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Intersections involving monopoles and waves in eleven dimensions

E Bergshoeff, M de Roo, E Eyras, B Janssen and J P van der Schaar
Institute for Theoretical Physics, Nijenborgh 4, 9747 AG Groningen, The Netherlands

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Abstract. We consider intersections in eleven dimensions involving Kaluza–Klein monopoles and Brinkmann waves. Besides these purely gravitational configurations, we also construct solutions to the equations of motion that involve additional $M2$- and $M5$-branes. The maximal number of independent objects in these intersections is nine, and such maximal configurations, when reduced to two dimensions, give rise to a 0-brane solution with dilaton coupling $a = -\frac{4}{9}$.

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1. Introduction

Eleven-dimensional supergravity has regained its prominent role in the search for a quantum theory of gravity. It is the low-energy limit of the conjectured M-theory, from which all five 10-dimensional string theories can be obtained.

One implication of this viewpoint is that all solutions of type IIA theory should have an 11-dimensional interpretation [1]. Indeed, the fundamental string ($F1$) [2] and the solitonic 5-brane ($S5$) [3, 4] are the double-dimensional reduction of the 11-dimensional $M2$-brane [5] and the direct dimensional reduction of the 11-dimensional $M5$-brane [6], respectively. The Dirichlet $D2$- and $D4$-branes can be obtained from $M2$ and $M5$ via direct and double-dimensional reduction, respectively. The $D0$- and $D6$-branes in the IIA theory are related to the purely gravitational Brinkmann wave [7] ($\mathcal{W}$) and the Kaluza–Klein monopole [8] ($\mathcal{KK}$) in eleven dimensions. These 11-dimensional solutions also have their counterparts in $D = 10$, which we denote by W and KK. Each of these solutions preserves $\frac{1}{2}$ of the $D = 11$ (or $D = 10$, $N = 2$) supersymmetry. In figure 1 we summarize the relationship between these $D = 10$ IIA and $D = 11$ solutions. The 11-dimensional interpretation of the type IIA 8-brane [9, 10] is still a mystery (see also below). Presumably, it is related to a 9-brane† in $D = 11$. The direct reduction of such a 9-brane is expected to lead to $D = 10$ Minkowski space.

The aim of this paper is to extend our recent work on intersections of $M2$- and $M5$-branes [15] by including the wave and monopole solutions indicated in figure 1. This paper is organized as follows. In section 2 we will first discuss the case of two intersecting 11-dimensional solutions. In section 3 we obtain all multiple intersections which are purely gravitational, i.e. which do not involve the 3-form gauge field of $D = 11$ supergravity. In section 4 we discuss multiple intersections involving $M2$- and $M5$-branes as well. We draw

† The conjectured 9-brane is also discussed in [10–14].

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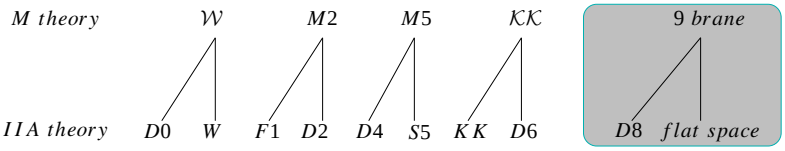

Figure 1. The relation between $D = 10$ IIA and $D = 11$ solutions. Vertical lines imply direct dimensional reduction, diagonal lines double-dimensional reduction. The shadowed area indicates the relationship between known 10-dimensional solutions and a conjectured 9-brane in $D = 11$.

our conclusions in section 5. In the remainder of this section we will summarize some relevant properties of the $\mathcal{W}$ and $\mathcal{KK}$ solutions.

The Brinkmann wave in D dimensions is given by the metric

$$\mathrm{d}s^2 = (2 - H)\,\mathrm{d}t^2 - H\,\mathrm{d}z^2 + 2(1 - H)\,\mathrm{d}t\,\mathrm{d}z - (\mathrm{d}x_2^2 + \cdots + \mathrm{d}x_{(D-1)}^2), \quad (1)$$

where H is a harmonic function in the variables $t + z, x_2, \ldots, x_{(D-1)}$. In ten dimensions the wave solution is T-dual to the fundamental string $F1$†, after assuming isometry in the z direction.

There are two ways to reduce the wave to $(D-1)$-dimensional spacetime. On imposing that z is an isometry direction, the solution becomes static and corresponds in $D - 1$ dimensions to a 0-brane. The charge is carried by a vector field of which only the time component does not vanish, and is given by $A_t = 1 - H^{-1}$. Alternatively, one can impose that H is independent of one of the x_μ $\big(\mu = 2, \ldots, (D-1)\big)$ coordinates. This results in a Brinkmann wave in $D - 1$ dimensions.

The metric for the Kaluza–Klein monopole reads $(i = 1, 2, 3)$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}x_1^2 - \cdots - \mathrm{d}x_{(D-5)}^2 - H^{-1}(\mathrm{d}z + A_i\,\mathrm{d}y_i)^2 - H\,\mathrm{d}y_i^2\,, \quad (2)$$

where H and A_i depend on y_i, and the relation between H and A_i is

$$F_{ij} \equiv \partial_i A_j - \partial_j A_i = \epsilon_{ijk}\partial_k H\,. \quad (3)$$

Here the directions t, x_μ $\big(\mu = 1, \ldots, (D-5)\big)$ and z are isometry directions. Reduction over x_μ leads to a Kaluza–Klein monopole in $D-1$ dimensions. Reduction over z leads to a $(D-5)$-brane in $D-1$ dimensions, where the y_i directions correspond to the transverse space. The solution (2) in ten dimensions is T-dual, with respect to the z direction, to the solitonic 5-brane $S5$.

On several occasions we will assume that one of the y_i, say y_1, corresponds to an isometry direction as well. In that case A_2 and A_3 can be gauged away, and the metric becomes (in $D - 1$ dimensions)

$$\mathrm{d}s^2 = \varphi^{-1/2}(\mathrm{d}t^2 - \mathrm{d}x_1^2 - \cdots - \mathrm{d}x_{(D-5)}^2) - \varphi^{1/2}(\mathrm{d}z^2 + (H^2 + A_1^2)(\mathrm{d}y_2^2 + \mathrm{d}y_3^2))\,, \quad (4)$$

where H, A_1 and

$$\varphi \equiv H/(H^2 + A_1^2) \quad (5)$$

are harmonic in y_2, y_3. The coordinate transformation to u, v, where

$$\mathrm{d}(u + \mathrm{i}v) = (H + \mathrm{i}A_1)\,\mathrm{d}(y_2 + \mathrm{i}y_3) \quad (6)$$

preserves the harmonic property of φ, and gives the usual metric, dilaton and a vector field with a non-vanishing component in the z direction for a magnetic $(D-5)$-brane. Of course

† Since the wave, and also the monopole solution considered below, involve only fields which IIA and IIB theories have in common, this duality transformation can be considered as a IIA transformation.

z remains an isometry direction in $D-1$ dimensions. The coordinate transformation (6) can also be done directly in D dimensions.

Sometimes we will consider monopoles which are truncated further, and for which the harmonic function H depends on only a single variable, say y_3. This implies that (locally) $H = my_3 + c$, $A_1 = -my_2$ for constant c. Note that this does not imply an additional isometry, and reduction over y_1 indeed gives, after the coordinate transformation (6), a $(D-5)$-brane for which φ again depends on u and v.

To obtain a $(D-5)$-brane in $D-1$ dimensions which has two additional isometries, we must choose for H and A_1 special functions that are harmonic in y_2, y_3. If the harmonic function φ depends only on u, it must be linear in u, and the coordinate transformation (6) then implies that H and A_1 must satisfy

$$\mathrm{d}\left(\frac{H - \mathrm{i}A_1}{H^2 + A_1^2}\right) = (H + \mathrm{i}A_1)\,\mathrm{d}(y_2 + \mathrm{i}y_3)\,, \tag{7}$$

which is solved by

$$(H + \mathrm{i}A_1)^2 = \frac{1}{2(y_2 + \mathrm{i}y_3 + \alpha)}\,, \tag{8}$$

where α is a complex integration constant. Reducing over the y_1 direction, we find that indeed the function φ, after the coordinate transformation (6), depends on u only, and that the only non-zero component of the gauge field is in the z direction, and is given by $v\mathrm{d}\varphi/\mathrm{d}u$. This form of the $(D-5)$-brane in $D-1$ dimensions was given in [1] for the case of the 6-brane in ten dimensions. There it is T-dual to the 8-brane [10]. Note that, strictly speaking, the $(D-5)$-brane does not have two additional isometries since the gauge field is linear in v. However, as discussed in [10], such linear dependence disappears after a further reduction over v to $D-2$ dimensions. Furthermore, the v-dependence also disappears in the $(D-1)$-dimensional dual formulation where the vector field has been replaced by a $(D-4)$-form gauge field. In this sense we may consider the v direction as a kind of 'generalized' isometry direction.

It is interesting to consider the uplifting of the truncated $(D-5)$-brane solution discussed above to D dimensions:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}x_1^2 - \cdots - \mathrm{d}x_{(D-5)}^2 - u^{-1}\big(\mathrm{d}y - v\,\mathrm{d}z\big)^2 - u\big(\mathrm{d}z^2 + \mathrm{d}u^2 + \mathrm{d}v^2\big)\,. \tag{9}$$

Since this solution has $D-2$ isometries and one 'generalized' isometry, it is similar to a $(D-2)$-brane solution in D dimensions. For $D = 11$ this would correspond to a 9-brane solution. Upon reduction to eight dimensions it leads to a solution which is identical to the ten-dimensional 8-brane when reduced to eight dimensions.

The Kaluza–Klein monopoles, for which additional isometry is imposed in the direction of the Kaluza–Klein vectors, are no longer asymptotically flat. Although this will disqualify them for certain applications, they are nevertheless solutions of the equations of motion, and reduce to (truncated) $D6$-branes in $D = 10$. Since in this paper we do not consider global properties of our solutions, we will include these truncated monopoles in multiple intersections.

2. Intersection rules

Intersections of a pair of branes are at the basis of the construction of multiple intersections. In a multiple intersection each pair obtained by setting all but two of the independent harmonic functions equal to one, must be one of the basic pairs described below. For the

Dp-branes and the NS–NS-branes $F1$ and $S5$ in $D = 10$, as well as for the $M2$ and $M5$ branes in $D = 11$, the allowed pair intersections are known [16–21]. For pair intersections involving waves and monopoles partial results were given in [17, 22, 23].

Table 1. Pair intersections in $D = 11$ and their reductions to $D = 10$ with dependence on overall transverse coordinates. The first column represents the pair intersections in $D = 11$. $(q|p_1, p_2)$ denotes an intersection of a p_1 and a p_2 brane over a common $(q + 1)$-dimensional world-volume. Reductions to non-trivial solutions in $D = 10$, obtained by compactification in different directions (common world-volume, relative transverse and overall transverse) with respect to the branes, are indicated in the remaining columns. The $D = 10$ solutions marked with * are not of the usual harmonic form.

	Common world-volume	Relative transverse	Overall transverse
$(0\|M2, M2)$	—	$(0\|F1, D2)$	$(0\|D2, D2)$
$(1\|M2, M5)$	$(0\|F1, D4)$	$(1\|F1, S5)$ $(1\|D2, D4)$	$(1\|D2, S5)$
$(3\|M5, M5)$	$(2\|D4, D4)$	$(3\|D4, S5)$	$(3\|S5, S5)$
$(1\|M2, \mathcal{W})$	$(0\|F1, D0)$	$(1\|F1, W)$	$(1\|D2, W)$
$(1\|M5, \mathcal{W})$	$(0\|D4, D0)$	$(1\|D4, W)$	$(1\|S5, W)$
$(2\|M2, \mathcal{KK})$	$(1\|F1, KK)$	$(2\|D2, KK)$	$(2\|D2, D6)$
$(5\|M5, \mathcal{KK})$	$(4\|D4, KK)$	$(5\|S5, KK)$	$(5\|S5, D6)$
$(0\|M2, \mathcal{KK})$	—	$(0\|F1, D6)$ $(0\|D2, KK)$	$(0\|D2, D6)^*$
$(3\|M5, \mathcal{KK})$	$(2\|D4, KK)$	$(3\|D4, D6)$ $(3\|S5, KK)$	$(3\|S5, D6)^*$
$(1\|\mathcal{W}, \mathcal{KK})$	$(0\|D0, KK)$	$(1\|W, KK)$	$(1\|W, D6)$
$(4\|\mathcal{KK}, \mathcal{KK})^a$	$(3\|KK, KK)^a$	$(4\|D6, KK)^*$	$(4\|D6, D6)$
$(4\|\mathcal{KK}, \mathcal{KK})^b$	$(3\|KK, KK)^b$	$(4\|D6, KK)$	$(4\|D6, D6)^*$

In tables 1 and 2 we summarize old and new results on the pair intersections. The two independent harmonic functions of the pairs in table 1 depend on the coordinates which are transverse to both branes (overall transverse)†. For the pairs in table 2 both harmonic functions must depend on the relative transverse coordinates. In sections 3 and 4, where we discuss multiple intersections, we will use only the pairs of table 1.

The first three rows of table 1 denote the intersections of $M2$- and $M5$-branes. As an example, which also explains our notation, consider $(1|M2, M5)$. Denoting a world-volume direction of a brane by $\times$, and a transverse direction by $-$, the metric for this pair can be represented by

$$(1|M2, M5) = \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & - & - & - & - & - & - & - & - \\ \times & \times & - & \times & \times & \times & \times & - & - & - & - \end{array} \right. . \tag{10}$$

The coordinates $t = x^0, x^1, \ldots, x^{10}$ are indicated from left to right. The common world-volume in this case is two dimensional (x^0, x^1), the overall transverse space four dimensional $(x^7, \ldots, x^{10})$, and there are five relative transverse directions $(x^2, \ldots, x^6)$. The spacelike directions $x^1, \ldots, x^6$ correspond to isometries. Reduction over x^1 gives $(0|F1, D4)$ in ten

† For some of the entries in table 1 another possibility exists, namely that one harmonic function depends on overall transverse, the other on directions which are transverse to only one brane in the pair (relative transverse) [18, 24]. We will not consider this option in this paper.

dimensions. For the relative transverse directions the possibilities are: either reduction over x^2, giving $(1|F1, S5)$, or reduction over one of the directions $x^3, \ldots, x^6$, giving $(1|D2, D4)$. Finally, one can impose an isometry in one of the overall transverse directions by restricting the dependence of the harmonic functions to three coordinates. Reduction over such a direction gives $(1|D2, S5)$. The next two rows represent the addition of a wave to the $D = 11$ M-branes. The z direction of the wave must be placed in the world-volume of the M-brane. The dependence of the harmonic functions is only on the directions transverse to the M-brane, so that the wave does not propagate. The metric for these two $D = 11$ pairs can be represented by†

$$(1|M2, \mathcal{W}) = \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & - & - & - & - & - & - & - & - \\ \times & z & - & - & - & - & - & - & - & - & - \end{array} \right. \tag{11}$$

$$(1|M5, \mathcal{W}) = \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & \times & \times & \times & - & - & - & - & - \\ \times & z & - & - & - & - & - & - & - & - & - \end{array} \right. . \tag{12}$$

The next four rows in table 1 denote the pairs involving one M-brane and one Kaluza–Klein monopole. The metric for these four cases takes on the form

$$(2|M2, \mathcal{KK}) = \left\{ \begin{array}{c|cccccccccc} \times & - & - & - & - & \times & \times & - & - & - & - \\ \times & A_1 & A_2 & A_3 & z & \times & \times & \times & \times & \times & \times \end{array} \right. \tag{13}$$

$$(5|M5, \mathcal{KK}) = \left\{ \begin{array}{c|cccccccccc} \times & - & - & - & - & \times & \times & \times & \times & \times & - \\ \times & A_1 & A_2 & A_3 & z & \times & \times & \times & \times & \times & \times \end{array} \right. \tag{14}$$

$$(0|M2, \mathcal{KK}) = \left\{ \begin{array}{c|cccccccccc} \times & \times & - & - & \times & - & - & - & - & - & - \\ \times & A_1 & A_2 & A_3 & z & \times & \times & \times & \times & \times & \times \end{array} \right. \tag{15}$$

$$(3|M5, \mathcal{KK}) = \left\{ \begin{array}{c|cccccccccc} \times & \times & - & - & \times & \times & \times & \times & - & - & - \\ \times & A_1 & A_2 & A_3 & z & \times & \times & \times & \times & \times & \times \end{array} \right. . \tag{16}$$

As we see, there are two possibilities. The z direction of the Kaluza–Klein monopole, the natural isometry direction which on compactification gives a magnetic $(D-5)$-brane, can be placed either in a direction transverse to $((2|M2, \mathcal{KK})$ and $(5|M5, \mathcal{KK}))$ or in the world-volume of the M-brane $((0|M2, \mathcal{KK})$ and $(3|M5, \mathcal{KK}))$. The solutions (13) and (14) were given in [17, 23]. For these, the reduction to $D = 10$ is straightforward. Note that the reduction over an overall transverse direction can be either over a direction indicated by z, or, by imposing an additional isometry, in the direction of a component of the vector field.

In the solutions (15) and (16) the harmonic functions depend only on the two overall transverse coordinates, so that the Kaluza–Klein monopole has one additional isometry direction (indicated by A_1). In both of these solutions the reduction over the relative transverse A_1 and z directions yields, after a coordinate transformation, the same result.

The last three rows of table 1 correspond to intersections of Kaluza–Klein monopoles and waves. The possibilities are shown in (17)–(19)‡. Note that there are two ways to intersect two Kaluza–Klein monopoles, both with a five-dimensional common world-volume. In solution (18) the two harmonic functions depend on a single coordinate (x^1), in (19) on two

† Note that we extend the notation $(q|p_1, p_2)$ to include waves and monopoles with the understanding that the world-volume directions of the '$\mathcal{W}$-brane' are given by t, z (see (1)), and the transverse directions of the '$\mathcal{KK}$-brane' are given by the isometry direction z and the coordinates in which the Kaluza–Klein vector is oriented. These directions (called y_i in (2)) will be denoted by A_i.

‡ Solution (17) was presented in [17].

coordinates (x^1, x^2)

$$(1|\mathcal{W}, \mathcal{KK}) = \left\{ \begin{array}{c|cccccccccc} \times & - & - & - & - & z_1 & - & - & - & - & - \\ \times & A_1 & A_2 & A_3 & z_2 & \times & \times & \times & \times & \times & \times \end{array} \right. \tag{17}$$

$$(4|\mathcal{KK}, \mathcal{KK})^a = \left\{ \begin{array}{c|cccccccccc} \times & A_1 & A_2 & A_3 & z & \times & \times & \times & \times & \times & \times \\ \times & B_1 & \times & \times & z & B_5 & B_6 & \times & \times & \times & \times \end{array} \right. \tag{18}$$

$$(4|\mathcal{KK}, \mathcal{KK})^b = \left\{ \begin{array}{c|cccccccccc} \times & A_1 & A_2 & A_3 & z_1 & \times & \times & \times & \times & \times & \times \\ \times & B_1 & B_2 & \times & \times & B_5 & z_2 & \times & \times & \times & \times \end{array} \right. . \tag{19}$$

For these solutions it may be useful to present the metric explicitly. We have:

$$\begin{aligned} (4|\mathcal{KK}, \mathcal{KK})^a \to \mathrm{d}s^2 = {} & \mathrm{d}t^2 - H_1 H_2\, \mathrm{d}x_1^2 - H_1\, \mathrm{d}x_{(2-3)}^2 - H_2\, \mathrm{d}x_{(5-6)}^2 - \mathrm{d}x_{(7-10)}^2 \\ & - (H_1 H_2)^{-1} (\mathrm{d}z + (A_1 + B_1)\, \mathrm{d}x_1 + A_2\, \mathrm{d}x_2 \\ & + A_3\, \mathrm{d}x_3 + B_5\, \mathrm{d}x_5 + B_6\, \mathrm{d}x_6)^2 , \end{aligned} \tag{20}$$

$$\begin{aligned} (4|\mathcal{KK}, \mathcal{KK})^b \to \mathrm{d}s^2 = {} & \mathrm{d}t^2 - H_1 H_2\, \mathrm{d}x_{(1-2)}^2 - H_1\, \mathrm{d}x_3 - H_2\, \mathrm{d}x_5 - \mathrm{d}x_{(7-10)}^2 \\ & - H_1^{-1} (\mathrm{d}z_1 + A_1\, \mathrm{d}x_1 + A_2\, \mathrm{d}x_2 + A_3\, \mathrm{d}x_3)^2 \\ & - H_2^{-1} (\mathrm{d}z_2 + B_1\, \mathrm{d}x_1 + B_2\, \mathrm{d}x_2 + B_5\, \mathrm{d}x_5)^2 . \end{aligned} \tag{21}$$

Note that in (20) the harmonic functions depend only on x_1. Therefore two of the components of each of the gauge fields A and B can be gauged to zero. For the reductions in table 1, different gauge choices are employed. In (21) the harmonic functions depend on x_1 and x_2. Here also different gauge choices can be made.

The solution (18) solves the equations of motion, since it is the known ten-dimensional solution $(4|D6, D6)$ lifted up to $D = 11$. The configuration (19) must be a solution because, after reduction over a common world-volume direction it can be related to a known solution involving two solitonic 5-branes via the following T-duality chain in $D = 10$:

$$(3|S5, S5) \to (3|S5, KK) \to (3|KK, KK)^b . \tag{22}$$

Note that it is possible to relate (18) and (19) by a chain of T-duality and one S-duality transformations in ten dimensions. This involves the S-duality transformation between $(3|D5, D5)$ and $(3|S5, S5)$.

Similarly, the intersection of a wave and a Kaluza–Klein monopole can be obtained from ten dimensions by first constructing an intersection in $D = 10$ of a $D0$-brane with the Kaluza–Klein monopole:

$$(0|D1, S5) \to (0|D0, KK) , \tag{23}$$

and by lifting this to eleven dimensions.

In table 1 there are four reductions to $D = 10$ that do not lead to solutions which are expressed in a standard form in terms of harmonic functions. As an example, consider the reduction of (18). The harmonic functions depend on x_1, the non-zero gauge field components can be chosen to be A_2 and B_5, which then depend on x_3 and x_6, respectively. Reduction over z gives $(4|D6, D6)$, but also reduction over x_2 is possible. This gives a $D = 10$ configuration which has the properties of $(4|D6, KK)$, but the fields do not have

the standard harmonic form. It is given by:

$$\begin{aligned} \mathrm{d}s^2 &= \varphi^{-1/2}(\mathrm{d}t^2 - \mathrm{d}x^2_{(7-10)} - H_2\,\mathrm{d}x^2_{(5-6)}) \\ &\quad - H_2^{-1}\varphi^{1/2}\big((\mathrm{d}z + B_5\,\mathrm{d}x_5)^2 + (H_1^2H_2 + A_2^2)(\mathrm{d}x_3^2 + H_2\,\mathrm{d}x_1^2)\big)\,, \\ \mathrm{e}^{2\phi} &= \varphi^{-3/2}\,, \\ C_z &= \frac{\varphi A_2}{H_1H_2}\,, \qquad C_5 = \frac{\varphi A_2 B_5}{H_1H_2}\,, \end{aligned} \tag{24}$$

where

$$\varphi = H_1H_2/(A_2^2 + H_1^2H_2)\,. \tag{25}$$

The non-zero components of the RR-vector field in $D = 10$ are denoted by C_μ. Note that φ is indeed not harmonic in x_1, x_3. If $H_2 = 1$ and $B_5 = 0$, φ does become harmonic, and we obtain a standard $D6$ solution, after the coordinate transformation (6). Conversely, for $H_1 = 1$, $A_2 = 0$ a standard Kaluza–Klein monopole is obtained in $D = 10$. These solutions show that the usual harmonic ansatz for intersecting pairs does not cover all possibilities. It will be interesting to investigate these non-harmonic solutions further (see also [24]).

Table 2. Pair intersections in $D = 11$ and their reductions to $D = 10$ with dependence on relative transverse coordinates. The reductions indicated by a * are not expressed in a standard way in terms of harmonic functions.

	Common world-volume	Relative transverse	Overall transverse
$(1\|M5, M5)$	$(0\|D4, D4)$	$(1\|D4, S5)$	$(1\|S5, S5)$
$(0\|M2, \mathcal{K}\mathcal{K})$	—	$(0\|D2, KK)$ $(0\|F1, D6)^*$	$(0\|D2, D6)$
$(1\|M5, \mathcal{K}\mathcal{K})$	$(0\|D4, KK)$	$(1\|S5, KK)$ $(1\|D4, D6)$	—
$(3\|M5, \mathcal{K}\mathcal{K})$	$(2\|D4, KK)$	$(3\|S5, KK)$ $(3\|D4, D6)^*$	$(3\|S5, D6)$
$(2\|\mathcal{K}\mathcal{K}, \mathcal{K}\mathcal{K})$	$(1\|KK, KK)$	$(2\|D6, KK)$	—
$(4\|\mathcal{K}\mathcal{K}, \mathcal{K}\mathcal{K})$	$(4\|KK, KK)$	$(4\|D6, KK)$ $(4\|D6, KK)^*$	$(4\|D6, D6)^*$

In table 2 we consider intersections in which the two harmonic functions depend on the relative coordinates. There is one pair involving only $M5$ [19], and five pairs involving Kaluza–Klein monopoles. Some of these configurations and their generalization to non-orthogonal intersections were discussed recently in [25].

Below we present the metric of these pairs in the usual way. The pairs involving Kaluza–Klein monopoles are each related to known solutions through $D = 10$, so that we can be sure that they solve the equations of motion. For example, $(2|\mathcal{K}\mathcal{K}, \mathcal{K}\mathcal{K})$ can be reduced to $(1|KK, KK)$ in ten dimensions, and applying T-duality twice, in the directions z_1 and z_2, we find

$$(1|KK, KK) \to (1|S5, KK) \to (1|S5, S5)\,, \tag{26}$$

and this can be oxidized to $(1|M5, M5)$, which is a known solution

$$(1|M5, M5) = \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & \times & \times & \times & - & - & - & - & - \\ \times & \times & - & - & - & - & \times & \times & \times & \times & - \end{array} \right. \tag{27}$$

$$(0|M2, \mathcal{K}\mathcal{K}) = \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & - & - & - & - & - & - & - & - \\ \times & A_1 & A_2 & A_3 & z & \times & \times & \times & \times & \times & \times \end{array} \right. \tag{28}$$

$$(1|M5, \mathcal{K}\mathcal{K}) = \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & \times & \times & \times & - & - & - & - & - \\ \times & A_1 & A_2 & A_3 & z & \times & \times & \times & \times & \times & \times \end{array} \right. \tag{29}$$

$$(3|M5, \mathcal{K}\mathcal{K}) = \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & - & - & - & - & - & \times & \times & \times \\ \times & A_1 & A_2 & A_3 & z & \times & \times & \times & \times & \times & \times \end{array} \right. \tag{30}$$

$$(2|\mathcal{K}\mathcal{K}, \mathcal{K}\mathcal{K}) = \left\{ \begin{array}{c|cccccccccc} \times & A_1 & A_2 & A_3 & z_1 & \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & z_2 & B_6 & B_7 & B_8 & \times & \times \end{array} \right. \tag{31}$$

$$(4|\mathcal{K}\mathcal{K}, \mathcal{K}\mathcal{K}) = \left\{ \begin{array}{c|cccccccccc} \times & A_1 & A_2 & A_3 & z_1 & \times & \times & \times & \times & \times & \times \\ \times & \times & \times & B_3 & B_4 & B_5 & z_2 & \times & \times & \times & \times \end{array} \right. . \tag{32}$$

In (27)–(32) the dependence is on the relative transverse coordinates, e.g. in (30) H_1 depends on $x_5, \ldots, x_7$ and H_2 on x_1, x_2. In the reduction of (30) to $D = 10$ we obtain $(3|S5, KK)$ when an isometry in one of the coordinates $x_5, \ldots, x_7$ is assumed, and $(3|D4, D6)^*$ when reducing over x_1 or x_2.

3. Purely gravitational solutions: monopoles and waves

In this section we will consider configurations involving several monopoles, with or without an additional wave, using the pair intersections of table 1. The interest of such solutions lies in the fact that they involve only the gravitational field. If the spacetime is of sufficient dimensionality, such solutions can always be present.

Configurations involving only monopoles differ in the way the z-isometry directions are related. In (33)–(35) we present three configurations to which no further monopole can be added

Type A:
$$\left\{ \begin{array}{c|cccccccccc} \times & \times & \times & \times & \times & \times & \times & A_7 & A_8 & A_9 & z \\ \times & \times & \times & \times & \times & B_5 & B_6 & \times & \times & B_9 & z \\ \times & \times & \times & C_3 & C_4 & \times & \times & \times & \times & C_9 & z \\ \times & D_1 & D_2 & \times & \times & \times & \times & \times & \times & D_9 & z \end{array} \right. \tag{33}$$

Type B:
$$\left\{ \begin{array}{c|cccccccccc} \times & \times & \times & \times & \times & \times & \times & A_7 & A_8 & A_9 & z_1 \\ \times & \times & \times & \times & \times & B_5 & B_6 & \times & \times & B_9 & z_2 \\ \times & \times & \times & \times & z_3 & C_5 & \times & \times & C_8 & C_9 & \times \\ \times & \times & \times & \times & z_4 & \times & D_6 & D_7 & \times & D_9 & \times \\ \times & \times & \times & z_5 & \times & E_5 & \times & E_7 & \times & E_9 & \times \\ \times & \times & \times & z_6 & \times & \times & F_6 & \times & F_8 & F_9 & \times \end{array} \right. \tag{34}$$

Type C:
$$\left\{ \begin{array}{c|cccccccccc} \times & z & A_2 & \times & \times & \times & \times & \times & \times & A_9 & A_{10} \\ \times & \times & \times & z & B_4 & \times & \times & \times & \times & B_9 & B_{10} \\ \times & \times & \times & \times & \times & z & C_6 & \times & \times & C_9 & C_{10} \\ \times & \times & \times & \times & \times & \times & \times & z & D_8 & D_9 & D_{10} \end{array} \right. . \tag{35}$$

In (33) there is a common isometry direction z, in (34) the six monopoles come in pairs with a common z-isometry, while the four monopoles in (35) have no common z-isometry. Note that in (33) and (34) the solution depends on only one coordinate, in (35) the harmonic functions may depend on two coordinates. To the solution (34) we can add a single wave in either the x^1 or the x^2 direction.

Table 3. Maximal number of monopoles and waves in $5 \leqslant D \leqslant 11$ dimensions. We indicate the maximum number of Kaluza–Klein monopoles in different dimensions, superimposed according to type A, B or C (see (33)–(35)). W means that a wave can be added.

D	Type A	Type B	Type C
5	1	1	1
6	1 + W	1 + W	1 + W
7	2	2	2
8	2 + W	4	2 + W
9	3	6	3
10	3 + W	6 + W	3 + W
11	4	6 + W	4

It is interesting to see how these purely gravitational solutions survive in lower dimensions. In table 3 we indicate the configurations with a maximum number of monopoles. Note that if we go to dimensions higher than eleven, configurations of type A and type C are naturally extended to an additional monopole in each odd-dimensional spacetime. The configurations of type B cannot be extended beyond six monopoles in higher dimensions. In some cases a single wave can be added to these monopole configurations. Note that the solution in $D = 5, 6$ is the same for type A, B and C. In $D = 7$ there is no difference between type A and type B.

The supersymmetry of these purely gravitational solutions, embedded in $D = 11$ supergravity and its toroidal compactifications, is $\frac{1}{16}$ of the $D = 11$ supersymmetry.

4. Multiple intersections

Having determined the 'no-force' condition between the basic 11-dimensional solutions in section 2 and the multiple intersections of waves and monopoles in section 3, we next consider multiple intersections that also involve $M2$- and $M5$-branes. Multiple intersections of D-branes in $D = 10$, and of $M2$- and $M5$-branes in $D = 11$ have only recently been classified [15]. The $D = 11$ result is given in table 1 of [15]. In this section we will generalize the result of [15] to intersections that also involve waves and monopoles. We will first restrict ourselves to configurations that can be reduced to intersections with only D-branes in $D = 10$. Looking back at table 1, we see that all pairs involving monopoles should then be of the form $(2|M2, \mathcal{KK})$, $(3|M5, \mathcal{KK})$ or $(4|\mathcal{KK}, \mathcal{KK})^a$, and that with a wave only $(1|M5, \mathcal{W})$ may be used. Thus only multiple monopoles of type A (see the previous section) will be used. At the end of this section we will relax these restrictions and consider the possibility of also using $(1|M2, \mathcal{W})$.

Our strategy will be to take table 1 of [15] as our starting point and then consider to which M-brane intersections waves and/or monopoles can be added. The rule for adding a wave is known [17, 26]. To each intersection involving at least a common string a wave can be added in such a way that the z-isometry direction of the wave lies in the spacelike common string direction. Furthermore, at most one wave can be added to any

given intersection.

From the intersection (13) we see that the world-volume of the $M2$-brane must lie in the world-volume directions of the monopole. Furthermore, two intersecting $M2$-branes have distinct (spacelike) world-volume directions. Since the monopole has six (spacelike) world-volume directions we conclude that monopoles may be added to configurations that contain at most three $M2$-branes [23]:

$$\left\{\begin{array}{c|cccccccccc} \times & \times & \times & - & - & - & - & - & - & - & - \\ \times & - & - & \times & \times & - & - & - & - & - & - \\ \times & - & - & - & - & \times & \times & - & - & - & - \\ \times & \times & \times & \times & \times & \times & \times & z & A_8 & A_9 & A_{10} \end{array}\right. . \tag{36}$$

We next consider the $M5$-branes. Using only the pair $(3|M5, \mathcal{K}\mathcal{K})$ we see that the z-isometry direction of the monopole should lie in a common world-volume direction of the $M5$-branes. One finds that to a single monopole one can add at most four $M5$-branes. An example of such a configuration is:

$$\left\{\begin{array}{c|cccccccccc} \times & - & \times & \times & - & \times & - & \times & - & \times & - \\ \times & \times & - & - & \times & \times & - & \times & - & \times & - \\ \times & \times & - & \times & - & - & \times & \times & - & \times & - \\ \times & \times & - & \times & - & \times & - & - & \times & \times & - \\ \times & \times & \times & \times & \times & \times & \times & A_7 & A_8 & z & A_{10} \end{array}\right. . \tag{37}$$

The harmonic functions depend only on the coordinate x_{10}. However, one may add more than one monopole to the four 5-branes. From (37) it is clear that the monopole could also have been placed with two components of the vector field in the (x_1, x_2), (x_3, x_4) or (x_5, x_6) directions. In fact, in this way one can combine four monopoles with the four $M5$-branes:

$$\left\{\begin{array}{c|cccccccccc} \times & - & \times & \times & - & \times & - & \times & - & \times & - \\ \times & \times & - & - & \times & \times & - & \times & - & \times & - \\ \times & \times & - & \times & - & - & \times & \times & - & \times & - \\ \times & \times & - & \times & - & \times & - & - & \times & \times & - \\ \times & \times & \times & \times & \times & \times & \times & A_7 & A_8 & z & A_{10} \\ \times & \times & \times & \times & \times & B_5 & B_6 & \times & \times & z & B_{10} \\ \times & \times & \times & C_3 & C_4 & \times & \times & \times & \times & z & C_{10} \\ \times & D_1 & D_2 & \times & \times & \times & \times & \times & \times & z & D_{10} \end{array}\right. . \tag{38}$$

One may verify that this intersection is consistent with the $M5$–$\mathcal{K}\mathcal{K}$ intersection rule (16) and the $\mathcal{K}\mathcal{K}$–$\mathcal{K}\mathcal{K}$ rule (18).

Having established the rule of how to add monopoles to an intersection of $M2$-branes and $M5$-branes or a mixture thereof, we are able to list all intersections involving $M2$-branes, $M5$-branes, waves and monopoles. It is enough to give only the intersection with the largest number of independent harmonics. All other intersections can be obtained from these by setting one or more of the harmonic functions equal to one†.

The result is given in table 4. The maximum number of intersecting objects N equals eight if we restrict ourselves to configurations which can be reduced to pure D-brane intersections in $D = 10$. We use the same notation as in [15]. In $D = 11$ a configuration is characterized by the number of $\times$'s (world-volume directions) in each of the spatial coordinates. In this notation, the four 5-branes in (37) or (38) are denoted by $[5^4]\{4, 0, 4, 1\}$, since there are four coordinates with one $\times$, zero with two $\times$'s etc. In $D = 10$ the same notation can be used, but then a convention can be chosen to avoid giving T-dual

† This is not the case if one considers multiple monopoles of type B and C.

Table 4. $N = 8$ intersections that reduce to pure D-brane intersections. The column headings indicate the 10-dimensional T-duality class. The notation $[2^k, 5^l] + n\mathcal{K}\mathcal{K}$ indicates that the intersections contain k $M2$-branes, l $M5$-branes and n monopoles. An additional wave is indicated by $+\mathcal{W}$.

$(0, 4, 0, 4)_{SUSY=1/32}$	$(1, 0, 7, 0)_{SUSY=1/32}$	$(0, 0, 0, 7)_{SUSY=1/16}$
$[2^4, 5^4]\{0, 4, 0, 5, 0, 0, 0\}$	$[2^4, 5^4]\{1, 0, 6, 1, 1, 0, 0, 0\}$	$[2^3, 5^4]\{0, 0, 6, 2, 0, 0, 0\} + \mathcal{K}\mathcal{K}$
$[2^3, 5^4]\{1, 2, 4, 1, 1, 0, 0\} + \mathcal{K}\mathcal{K}$	$[2^3, 5^4]\{1, 3, 1, 4, 0, 0, 0\} + \mathcal{K}\mathcal{K}$	$[2^1, 5^4]\{1, 6, 0, 1, 1\} + 3\mathcal{K}\mathcal{K}$
$[2^2, 5^4]\{2, 2, 2, 3, 0, 0\} + 2\mathcal{K}\mathcal{K}$	$[2^2, 5^4]\{1, 4, 2, 1, 1, 0, 0\} + 2\mathcal{K}\mathcal{K}$	$[5^7]\{0, 0, 0, 7, 0, 0, 1\} + \mathcal{W}$
$[2^1, 5^4]\{0, 4, 2, 2, 0\} + 3\mathcal{K}\mathcal{K}$	$[2^2, 5^4]\{0, 2, 4, 2, 0, 0\} + 2\mathcal{K}\mathcal{K}$	
$[5^4]\{4, 0, 4, 1\} + 4\mathcal{K}\mathcal{K}$	$[2^1, 5^4]\{3, 1, 3, 2, 0\} + 3\mathcal{K}\mathcal{K}$	
$[5^7]\{0, 3, 0, 4, 0, 1, 1\} + \mathcal{W}$	$[5^4]\{0, 6, 0, 2\} + 4\mathcal{K}\mathcal{K}$	
	$[5^7]\{0, 0, 7, 0, 0, 0, 2\} + \mathcal{W}$	
	$[5^7]\{1, 0, 4, 0, 3, 0, 1\} + \mathcal{W}$	

solutions. The convention is that in each coordinate T-duality should be used to minimize the number of world-volume directions. Then for $N = 8$ only four numbers need to be specified to characterize a $D = 10$ class of (duality) equivalent solutions. In table 4 we have also indicated the unbroken supersymmetry which directly follows from the unbroken supersymmetry of the corresponding D-brane intersection.

Now consider using also the pair $(1|M2, \mathcal{W})$. The reduction to $D = 10$ will then necessarily include also NS/NS branes†. It turns out that there are three such maximum intersections. All other intersections follow by truncation of these. We find one intersection with $N = 8$ and two intersections with $N = 9$ independent harmonics:

$$\begin{aligned} N = 8: &\qquad [2^1, 5^6]\{1, 0, 4, 3, 0, 0, 1\} + \mathcal{W}, \\ N = 9: &\qquad [2^1, 5^7]\{1, 0, 0, 7, 0, 0, 0, 1\} + \mathcal{W}, \\ &\qquad [2^1, 5^4]\{1, 6, 0, 1, 1\} + 3\mathcal{K}\mathcal{K} + \mathcal{W}. \end{aligned} \tag{39}$$

All three solutions have $\frac{1}{32}$ unbroken supersymmetry. Interestingly enough we find intersections with *nine* independent harmonics. These intersections have one common time direction, nine relative transverse directions and one overall transverse direction. They therefore naturally reduce, upon identifying all harmonics, to a supersymmetric dilatonic 0-brane solution in two dimensions. Since this solution involves the newly constructed $N = 9$ intersection given above, it did not occur in our previous paper [15]. The specific dilaton coupling in two dimensions is the same for each of the two $N = 9$ intersections since it only depends on the number of independent harmonics (= field strengths in two dimensions) [27]. We find that the dilaton coupling is given by $a = -\frac{4}{9}$.

The two intersections with $N = 9$ are extensions of $N = 8$ intersections with $\frac{1}{16}$ supersymmetry in table 4. The remaining intersection with $\frac{1}{16}$ supersymmetry, $[2^3, 5^4]+\mathcal{K}\mathcal{K}$ can also be extended to $N = 9$, but this necessarily requires the use of a pair from table 2. For example, an additional 5-brane can be added, giving $\frac{1}{32}$ supersymmetry.

† Such intersections were indicated by grey colour in the tables of [15].

5. Conclusions

In this paper we have considered intersections of $M2$-branes, $M5$-branes, waves and monopoles. We first considered the pair intersections, which fall in two groups (tables 1 and 2) depending on the coordinates on which the intersecting branes depend. Using only the pairs of table 1, where the branes depend on overall transverse coordinates, we then considered purely gravitational solutions with only monopoles and waves. We found three types of such intersections (see table 3) consisting of multiple monopoles and in one case an additional wave. We next included the $M2$- and $M5$-branes and gave all intersections that can be reduced to 10-dimensional intersections involving only D-branes. This restriction is implemented by using only a limited number of the pair intersections of table 1. This was completed by adding additional waves. As a new result we found two new configurations with *nine* independent harmonic functions. Upon reduction they lead to a new supersymmetric 0-brane solution in two dimensions with dilaton coupling $a = -\frac{4}{9}$.

The pair intersections in section 2 show the interesting feature that in some cases the reduction to $D = 10$ gives rise to a solution which is not expressed in the standard way in terms of harmonic functions. In much of the previous work on pair intersections in $D = 10$ the possibility of such solutions, which interpolate between standard harmonic single-brane solutions, but cannot themselves be expressed in terms of two harmonic functions, was not considered (see, however, [24]). These solutions may provide a useful hint in a search for more general, non-harmonic, pair intersections. In particular, it may well be that the structure of completely localized brane intersections can be clarified in this way.

In this paper we did not consider intersections containing multiple monopoles of type B and C where the z-isometry direction is not the same for all monopoles. Such configurations are characterized by the fact that, upon reduction to ten dimensions, they always lead to an intersection involving at least one monopole. Although the result can be derived in a straightforward manner it turns out that the answer is involved. This is due to the fact that for these cases not all possible configurations follow by truncation from the intersections with the maximum number of harmonics.

We finally note that we did not consider 11-dimensional intersections involving 9-branes. In order to do that, one should first be able to construct such a 9-brane solution. We nevertheless found a hint in our calculations that the addition of such would-be 9-branes would be consistent with supersymmetry† in the following sense. Assuming that the unbroken supersymmetry of the 9-brane is determined by

$$\left(1 + \gamma_{01\cdots 9}\right)\epsilon = 0\,, \tag{40}$$

we found that such a projection operator naturally follows by taking products of similar projection operators corresponding to the other 11-dimensional solutions. This suggests that to specific combinations of $M2$-, $M5$-branes, waves and monopoles a 9-brane can be added without breaking supersymmetry [19]. It would be interesting to clarify the role of this would-be 11-dimensional 9-brane.

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† The use of supersymmetry in constructing multiple intersections is discussed in more detail in [28].

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