ELSEVIER
25 September 1997
PHYSICS LETTERS B
Physics Letters B 410 (1997) 13–21

Massive IIA supergravity from the topologically massive D-2-brane

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Received 28 June 1997
Editor: P.V. Landshoff

Abstract

The superfield equations of massive IIA supergravity, in the form of constraints on the superspace geometry, are shown to be implied by κ-symmetry of the topologically massive D-2-brane. © 1997 Elsevier Science B.V.

1. Introduction

The problem of the determination of the full κ-symmetric action for type II super D-branes in general supergravity backgrounds has now been largely solved by the concerted efforts of several groups [1–7]. Most of this work has concentrated on the verification of κ-symmetry in backgrounds of varying generality, but it is known from earlier work on super p-branes [8–11] that the requirement of κ-symmetry constrains the possible backgrounds. For example, κ-symmetry of the $D=11$ supermembrane requires the background to satisfy the field equations of $D=11$ supergravity. Moreover, since κ-symmetry is necessary for the consistency of the worldvolume field equations, i.e. the 'branewave' equations, one can view the equations of $D=11$ supergravity as branewave integrability conditions. The field equations of $D=10$ IIA supergravity similarly follow from κ-symmetry of the IIA D-2-brane; we shall verify this here, but it is an immediate consequence of the fact that the super D-2-brane action is dual to the $D=11$ supermembrane action in $D=11$ backgrounds with a $U(1)$ isometry [5]. Such a background is equivalent to one of $D=10$ IIA supergravity. However, not all IIA backgrounds can be viewed as reductions of $D=11$ supergravity backgrounds. Specifically, only the usual, 'massless', IIA supergravity is obtainable in this way. The 'massive' IIA theory, which has a cosmological constant proportional to a mass parameter m [12], has no known interpretation of this type, although one might expect the field equations to be required by κ-symmetry of some generalization of the super D-2-brane action. In fact, it was shown in [5] that κ-symmetry of the D-2-brane action in a purely bosonic IIA background requires the inclusion of a worldvolume Chern-Simons term when $m \neq 0$, as expected from earlier T-duality considerations [13,14]. We shall refer to this as the 'topologically massive' D-2-brane, since the CS term constitutes a topological mass term for the Born-Infeld field [15].

PII S0370-2693(97)00911-8

The main purpose of this letter is to show that the massive IIA field equations are a *consequence* of κ-symmetry of the topologically massive D-2-brane action; similar results then follow for lower-dimensional massive supergravity theories [16,17] by dimensional reduction. To establish this requires consideration of general IIA supergravity backgrounds, including fermions. It is notable that the massive field equations obtained in this way arise as a particularly simple set of superfield constraints that are formally the same as those of the massless IIA theory, differing only in the m-dependence of the field strengths. Superspace constraints for both massless and massive IIA supergravity have been proposed previously [18]. It is not clear to us whether our results are in complete agreement with those of [18]. In any case, we think it worthwhile to have an independent derivation of these constraints in view of the fact that invariance under supersymmetry was not completely established in [12] because terms quartic in fermions were omitted from the action.

The coupling of D-branes to a supergravity background leads to a particular basis of supergravity field variables. As seen in [14], and as we shall see again here, this basis leads to a number of simplifications as compared to the 'canonical' basis used in the supergravity literature (e.g. [19,20]). We conclude this paper with an examination of the details of the map from the old variables to the new 'D-brane inspired' ones.

2. The D-2-brane in a general IIA background

Let Z^M be local coordinates on $D=10$ IIA superspace, with E_M^A the superspace vielbein, so $E^A \equiv dZ^M E_M^A$ is a basis of one-forms on superspace. We define a worldvolume metric, in local coordinates ξ^i, by

$$g_{ij} = E_i^a E_j^b \,\eta_{ab}\,, \tag{2.1}$$

where $E_i^A = \partial_i Z^M E_M^A$ and η is the $D=10$ Minkowski metric. We introduce a scalar superfield ϕ and two-form superspace gauge potential B, the lowest components of which are, respectively, the dilaton and the Neveu-Schwarz/Neveu-Schwarz $(NS \otimes NS)$ two-form gauge potential. We also introduce a Born-Infeld 1-form gauge potential V with 'modified' field strength

$$\mathscr{F} = dV - B\,. \tag{2.2}$$

Whereas V is defined directly on the worldvolume the two form B is here the pullback of the two-form on superspace; we use the same letter to denote the superspace and worldvolume forms since it should be clear from the context which is intended. Finally, we introduce the superspace 1-form C and 3-form A, the lowest components of which are the Ramond/Ramond $(R \otimes R)$ gauge potentials. Again, we shall use the same letters to denote their pullbacks to the worldvolume. With these ingredients we can write down the action for the super D-2-brane in a general IIA supergravity background. Setting the tension to unity we have

$$S = -\int d^3\xi \left[e^{-\phi}\sqrt{-\det(g+\mathscr{F})} \right. \\ \left. + \tfrac{1}{6}\varepsilon^{ijk}\left(A_{ijk} + 3C_i\mathscr{F}_{jk} + \tfrac{3}{2}mV_iF_{jk}\right)\right], \tag{2.3}$$

where m is the mass parameter.

The structure group of the superspace tangent bundle is taken to be the Lorentz group, with respect to which E^A decomposes into $E^A = (E^a, E^\alpha)$ where E^a is a Lorentz vector and E^α a Majorana spinor. The spacetime Dirac matrices Γ_a can be pulled back to the worldvolume to yield

$$\gamma_i = E_i^a \Gamma_a\,, \tag{2.4}$$

which behave like three-dimensional Dirac matrices except for the fact that the product of all three is not the identity matrix. Instead, the matrix

$$\Gamma_{(0)} = \frac{1}{6\sqrt{-\det g}}\,\varepsilon^{ijk}\gamma_{ijk} \tag{2.5}$$

is traceless, commutes with γ_i matrices, and satisfies

$$\Gamma_{(0)}^2 = 1\,. \tag{2.6}$$

We refer to [5] for further details of the notation and conventions, but we note here that the exterior derivative of a scalar superfield ϕ can be expanded on the basis of 1-forms $E^A = dZ^M E_M^A$ as

$$d\phi = E^A D_A \phi\,, \tag{2.7}$$

which defines the supercovariant derivative $D_A \phi$ of ϕ.

To present the κ-symmetry variations it will be convenient to define $\delta E^A := \delta Z^M E_M^A$. The variation of the worldvolume fields Z^M is always such that $\delta_\kappa E^a = 0$. Making use of various lemmas presented in [5] we then find that

$$\delta_\kappa \phi = \delta_\kappa E^\alpha D_\alpha \phi$$

$$\delta_\kappa g_{ij} = -2\delta_\kappa E^\alpha E^a_{(i} E^A_{j)} T^b_{A\alpha} \eta_{ab}$$

$$\delta_\kappa C_i = \partial_i(\delta_\kappa E^\alpha C_\alpha) - \delta_\kappa E^\alpha E_i^B (K - mB)_{B\alpha}$$

$$\delta_\kappa A_{ijk} = \delta_\kappa E^\alpha E_i^B E_j^C E_k^D \{F_{DCB\alpha} - 4C_{[D} H_{CB\alpha]}$$

$$-3mB_{DC}B_{B\alpha}\} + \quad \text{derivative} \tag{2.8}$$

where T_{AB}^C is the superspace torsion, $H = dB$ is the $NS \otimes NS$ two-form field strength, and

$$K = dC + mB$$

$$F = dA + H \wedge C + \tfrac{1}{2} mB \wedge B \tag{2.9}$$

are the $R \otimes R$ superspace field strengths [14,5]. The square brackets around suffices indicate super-anti-symmetrization on the enclosed indices. Note that we adopt the conventions

$$P = \frac{1}{p!} E^{A_p} \ldots E^{A_1} P_{A_1 \ldots A_p}$$

$$d(PQ) = PdQ + (-1)^q (dP) Q \tag{2.10}$$

for superspace p-form P and q-form Q, where the exterior product of forms is understood. These conventions lead to some sign differences relative to [14]. We adopt the same conventions for worldvolume forms, e.g.

$$\mathscr{F} = \tfrac{1}{2} d\xi^j d\xi^i \mathscr{F}_{ij}, \tag{2.11}$$

which leads to some sign differences relative to [5].

Apart from the specification of $\delta_\kappa E^\alpha$, which will be postponed until later, we must also specify $\delta_\kappa V$. This must be such as to ensure that the variation of $\mathscr{F}$ is 'supercovariant', i.e. appears without derivatives of the parameter κ, and this property essentially fixes it uniquely to be

$$\delta V_i = E_i^A \delta_\kappa E^B B_{BA}. \tag{2.12}$$

The resulting transformation of $\mathscr{F}$ is [1]

$$\delta_\kappa \mathscr{F}_{ij} = E_i^A E_j^B \delta_\kappa E^\alpha H_{\alpha BA}. \tag{2.13}$$

With these variations in hand, and discarding a surface term, we compute that

$$\delta_\kappa S = \int d^3\xi\, \delta_\kappa E^\alpha \Big\{ e^{-\phi} \sqrt{-\det(g + \mathscr{F})}$$
$$\times \Big[D_\alpha \phi + (g + \mathscr{F})^{ij} \big(E^a_{(i} E^B_{j)} T^c_{B\alpha} \eta_{ac}$$
$$+ \tfrac{1}{2} E_i^A E_j^B H_{BA\alpha} \big) \Big]$$
$$+ \tfrac{1}{6} \varepsilon^{ijk} \big(E_k^A E_j^B E_i^C F_{CBA\alpha} + 3 E_i^A \mathscr{F}_{jk} K_{A\alpha} \big) \Big\}, \tag{2.14}$$

where $(g + \mathscr{F})^{ij}$ are the entries of the inverse of the matrix $(g + \mathscr{F})$. Note that all m-dependence of this variation is now implicit in the $R \otimes R$ field strengths.

Following [5], it is convenient to introduce the matrix X by $X = g^{-1} \mathscr{F}$, or

$$X^i_j = g^{ik} \mathscr{F}_{kj} \tag{2.15}$$

Because of the antisymmetry of $\mathscr{F}$, this matrix satisfies the identity

$$X^3 \equiv \tfrac{1}{2} (\mathrm{tr} X^2) X. \tag{2.16}$$

We now rewrite (2.14) as a sum of terms in which each term involves a different number of worldvolume fermions. Thus,

$$\delta_\kappa S = \int d^3\xi \sqrt{-\det g}$$
$$\times e^{-\phi}\, \delta_\kappa E^\alpha \left[\Delta_0 + \Delta_1 + \Delta_2 + \Delta_3 \right]_\alpha \tag{2.17}$$

where

$$(\Delta_0)_\alpha = \sqrt{\det(1 + X)} \Big\{ \big[(1 + X)^{-1} \big]^i_k$$
$$\times g^{kj} \big[E^a_{(i} E^b_{j)} T^c_{b\alpha} \eta_{ca} + \tfrac{1}{2} E_i^a E_j^b H_{ba\alpha} \big]$$
$$+ D_\alpha \phi \Big\} + \frac{e^\phi}{6\sqrt{-\det g}}$$
$$\times \varepsilon^{ijk} \big[E_k^a E_j^b E_i^c F_{cba\alpha} + 3 E_i^a (gX)_{jk} K_{a\alpha} \big]$$

[1] The sign differs from [5] as a result of the 'reverse order' convention (2.11) for the components of worldvolume forms.

$$
\begin{aligned}
(\Delta_1)_\alpha = & \sqrt{\det(1+X)}\,\left[(1+X)^{-1}\right]^i_{\ k} \\
& \times g^{kj}\left[E^a_{(i}E^\beta_{j)}T^c_{\beta\alpha}\eta_{ca} - E^a_{[i}E^\beta_{j]}H_{a\beta\alpha}\right] \\
& + \frac{e^\phi}{2\sqrt{-\det g}} \\
& \times \varepsilon^{ijk}E^\beta_i\left[E^a_jE^b_kF_{ab\beta\alpha} + (gX)_{jk}K_{\beta\alpha}\right] \\
(\Delta_2)_\alpha = & \tfrac{1}{2}E^\beta_iE^\gamma_j\left\{\sqrt{\det(1+X)}\,\left[(1+X)^{-1}\right]^i_{\ k}\right. \\
& \left.\times g^{kj}H_{\gamma\beta\alpha} - \frac{e^\phi}{\sqrt{-\det g}}\varepsilon^{ijk}E^c_kF_{c\beta\gamma\alpha}\right\} \\
(\Delta_3)_\alpha = & \frac{e^\phi}{6\sqrt{-\det g}}\varepsilon^{ijk}E^\beta_kE^\gamma_jE^\delta_iF_{\delta\gamma\beta\alpha}\,. \qquad (2.18)
\end{aligned}
$$

The subscript on Δ indicates the number of worldvolume fermions. Each of these terms must vanish separately, for some choice of $\delta_\kappa E^\alpha$.

We know that $\delta_\kappa E^\alpha$ must take the form

$$\delta_\kappa E^\alpha = \left[\bar\kappa(1-\hat\Gamma)\right]^\alpha , \qquad (2.19)$$

where the matrix $\hat\Gamma$ is tracefree and squares to the identity matrix. From [5] we know that for backgrounds that are purely bosonic solutions of IIA supergravity we can choose [2]

$$\hat\Gamma = \sqrt{\left(1-\tfrac{1}{2}\mathrm{tr}X^2\right)}\;\Gamma_{(0)} + \tfrac{1}{2}X_{ij}\Gamma^{ij}\Gamma_{11}\,. \qquad (2.20)$$

We do not wish to assume here that this is the form of $\hat\Gamma$ in general backgrounds, although this will turn out to be the case. We shall need only the expansion to first order in X, which is

$$\hat\Gamma = \Gamma_{(0)} - \tfrac{1}{2}\gamma^{ij}X_{ij}\Gamma_{11} + \mathscr{O}(X^2)\,. \qquad (2.21)$$

The leading term, independent of X is one of only two possibilities consistent with spacetime and worldvolume Lorentz invariance; the other possibility is Γ_{11} but this choice leads immediately to much stronger constraints on the background so we may discard it. The term linear in X is also effectively unique; there is a freedom to replace Γ_{11} by $\Gamma_{(0)}\Gamma_{11}$ since this affects only the $\mathscr{O}(X^2)$ terms, but this leads to equivalent results [5].

We now turn to an examination of each of the four Δ terms in (2.17). The term involving Δ_3 can cancel only if

$$F_{\alpha\beta\gamma\delta} = 0\,. \qquad (2.22)$$

Consideration of the terms independent of and linear in X in the Δ_2 term leads directly to the conclusion that

$$H_{\alpha\beta\gamma} = 0\,, \quad F_{\alpha\beta\gamma a} = 0\,. \qquad (2.23)$$

We turn next to Δ_1. The vanishing of $\delta_\kappa E\Delta_1$ to zeroth order in X requires

$$
\begin{aligned}
&(1-\Gamma_{(0)})^{\gamma\beta}\left[\sqrt{-\det g}\;g^{kj}E^a_jT^c_{\beta\alpha}\eta_{ca}\right. \\
&\left. + \tfrac{1}{2}\varepsilon^{ijk}\,E^a_iE^b_je^\phi F_{ab\beta\alpha}\right] = 0\,. \qquad (2.24)
\end{aligned}
$$

Without the $(1-\Gamma_{(0)})$ factor, terms with different numbers of E^a_i factors would have to cancel separately. This would impose very strong constraints on the background. In fact, the constraints are weaker because the identity

$$(1-\Gamma_{(0)})\left[\sqrt{-\det g}\;\gamma^i + \tfrac{1}{2}\varepsilon^{ijk}\gamma_{jk}\right] \equiv 0 \qquad (2.25)$$

allows a cancellation between terms, but this can happen only if [3]

$$T^c_{\beta\alpha} = i(\Gamma^c)_{\alpha\beta}\,, \quad F_{ab\beta\alpha} = ie^{-\phi}(\Gamma_{ab})_{\alpha\beta}\,. \qquad (2.26)$$

In principle, these expressions could come multiplied by some scalar function but this could be removed by a rescaling of the component of the spin connection that these 'conventional' constraints allow us to solve for. We may now use (2.26) in the terms linear in X in the expansion of $\delta_\kappa E\Delta_1$ to find

$$
\begin{aligned}
0 = & \tfrac{1}{2}(gX)_{ij}\left[(1-\Gamma_{(0)})\gamma^{ijk}\right]^{\gamma\beta}\left[e^\phi K_{\beta\alpha} - (\Gamma_{11})_{\beta\alpha}\right] \\
& + X^{ik}(1-\Gamma_{(0)})^{\gamma\beta}\left[E^a_iH_{a\beta\alpha} - (\gamma_i\Gamma_{11})_{\beta\alpha}\right], \qquad (2.27)
\end{aligned}
$$

from which we deduce that

$$H_{a\alpha\beta} = i(\Gamma_a\Gamma_{11})_{\alpha\beta}\,, \quad K_{\alpha\beta} = ie^{-\phi}(\Gamma_{11})_{\alpha\beta}\,. \qquad (2.28)$$

[2] We refer to [5] for the relation of $\hat\Gamma$ to the 'standard' matrix Γ that arises in the proof of κ-invariance for general p.

[3] The factor of i is needed for reality of $\delta_\kappa S$ with standard conventions for complex conjugation of products of anticommuting spinors.

We have now determined that the superspace constraints (2.26) and (2.28) are necessary for κ-symmetry. Since the Δ_1 terms involve no background fermions it follows from the results of [4,5] that the these constraints are also sufficient for the cancellation of the Δ_1 terms to all orders in X if $\hat{\Gamma}$ is given by (2.20).

We now turn to the Δ_0 terms in (2.17), which involve background fermion fields. We first expand expand $\delta_\kappa E\Delta_0$ in powers of X. To zeroth order we find that

$$(1-\Gamma_{(0)})\left[\lambda + g^{ij}E_i^aE_j^bT_{b\alpha}^c\eta_{ca} + \frac{e^\phi}{6\sqrt{-\det g}}\varepsilon^{ijk}E_k^aE_j^bE_i^cF_{cba\alpha}\right]=0, \tag{2.29}$$

where we have introduced the dilatino superfield

$$\lambda_\alpha = D_\alpha\phi. \tag{2.30}$$

As before, terms with different numbers of E_i^a factors would have to cancel separately were it not for the possibility of combining them by means of the identity (2.25) and the further identity $E_i\cdot E_j \equiv g_{ij}$. We thereby deduce that the vanishing of the Δ_0 terms requires

$$T_{c\alpha}^b = \delta_c^b\,\chi_\alpha, \quad F_{abc\gamma} = e^{-\phi}[\Gamma_{abc}(\lambda+3\chi)]_\gamma, \tag{2.31}$$

where χ is some background spinor field. We may choose χ at will since the torsion constraint defining χ is a 'conventional' one that just determines some components of the spin connection. Obvious choices are $\chi=0$ and $\chi=-3\lambda$, but neither of these turns out to be the simplest one so we leave χ free at present.

If we now use (2.31) in the terms in $\delta_\kappa E\Delta_0$ linear in X we find that

$$\begin{aligned}0 &= \sqrt{-\det g}\,(gX)_{ij}\left[(1-\Gamma_{(0)})\gamma^{ij}\Gamma_{11}(\lambda+3\chi)\right]^\beta \\ &\quad - \sqrt{-\det g}\,X_k^i g^{kj}E_i^aE_j^b(1-\Gamma_{(0)})^{\beta\alpha}H_{ba\alpha} \\ &\quad + e^\phi\varepsilon^{ijk}E_i^a(gX)_{jk}(1-\Gamma_{(0)})^{\beta\alpha}K_{a\alpha}.\end{aligned} \tag{2.32}$$

It follows by a reasoning similar to that used previously that

$$H_{ab\gamma} = [\Gamma_{ab}\zeta]_\gamma, \quad K_{a\beta} = e^{-\phi}[\Gamma_a\xi]_\beta, \tag{2.33}$$

where ζ and ξ are two further spinor fields. If this information is now used in (2.32) one finds that

$$\xi+\zeta = -\Gamma_{11}(\lambda+3\chi). \tag{2.34}$$

At this point we have found the general form of the constraints in terms of the dilatino superfield and two other undetermined spinor superfields, one combination of which must be fixed by the cancellation of terms higher order in X in the *kappa*-symmetry variation (for consistency with known results for $D=11$ supermembrane. Indeed, with $\hat{\Gamma}$ given by (2.20) one finds that the relation

$$\zeta = -2\Gamma_{11}\chi \tag{2.35}$$

is needed for cancellation of terms quadratic in X, and that all higher order terms then cancel. Thus

$$\begin{aligned}H_{ab\gamma} &= -2[\Gamma_{ab}\Gamma_{11}\chi]_\gamma \\ K_{a\beta} &= -e^{-\phi}[\Gamma_a\Gamma_{11}(\lambda+\chi)]_\beta.\end{aligned} \tag{2.36}$$

We now see that there is another obvious choice for χ, namely

$$\chi = -\lambda \tag{2.37}$$

since $K_{a\alpha}$ then vanishes. This choice greatly simplifies the analysis of the Bianchi identities, to which we now turn.

3. Bianchi identities

The superspace constraints derived above are all m-independent. The m-dependence is implicit in the $R\otimes R$ field strengths K and F, defined in (2.9), which results in an m-dependence of the Bianchi identities. These are

$$\begin{aligned}&dT^A = E^BR_B^A, \quad dH=0, \quad dF = H\wedge K, \\ &dK = mH,\end{aligned} \tag{3.1}$$

where R_B^A is the curvature 2-form. At dimension zero or less the Bianchi identities are indeed satisfied by superspace tensors satisfying the constraints found above. In particular, the F Bianchi identity at dimension zero is satisfied by virtue of the gamma-matrix identity

$$(\Gamma^a)_{(\alpha\beta}(\Gamma_{ab})_{\gamma\delta)} + (\Gamma_{11})_{(\alpha\beta}(\Gamma_{11}\Gamma_b)_{\gamma\delta)} \equiv 0 \tag{3.2}$$

which is clearly the dimensional reduction to $D=10$

of the $D=11$ identity required for κ-symmetry of the $D=11$ supermembrane [10].

Because the structure group of the frame bundle is taken to be the Lorentz group, the Bianchi identities determine the only remaining torsion component at dimension $1/2$, $T_{\alpha\beta}^{\gamma}$. The result given in [4], where the choice $\chi=0$ was made, is rather complicated. Here we shall see that the choice $\chi=-\lambda$ leads to considerable simplifications. With this choice, the Bianchi identity for K at dimension $1/2$ (which is m-independent since $H_{\alpha\beta\gamma}=0$) implies that

$$(\Gamma_{11})_{\epsilon(\gamma}T_{\alpha\beta)}^{\epsilon}=(\Gamma_{11})_{(\alpha\beta}\lambda_{\gamma)}, \tag{3.3}$$

while the torsion Bianchi identity at dimension $1/2$ implies that

$$(\Gamma^{a})_{\epsilon(\gamma}T_{\alpha\beta)}^{\epsilon}=(\Gamma^{a})_{(\alpha\beta}\lambda_{\gamma)}. \tag{3.4}$$

These are solved by

$$T_{\alpha\beta}^{\gamma}=\delta_{(\alpha}^{\gamma}\lambda_{\beta)}. \tag{3.5}$$

We have now arrived at a set of constraints on all superspace tensors of dimension $1/2$ or less in terms of the dilatino superfield ϕ (since $\lambda=D\phi$). These constraints are as follows, in order of increasing dimension. At dimension -1:

$$F_{\alpha\beta\gamma\delta}=0. \tag{3.6}$$

At dimension $-1/2$:

$$H_{\alpha\beta\gamma}=0, \qquad F_{\alpha\beta\gamma a}=0. \tag{3.7}$$

At dimension 0:

$$\begin{aligned} &T_{\beta\alpha}^{a}=i(\Gamma^{a})_{\beta\alpha}, \quad H_{\alpha\beta c}=i(\Gamma_{c}\Gamma_{11})_{\alpha\beta},\\ &K_{\beta\alpha}=ie^{-\phi}(\Gamma_{11})_{\beta\alpha}, \quad F_{\alpha\beta ba}=ie^{-\phi}(\Gamma_{ba})_{\alpha\beta}. \end{aligned} \tag{3.8}$$

At dimension $1/2$:

$$\begin{aligned} &T_{a\beta}^{c}=-\delta_{a}^{c}\lambda_{\beta}, \quad T_{\beta\alpha}^{\gamma}=\delta_{(\beta}^{\gamma}\lambda_{\alpha)},\\ &H_{\alpha bc}=2(\Gamma_{bc}\Gamma_{11}\lambda)_{\alpha}, \quad K_{a\beta}=0,\\ &F_{\alpha abc}=-2e^{-\phi}[\Gamma_{abc}\lambda]_{\alpha}. \end{aligned} \tag{3.9}$$

The only undetermined components of the torsion and field strengths are now those of dimension 1 or higher. These include the bosonic field strengths K_{ab}, F_{abcd} and H_{abc} and the torsion component $T_{\alpha b}^{\gamma}$ at dimension 1. These will be m-dependent, in general, because of the m-dependence of the Bianchi identity for K. For example, the Bianchi identity for K at dimension 1 is

$$\begin{aligned} &(D_{a}\phi)(\Gamma_{11})_{\beta\gamma}+2T_{a(\beta}^{\alpha}(\Gamma_{11})_{\beta)\alpha}-e^{\phi}(\Gamma^{b})_{\beta\gamma}K_{ba}\\ &\quad+me^{\phi}(\Gamma_{a}\Gamma_{11})_{\beta\gamma}=0. \end{aligned} \tag{3.10}$$

This implies that

$$T_{\alpha\beta}^{\gamma}=\bar{T}_{\alpha\beta}^{\gamma}-\tfrac{1}{2}me^{\phi}(\Gamma_{a})_{\beta}^{\gamma}, \tag{3.11}$$

where $\bar{T}^{A}$ is the torsion 2-form for $m=0$. This m-dependent modification of the torsion tensor was first found in [18], in which a complete set of constraints for massless and massive IIA supergravity were proposed. As far as we can tell, our results are in agreement with those of [18], but it is not clear to us whether the m-dependence of the 4-form field strength was taken into account by these authors.

When $m=0$ the IIA superspace constraints found above are just those obtained by dimensional reduction of the standard $D=11$ superspace constraints. In fact, they were deduced in this way in [11], independently of [18]. These constraints are known to imply the field equations of $D=11$ supergravity [21]. Thus, the $m=0$ constraints imply the field equations of massless IIA supergravity. It follows that the $m\neq 0$ constraints imply the field equations of the massive IIA theory. Note that by 'constraints' we mean the specification of the components of all superspace tensors of dimension $1/2$ or less. The massive IIA constraints are therefore formally identical to those of the massless theory, differing only in the m-dependence of the $R\otimes R$ field strength superforms. This is a consequence of the 'natural' choice of basis of IIA supergravity field variables selected by the coupling to the super D-2-brane. We shall now conclude with a discussion of how this basis is related to the 'canonical' one, and why the new basis is simpler.

4. Field variables in IIA/IIB supergravity

We first recall what the canonical variables are. To simplify the notation we use form notation and indicate the $NS\otimes NS$ 2-form by B with corresponding gauge transformation $\delta B=d\Lambda$. All other gauge fields are $R\otimes R$ potentials which we denote by $C^{(r)}$ $(r=1,\cdots,9)$. We use the notation and conventions of [19] but have renamed the fields of IIA/IIB supergravity as follows:

$$A^{(1)} \to C^{(1)}, \quad \mathscr{B}^{(2)} \to C^{(2)}, \quad C \to C^{(3)}, \quad D \to C^{(4)}. \tag{4.1}$$

The potentials with $r \geq 5$ are the corresponding dual potentials. The fields $C^{(r)}$ are potentials of IIA (IIB) supergravity for r odd (r even). In the canonical basis the $R \otimes R$ potentials transform under the following $R \otimes R$ gauge transformations with parameters $\Lambda^{(r)}(r = 0, \cdots, 8)$ [19] [4]:

$$\begin{aligned}
\delta C^{(1)} &= d\Lambda^{(0)} - \frac{m}{2}\Lambda, \\
\delta C^{(2)} &= d\Lambda^{(1)}, \\
\delta C^{(3)} &= d\Lambda^{(2)} + 2d\Lambda^{(0)}B - m\Lambda B, \\
\delta C^{(4)} &= d\Lambda^{(3)} + \tfrac{3}{4}d\Lambda^{(1)}B - \tfrac{3}{4}d\Lambda C^{(2)}, \\
\delta C^{(5)} &= d\Lambda^{(4)} - \tfrac{15}{2}d\Lambda^{(2)}B + \tfrac{15}{2}d\Lambda C^{(3)}, \\
\delta C^{(6)} &= d\Lambda^{(5)} + \Lambda^{(3)}dB + \Lambda^{(1)}BdB - \tfrac{1}{2}d\Lambda BC^{(2)}.
\end{aligned} \tag{4.2}$$

With respect to [19] we have renamed the parameters as follows:

$$\Lambda^{(1)} \to \Lambda^{(0)}, \quad \Sigma^{(2)} \to \Lambda^{(1)}, \quad \chi \to \Lambda^{(2)}, \quad \rho \to \Lambda^{(3)}. \tag{4.3}$$

The gauge transformations of the dual potentials $C^{(5)}$ and $C^{(6)}$ have been taken from [20] and [22], respectively.

The new basis presented in [14] has the following distinguishing features:

1. None of the $R \otimes R$ potentials transform under the gauge transformation of the $NS \otimes NS$ 2-form B (with parameter Λ) except for the m-dependent terms in the IIA case.

2. All $R \otimes R$ gauge transformations are written in a canonical way such that in the terms containing the $NS \otimes NS$ 2-form B the $R \otimes R$ parameter $\Lambda^{(r)}$ always occurs undifferentiated.

It is now straightforward to show that by performing a suitable redefinition of the fields $C^{(r)}(r \geq 4)$ and the parameters $\Lambda^{(r)}(r \geq 2)$ the canonical basis (4.1), (4.2) can be transformed into the new basis defined above. More precisely, the following redefinitions are needed:

$$\begin{aligned}
C^{(4)\prime} &= C^{(4)} + \tfrac{3}{4}BC^{(2)}, \\
C^{(5)\prime} &= C^{(5)} - \tfrac{15}{2}BC^{(3)}, \\
C^{(6)\prime} &= C^{(6)} + \tfrac{1}{4}B^2C^{(2)},
\end{aligned} \tag{4.4}$$

and

$$\begin{aligned}
\Lambda^{(2)\prime} &= \Lambda^{(2)} + 2B\Lambda^{(0)}, \\
\Lambda^{(3)\prime} &= \Lambda^{(3)} + \tfrac{3}{2}B\Lambda^{(1)}, \\
\Lambda^{(4)\prime} &= \Lambda^{(4)} - 15B\Lambda^{(2)} - 15B^2\Lambda^{(0)}, \\
\Lambda^{(5)\prime} &= \Lambda^{(5)} + \tfrac{1}{4}B^2\Lambda^{(1)}.
\end{aligned} \tag{4.5}$$

In the new basis the $R \otimes R$ gauge transformations are given by (omitting the primes)

$$\begin{aligned}
\delta C^{(1)} &= d\Lambda^{(0)} - \frac{m}{2}\Lambda, \\
\delta C^{(2)} &= d\Lambda^{(1)}, \\
\delta C^{(3)} &= d\Lambda^{(2)} - 2\Lambda^{(0)}dB - m\Lambda B, \\
\delta C^{(4)} &= d\Lambda^{(3)} + \tfrac{3}{2}\Lambda^{(1)}dB, \\
\delta C^{(5)} &= d\Lambda^{(4)} + 15\Lambda^{(2)}dB + \tfrac{15}{2}m\Lambda B^2, \\
\delta C^{(6)} &= d\Lambda^{(5)} + \Lambda^{(3)}dB.
\end{aligned} \tag{4.6}$$

As an example of the simplicity inherent to the new basis we will give the T-duality rules of [19] in this basis. First, to keep the calculations simple, we make the same assumption about the background fields as [14], i.e. [5]

$$g_{x\mu} = B_{x\mu} = 0. \tag{4.7}$$

Here x refers to the isometry direction. Under this assumption the T-duality rules of [19] simplify as follows. The T-duality rules for the $NS \otimes NS$ fields reduce to

$$\begin{aligned}
&\tilde{g}_{\mu\nu} = g_{\mu\nu}, \quad \tilde{g}_{xx} = 1/g_{xx}, \quad \tilde{B}_{\mu\nu} = B_{\mu\nu}, \\
&e^{2\tilde{\phi}} = e^{2\phi}/|g_{xx}|,
\end{aligned} \tag{4.8}$$

while those of the $R \otimes R$ potentials are given by

$$\begin{aligned}
&\tilde{C}^{(0)} = C^{(1)}_x, \quad \tilde{C}^{(1)}_x = C^{(0)}, \\
&\tilde{C}^{(1)}_\mu = -C^{(2)}_{x\mu}, \quad \tilde{C}^{(2)}_{x\mu} = -C^{(1)}_\mu, \\
&\tilde{C}^{(2)}_{\mu\nu} = \tfrac{3}{2}C^{(3)}_{\mu\nu x}, \quad \tilde{C}^{(3)}_{\mu\nu x} = \tfrac{2}{3}C^{(2)}_{\mu\nu}, \\
&\tilde{C}^{(3)}_{\mu\nu\rho} = \tfrac{8}{3}C^{(4)}_{x\mu\nu\rho} - C^{(2)}_{x[\mu}B_{\nu\rho]}, \\
&\tilde{C}^{(4)}_{x\mu\nu\rho} = \tfrac{3}{8}\left[C^{(3)}_{\mu\nu\rho} - C^{(1)}_{[\mu}B_{\nu\rho]}\right].
\end{aligned} \tag{4.9}$$

[4] For simplicitly we only give the rules for $r = 1, \cdots, 6$. The remaining $R \otimes R$ potentials can be dealt with similarly.

[5] This assumption is not essential to the simplifications discussed below.

We see that under T-duality the $R\otimes R$ potentials $C^{(r)}$ transform to the potentials $C^{(r\pm 1)}$ except for $C^{(3)}$ for which the T-duality rule involves the $NS\otimes NS$ 2-form B. We find that in the new basis all dependence on B disappears. In particular the rules involving $C^{(3)}$,$C^{(4)}$ are given by (omitting the primes)

$$\tilde{C}^{(3)}_{\mu\nu\rho}=\tfrac{8}{3}C^{(4)}_{x\mu\nu\rho}\,,\quad \tilde{C}^{(4)}_{x\mu\nu\rho}=\tfrac{3}{8}C^{(3)}_{\mu\nu\rho}\,. \tag{4.10}$$

It is not too difficult to understand why, in the new basis, the T-duality rules of the $R\otimes R$ potentials are of the simple form given above. The point is that, using an appropriate normalization, the kinetic term of any of the $R\otimes R$ potentials takes the form (using the string-frame metric)

$$\mathcal{L}_{R\otimes R}=\sqrt{|\hat{g}|}\,\frac{(-1)^{p+1}}{2(p+2)!}\,\hat{R}^2_{p+2}(C)\,, \tag{4.11}$$

where the hatted fields are ten-dimensional and $R(C)$ is defined as [14]

$$R(C)=dC-dB\wedge C+me^{B}\,. \tag{4.12}$$

Because of the assumption (4.7), the reduction rules for the $R\otimes R$ potentials are particularly simple:

$$\hat{C}_{\mu_1\cdots\mu_{p+1}}=C_{\mu_1\cdots\mu_{p+1}}\,,\quad \hat{C}_{x\mu_1\cdots\mu_{p+1}}=C_{\mu_1\cdots\mu_p}\,. \tag{4.13}$$

Similar simple reduction rules apply to the curvatures. Consider now the kinetic term for a IIA potential for fixed (even) p. Reduction in the isometry direction x leads to

$$\mathcal{L}_{\rm IIA}=\sqrt{|g|}\,\frac{(-1)^{p+1}}{2(p+2)!}e^{x/4}\,R^2_{p+2}(C)+\sqrt{|g|}\,\frac{(-1)^{p}}{2(p+1)!}e^{-x/4}\,R^2_{p+1}(C)\,, \tag{4.14}$$

with $\hat{g}_{xx}=-e^{x/2}$. Similarly, reducing the kinetic term for a IIB potential for fixed (odd) q leads to

$$\mathcal{L}_{\rm IIB}=\sqrt{|g|}\,\frac{(-1)^{q+1}}{2(q+2)!}e^{-x/4}\,R^2_{q+2}(C)+\sqrt{|g|}\,\frac{(-1)^{q}}{2(q+1)!}e^{x/4}\,R^2_{q+1}(C)\,. \tag{4.15}$$

with $\hat{g}_{xx}=-e^{-x/2}$. Comparing these two expression for the two cases $q=p\pm 1$ immediately leads to the following simple T-duality rules for the $R\otimes R$ potentials (together with the usual Buscher's rules for the $NS\otimes NS$ fields)

$$\hat{C}_{\mu_1\cdots\mu_p}=\hat{C}_{x\mu_1\cdots\mu_p}\,,\quad \hat{C}_{x\mu_1\cdots\mu_p}=\hat{C}_{\mu_1\cdots\mu_p}\,. \tag{4.16}$$

These are exactly the same T-duality rules as those given in [14].

Acknowledgements

We are pleased to thank Martin Cederwall, Bengt Nilsson and Anders Westerberg for helpful discussions on issues related to the work reported here.

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