Telluric and Ocean Current Effects on Buried Pipelines and Their Cathodic Protection Systems

Contract PR-262-0030

Prepared for the
Pipeline Corrosion Supervisory Committee
Pipeline Research Committee
of
Pipeline Research Council International, Inc.

Prepared by the following Research Agencies:
CORRENG Consulting Service Inc.
Geological Survey of Canada

Authors:
R.A. Gummow – Correng
D.H. Boteler, Ph.D. and L. Trichtchenko, Ph.D. – GSC

Publication Date:
December 2002
“This report was furnished to the Pipeline Research Council International, Inc. (PRCI) under the terms of PRCI Project PR-262-0030, between PRCI and CORRENG Consulting Service Inc. The contents of this report are published as received from CORRENG Consulting Service Inc. The opinions, findings, and conclusions expressed in the report are those of the author and not necessarily those of PRCI, its member companies, or their representatives. Publication and dissemination of this report by PRCI should not be considered an endorsement by PRCI or CORRENG Consulting Service Inc., or the accuracy or validity of any opinions, findings, or conclusions expressed herein.

In publishing this report, PRCI makes no warranty or representation, expressed or implied, with respect to the accuracy, completeness, usefulness, or fitness for purpose of the information contained herein, or that the use of any information, method, process, or apparatus disclosed in this report may not infringe on privately owned rights. PRCI assumes no liability with respect to the use of, or for damages resulting from the use of, any information, method, process or apparatus disclosed in this report.

The text of this publication, or any part thereof, may not be reproduced or transmitted in any form by any means, electronic or mechanical, including photocopying, recording, storage in an information retrieval system, or otherwise, without the prior written approval of PRCI.”
ACKNOWLEDGEMENT

The advice provided by Jules Chorney of TransGas on the structure and content of this report is gratefully acknowledged.
Ad-Hoc Group Committee

J. Chorney, TransGas
S. Olsen, Statoil
R. Reid, Union Gas Limited
K. Keith, Foothills Pipe Lines Ltd.
R. Worthingham, TransCanada PipeLines Limited
Corrosion Supervisory Committee

D.A. DesNoyer, Consumers Energy Co.
T.A. Widin, BP Amoco Pipeline
D.A. Bacon, Enron Corp.
R.M. Bass, Equilon Enterprises LLC (Shell)
M.A. Brockman, El Paso Energy – Tennessee Gas Pipeline
J.A. Card, Great Lakes Transmission Company
D.R. Catte, Saudi Aramco
J.E. Chorney, TransGas Ltd.
B.A. Cookingham, ANR Pipeline Company
J.K. Keith, Foothills Pipe Lines Ltd.
K. Krist, Gas Research Institute
V.B. Lawson, Westcoast Energy Inc.
K. Leewis, Gas Research Institute
M.E. Linville, CNG Corporate
H. Mitschke, Shell E & P Technology
D.P. Moore, ARCO Technology and Operations
P.R. Nichols, Equilon Enterprises, LLC
S. Olsen, Statoil
R.N. Parkins, University of Newcastle
L. Perry, Southern California Gas Company
W.P. Pickard, Southern Natural Gas Company
M. Pitkanen, Gasum Oy
M.D. Platzke, ANR Pipeline Company
J.F. Rau, CMS Panhandle Pipe Line Companies
R.G. Reid, Union Gas Limited
S.B. Rigling, Williams Gas Pipeline – Texas Gas
J.T. Schmidt, Duke Energy
H.U. Schutt, Consultant
W.J. Sisak, Exxon Production Research Company
W. Sloterdijk, N.V. Nederlandse Gasunie
A. Teitsma, Gas Research Institute
H.H. Wang, Columbia Gulf Transmission Co.
R.G. Worthingham, TransCanada Transmission
T. Yamagishi, Osaka Gas Company, Ltd.
B. Dutton, Pipeline Research Council International, Inc.
This Page is Intentionally Blank
Executive Summary

Pipelines are subjected to telluric current activity due to the modulation of the earth’s magnetic field by solar particles. This changing magnetic field produces an electric field that causes charges to flow in the earth and in metallic networks located on the earth such as pipelines, electric powerlines, and communication cables. This electrical disturbance is observed on pipelines as potential and current fluctuations that can vary with time due to the earth’s rotation, tidal cycles, the sun’s rotation, eleven year solar cycles, and solar storms. The magnitude and location of these disturbances depend on the pipeline’s proximity to the earth’s magnetic poles, on its length, on its orientation, on changes in direction, on the coating resistance, on electrical continuity along its length, on soil resistivity and the presence of abrupt changes in earth conductivity, and proximity to a sea coast.

Historically the effects of telluric currents on pipelines have been considered a curiosity and an inconvenience when conducting cathodic protection surveys for compliance with the pipeline codes and regulations. Recently however, as more pipelines have been constructed at higher latitudes and in higher resistivity soils, and as higher quality coatings have been used, the resulting telluric potential and current variations, being more severe, have prompted concerns about the following issues;

- whether or not the pipe is corroding during periods of telluric current discharges, and
- will the coating be stressed and possibly disbonded during periods of pick-up, and
- how can the effects of telluric current activity be mitigated, and
- what techniques are available to measure accurate pipe-to-soil potentials during periods of telluric activity

This report describes two methods for modeling telluric currents in pipelines wherein the distributed source transmission line (DSTL) model has been used successfully to predict the magnitude of telluric currents and the associated potential variations throughout pipeline networks. The model addresses typical pipeline situations including pipe bends, pipe junctions, branch lines, insulating flanges, grounding points, changes in pipe dimensions, and changes of coating conductance.

Although an AGA study in the late 1960’s concluded that the corrosive effects of telluric currents were “insignificant” on protected pipelines, more recent field reports suggest a small but measurable corrosion rate. Corrosion rates on cathodically protected pipe
attributed to telluric currents appear to be low (<0.1 mm/yr) based on a few investigations, and there have been no reported corrosion failures. Nevertheless, over a long time period these corrosion rates could produce a significant corrosion problem. Measures to mitigate and control telluric currents to minimize the possibility of small residual corrosion rates have been undertaken by a number of pipeline operators. Providing a path to ground, as is done to reduce AC induced voltages, is also effective in decreasing the magnitude of telluric voltage fluctuations. Cathodic protection systems, which can compensate for the telluric current discharge also, to some extent provide a path to earth for the telluric current depending on the type of system. Use of galvanic cathodic protection systems can be very effective on well-coated pipe, because they significantly increase the leakage conductance and they may also enhance the cathodic protection of the pipeline by creating a net pick-up of telluric current on the pipe, due to their offset in potential with respect to the pipe.

Potential controlled impressed current systems have also been used successfully to control telluric currents, whereas transformer-rectifiers operating in constant current mode, may actually hinder mitigation of telluric current effects. Using constant voltage or constant current transformer-rectifiers at increased output to counterbalance positive telluric voltage fluctuations, besides being inefficient, can result in over-protection and increased risk of coating damage.

A number of methods have been used to improve the accuracy of pipe-to-soil potential measurements in the presence of telluric voltage fluctuations. Coupons installed at test stations provide an economical and effective means of measuring a polarized potential, and coupons that do not have to be disconnected to measure a polarized potential have a decided advantage, since monitoring usually requires recording potentials with time.

Correcting for the telluric activity while conducting close interval potential surveys is somewhat more complex. Most of the techniques that have been developed require multiple recorders to collect data from stationary electrodes, prior to or during the survey, so that a correction factor can be derived and applied to the potential measured by the moving electrode. This operation requires accurate time stamping, usually by reference to the global positioning system. No single technique corrects for all the possible voltage drops in the measurement circuit.

Seven recommendations for further research attention have been made.
# TABLE OF CONTENTS

**Executive Summary**

1.0 **Introduction** ............................................................... 1

2.0 **Modeling** ........................................................................ 7
   2.1 Geomagnetic Induction........................................................... 8
   2.2 Tidal Induction and Coast Effect........................................... 12
       2.2.1 Coast Effect................................................................. 14
       2.2.2 Coastal Modeling......................................................... 14
   2.3 Geomagnetic Interaction with a Pipeline............................... 19
   2.4 DSTL Modeling................................................................... 24
       2.4.1 Electrically-Long Pipeline........................................... 25
       2.4.2 Electrically-Short Pipeline......................................... 27
       2.4.3 Pipeline Bend ............................................................ 28
       2.4.4 Modeling Pipeline Networks...................................... 29

3.0 **Impact of Telluric Currents on Corrosion Control Systems** ............... 33
   3.1 General............................................................................... 33
   3.2 Corrosion Impact of Telluric Current on Cathodically Protected Pipelines .... 34
       3.2.1 Background................................................................. 34
       3.2.2 Reported Instances of Corrosion Caused by Telluric Currents .... 35
       3.2.3 Theoretical Corrosion Considerations............................ 37
   3.3 Effect of Telluric Voltage Fluctuations on Coatings....................... 46
       3.3.1 General....................................................................... 46
   3.4 Cathodic Protection Performance Monitoring in the Presence of
       Telluric Activity..................................................................... 47
       3.4.1 Pipe-to-Soil Potential Measurement at a Test Station........... 47
       3.4.2 Close Interval Potential Surveys..................................... 50

4.0 **Mitigating Telluric Current Effects** ........................................ 55
   4.1 By Grounding....................................................................... 55
   4.2 Using Cathodic Protection..................................................... 56
       4.2.1 Sacrificial Systems......................................................... 56
       4.2.2 Impressed Current Systems.......................................... 58

5.0 **Summary** .......................................................................... 61

6.0 **Recommendations for Further Research on Telluric Current Effects on Pipelines** ......... 63

**References** ............................................................................. 65
List of Figures

Section 1

Figure 1-1 Recordings of Electric Fields at College, Alaska and Telluric Currents at Chena on the Alaska Pipeline ................................................................. 1

Figure 1-2 History of Geomagnetic Effects on Ground Technology .................. 2

Figure 1-3 Diagram of the Ionospheric Currents on the Day-Side of the Earth that Create the “Quiet Day” Variation of the Magnetic Field and Corresponding Variations in Pipe-to-Soil Potentials ........................................ 3

Figure 1-4 Schematic of the Electric Currents Associated with the Aurora that are Responsible for Magnetic Substorms and Associated Changes in Pipe-to-Soil Potential ................................................................. 4

Figure 1-5 Oscillations of the Earth’s Magnetic Field Lines that Cause Changes In Pipe-to-Soil Potential ................................................................. 5

Section 2

Figure 2-1 Schematic Showing the Ionospheric Currents, Tidal Water Movements and Telluric Currents in Pipelines ................................................................. 7

Figure 2-2 Magnetic Fields Recorded at the Ottawa Magnetic Observatory and Electric Fields Calculated Using a Layered Earth Model .......................... 8

Figure 2-3a Magnetic Field Spectrum .......................................................................... 9

Figure 2-3b Earth Surface Impedance .......................................................................... 9

Figure 2-3c Electric Field Spectrum .......................................................................... 9

Figure 2-4 Horizontal (Bx) and Vertical (Bz) Magnetic Fields and Horizontal (Ey) Electric Field with the Cauchy Distribution Produced by a Line Current of 1 Million Amps 100 km Above the Earth’s Surface. The calculations are made for a period of 5 min. and an earth model for the Canadian Shield. 11

Figure 2-5 Electric Field, E, Generated by Seawater Moving with Velocity, V, Through the Earth’s Magnetic Field, B ................................................................. 12

Figure 2-6 Potentials Produced by Tidal Flow in the Bay of Fundy ....................... 13

Figure 2-7 Current continuity for currents flowing across the coast is produced by charge accumulation that modifies the electric fields near the coast... 14

Figure 2-8 Generalized Thin Sheet Model of the Land – Ocean Boundary ............ 16

Figure 2-9 Effect of a Coast on the Ground Potential and Electric Field ............ 17
Figure 2-10 Potential and Electric Field Produced at a Geological Boundary .................. 17

Figure 2-11 Electric Field Generated by Tidal Water Movement and the Resultant Potential in the Land and the Sea ................................................................. 18

Figure 2-12 Contour Lines of the Earth Surface Potential (Volts) Produced by an Induced Electric Field of 1 V/km Parallel to the Atlantic Coast of Nova Scotia Determined Using Analogue Modeling. Also shown is the route of the Maritimes and Northeast Pipeline ......................................................... 19

Figure 2-13 Diagram of the Incident Plane Waves of the Geomagnetic Field Producing Reflected Cylindrical Waves by Their Interaction with a Pipeline ........................................................................ 20

Figure 2-14 Distortion of Electric Field in the Earth Produced by a Pipeline ................. 21
   a. Contours of electric field strength
   b. Vertical sections showing the decrease of the Electric Field that occurs away from the pipe (BB') and the additional attenuation produced by the pipe (AA')
   c. Horizontal section showing the attenuation of the field near the pipe

Figure 2-15 Close-up View of the Electric Fields in and Around a Pipeline....................... 22
   a. Contours of electric field strength around the pipe
   b. Vertical sections through (AA') and to the Side (BB') of the Pipe
   c. Close-up View of the Electric Field Inside the Pipe Wall and the Coating

Figure 2-16 Frequency Dependence for Different Resistivities (in S/m)............................ 23
   a. Amplitude
   b. Phase

Figure 2-17 Transmission Line Model of Pipeline Including Distributed Voltage Sources Representing the Induced Electric Field ..................................................... 24

Figure 2-18 DSTL Model Results (V, dV/dx, and I) for a Long Pipeline .......................... 26

Figure 2-19 Linear Variation in Potential Produced by Telluric Electric Fields in a Short Pipeline .................................................................................................................. 27

Figure 2-20 Bend in a Pipeline
   a) Pipe-to-soil Potentials Produced Around a Bend, at x = 0, with \( \alpha - \beta = 120^\circ \) .... 28

Figure 2-21 Route of the Maritimes and Northeast Pipeline ............................................ 29

Figure 2-22 Maritimes DSTL Results with Insulating Flanges ....................................... 30

Figure 2-23 Maritimes DSTL Results without Flanges .................................................... 31

Figure 2-24 Maritimes DSTL Results with 0.1 ohm Terminations .................................... 32
### Section 3

| Figure 3-1 | Calculated Telluric Induced Voltage at the End of the Maritimes & Northeast Pipeline as a Function of Coating Conductance for an East-West Electric Field of 0.1 V/km | 34 |
| Figure 3-2 | Current Flow & Calculated OFF Potentials during a GIC Incident | 37 |
| Figure 3-3 | Oxidation Reaction at Telluric Pipe Surface During Telluric Current Discharge in the Absence of Cathodic Protection | 37 |
| Figure 3-4 | Reduction Reactions During Negative Cycle Telluric and Cathodic Protection Current Pick-Up | 38 |
| Figure 3-5 | Steel Surface pH versus Applied C.P. Current Density | 38 |
| Figure 3-6 | Polarization Curves after Several Days of Potentiostatic Polarization | 39 |
| Figure 3-7 | Telluric Current Discharge from a Cathodically Protected Pipe | 39 |
| Figure 3-8 | Experimental Anodic Polarization Curve of Steel in Hydroxide (pH 12.0) | 40 |
| Figure 3-9 | Coefficient of Corrosion at Different Frequencies for Iron Electrodes | 40 |
| Figure 3-10 | Effect on Corrosion Rate of Reversing Direction of Current Compared to Steady State Direct Current and Length of Time between Reversals | 41 |
| Figure 3-11 | Corrosion Current Density at a Coating Defect having an Applied Voltage of 1.0V in 1000 ohm-cm Soil for Various Coating Thicknesses | 42 |
| Figure 3-12 | Peak Electric Field Magnitudes as a Function of Kp | 43 |
| Figure 3-13 | Average Occurrence of 3-Hour Intervals with Kp ≥ A Specified Value | 44 |
| Figure 3-14 | Corrosion Rate for Unprotected Steel vs. Telluric Potential Change at a 1cm Holiday in 1000 ohm-cm Soil for Various Telluric Intensities (Kp Indexes) Having a Period of 1 Hour | 45 |
| Figure 3-15 | Typical Pipe-to-Soil Potential Measurements at a Test Station | 47 |
| Figure 3-16 | Potential vs. Time Recorded on the Maritimes & Northeast Pipeline near the Canada/United States Border | 48 |
| Figure 3-17 | Typical Pipe-to-Soil Potential Measurement at Test Station Having a Steel Coupon and Soil Tube | 49 |
| Figure 3-18 | Typical Pipe-to-Soil Potential Recording at a Test Station Using a Coupon/Reference Probe | 49 |
Figure 3-19  Comparison between Pipe/Coupon Potential with Time between a Copper-Copper Sulphate Reference on Grade and a Zinc Reference Electrode Located inside the Coupon ......................................................... 50

Figure 3-20  Pipe-to-Soil Potential Measurement Method to Compensate for Telluric Current Effects During a Close Interval CP Survey .......................................................... 51

Figure 3-21  CIPS Method using One Moving and Two Stationary Data Loggers .............. 52

Figure 3-22  Comparison of Raw Pipe-to-Soil Potential Data to Compensated Data ......... 53

Figure 3-23  Pipe Potential/Telluric Current Relationship at a Test Station ....................... 53

Figure 3-24  Four Wire Test Lead Arrangement for Measuring Pipe Current .................. 54

Section 4

Figure 4-1  Schematic of a Telluric Bond Switch .............................................................. 55

Figure 4-2  Mitigation of Telluric Current Discharge using Galvanic Anodes .................. 56

Figure 4-3  Effect of Connecting and Disconnecting Groups of Galvanic Anodes to a Pipeline Subjected to Telluric Current ................................................................. 57

Figure 4-4  Electrical Schematic at a Constant Voltage Transformer Rectifier During a Positive Telluric Voltage Fluctuation ................................................................. 58

Figure 4-5  Schematic of Potentially Controlled Cathodic Protection System used to Mitigate Telluric Current Effects ................................................................. 59

Figure 4-6  Pipe Potential and Rectifier Current Output vs. Time for an Impressed Current System Operating in Potential Control ..................................................... 60

Appendices

Figure A.1  Electric and Magnetic Fields at the Top and Bottom Surfaces of A Layer ......................................................................................................................... A3

Figure A.2  Line Current Source Above the Earth and the Position of an Image Current at Complex Depth Used to Represent the Effect of Induced Currents ................................................................. A8

Figure B.1  Tidal Water Flow in a Semi-Elliptical Channel ................................................ B1

Figure B.2  Generalized Thin Sheet Model ................................................................. B3

Figure B.3  Thin Sheet Model of a Conductivity Boundary with an H-Polarized Source Field, i.e. the Magnetic Field Parallel to the Boundary ............................................ B4

Figure C.1  Coordinate System and Characteristics of the Pipeline Layers ................. C1
Figure D.1  Distributed-Source Transmission Line Model of a Pipe Section .................  D1

Figure D.2  Transmission Line Terminated at Each End by a Thevenin Equivalent Circuit ...........................................................................................................  D3

Figure D.3  a) Transmission Line of Length L Terminated by a Thevenin Equivalent Circuit ...........................................................................................................  D5
             b) Thevenin Equivalent Circuit for the Whole System shown in (a) ...............  D5
1.0 Introduction

Telluric current effects on pipelines have been observed for nearly half a century.\cite{1,2} The first extensive investigation of telluric currents was made in the mid-west US by Gideon and co-workers, for the American Gas Association.\cite{3,4} Construction of the Alaska pipeline in the high latitude region noted for enhanced telluric current activity (see Figure 1-1) prompted further investigations.\cite{5,6,7,8,9} Subsequently, other high latitude pipelines were shown to be affected\cite{10,11} and reports of telluric current effects were also obtained for pipelines in New Zealand,\cite{12,13} Africa\cite{14} and Germany,\cite{15} as well as on the seafloor.\cite{16} Most telluric currents are produced by geomagnetic disturbances,\cite{17,18} although tidally-induced effects have also been reported.\cite{19} As well as pipelines, other systems such as power lines and phone cables are affected (Figure 1-2)\cite{20,21,22,23,24} and these are just one class of technological system that is affected by geomagnetic disturbances.\cite{25,26}

![Figure 1-1 – Recordings of Electric Fields at College, Alaska and Telluric Currents at Chena on the Alaska Pipeline](image-url)
The sequences of phenomena that give rise to geomagnetic disturbances and telluric currents originate on the Sun. The simplest starts with the normal electromagnetic radiation given off by the Sun. As well as illuminating and heating the day-side of the earth, this radiation also heats the ionosphere and creates a dynamo action that drives ionospheric electric currents above the equator and up to mid latitudes. These currents produce a magnetic field that, viewed from space, appears fixed on the day-side of the earth. The rotation of the earth carries pipelines in and out of this magnetic field creating a 12-hour variation (Figure 1-3).
Figure 1-3 – Diagram of the ionospheric currents on the day-side of the Earth that create the "quiet day" variation of the magnetic field and corresponding variations in pipe-to-soil potentials
Figure 1-4 – Schematic of the electric currents associated with the aurora that are responsible for magnetic substorms and associated changes in pipe-to-soil potential

The Sun also radiates particles out into space, and upon reaching the Earth they can be guided down the magnetic field lines into the upper atmosphere where they create the aurora (northern lights). Strong electric currents are also produced and flow down to the ionosphere in the auroral zones where an intense east-west current is produced called the auroral electrojet (Figure 1-4). The magnetic disturbances observed at high latitudes are the magnetic field of this auroral electrojet, and are the principal cause of telluric currents in higher latitude pipelines.
The interaction of solar particles with the Earth’s magnetic field can also excite oscillations of the magnetic field lines (analogous to a bow going across violin strings). These field line oscillations are seen on the ground as small oscillations of the magnetic field with periods of a few minutes. Although small, their higher frequency (relative to the other magnetic variations) means that they can generate significant electric fields that affect pipelines (Figure 1-5).

![Field Line Oscillations](image)

**Figure 1-5 – Oscillations of the earth’s magnetic field lines that cause changes in pipe-to-soil potential**
To provide a quantitative understanding of the telluric currents that will be produced during geomagnetic disturbances two modeling approaches have been used. In one, the electrical characteristics of the pipeline are modeled as a lossy transmission line and the induced electric field is represented by voltage sources distributed along the line. This distributed source transmission line (DSTL) model allows the telluric current and pipe-to-soil potentials along a finite-length pipe to be calculated for a specified electric field in the pipeline. In the second method the pipe is represented as an infinitely long cylinder (ILC) and a solution is derived in terms of Bessel functions describing the induced current as a function of the incident magnetic field. This method includes the distortion of the fields caused by the presence of the pipe but does not consider how the fields behave at the ends of the pipeline.

In more recent times engineers have continued to observe telluric current effects on pipelines from the arctic to Australia and tried to find ways of dealing with the problem. The ILC modeling approach has been used to study telluric currents on pipelines in Argentina, while the DSTL modeling has been shown to agree with observations on both long and short pipelines. The DSTL modeling has also been extended to include multiple pipeline sections so that features such as bends in the pipe and changes of pipe characteristics can be modeled. This now brings the DSTL technique up to the level at which it can be used as a design tool when planning CP systems that can cope with telluric fluctuations.

Two further studies have recently advanced the knowledge of telluric currents. Past DSTL model results had shown that the pipe-to-soil potential variations produced by telluric currents would vary considerably in size along the pipe and would even be out of phase at opposite ends of the pipe. However, few pipeline recordings were made with sufficient temporal or spatial resolution to show these features. Therefore an international study was undertaken to make multi-site recordings of telluric current effects on a number of Canadian and Scandinavian pipelines. These observations, combined with modeling of each pipeline, provide the most comprehensive study of telluric currents in pipelines. The second study was a re-examination of the ILC modeling of pipelines and resulted in a more straightforward derivation leading to a more rigorous set of equations that show the complete interaction of a geomagnetic disturbance with a pipeline. This has been used to investigate the frequency dependence of the pipeline response to geomagnetic field variations.
2.0 Modeling

A test of knowledge of natural phenomena is the degree to which the processes can be modeled. In this section we present the techniques that have been developed for calculating the electric fields at the earth’s surface that drive telluric currents, and then present the methods of modeling the pipeline response to these electric fields. Electric fields are produced by geomagnetic variations due to ionospheric currents and by tidal water movements as illustrated in Figure 2-1. Geomagnetic induction is examined in Section 2.1, while Section 2.2 considers the electric fields generated by tidal water movements and also how the coastal boundary influences the induced electric fields. Section 2.3 shows how the interaction of the geomagnetic disturbance with a pipeline modifies the fields both inside the pipe and in the surrounding medium. Section 2.4 shows how the variations in pipe-to-soil potential produced by induced electric fields are influenced by different pipeline characteristics.

Figure 2-1 – Schematic showing the ionospheric currents, tidal water movements and telluric currents in pipelines.
2.1 Geomagnetic Induction

Electric fields are induced in pipelines by variations of the geomagnetic field at the earth’s surface. The primary causes of these geomagnetic field variations are electric currents in the ionosphere above the earth. However, currents induced in the earth also create magnetic fields that contribute to the disturbance seen at the earth’s surface and influence the electric fields produced in pipelines. The size of the electric fields produced depends on the size of the geomagnetic disturbance, the rate of change of the magnetic field (i.e. the frequency content) and the resistivity of the earth. At the frequencies involved in geomagnetic disturbances (periods from minutes to hours) the geomagnetic field variations, and hence the induced currents, penetrate hundreds of kilometers into the earth so the resistivity down to these depths needs to be included in the calculations.

The electric field variations at a particular frequency, \( \omega \), are related to the magnetic field variations \( B(\omega) \), and the surface impedance of the earth \( Z(\omega) \).

\[
E(\omega) = Z(\omega) B(\omega)
\]  

[2.1]

The surface impedance for a uniform earth is given by

\[
Z_s = \sqrt{\frac{i \omega \mu}{\sigma}}
\]  

[2.2]

where \( i = \sqrt{-1} \), \( \sigma \) is the conductivity of the earth, and \( \mu \) is the magnetic permeability (which, in the earth, usually has its free value of \( 4\pi \times 10^{-7} \text{ H/m} \)). For the more realistic case where the resistance of the earth varies with depth, the surface impedance can be calculated by modeling the earth as a stack of layers with different conductivities, as shown in Appendix A.

![Figure 2.2 – Magnetic fields recorded at the Ottawa magnetic observatory and electric fields calculated using a layered earth model.](image)
An example of a practical calculation is shown in Figures 2-2 and 2-3. The recordings of the magnetic field (Figure 2-2) were Fourier transformed to give the frequency spectrum, i.e. the amplitudes at each component frequency. The magnetic field at each frequency was then multiplied by the corresponding surface impedance value to produce the electric field spectrum (Figure 2-3). An inverse Fourier transform was then used to obtain the time variation of the electric field (Figure 2-2). These calculated electric fields can then be compared with nearby pipeline recordings or used as input to pipeline models.
To evaluate the electric fields over a long pipeline it is necessary to take account of the changing size of the magnetic disturbance with distance from the ionospheric source currents. This is especially important at high latitudes or near the equator where the pipeline can be directly underneath the ionospheric currents causing the disturbance. The magnetic field from a current, $I$, in the ionosphere can be calculated from the Biot Savart law. However, as mentioned above, currents induced in the earth also contribute to the magnetic fields at the earth’s surface and need to be included in the calculations. Appendix A shows that these calculations are greatly simplified if the induced currents are represented by an image current at a complex depth, $h + a + 2p$ where $h$ and $a$ are the height and half-width of the ionospheric current respectively and $p$ is the complex skin depth in the earth which is related to the surface impedance $Z$.

$$p = \frac{Z}{i\omega}$$  \[2.3\]

The horizontal $B_x$ and vertical $B_z$ components of the magnetic field and the electric field, $E_y$, at the earth’s surface then vary with horizontal distance, $x$, from the source according to the simple formulas

$$B_x = \frac{\mu_0 I}{2\pi} \left( \frac{h + a}{(h + a)^2 + x^2} + \frac{h + a + 2p}{(h + a + 2p)^2 + x^2} \right)$$  \[2.4\]

$$B_z = -\frac{\mu_0 I}{2\pi} \left( \frac{x}{(h + a)^2 + x^2} - \frac{x}{(h + a + 2p)^2 + x^2} \right)$$  \[2.5\]

$$E_y = -\frac{j\omega \mu_0 I}{2\pi} \ln \left[ \frac{\sqrt{(h + a + 2p)^2 + x^2}}{\sqrt{(h + a)^2 + x^2}} \right]$$  \[2.6\]

Plots for a current of 1 million amps above central Canada (Figure 2-4) show that the magnetic and electric field can change over distances of hundreds of kilometres.
Figure 2-4 – Horizontal (Bx) and vertical (Bz) magnetic fields and horizontal (Ey) electric field with the Cauchy distribution produced by a line current of 1 million amps 100 km above the earth’s surface. The calculations are made for a period of 5 min. and an earth model for the Canadian Shield.

Asterisks show results from complex image method. Solid lines show results of exact calculations.
2.2 Tidal Induction and Coast Effect

Electric fields in the earth are also produced by the dynamo action arising from the tidal movement of conducting seawater through the earth’s magnetic field. For water flowing through a channel or into a bay, the “tidal dynamo” produces an electric field across the channel as shown in Figure 2.5. The dynamo electric field drives an electric current (i.e. a flow of electrical charge) through the seawater and produces an accumulation of charge on either shore of the channel. This build-up of charge raises (and lowers) the electrical potential on opposite sides of the channel which then causes a potential gradient in the land on either side of the channel and in the seabed beneath the channel.

![Figure 2-5 – Electric Field, E, generated by seawater moving with velocity, v, through the earth’s magnetic field, B](image)

A formula for the potential difference across a channel was derived by Longuet-Higgins and includes the partial ‘shorting-out’ of voltages by return currents flowing in the seabed. In cases where the ‘shorting-out’ by return currents is negligible, Longuet-Higgins’ expression can be simplified and shows (see Appendix B) that, assuming a constant flow rate across the channel, the potential difference \( V_{12} \) generated is proportional to the speed of the water flow, \( v_w \), the strength of the vertical magnetic field, \( B_Z \), and the width of the channel, \( W \).

\[
V_{12} = v_w B_Z W
\]  

[2.7]

To give an example of the voltages that can be produced, calculations were made for the Bay of Fundy in the Maritimes region of Canada. The Bay of Fundy is famous for having the highest rise and fall of tide in the world, and the peak flow rates range from 2.2 knots (11.3 m/sec) during neap tides to 4.0 knots (20.6 m/sec) during spring tides.
In the Maritimes region the vertical component of the earth’s magnetic field is approximately $50.1 \times 10^{-6}$ Tesla. Using a width of 50 km for the Bay of Fundy then gives the potential difference (in volts) across the Bay of Fundy

$$V_{BF} = 2.5 v_{BF}$$

where $v_{BF}$ is the water velocity in the Bay of Fundy in metre/sec.

The ebb and flow of the tides in the Bay of Fundy will produce a sinusoidal variation in the earth potentials with a period of approximately 12.5 hours. Substituting the peak flow values into the equations above gives a maximum potential difference across the bay of 52V at spring tides and 28V at neap tides.

The potential difference across the channel $V$ produces an earth potential $V/2$ on one side of the channel and an earth potential $-V/2$ on the other side as shown in Figure 2.6. Thus the peak earth potentials will be half of the voltage values given above. For the Bay of Fundy, the flow rate diminishes as one moves further up the Bay, and the earth potentials will be similarly reduced.
2.2.1 Coast Effect

Another effect that happens at the coast is an increase in the electric fields produced by geomagnetic disturbances. This effect can be understood by considering a magnetic disturbance that produces an induced electric field perpendicular to the coast. Current continuity across the coast requires a step change in the electric field due to the difference in the conductivity of the land and the sea. This is achieved by charges at the coast that produce potential gradients which modify the electric field, decreasing it in the sea and increasing it in the land (see Figure 2-7). Pipelines on the land near the coast would therefore be in a region of higher than normal electric fields.

![Figure 2-7](image)

Figure 2-7 – Current continuity for currents flowing across the coast is produced by charge accumulation that modifies the electric fields near the coast

The tidal dynamo and the geomagnetic coast effect both result in an increase in earth potential at the coast, and the same modeling techniques can be used for calculating the voltage profile in the land near the coast.

2.2.2 Coastal Modeling

Examining the electric fields produced in the vicinity of a conductivity anomaly requires three-dimensional modeling of the electromagnetic fields in the earth. To do this there are a variety of mathematical techniques based on solving sets of integral equations or differential equations or a combination of the two. The integral equation approach involves the more complex mathematics, but only needs to calculate the electric field in small anomalous regions instead of throughout the whole region, so has less demanding computing requirements. An alternative approach is to use differential equations solved either by the finite difference method or the finite element method. The differential equations are simpler to set up but require the solution of large matrices and therefore need sufficient computing resources.
The most successful method for investigating near-surface features is the thin sheet approximation. This is less computationally demanding than a complete three-dimensional model as it only requires the calculation of the horizontal electric fields across a surface grid. The use of thin sheet modeling to calculate the electric fields near a coastline is explained in Appendix B.2. In the thin sheet modeling, the surface layer of the earth (i.e. ocean or surface rock layer) is represented by a conductance upper layer with $\sigma_s = \sigma t_U$, where $\sigma$ and $t_U$ are the conductivity and thickness of the upper layer respectively. Underneath this is a lower layer, representing the resistive crust of the earth, with integrated resistivity $\rho_s = \rho t_L$ where $\rho$ and $t_L$ are the resistivity and thickness of the lower layer. Below this is a more conductive region representing the earth’s mantle for which we can specify a terminating impedance $Z_T$. This can be a uniform region as defined by equation [A.20] or a multi-layer impedance as described in Appendix A.2.2.

The potential at the surface is then given by

$$V(x) = V_b \exp(\psi_1 x) \quad x < 0$$  \hspace{1cm} [2.9]$$

$$V(x) = V_b \exp(-\psi_2 x) \quad x > 0$$  \hspace{1cm} [2.10]$$

where:

$$V_b = \frac{\sigma_{s2} E_2^0 - \sigma_{s1} E_1^0}{\sigma_{s1} \psi_1 + \sigma_{s2} \psi_2}$$  \hspace{1cm} [2.11]$$

$$\psi_1 = \frac{1}{\sqrt{\sigma_{s1} \rho_{s1}}} \quad \text{and} \quad \psi_2 = \frac{1}{\sqrt{\sigma_{s2} \rho_{s2}}}$$  \hspace{1cm} [2.12]$$

and $E_1^0$ and $E_2^0$ are the electric fields, well away from the coast, in the land and sea respectively.

The resulting potential gradient adds to the induced electric field on the land side and subtracts from it on the sea side giving a total electric field:

$$E_x = E_1^0 + \psi_1 V_b \exp(\psi_1 x) \quad x < 0$$  \hspace{1cm} [2.13]$$

$$E_x = E_2^0 - \psi_2 V_b \exp(-\psi_2 x) \quad x > 0$$  \hspace{1cm} [2.14]$$
To illustrate the coast effect, calculations have been made for a coastal boundary as shown in Figure 2-8. The seawater is assumed to be 50 m deep and to have a resistivity of 0.25 Ω-m (conductivity of 4.0 S/m) which gives a conductance of 200 S. Beneath this are the more resistive rocks of the crust for which we assume a resistivity of 2,000 Ω-m and a thickness of 15 km, which gives a resistance of 3.10^7 Ω. On the land side of the coast, the surface layer is assumed to be 100 m thick and have a resistivity of 100 Ω-m (conductivity of 10^-2 S/m) which gives a conductance of 1.0 S. The underlying crust is assumed to be the same as under the sea. The underlying mantle is assumed to have a resistivity of 10 ohm-m.

These values were used in equations [2.9] and [2.10] to determine the variation in earth potential perpendicular to the coast shown in Figure 2-9. This shows that, for an induced electric field of 1 V/km perpendicular to the coast, a voltage of 72 V is produced at the coast and falls off exponentially with distance from the coast. The change of potential represents a potential gradient that adds to the induced electric field on the land side of the coast, producing a peak electric field of 11 V/km at the coast.
Similar but smaller earth voltages are produced at conductivity boundaries within the land, such as at geological boundaries. These can be modeled the same way as the coast effect by substituting appropriate values. Figure 2-10 shows a boundary between a region of high resistance granite and a shallow sedimentary basin. The Granite region is modelled as a thin (1 metre) soil layer over a 15 km high resistance (20,000 $\Omega$-m) crust. On the right side in Figure 2-8, the sedimentary basin is represented by a more conductive layer with resistivity of 100 $\Omega$-m and 100 metre thickness. For an electric field of 1 V/km, perpendicular to the boundary, the thin sheet modeling shows that the surface potential has a peak value of 15 V at the boundary and falls off exponentially on either side of the boundary (Figure 2-10).
The thin sheet modeling can also be used to examine the electric fields produced by the tidal dynamo. In this case there is only an electric field in the seawater. Using the value for the Bay of Fundy, $E \approx 1 \text{ V/km}$ as shown in Figure 2-11, the modeling results show that this produces a potential of $\pm 26 \text{ V}$. In the seawater there is a linear change of potential across the bay. In the land on either side, the earth surface potential falls off exponentially, dropping to about 10% of the coastal value about 10 km inland.

![Figure 2-11 – Electric field generated by tidal water movement and the resultant potential in the land and the sea.](image)

These simple models show the order of magnitude of the earth potentials that can be produced at the coast and other conductivity boundaries. To accurately assess the earth potentials that can be produced in particular locations is more difficult and requires models that can take account of the complex shape of the coastline and the varying depth of the seawater. Developments in 3-dimensional computer modeling are making it possible to examine the electromagnetic response of more complicated structures. However, to represent realistic coast lines the best option is analogue modeling. Several research groups throughout the world operate test tanks in which they can put scale models of coastal regions and apply suitably scaled magnetic fields and measure the electric fields produced. An example of analogue model results for the east coast of Canada (Figure 2-12) shows the increased earth surface potential produced by charge accumulation associated with electric currents in the sea flowing into the Bay of Fundy.
We have shown that simple expressions can be derived for the earth voltages produced at a straight coastline. This modeling can also be used for conductivity boundaries inland and for looking at the potentials produced by the tidal dynamo. These simple models provide a useful guide to where changes in earth potential can be expected and the size of the voltages produced. Modeling more realistic coastlines requires more elaborate 3-dimensional computer or analogue models.

2.3 Geomagnetic Interaction with a Pipeline

In the above sections we have considered the production of electric fields at the earth’s surface in the absence of a pipeline. Where there is a pipeline, reflections from the pipe modify the electric and magnetic fields in the surrounding soil and also mean that the electric field inside the pipe is not necessarily the same as the electric field away from the pipe (Figure 2-13). To determine the relationship between the electric field in the pipe steel and the incident electric field in the earth calculated in the previous sections, a pipeline can be modeled as an infinitely-long multi-layered cylinder (see Appendix C). Two cases can be considered: when the electric field is parallel to the pipeline (E-polarization), and when the electric field is perpendicular to the pipeline and the magnetic field is parallel to the pipeline (H-polarization) (see Appendix C).
In the case of E-polarization, the electric field in the pipe steel (layer 3) is given by

\[ E_3 = \sum_{n=0}^{\infty} A_{3n} \left( I_n(k_3r) + R_{3n} K_n(k_3r) \right) \cos n\phi \]

[2.15]

where \( I_n(kr), K_n(kr) \) are modified Bessel functions of the first and second kind.

In this equation the first term represents the incident wave coming in from the outer layer and the second term represents the wave reflected from the boundary with the inner layer. Corresponding expressions (see Appendix C) can be written for the other layers, i.e. the surrounding earth, the pipe coating, and the oil or gas in the pipe. In these expressions the amplitudes, \( A \), and reflection coefficients, \( R \), can be found from the boundary conditions\(^{[50]}\) and are defined in terms of the amplitude of the incident electric field and the conductivity, \( \sigma \), and magnetic permeability, \( \mu \), of each layer.

Figure 2-14 shows an example of the distortion of the electric field in the earth that can be caused by the presence of a pipeline (in this example, for a pipeline in seawater with a 1 Hz incident field). The electric field changes from a plane wave away from the pipe where the incident field dominates to a nearly cylindrical wave close to the pipe where the reflected field is largest (Figure 2-14a). The effect on the electric field amplitude can be most clearly seen by examining the cross-sections shown in Figures 2-14b and 2-14c. The cross-section BB’ shows the normal exponential decay of the field that occurs away from the pipe. Cross-section AA’ shows the additional attenuation of the electric field produced by the pipe.
Figure 2-14c shows that, in this example, the electric field just outside the pipe is reduced to approximately 0.4 of its value away from the pipe. (The results are normalized by dividing by the electric field $E_0$ that would occur at the same depth but in the absence of the pipe.)

Figure 2-14c – Distortion of electric field in the earth produced by a pipeline.

a) contours of electric field strength
b) vertical sections showing the decrease of the electric field that occurs away from the pipe (BB') and the additional attenuation produced by the pipe (AA')
c) horizontal section showing the attenuation of the field near the pipe
The electric field close to and inside the pipe, for the same electromagnetic and frequency values as in Figure 2-14, is shown in Figure 2-15. Close to the pipe contour lines are nearly circular (Figure 2-15a), in contrast to the horizontal contour lines produced further from the pipe. Figure 2-15b shows the attenuation of the electric field outside the pipeline. Figure 2-15c shows an expanded view of the field within the pipe steel and coating. In this case the electric field is constant over the depth of the pipe steel. On the outer edge of the steel, the thin insulating coating does not attenuate the electric fields.
The interaction between the geomagnetic disturbance and a pipeline depends on the frequencies of geomagnetic fluctuations and on the electromagnetic characteristics of the pipe and surrounding soil. Figure 2-16 shows the amplitude and phase of total current for different conductivities of the surrounding medium, representing materials ranging from air \(10^6\) S/m, to rock \(10^3\) to \(10^1\) S/m, to seawater \(1\) S/m. These results show that the total current produced in the pipe drops off significantly at periods below 100 seconds. This frequency dependence will be significant as studies of telluric currents move to higher frequency recordings. The drop off with frequency could also lead to an upper limit being specified for the frequencies that need to be considered when investigating geomagnetic effects on pipelines.

![Figure 2-16](image_url)

*Figure 2-16 – Frequency dependence for different resistivities (in S/m)*

a)  amplitude  
b)  phase
2.4 DSTL Modeling

The effect of electric fields induced in pipelines can be modeled using distributed-source transmission line (DSTL) theory first described by Schelkunoff.\[^{51}\] DSTL theory has been used extensively for modeling AC induction in pipelines,\[^{52}\] and was applied to geomagnetic induction in pipelines by Boteler and Cookson.\[^{53}\] This modeling was useful for a single straight pipeline. To extend the modeling to include multiple pipeline sections, the DSTL theory has been extended to include the general case of induction in a transmission line which is terminated at each end by transmission lines themselves subject to electromagnetic induction (see Appendix D). This development allows model calculations to be made for realistic pipelines including bends, laterals, or even whole networks.

![Transmission line model of pipeline including distributed voltage sources representing the induced electric field.](image)

In the DSTL approach, the pipeline is represented by a transmission line with a series impedance given by the resistance of the pipeline steel and the inductance of the pipeline, and a parallel admittance given by the conductance through the pipeline coating and the capacitance of the pipe to earth. The induced electric field is represented by voltage sources distributed along the transmission line (Figure 2-17). The series impedance and parallel admittance can be used to determine the characteristic impedance, $Z_0$, and the propagation constant, $\gamma$. 


\[ \gamma = \sqrt{ZY} \quad \text{and} \quad Z_o = \frac{Z}{Y} \]  

These are the key parameters that describe the electrical response of the pipeline. Another useful parameter is the adjustment distance, which is the inverse of the propagation constant, and is a measure of the distance along the pipe for the potential to adjust to a change in characteristics.

The fundamental equations describing the current and voltage produced in a pipeline by an induced electric field, \( E \), are

\[
V = \frac{E}{\gamma} (A e^{-\gamma \cdot \chi_1} - Be^{-\gamma \cdot \chi_2}) \quad \text{[2.17]}
\]

\[
I = \frac{E}{Z} \left(1 + (A e^{-\gamma \cdot \chi_1}) + (Be^{-\gamma \cdot \chi_2})\right) \quad \text{[2.18]}
\]

and where: \( x_1 \) and \( x_2 \) are the positions of the ends of the pipeline and \( A \) and \( B \) are constants dependent on the conditions at the ends of the pipeline (see Appendix D).

It is found that the solutions obtained from these equations depend on whether the pipeline is long or short compared to the adjustment distance, \( 1/\gamma \), defined by the pipeline’s electrical characteristics.

### 2.4.1 Electrically-Long Pipeline

When a pipeline is long enough that the exponential components at either end do not significantly overlap, i.e. when the pipeline is longer than four adjustment distances, constants \( A \) and \( B \) simplify to

\[
A = - \frac{Z_1}{Z_o + Z_1} 
\quad B = - \frac{Z_2}{Z_o + Z_2} \quad \text{[2.19]}
\]

and the current and voltage for a pipeline extending from \( \chi = 0 \) to \( \chi = L \) are now given by

\[
I = \frac{E}{Z} \frac{V_1}{Z_o} e^{-\gamma \cdot \chi} - \frac{V_2}{Z_o} e^{-\gamma (L \cdot \chi)} \quad \text{[2.20]}
\]
\[ V = -V_1 e^{\gamma z} + V_2 e^{\gamma(L - z)} \]  \[2.21\]

and where:

\[ V_1 = \frac{E}{\gamma} \frac{Z_1}{Z_0 + Z_1} \quad \text{and} \quad V_2 = \frac{E}{\gamma} \frac{Z_2}{Z_0 + Z_2} \]  \[2.22\]

are the voltages at the ends of the pipeline.

These expressions show that for a long straight pipeline, the maximum pipe-to-soil potential variations occur at the ends of the pipeline and are independent of pipeline length. The pipeline potentials fall off exponentially with distance from either end and cross through zero in the middle of the pipeline as shown in Figure 2-18. The theory also shows that the voltage variations at opposite ends of the pipeline are out of phase. In contrast, the current is in phase all along the pipeline and reaches its maximum value in the center of the pipeline.

![Graph showing voltage, dV/dx, and current vs. distance](image)

Figure 2-18 – DSTL model results (V, dV/dx, and I) for a long pipeline.
2.4.2 Electrically-Short Pipeline

For an electrically-short pipeline with a high resistance to ground at the ends of the pipe (or no end connection at all) the constants $A$ and $B$ are

$$A = B = -\frac{e^{\gamma L} - 1}{e^{\gamma L} - e^{\gamma L_x}} \quad [2.23]$$

and the expressions for voltage and current reduce to (see Appendix D)

$$I = \frac{E}{Z} \left[ 1 - \left( 1 - \left( \frac{\gamma L}{2} \right)^2 \right) \left( 1 + \frac{\gamma \left( \chi - \frac{L}{2} \right)^2}{2} \right) \right] \quad [2.24]$$

and

$$V = E \left( 1 - \frac{\gamma L}{8} \right) \left( x - \frac{L}{2} \right) \quad [2.25]$$

When the pipeline is considerably shorter than the adjustment distance, the current goes to zero and the voltage is given by

$$V = E \left( x - \frac{L}{2} \right) \quad [2.26]$$

This equation shows that the voltage varies linearly from $-EL/2$ at $x = 0$ to $EL/2$ at $x = L$ (Figure 2-19). Thus the maximum voltage occurs at the ends of the pipeline and is equal to half the product of the electric field and the pipeline length. Opposite voltages occur at opposite ends of the pipeline.

![Figure 2-19 – Linear variation in potential produced by telluric electric fields in a short pipeline](image-url)
2.4.3 Pipeline Bend

A feature found on all pipelines is a change in direction (bend) in the pipeline. For a uniform incident electric field, the change in pipe direction means that the component of the electric field parallel to the pipeline will be different on either side of the bend.

For pipelines directed away from the bend with angles $\alpha$ and $\beta$ relative to the direction of the electric field, as shown in Figure 2-20a, the voltage at the bend is given by

$$V_b = \frac{E \cos \alpha + E \cos \beta}{2 \gamma}$$  \[2.27\]

The pipeline potential on either side of the bend falls off exponentially with distance, $x$, from the bend with an adjustment distance $1/\gamma = 1/\sqrt{ZY}$.

$$V = \begin{cases} V_b e^{\gamma x} & \text{for } x < 0 \\ V_b e^{-\gamma x} & \text{for } x > 0 \end{cases}$$  \[2.28\]

For a practical example, consider a bend with $\beta - \alpha = 120^\circ$, and an electric field of 1 V/km in a direction that bisects the angle between the pipe sections. The pipe-to-soil potentials around the bend then vary as shown in Figure 2-20.

![Figure 2-20:](image)
2.4.4 Modeling Pipeline Networks

For more realistic pipeline networks, solutions can be obtained using the full DSTL modeling as shown in Appendix D. This allows the influence of the pipe coating, ground bed resistance, and the geometry of the pipe (length, bends, flanges) on the pipe-to-soil potentials to be examined. Here examples are presented of the DSTL modeling using results obtained as part of the cathodic protection design work for the Maritimes and Northeast Pipeline (Figure 2.21).

Figure 2-21 – Route of the Maritimes and Northeast Pipeline
Figure 2-22 shows the pipeline potentials calculated for an eastward electric field, for the pipeline with insulating flanges at the bends 300 km, 400 km, and 800 km from the eastern end. It can be seen that the maximum pipeline potentials occur on either side of the flanges and that the potentials are greater with lower coating conductances. The pipeline potentials are also larger at the ends of the longer pipe sections. With the higher conductance coating (100 $\mu$S/m$^2$) the adjustment distance is small, and the pipeline is long enough that the regions of increased potential at the opposite ends of a section do not overlap. As long as this condition is maintained, the end potential is independent of pipeline length. With the medium coating conductance (10 $\mu$S/m$^2$) the adjustment distance is greater and it can be seen that the end potential regions start to overlap. For the low conductance coating (1 $\mu$S/m$^2$) the adjustment distance is much larger, and the end potentials overlap significantly. This produces a nearly linear variation in pipe potential with the end potentials proportional to pipeline length.
If the insulating flanges at the bends are removed, the pipeline potentials now only have single maximum and minimum potentials located at opposite ends of the pipeline (Figure 2-23). For the high conductance coating the end potentials are no bigger than for the short sections, consistent with the small adjustment distance in this case. With the middle value of coating conductance there is a slight increase in the end potential compared to the pipe with the insulating flanges. However, in this case there are only higher potential regions at the ends of the pipe and not at each intermediate site when flanges are inserted. For the low conductance coating, the pipeline potentials at the ends of the long pipeline (without flanges) are larger than the potentials at the ends of the shorter sections with the flanges, consistent with the longer adjustment distance in this case.

![Figure 2-23 – Maritimes DSTL results without flanges](image_url)
The above model calculations were made assuming that there were no low resistance groundbeds connected to the pipe. If a groundbed with a resistance of 0.1 ohm is connected at each end of the pipeline, the currents induced in the pipe can flow harmlessly in and out of the pipe at the ends. The end voltages in this case are considerably reduced as shown in Figure 2-24. The peak potentials now occur at the pipeline bends where, because of the different alignment of the pipe section relative to the electric field, pipe-to-soil potentials are produced as shown in section 2.4.3.

Figure 2-24 – Maritimes DSTL results with 0.1 ohm terminations
3.0 Impact of Telluric Currents on Pipeline Corrosion Control Systems

3.1 General

Potential and current fluctuations on oil and gas pipelines, attributed to geomagnetic activity, have been observed for many years by corrosion control personnel when conducting routine cathodic protection performance surveys. The impact of these telluric currents has generally been considered more of a nuisance during periods when cathodic protection parameters were being measured rather than a serious corrosion concern. In fact it was common practice in the pipeline industry to temporarily discontinue cathodic protection surveys until telluric fluctuations had ceased. With the advent of close interval potential survey programs and the need to survey long lengths of pipeline over a predictable period of time, the temporary shutdown of survey crews is not a practical option. Moreover, pipelines in the higher latitudes are subjected to telluric activity much of the time. Russell and Nelson\textsuperscript{[54]} commented that “The stray currents were always present during a 10 week period of continuous surveying over a 380 mile section which was completely tested...Until a means is found to eliminate or at least limit these current fluctuations, future surveys will be arduous and it will be difficult to determine if a cathodic protection system is fully effective”.

Boteler\textsuperscript{[55]} has shown that the telluric voltage induced on a pipeline can be calculated using distributed source transmission line (DSTL) equations, and that the magnitude of the telluric voltage ($V_t$) is not only a function of the direction and magnitude of the electric field, but is also directly dependent on the pipe’s length and resistance to earth. These calculations when applied to modern well coated pipelines, suggest that telluric current effects may not be as innocuous as originally thought, especially for long pipelines located at higher latitudes.

As the number of pipelines located in higher latitudes has grown, as the pipeline coating quality has improved, and as the intensity of geomagnetic storms increase, a number of concerns about the impact of telluric current interference on pipeline corrosion control systems arise as follows:

- corrosion during the positive half cycles of the telluric waveform;
- accuracy of pipeline current and potential measurements when determining the level of cathodic protection for comparison with industry criteria;
- coating damage caused by excessively negative potentials during the negative half cycles of the telluric waveform; and
- effectiveness of techniques to mitigate telluric voltage fluctuations.
3.2 Corrosion Impact of Telluric Current on Cathodically Protected Pipelines

3.2.1 Background

There have been no reported instances of pipeline corrosion failures due to telluric currents, but there have been a number of investigations that have evaluated the relative corrosion risks of telluric currents on pipelines.

An early research study[^56] on “Earth Current Effects on Buried Pipelines” was sponsored by the American Gas Association (AGA). Phase 3 of this project was an “analysis of observations of telluric gradients and their effects” recorded on four widely separated pipelines in the U.S.A. The investigators concluded that “the effects are insignificant, both for coated, protected lines and for bare lines”. This conclusion was based on the analysis of field data recorded on four pipelines between the summer of 1968 and October 1969, close to a peak in the 11 year sunspot activity cycle, when a reasonably high level of geomagnetic activity was expected. The conclusion drawn from this investigation, undertaken more than 30 years ago, may not however be as relevant for some modern pipeline networks, especially those in latitudes closer to the magnetic poles.

The coated pipelines chosen in the AGA study had relatively low leakage resistances, compared to pipelines with modern coatings for which values of greater than 100K-ohm-m[^57,58] are common. Geomagnetically induced voltages would, therefore, be greater on pipelines with a high coating quality, since the level of induced voltage is directly proportional to coating resistance as shown in Figure 3-1.

![Figure 3-1 – Calculated Telluric Induced Voltage at the End of the Maritimes and Northeast Pipeline as a Function of Coating Conductance for an East-West Electric Field of 0.1V/km[^59]"](image-url)

[^56]: Reference to the study
[^57]: Reference to leakage resistance values
[^58]: Reference to modern coatings
[^59]: Reference to induced voltage calculation
Furthermore, all the pipelines under test were electrically short (only one greater than 65 km) which, according to the DSTL equations, would produce lower amplitude fluctuations than on longer pipelines.

The pipelines were also located at mid-latitudes (all were at latitudes lower than 46° N), where the probability of a large storm is up to 100 times less than in Canada and Alaska.\[60\]

Even though the study spanned a time period which was near the peak of solar cycle 20, the number of magnetic disturbances was unusually low compared to the following 30 years\[61\] as is apparent in Figure 1-2.

Finally, the longest pipeline in the study (190 km) and the one that exhibited the largest telluric pipe-to-soil potential amplitudes was located about 35°N, where the probability of a large geomagnetic storm, as previously defined, is only 0.003%. It is apparent therefore that the results of the AGA study may not be applicable to long pipelines, to very well coated pipelines, to pipelines located at higher latitudes, and even to similar pipelines today since the telluric intensity, as represented by the ‘aa’ index (i.e. >60 nT), has generally increased with time.\[62\]

Hessler,\[63\] in a 1974 paper presenting earth gradient test results for telluric disturbances in the arctic, commented “I do not believe that anyone, at this state of the art, has the capability of quantifying the contribution of telluric currents to corrosion rates on a pipeline with any degree of certainty”. He felt that, because of the alternating frequency, the wide range of frequencies, and the considerable variation in intensity, such a corrosion study would require careful evaluation over a long period of time.

Osella et al.\[64\] used Bessel functions to model telluric current activity on a pipeline system and to quantify the corrosion rate where there was an abrupt change in soil resistivity. They considered that only telluric periods of 1 hour or greater produced a corrosion concern. The Tierra del Fuego gas pipeline system modeled in the calculations was located across the southern tip of Argentina at about 55° degrees latitude. They found that the mass of steel lost increased by a factor of 10 (0.1g/day to 1.0g/day) during moderate to strong geomagnetic events, of which there were 32 during 1994. Corrosion weight loss was shown to be dependent only on the ratios of the two soil resistivities involved in the abrupt change and this effect increased as the pipe diameter increased. Faraday’s law was used in the calculations, which probably overstates the weight loss, particularly on cathodically protected pipe.

### 3.2.2 Reported Instances of Corrosion Caused by Telluric Currents

Pipe-to-soil potential measurements on a cathodically protected pipeline in Northern Norway were recorded over a 2-3 month period in 1971 and analyzed by Henriksen et al.\[65\] with respect to the probable corrosion impact of telluric
disturbances. By correlating the duration and magnitude of potential excursions more positive than the –850mV_{cse} criterion with corrosion rate versus potential data obtained in laboratory tests, they concluded “that telluric current corrosion in auroral zones has about the same magnitude as the normal corrosion is (sic) soil where telluric corrosion is lacking”. This conclusion assumed that the telluric discharge involved purely metal dissolution rather than oxidation of any other species, which therefore probably overstates the corrosion activity.

In 1986, Seager\(^\text{[66]}\) conducted a study on a 522 km cathodically protected oil transmission pipeline, located in Canada, between 55°N and 70°N geomagnetic latitude. He used small steel coupons installed at test stations along the pipe length, and concluded “…telluric related corrosion can override any standard corrosion prevention system and cause pipe perforation in unacceptably short periods of time…”.

By measuring each coupon’s potential instantaneously after disconnecting it from the pipeline (i.e. an ‘instant off’ potential), the ‘polarized’ potential was determined, free of IR drop caused by both the cathodic protection and the telluric current. This showed that there were periods of time when the polarized potential was more electropositive than the generally accepted –850mV_{cse} cathodic protection criterion\(^\text{[67]}\) and other periods of time when it was more positive than –650mV_{cse}, prompting Seager to conclude that corrosion would occur for an estimated 15% and 4% of these periods respectively. Based on this pattern of activity, he calculated that the pipe could be perforated in less than four years at a circular coating flaw having a 0.6cm diameter.

Martin\(^\text{[68]}\) has also reported telluric corrosion on a 515 km gas pipeline in northeastern Australia, where the cathodic protection monitoring criterion was being met, but the buried resistance probes indicated corrosion rates in excess of .010 mm/a. In one location the corrosion rate was .038 mm/a, a rate that would cause a 10% pipe wall penetration in about 14 years.

In a Norwegian study\(^\text{[69]}\) corrosion rates were calculated based on current measurements made on coupons connected to three parallel pipelines that were subjected to telluric activity. On these cathodically protected pipelines, the corrosion rate was estimated conservatively at 0.04 mm/a for an average of 300 telluric events of similar magnitude each year. The telluric current discharge from a coupon occurs simultaneously when the ‘on’ potential becomes more positive than the ‘off’ potential as shown in Figure 3-2. The \(E_{\text{OFF}}\) potential was calculated using the \(E_{\text{ON}}\) potential and applying a connection factor determined by multiplying the coupon current by the coupon resistance to earth, which had been previously measured.
For approximately 1/3 of the recording time the coupon was discharging current, and these sustained current discharges resulted in significant depolarization. Discharge current densities were comparable in magnitude to the pick up current densities.

3.2.3 Theoretical Corrosion Considerations

During the time when telluric current transfers from the pipe to earth (positive portion of the telluric cycle) the charges must transfer through an oxidation reaction. For a pipe without cathodic protection, the primary oxidation reaction is corrosion of the steel as illustrated in Figure 3-3 and as expressed by the following reaction:

$$\text{Fe}^0 \Rightarrow \text{Fe}^{++} + 2e^- \quad \text{(corrosion)}$$

\[ [3.1] \]
Theoretically, approximately 10 kg of steel will be lost in 1 year for every ampere of continuous direct current that discharges.

When a pipeline is being cathodically protected or is receiving telluric current, as illustrated in Figure 3-4, the charge transfer reactions can be one or both of the following depending on the soil conditions:

\[
\text{H}_3\text{O}^+ + e^- \rightarrow \text{H}_2 + \text{OH}^- \quad \text{(in de-aerated or acidic soils)} \quad [3.2]
\]

or

\[
2\text{H}_2\text{O} + \text{O}_2 + 4e^- \rightarrow 4\text{OH}^- \quad \text{(in alkaline or neutral aerated soils)} \quad [3.3]
\]

These reduction reactions produce a high pH environment, typically in the range of 10-13, at coating flaws (holidays) regardless of which reduction reaction transfers the charges. The magnitude of the pH has been shown to be proportional to the logarithm of the current density \([70]\) as shown in Figure 3-5.
When positive charges transfer from a surface that has been cathodically protected, the initial oxidation reaction is therefore likely to result in the formation of a passive film as illustrated in Figure 3-6. Here it can be seen that as the steel becomes progressively more cathodically polarized, the anodic polarization curve exhibits progressively more passive behavior.

![Figure 3-6 – Polarization Curves after Several Days of Potentiostatic Polarization](redrawn from Hesjevik, S.M. and Birketveit, O., Telluric Current on Short Gas Pipelines in Norway – Risk of Corrosion on Buried Gas Pipelines, NACE Corrosion 2001, Paper #01313)

If the telluric current discharge is sustained, and the residual pH remains high, then the oxidation reaction could be Equation [3.4], the oxidation of hydroxyl ions, or by Equation [3.5], the hydrolysis of water, as illustrated in Figure 3-7. Neither of these oxidation reactions results in metal loss.

\[
\begin{align*}
4\text{OH}^- & \rightarrow 2\text{H}_2\text{O} + \text{O}_2\uparrow + 4e^- \quad [3.4] \\
2 \text{H}_2\text{O} & \rightarrow \text{O}_2\uparrow + 4\text{H}^+ + 4e^- \quad [3.5]
\end{align*}
\]

![Figure 3-7 – Telluric Current Discharge from a Cathodically Protected Pipe](grade diagram)
As noted in Figure 3-8, a current discharge from steel, exposed to a pH of 12, can produce passivity and hence a very low corrosion rate up to a potential of +600 mV_{SCE}.

![Figure 3-8 – Experimental Anodic Polarization Curve of Steel in Hydroxide (pH 12.0)\(^{[71]}\)](image)

Accordingly, the total corrosion that occurs at a coating defect as a result of current discharge is not strictly proportional to the charge transferred as would be predicted by Faraday’s Law for a steady state direct current. Cyclic variations in telluric current of equal amplitude and period will corrode steel less than a steady state direct current of the same magnitude applied for the same time period, as discovered in a National Bureau of Standards investigation\(^{[72]}\) and as illustrated in Figure 3-9.

![Figure 3-9 – Coefficient of Corrosion at Different Frequencies for Iron Electrodes](image)
This study, which was commissioned to determine the relative corrosivity of stray currents arising from DC transit systems, has some merit with respect to telluric stray currents because the periods of activity are somewhat similar. In fact Campbell\textsuperscript{[73]} produced the following mathematical relationship using the NBS findings:

\[ C = (4.7 \pm 1.3) T^{-0.186} \]

Where: 
- \( C \) is percent of direct current corrosion that would occur at the same amplitude,
- \( T \) is the period of the current cycle in seconds

Peabody\textsuperscript{[74]} as shown in Figure 3-10 also summarized the NBS findings graphically, which demonstrates a relationship between the logarithm of the period and the logarithm of the percentage of corrosion compared to an equal amount of direct current.

![Figure 3-10 – Effect on Corrosion Rate of Reversing Direction of Current Compared To Steady State Direct Current and Length of Time Between Reversals](redrawn from Peabody, 1979)
Although telluric frequencies cover a wide spectrum, the induced electric field is typically maximized at periods of from between 30 minutes to 2 hours.[75] This corresponds to corrosion activity that would be about 22-29% of an equivalent direct current. It should be noted however that diurnal telluric activity, although typically less intense than other types of activity having shorter periods, would produce a corrosion rate of approximately 50% of an equivalent direct current because it would have a 12 hour time between reversals.

The amount of stray telluric current during the positive period also depends on the intensity of the telluric disturbances. On very well coated modern pipelines, current transfer between the pipe and soil occurs at small coating defects. Relatively small potential fluctuations in the order of 0.5 to 1.0V can produce a large current density as shown in Figure 3-11.[76] For the case of a 1 cm diameter circular holiday in a 0.3mm thick coating (a typical thickness for fusion bonded epoxy coatings), the current density, for a soil resistivity of 1000 ohm-cm and a telluric voltage change of 1.0V, would be approximately 2500µA/cm² producing a corrosion rate of approximately 31.3mm/a.

![Figure 3-11 – Corrosion Current Density at a Coating Defect having an Applied Voltage of 1.0V in 1000 ohm-cm Soil for Various Coating Thicknesses](redrawn from Von Baeckman & Schwenk, 1975)

The corrosion rate arising from Figure 3-11 for a 1cm diameter defect in a 0.3mm thick coating with a steady state voltage of +1V applied in 1,000 ohm-cm soil can be expressed as follows:

$$CR = K_i \cdot P$$

where:

- $K_i$ = corrosion current density factor (2.5 x 10⁻³ A/cm per volt)
- $P$ = corrosion penetration factor (12.5 x 10⁻³ mm/a per 10⁻⁶ A/cm²)
- $CR$ = corrosion rate (mm/a)
This corrosion rate formula must be modified however to account for the cyclic variations in the telluric wave form (Fp) and the duration of time that the activity is present (Ft):

$$\text{Corrosion Rate (Fe)} = \frac{2.5 \times 10^{-3} \, A}{cm^2 V} \times \frac{12.5 \times 10^{-3} \, mm/a}{10^{-6} \, A/cm} \times \Delta V_t \times F(p) \times F(t)$$

$$\Delta V_t = \text{change in potential of the pipe caused by telluric activity}$$
$$F(p) = \text{fraction of steady state corrosion due to alternating period of the telluric current}$$
$$F(t) = \text{fraction of time that telluric activity is present}$$

The magnitude of the voltage change ($\Delta V_t$) has been shown to be directly proportional to the magnitude of the electric field. This can be assessed by recording the pipe-to-soil potential over time or can be estimated using the DSTL model for specific electric field intensities. The electric field intensity can be determined from a geomagnetic Kp index as shown in Figure 3-12. This Kp index was developed by the Geomagnetic Laboratory in Ottawa, and has been used to calculate the telluric voltage profile for the Maritime and Northeast Pipeline system.

![Figure 3-12 – Peak Electric Field Magnitudes as a Function of Kp](image-url)
The Kp index ranges from 0 (quiet) to 9 (severe storm) for a corresponding electric field intensity of 1 to 1000 mV/km. This relationship varies with geographic location and is based on assessing the electric field intensity over 3-hour periods.

The Kp index can also be used to determine the fraction of time [F(t)] that telluric activity is present using Figure 3-13. The relationship between the probability of the specific telluric activity and the Kp index is generic for any location once the Kp index is determined for the location of interest. Information on geomagnetic activity is available through various sources located throughout the world as listed in Appendix G.

![Figure 3-13 – Average Occurrence of 3-Hour Intervals with Kp ≥ a Specified Value](image)

The corresponding corrosion rate based on a 0.5V potential change (V_{t} = 0.5V) caused by a telluric current occurring for 6% of the time (Kp ≅ 5), in the absence of any cathodic protection current, is calculated to be 0.152 mm/a (6mpy) for a telluric period of 2 hours. This resultant corrosion rate exceeds 0.025 mm/a (1mpy), which is generally considered the maximum acceptable corrosion rate for oil or gas transmission pipelines when cathodically protected.
The range of corrosion rates for unprotected steel in 1000 ohm-cm soil at a 1 cm diameter holiday can therefore be calculated for various geomagnetic intensity levels (Kp indexes) and telluric voltage effects ($V_t$) as shown in Figure 3-14.

![Figure 3-14 – Corrosion Rate for Unprotected Steel vs. Telluric Potential Change at a 1 cm Holiday in 1000 ohm-cm Soil for Various Telluric Intensities (Kp indexes) having a Period of 1 Hour](image_url)

It can be seen that even modest telluric voltage shifts of 0.10V can have a significant corrosion impact if produced by a Kp 3 magnetic disturbance in the absence of cathodic protection. It is generally perceived that the large potential shifts caused by the more severe telluric disturbances (e.g. Kp 8) cause the most corrosion damage, but this is not the case for piping without cathodic protection.

Cathodic protection will of course reduce the telluric corrosion depending on the level of protection on the pipeline at the time of the telluric activity. Cathodic protection systems often provide a minimum polarized potential of $-850mV_{cse}$, which depending on soil conditions is typically 100 to 300mV more electronegative than the corrosion potential. Since the corrosion potential is not usually known at any location on a pipeline, the corrosion rate can be conservatively estimated, from Figure 3-12, on the basis of the change in potential ($\Delta V_t$) more electropositive than the $-850mV$ criterion. For example, if the telluric voltage fluctuations due to diurnal telluric activity (Kp 3) produce a 0.1V shift more positive than the criterion, then a corrosion rate of 0.3mm/a (12mpy) might be possible. In contrast, a similar telluric discharge produced by a Kp 8 severity storm might produce a corrosion rate...
of only 0.002mm/a (0.08mpy). The corrosion rates used in constructing Figure 3-14 were based on a soil resistivity of 1000 ohm-cm. For a higher resistivity, the current density and therefore the corrosion rate would be proportionately lower. A knowledge of the soil resistivity, corrosion potential, and recorded telluric voltage fluctuations on a pipeline should allow for a reasonable estimate of the corrosion consequences of that activity. When determining the soil resistivity however, it is the resistivity of the electrolyte within and immediately adjacent to the holiday that is more important than the bulk resistivity.

3.3 Effect of Telluric Voltage Fluctuations on Coatings

3.3.1 General

It is generally recommended\[79\] that cathodic protection systems be designed to produce a minimum of −0.3V change in potential at the pipe/soil interface. Hence the corrosion impact of a +0.3V potential change created by a telluric current would be largely negated by a properly operating cathodic protection system, even though the −850mV\(_{\text{cse}}\) minimum potential criterion would not be met. To ameliorate telluric voltage shifts and maintain the minimum potential criterion, requires either a proportionate steady state increase in the output of the cathodic protection system, or an increase in the output of the cathodic protection system in response to a telluric current discharge. Where large telluric voltage fluctuations are anticipated, as at isolated fittings or at changes in direction of the piping system, cathodic protection systems might require such high steady state current outputs as to over-stress the coating.

The NACE RP0169-96 standard\[80\] cautions that “the use of excessive polarized potentials on externally coated pipelines should be avoided to minimize cathodic disbondment of the coating”. This caution is increasingly being interpreted as a maximum of −1200mV\(_{\text{cse}}\) polarized potential. Figure 3-2 indicates that polarized potentials around this value can occur during telluric current pick-up periods. This is a particularly difficult limit to avoid especially close to a rectifier drain point, let alone having to increase the rectifier output in anticipation of any telluric discharge activity.

It appears from a coating integrity point of view that operating impressed current systems at higher steady state current outputs simply to compensate for telluric current activity should be avoided. Also, telluric current activity can regularly produce polarized potentials at the upper recommended electronegative limit. This is less of a problem with sacrificial systems, as the anode current output is self-limiting due to the diminishing driving voltage as the pipe polarizes. Furthermore, it has been shown\[81\] that once the open circuit potential of the galvanic anode has been exceeded, the anode will pick-up much of the telluric current, thereby limiting the pipeline potential.
3.4 Cathodic Protection Performance Monitoring in the Presence of Telluric Activity

3.4.1 Pipe-to-Soil Potential Measurement at a Test Station

It is usual and required by codes\(^{[82,83]}\) to measure the pipe-to-soil potential on a routine basis to ensure that a minimum level of cathodic polarization is being maintained. This involves taking potential measurements at test station locations as illustrated in Figure 3-15.

Here the pipe-to-soil potential \(V_{ps}\) is measured using a high resistance voltmeter connected between a pipe test lead and a reference electrode placed in contact with the soil such that

\[
V_{ps} = E_p + V_e
\]

where:

- \(E_p\) = the pipe polarized potential across the pipe/soil interface (V)
- \(V_e\) = \(I_{cp} \cdot R_e\) = the voltage drop in the earth caused by the cathodic protection current in the earth between the point in the earth where the reference is placed and the pipe surface (V)
- \(V_{ps}\) = voltage appearing on the voltmeter (V)

![Figure 3-15 – Typical Pipe-to-Soil Potential Measurement at a Test Station](image-url)
A pipeline is considered effectively protected from corrosion when the pipe polarized potential ($E_p$) is equal to or more negative than $-850 \text{ mV}_{\text{cse}}$. To obtain an accurate measurement of the polarized potential requires that the voltage drop ($V_e$) in the earth be removed from the voltmeter reading. Perhaps the most common technique for eliminating this IR drop from the measurement is by cyclically interrupting the cathodic protection current and measuring the ‘instant-off’ potential.

When telluric current is present however, the voltmeter measures an additional potential difference ($V_t$) between the pipe and reference whose polarity alternates with time and whose magnitude fluctuates with time and location on the pipeline.

$$Hence \quad V_{ps} = E_p + V_e \pm V_t$$

Figure 3-16 is an example of a pipe potential recording over a 10 day period with telluric activity present and the cathodic protection current applied. Note that there is a repeating pattern that results in less negative potentials at the same time each day. This is attributed to the diurnal effects resulting from the rotation of the earth.
Since the geomagnetically induced current cannot be arbitrarily interrupted, alternative methods of compensating for the voltage drop in the earth ($V_t$) caused by the telluric current have been employed.[85,86] One method involves the use of a small steel coupon installed next to the pipe, which is normally interconnected with the pipe at the test station terminal strip. The coupon simulates the pipe/soil surface at a holiday in the coating. When the coupon is temporarily disconnected and the reference electrode is placed in the soil tube, as illustrated in Figure 3-17, both the telluric and cathodic protection voltage drops in the earth are removed from the measurement and the ‘instant off’ potential ($E_{ip}$) of the coupon is measured. This value can be compared to the $-850 \text{ mV}_{cse}$ criterion or the coupon can be left disconnected for comparison to the $100 \text{ mV}$ of polarization decay criterion.

This test arrangement however is not convenient for recording the polarized potential with time, since the coupon has to be disconnected for each measurement. The use of a reference/coupon combination, as illustrated in Figure 3-18, has proved to be an excellent method of recording a polarized potential with time. The coupon in this device does not require disconnection, since a reference electrode is located inside the pipe coupon, so that neither a cathodic protection nor telluric voltage gradient exist between the coupon and the reference. Coupons designed with a sensing port through the middle of the coupon are also available[87] for use with a portable reference electrode and soil tube which eliminates the need to incorporate a permanent reference electrode.
Figure 3-19 is a comparison of the pipe/coupon potentials recorded to a CSE reference placed on grade, and to the zinc reference located inside the coupon. The difference between the potential values is the soil voltage gradient caused by both the telluric and cathodic protection currents.

![Figure 3-19 – Comparison between Pipe/Coupon Potential with Time between a Copper-Copper Sulphate Reference on Grade and a Zinc Reference Electrode Located inside the Coupon](image)

### 3.4.2 Close Interval Potential Surveys

The measurement of telluric free potentials is more complex for close interval potential surveys (CIPS) where the reference is moved and placed over the pipe at regular intervals (typically < 3m) along the route of the pipeline.

Proctor\(^{[88]}\) proposed a measurement method to compensate for the telluric induced voltage that involved the correction of the measured potential (V\(m\)) with respect to the moving reference, by the change in potential (ΔV\(f\)) measured with respect to a fixed reference located at a nearby test station such that

\[
V_{ps} = V_m \pm \Delta V_f
\]

where:

\[
\Delta V_f = V_{f_{ave}} - V_f
\]
This measurement technique is illustrated in Figure 3-20 in which two separate data loggers are used to record the potentials synchronously with respect to the fixed and moving electrodes. This technique can also be used with synchronous interruption of the rectifiers such that a telluric compensated ‘instant off’ potential can be calculated in software from the recorded data.

![Figure 3-20 – Pipe-to-Soil Potential Measurement Method to Compensate for Telluric Current Effects During a Close Interval CP Survey](image)

The validity of this technique depends on how accurately the average potential ($V_{f,ave}$) represents a ‘telluric free’ condition, on the proximity of the fixed location to the moving electrode (since large separation distances can introduce errors due to potential differences in the earth parallel to the pipe route), and on the magnitude and direction of the pipe voltage drop caused by the telluric current.

Place and Sneath\[^{89}\] have used a variation of the foregoing technique in combination with cathodic protection current interruption to produce close interval survey data that is telluric compensated. Their test arrangement, which is illustrated in Figure 3-19, uses two stationary data loggers, one at the start of the CIPS ($V_{rs}$) and one at the end of the span ($V_{rt}$).
All data loggers are synchronized by referencing the global positioning system (GPS) and the telluric compensation is a linear extrapolation of the telluric shift at each data logger relative to the moving reference’s proximity to each stationary reference. This correction routine, done in software, is expressed as follows;

$$V_{ps} = V_m + \Delta V_{rs} \cdot \frac{y}{x+y} \pm \Delta V_{rt} \cdot \frac{x}{x+y}$$

*Where:* the $\Delta V_{rs}$ and $\Delta V_{rt}$ are the differences in potential compared to the average potential [$V_{rfave}$ and $V_{rsave}$] recorded at each location over a period of time prior to the survey.

This technique tends to minimize the error inherent in the previous method when the distance between the moving reference and the single stationary data logger increases significantly. Both techniques assume that the telluric voltage amplitude is linear over the relatively short distances surveyed, and that pipeline voltage drop error created by the telluric current in the pipe between the start and finish test stations is negligible. Also, each method is dependent on the validity of the prerecorded data that establishes the average potential with time at the start and finish test stations. The shorter this period is prior to the survey, the greater will be the influence of short duration telluric activity and the less will be the effect of any diurnal telluric activity.
Figure 3-22 compares the typical before and after correction pipe-to-soil potential data.

![Telluric Affected Close Interval Survey (CIS) Data](image1)

![Telluric Compensated Close Interval Survey (CIS) Data](image2)

Figure 3-22 – Comparison of Raw Pipe-to-Soil Potential Data to Compensated Data

Degerstedt et al. [90] have used a ‘telluric null’ technique on the Trans Alaska Pipeline System. They recorded the potential and pipe current parameters at test stations with time to produce a potential vs. telluric current relationship at each test station location as illustrated in Figure 3-23.

![Pipe Potential/Telluric Current Relationship at a Test Station](image3)

Figure 3-23 – Pipe Potential/Telluric Current Relationship at a Test Station
The telluric current was measured using magnetometers placed on grade on each side of the pipeline. It can be seen that there is a linear relationship between the telluric current and the pipe potential, and through regression analysis the ‘telluric null’ potential is identified as the intercept with the pipe potential axis.

With a historical characteristic established at each test station, the CIPS is conducted using GPS time stamping to record both pipe current magnitude and potential with respect to a moving reference, and this potential is corrected relative to the voltage at the fixed electrodes at the adjacent test stations by an appropriate correction factor.

In lieu of magnetometers, the pipe current can also be determined by measuring the voltage drop along the pipe as illustrated in Figure 3-24, although this arrangement would require installation of pipe test leads at each test station location. Where telluric current activity is anticipated on a new pipeline system, this four wire test arrangement should be installed at each test station location if the telluric null method is to be utilized. In addition, each test station should also incorporate a coupon/reference probe to facilitate the recording of pipe-to-soil polarized potentials with time.

Another telluric compensation method was applied to a CIPS survey on offshore pipelines in the North Sea by Weldon et al. They installed a series of matched silver-silver chloride reference electrodes on the sea bottom at about 1km spacings near the production platform and in the intertidal region perpendicular to the shoreline. The telluric gradient parallel to the pipeline(s) was recorded over time, and the shipboard recorded potential obtained in the CIPS was corrected by subtracting the normalized and distorted telluric potential. They concluded that although the technique produced a potential profile that was indicative of the general level of cathodic protection, better results would have been obtained with more accurate time stamping of data.
4.0 Mitigating Telluric Current Effects

4.1 By Grounding

Telluric voltages on pipelines arise from electromagnetic induction and are therefore analogous to induced AC voltages. Similarly, grounding the pipeline can be an effective method of mitigating telluric voltages just as it is with AC voltages. Telluric voltages, which appear across an insulated flange, can be reduced by electrically bonding around the isolating joint. As with AC mitigation however, the bond must be designed to maintain the performance of the cathodic protection system. A telluric bond switch, as illustrated in Figure 4-1, has been used\(^{[92]}\) to pass telluric current across an insulator separating the onshore and offshore portions of a cathodically protected pipeline.

![Figure 4-1 – Schematic of a Telluric Bond Switch](image)

Back-to-back diodes provide a fault path for the large telluric currents once the breakover voltage of the diodes (typically 0.8V) has been breached. These diodes are therefore rated to handle the largest expected telluric current typically arising from a once per year severe storm (i.e. Kp 9). Adjustment of the variable resistor allows for a steady-state drain of current to balance the cathodic protection systems between the onshore and offshore sections of the pipeline. Lightning protection is provided by the varistor.

It may also be possible to mitigate telluric effects by connecting the pipeline to electrical ground using AC coupling-DC isolating devices such as isolating surge protectors and polarization cells, although the use of such devices for this purpose has not been reported in the literature.
4.2 Using Cathodic Protection

Cathodic protection systems can be designed and operated to mitigate telluric voltage fluctuations by a combination of two related mechanisms. The cathodic protection current output can be increased to compensate for a telluric current discharge, or galvanic anodes can provide a grounding path for the telluric current to pass to earth. The capacity to perform these functions varies with the type and operating characteristics of the cathodic protection system relative to the operating characteristics of the pipeline system.

4.2.1 Sacrificial Systems

Sacrificial cathodic protection systems have a limited voltage capacity to compensate for a telluric potential shift since they have a relatively small fixed output voltage. They do however, offer an alternative path to earth for the telluric current (I\textsubscript{t}) because of their low resistance to earth compared to a coated pipeline. Some proportion of the telluric current (I\textsubscript{t}) will transfer to earth via the anode as shown in Figure 4-2, depending primarily on the anode-to-earth resistance compared to the pipe-to-earth resistance, both locally and looking down the pipe in the direction of the current. If the cathodic protection current (I\textsubscript{cp}) is equal to or greater than the residual telluric discharge current (I\textsubscript{t} \textsuperscript{II}), then corrosion cannot occur in the vicinity of the anode.

![Diagram of Mitigation of Telluric Current Discharge Effects using Galvanic Anodes](image)

where: \[ I_t = I_t^I + I_t^II + I_t^III \]

**Figure 4-2 – Mitigation of Telluric Current Discharge Effects using Galvanic Anodes**

This cathodic protection method, which makes the pipeline electrically lossy, has been used on the Trans-Alaska pipeline\textsuperscript{[93]} in the form of a zinc ribbon anode which was placed at pipe invert elevation on each side of the pipe for the full extent of the underground portion of the pipeline. Grouping of zinc and magnesium sacrificial anodes at selected intervals has also been shown to be effective by Henriksen et al.\textsuperscript{[94]} when used on a pipeline in northern Norway where the telluric potential...
fluctuations were reduced from ± 5 V to ± 0.1 V as illustrated in Figure 4-3. Just as with induced AC mitigation, the more electrically lossy a pipeline is, the lesser the magnitude of the telluric voltage fluctuations.

For instance, if a 0.5m diameter coated pipeline has a conductance of $10^{-6}$ S/m$^2$ in 10,000 ohm-cm soil (a reasonable expectation for modern coatings) then it has a conductance per 100m of $0.157 	imes 10^{-3}$ S.

If a 1.5m long magnesium anode is attached to the pipe every 100m, the pipe conductance would increase by a factor of 167 times to $26.3 	imes 10^{-3}$ S, due principally to the anode conductance (see detailed calculation in Appendix E). This would reduce the telluric voltage for a given electric field intensity by over 90% while also providing adequate cathodic protection. In addition, there is some belief that the telluric current seen on pipelines is due primarily to current transfer (conductance) between the pipe and earth, rather than from inductance directly. If this were the case, then magnesium anodes would be preferred over zinc anodes since they would not pick-up current until their open circuit potential (approximately $-1.750V_{cse}$) was exceeded, whereas zinc would accept telluric current when a potential of $-1.100V_{cse}$ was exceeded. Magnesium anodes would therefore lessen the amount of current pick-up, and provide more cathodic protection current compared to zinc.

There may also be net cathodic protection benefit with the use of sacrificial anodes in the presence of a telluric current. Results from an experiment[95] that applied a signal simulating a telluric wave form to a combination of a steel pipe and a zinc
ribbon found that there was a net pick-up of the alternating signal on the pipe. Conversely, there was a net increase in the amount of current discharged from the anode. This may be due to the fact that the anode does not pick-up the alternating current until its open circuit potential is exceeded, and the pipe does not discharge current until the anode potential is polarized electropositively to the pipe polarized potential.

4.2.2 Impressed Current Systems

Impressed current cathodic protection (ICCP) systems can theoretically be designed with unlimited voltage capacity, although it is inefficient to continuously operate the system at higher voltages just to provide a buffer for the anticipated telluric positive voltage shift. In addition, the very high negative potentials produced, as a result of operating ICCP systems at high current outputs, can cause cathodic disbondment of the coating. Martin[96] found that operating rectifiers in constant voltage or constant current mode had “little mitigative effect” since they caused “overprotection during local negative transients and underprotection during local positive transients”.

Interestingly, there have been reports[97,98] that telluric voltage fluctuations are more pronounced near rectifier locations than between them. This may be due to telluric currents passing to earth through the rectifying element in the transformer-rectifier. The total current output (I_o) will be the sum of the cathodic protection current (I_{cp}) and the telluric current (I_t) as follows:

\[ I_o = I_{cp} + I_t \]

For a transformer-rectifier operating in constant voltage mode, the current output will increase when the telluric current passes through the rectifying element to ground (during the electropositive half-cycle) and the cathodic protection current component will not change significantly. During a telluric voltage positive fluctuation on the pipe, the telluric driving voltage is in series with the transformer rectifier voltage as shown in Figure 4-4. This results in an increase in the current output by an amount I_t.

\[ V_T + V_R = V_O \]

\[ I_O = I_{cp} + I_t \]

**Figure 4-4 – Electrical Schematic at a Constant Voltage Transformer Rectifier During a Positive Telluric Voltage Fluctuation**
When operating in constant current mode, where $I_o$ is kept constant, cathodic protection current will be reduced by the amount of the telluric current through the rectifier, thereby diminishing the amount of cathodic protection available to counteract the residual telluric current discharge from the pipe. It would seem therefore, from a telluric current mitigation point of view, that impressed current systems should not be operated in constant current mode.

Martin\(^{99}\) and other operators\(^{100,101}\) have used the potential control mode to successfully ameliorate telluric currents even though Proctor\(^{102}\) concluded that “the value of constant potential impressed current power sources in compensating for telluric current interference is questionable”. The voltage and current output of these units change automatically in response to the pipe potential, as measured to a local reference electrode, as illustrated schematically in Figure 4-5.

![Figure 4-5 – Schematic of Potentially Controlled Cathodic Protection System used to Mitigate Telluric Current Effects](image)

Here the coupon potential is measured continuously with respect to the permanent reference electrode and compared to a pre-set potential in the controller of the DC power supply. When a telluric current attempts to discharge from the pipe/coupon, the reference senses the positive potential shift and the power supply immediately increases its output to maintain the set potential value. The impressed current system therefore presents a negative resistance path for the telluric current to earth and there is no residual discharge of telluric current from the pipe as long as the voltage or current limit of the power supply has not been reached. A coupon is used to minimize IR drop between the reference electrode and the nearest holiday so that the rectifier can control to a potential that has minimal IR drop component.
The power supply voltage and current capacity must be sized to provide the needed cathodic protection current plus the amount of telluric current to be drained. This type of cathodic protection system functions as a telluric current ‘forced drainage’ system, and its mitigating effect is illustrated in Figure 4-6 which compares typical rectifier current output and pipe potential over time.

![Figure 4-6 – Pipe Potential and Rectifier Current Output versus Time for an Impressed Current System Operating in Potential Control](image)

Note that, in this example, the rectifier operates only when the pipe potential attempts to go more electropositive than the set potential of $-100 \text{ mV}_{ZRE}$ ($-1200 \text{ mV}_{cse}$). Telluric current is drained to earth during periods of telluric current discharge. During periods of telluric current pick-up the current output goes to zero, thus limiting the magnitude of the negative potential applied across the coating. This mode of operation effectively eliminates the positive telluric voltage fluctuations while minimizing excessively negative potentials and maximizing the life of the groundbed.
5.0 Summary

Pipelines are subjected to telluric current activity due to the modulation of the earth’s magnetic field by solar particles. The changing magnetic field produces an electric field that causes charges to flow in the earth and in metallic networks located on the earth such as pipelines, electric powerlines, and communication cables. This electrical disturbance is observed on pipelines as potential and current fluctuations that can vary with time due to the earth’s rotation, tidal cycles, the sun’s rotation, eleven-year solar cycles, and solar storms.

The magnitude and location of these disturbances are dependent on the pipeline’s proximity to the earth’s magnetic poles, on its length, on its orientation, on changes in direction, on the coating resistance, on electrical continuity along its length, on soil resistivity and the presence of abrupt changes in earth conductivity, and proximity to a sea coast. Telluric effects, although more pronounced on pipelines located at higher latitudes, have also been observed near equatorial regions in Panama\textsuperscript{[103]} and Kenya\textsuperscript{[104]}.

Historically the effects of telluric currents on pipelines have been considered a curiosity and an inconvenience when conducting cathodic protection surveys for compliance with the pipeline codes and regulations. Recently however, as more pipelines have been constructed at higher latitudes, in higher resistivity soils, and with better quality coatings, the resulting telluric potential and current variations, being more severe, have prompted concerns about the following issues:

- whether or not the pipe is corroding during periods of telluric current discharges, and
- will the coating be stressed and possibly disbonded during periods of pick-up, and
- how can the effects of telluric current activity be mitigated, and
- what techniques are available to measure accurate pipe-to-soil potentials during periods of telluric activity

This report described two methods for modeling telluric currents in pipelines. The infinitely long cylinder (ILC) method is appropriate for examining how reflections from the pipe surface affect the electric field inside the pipeline. Initial ILC modeling has shown that this effect is small at low frequencies (periods > 5 minutes) but is significant at higher frequencies (periods < 5 minutes). A limitation of the ILC method is that it cannot be applied to a pipeline network.

Distributed source transmission line (DSTL) theory has been used successfully to model the flow of telluric currents and the associated potential variations throughout pipeline networks. The model addresses typical pipeline situations including pipe bends, pipe junctions, branch lines, insulating flanges, grounding points, changes in pipe dimensions, and changes of coating conductance. The input
to the DSTL model is the electric field in the pipe which is assumed to be the same as in the earth although this is not strictly true at higher frequencies.

Although an AGA study in the late 1960’s concluded that the corrosive effects were “insignificant” on protected pipelines, more recent field reports suggest a small but measurable corrosion rate. Corrosion rates on cathodically protected pipe attributed to telluric currents appear to be low (< 0.1 mm/yr) based on a few investigations, and there have been no reported corrosion failures. Nevertheless, over a long time period these corrosion rates could produce a significant corrosion problem. Because of the variation in telluric current intensity and magnitude with time, it is difficult to calculate an accurate corrosion rate, although it is clear from the literature that the corrosion rate is less than would be calculated using Faraday’s relationship.

In view of these uncertainties, measures to mitigate and control telluric currents, in order to minimize the possibility of small residual corrosion rates, have been undertaken by a number of pipeline operators. Providing a path to ground, as is done to reduce AC induced voltages, is one effective measure to decrease the magnitude of telluric voltage fluctuations. Cathodic protection systems, which can compensate for the telluric current discharge, also to some extent provide a path to earth for the telluric current depending on the type of system. Use of galvanic cathodic protection systems can be very effective on well-coated pipe, because they significantly increase the pipe leakage conductance. They may also enhance the cathodic polarization of the pipeline by creating a net pick-up of telluric current on the pipe, due to their offset in potential with respect to the pipe.

Potential controlled impressed current systems have also been used successfully to control telluric currents, whereas transformer-rectifiers, operating in constant current mode, may actually hinder mitigation of telluric current effects. Using constant voltage or constant current transformer-rectifiers at increased outputs to counterbalance positive telluric voltage fluctuations, besides being inefficient, can result in over-protection and increased risk of coating damage.

A number of methods have been developed to improve the accuracy of pipe-to-soil potential measurements in the presence of telluric voltage fluctuations. Coupons installed at test stations provide an economical and effective means of measuring a polarized potential. Coupons that do not have to be disconnected to measure a polarized potential have a decided advantage since proper monitoring usually requires that potentials be recorded over time. In addition, coupon current magnitude and direction can also be monitored to assist in evaluating the risk of corrosion.

Conducting close interval surveys, which correct for telluric induced voltage fluctuations, is somewhat more complicated. Here, most of the techniques that have been developed require multiple recorders to collect data from stationary electrodes so that a correction factor can be derived and applied to the potential measured with respect to the moving electrode. This operation requires accurate time stamping, usually by reference to the global positioning system. None of the techniques, however, correct for all the possible voltage drops in the measurement circuit.
6.0 Recommendations for Further Research on Telluric Current Effects on Pipelines

Although there are methods of predicting and controlling the adverse effects of telluric current activity on the operation and monitoring of cathodic protection systems, there remain several areas where more research and development activity is needed.

6.1 The DSTL model for calculating tellurally induced voltage and current on pipelines presently does not accommodate the wide range of frequencies inherent in geomagnetic disturbances. Moreover, it assumes that the electric field inside the pipe is the same as in the ground, which is not a valid assumption as frequency increases. The ILC and DSTL methods are however complementary and could be integrated to provide a unified model that accommodates the full frequency range of the telluric spectrum.

6.2 Long term monitoring of pipe-to-soil potential variations is needed to provide statistics on the observed occurrence of large pipe-to-soil variations. This would be essential for assessing the corrosion hazard of telluric currents, and for validating the statistical prediction. A study linking the frequency dependent DSTL model to long term field measurements of pipe-to-soil potentials and the electric and magnetic fields at multiple locations is therefore needed to verify the program. The typical field locations should include pipe bends, areas of geological discontinuities, coastal areas, and insulated flanges.

6.3 A laboratory investigation of the corrosion rates associated with the telluric current spectrum is required to allow pipeline operators to better estimate corrosion activity in the field, based on the magnitude and frequency of positive shifts that are sub-criterion. This should be conducted in various soil conditions, should involve a range of current densities, and include wave shapes typical of telluric current activity. The measured corrosion rates should be compared to the theoretical corrosion rates derived from the total charge transferred.

6.4 A comparative field evaluation of the various techniques that have been used to correct pipe-to-soil potentials for telluric effects is required. This study should be augmented with an assessment of other interference testing methods, such as side-drain measurements, that have not previously been applied to telluric current situations.

6.5 The possibility that a galvanic cathodic protection system might enhance the level of cathodic polarization (in effect using some of the telluric current as net cathodic protection current) should be investigated in the field and laboratory for both zinc and magnesium anode materials.
6.6 A field study needs to be undertaken to determine the telluric mitigating effect of operating transformer-rectifiers in the constant voltage and constant current modes. Presently, the assumed difference in performance between these operating modes is anecdotal and theoretical.

6.7 Telluric current interference may be pronounced at locations where a pipeline parallels an AC powerline, because both structures are subject to induced voltages. This possibility should be examined using the improved DSTL model, and if there appears to be an area of concern, then field tests should ensue.

The foregoing list is by no means complete, but is rather a compilation of the most important areas to focus research effort and resources.
REFERENCES


27. Ibid. [19]

28. Ibid. [26]


50. Ibid. [48]


53. Ibid. [19]

54. Ibid. [2], pp.400.


62. Ibid. [22], pp.19.


64. Ibid. [37]


66. Ibid [31], pp.7.


68. Ibid. [32], pp.349.


73. Ibid. [5], pp.1167.

74. Ibid. [11], pp.30.

75. Ibid. [5], pp.1164.


82. CSA Standard Z662-94, Oil and Gas Pipeline Systems, Canadian Standards Association, Section 9.2.11.2, pp.165.


84. Ibid. [66]


88. Ibid. [12], pp.29.

89. Place, T., and Sneath, O., Practical Telluric Compensation for Pipelines, Proceedings, NACE Northern Area Western Conference, Saskatoon, Feb. 2000.

90. Ibid. [34]


92. Ibid. [43]

93. Ibid. [32]

94. Ibid. [10]

95. Unpublished results from research to determine the potential and current effects on a steel pipe/zinc ribbon couple, CORRENG Consulting Service Inc., Downsview, ON, Canada 1993.

96. Ibid. [32], pp.345.


100. Ibid. [13], pp.27.

101. Ibid. [90]

102. Proctor, T.G., Pipeline Telluric Current Difference as one Phase of a Wider Interdisciplinary Technological Problem, NACE, Corrosion /74, Paper No.60, pp.16.

103. Soto, Gonzalo, M., Control de Corrosion en el Oleoducto Transsisto de Panama and Kenya, Plenary Lecture, VII Seminario Latinoamericano de corrosion y Electroquimica, 1985, Ciudad de Panama.

A

Geomagnetic Induction
in the Earth
A.1 General Equations

To determine the electric fields induced at the earth’s surface by magnetic field variations it is necessary to start with Maxwell’s equations:

\[ \text{curl } E = \frac{\partial B}{\partial t} \]  \[\text{(A.1)}\]

\[ \text{curl } H = J + \frac{\partial D}{\partial t} \] \[\text{(A.2)}\]

\[ \text{div } B = 0 \] \[\text{(A.3)}\]

\[ \text{div } D = \rho \] \[\text{(A.4)}\]

and the properties of the medium

\[ D = \varepsilon E \quad B = \mu H \quad J = \sigma(z) E \] \[\text{(A.5)}\]

When there are no lateral variations in conductivity there is no charge accumulation so

\[ \text{div } E = 0 \] \[\text{(A.6)}\]

At the frequencies of concern for geomagnetic induction, the ‘displacement current’ term, \( \frac{\partial D}{\partial t} \), is small compared to \( J \) in equation \( \text{(A.2)} \). Then, with a time dependence of the form \( e^{i\omega t} \), so that \( \frac{\partial B}{\partial t} \) can be written \( i\omega B \), equations \( \text{(A.1)} \) and \( \text{(A.2)} \) become:

\[ \text{curl } E = -i\omega \mu H \] \[\text{(A.7)}\]

\[ \text{curl } H = \sigma \text{ } E \] \[\text{(A.8)}\]

To determine the electric field, taking the curl of \( \text{(A.7)} \) and substituting from \( \text{(A.8)} \) for \( \text{curl } H \) gives

\[ \nabla (\nabla \cdot E) - \nabla^2 E = -i\omega \mu \sigma E \] \[\text{(A.9)}\]

and, because \( \nabla \cdot E = 0 \) (equation \( \text{A.6} \)), this gives the diffusion equation

\[ \nabla^2 E = i\omega \mu \sigma E \] \[\text{(A.10)}\]

Similarly, taking the curl of \( \text{(A.8)} \) and substituting from \( \text{(A.7)} \) for \( \text{curl } E \), because \( \nabla \cdot B = 0 \) [equation \( \text{A.3} \)], leads to an equivalent diffusion equation for the magnetic field

\[ \nabla^2 H = i\omega \mu \sigma H \] \[\text{(A.11)}\]
A.2 Uniform Source Fields

For a vertically incident plane wave source, the vertical electric field $E_z$ is zero and there are no variations in the horizontal directions, i.e.

$$\frac{\partial E_z}{\partial x} = \frac{\partial E_z}{\partial y} = 0 \quad \text{and} \quad \frac{\partial H_z}{\partial x} = \frac{\partial H_z}{\partial y} = 0$$

Therefore, expanding curl $E$ in equation [A.7] in cartesian coordinates and equating $x$ components gives the relation between orthogonal horizontal components of the magnetic and electric fields

$$-i\omega \mu H_x = \frac{\partial E_y}{\partial z} \quad [A.12]$$

Also the diffusion equations [A.10] and [A.11] reduce to

$$\frac{\partial^2 E}{\partial z^2} = k_z^2 E \quad [A.13]$$

$$\frac{\partial^2 H}{\partial z^2} = k_z^2 H \quad [A.14]$$

where:

$$k_z = \sqrt{i\omega \mu \sigma} \quad [A.15]$$

The solution to equation [A.13] is

$$E = S e^{k_z z} + R e^{k_z z} \quad [A.16]$$

where $S$ and $R$ are complex constants representing the amplitude and phase of waves travelling down and up respectively. Differentiating with respect to $z$ and substituting into equation [A.12] gives

$$H = \frac{k}{i\omega \mu} S e^{k_z z} - \frac{k}{i\omega \mu} R e^{k_z z} \quad [A.17]$$

This is usually rewritten

$$H = \frac{S e^{k_z z}}{Z_C} - \frac{R e^{k_z z}}{Z_C} \quad [A.18]$$

where $Z_C = \frac{i\omega \mu}{k} = \sqrt{\frac{i\omega \mu}{\sigma}}$ is called the characteristic impedance.

The ratio of the horizontal and electric fields at $z = 0$ is known as the surface impedance of the earth, i.e.

$$Z_s(\omega) = \frac{E(\omega)}{H(\omega)} \quad [A.19]$$
A.2.1 Surface Impedance for a Uniform Earth

If the conductivity is uniform there are no reflected waves (i.e. $R = 0$) and the relation between the electric and magnetic fields at the earth’s surface can be found from equations [A.16] and [A.18] by setting $z = 0$.

$$Z_s = \frac{E}{H} = \sqrt{\frac{i\omega \mu}{\sigma}}$$ \[A.20\]

A.2.2 Surface Impedance for a Multi-Layer Earth

In practice the conductivity of the earth varies significantly with depth. This can be modeled by representing the earth as a series of layers above a uniform semi-infinite region, and the electrical response calculated in a way that is analogous to transmission line theory.

Consider a single layer with propagation constant $k_z$ and characteristic impedance $Z_C$ as shown in Figure A.1.

![Figure A.1 – Electric and magnetic fields at the top and bottom surfaces of a layer.](image)

Within this layer the electric and magnetic fields are given by

$$E = S_1 e^{k_z z} + R_1 e^{k_z z}$$ \[A.21\]

$$H = \frac{S_1 e^{k_z z}}{Z_C} - \frac{R_1 e^{k_z z}}{Z_C}$$ \[A.22\]

where $S_1$ and $R_1$ are amplitudes of the downward and upward travelling waves at the top surface.
The ratio of the electric and magnetic fields at the top surface of the layer can be obtained by combining equations [A.21] and [A.22] with \( z = 0 \) to give

\[
\frac{E_t}{H_t} = \left( \frac{l + R_t / S_t}{l - R_t / S_t} \right) Z_C \tag{A.23}
\]

Similarly, the ratio of the electric and magnetic fields at the bottom edge of the layer of thickness, \( l \), can be found by combining equations [A.21] and [A.22] with \( z = l \). This represents the terminating impedance, \( Z_T \), seen by the layer

\[
Z_T = \frac{E_2}{H_2} = \frac{S_t e^{k_z l} + R_t e^{k_z l}}{S_t e^{k_z l} - R_t e^{k_z l}} Z_C \tag{A.24}
\]

Rearranging equation [A.24] gives

\[
Z_T = \left( \frac{e^{2k_z l} + R_t / S_t}{e^{2k_z l} - R_t / S_t} \right) Z_C \tag{A.25}
\]

Collecting terms in \( R_t / S_t \) and rearranging gives the ratio of the reflected and incident waves

\[
\frac{R_t}{S_t} = \left( \frac{Z_T - Z_C}{Z_T + Z_C} \right) e^{2k_z l} \tag{A.26}
\]

Substituting this into equation [A.23], and rearranging, gives

\[
\frac{E_t}{H_t} = \left( \frac{(Z_T + Z_C) e^{k_z l} + (Z_T - Z_C) e^{k_z l}}{(Z_T + Z_C) e^{k_z l} - (Z_T - Z_C) e^{k_z l}} \right) Z_C \tag{A.27}
\]

showing that the ratio of the electric and magnetic fields at the top surface of the layer can be expressed in terms of the characteristics of the layer and the terminating impedance at the bottom edge of the layer.

For a multi-layer model of the earth, the terminating impedance seen by the bottom layer is given by the surface impedance of the lower half-space with the appropriate value of conductivity inserted

\[
Z_T = \sqrt{\frac{i\omega\mu}{\sigma}} \tag{A.28}
\]
This can be used in equation [A.27] to calculate the ratio of the fields at the upper surface of the bottom layer. This then represents the terminating impedance seen by the next layer up and again equation [A.27] can be used to calculate the field ratio at the upper surface of this layer, remembering that $Z_C$ is dependent on the conductivity and is different for each layer. These calculations are repeated for as many layers as there are in the conductivity model to give the ratio of the electric and magnetic fields at the surface of the earth, i.e. the surface impedance of the earth, $Z_s$. This surface impedance can then be used to calculate the propagation constant, $k_z$, and complex skin depth, $p$, for the multi-layer model of the earth.

$$k_z = \frac{1}{p} = \frac{i \omega \mu}{Z_s} \quad [A.29]$$

### A.3 Non-Uniform Source Fields

When the source field is not uniform the spatial variations of the field in the horizontal directions have to be taken into account, i.e. the differential terms

$$\frac{\partial E}{\partial x}, \frac{\partial E}{\partial y} \quad \text{and} \quad \frac{\partial H}{\partial x}, \frac{\partial H}{\partial y}$$

are now not necessarily zero and have to be included.

Consider a source field with a horizontal spatial dependence given by $E = E_0 \cos(\nu x)$ where $\nu$ is the horizontal wavenumber. Differentiating twice with respect to $x$ then gives

$$\frac{\partial^2 E}{\partial x^2} = -\nu^2 E \quad [A.30]$$

The diffusion equation [A.10] now becomes

$$\frac{\partial^2 E}{\partial z^2} - \nu^2 E = i \omega \mu \sigma \quad E \quad [A.31]$$

which can be re-written

$$\frac{\partial^2 E}{\partial z^2} = \left( \nu^2 + i \omega \mu \sigma \right) E \quad [A.32]$$

This has solutions of the form

$$E = \left( A e^{\theta z} + B e^{-\theta z} \right) \cos(\nu x) \quad [A.33]$$

where

$$\theta = \left( \nu^2 + i \omega \mu \sigma \right)^{1/2} \quad [A.34]$$
A.3.1 Line Current Source

For a line current source, the fields can be expressed as a Fourier integral over all spatial wavenumbers. The horizontal and vertical components of the magnetic field and horizontal electric field are then given by

\[ B_x = \frac{\mu_0 I}{2\pi} \int_0^\infty \left[ 1 + R_S \right] e^{i\nu x} \cos \nu y \, d\nu \]  \hspace{1cm} \text{(A.35)}

\[ B_z = -\frac{\mu_0 I}{2\pi} \int_0^\infty \left[ 1 - R_S \right] e^{i\nu x} \sin \nu y \, d\nu \]  \hspace{1cm} \text{(A.36)}

and

\[ E_y = -\frac{\mu_0 I}{2\pi} i\omega \int_0^\infty \left[ 1 - R_S \right] e^{-i\nu x} \cos \nu y \, d\nu \]  \hspace{1cm} \text{(A.37)}

where \( h \) is the height of the line current, \( I \), and \( x \) is the horizontal distance from the line current. \( R_S \) is the reflection coefficient at the earth’s surface and is dependent on the conductivity structure of the earth and on the frequency \( \omega \) and the wavenumber \( \nu \).

The incident parts in expressions [A.35] and [A.36] can be written\(^1\) in the simpler forms

\[ B_x^{inc} = \frac{\mu_0 I}{2\pi} \frac{h}{h^2 + x^2} \]  \hspace{1cm} \text{(A.38)}

and

\[ B_z^{inc} = -\frac{\mu_0 I}{2\pi} \frac{x}{h^2 + x^2} \]  \hspace{1cm} \text{(A.39)}

which are simply the Biot-Savart expressions for the external line current. However, the expression for the reflected part cannot be simplified in the same way because of the \( R_S \) term.

The surface reflection coefficient, \( R_S \), is related to the propagation constants above and below the surface

\[ R_S = \frac{k_z - \nu}{k_z + \nu} \]  \hspace{1cm} \text{(A.40)}

Substituting \( k_z = 1/p \) gives

\[ R_S = \frac{1/p - \nu}{1/p + \nu} \]  \hspace{1cm} \text{(A.41)}

This can be rewritten in the form

\[ R_S = I - 2 \, p \nu \left( \frac{1}{1 + p \nu} \right) \]  \hspace{1cm} [A.42]

which, using the expansion \( \frac{1}{1 + x} = 1 - x + x^2 - x^3 + \ldots \), becomes

\[ R_S = I - 2 \, p \nu + 2(p \nu)^2 - 2(p \nu)^3 + 2(p \nu)^4 \ldots \]  \hspace{1cm} [A.43]

Compare this to the expansion of the exponential function

\[ e^{-2p \nu} = I - 2 \, p \nu + 2(p \nu)^2 - \frac{4}{3}(p \nu)^3 + \frac{2}{3}(p \nu)^4 \ldots \]  \hspace{1cm} [A.44]

It will be seen that, for \((p \nu)^3 << 1\), these two expressions are identical and we can use the approximation:

\[ R_S = e^{-2p \nu} \]  \hspace{1cm} [A.45]

The reflected parts of the magnetic field can now be written in the form

\[ B_x^{\text{refl}} = \frac{\mu_0 I}{2\pi} \int_0^\infty e^{-i(h+2p \nu)} \cos \nu x \ d\nu \]  \hspace{1cm} [A.46]

\[ B_z^{\text{refl}} = \frac{\mu_0 I}{2\pi} \int_0^\infty e^{-i(h+2p \nu)} \sin \nu x \ d\nu \]  \hspace{1cm} [A.47]

which can be simplified, in the same way as the incident parts, to give

\[ B_x^{\text{refl}} = \frac{\mu_0 I}{2\pi} \frac{h + 2p}{(h + 2p)^2 + x^2} \]  \hspace{1cm} [A.48]

\[ B_z^{\text{refl}} = \frac{\mu_0 I}{2\pi} \frac{x}{(h + 2p)^2 + x^2} \]  \hspace{1cm} [A.49]
Thus the magnetic fields at the surface of the earth due to a line current at height, \(h\), are given by

\[
B_z = \frac{\mu_0 I}{2\pi} \left( \frac{h}{h^2 + x^2} + \frac{h + 2p}{(h + 2p)^2 + x^2} \right) \quad \text{(A.50)}
\]

and

\[
B_z = \frac{-\mu_0 I}{2\pi} \left( \frac{x}{h^2 + x^2} - \frac{x}{(h + 2p)^2 + x^2} \right) \quad \text{(A.51)}
\]

which is the field of the line current plus the field of an image current at complex depth \(h + 2p\), as illustrated in Figure A.2.

A simpler expression can also be obtained for the electric field. Using the approximation that \(R_S = e^{-2\pi v}\) in equation [A.37] and expressing the \(\cos vx\) in terms of exponential functions we obtain

\[
E_y = -\frac{i\omega \mu_0 I}{2\pi} \left[ \int_0^\infty \frac{e^{-(h-ix)v} - e^{-(h-2p-ix)v}}{v} dv + \int_0^\infty \frac{e^{-(h-ix)v} - e^{-(h-2p-ix)v}}{v} dv \right] \quad \text{(A.52)}
\]

Using the integral relation \(\int_0^\infty \frac{e^{-\alpha x} - e^{-\beta x}}{x} dx = \ln \frac{\beta}{\alpha}\) this becomes, after minor algebra

\[
E_y = -\frac{i\omega \mu_0 I}{2\pi} \ln \left[ \frac{\sqrt{(h + 2p)^2 + x^2}}{\sqrt{h^2 + x^2}} \right] \quad \text{(A.53)}
\]
A.3.2 Wide Electrojet Source

The real ionospheric current systems, such as the auroral electrojet and the equatorial electrojet, are hundreds of kilometres wide. The width of the current system can be included in the magnetic field calculations if we assume that the current has a Cauchy distribution:

\[
j_c(x) = \frac{I}{\pi} \frac{a}{x^2 + a^2}
\]  

[A.54]

In this case the expressions for the magnetic and electric fields are

\[
B_x(x) = \frac{\mu_0 I}{2\pi} \int_0^\infty (1 + R_S) e^{-x(h-a)} \cos \nu x \, d\nu
\]  

[A.55]

\[
B_z(x) = -\frac{\mu_0 I}{2\pi} \int_0^\infty (1 - R_S) e^{-x(h-a)} \sin \nu x \, d\nu
\]  

[A.56]

\[
E_y(x) = -\frac{\mu_0 I}{2\pi} \int_0^\infty \frac{i\nu}{\nu} (1 - R_S) e^{-x(h-a)} \cos \nu x \, d\nu
\]  

[A.57]

Comparing equations [A.55], [A.56], [A.57] with equations [A.35], [A.36], [A.37] shows that the expressions for a Cauchy distribution are identical to those for a line current except for the change in the exponential term. This is equivalent to an increase in height of the line current. Thus the magnetic fields produced by a Cauchy distribution with half-width \(a\) at a height \(h\) are exactly the same as the fields produced by a line current at a height \(h + a\). Using the approximation for the reflection coefficient, as for the line current, now gives the expressions\(^2\):

\[
B_x = \frac{\mu_0 I}{2\pi} \left( \frac{h+a}{(h+a)^2 + x^2} + \frac{h+a+2p}{(h+a+2p)^2 + x^2} \right)
\]  

[A.58]

\[
B_z = -\frac{\mu_0 I}{2\pi} \left( \frac{h}{(h+a)^2 + x^2} - \frac{x}{(h+a+2p)^2 + x^2} \right)
\]  

[A.59]

\[
E_y = -\frac{i\nu \mu_0 I}{2\pi} \ln \left[ \frac{\sqrt{(h+a+2p)^2 + x^2}}{\sqrt{(h+a)^2 + x^2}} \right]
\]  

[A.60]

---

B

Tidal and Coast Effects
### B.1 The Tidal Dynamo

Any electrical conductor moving through a magnetic field has an electric field generated in it. As well as its frequent application in electrical machinery, this physical law also applies to the tidal movement of conducting seawater through the earth’s magnetic field.

Longuet-Higgins\(^1\) has calculated the voltage generated by water flow through a long straight shallow channel of semi-elliptical cross-section (Figure B.1), taking into account the partial ‘shorting-out’ of voltages by return currents flowing in the saturated sediments of the seabed. He showed that the potential gradient in volts/metre is given by

\[
\frac{dV}{dx} = \frac{v_w B_Z}{1 + \frac{\rho_w W}{\rho_e 2 D}}
\]

where:
- \(v_w\) = average velocity of the water (metres/sec)
- \(W\) = width of the channel (metres)
- \(D\) = depth of the channel (metres)
- \(B_Z\) = vertical component of the magnetic field (Tesla)
- \(\rho_w\) = resistivity of the water (ohm-metres)
- \(\rho_e\) = resistivity of the earth (ohm-metres)

\[
\text{Figure B.1 – Tidal water flow in a semi-elliptical channel}
\]

The circuit is completed by return currents that extend below the sea and inland from the shorelines to distances comparable with the width of the channel.

In equation [B.1] \( v_w B_z \) is the electromotive force generated by the water movement through the Earth’s magnetic field. The fractional term represents the effect of short-circuiting of currents through the Earth beneath the channel.

The total potential difference between opposite sides of the channels can then be found by integrating equation [B.1]. Assuming a uniform flow rate in the channel, the potential difference is given by

\[
V_{12} = v_w B_z W \left( \frac{1}{1 + \frac{\rho_w W}{\rho_e 2D}} \right) \quad \text{[B.2]}
\]

In cases where the ‘shorting-out’ by return currents was negligible i.e.,

\[
\frac{\rho_e W}{\rho_e 2D} \ll 1 \quad \text{[B.3]}
\]

the potential difference across the channel is:

\[
V_{12} = v_w B_z W \quad \text{[B.4]}
\]

This equation shows, not surprisingly, that the potential generated is proportional to the speed of the water flow, \( v \), the strength of the vertical magnetic field \( B_z \), and the width of the channel, \( W \).

Equation [B.4] represents the maximum potential difference that can be produced across a channel. However, in many cases the condition [B.3] is not satisfied and equation [B.2] should be used.
B.2 Coastal Modeling

The simplest situation to consider is a long straight coastline where the conductivity of the earth changes in only 1 dimension; that being the direction perpendicular to the coast. This 1-dimensional coast effect has been analysed using a generalized thin sheet approach by Ranganayaki and Madden.\(^2\) The thin sheet model consists of two thin layers above a basement with a surface impedance \(Z_T\). The thin sheet comprises a conducting upper layer above a resistive lower layer (Figure B.2). These represent the more conductive crustal sediments or ocean layer above the more resistive lower crust. The basement represents the mantle of the earth.

In the thin sheet approximation it is assumed that the skin depth in the upper layer is large compared to the layer thickness so that electric field is constant over this layer. Most of the current flows in this upper conducting layer and this produces a change in the magnetic field. In contrast, in the more resistive lower layer the current is small and the magnetic field is assumed to be constant over this layer and instead there is a change in the electric field. Thus, for electric and magnetic fields, \(E_U\) and \(H_U\) at the upper surface and \(E_L\) and \(H_L\) at the lower surface, as shown in Figure B.2, the electric and magnetic fields at the boundary between the two layers will be \(E_U\) and \(H_L\)

\[
\begin{array}{c|c|c}
\text{Conductivity, } \sigma & \Delta z_1 & \text{E constant, H changes} \\
\hline
\text{Resistivity, } \rho & \Delta z_2 & \text{E changes, H constant} \\
\hline
E_U & \Delta z_1 \\
H_U & \\
E_L & \Delta z_2 \\
H_L & \\
\end{array}
\]

Figure B.2 – Generalized thin sheet model

The two layers respectively have an integrated conductivity and integrated resistivity:

\[
\sigma_s = \sigma \Delta z_1 \quad \text{[B.5]}
\]

\[
\rho_s = \rho \Delta z_2 \quad \text{[B.6]}
\]

To simplify the mathematics we will consider a simple two-dimensional situation, such as an electric field perpendicular to a coastline as shown in Figure B.2. This is represented by a conductivity boundary along the y axis, with integrated conductivity and resistivity in each region given by

\[ \sigma_x = \sigma_x, 1 \quad \rho_x = \rho_x, 1 \quad \text{for } x > 0 \quad [B.7] \]

\[ \sigma_x = \sigma_x, 2 \quad \rho_x = \rho_x, 2 \quad \text{for } x > 0 \quad [B.8] \]

\[ \nabla \times \mathbf{E} = \mu_0 \mathbf{H} \quad [B.9] \]

\[ \nabla \times \mathbf{H} = \sigma \mathbf{E} \quad [B.10] \]

Writing out the curl gives

\[ i \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) + j \left( \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) + k \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) = i J_x + j J_y + k J_z \quad [B.11] \]

where \( i, j \) and \( k \) are the unit vectors in the \( x, y \) and \( z \) directions respectively.
For an H-polarised source field the electric field has component \( E_0^x \) and the magnetic field has component \( H_0^y \). Thus \( H_y = 0 \) and \( H_z = 0 \). Equating the \( j \) components gives \( J_j = 0 \). Equating the \( i \) and \( k \) components gives

\[
-\frac{\partial H_x}{\partial z} = J_x \tag{B.12}
\]

and

\[
\frac{\partial H_y}{\partial z} = J_x \tag{B.13}
\]

Using the thin sheet conductivity and resistivity these equations become

\[
-\frac{\partial H_y}{\partial z} = \sigma \ E_x \tag{B.14}
\]

and

\[
\frac{\partial H_x}{\partial z} = \frac{l}{\rho} \ E_z \tag{B.15}
\]

Similarly, expanding equation (B.10) gives

\[
i \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + j \left( \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + k \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = (iH_x + jH_y + kH_z) \ i\omega \mu \tag{B.16}
\]

For the 2 dimensional case, \( H_z = 0 \) and \( H_y = 0 \). Equating \( j \) components gives

\[
\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = i\omega \mu \ H_y \tag{B.17}
\]

Re-arranging and substituting for \( E_z \) from equation [B.15] gives

\[
\frac{\partial E_x}{\partial z} = i\omega \mu \ H_y + \frac{\partial}{\partial x} \left( \rho \frac{\partial H_y}{\partial x} \right) \tag{B.18}
\]

Equations [B.14], [B.15] and [B.18] describe the field relations in the thin sheet.
Applying equation [B.14] to the change of the magnetic field across the top layer of the thin sheet gives

\[
\frac{\Delta H_z}{\Delta z_1} = \frac{\partial H_y}{\partial z} = -\sigma E_x^{\prime}\mu
\]

This gives

\[
\Delta H_y = H_y^{\prime\prime} - H_y^L = -\sigma \Delta z_1 E_x^{\prime}\mu
\]

Applying equation [B.18] to the change of electric field across the lower layer of the generalized thin sheet where the magnetic field is constant and equal to \(H_y^L\) gives

\[
\frac{\Delta E_x}{\Delta z_2} = \frac{\partial E_x}{\partial z} = \rho \frac{\partial^2}{\partial x^2} H_y^L + i\omega \mu \Delta z_2 H_y^L
\]

\[
\Delta E_x = E_x^{\prime\prime} - E_x^L = \rho \Delta z_2 \frac{\partial^2}{\partial x^2} H_y^L + i\omega \mu \Delta z_2 H_y^L
\]

\[
E_x^{\prime\prime} = \rho \Delta z_2 \frac{\partial^2}{\partial x^2} H_y^L + E_x^L + i\omega \mu \Delta z_2 H_y^L
\]

The electric and magnetic fields at the lower edge of the thin sheet are related by the surface impedance of the basement, ie \(E_x^L = Z_T H_y^L\). Substituting this into equation [B.23] gives

\[
E_x^{\prime\prime} = \rho \Delta z_2 \frac{\partial^2}{\partial x^2} H_y^L + \left( Z_T + i\omega \mu \Delta z_2 \right) H_y^L
\]

\[
E_x^{\prime\prime} = \rho \Delta z_2 \frac{\partial^2}{\partial x^2} H_y^L + \left( Z_T + i\omega \mu \Delta z_2 \right) H_y^L
\]

In the absence of any boundaries \(\frac{\partial H_y^L}{\partial x} = 0\) so the first term is zero and the electric field is given by the last term which represents the incident electric field \(E_x^0\) produced at the surface of the layered model. Thus equation [B.24] can be written

\[
E_x^{\prime\prime} = \rho \Delta z_2 \frac{\partial^2}{\partial x^2} H_y^L + E_x^0
\]
Substituting for $H^U_y$ from equation [B.20] gives

$$E_x^U = \rho \Delta z_2 \frac{\partial^2}{\partial X^2} \left( \sigma \Delta Z E_x^U + H^U_y \right) + E_x^0 \tag{B.26}$$

$H^U_y$ is constant along the surface, so the term involving $\frac{\partial^2 H^U_y}{\partial X^2}$ is zero. Also substituting for $\rho \Delta z_2$ and $\sigma \Delta Z$ gives

$$E_x^U = \rho_s \frac{\partial^2}{\partial x^2} (\sigma_s E_x^U) + E_x^0 \tag{B.27}$$

For regions where $\sigma_s$ is constant

$$E_x^U = \rho_s \sigma_s \frac{\partial^2}{\partial x^2} E_x^U \equiv E_x^0 \tag{B.28}$$

This has solutions of the form

$$E_x^U = E_x^0 + A_{\pm} \exp \left( \frac{\pm x}{\sqrt{\rho_s \sigma_s}} \right) \tag{B.29}$$

General solutions, for multiple boundaries, can be built up from this expression but the constants $A$ depend on conditions at more than one boundary.

The last term in equation [B.29] can be written as a gradient of a scalar potential, i.e.

$$E_x^U = E_x^0 + \frac{\partial V}{\partial x} \tag{B.30}$$

where

$$V = A' \exp \left( \frac{\pm x}{\sqrt{\rho_s \sigma_s}} \right) \tag{B.31}$$

and hence

$$\frac{\partial V}{\partial x} = \pm \sqrt{\rho_s \sigma_s} A' \exp \left( \frac{\pm x}{\sqrt{\rho_s \sigma_s}} \right) \tag{B.32}$$
For the simple case of a single conductivity boundary

\[ E_x = E_x^0 + \psi_1 V_b \exp(\psi_1 x) \quad x < 0 \]  \hspace{1cm} \text{[B.33]}

\[ E_x = E_x^0 - \psi_2 V_b \exp(-\psi_2 x) \quad x > 0 \]  \hspace{1cm} \text{[B.34]}

where

\[ \psi_1 = \frac{1}{\sqrt{\sigma_1 \rho_1}} \quad \text{and} \quad \psi_2 = \frac{1}{\sqrt{\sigma_2 \rho_2}} \]  \hspace{1cm} \text{[B.35]}

and \( E_x^0 \) and \( E_x^0 \) are the electric fields, well away from the boundary, in each region.

The boundary voltage \( V_b \) is determined from current continuity at the boundary

\[ \sigma_1 I (E_x^0 + \psi_1 V_b) = \sigma_2 I (E_x^2 + \psi_2 V_b) \]  \hspace{1cm} \text{[B.36]}

This gives

\[ V_b = \frac{\sigma_2 E_x^2 \psi_2 - \sigma_1 E_x^1 \psi_1}{\sigma_1 \psi_1 + \sigma_2 \psi_2} \]  \hspace{1cm} \text{[B.37]}

The earth voltage near the boundary is then given by

\[ V(x) = V_b \exp(\psi_1 x) \quad x < 0 \]  \hspace{1cm} \text{[B.38]}

\[ V(x) = V_b \exp(-\psi_2 x) \quad x > 0 \]  \hspace{1cm} \text{[B.39]}
C
Interaction of Electromagnetic Waves with Pipelines
C.1 Interaction of Electromagnetic Waves with Pipelines

In this appendix general expressions are derived that fully describe the electromagnetic response of a pipeline to an incident geomagnetic field. It is shown how the problem can be set up to include an arbitrary number of layers in the pipeline. This easily leads to expressions for a pipeline that includes a hollow (gas or oil filled) conducting cylinder with an insulating coating, in contrast to earlier studies that sometimes only considered solid pipes or neglected the pipeline coating. The full expressions can be used to calculate the electric and magnetic fields in any region of the pipe and in the surrounding medium.

C.2 The Mathematical Model

Induction in a pipeline is modeled as excitation of a multi-layered infinitely-long cylinder in a uniform surrounding medium. The mathematical basis for this model is given by Kaufman and Keller. We use the geomagnetic coordinate system with horizontal components $x$ and $y$, and $z$ vertically downwards. The primary field outside the cylinder has the form of plane waves with the direction of propagation downward (in positive $z$ direction). The multi-layered cylinder is aligned with the $y$ axis and is placed in the conducting earth (layer 1). To apply this model to pipelines use a cylinder with three layers: the outside insulating coating (layer 2), the pipe steel (layer 3), and the gas inside the pipe (layer 4), each with its corresponding conductivity as shown in Figure C.1.

![Figure C.1 – Coordinate system and characteristics of the pipeline layers.](image)

---


C.3 E-polarization

For an electromagnetic field with \( E \) parallel to the infinitely long cylinder, the electric field in any layer satisfies the equation

\[
\nabla^2 E - k^2 E = 0 \tag{C.1}
\]

where \( k \) is the propagation constant of the particular layer given by

\[
k^2 = i\mu\sigma (\omega + i\omega\mu) \tag{C.2}
\]

In cylindrical coordinates, equation [C.1] can be written

\[
\frac{\partial^2 E}{\partial r^2} + \frac{1}{r} \frac{\partial E}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E}{\partial \phi^2} + k^2 E = 0 \tag{C.3}
\]

Expressing the electric field as \( E = P(r)\Phi(\phi) \), and through the separation of variables, equation (C.3) transforms into two ordinary equations

\[
\frac{d^2 P}{dr^2} + \frac{1}{r} \frac{dP}{dr} - \left( k^2 + \frac{n^2}{r^2} \right) P = 0 \tag{C.4}
\]

and

\[
\frac{d^2 \Phi}{d\phi^2} + n^2 \Phi = 0 \tag{C.5}
\]

where \( n \) is the constant of separation.

Solutions for [C.4] and [C.5] are

\[
P_n(kr) = C_n I_n + D_n K_n(kr) \tag{C.6}
\]

and

\[
\Phi_n(\phi) = A_n^+ \cos(n\phi) + B_n^+ \sin(n\phi) \tag{C.7}
\]

where \( I_n(kr), K_n(kr) \) are modified Bessel functions of the first and second kind.
The electric field in any layer except the inner can be represented as the sum of an incident part \( E_{\text{inc}} \) and a reflected part \( E_{\text{refl}} \).

\[
E = E_{\text{inc}} + E_{\text{refl}}
\]  \hspace{1cm} \text{[C.8]}

For the reflected wave we have to use Bessel function of the second kind, because it goes to 0 in infinity, but it has singularity at \( kr = 0 \) and cannot also describe the incident part. For the incident part we have to use the Bessel function of the first kind, that goes to a constant at \( kr = 0 \) and to infinity at \( kr \to \infty \).

For the outer layer (i.e. the surrounding medium) the incident part is the field that would exist in an area undisturbed by the pipeline. In the case of an incident plane wave, the incident electric field in the earth (layer 1) can be represented in cylindrical coordinates in terms of a Fourier series

\[
E_{\text{inc}} = E_0 e^{kz} = E_0 \sum_{n=0}^{\infty} \alpha_n I_n(k_1 r) \cos n\phi
\]  \hspace{1cm} \text{[C.9]}

For the reflected part, in layer 1, we have

\[
E_{\text{refl}} = E_0 \sum_{n=0}^{\infty} \alpha_n R_{1n} K_n(k_1 r) \cos n\phi
\]  \hspace{1cm} \text{[C.10]}

where \( \alpha_n = 1 \) if \( n = 0 \), and \( \alpha_n = 2 \) if \( n \neq 0 \),

\( E_0 \) is the amplitude of incident wave,

\( R_{1n} \) is the reflection coefficient in the earth (first layer),

\( k_1 = \sqrt{i \mu_\omega \omega (\sigma_1 + i \omega \epsilon_1)} \) is the propagation constant in the first layer.

Therefore, the electric field in the earth (layer 1) can be expressed as

\[
E_1 = E_0 \sum_{n=0}^{\infty} \alpha_n \left( I_n(k_1 r) + R_{1n} K_n(k_1 r) \right) \cos n\phi
\]  \hspace{1cm} \text{[C.11]}

The electric field in the coating (layer 2) is given by

\[
E_2 = \sum_{n=0}^{\infty} A_{2n} \left( I_n(k_2 r) + R_{2n} K_n(k_2 r) \right) \cos n\phi
\]  \hspace{1cm} \text{[C.12]}
The electric field in the pipe steel (layer 3) is given by

\[
E_3 = \sum_{n=0}^{\infty} A_{3n} \left( I_n (k_3 r) + R_{3n} K_n (k_3 r) \right) \cos n\phi
\]  

[C.13]

The electric field in the oil or gas (layer 4) is given by

\[
E_4 = \sum_{n=0}^{\infty} A_{4n} I_n (k_4 r) \cos n\phi
\]  

[C.14]

In equations [C.4], [C.5], and [C.6] the first term represents the incident wave coming in from the outer layer and the second term represents the wave reflected from the boundary with the inner layer. In equation [C.7] for the electric field in the inner-most layer, there is only the incident part.

The amplitudes, A, and reflection coefficients, R, can be found from the boundary conditions and are defined in terms of the amplitude of the incident electric field and the electromagnetic properties of the layers.

The amplitude of the telluric current flowing along the pipe steel (layer 3) can be derived from the differential form of Ohm’s law \( j = \sigma E \) as

\[
I_3 = \sigma_3 \int_0^{2\pi} d\phi \int_{r_3^0}^{r_3^1} rE_3 dr
\]  

[C.15]

It follows from the formula for the electric field [C.6], that all harmonics with order more than 0 will give no current (\( \int_0^{2\pi} \cos n\phi d\phi = 0 \)) and that the net current depends only on the 0-harmonic of the electric field. After integrating, the final formula for the telluric current is:

\[
I_3 = \frac{2\pi \sigma_3}{k_3} A_{3,0} \left( I_3^\text{out} K_3 (k_3 r_3^0) - I_3^\text{in} K_3 (k_3 r_3^0) \right)
\]  

[C.16]

where \( A_{3,0} \) is the amplitude of the electric field in layer 3.
C.4 H-polarization

When the magnetic field is parallel to the infinitely long cylinder, the magnetic field is given by

$$\nabla^2 H - k^2 H = 0$$  \[C.17\]

where $k$ is the propagation constant defined by [C.2].

This is in the same form as the expression for the electric field in the E-polarization case and has solutions of the same form only in terms of the magnetic field parallel to the pipe. The expressions for the electric field in the H-polarization case can then be obtained from Maxwell’s equations as

$$E_\phi = -\frac{1}{\sigma} \frac{\partial H}{\partial r}$$  \[C.16\]

$$E_r = \frac{1}{\sigma} \frac{\partial H}{\partial \phi}$$  \[C.17\]

The electric field radial, $E_r$, and tangential, $E_\phi$, components in the earth surrounding the pipe are

$$E_{r1} = -\frac{1}{i\sigma_1 \omega r} H_0 \sum_{n=0}^{\infty} \alpha_n n \left( I_n(k_1 r) + R_{(H)1n} K_n(k_1 r) \right) \sin n\phi$$  \[C.18\]

$$E_{\phi 1} = \frac{k_1}{i\sigma_1 \omega} H_0 \sum_{n=0}^{\infty} \alpha_n \left( I_n'(k_1 r) + R_{(H)1n} K_n'(k_1 r) \right) \cos n\phi$$  \[C.19\]

The electric field components in the coating (layer 2) are given by

$$E_{r2} = -\frac{1}{i\sigma_2 \omega r} \sum_{n=0}^{\infty} \alpha_n \left( I_n(k_2 r) + R_{(H)2n} K_n(k_2 r) \right) \sin n\phi$$  \[C.20\]

$$E_{\phi 2} = \frac{k_2}{i\sigma_2 \omega} \sum_{n=0}^{\infty} \alpha_n \left( I_n'(k_2 r) + R_{(H)2n} K_n'(k_2 r) \right) \cos n\phi$$  \[C.21\]
The electric field components in the pipe steel (layer 3) are given by

\[ E_{r3} = \frac{-i}{i \sigma \omega r} \sum_{n=0}^{\infty} A_{(H)3n} \ n \left( I_n(k_J r) + R_{(H)3n} K_n(k_J r) \right) \sin n\phi \]

\[ E_{\phi3} = \frac{k_2}{i \sigma \omega} \sum_{n=0}^{\infty} A_{(H)3n} \left( I_n'(k_J r) + R_{(H)3n} K_n'(k_J r) \right) \cos n\phi \]  

The electric field components in the oil or gas (layer 4) are given by

\[ E_{r4} = \frac{-i}{i \sigma \omega r} \sum_{n=0}^{\infty} A_{(H)4n} \ n \left( I_n(k_J r) \right) \sin n\phi \]

\[ E_{\phi4} = \frac{k_2}{i \sigma \omega} \sum_{n=0}^{\infty} A_{(H)4n} \ I_n'(k_J r) \cos n\phi \]

Unknown amplitudes \( A_{(H)} \) and reflection coefficients \( R_{(H)} \) can be found from the boundary conditions\(^3\). Equations [C.11] to [C.18] thus describe the electric fields anywhere in the pipe and the surrounding earth produced by an H-polarized incident wave.

---

DSTL Modeling of Pipelines
This Page is Intentionally Blank
D.1 DSTL Modeling of Pipelines

The effect of electric fields induced in pipelines can be modeled using distributed-source transmission line (DSTL) theory first described by Schelkunoff. DSTL theory has been used extensively for modeling AC induction in pipelines, and was applied to geomagnetic induction in pipelines by Boteler and Cookson. This modeling was useful for a single straight pipeline. To extend the modeling to include multiple pipeline sections, the DSTL theory has been extended to include the general case of induction in a transmission line which is terminated at each end by transmission lines themselves subject to electromagnetic induction. This development allows model calculations to be made for realistic pipelines including bends, branch points, or even whole networks.

In the DSTL approach the pipeline is represented by a transmission line with a series impedance given by the resistance of the pipeline steel, and a parallel admittance given by the conductance through the pipeline coating. The induced electric field is represented by voltage sources distributed along the transmission line (Figure D.1). The series resistance and parallel conductance can be used to determine the characteristic impedance, $Z_0$, and the propagation constant, $\gamma$,

\[
\gamma = \sqrt{ZY} \quad \text{and} \quad Z_0 = \sqrt{\frac{Z}{Y}} \quad \quad [D.1]
\]

These are the key parameters that describe the electrical response of the pipeline. Another useful parameter is the inverse of the propagation constant which is a measure of the distance along the pipe for the potential to adjust to a change in characteristics.

![Figure D.1 – Distributed-source transmission line model of a pipe section.](image)

---

The transmission line equations can be written

\[ \frac{dV}{dx} + Z I = E_x \]  \hspace{1cm} [D.2]

\[ \frac{dI}{dx} + YV = 0 \]  \hspace{1cm} [D.3]

where \( E_x \) represents the line density of constant-voltage generators along the line.

\[ \frac{d^2 V}{dx^2} \gamma^2 V = 0 \]  \hspace{1cm} [D.4]

\[ \frac{d^2 I}{dx^2} \gamma^2 I = -Y E_x \]  \hspace{1cm} [D.5]

Differentiation and substitution leads to the equations for a uniform electric field

\[ I = \frac{E}{\gamma Z_0} \left( 1 + A e^{\gamma(x-x_1)} + B e^{\gamma(x_2-x)} \right) \]  \hspace{1cm} [D.6]

These have solutions of the form

\[ V = \frac{E}{\gamma} \left( A e^{\gamma(x-x_1)} - B e^{\gamma(x_2-x)} \right) \]  \hspace{1cm} [D.7]

and where A and B are constants dependent on the conditions at the ends of the line.
D.2 Equations for an Active Line with Active Terminations

Consider a uniform section of transmission line, extending from $x = 0$ to $x = L$, terminated at each end by a Thevenin equivalent circuit, as shown in Figure D.2.

![Figure D.2 – Transmission line terminated at each end by a Thevenin equivalent circuit.](image)

The voltage and current at $x = 0$ are

$$V(0) = \frac{E}{\gamma} \left( A - B e^{\gamma L} \right)$$  \[D.8\]

$$I(0) = \frac{E}{\gamma Z_o} \left( I + A + B e^{\gamma L} \right)$$  \[D.9\]

and are linked by the expression

$$V(0) = V_1 - Z_1 I(0)$$  \[D.10\]

Similarly the voltage and current at $x = L$ are

$$V(L) = \frac{E}{\gamma} \left( A e^{\gamma L} - B \right)$$  \[D.11\]

$$I(L) = \frac{E}{\gamma Z_o} \left( I + A e^{\gamma L} + B \right)$$  \[D.12\]

and are linked by the expression

$$V(L) = V_2 + Z_2 I(L)$$  \[D.13\]
Thus there are two independent equations

\[
\frac{E}{\gamma} \left( A - B e^{\gamma l} \right) = V_1 - Z_1 \frac{E}{\gamma Z_o} \left( I + A + B e^{\gamma l} \right) \tag{D.14}
\]

\[
\frac{E}{\gamma} \left( A e^{\gamma l} - B \right) = V_2 + Z_2 \frac{E}{\gamma Z_o} \left( I + A e^{\gamma l} + B \right) \tag{D.15}
\]

from which \( A \) and \( B \) can be found. Collecting terms in \( A \) and \( B \) gives the equations

\[
(Z_o + Z_1) A + (Z_o - Z_1) B e^{\gamma l} = Z_o \frac{\gamma}{E} V_1 - Z_1 \tag{D.16}
\]

\[
(Z_o - Z_2) A e^{\gamma l} + (Z_o + Z_2) B = Z_o \frac{\gamma}{E} V_2 + Z_2 \tag{D.17}
\]

which can be solved to give

\[
A = \frac{(Z_1 - Z_o) (Z_2 + Z_o \frac{\gamma}{E} V_2) - (Z_o + Z_2) (Z_1 - Z_o \frac{\gamma}{E} V_1) e^{\gamma l}}{(Z_o + Z_1) (Z_o + Z_2) e^{\gamma l} - (Z_1 - Z_o) (Z_2 - Z_o) e^{\gamma l}} \tag{D.18}
\]

\[
B = \frac{(Z_2 - Z_o) (Z_1 + Z_o \frac{\gamma}{E} V_1) - (Z_1 + Z_o) (Z_2 - Z_o \frac{\gamma}{E} V_2) e^{\gamma l}}{(Z_o + Z_1) (Z_o + Z_2) e^{\gamma l} - (Z_1 - Z_o) (Z_2 - Z_o) e^{\gamma l}} \tag{D.19}
\]

The active terminations of the line can be an equivalent circuit representing the rest of the system. To determine this, it is necessary to be able to calculate the equivalent circuit for an active line with an active termination (that may itself be the equivalent circuit for a further active line).
D.3 Equivalent Circuit for an Active Line with an Active Termination

Consider a transmission line of length $L$ with a distributed source, terminated by a Thevenin equivalent circuit, as shown in Figure D.3a. This can be represented by a new Thevenin equivalent circuit, as shown in Figure D.3b, where the constant-voltage source and the series impedance can be expressed in terms of the open-circuit voltage and short-circuit current at the end $x = 0$ of the transmission line:

\[ V_{Th} = V_{oc} \quad Z_{Th} = \frac{V_{oc}}{I_{sc}} \quad [D.20] \]

The voltage at $x = 0$ is given by:

\[ V(0) = \frac{E}{\gamma}(A - Be^{\gamma L}) \quad [D.21] \]

and when the end is short-circuit so that $V(0) = 0$ then $B_{sc} = A_{sc}e^{\gamma L}$.

Substituting this into equations [D.11] and [D.12] and combining them as in equation [D.13] gives:

\[ \frac{E}{\gamma}(A_{sc}e^{\gamma L} - A_{sc}e^{\gamma L}) = V_2 + Z_2 \frac{E}{\gamma Z_o} \left( 1 + A_{sc}e^{\gamma L} + A_{sc}e^{\gamma L} \right) \quad [D.22] \]
Collecting terms in $A_{sc}$ and then dividing gives

$$A_{sc} = \frac{-\left(\frac{\gamma}{E} V_2 Z_o + Z_2\right)}{(Z_o + Z_2) e^{\gamma L} - (Z_o - Z_2) e^{-\gamma L}} \tag{D.23}$$

which, through equation [D.21], also gives $B_{sc}$. Substituting these into equation [D.9] then gives the short-circuit current

$$I(0)_{sc} = \frac{E}{\gamma L_o} \left(2V_2 \frac{\gamma Z_o}{E} + 2Z_2 - (Z_2 + Z_o) e^{\gamma L} - (Z_2 - Z_o) e^{-\gamma L}\right) \left(\frac{Z_2 + Z_o}{e^{\gamma L} + (Z_2 - Z_o) e^{-\gamma L}}\right) \tag{D.24}$$

Similarly the current at $=0$ is given by

$$I(0) = \frac{E}{\gamma L_o} \left(I + A + Be^{-\gamma L}\right) \tag{D.25}$$

and when this end is open circuit so that $I(0)=0$ then $B_{oc} = -(I + A_{oc}) e^{\gamma L}$.

again substituting this into equations [D.11] and [D.12] and combining them as in equation [D.13] gives

$$\frac{E}{\gamma} (A_{oc} e^{\gamma L} + (I + A_{oc}) e^{\gamma L}) = V_2 + Z_2 \frac{E}{\gamma L_o} \left(I + A_{oc} e^{\gamma L} - (I + A_{oc}) e^{\gamma L}\right) \tag{D.25}$$

Collecting terms in $A_{oc}$ and then dividing gives

$$A_{oc} = \frac{\gamma}{E} V_2 Z_o + Z_2 - \left(Z_o + Z_2\right) e^{\gamma L} \tag{D.27}$$

which, through equation [D.25], also gives $B_{oc}$. 
Substituting these into equation (D.8) then gives the open-circuit voltage (equal to the Thevenin equivalent circuit voltage):

\[
V(0)_{oc} = \frac{E}{\gamma} \left( \frac{2V_2 \gamma Z_o + 2 Z_2 - (Z_2 + Z_o) e^{\gamma L} - (Z_2 - Z_o) e^{-\gamma L}}{(Z_2 + Z_o) e^{\gamma L} - (Z_2 - Z_o) e^{-\gamma L}} \right)
\]  \[\text{[D.28]}\]

Then combining this with equation [D.24] gives the Thevenin equivalent circuit impedance

\[
Z_{th} = Z_o \left( \frac{(Z_2 + Z_o) e^{\gamma L} + (Z_2 - Z_o) e^{-\gamma L}}{(Z_2 + Z_o) e^{\gamma L} - (Z_2 - Z_o) e^{-\gamma L}} \right)
\]  \[\text{[D.29]}\]

Equations [D.28] and [D.29] represent the active termination at the right-hand end of the line where a positive electric field is directed towards the termination. For the active termination at the left-hand end of the line, where the electric field is directed away from the termination, the sign of \(E\) is reversed and we use \(V_1\) and \(Z_1\) instead of \(V_2\) and \(Z_2\). This gives

\[
V_{th}(L) = \frac{-E}{\gamma} \left( \frac{2V_1 \gamma Z_o + 2 Z_1 - (Z_1 + Z_o) e^{\gamma L} - (Z_1 - Z_o) e^{-\gamma L}}{(Z_1 + Z_o) e^{\gamma L} - (Z_1 - Z_o) e^{-\gamma L}} \right)
\]  \[\text{[D.30]}\]

and

\[
Z_{th}(L) = Z_o \left( \frac{(Z_1 + Z_o) e^{\gamma L} + (Z_1 - Z_o) e^{-\gamma L}}{(Z_1 + Z_o) e^{\gamma L} - (Z_1 - Z_o) e^{-\gamma L}} \right)
\]  \[\text{[D.31]}\]

Thus the ability to calculate the equivalent circuit for an active line with an active termination and use this as the termination in the equations [D.18] and [D.19] for an active line provides a completely general solution for electromagnetic induction in any transmission line system.
Calculation of Effect of Galvanic Anodes on Pipe-to-Earth Conductance
This Page is Intentionally Blank
E.1 Consider a coating having a specific leakage conductance (G) of $10^{-6}$ S/m$^2$ in 10,000 ohm-cm soil.

For 100m of 0.5m diameter pipe the leakage conductance (g) would be:

$$g_{pipe} = G_{10,000} \times A_p$$
$$= 10^{-6} \text{ S/m}^2 \times 157 \text{ m}^2$$

$$g_{pipe} = 1.57 \times 10^{-4} \text{ S}$$

Assume a magnesium anode (20 lb. x 60” lg) is attached to the piping for each 100m length.

The conductance (g) of the anode to earth in 10,000 ohm-cm soil is given by:

$$g_{anode} = \frac{1}{R_{anode}} = \frac{2\pi L}{\rho} \times \frac{1}{\ln \frac{8L}{d} - 1}$$

where:
- $L = 1.52$ m
- $d = 0.12$ m
- $\rho = 100 \Omega \cdot \text{m}$

$$= \frac{6.28 \times 1.52}{100 \Omega \cdot \text{m}} \times \frac{1}{\ln \frac{12.16}{0.12} - 1}$$

$$= 0.095 \times \frac{1}{3.61} = 0.0263 \text{ S}$$

$g = 26.3 \times 10^{-3} \text{ S}$

The net conductance ($g_n$) for a 100m of pipe with the anode attached is therefore:

$$g_n = g_{anode} + g_{pipe}$$
$$= 26.3 \times 10^{-3} \text{ S} + 0.157 \times 10^{-3} \text{ S}$$

$$= 26.5 \times 10^{-3} \text{ S}$$

This is an increase in conductance of 167 times, well over 2 orders of magnitude.

As seen previously in Figure 2-23, an increase in conductance of this order can significantly reduce the magnitude of telluric induced voltage (i.e. 90% reduction).
This Page is Intentionally Blank
Bibliography
F.1 Telluric Current Interference


Peabody, A.W., Corrosion Aspects of Arctic Pipelines, Materials Performance, Vol. 18, No.5, May 1979, pp. 27.


F.2 Geomagnetic Effects on Ground-based Systems


Hessler, V.P., Causes, Recording Techniques and Characteristics of Telluric Currents, Materials Performance, Vol. 15, No.4, April 1976, pp.38.


Sources of Geomagnetic Activity Information
This Page is Intentionally Blank
G.1 Sources for Geomagnetic Activity Information

- Regional Warning Center (RWC) for Space Weather Canada
  http://www.geolab.nrcan.gc.ca/

- Space Environment Center, USA
  http://www.sec.noaa.gov/Data/

- RWC for Western Europe, Solar Influences Data Analysis Center, Brussels, Belgium

- Australian Space Weather Forecast Center, Sydney

There are other more general space physics sites which can also give space weather forecasts.
This Page is Intentionally Blank
List of Symbols
This Page is Intentionally Blank
H.1 General

\( j \) \( \sqrt{-1} \)

\( f \) frequency (Hz)

\( \omega \) angular frequency (\( \omega = 2\pi f \))

\( \nu \) wavenumber

\( B \) magnetic field (nT)
- \( B_x \) horizontal (northwards) component
- \( B_y \) horizontal (eastwards) component
- \( B_z \) vertical (downwards) component

\( E \) electrical field (V/km)
- \( E_x \) horizontal (northwards) component
- \( E_y \) horizontal (eastwards) component
- \( E_z \) vertical (downwards) component

\( \rho \) resistivity

\( \sigma \) conductivity (= 1/resistivity)

\( \mu \) magnetic permeability
- Usually has its free space, \( \mu_0 = 4\pi . 10^{-7} \) H/m

H.2 Geomagnetic Induction

\( Z_S \) surface impedance of the earth

\( R_S \) reflection coefficient at the earth’s surface

\( k_z \) vertical propagation constant within an earth layer

\( Z_C \) characteristic impedance within an earth layer

\( h \) height of the ionospheric current

\( a \) half-width of ionospheric current density with a Cauchy distribution

\( p \) complex skin depth in the earth \( p = Z_S/j\omega \)
H.3 Tidal Dynamo and Coast Effect

\( \Phi_{12} \) Tidal potential difference generated across channel

\( v \) speed of the water flow

\( W \) width of the channel

\( \sigma_S \) integrated conductivity of upper surface layer of the earth

\( \sigma_S = \sigma t_L \) where \( \sigma \) and \( t_L \) are conductivity and thickness of the lower layer

\( \rho_S \) integrated conductivity of upper surface layer of the earth

\( \rho_S = \rho t_L \) where \( \rho \) and \( t_L \) are resistivity and thickness of the lower layer

\( V \) earth potential at the surface

\( V_b \) earth potential at a boundary

\( \psi \) attenuation constant in the horizontal direction \( \Psi = \frac{1}{\sqrt{\sigma_s \rho_s}} \)

H.4 Geomagnetic Interaction with a Pipeline

\( A_i \) amplitudes at the surface of layers \( i = 1,2,3 \)

\( R_i \) reflection coefficients at the surface of layers \( i = 1,2,3 \)

\( I_i(kr) \) modified Bessel functions of the first kind

\( K_i(kr) \) modified Bessel functions of the second kind

\( E_3 \) electric field in pipe steel (layer 3 of cylinder model)

H.5 Distributed Source Transmission Line Model

\( Z \) series resistance along steel of a pipeline

\( Y \) parallel conductance through the coating of a pipeline

\( Z_0 \) characteristic impedance of a pipeline \( Z_0 = \frac{Z}{\sqrt{Y}} \)

\( \gamma \) propagation constant of a pipeline \( \gamma = \sqrt{ZY} \)

\( 1/\gamma \) pipeline adjustment distance