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Super D-branes

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Abstract

We present a manifestly Lorentz invariant, space-time supersymmetric, and 'κ-invariant' world-volume action for all type II Dirichlet p-branes, $p \leqslant 9$, in a general type II supergravity background, including massive backgrounds in the IIA case. The $p = 0, 2$ cases are rederived from $D = 11$. The $p = 9$ case provides a supersymmetrization of the $D = 10$ Born–Infeld action. © 1997 Elsevier Science B.V.

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1. Introduction

The world-volume actions for p-brane solutions of supersymmetric field theories may be viewed as $(p+1)$-dimensional non-linear sigma-models with a superspace as the target space, i.e. the world-volume fields $Z^M(\xi)$ define a map $W \to \Sigma$ from the world-volume W with coordinates ξ^i $(i = 0, 1, \dots, p)$ to a superspace Σ with coordinates $Z^M = (x^m, \theta^\mu)$. Many p-brane solutions of supergravity theories do not quite fit this pattern, however, because their world-volume fields include vectors or antisymmetric tensors. Examples are provided by the Ramond–Ramond (R–R) p-branes of ten-dimensional $(D = 10)$ type II supergravity theories. Because these have an interpretation as Dirichlet p-branes, or D-p-branes, of type II superstring theory [1], their bosonic world-volume actions (to leading order in α') can be computed by standard superstring methods [2–4]. The world-volume action includes a 1-form gauge potential V, the so-called Born–Infeld (BI) field, which couples to the endpoints of type II strings 'on the brane'. For reasons explained elsewhere (see, e.g. Ref. [5]) the $D = 10$ Lorentz covariant form of this action must have a fermionic gauge invariance, usually called 'kappa-symmetry'. Given the bosonic D-brane action, it is not difficult to guess the form of the super

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D-brane action, even in a general supergravity background, but the complications due to the BI gauge field have so far prevented the construction of the complete κ-symmetry transformations for general p, and the verification of κ-invariance, although a number of partial results have been obtained. A form of the super D-2-brane action and its κ-symmetry transformations, in a Minkowski background, was deduced from that of the $D = 11$ supermembrane by means of IIA/M-theory duality [6]. More recently, the super D-3-brane action and its κ-symmetry in a general IIB supergravity background has been presented [7]. Also, an action for general p in a Minkowski background has been given and a strategy for verifying its κ-symmetry proposed [8]. We should also mention that super D-p-brane actions with simultaneous world-volume and space-time supersymmetry have been found [9], but the relation to the Green–Schwarz-type action considered here has not yet been spelled out.

Here, we present a Lorentz invariant and supersymmetric world-volume action for all type II Dirichlet p-branes, $p \leqslant 9$, in a general type II supergravity background. We also give the explicit form of the κ-symmetry transformations, which we have fully verified in bosonic backgrounds for $p \leqslant 6$ and partially verified for $p > 6$. We also show how the $p = 0, 2$ actions can be obtained from $D = 11$. The $p = 0$ action is obtained by reduction of the $D = 11$ massless superparticle. The $p = 2$ action is obtained by reduction of the $D = 11$ supermembrane action, followed by a scalar/vector duality transformation, as in [6] but incorporating the results of [10,11] so as to arrive at the standard Born–Infeld form of the action. The κ-symmetry transformations of the super D-2-brane action can also be deduced from $D = 11$, but the form of these transformations do not obviously generalize to higher p. A redefinition of the κ-symmetry parameter is needed to put them in the form used in this paper.

Although the focus of this paper is on the D-p-branes for $p \leqslant 8$, our main result applies equally when $p = 9$. This special case is equivalent to a supersymmetrization of the $D = 10$ Born–Infeld action, which was only partially known previously [12]. Thus, subject to a full verification of κ-symmetry for $p = 9$, this problem is now solved, at least in principle. A curious feature of this approach to the super Born–Infeld action is that it makes essential use of an 11-form superspace field strength, which vanishes identically when restricted to space-time! The possibility of such superspace gauge fields has been explored in the past [13] and our work provides a nice example of their utility.

The organization of the remainder of this paper is as follows. We first give the super D-p-brane action and explain some of the superspace conventions. We then present some preliminary results needed to compute the κ-variation of this action. We then perform the calculation and verify κ-symmetry in detail for $p \leqslant 6$ and partially for $p = 7, 8, 9$. We then rederive the $p = 0, 2$ results from known results in $D = 11$, including the generalization to fermionic and 'massive' backgrounds.

2. The super D-brane action

Our proposed super D-brane action, in a general $N = 2$ supergravity background, (for the string-frame metric) takes the form

$$I = I_{\rm DBI} + I_{\rm WZ}\,, \tag{2.1}$$

where

$$I_{\rm DBI} = -\int d^{p+1}\xi\, e^{-\phi}\sqrt{-\det(g_{ij} + \mathcal{F}_{ij})} \tag{2.2}$$

is a Dirac–Born–Infeld type action and $I_{\rm WZ}$ is a Wess–Zumino (WZ) type action to be discussed below; $\mathcal{F}_{ij}$ are the components of a 'modified' 2-form field strength

$$\mathcal{F} = F - B\,, \tag{2.3}$$

where $F = dV$ is the usual field strength 2-form of the BI field V and B is the pullback to the world-volume of a 2-form potential B on superspace, whose leading component in a θ-expansion is the 2-form potential of Neveu–Schwarz/Neveu–Schwarz (NS–NS) origin in type II superstring theory. We use the same letter for superspace forms and their pullbacks to the world-volume since it should be clear from the context which is meant. Superspace forms may be expanded on the coordinate basis of 1-forms dZ^M or on the inertial frame basis $E^A = dZ^M E_M{}^A$, where $E_M{}^A$ is the supervielbein. The basis E^A decomposes under the action of the Lorentz group into a Lorentz vector E^a and a Lorentz spinor. The latter is a 32-component Majorana spinor for IIA superspace and a pair of chiral Majorana spinors for IIB superspace. Thus

$$E^A = \begin{cases} (E^a, E^\alpha) & \text{(IIA)} \\ (E^a, E^{\alpha\, I}) \quad I = 1,2 & \text{(IIB)} \end{cases} \tag{2.4}$$

In the IIB case we still allow α to run from 1 to 32 but include a chiral projector as appropriate. The world-volume metric g_{ij} appearing in (2.2) is defined in the standard way as

$$g_{ij} = E_i^a E_j^b \eta_{ab}\,, \tag{2.5}$$

where η is the $D = 10$ Minkowski metric and

$$E_i^A = \partial_i Z^M E_M{}^A\,. \tag{2.6}$$

Thus $I_{\rm DBI}$ is a straightforward extension to superspace of the corresponding term in the bosonic action. The same is true for the WZ term except for one new feature of relevance to the 9-brane. We introduce a R-R potential C as a formal sum of r-form superspace potentials $C^{(r)}$, i.e.

$$C = \sum_{r=0}^{10} C^{(r)}\,. \tag{2.7}$$

The even potentials are those of IIB supergravity while the odd ones are those of IIA supergravity. In the bosonic case one could omit the 10-form gauge potential $C^{(10)}$ on the grounds that its 11-form field strength is identically zero. But an 11-form field strength on *superspace* is not identically zero; in fact we shall see that it is non-zero even in a Minkowski background, a fact that is crucial to the κ-symmetry of the super 9-brane action.

The WZ term can now be written as [3,4,14,15]

$$I_{\rm WZ} = \int_W C\, e^{\mathcal{F}} + m I_{\rm CS}\,, \tag{2.8}$$

where, in the first term, the product is understood to be the exterior product of forms and the form of appropriate degree is chosen in the 'form expansion' of the integrand, i.e. $(p+1)$ for a D-p-brane. The $I_{\rm CS}$ term is a $(p+1)$-form Chern–Simons (CS) action that is present (for odd p) in a *massive* IIA background; its coefficient m is the IIA mass parameter. This WZ term is formally the same as the known *bosonic* D-brane WZ action, but here the forms $C^{(r)}$ and B are taken to be forms on *superspace*, e.g.

$$C^{(r)} = \frac{1}{r!} dZ^{M_1} \dots dZ^{M_r} C_{M_r \dots M_1}\,. \tag{2.9}$$

This illustrates the standard normalization and the 'reverse order' convention for components of superspace forms. This convention goes hand in hand with the convention for exterior differentiation of superspace forms in which the exterior derivative acts 'from the right'. Thus,

$$dC^{(r)} = \frac{1}{r!} dZ^{M_1} \dots dZ^{M_r} dZ^N \partial_N C_{M_r \dots M_1}\,. \tag{2.10}$$

As explained in [15], the field strength for the RR field C is

$$R(C) = dC - HC + m\, e^{B}\,, \tag{2.11}$$

where m is the mass parameter of the IIA theory. $R(C)$ can be written as the formal sum

$$R(C) = \sum_{n=1}^{11} R^{(n)}\,. \tag{2.12}$$

Note that the top form is an 11-form because we included a 10-form $C^{(10)}$ in the definition of C. The field strengths $R^{(n)}$ will be subject to superspace constraints, to be given below for bosonic backgrounds, in addition to the constraint described in [15] relating the bosonic components of $R^{(n)}$ to the Hodge dual of the bosonic components of $R^{(10-n)}$. This concludes our presentation of the super D-brane action.

3. κ-symmetry preliminaries

Given variations δZ^M of the world-volume fields Z^M, we define

$$\delta E^A \equiv \delta Z^M E_M{}^A ,$$
$$\delta \Omega_B{}^A = \delta Z^M \Omega_{M\,B}{}^A , \tag{3.1}$$

where Ω is the connection on the tangent bundle of superspace. As usual, we take the structure group of the tangent bundle to be the Lorentz group so that, in particular,

$$\Omega_b{}^\alpha = \Omega_\beta{}^a = 0 ,$$
$$\Omega_\alpha{}^\alpha = \Omega_{(b}{}^a \eta_{c)a} = 0 . \tag{3.2}$$

A useful lemma is

$$\delta E_i^A = \partial_i(\delta E^A) - \delta E^B E_i^C T_{CB}{}^A - E_i^B \delta\Omega_B{}^A + \delta E^B \partial_i Z^N \Omega_{N\,B}{}^A , \tag{3.3}$$

where T^A is the torsion 2-form, i.e. $T^A = \frac{1}{2} E^B \wedge E^C T_{BC}{}^A$. This corrects a result quoted in [5,16] in which the last term was missing.

The IIA torsion tensor is subject to the constraints [17]

$$T_{\beta\gamma}{}^a = i\Gamma^a_{\alpha\beta} ,$$
$$T_{b\gamma}{}^a = \delta^b_a \chi_\gamma , \tag{3.4}$$

where χ is a spinor proportional to the dilatino of the supergravity background, i.e. $\chi \propto D\phi$ [18]. The IIB torsion tension is subject to the constraints [19]

$$T_{\alpha\,I\,\beta\,J}{}^a = i\delta_{IJ}\left(\Gamma^a \mathcal{P}\right)_{\alpha\beta} ,$$
$$T_{b\,\gamma\,K}{}^a = \delta^b_a \chi_{\gamma\,K} \tag{3.5}$$

where $\mathcal{P}$ is a chiral projection operator and χ^I is a pair of chiral spinors, i.e. $\mathcal{P}\chi^I = \chi^I$.

A universal feature of κ-symmetry is that

$$\delta_\kappa E^a = 0 . \tag{3.6}$$

Using this fact in the lemma (3.3), and taking into account the restrictions (3.2) on the form of Ω, we have that

$$\delta_\kappa g_{ij} = -2E_i^a \delta_\kappa E^B E_j^C T_{CB}{}^b \eta_{ab} . \tag{3.7}$$

Using (3.6) again, and the torsion constraints, we find that

$$\delta_\kappa g_{ij} = \begin{cases} -2i\delta_\kappa E \Gamma_{11} \gamma_{(i} E_{j)} - 2g_{ij}\delta_\kappa E \chi & \text{(IIA)} \\ -2i\delta_\kappa E^I \gamma_{(i} E^I_{j)} - 2g_{ij}\delta_\kappa E^I \chi^I & \text{(IIB)} . \end{cases} \tag{3.8}$$

Here, $\delta_\kappa E$ and $\delta_\kappa E^I$ are the spinor components of $\delta_\kappa E^A$ for IIA and IIB, respectively, the vector components vanishing by hypothesis, and E_i and E_i^I are the spinor components of

E_i^A. Since all IIB spinors are chiral, the chirality projection operator $\mathcal{P}$ may be omitted in the IIB expression. We have also suppressed spinor indices. The matrices γ_i are defined as

$$\gamma_i = E_i{}^a \Gamma_a \,. \tag{3.9}$$

The round brackets enclosing indices indicate symmetrization with 'strength one'; we use square brackets for antisymmetrization. We also adopt the convention by which spinor indices are raised and lowered with the charge conjugation matrix C, as explained e.g. in [16]. Thus $\Gamma^a_{\alpha\beta}$ are the components of the symmetric matrix formed from the product $C\Gamma^a$.

Another useful lemma concerns the variation of a $(p+1)$-form A induced by the variation δZ^M. Defining $F(A) = dA$, we have

$$\begin{aligned}\delta A = &\frac{1}{(p+1)!} E^{A_{p+2}} \ldots E^{A_2} \delta E^{A_1} F(A)_{A_1 A_2 \ldots A_{p+2}} \\ &+ \frac{1}{p!} d\left(E^{A_p} \ldots E^{A_1} \delta E^B A_{BA_1 \ldots A_p} \right).\end{aligned} \tag{3.10}$$

This corrects a result quoted in [16] in which the total derivative term is missing. An application of this result to the NS–NS 2-form B with field strength $H = dB$ leads to

$$\delta B = \frac{1}{2} E^A E^B \delta E^C H_{CBA} + d\left(E^A \delta E^B B_{BA}\right). \tag{3.11}$$

One useful piece of information that one learns from the $D = 11$ origin of the D-2-brane is that

$$\delta_\kappa V_i = E_i^A \delta_\kappa E^B B_{BA} \tag{3.12}$$

so that

$$\delta_\kappa \mathcal{F}_{ij} = -E_{[i}^A E_{j]}^B \delta_\kappa E^C H_{CBA} \,. \tag{3.13}$$

A feature of this result is that $\mathcal{F}$ is 'κ-covariant' in the sense that its κ-variation does not involve derivatives of κ, a property that is crucial for κ-invariance.

The IIA superspace constraints on H are [17]

$$\begin{aligned} H_{\alpha\beta\gamma} &= 0 \,, \\ H_{\alpha\beta c} &= -i\left(\Gamma_{11}\Gamma_a\right)_{\alpha\beta} \,, \\ H_{\alpha bc} &= (\Gamma_{bc}\zeta)_\alpha \,, \end{aligned} \tag{3.14}$$

where ζ is another spinor proportional to $D\phi$. The IIB superspace constraints on H are [17]

$$\begin{aligned} H_{\alpha I \beta J \gamma K} &= 0 \,, \\ H_{\alpha I \beta J c} &= i(\sigma_3)_{IJ}\left(\Gamma_a \mathcal{P}\right)_{\alpha\beta} \,, \\ H_{\alpha I bc} &= (\Gamma_{bc}\zeta)^I_\alpha \,. \end{aligned} \tag{3.15}$$

If these constraints are now used in (3.13) we have, for IIA,

$$\delta_\kappa \mathcal{F}_{ij} = -2i\delta_\kappa E\Gamma_{11}\gamma_{[i}E_{j]} + \delta_\kappa E\gamma_{ij}\zeta \tag{3.16}$$

and for IIB,

$$\delta_\kappa \mathcal{F}_{ij} = 2is_{IJ}\delta_\kappa E^I\gamma_{[i}E^J_{j]} + \delta_\kappa E^I\gamma_{ij}\zeta^I\,. \tag{3.17}$$

It remains to give the superspace constraints on the R–R field strengths. These constraints imply that all components of $R^{(n)}$ with more than two spinor indices vanish. The components with no spinor indices are unconstrained apart from the Hodge dual relation between the components of $R^{(n)}$ and $R^{(10-n)}$. The components with one spinor index are all proportional to the dilatino and hence vanish for bosonic backgrounds. The only other non-vanishing components are those with precisely two spinor indices; the constraints on these components are needed for verification of κ-symmetry. They are

$$R^{(n+2)}_{\alpha\beta a_1\ldots a_n} = \begin{cases} ie^{-\phi}(\Gamma_{a_1\ldots a_n}\Gamma_{11})_{\alpha\beta} & (n=0,4,8) \\ ie^{-\phi}(\Gamma_{a_1\ldots a_n})_{\alpha\beta} & (n=2,6) \end{cases} \tag{3.18}$$

for the IIA R–R field strengths, while

$$R^{(n+2)}_{\alpha I\beta J a_1\ldots a_n} = \begin{cases} ie^{-\phi}(\Gamma_{a_1\ldots a_n}\mathcal{P})_{\alpha\beta}(\sigma_1)_{IJ} & (n=1,5,9) \\ ie^{-\phi}(\Gamma_{a_1\ldots a_n}\mathcal{P})_{\alpha\beta}(i\sigma_2)_{IJ} & (n=3,7) \end{cases} \tag{3.19}$$

for the IIB R–R field strengths.

Finally, It is convenient to introduce the matrix

$$\Gamma_{(0)} = \frac{1}{(p+1)!\sqrt{-\det g}}\,\varepsilon^{i_1\ldots i_{(p+1)}}\gamma_{i_1\ldots i_{(p+1)}}\,, \tag{3.20}$$

which has the properties

$$(\Gamma_{(0)})^2 = (-1)^{(p-1)(p-2)/2}\,, \qquad \Gamma_{(0)}\gamma^i = (-1)^p\gamma^i\Gamma_{(0)}\,. \tag{3.21}$$

A useful identity involving $\Gamma_{(0)}$ is

$$\varepsilon^{i_1\ldots i_k j_{k+1}\ldots j_{p+1}}\gamma_{j_{k+1}\ldots j_{p+1}} = (-1)^{k(k-1)/2}(p+1-k)!\sqrt{-\det g}\,\gamma^{i_1\ldots i_k}\Gamma_{(0)}\,. \tag{3.22}$$

4. Proof of κ-invariance

We are now in a position to compute the κ-variation of the proposed super D-p-brane action in a bosonic background. We begin with the variation of the DBI term, for which the Lagrangian is

$$\mathcal{L}_{\rm DBI} = -e^{-\phi}\sqrt{-\det(g+\mathcal{F})}\,. \tag{4.1}$$

Our results for $\delta_\kappa\mathcal{L}_{\rm DBI}$ are essentially the same as those obtained in [7,8], but we need them in our conventions. Using the results of the previous section for the κ-variations of g_{ij} and $\mathcal{F}_{ij}$, but now omitting terms which vanish in a bosonic background, we have

$$\delta_\kappa(g_{ij}+\mathcal{F}_{ij}) = -2i\delta_\kappa E\left[P_+\gamma_i E_j + P_-\gamma_j E_i\right], \tag{4.2}$$

where $P_\pm$ are the following projection operators:

$$P_\pm = \begin{cases} \frac{1}{2}(1\pm\Gamma_{11}) & \text{IIA} \\ \frac{1}{2}(1\pm\sigma_3) & \text{IIB} \end{cases} \tag{4.3}$$

We have here suppressed the I, J indices on IIB spinors. It follows that

$$\delta_\kappa\mathcal{L}_{\rm DBI} = -i\mathcal{L}_{\rm DBI}\,\delta_\kappa E\left[(g+\mathcal{F})^{ij}\gamma_j P_+ + (g-\mathcal{F})^{ij}\gamma_j P_-\right]E_i\,, \tag{4.4}$$

where $(g+\mathcal{F})^{ij}$ is the inverse of $(g+\mathcal{F})_{ij}$ and we have used the fact that

$$(g+\mathcal{F})^{ji} = (g-\mathcal{F})^{ij}\,. \tag{4.5}$$

Note that the κ-variation of ϕ is proportional to the dilatino (as $\delta_\kappa\phi = \delta_\kappa E^A D_A\phi$ and $\delta_\kappa E^a$ vanishes) and the dilatino vanishes in a bosonic background, by definition.

It will prove convenient to rewrite (4.4) in terms of the $(p+1)\times(p+1)$ matrix

$$X^i{}_j = g^{ik}\mathcal{F}_{kj}\,. \tag{4.6}$$

Thus,

$$\delta_\kappa\mathcal{L}_{\rm DBI} = -i\mathcal{L}_{\rm DBI}\,\delta_\kappa E\{\left[(1+X)^{-1}\right]^i{}_j\gamma^j P_+ + \left[(1-X)^{-1}\right]^i{}_j\gamma^j P_-\}E_i\,. \tag{4.7}$$

Using the fact that

$$(1\pm X)^{-1} = (1-X^2)^{-1}(1\mp X) \tag{4.8}$$

we can rewrite (4.7) in the alternative form

$$\delta_\kappa\mathcal{L}_{\rm DBI} = -i\mathcal{L}_{\rm DBI}\,\delta_\kappa E N^i E_i\,, \tag{4.9}$$

where

$$N^i = \begin{cases} \left[(1-X^2)^{-1}\right]^i{}_j\gamma^j + \left[(1-X^2)^{-1}X\right]^i{}_j\gamma^j\Gamma_{11} & \text{(IIA)} \\ \left[(1-X^2)^{-1}\right]^i{}_j\gamma^j\otimes\mathbf{1}_2 - \left[(1-X^2)^{-1}X\right]^i{}_j\gamma^j\otimes\sigma_3 & \text{(IIB)} \end{cases} \tag{4.10}$$

We now turn to the κ-variation of the WZ term (2.8). Using (3.10) and (3.13), and omitting a surface term, we find (for a bosonic background) that

$$\delta_\kappa I_{\rm WZ} = \int_W i_{\delta Z}R(C)e^{\mathcal{F}}\,, \tag{4.11}$$

where $i_{\delta Z}R(C)$ indicates a contraction of the differential form $R(C)$ with the vector field $\delta_\kappa Z^M$, i.e.

$$i_{\delta Z}R^{(n)} = \frac{1}{(n-1)!}E^{A_{p+2}}\ldots E^{A_2}\delta E^{A_1}R^{(n)}_{A_1A_2\ldots A_n}\,. \tag{4.12}$$

Using the superspace constraints (3.18) and (3.19), together with the identity (3.22), we find that

$$\delta\mathcal{L}_{\mathrm{WZ}} = \begin{cases} i\mathcal{L}_{\mathrm{DBI}}\, \delta_\kappa E M^i_p \Gamma_{(0)} E_i & \text{(IIA)} \\ i\mathcal{L}_{\mathrm{DBI}}\, \delta_\kappa E M^i_p\, i\sigma_2 \otimes \Gamma_{(0)} E_i & \text{(IIB)} \end{cases} \tag{4.13}$$

where

$$M^i_{(p)} = \frac{1}{\sqrt{\det(1+X)}} \sum_{n=0}^{\infty} \frac{1}{2^n n!} \gamma^{ij_1k_1\ldots j_nk_n} X_{j_1k_1} \ldots X_{j_nk_n} Q^{(n)}_{(p)} \tag{4.14}$$

with

$$Q^{(n)}_{(p)} = \begin{cases} (-\Gamma_{11})^{n+(p-2)/2} & \text{(IIA)} \\ (-\sigma_3)^{n+(p-3)/2} & \text{(IIB)} \end{cases} \tag{4.15}$$

The infinite sum in (4.14) is of course truncated automatically as soon as n exceeds $p/2$.

Combining the variations of both the DBI and WZ terms in the action we deduce that

$$\delta_\kappa I = -\int d^{p+1}\xi\, \mathcal{L}_{\mathrm{DBI}}\, i\delta_\kappa E K^i_{(p)} E_i\,, \tag{4.16}$$

where

$$K^i_{(p)} = N^i + M^i_{(p)}$$

with N^i and $M^i_{(p)}$ as given in (4.10) and (4.14), respectively.

We still have to specify the spinor variation $\delta_\kappa E$. On general grounds it must take the form

$$\delta_\kappa E = \bar{\kappa}(1+\Gamma)\,, \tag{4.17}$$

where Γ is a matrix with the properties

$$\Gamma^2 = 1\,, \qquad \mathrm{tr}\,\Gamma = 0\,. \tag{4.18}$$

For κ-symmetry of the D-p-brane action we require also that

$$(1+\Gamma)K^i_{(p)} \equiv 0\,. \tag{4.19}$$

Given this identity one could deduce that

$$K^i_{(p)} = (1-\Gamma)T^i_{(p)} \tag{4.20}$$

for some matrix $T^i_{(p)}$, which would make the κ-symmetry manifest. This was the basis of the strategy for proving κ-symmetry proposed in [8], but since it involves the simultaneous determination of both Γ and $T^i_{(p)}$, we chose instead to determine Γ directly from (4.19).

Consider the matrix

$$\Gamma = \frac{1}{\sqrt{\det(1+X)}} \sum_{n=0}^{\infty} \frac{1}{2^n n!} \gamma^{j_1 k_1 \ldots j_n k_n} X_{j_1 k_1} \ldots X_{j_n k_n} J^{(n)}_{(p)}, \tag{4.21}$$

where

$$J^{(n)}_{(p)} = \begin{cases} (\Gamma_{11})^{n+(p-2)/2} \Gamma_{(0)} & \text{(IIA)} \\ (-1)^n (\sigma_3)^{n+(p-3)/2} i\sigma_2 \otimes \Gamma_{(0)} & \text{(IIB)} \end{cases} \tag{4.22}$$

Clearly, Γ thus defined has vanishing trace and, for a given value of p, standard gamma matrix algebra suffices to establish that $\Gamma^2 = 1$; we have verified this for $p \leqslant 6$. We now claim that this matrix Γ is also such that (4.19) is true. To establish this, we begin by separating those terms on the left-hand side of (4.19) with a factor of $\sqrt{\det(1+X)}$ from those terms without this factor. The former cancel straightforwardly, the only subtle point being that for even p the following identity is needed:

$$X^i{}_{[j} X_{k_1 k_2} \ldots X_{k_{p-1} k_p]} \equiv 0. \tag{4.23}$$

The cancellation of the terms *without* a factor of $\sqrt{\det(1+X)}$ is more involved. Firstly, we may separate these terms into four distinct matrix structures according to whether there is a factor of $\Gamma_{(0)}$ and/or a factor of Γ_{11}, in the IIA case, and σ_3 in the IIB case (possibly after multiplication by σ_1 or σ_2). Each of these subcalculations involves the reduction of a sum of products of an odd number of γ matrices to the form

$$\sum_{n=0}^{\infty} A^i_{(n) j_1 \ldots j_{2k+1}}(X)\, \gamma^{j_1 \ldots j_{2k+1}}, \tag{4.24}$$

where the coefficients $A_{(n)}$ are polynomials in the entries of X. The validity of the identity (4.19) requires that $A_{(n)}$ vanish unless $n = 0$. Using straightforward gamma-matrix algebra we have verified in detail for $p \leqslant 6$ that this condition is indeed satisfied. The verification of κ-symmetry is thereby reduced to establishing a relation of the form

$$\det(1+X)\gamma^i = A^i_{(0)j}(X)\gamma^j, \tag{4.25}$$

where $A_{(0)}$ is the matrix with entries $A^i_{(0)j}$. Equivalently,

$$A_{(0)}(X) = \det(1+X)\mathbf{1}, \tag{4.26}$$

where $\mathbf{1}$ is the $(p+1) \times (p+1)$ identity matrix. One also requires a relation similar to (4.25) in which the left-hand side involves $X^i{}_j \gamma^j$ instead of γ^i, but this turns out to be an immediate consequence of (4.25).

The left-hand side of (4.26) is a polynomial in the matrix X, so the validity of the relation clearly requires that X satisfy some polynomial identity. The identities satisfied by the $(p+1) \times (p+1)$ matrix X can be obtained as follows. Let us suppose that X satisfies the identity $P_{p+1}(X) \equiv 0$, where P_{p+1} is a $(p+1)$th order polynomial; then the $(p+2) \times (p+2)$ matrix X satisfies the identity $P_{p+2}(X) \equiv 0$, where

$$P_{p+2} = \begin{cases} P_{p+1}(X)X & p = 1,3,5,7,9 \\ P_{p+1}(X)X - \frac{1}{(p+2)}\mathrm{tr}\big(\mathrm{P_{p+1}(X)X}\big) & p = 2,4,6,8. \end{cases} \tag{4.27}$$

Thus all polynomial identities follow from the $p = 1$ identity for which

$$P_2(X) = X^2 - \frac{1}{2}\mathrm{trX}^2 . \tag{4.28}$$

For example, for $p = 4$ one has $P_5(X) \equiv 0$, where

$$P_5(X) = X^5 - \frac{1}{2}X^3\mathrm{trX}^2 + \frac{1}{8}\mathrm{X}(\mathrm{trX}^2)^2 - \frac{1}{4}\mathrm{X}(\mathrm{trX}^4) . \tag{4.29}$$

Using these identities, we have verified for $p \leqslant 6$ that $A_{(0)}(X)$ is indeed proportional to the identity matrix, and then that the coefficient of proportionality is indeed just $\det(1+X)$. For this final step one needs the expansion of the determinant. Since $p \leqslant 9$ this expansion is never needed to beyond the X^{10} level. We record here the expansion to this level:

$$\begin{aligned}\det(1+X) = 1 &- \frac{1}{2}\mathrm{trX}^2 - \frac{1}{4}\mathrm{trX}^4 + \frac{1}{8}(\mathrm{trX}^2)^2 - \frac{1}{6}\mathrm{trX}^6 + \frac{1}{8}\mathrm{trX}^2\mathrm{trX}^4 \\ &- \frac{1}{48}(\mathrm{trX}^2)^3 - \frac{1}{8}\mathrm{trX}^8 + \frac{1}{12}\mathrm{trX}^2\mathrm{trX}^6 + \frac{1}{32}(\mathrm{trX}^4)^2 - \frac{1}{32}\mathrm{trX}^4(\mathrm{trX}^2)^2 \\ &+ \frac{1}{4!2^4}(\mathrm{trX}^2)^4 - \frac{1}{10}\mathrm{trX}^{10} + \frac{1}{16}\mathrm{trX}^2\mathrm{trX}^8 + \frac{1}{24}\mathrm{trX}^4\mathrm{trX}^6 \\ &- \frac{1}{48}\mathrm{trX}^6(\mathrm{trX}^2)^2 - \frac{1}{64}\mathrm{trX}^2(\mathrm{trX}^4)^2 + \frac{1}{4!2^5}\mathrm{trX}^4(\mathrm{trX}^2)^3 \\ &- \frac{1}{5!2^5}(\mathrm{trX}^2)^5 + \mathcal{O}(\mathrm{X}^{12}) .\end{aligned} \tag{4.30}$$

5. The super D-0-brane from $D = 11$

We now rederive the super D-0-brane action and its κ-transformations from $D = 11$. We start with the *massless* $D = 11$ superparticle, for which the action is

$$I = -\int dt\, \frac{1}{2\hat{v}}\hat{E}_t \cdot \hat{E}_t , \tag{5.1}$$

where $\hat{v}$ is an independent world-line density and the hats indicate $D = 11$ quantities. This action is invariant under the κ-transformations

$$\delta_\kappa \hat{E}^\alpha = \hat{\gamma}_t \hat{\kappa} , \qquad \delta_\kappa \hat{v} = 2i\hat{v}(\hat{\kappa}E_t) , \tag{5.2}$$

where $\hat{\kappa}(t)$ is a $D = 11$ spinor parameter and $\hat{\gamma}_t = \hat{E}_t^{\hat{a}}\Gamma_{\hat{a}}$. The dimensional reduction to $D = 10$ string-frame fields of the $D = 11$ background fields is achieved by adopting the notation $dx^{\hat{m}} = (dx^m, dy)$, $\hat{E}^{\hat{a}} = (\hat{E}^a, \hat{E}^{11})$ and then taking the supervielbein to be such that

$$\hat{E}_y{}^a = 0\,, \qquad \hat{E}_y{}^{11} = e^{\frac{2}{3}\phi}\,,$$
$$\hat{E}_M{}^{11} = e^{\frac{2}{3}\phi} C_M\,, \qquad \hat{E}_M{}^a = e^{-\frac{1}{3}\phi} E_M{}^a\,. \tag{5.3}$$

We also choose

$$\hat{E}_M{}^\alpha = E_M{}^\alpha\,.$$

It then follows that

$$\hat{E}_t{}^a = e^{-\frac{1}{3}\phi}\, E_t{}^a\,,$$
$$\hat{E}_t{}^{11} = e^{\frac{2}{3}\phi}(\dot{y} + C_t)\,, \tag{5.4}$$

where $C_t = \dot{Z}^M C_M$. It is convenient to define a new world-line density v by

$$v = e^{\frac{2}{3}\phi}\hat{v}\,. \tag{5.5}$$

With this notation, and in this Kaluza–Klein (KK) background, the $D = 11$ massless 0-brane Lagrangian is

$$L = -\frac{1}{2v}E_t^2 - \frac{1}{2v}e^{2\phi}(\dot{y} + C_t)^2\,. \tag{5.6}$$

We remark that $\delta_\kappa \hat{E}^{11} = 0$ implies that $\delta_\kappa y = -\delta_\kappa Z^M C_M$, so that $(\dot{y} + C_t)$ transforms 'covariantly' under κ-symmetry, i.e. without time derivatives of κ. The Euler–Lagrange equations of this Lagrangian allow the solution

$$(\dot{y} + C_t) = \mu e^{-2\phi} v \tag{5.7}$$

for arbitrary constant μ (which has dimensions of mass). This equation can then be used to eliminate $\dot{y}$ from the remaining field equations, which thereby acquire a μ-dependence. It is important to appreciate that these equations are *not* the same as those found by first substituting for $\dot{y}$ in (5.6) and then varying with respect to Z^M. This subtlety has been addressed elsewhere in a different context [20]. It will suffice here to state that the substitution is permissible if one first adds to the Lagrangian the total derivative $\mu\dot{y}$.[1] One then finds the new Lagrangian

$$L = -\frac{1}{2v}E_t^2 + \frac{1}{2}\mu^2 v e^{-2\phi} - \mu \dot{Z}^M C_M\,. \tag{5.8}$$

Elimination of the auxiliary variable v then yields the action

$$I = -\mu \int dt \left\{ e^{-\phi}\sqrt{-E_t^2} + \dot{Z}^M C_M \right\} \tag{5.9}$$

which is the D-0-brane action with mass μ (set to unity previously) in a IIA supergravity background with mass parameter m set to zero. The non-vanishing κ-variations are

[1] An alternative procedure is to rewrite the action in Hamiltonian form and then set $P_y = \mu$, where P_y is the momentum canonically conjugate to y.

$$\delta E^{\alpha} = \delta\hat{E}^{\alpha} = \bar{\kappa}(e^{-\frac{1}{3}\phi}\gamma_t + e^{-\frac{4}{3}\phi}\mu v\Gamma_{11})^{\alpha}. \tag{5.10}$$

Now $\gamma_t = \sqrt{-E_t^2}\,\Gamma_{(0)}$, and the equation for v is $\mu v = e^{\phi}\sqrt{-E_t^2}$, so defining a new parameter κ by

$$\kappa = \frac{1}{\sqrt{-E_t^2}} e^{-\frac{1}{3}\phi}\Gamma_{(0)}\hat{\kappa}, \tag{5.11}$$

we find that

$$\delta E^{\alpha} = \bar{\kappa}(1 + \Gamma_{(0)}\Gamma_{11})^{\alpha}, \tag{5.12}$$

which is precisely the $p = 0$ case of the κ-symmetry transformation derived earlier for the general D-p-brane.

We learn from this exercise that the D-0-brane action is invariant under the κ-symmetry transformation (5.12) for general backgrounds, i.e. including fermions, because the $D = 11$ particle action was. This can be checked directly; imposing the constraint

$$R^{(2)}_{\alpha\beta} = e^{-\phi}(\Gamma_a\xi)_{\beta} \tag{5.13}$$

for some spinor ξ, one finds that the action (5.9) is κ-symmetric in a general background if

$$\chi = -D\phi + \Gamma_{11}\xi, \tag{5.14}$$

where χ is the fermion field appearing in the torsion constraints (3.4). One might have expected κ-symmetry to fix both χ and ξ in terms of the dilatino $D\phi$; the additional freedom arises because of the freedom in the choice of the 'conventional' superspace constraints. The usual choice in $D = 11$ is $T_{\alpha\beta}{}^{c} = 0$, and this corresponds in $D = 10$ to

$$\chi = -\frac{1}{3}D\phi, \qquad \xi = \frac{2}{3}\Gamma_{11}D\phi. \tag{5.15}$$

Thus, the D-0-brane action is invariant under the κ-symmetry transformations given above in a general background of massless IIA supergravity, since it is the massless IIA theory that is obtained from $D = 11$, but the result can be immediately generalized to allow for massive IIA backgrounds. First, one must include the 'CS' term in the D-0-brane action (so $C^{(1)} \to C^{(1)} + mV$, where m is the IIA mass parameter). One then need only impose the same superspace constraints as before but allow for the m-dependent modifications of the R–R field strengths; specifically, $R^{(2)}$ should now include the term mB.

6. The super D-2-brane from $D = 11$

We turn now to the derivation of the super D-2-brane action and its κ-transformations from $D = 11$. We again use hats to distinguish $D = 11$ quantities from their $D = 10$ counterparts. The $D = 11$ supermembrane action has the Lagrangian [5]

$$L = \frac{1}{2\hat{v}} \det \hat{g}_{ij} - \frac{1}{2}\hat{v} + \frac{1}{6}\varepsilon^{ijk}\hat{A}_{ijk} . \tag{6.1}$$

We shall use the same KK ansatz as in the previous section, which yields

$$\begin{aligned} \hat{E}_i^a &= e^{-\frac{1}{3}\phi} E_i^a , \\ \hat{E}_i^{11} &= e^{\frac{2}{3}\phi} (\partial_i y + C_i) \end{aligned} \tag{6.2}$$

and hence

$$\hat{g}_{ij} = e^{-\frac{2}{3}\phi}\left[g_{ij} + e^{2\phi}(\partial_i y + C_i)(\partial_j y + C_j)\right] . \tag{6.3}$$

In addition,

$$\frac{1}{6}\varepsilon^{ijk}\hat{A}_{ijk} = \frac{1}{6}\varepsilon^{ijk}A_{ijk} + \frac{1}{2}B_{ij}\partial_k y . \tag{6.4}$$

It will prove convenient to introduce the 1-form field strength

$$Y = dy + C , \tag{6.5}$$

which has the Bianchi identity

$$dY \equiv K , \tag{6.6}$$

where $K \equiv R^{(2)} = dC$. We shall also need the identity for 3×3 matrices

$$\det(g_{ij} + e^{2\phi}Y_iY_j) \equiv (\det g)\,[1 + e^{2\phi}|Y|^2] , \tag{6.7}$$

where $|Y|^2$ is shorthand for $g^{ij}Y_iY_j$.

We can now rewrite the $D = 11$ supermembrane Lagrangian in the KK background as

$$\begin{aligned} L = {} & \frac{1}{2\hat{v}} e^{-2\phi}(\det g)\,[1 + e^{2\phi}|Y|^2] - \frac{1}{2}\hat{v} \\ & + \frac{1}{6}\varepsilon^{ijk}(A_{ijk} - 3B_{ij}C_k) + \frac{1}{2}\varepsilon^{ijk}B_{ij}Y_k . \end{aligned} \tag{6.8}$$

The strategy will now be to replace the scalar y by its dual, a 1-form gauge potential. To this end we promote the field strength Y to the status of an independent variable, which we can do provided that we add to the action a Lagrange multiplier term for the Bianchi identity (6.6), i.e. a term proportional to $V \wedge (dY - K)$, where V is a 1-form Lagrange multiplier field. Integrating by parts, we see that this is equivalent to

$$\begin{aligned} L_{\rm LM} &= -\frac{1}{2}\varepsilon^{ijk}F_{ij}(Y - C)_k \\ &= -\frac{1}{2}\varepsilon^{ijk}[\mathcal{F}_{ij}(Y - C)_k + B_{ij}(Y - C)_k] , \end{aligned} \tag{6.9}$$

where we have defined $F = dV$ and $\mathcal{F} = dV - B$.

Adding $L_{\rm LM}$ to the Lagrangian of (6.8) we arrive at the new, dual, Lagrangian

$$\tilde{L} = \frac{1}{2\hat{v}} e^{-2\phi}(\det g) - \frac{1}{2}\hat{v} + \frac{1}{6}\varepsilon^{ijk}(A_{ijk} + 3\mathcal{F}_{ij}C_k) + \frac{1}{2\hat{v}}(\det g)|Y|^2 - \frac{1}{2}\varepsilon^{ijk}\mathcal{F}_{ij}Y_k . \tag{6.10}$$

The one-form Y is now an auxiliary field which may be eliminated by its field equation

$$Y^i = \frac{\hat{v}}{2\det g}\varepsilon^{ijk}\mathcal{F}_{jk} . \tag{6.11}$$

Back substitution then yields

$$\tilde{L} = \frac{1}{2\hat{v}} e^{-2\phi}(\det g) - \frac{1}{2}\hat{v}(\det g)[1 + |\mathcal{F}|^2] + L_{\rm WZ} , \tag{6.12}$$

where

$$L_{\rm WZ} = \frac{1}{6}\varepsilon^{ijk}(A_{ijk} + 3\mathcal{F}_{ij}C_k) \tag{6.13}$$

$$|\mathcal{F}|^2 \equiv g^{ij}g^{kl}\mathcal{F}_{ik}\mathcal{F}_{jl} . \tag{6.14}$$

We now use the further identity for 3×3 matrices

$$(\det g)\left[1 + \frac{1}{2}|\mathcal{F}|^2\right] \equiv \det(g + \mathcal{F}) , \tag{6.15}$$

and define the new world-volume density

$$v = -(\hat{v})^{-1} e^{-2\phi}\det g , \tag{6.16}$$

to rewrite (6.12) as

$$\tilde{L} = \frac{1}{2v} e^{-2\phi}\det(g + \mathcal{F}) - \frac{1}{2}v + L_{\rm WZ} . \tag{6.17}$$

This new $D = 10$ supermembrane Lagrangian is in a form similar to that with which we started in $D = 11$. If v is now eliminated by its Euler–Lagrange equation we obtain the equivalent Lagrangian

$$L = -e^{-\phi}\sqrt{\det(g + \mathcal{F})} + L_{\rm WZ} , \tag{6.18}$$

for which the corresponding action is

$$I = I_{\rm DBI} + \int [A + \mathcal{F}\wedge C] . \tag{6.19}$$

This is precisely the super D-2-brane action given previously with $C^{(2)} = A$ and $C^{(1)} = C$.

As shown in [6], the κ-symmetry variations of Z^M and V that leave this action invariant can also be deduced from its $D = 11$ origin. The κ-variation of Z^M is encoded in the $D = 11$ supermembrane κ-variation

$$\delta_\kappa \hat{E}^\alpha = [\bar{\kappa}(1+\hat{\Gamma})]^\alpha , \tag{6.20}$$

where $\hat{\Gamma}$ is the matrix

$$\hat{\Gamma} = \frac{1}{6\sqrt{-\det \hat{g}}} \varepsilon^{ijk} \hat{E}_i^{\hat{a}} \hat{E}_j^{\hat{b}} \hat{E}_k^{\hat{c}} \Gamma_{\hat{a}\hat{b}\hat{c}} . \tag{6.21}$$

When expressed in terms of $D = 10$ variables this becomes

$$\hat{\Gamma} = \sqrt{\det(1+X)}\, \Gamma_{(0)} - \frac{1}{2}\gamma^{ij} X_{ij} \Gamma_{11} , \tag{6.22}$$

where

$$\Gamma_{(0)} = \frac{1}{6\sqrt{-\det g}} \varepsilon^{ijk} \gamma_{ijk} . \tag{6.23}$$

Note that $\hat{\Gamma}$ has the properties required for κ-invariance, i.e.

$$\hat{\Gamma}^2 = 1 , \qquad \operatorname{tr} \hat{\Gamma} = 0 . \tag{6.24}$$

The κ-symmetry variation of V, and hence of $\mathcal{F}$, can be deduced from $D = 11$ as follows. The variation of Y in (6.10) is determined in terms of the variations $\delta_\kappa Z^M$ via its definition (6.5); so as long as Y satisfies its Bianchi identity $dY = K$ the action L is κ-invariant. It then follows, when Y is taken to be an independent variable, that the κ-variation of L must have $(dY - K)$ as a factor. Since $dY - K$ is itself κ-invariant, the κ-invariance of $\tilde{L}$ is ensured by an appropriate variation of the Lagrange multiplier V. Now, the Bianchi identity $dY = K$ was needed for κ-invariance of L only to justify the neglect of a term arising after integration by parts in one term of $\delta_\kappa L_{\rm WZ}$. This term is easily isolated:

$$\delta_\kappa L = \frac{1}{2} \varepsilon^{ijk} \delta_\kappa E^A E_i^B B_{BA} (2\partial_j Y_k - K_{jk}) . \tag{6.25}$$

This variation of L may be cancelled in $\delta_\kappa \tilde{L}$ by the variation

$$\delta_\kappa V = \delta_\kappa E^A E_i^B B_{BA} . \tag{6.26}$$

Together with the variation of B, given in (3.11) for general variations δE^A, this allows us to determine the κ-variation of $\mathcal{F} \equiv dV - B$. The result is

$$\delta_\kappa \mathcal{F} = -\frac{1}{2} E^A E^B \delta_\kappa E^C H_{CBA} . \tag{6.27}$$

Observe that the variation of V has simply removed the total derivative term in the variation of $B - dV$. As we saw earlier, this result generalizes to all p. We have now recovered the results obtained earlier for the D-2-brane action, but without the restriction to bosonic backgrounds, so the κ-symmetry transformations derived earlier are equally valid in a general background. We expect that this is equally true for all p.

A curious feature of the above results is that the matrix $\hat{\Gamma}$ appearing in the κ-transformations deduced from $D = 11$ is *not* the same as the specialization to $p = 2$

of the matrix Γ appearing earlier in our discussion of the κ-transformations of the general D-p-brane action. Thus there are *two* matrices satisfying the conditions needed to establish κ-symmetry! In fact, these two matrices are related as follows:

$$(1+\Gamma) = \frac{1}{\sqrt{\det(1+X)}}\Gamma_{(0)}(1+\hat{\Gamma}) . \tag{6.28}$$

Thus, a spinor annihilated by $(1+\Gamma)$ is also annihilated by $(1+\hat{\Gamma})$, and vice versa.

7. Conclusions

We have presented the super D-p-brane action for all $p \leqslant 9$ in general supergravity backgrounds, including massive supergravity backgrounds in the IIA case. We have also presented the full κ-symmetry transformations. We have fully verified κ-invariance for bosonic backgrounds for $p \leqslant 6$, and general backgrounds for $p = 0, 2$. The calculations required for verification of κ-symmetry of the D-p-brane action are embedded in those required for verification of κ-symmetry of the D-$(p+1)$-brane, so our results for $p \leqslant 6$ also provide a partial verification of κ-symmetry for the remaining $p \geqslant 7$ cases. We have no doubt that κ-symmetry also holds in these cases.

We have also shown in detail how the $p = 0$ and $p = 2$ actions follow from the massless superparticle and supermembrane actions in $D = 11$. It is believed that the D-4-brane action should similarly be derivable from the M-theory super-fivebrane action [6], but the latter is not yet known. We hope that the results of this paper will provide some clues to the solution of this outstanding problem.

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Note added

Upon completion of this work a paper appeared [21] with similar results to those obtained here. These authors have established κ-symmetry for general backgrounds involving fermions but did not consider massive IIA backgrounds, nor the relation of the D-0-brane to $D = 11$.

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