (18) we will use the string frame:

$$\begin{aligned}S_{4D,N=4}=\tfrac{1}{2}\int d^4x\sqrt{|g|}\,e^{-2\phi}\Big\{&R-4(\partial\phi)^2+\left(\eta_{RS}+\eta_{RT}\eta_{SU}Z^{TU}\right)\mathscr{D}_\mu Z_a^R\mathscr{D}^\mu Z_a^S\\&+Z^{RU}Z^{SV}\left(\eta^{TW}+\tfrac{2}{3}Z^{TW}\right)f_{RST}f_{UVW}\Big\}\,.\end{aligned} \tag{26}$$

[9] The Heisenberg algebra is the algebra of three operators Q, P and E that fulfill the canonical conmutation relations $[Q,P]=E$, $[Q,E]=0$ and $[P,E]=0$.

[10] These variables were introduced in [13].

The structure constants are defined as $f_{RST} = f^U_{RS}\eta_{TU}$. We must now show that these terms are equivalent to the corresponding terms in (18) for the case of the group (24). First of all note that (25) implies that $W^{RS} = \eta^{RS} + 2Z^{RS}$ is an element of $O(6,6+n)$:

$$W^{RS}\eta_{ST}W^{TU} = \eta^{RU}. \tag{27}$$

Introducing W in (26) we obtain

$$S_{4D,\,N=4} = \tfrac{1}{2}\int d^4x\sqrt{|g|}\,e^{-2\phi}\Big\{R - 4(\partial\phi)^2 - \tfrac{1}{8}\mathscr{D}_\mu W^{RS}\mathscr{D}^\mu W_{SR}$$
$$+ \tfrac{1}{12}\{2\eta^{RU}\eta^{SV}\eta^{TW} + 3\eta^{RU}\eta^{SV}W^{TW} + W^{RU}W^{SV}W^{TW}\}f_{RST}f_{UVW}\Big\}. \tag{28}$$

We can now identify $W^{RS} = \mathscr{N}_{RS}$, which gives the correct kinetic term. In the potential we use that the only nonzero structure constants with lower indices are $f_{a13} = m^b L_{ba}$. Using the explicit form of $\mathscr{N}$ (14) and η (16) we recover exactly the potential of (18).

This shows that our massive theory (18), obtained by the Scherk-Schwarz method, is equivalent to a gauged $N=4$ supergravity theory.

5. Conclusions

In this letter we perform a generalised Scherk-Schwarz reduction of the effective action of the heterotic string on T^6 to obtain a massive supergravity theory in four dimensions. Our implementation of the Scherk-Schwarz method uses a noncompact subgroup of $O(5,5+n)$ in the step from five to four dimensions. We have shown explicitly that this theory can be obtained by gauging a Heisenberg subgroup of the global $O(6,6+n)$ symmetry group of $d=4$, $N=4$ matter coupled supergravity.

In the Einstein frame, with the dilaton kinetic term normalised to $1/2(\partial\phi)^2$, the scalar potential in (18) appears with a factor $\exp(\phi)$, which would be $\exp(\sqrt{2/(d-2)}\;\phi)$ for a similar analysis in d dimensions. The presence of such a scalar potential, with the same dilaton dependence, had to be assumed in [14] in order to obtain maximally symmetric black hole solutions in a $d=5$ context. It is interesting to note that the Scherk-Schwarz procedure, as well as gauging supergravity (see [15] for an example in $d=7$), generates such potentials. In this respect it would be interesting to elucidate further the relationship between the Scherk-Schwarz procedure and stringy reduction methods [11].

Clearly, by breaking the global $O(4,4+n)$ symmetry in five dimensions, there are further possible shift symmetries that can be used in the dimensional reduction. A study of these possibilities has been made in [2]. It would be interesting to see whether they also lead to gauged supergravities in four dimensions and to determine the corresponding gauge group. In this way, a whole class of gauged supergravities could be given a higher-dimensional interpretation. We do not expect that this can be done for all gauged supergravity theories. For instance, it cannot be done for the massive $d=10$ supergravity theory of Romans (other exceptions have been given in [2]). It would be interesting to see whether these exceptional cases can be given a higher-dimensional interpretation by some other techniques.

Acknowledgements

E.E. thanks H. Boonstra, B. Janssen and J. Maharana for useful remarks. This work is part of the Research program of the "Stichting voor Fundamenteel Onderzoek der Materie"(FOM). It is also supported by the European Commission TMR programme ERBFMRX-CT96-0045, in which E.B. and M. de R. are associated to the University of Utrecht.

[11] For recent work on this, see [16] and references therein.

References

[1] E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos, P.K. Townsend, Nucl. Phys. B 470 (1996) 113, hep-th/9601150.
[2] P.M. Cowdall, H. Lu, C.N. Pope, K.S. Stelle, P.K. Townsend, Nucl. Phys. B 486 (1997) 49, hep-th/9608173.
[3] J. Scherk, J.H. Schwarz, Phys. Lett. B 82 (1979) 60;
J. Scherk, J.H. Schwarz, Nucl. Phys. B 153 (1979) 61.
[4] A.H. Chamseddine, Phys. Rev. D 24 (1981) 3065.
[5] S. Thomas, P.C. West, Nucl. Phys. 245 (1984) 45;
M. Porrati, F. Zwirner, Nucl. Phys. B 326 (1989) 162.
[6] E. Cremmer, J. Scherk, J.H. Schwarz, Phys. Lett. B 84 (1979) 83.
[7] E. Dudas, C. Grojean, Four-dimensional M-theory and supersymmetry breaking, hep-th/9704177.
[8] E. Cremmer, B. Julia, Phys. Lett. B 80 (1978) 48.
[9] L.J. Romans, Phys. Lett. B 169 (1986) 374.
[10] J. Maharana, J.H. Schwarz, Nucl. Phys. B 390 (1993) 3, hep-th/9207016.
[11] M. de Roo, Nucl. Phys. B 255 (1985) 515;
M. de Roo, Phys. Lett. B 156 (1985) 331;
E. Bergshoeff, I.G. Koh, E. Sezgin, Phys. Lett. 155 (1985) 71.
[12] K. Behrndt, E. Bergshoeff, B. Janssen, Nucl. Phys. B 467 (1996) 100, hep-th/9512152.
[13] P. Wagemans, Phys. Lett. B 206 (1988) 241;
P. Wagemans, Aspects of $N = 4$ supergravity, Thesis, Groningen (1990), unpublished.
[14] J. Maharana, H. Singh, On the Compactification of type IIA String Theory, hep-th/9705058.
[15] P.K. Townsend, P. van Nieuwenhuizen, Phys. Lett. B 125 (1983) 41.
[16] I. Shah, S. Thomas, Finite soft terms in string compactifications with broken supersymmetry, hep-th/9705182