Intersecting branes in matrix theory

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Abstract

We construct BPS states in the matrix description of *M*-theory. Starting from a set of basic *M*-theory branes, we study pair intersections which preserve supersymmetry. The fractions of the maximal supersymmetry obtained in this way are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{16}$ and $\frac{1}{16}$. In explicit examples we establish that the matrix BPS states correspond to (intersecting) brane configurations that are obtained from the $d = 11$ supersymmetry algebra. This correspondence for the $1/2$ supersymmetric branes includes the precise relations between the charges. © 1998 Elsevier Science B.V. All rights reserved.

1. Introduction

Recently, the matrix model formulated in [1] for the microscopic description of M-theory [2,3] has drawn a considerable amount of attention. This model may be taken as a quantum mechanical framework for non-perturbative string theory (see [4] for a recent review). In this model the only degrees of freedom are the zerobranes. However, various authors have successfully demonstrated how the dynamics of strings, membranes and higher branes can arise in this model. The matrix description of a membrane can be found in [1], while the open membrane is described in [5]. A proposal for the description of a fourbrane (the wrapped fivebrane of *M*-theory) is provided in [6].

In [7] these higher dimensional objects are studied through the supersymmetry algebra of the matrix description of *M*-theory. Thus, the existence of conserved charges associated with the membrane and fivebrane is established. Interactions involving different branes have been studied in, e.g., [8–11].

In this note we provide additional evidence in support of the matrix model from an investigation of matrix configurations that preserve some fraction of the maximal supersymmetry, and correspond to intersecting branes. We start from a small number of basic objects with $1/2$ supersymmetry, which have nonzero 2-, 4-, 6- and 8-form charges. Besides these, some basic objects with less supersymmetry can be obtained. These configurations and their intersections should correspond to BPS solutions of the $d = 11$ supergravity theory. By an explicit analysis we establish this correspondence for the basic objects and their pair intersections.

The fractions of maximal supersymmetry which can be obtained in this way are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{16}$ and $\frac{1}{16}$. In

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the next section we discuss the basic objects. Pair intersections, starting with an explicit construction of the matrix configuration corresponding to two intersecting membranes, are discussed in Section 3. The correspondence of the matrix configurations to $d = 11$ supergravity is discussed in Section 4 through an analysis of the $d = 11$ supersymmetry algebra.

2. Basic solutions and residual supersymmetry

The supersymmetric quantum mechanical theory which corresponds to the matrix version of M-theory can be written, in a suitable parametrisation [12–14,1]

$$L = \frac{1}{2g} \text{tr} \left\{ (\partial_0 X^a)^2 + 2\theta_\alpha \partial_0 \theta_\alpha - \frac{1}{2}[X^a, X^b]^2 \right.$$
$$\left. - 2\theta_\alpha (\gamma_a)_{\alpha\beta} [\theta_\beta, X^a] \right\}. \qquad (1)$$

Here $a,b = 1, \ldots, 9$ correspond to the nine transverse directions in the matrix model, $\alpha, \beta = 1, \ldots, 16$ are nine-dimensional spinor indices [2], X and θ_α are hermitian $N \times N$-matrices. It is understood that the limit $N \to \infty$ has to be taken, although one can also give meaning to the finite N models [15].

The action (1) is invariant under the supersymmetry transformations

$$\delta X^a = -2\bar{\epsilon}\gamma^a\theta,$$
$$\delta \theta = \frac{1}{2} \left\{ \partial_0 X^a \gamma_a + \frac{i}{2}[X^a, X^b]\gamma_{ab} \right\} \epsilon + \tilde{\epsilon}, \qquad (2)$$

where ϵ and $\tilde{\epsilon}$ are independent supersymmetry parameters. The algebra of supersymmetry transformations is given in [12,7], and contains besides the usual translational term contributions of 2-form and 4-form charges [3]

$$Z_2^{a_1 a_2} = i \, \text{tr} \, X^{[a_1} X^{a_2]},$$
$$Z_4^{a_1 a_2 a_3 a_4} = R_{11} \text{tr} \, X^{[a_1} X^{a_2} X^{a_3} X^{a_4]}. \qquad (3)$$

We also define 6- and 8-form charges:

$$Z_6^{a_1 \cdots a_6} = iR_{11}^2 \, \text{tr} \, X^{[a_1} X^{a_2} \cdots X^{a_6]},$$
$$Z_8^{a_1 \cdots a_8} = R_{11}^3 \text{tr} \, X^{[a_1} X^{a_2} \cdots X^{a_8]}. \qquad (4)$$

Here R_{11} is the radius of the compact direction X^{11} in the matrix model. The momentum P_{11} in that direction is given by $P_{11} = N/R_{11}$. Nonzero charges can only occur in the limit $N \to \infty$.

To see how objects with non-vanishing charges Z_n can be constructed in matrix theory, we start with the single branes preserving $1/2$ of supersymmetry. These are [7]

· W, the wave in the a-direction: here we have $\partial_0 (X^a)_{ij} = p^a \delta_{ij}$.

· $M2$, the membrane: in this case the X^a are time-independent, and for a membrane in the 12-direction we require $[X^1, X^2]_{ij} = ic_1 \delta_{ij}$, where c_1 is a real parameter. To obtain a finite membrane charge Z_2, c_1 should scale as N^{-1} for $N \to \infty$.

· $M5$, the fivebrane which is wrapped around the longitudinal direction. For an $M5$ in the 1234-direction, we have $[X^1, X^2]_{ij} = ic_1 \delta_{ij}, [X^3, X^4]_{ij} = ic_2 \delta_{ij}$. The charge of $M5$ is built up out of membrane charges. This object can be thought of as infinite stacks of membranes in both the 12-, and the 34-direction. Finite Z_4 again requires that the c_i scale appropriately as $N \to \infty$.

· $M6$, the sixbrane: in this case $[X^1, X^2]_{ij} = ic_1 \delta_{ij}, [X^3, X^4]_{ij} = ic_2 \delta_{ij}, [X^5, X^6]_{ij} = ic_3 \delta_{ij}$. This is built up out of membranes in the 12-, 34-, and 56-directions, but there are also non-vanishing fivebrane charges. Presumably $M6$ is related to the Kaluza-Klein monopole in $d = 11$, although this correspondence has not been established.

· $M9$, the ninebrane, which is wrapped around the longitudinal direction. Here $[X^1, X^2]_{ij} = ic_1 \delta_{ij}$, $[X^3, X^4]_{ij} = ic_2 \delta_{ij}$, $[X^5, X^6]_{ij} = ic_3 \delta_{ij}$, $[X^7, X^8]_{ij} = ic_4 \delta_{ij}$. Again we have infinite stacks of membranes, as well as nonzero five- and sixbrane charges.

Since higher dimensional objects are built out of stacks of membranes, the charges Z_{2n} can, for any n, be related to membrane charges. We find, inde-

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[2] The γ-matrices satisfy $\{\gamma_a, \gamma_b\} = 2\delta_{ab}$. We use the notation $\gamma_{(n)} \equiv \gamma_{i_1 \ldots i_n} = \gamma_{i_1} \gamma_{i_2} \cdots \gamma_{i_n}$. Note that $\gamma_{(n)}^2 = 1$ for $n = 1,4,5,8,9$.

[3] We normalize the charges Z_{2n} with factors of R_{11}^{n-1}. For Z_2 and Z_4 this is in agreement with [7]. The factors in Z_6 and Z_8 ensure that all charges have the same dimension. They are in agreement with the analysis of the $d = 11$ supersymmetry algebra in Section 4.

pendently of the choice of the scaling of $[X^a, X^b]$, the behaviour

$$Z_{2n} = P_{11}^{1-n} \prod_{i=1}^{n} Z_{2,i}, \quad n = 1, \dots, 4. \qquad (5)$$

The result (5) is in agreement with the results of Section 4 when considering $1/2$ supersymmetric non-threshold states.

These solutions to the matrix model equations of motion have $\theta = 0$ and preserve $1/2$ supersymmetry. The vanishing of $\delta\theta$ (for static solutions, the preservation of supersymmetry for W is shown in a similar way) implies that

$$\delta_{ij}\tilde{\epsilon} = -\frac{i}{4}[X^a, X^b]_{ij}\gamma_{ab}\epsilon, \qquad (6)$$

where the indices $i, j = 1, \dots, N$ have been made explicit. The relation (6) can only be satisfied if

$$[X^a, X^b]_{ij} = iF^{ab}\delta_{ij}. \qquad (7)$$

A representation of (7) can be given in terms of a pair of operators p and q satisfying canonical commutation relations $[q, p] = i$. As long as the commutator of the matrices X is proportional to the unit matrix $\tilde{\epsilon}$ is determined in terms of ϵ, so that $1/2$ of supersymmetry is preserved.

Other basic solutions have less supersymmetry. We will consider the following ones:

· $P5$, the pure fivebrane [7]. This has the following structure: $[X^1, X^2]_{ij} = [X^3, X^4]_{ij} = ic_1(\mathbb{1} \otimes \sigma_3)_{ij}$. We call it the pure fivebrane since the membrane charges vanish. Here $1/4$ supersymmetry is preserved (see below).

· $P9$, the ''pure'' ninebrane. Here we have $[X^1, X^2]_{ij} = ic_1(\mathbb{1} \otimes \sigma_3)_{ij}$, $[X^3, X^4]_{ij} = ic_2 \mathbb{1} \otimes \sigma_3)_{ij}$, $[X^5, X^6]_{ij} = ic_3(\mathbb{1} \otimes \sigma_3)_{ij}$, $[X^7, X^8]_{ij} = ic_4(\mathbb{1} \otimes \sigma_3)_{ij}$. This object is not entirely pure, since the constituent $P5$-charges do not vanish. However, there is no $M2$ or $M6$ charge. Depending on the values of the coefficients, $2n$, $n = 1,2,3$ of the 32 supersymmetry charges are unbroken (see below).

Note that we cannot define a ''pure'' $M6$ ($P6$) in the same way, since then the charge of rank six vanishes. By using a more complicated tensor structure for the matrices we can form a $P6$ and $P9$, but these configurations do not preserve supersymmetry.

Let us now discuss the residual supersymmetry of the two solutions $P5$ and $P9$. We first consider $P9$. There are two equations that must have a solution to preserve some supersymmetry ($c_i \neq 0$), which imply $\tilde{\epsilon} = 0$ and

$$(c_1\gamma_{12} + c_2\gamma_{34} + c_3\gamma_{56} + c_4\gamma_{78})\epsilon = 0. \qquad (8)$$

We rewrite this as $(1 - P)\epsilon = 0$, with

$$P = (c_2\gamma_{1234} + c_3\gamma_{1256} + c_4\gamma_{1278})/c_1. \qquad (9)$$

The γ-matrices in P all square to one, and commute with each other. Also their trace, and the trace of their products, vanishes. These conditions determine the eigenvalues of P. Depending on the values of the coefficients, $2n$, $n = 1,2,3$ of the eigenvalues of P can be equal to 1. We find $n = 1$, or preservation of $1/16$ of the maximal supersymmetry, if, e.g., $c_1 = \pm(c_2 + c_3 + c_4)$. For $n = 2$, or $1/8$, we need more stringent conditions: $c_1 = c_2, c_3 = c_4$. In that case the fivebrane charges in the directions 1234 and 5678 are still arbitrary (proportional to c_1^2 and c_3^2, respectively), but the other fivebrane charges (1256, 1278, 3456, 3478) are equal and proportional to $2c_1c_3$. The amount of preserved supersymmetry can be further increased by setting all coefficients equal: $c_1 = c_2 = c_3 = c_4$. This corresponds to equal fivebrane charges in all six directions, and $3/16$ of the maximal supersymmetry.

If one of the coefficients, say c_4, in (8) vanishes, and we choose $c_1 = \pm(c_2 + c_3)$, then $1/8$ supersymmetry is preserved. This can be interpreted as an intersection of oppositely charged sixbranes, a configuration which also has nonzero fivebrane charges. If two coefficients vanish, the remaining two must be equal to preserve $1/4$ supersymmetry. This last case corresponds to $P5$.

This supersymmetry analysis is very similar to that occurring in the analysis of branes which intersect at angles [16–18].

3. Pair intersections

The fact that for $P5$ and $P9$ the commutators of the X^a are not proportional to the unit matrix is the cause of the additional supersymmetry breaking. For intersecting pairs we split the matrices in two blocks, each representing a brane, of size N_1 and N_2, with $N_1 + N_2 = N$. We will limit ourselves in this paper to

Table 1

Supersymmetric pair intersections involving W. The notation $(p|A,B)$ indicates that the objects A and B have p common spacelike worldvolume directions. The second column gives the amount of residual supersymmetry that can be obtained

Matrix configuration	SUSY
$(1|W,M2)$	$\frac{1}{4}$
$(1|W,M5)$	$\frac{1}{4}$
$(1|W,M6)$	$\frac{1}{8}$
$(1|W,M9)$	$\frac{1}{8}, \frac{1}{16}, \frac{3}{16}$

pair intersections, starting with those involving the wave W.

It is easy to see that only in the case of the branes $M2$, $M5$, $M6$ and $M9$ a supersymmetric intersection with a wave can be constructed [4]. In these cases the direction of the wave necessarily must be in the worldvolume of the brane [5]. The same analysis we did in Section 2 reveals that the possible fractions are 1/4, 1/8, 3/16 and 1/16. This case is summarized in Table 1.

Let us now look at the pair intersections of $M2$, $M5$, $M6$ and $M9$. In the general case, the condition (8) will be of the form

$$F^{ab}\gamma_{ab}\epsilon = 0, \quad F^{ab} \equiv F_1^{ab} - F_2^{ab}, \quad (10)$$

where F_i, $i=1,2$ come from the commutators $[X^a, X^b]$ for the two branes. Using nine-dimensional rotations a generic antisymmetric matrix can be put into a canonical form, in which only F^{12}, F^{34}, F^{56} and F^{78} are nonzero. Thus the analysis reduces to that of (8). Therefore the only fractions of maximal supersymmetry [6] in pair intersections are $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{3}{16}$.

We will limit ourselves to those cases for which the only nonzero commutators used in constructing the branes are $[X^{2n-1}, X^{2n}]$ for $n = 1, \ldots, 4$. For such configurations the pair intersections are summarised in Table 2. As an illustration we will work out one particular case, the intersection of a membrane $M2$ with the $M6$-brane, in detail.

Splitting up the matrices appropriately and using (2) we get the following equations for the supersymmetry parameters

$M6$: $\quad \tilde{\epsilon} = (c_1\gamma_{12} + c_2\gamma_{34} + c_3\gamma_{56})\epsilon$,

$M2$: $\quad \tilde{\epsilon} = c_4\gamma_{12}\epsilon$. $\quad (11)$

The first equation breaks half of the supersymmetry and for the second equation to be consistent with the first we find that

$$((c_1 - c_4)\gamma_{12} + c_2\gamma_{34} + c_3\gamma_{56})\epsilon = 0. \quad (12)$$

When $c_1 = c_4$ we must have $c_2 = \pm c_3$, and 1/4 supersymmetry is preserved. If $c_1 \neq c_4$, we must require $c_2 + c_3 = c_1 - c_4$ (up to choices of signs) to preserve $1/8$ of the maximal supersymmetry.

We can also have $(0|M2,M6)$, with the membrane directions outside the $M6$. This leads to Eq. (8), and can preserve $n/16$, $n = 1,2,3$ of the maximal supersymmetry.

When one brane in the pair is a 'pure' brane ($P5$ or $P9$) the analysis changes. Because a 'pure' brane makes $\tilde{\epsilon} = 0$, for every brane in the pair we get an equation $R\epsilon = 0$ where R is the sum of one or more matrices $\gamma_{(2)}$. So we have to look for zero eigenval-

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[4] For $P5$ and $P9$ one finds the requirement $\gamma_1\epsilon = 0$ for a wave in the 1-direction (since $\tilde{\epsilon}$ vanishes). However, γ_1 has no zero eigenvalues.

[5] Consider a membrane in the 12-direction. If the wave is not in the worldvolume of the brane, the condition on ϵ is of the form $(c_1\gamma_{12} - p\gamma_9)\epsilon = 0$ for a wave in the 9-direction. The γ matrices can be simultaneously diagonalised. Since γ_9 has real, and γ_{12} imaginary eigenvalues, their linear combination cannot have eigenvalue zero. For branes of higher dimension the same argument holds.

[6] Note that in [17] also fractions $\frac{1}{32}$, $\frac{3}{32}$ and $\frac{5}{32}$ are obtained for pair intersections at angles, but these require ten spatial dimensions. For orthogonal intersection of branes also $\frac{1}{32}$ can occur [19], but this requires at least five branes.

Table 2

Pair intersections of $M2$, $M5$, $M6$, $M9$. Only branes are considered which are built up out of membranes in the 12, 34, 56 and 78 directions

Configuration	SUSY	Configuration	SUSY
$(0|M2,M2)$	$\frac{1}{4}$	$(4|M5,M5)$	$\frac{1}{2}, \frac{1}{4}$
$(2|M2,M2)$	$\frac{1}{2}$	$(2|M5,M6)$	$\frac{1}{8}, \frac{1}{16}, \frac{3}{16}$
$(0|M2,M5)$	$\frac{1}{8}$	$(4|M5,M6)$	$\frac{1}{4}, \frac{1}{8}$
$(2|M2,M5)$	$\frac{1}{4}$	$(4|M5,M9)$	$\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{3}{16}$
$(0|M2,M6)$	$\frac{1}{8}, \frac{1}{16}, \frac{3}{16}$	$(4|M6,M6)$	$\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{3}{16}$
$(2|M2,M6)$	$\frac{1}{4}, \frac{1}{8}$	$(6|M6,M6)$	$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$
$(2|M2,M9)$	$\frac{1}{8}, \frac{1}{16}, \frac{3}{16}$	$(6|M6,M9)$	$\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{3}{16}$
$(0|M5,M5)$	$\frac{1}{8}, \frac{1}{16}, \frac{3}{16}$	$(8|M9,M9)$	$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{3}{16}$
$(2|M5,M5)$	$\frac{1}{4}, \frac{1}{8}$		

ues of the matrix R. This means that we cannot add an $M2$ or W to a $P5$ or $P9$, because in those cases R has no zero eigenvalues. The preserved supersymmetry depends on the relative orientation and on the number of γ-matrices in each R. The fractions of supersymmetry that can be obtained are the same as in the cases considered previously.

4. Relation with eleven-dimensional supergravity

The supersymmetry algebra in $d = 11$, including all allowed central charges, takes on the following form [20,21]:

$$\{Q_\alpha, Q_\beta\} = (C\Gamma^m)_{\alpha\beta} P_m + \tfrac{1}{2}(C\Gamma^{mn})_{\alpha\beta} Z_{mn}$$
$$+ \tfrac{1}{5!}(C\Gamma^{m_1\ldots m_5})_{\alpha\beta} Z_{m_1\ldots m_5}, \qquad (13)$$

where $\alpha, \beta = 1, \ldots, 32$. The charges Z_{mn} and $Z_{m_1\ldots m_5}$ correspond, in the case of spacelike indices, to the membrane charge and fivebrane charge. If one of the indices is timelike, Z_{0m} and $Z_{0m_1\ldots m_4}$ correspond to the dual of a ninebrane and a sixbrane charge, respectively [22]. We choose $C = \Gamma^0$ and write

$$\{Q, Q\} = P^0(\mathbb{1} + \overline{\Gamma}). \qquad (14)$$

In matrix theory in the infinite momentum frame there is always a wave present, which, in this section, we place in the direction 9. The basic $M5$ configuration corresponds to nonzero $P_9 = p$, $Z_{12} = z_1$ and $Z_{34} = z_2$ (because $M5$ has nonzero membrane charges) and $Z_{12349} = y$, with of course a component in the direction of the boost. We find

$$(\overline{\Gamma})^2 = (P^0)^{-2}$$
$$\times (p^2 + z_1^2 + z_2^2 + y^2 + 2(py - z_1 z_2)\Gamma^{1234}). \qquad (15)$$

If the charges are such that $py = z_1 z_2$ then we can choose P^0 to set $(\overline{\Gamma})^2 = \mathbb{1}$, which implies that $1/2$ of the maximal supersymmetry is preserved. This relation between the momentum and the charges is what we expect from the matrix theory ((5) for $n = 2$).

The pure fivebrane, $P5$, has no membrane charges, and therefore

$$(\overline{\Gamma})^2 = (P^0)^{-2}(p^2 + y^2 + 2py\Gamma^{1234}). \qquad (16)$$

Now we cannot set $(\overline{\Gamma})^2 = \mathbb{1}$, but we can set 16 of the eigenvalues of $(\overline{\Gamma})^2$ equal to one by choosing P^0 appropriately. This means that $\overline{\Gamma}$ has 8 eigenvalues equal to -1, and $1/4$ supersymmetry is unbroken. This $d = 11$ configuration corresponds to a fivebrane and a wave.

In this way the matrix configurations of Section 2 can be identified with supergravity solutions. $M5$ corresponds to a bound state of two membranes and a fivebrane, boosted in the 9 direction (see also the discussion in [23]). With $1/2$ supersymmetry this is a non-threshold solution, which is not yet known as a solution of the $d = 11$ supergravity equations. The result (16) for $P5$ corresponds to a threshold solution, and is the known intersection of a fivebrane and a wave [24].

To find corresponding BPS states for the $1/2$ supersymmetric matrix $M6$ and $M9$ the same analysis can be done as for the $M5$. The result is that these (non-threshold) states do exist in the supersymmetry algebra, but we have to impose constraints on the charges. These constraints however are exactly the relations between the different charges (5) in matrix theory.

As is clear from Table 2, there are configurations preserving $3/16$ of the maximal supersymmetry. The case of $(0|M5, M5)$ [7] was studied in detail in [25]. These authors show that this configuration in $d = 10$ is T-dual to two $D4$ branes at angles.

As an example of a state preserving $3/16$ of the supersymmetry we analyse $P9$. There is one ninebrane charge, mixed with 6 fivebrane charges and momentum in the 9th direction. The ninebrane charge corresponds to [8] $Z_{0\natural} = m$. Including all charges we obtain

$$P^0\overline{\Gamma} = \Gamma^{09}p + \Gamma^{012349}y_1 + \Gamma^{012569}y_2 + \Gamma^{012789}y_3$$
$$+ \Gamma^{034569}y_4 + \Gamma^{034789}y_5 + \Gamma^{056789}y_6 + \Gamma^{\natural}m, \qquad (17)$$

In $(P^0\overline{\Gamma})^2$ there are three independent commuting Γ-matrices so that in the generic case this configura-

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[7] This configuration corresponds to two fivebranes intersecting over a string. In matrix theory the common direction corresponds to the longitudinal direction and that is why we write $(0|M5, M5)$.
[8] $\natural$ indicates the direction 10. Note that $\Gamma^{\natural} = \Gamma^{0123456789}$.

tion will preserve $1/16$ of the supersymmetry. This corresponds to a threshold bound state of six five-branes, a ninebrane and a wave. We can also obtain configurations which preserve $1/8$ and $3/16$, by restricting the coefficients. If we set $y_2 = y_3 = y_4 = y_5 = y$, leaving y_1 and y_6 arbitrary, we find that $(P^0 \bar{\Gamma})^2$ has the following eigenvalues: $(p - m \pm (y_1 - y_6))^2$ with multiplicity 8 for each choice of sign, $(p + m + y_1 + y_6)^2$ with multiplicity 8, and $(p + m - y_1 - y_6 \pm 4y)^2$ with multiplicity 4 for each sign. Therefore, by choosing P^0 appropriately, we preserve $1/8$ supersymmetry, for each of the eigenvalues of multiplicity 8. If we also set $y_1 = y_6 = y$, the eigenvalues simplify further. There are then 12 eigenvalues equal to $(p + m + 2y)^2$, leading to $3/16$ of the maximal supersymmetry. For six equal charges we find that for $(P^0)^2 = (p - m)^2$ then $1/4$ of the supersymmetry charges are preserved. Thus the $d = 11$ supersymmetry algebra seems to support a boosted longitudinal ninebrane with $1/4$ supersymmetry. In Section 2 we showed that such an object is absent in the matrix model.

We believe that the supersymmetric configurations in matrix theory that we constructed in Section 2 all correspond to supersymmetric states in the $d = 11$ supersymmetry algebra (13). We have verified this in a number of cases, and always found agreement. Presumably, solutions of the $d = 11$ supergravity equations of motion for such states can be constructed. A lot of work has been done on non-threshold states involving membranes and fivebranes (see for instance [26,27]). In the case of the sixbrane or Kaluza-Klein monopole much less is known, while of course the status of the ninebrane as a solution in $d = 11$ supergravity is uncertain.

However, not all supersymmetric configurations constructed in the $d = 11$ supersymmetry algebra can be obtained from the matrix model. For instance, in the $d = 11$ algebra the sixbrane together with a transverse wave gives a state with 16 preserved supersymmetry charges [9]. This we do not find in the matrix model. In the analysis of $P9$ given above the result in the $d = 11$ algebra suggests a pure nine-

brane with $1/4$ supersymmetry in the matrix model. This is also absent in Section 2.

So, concerning the basic branes there seems to be a problem involving the absence of pure sixbranes and ninebranes. In the matrix model they can be constructed but break all of the supersymmetries, while the $d = 11$ supersymmetry algebra seems to support supersymmetric configurations of this type.

5. Conclusion

Although the missing transversal fivebrane, as well as the problems involving six- and ninebranes, indicate that something is still poorly understood in the matrix model, many BPS states and their pair intersections seem to be in agreement with what we expect if the matrix model is to describe M theory. We believe that any matrix BPS state has an analogue as a threshold or non-threshold intersecting brane configuration in the $d = 11$ supersymmetry algebra.

In this paper we establish a (partial) correspondence between supersymmetric branes in the matrix model and in the $d = 11$ supersymmetry algebra. Especially, the $1/2$ supersymmetric basic $M6$ and $M9$ in matrix theory correspond exactly to $1/2$ supersymmetric non-threshold states in the supersymmetry algebra carrying the same charges. Interesting open questions concerning the existence of non-threshold solutions to the $d = 11$ supergravity equations of motion corresponding to matrix model states remain. The relations presented here between matrix theory and the supergravity limit can be considered additional evidence for matrix theory. The fact that the correspondence is not complete implies that further work needs to be done, and hopefully this will lead to a better understanding of matrix theory.

Acknowledgements

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[9] In this case we have $\bar{\Gamma} = (\Gamma^{09} p + \Gamma^{0123456} m)/P^0$, which corresponds to $\bar{\Gamma}^2 = (p^2 + m^2)/(P^0)^2$.

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