The composition of semi finished inventories at a solid board plant

Henrico L.T. Wanders [*] Gerard J.C. Gaalman [†] Gerard Sierksma [‡]

SOM-theme A **Primary processes within firms**

Abstract

A solid board factory produces rectangular sheets of cardboard in two different formats, namely large formats and small formats.

The production process consists of two stages separated by an inventory point. In the first stage a cardboard machine produces the large formats. In the second stage a part of the large formats is cut into small formats by a separate rotary cut machine. Due to very large setup times, technical restrictions, and trim losses, the cardboard machine is not able to produce these small formats.

The company follows two policies to satisfy customer demands for rotary cut format orders. When the company applies the first policy, then for each customer order an 'optimal' large format (with respect to trim loss) is determined and produced on the cardboard machine. In case of the second policy, a stock of a restricted number of large formats is determined in such a way that the expected trim loss is minimal. The rotary cut format order then uses the most suitable standard large format from the stock. Currently, the dimensions of the standard large formats in the semi finished inventory are based on intuitive motives, with an accent on minimizing trim losses.

From the trim loss perspective it is most efficient to produce each rotary cut format from a

[*] Faculty of Management and Organization, Product Systems Design, University of Groningen, PO Box 800, NL-9700 AV GRONINGEN

[†] Faculty of Management and Organization, Product Systems Design, University of Groningen, PO Box 800, NL-9700 AV GRONINGEN, The Netherlands

[‡] Faculty of Economic Sciences, University of Groningen, PO Box 800, NL-9700 AV GRONINGEN, E-mail: g.sierksma@eco.rug.nl

specific large format. On the other hand, if there is only one large format in each caliper, the variety is minimal, but the trim loss might be inacceptably high.

On average, the first policy results in a lower trim loss. In order to make efficiently use of the two machines and to meet customer's due times the company applies both policies.

In this paper we concentrate on the second policy, taking into account the various objectives and restrictions of the company. The purpose of the company is to have not too many different types of large formats and an acceptable amount of trim loss. The problem is formulated as a minimum clique covering problem with alternatives (MCCA), which is presumed to be NP-hard**.** We solve the problem by using an appropriate heuristic, which is built into a decision support system. Based on a set of real data, the actual composition of semi finished inventories is determined. The paper concludes with computational experiments.

(also downloadable) in electronic version: http://som.rug.nl/

1. Introduction

A solid board factory produces sheets of cardboard in two formats, namely large and small. A large format is a large sheet of cardboard, whereas a small format has smaller dimensions. To produce both formats, the factory uses a cardboard machine, a die cut machine, and a rotary cut machine. A large format is ready after production on the cardboard machine, whereas a small format needs an additional processing step. Because of technical restrictions (*e.g.* dimension tolerances), very large setup times, and trim losses, small formats cannot be produced directly on the cardboard machine. Small formats are cut from large formats. There exist two types of small formats; rotary cut formats and die cut formats. We focus our attention in this article to the rotary cut formats. The rotary cut machine cuts batches of one rotary cut format order from one type of large formats.

The cardboard production process as a whole is rather complex. Figure 1.1 shows a schematic reproduction of the main processing steps on the machines. The inputs of the process are: waste paper (mostly old newspapers), water, additives or substances, and, if necessary, lining paper, and other adhesives.

The production process consists successively of the following integrated steps. The first step is the transformation of input material (waste paper) into pulp by mixing the waste paper with hot water. After filtering, sieving, and pressing the liquid percentage drops to 50%. After the remaining water is evaporated, the layer on the cardboard machine is ready. If the caliper of the cardboard needs to be above a certain value ($1.5mm$) the layer is laminated with lining paper. The lining paper production process is similar to that on the cardboard machine. Most of the lining paper comes from stock, and is manufactured by the company itself. The product is then cut into large formats, which are stored on pallets, and after a quality control in the wrapping street, the product is either ready or directed to the converting department.

The company produces the cardboard in several calipers. In case of laminated cardboard, the cardboard consists of three layers. The so-called inner layer is produced on the cardboard machine and the other layers, which are called lining papers, are produced on the paper machine. During the production process the lining paper is laminated on the inner layer. The production department can vary the caliper of the layers as long as the total caliper of a cardboard sheet stays equal. If the capacity on the paper machine (cardboard machine) is too low, the company may decide to use thinner (thicker) lining paper and a thicker (thinner) inner layer. In Sierksma and Wanders (2000) the laminating process and related aspects are studied in more detail.

In the converting department the large formats are cut into small formats. The department has two different types of machines, namely four rotary cut machines and one die cut machine. Figure 1.2 shows the relationship between a large and small format sheet. Figure 1.2a shows the large format, 1.2b the rotary cut formats in a large format, and 1.2c the die cut formats in a large format. A rotary cut machine only manufactures rectangular small format sheets, whereas a die cut machine is able to make sheets with rounded corners. Another difference is that at the die cut machine a *set of sheets* can be cut from one large format (the dashed rectangles in Figure 1.2c). A set normally consists of three sheets with different format dimensions, for instance the front, back and spin side of a book.

In this paper we only consider customer orders that are processed on the rotary cut machine; the rotary cut formats. The restrictions on the rotary cut machine are determined by its physical dimensions, and the number of knives that can be used. In the rest of this text we use both the term 'small format' and 'rotary cut format' to refer to a rotary cut format.There are two types of cuts, namely horizontal and vertical cuts. The meaning of 'horizontal' and 'vertical' is related to the direction in which the paper feeds at the cardboard machine. Both large and rotary cut formats have a rectangular shape. The shaded region of Figure 1.2b and 1.2c is the trim loss on the cutting machine, whereas the cutting losses in a particular cutting direction are defined as the rim cuts (in *mm*). We only consider the trim loss from the rotary cut machines.

From the trim loss perspective it is most efficient to produce each rotary cut format from a specific large format. On the other hand, if there is only one large format in each caliper, the variety is minimal, but the trim loss might be inacceptably high. The determination of an acceptable variety of stock large formats is the purpose of this paper: not too many different types of large formats and an acceptable amount of trim loss.

The converting department uses two categories of inventory. The first category is customer order specific: each customer order has its own 'optimal' large format with a minimal amount of trim loss on the cutting machine. The second category consists of a restricted number of large formats in every relevant caliper that is held to stock. A rotary cut format order uses then the most suitable standard large format. This inventory is defined as the *large format stock*.

The company only cuts a rotary cut format order from the large format stock, either if the order demand is less than $3000kg$, or if the delivery times are too short to pass through all processing steps, including production on the cardboard machine.

Currently, about 25% of the production volume consists of small format orders (both rotary cut and die cut) and this percentage is still increasing. Besides the shift from large to small format orders the due dates and the order sizes are decreasing. These trends in customer demand cause a deteriorating performance and exceeding due dates, if all large formats have to be manufactured customer specific. The large format stock will become more important, because an appropriate composition of the large format stock might cushion the disturbances in the production process.

Given the above mentioned trends, the control of the decoupling inventory between the cardboard and rotary cut machine will become more and more important. At the moment the dimensions of the stock formats are based on intuitive motives, with an accent on minimizing trim losses on the cutting machines. The company does not use any means that considers both the overall trim loss and the number of large formats within a caliper; only the trim loss is used as an objective to be minimized. In this paper we concentrate on the problem of determining a restricted number of large formats in the most important calipers, such that the expected trim loss is minimized, taking into account the various objectives and restrictions of the company.

The organization of the paper is as follows. We start the analysis in Section 2 and use a simple example to explain the problem and its structure. The example gives an impression in which direction the problem can be formalized. In Section 3 we formalize the problem as a minimum clique covering problem with alternatives (MCCA). Because of the specific structure of the problem, the general heuristics known from literature do not perform well. Therefore in Section 4 we formulate a 5-stage heuristic, which will be labeled as large sheet-set covering heuristic. The heuristic is tested for a variety of calipers based on a set of real data. Section 6 gives some conclusions and indications for future research.

2. The minimal large format stock problem; an example

Recall that the problem is to determine, for each specified caliper, a minimal set of inventory large formats. We define this problem as the *minimal large format stock* problem; notation MLFS-problem. Actually the company wants to satisfy two objectives, namely:

- an average trim loss over all large formats in the stock below a certain value, and
- a stock level, such that the inventory costs are acceptable.

In practice, the company reduces the number of different types of large formats in a specific caliper to say 3 or 4, so that most rotary cut format orders can be cut from these large formats with an acceptable trim loss. In our solution procedure we also start with calculating the inventory of large formats and adjust, if desired, the trim loss.

Throughout this paper expressions of the form $s_1 \times s_2$ always mean the dimension of a cardboard sheet, with s_1 the horizontal dimension and s_2 the vertical dimension. Let J be the set of large format types in stock to be determined. Consider a large format with label j. Let $g_1^j \times g_2^j$ be the dimension of large format j. I is the set of rotary cut format orders. In each specific order, all rotary cut sheets have the same dimensions. Let $k_{1,i} \times k_{2,i}$ be the dimension of the rotary cut sheets in order $i \in I$ (shortly, rotary cut format i). Moreover, by $m_{1,i}^j \times m_{2,i}^j$ is denoted the maximal number of rotary cut sheets in rotary cut format i that can be cut from large format j. Furthermore, $a_{1,i}^j$ and $a_{2,i}^j$ are the rim cuts. Clearly, $m_{\alpha,i}^j \times k_{\alpha,i} + a_{\alpha,i}^j = g_\alpha^j \quad \text{for } i \in I,\ j \in J,\ \alpha \in \{1,2\}$, *i.e.* the number of strokes times the rotary cut format dimension plus the rim cut is equal to the large sheet dimension. Define

$$m_{\alpha,i}^j = \left\lfloor \frac{g_\alpha^j - a_\alpha^{\min}}{k_{\alpha,i}} \right\rfloor \quad \text{for } \alpha \in \{1,2\}\,, \tag{1}$$

being the number of strokes in a specific direction.

Due to technical restrictions, the rim cuts are bounded as follows:

$$a_\alpha^{\min} \leq a_{\alpha,i}^j \leq a_\alpha^{\max} \qquad \text{for } i \in I,\ \alpha \in \{1,2\}\,, \tag{2}$$

and the large sheet dimensions in the following way:

$$g_\alpha^{\min} \leq g_\alpha^j \leq g_\alpha^{\max} \qquad \text{for } j \in J,\ \alpha \in \{1,2\}\,. \tag{3}$$

The following example shows that the set of large formats is in general not unique. Suppose we have four rotary cut format orders, labelled 1, 2, 3, and 4, in a given caliper. Table 2.1 shows the data.

For simplicity, assume that the large sheet bounds are $g_1^{\min} = g_2^{\min} = g^{\min} = 1000mm$ and $g_1^{\max} = g_2^{\max} = g^{\max} = 1350mm$, whereas the rim cut bounds are $a_1^{\min} = a_2^{\min} = a^{\min} = 30mm$ and $a_1^{\max} = a_2^{\max} = a^{\max} = 100mm$.

For each order we have calculated all possible large sheet dimensions, the so-called large format feasible region (LFF-region). Such a region can be represented as a rectangle, of which the left-lower vertex and the right-upper vertex are the minimum and maximum values, respectively, of the large formats allowed.

Tabel 2.1: Data sample

Rotary cut format		***Demand***
Label	***Dimensions***	**(*sheets*)**
1	$275 \times 224mm$	75750
2	$214 \times 267.5mm$	11000
3	$280 \times 211mm$	14600
4	$234 \times 255mm$	62100

In general, the rectangles can be represented by its four vertices in the following way. Let $\alpha \in \{1, 2\}$, where $\alpha = 1$ denotes the horizontal direction and $\alpha = 2$ the vertical direction in Figure 2.2.

Each rectangle is denoted by its left-lower vertex (**A**, **B**, $\ldots$, **I**). Table 2.2 shows the LFF-regions for all rotary cut format orders. The *large format set* of a certain small format order is the collection of all LFF-regions allowed for the small format order.

Tabel 2.2: Vertices of the feasible regions for the rotary cut format sample

Rotary cut format		***Vertex***	***Large format rectangle***			***# of***
Label	***Size***		***Label***	***Min. value***	***Max. value***	***sheets***
1	275×224	**A**	1.1	1130×1150	1200×1220	4×5
2	214×267.5	**B**	2.1	1100×1100	1170×1170	5×4
		C	2.2	1314×1100	1350×1170	6×4
3	280×211	**D**	3.1	1150×1085	1220×1155	4×5
		E	3.2	1150×1296	1220×1350	4×6
4	234×255	**F**	4.1	1000×1050	1046×1120	4×4
		G	4.2	1200×1050	1270×1120	5×4
		H	4.3	1000×1305	1046×1350	4×5
		I	4.4	1200×1305	1270×1350	5×5

Consider for example the order with label 2. The large format set of rotary cut format 2 consists of the two rectangles 2.1 and 2.2. They can be calculated as follows. We may cut either 6×4 (6 horizontal and 4 vertical), or 5×4 small sheets from a large sheet within the large sheet dimension restrictions. Taking the smallest rim cut ($d_\alpha^{\min} = 30mm, \quad \alpha \in \{1, 2\}$) we obtain the large formats corresponding to the vertices **B** and **C**, respectively. Namely, $\mathbf{B} = (5 \times 214 + 30,\ 4 \times 267.5 + 30) = (1100,\ 1100)$, and $\mathbf{C} = (6 \times 214 + 30,\ 4 \times 267.5 + 30) = (1314,\ 1100)$.

The largest rim cut ($a_\alpha^{\max} = 100mm, \quad \alpha \in \{1, 2\}$) gives the right-upper vertex of the rectangle. For rotary cut format 2 this results in
$\mathbf{B^1} = (5 \times 214 + 100,\ 4 \times 267.5 + 100) = (1170,\ 1170)$ and
$\mathbf{C^1} = (6 \times 214 + 100,\ 4 \times 267.5 + 100) = (1384,\ 1170)$, respectively.

In Figure 2.2 we have depicted all possible large formats for this example. The large dotted square is determined by $g^{\min}$ and $g^{\max}$, which are the technical restrictions on the large sheet dimensions. Hence each point in this square refers to an allowed large sheet size. For instance, the point **G** with coordinates $(1200, 1050)$ refers to a large format with horizontal dimension $g_1 = 1200mm$ and vertical dimension $g_2 = 1050mm$.

We prune values outside the dotted large square determined by $g^{\min}$ and $g^{\max}$. In our example, $\mathbf{C^1} = (1384,\ 1170)$ is not allowed. Therefore, the rectangle prunes at $\mathbf{C^2} = (1350,\ 1170)$.

In order to serve rotary cut format 2, a point in *either* rectangle 2.1 *or* 2.2 must be selected. For rotary cut format 3 a point has to be selected in either 3.1, or 3.2; for rotary cut format 4 a point in either 4.1, 4.2, 4.3, or 4.4 has to be selected.

If we restrict the selection to large formats with a minimal trim loss, we can simply take the large format represented by the lower left corner of the intersection of the corresponding rectangles. For instance, to serve the rotary cut formats 2 and 3, we may take the intersection of the rectangles 2.1 and 3.1, which is the rectangle determined by the points $(1150, 1100)$ and $(1170, 1150)$ respectively. Vertex **K** is the lower left corner of this intersection rectangle, and corresponds to the large format that cuts both orders with a minimal (average) trim loss.

Figure 2.2 shows that it is not possible to use only one large format type for covering all rotary cut formats. The vertex **L** covers all rotary cut format orders, except for order 4, so that at least two large formats are needed in order to cut all rotary cut formats. From Figure 2.2 it is not immediately clear which large formats result in a minimum total trim loss. We have to calculate and compare different sets of large format types. Table 2.3 lists all collections of large format types covering all rotary cut formats together with the corresponding trim losses.

Tabel 2.3: Possible sets of large formats

1^{st} ***large format***		2^{nd} ***large format***		***Rotary cut format***			
Label	***Dimensions***	***Label***	***Dimensions***	***Label***	***Cut from***	***Trim loss***	***Net need***
I	$1200 \times 1305mm$	**L**	$1150 \times 1150mm$	1	**L**	6.8%	3788
				2	**L**	13.4%	550
				3	**I**	9.5%	609
				4	**I**	4.7%	2484
Average trim loss						$\mathbf{6.8}\%$	
I	$1200 \times 1305mm$	**A**	$1130 \times 1150mm$	1	**A**	5.1%	3788
				2	**A**	11.9%	550
				3	**I**	9.5%	609
				4	**I**	4.7%	2484
Average trim loss						$\mathbf{5.9}\%$	
G	$1200 \times 1050mm$	**L**	$1150 \times 1150mm$	1	**L**	6.8%	3788
				2	**L**	13.4%	550
				3	**L**	10.7%	730
				4	**G**	5.3%	3105
Average trim loss						$\mathbf{7.0}\%$	
J	$1200 \times 1085mm$	**A**	$1130 \times 1150mm$	1	**A**	5.1%	3788
				2	**A**	11.9%	550
				3	**J**	9.1%	730
				4	**J**	8.3%	3105
Average trim loss						$\mathbf{7.1}\%$	

The first four columns of Table 2.3 show the set of large formats that is considered, together with their dimensions. The other five columns show the results on the rotary cut format level. The column with the label ‘Cut from’ shows which large format is used to cut the rotary cut format order such with minimal trim loss. The ‘trim loss’ column, shows the trim loss if the rotary cut format order is cut from the corresponding large format. The last column shows how many large sheets are necessary in order to cut the demanded number of rotary cut sheets. Besides the individual trim losses we also show the average trim loss. The average trim loss shows the trim loss, if each rotary cut format order is cut from the corresponding large format with minimal trim loss.

In order to calculate the trim loss and net need of large sheets, we use the following formulas. The trim loss T_i^j denotes the percentage of waste if order i is cut from large format j $(i \in I, j \in J)$. One can easily verify that:

$$T_i^j = \left(1 - \frac{\left(m_{1,i}^j \times k_{1,i}\right) \times \left(m_{2,i}^j \times k_{2,i}\right)}{g_1^j \times g_2^j}\right) \times 100\% \qquad \text{for } i \in I,\ j \in J. \quad (4)$$

For any $i \in I$ let q_i be the number of demanded rotary cut format sheets in order i. Then the net need Q_i^j is the number of sheets in large format j needed for the production of order i. Clearly,

$$Q_i^j = \left\lceil \frac{q_i}{m_{1,i}^j \times m_{2,i}^j} \right\rceil \qquad \text{for } i \in I,\ j \in J. \quad (5)$$

Thus, if a customer demands 14600 rotary cut format sheets with dimensions 280×211 (Label 3 in Table 2.1), and if it is cut from large format **I**, with 4 strokes in horizontal and 6 in vertical direction, then the trim loss T_i^j is equal to $\left(1 - \frac{(4\times280)\times(6\times211)}{1200\times1305}\right) \times 100\% = 9.5\%$; the net need is equal to $\frac{14600}{4\times6} = 609$ large sheets (See Table 2.3). If we cut this order from large format **L**, with 4×5 strokes, the trim loss is equal to 10.65%, and the net need is equal to 730 large sheets. From these two alternatives, large format **I** shows a lower trim loss; it is the best choice. In Table 2.3 only the best results for each small format with respect to the trim loss order are presented.

From Table 2.3, it follows that the combination $\{\mathbf{I}, \mathbf{A}\}$ is the best possible when the trim loss is minimized. On the other hand, the gap with the other combinations is around 1%; namely 5.9% for $\{\mathbf{I}, \mathbf{A}\}$ and 6.8%, 7.0%, and 7.1% for $\{\mathbf{I}, \mathbf{L}\}$, $\{\mathbf{G}, \mathbf{L}\}$, and $\{\mathbf{J}, \mathbf{A}\}$ respectively. So one might consider one of these three combinations when objectives different from the trim loss are dominant. For instance, if we consider maximal flexibility of the use of input, $\{\mathbf{I}, \mathbf{L}\}$ might be better than $\{\mathbf{I}, \mathbf{A}\}$ because large format **L** can be used for three rotary cut formats orders, whereas large format **A** only cuts two rotary cut format orders.

3. A generalized minimal clique covering problem

In this section we will show how the minimal large format stock problem, can be formulated as a minimal clique covering problem with alternatives. To that end we first give the general formulation of the problem. Let $G = (V, E)$ be a graph with vertex set V and edge set E. For any $S \subseteq V$, let $E(S)$ be the set of edges from E that have both end points in S. A clique of G is a subset S of V for which $(S, E(S))$ is a complete graph.

Let $r \geq 1$. An r-partition of V, say $\mathcal{V} = \{V_1, \ldots, V_r\}$, is a collection of subsets of V such that $V_1 \cup V_2 \cup \cdots \cup V_r = V$ and $V_i \cap V_j = \emptyset$ for each $i, j \in \{1, 2, \ldots, r\}$, $i \neq j$, and $E(V_i) = \emptyset$ for each $i \in \{1, 2, \ldots, r\}$. Note that if $G = K_n$ (the complete graph on n vertices), then the only r-partition is the singleton partition (*i.e.* $r = n$). For any r-partition $\mathcal{V} = \{V_1, \ldots, V_r\}$ of V, a $\mathcal{V}$-cover $\mathcal{C}$ is a collection of subsets of V such that

$$V_i \cap (\cup \mathcal{C}) \neq \emptyset \quad \text{for each } i \in \{1, 2, \ldots, r\},$$

where $\cup \mathcal{C}$ denotes the set of all vertices in $\mathcal{C}$. Hence, for each i at least one vertex of V_i is in at least one element of $\mathcal{C}$. A minimal $\mathcal{V}$-cover $\mathcal{C}'$ with respect to $\mathcal{C}$ is a $\mathcal{V}$-cover $\mathcal{C}' \subseteq \mathcal{C}$, such that the number of elements in $\mathcal{C}'$ is minimal. The problem of determining a minimal $\mathcal{V}$-cover with respect to some given collection $\mathcal{C}$ is called a minimum covering problem with alternatives. If $\mathcal{C}$ consists of all cliques in G, then the problem is called a *minimal clique covering problem with alternatives*; notation MCCA-problem. Note that elements of $\mathcal{C}$ that are completely contained in one of the V_i's need not be considered in a MCCA-problem.

The MLFS-problem can now be formulated as an MCCA-problem in the following way. In the underlying graph, the LFF-regions from Figure 2.2 are the vertices, while there is an edge if two LFF-regions intersect. In order to calculate these edges we consider the horizontal and vertical projections of the LFF-regions: two LFF regions intersect if and only if both the horizontal and vertical projections intersect. The projections are denoted by $I_{\alpha,i}^{n_{\alpha,i}}$, and are defined as follows.

Take $\alpha = 1, 2$ and $i \in I$. Define

$$\begin{aligned} n_{\alpha,i}^{\min} &= \min\left\{n \left| n \times k_{\alpha,i} + a_\alpha^{\max} \geq g_\alpha^{\min}\right.\right\} \\ n_{\alpha,i}^{\max} &= \max\left\{n \left| \max\left\{g_\alpha^{\min},\, n \times k_{\alpha,i} + a_\alpha^{\min}\right\} \leq \min\left\{n \times k_{\alpha,i} + a_\alpha^{\max},\, g_\alpha^{\max}\right\}\right.\right\} \end{aligned}$$

Then the projection interval corresponding to the 'projection' α and the order i is

$$\begin{aligned} I_{\alpha,i}^{n_{\alpha,i}} = \Big[&\max\left\{g_\alpha^{\min},\, \left(n_{\alpha,i}^{\min} + n_{\alpha,i}\right) \times k_{\alpha,i} + a_\alpha^{\min}\right\}, \\ &\min\left\{\left(n_{\alpha,i}^{\min} + n_{\alpha,i}\right) \times k_{\alpha,i} + a_\alpha^{\max},\, g_\alpha^{\max}\right\}\Big] \end{aligned}$$

where $n_{\alpha,i} = 1, \ldots, n_{\alpha,i}^{\max} - n_{\alpha,i}^{\min} = N_{\alpha,i}$. So for each order $i \in I$, there are $N_{1,i} \times N_{2,i}$ LFF-regions $V_i^{n_{1,i}, n_{2,i}} = \left\{(g_1, g_2) \left| g_1 \in I_{1,i}^{n_{1,i}},\, g_2 \in I_{2,i}^{n_{2,i}}\right.\right\}$, with $\alpha = 1, 2$ and $n_{\alpha,i} =$

$1,\ldots,N_{\alpha,i}$. These are the $\sum_{i=1}^{I} N_{1,i} \times N_{2,i}$ vertices of the graph G. There exists an edge $\left\langle V_i^{n_{1,i},n_{2,i}}, V_h^{q_{1,h},q_{2,h}} \right\rangle$ if two rectangles intersect, *i.e.*, define

$$\left\langle V_i^{n_{1,i},n_{2,i}}, V_h^{q_{1,h},q_{2,h}} \right\rangle = \begin{cases} 1, \text{if } V_i^{n_{1,i},n_{2,i}} \cap V_h^{q_{1,h},q_{2,h}} \neq \emptyset \\ 0, \text{otherwise} \end{cases} .$$

for $n_{\alpha,i}, q_{\alpha,h} = 1,\ldots,N_{\alpha,i}$, $\alpha = 1,2$, $i,h \in I$, $i \neq h$. Since there is an edge between two 'LFF-regions' if for both projections the corresponding projection intervals intersect, this means that the so-constructed graph is an interval graph; see *e.g.* Golumbic (1980).

The graph $G = (V,E)$ corresponding to Figure 2.2 is depicted in Figure 3.1. Since for each $i \in I$, $\alpha = 1,2$, and $n_{\alpha,i} = 1,\ldots N_{\alpha,i}$ at least one vertex $V_i^{n_{1,i},n_{2,i}}$ needs to be covered by a clique, we have the problem with alternatives.

No two LFF-regions from one order intersect, meaning that no rotary cut formats can be cut from the rim, because $k_{1,i} > \frac{1}{2}a_{1,i}^j$ and $k_{2,i} > \frac{1}{2}a_{2,i}^j$ for each order i and each large format j.

The $\mathcal{V}$-cover in case of Figure 2.2 reads

$$\mathcal{V} = \left\{ \left\{ V_1^{1,1} \right\}, \left\{ V_2^{1,1}, V_2^{1,2} \right\}, \left\{ V_3^{1,1}, V_3^{2,1} \right\}, \left\{ V_4^{1,1}, V_4^{1,2}, V_4^{2,1}, V_4^{2,2} \right\} \right\}.$$

The cliques in Figure 3.1 form the collection

$$\begin{aligned} \mathcal{C} \;=\; & \left\{ \left\{ V_1^{1,1} \right\}, \left\{ V_2^{1,1} \right\}, \left\{ V_2^{1,2} \right\}, \left\{ V_3^{1,1} \right\}, \left\{ V_3^{2,1} \right\}, \left\{ V_4^{1,1} \right\}, \right. \\ & \left\{ V_4^{1,2} \right\}, \left\{ V_4^{2,1} \right\}, \left\{ V_4^{2,2} \right\}, \left\{ V_3^{2,1}, V_4^{2,2} \right\}, \left\{ V_1^{1,1}, V_2^{1,1}, V_3^{1,1} \right\}, \\ & \left. \left\{ V_3^{1,1}, V_4^{1,2} \right\}, \left\{ V_1^{1,1}, V_2^{1,1} \right\}, \left\{ V_1^{1,1}, V_3^{1,1} \right\}, \left\{ V_2^{1,1}, V_3^{1,1} \right\} \right\} \end{aligned}$$

The MCCA-problem corresponding to Figure 3.1 has eight solutions, namely

$$\begin{aligned} \mathcal{C}' \;=\; & \left\{ \left\{ \left\{ V_1^{1,1}, V_2^{1,1}, V_3^{1,1} \right\}, \left\{ V_4^{1,1} \right\} \right\}, \left\{ \left\{ V_1^{1,1}, V_2^{1,1}, V_3^{1,1} \right\}, \left\{ V_4^{1,2} \right\} \right\}, \right. \\ & \left\{ \left\{ V_1^{1,1}, V_2^{1,1}, V_3^{1,1} \right\}, \left\{ V_4^{2,1} \right\} \right\}, \left\{ \left\{ V_1^{1,1}, V_2^{1,1}, V_3^{1,1} \right\}, \left\{ V_4^{2,2} \right\} \right\}, \\ & \left\{ \left\{ V_1^{1,1}, V_2^{1,1}, V_3^{1,1} \right\}, \left\{ V_3^{1,1}, V_4^{1,2} \right\} \right\}, \left\{ \left\{ V_1^{1,1}, V_2^{1,1}, V_3^{1,1} \right\}, \right. \\ & \left. \left\{ V_3^{2,1}, V_4^{2,2} \right\} \right\} \left\{ \left\{ V_1^{1,1}, V_2^{1,1} \right\}, \left\{ V_3^{1,1}, V_4^{1,2} \right\} \right\}, \left\{ \left\{ V_1^{1,1}, V_2^{1,1} \right\}, \right. \\ & \left. \left. \left\{ V_3^{2,1}, V_4^{2,2} \right\} \right\} \right\}. \end{aligned}$$

Obviously, the singleton cliques $\left\{V_2^{2,1}\right\}$, $\left\{V_4^{1,1}\right\}$, and $\left\{V_4^{2,1}\right\}$ are redundant with respect to the MCCA-problem. Clearly, the in this example unique optimal solution, when minimal trim loss is demanded, reads $\left\{\left\{V_1^{1,1}, V_2^{1,1}\right\}, \left\{V_3^{2,1}, V_4^{2,2}\right\}\right\}$. This solution has a trim loss of 5.9%.

When considering the complexity of the MLFS-problem, we first note that the general MCCA-problem is NP-hard, since the MCC-problem is NP-hard; see Garey and Johnson (1979). For the specific case that the MCCA-problem is based on the intersection graph of an MLFS-problem, we do not know the answer, although we expect it to be NP-hard as well. This conjecture is also motivated by the fact that similar problems on general box graphs are NP-hard; see e.g. McKee and McMorris (1999) and Roberts (1989).

In the following section we present a heuristic that solves MLFS-problems. The heuristic is based on the LFF-regions representation of the problem as formulated in this section and includes solving a set covering problem.

4. The Large Sheet-Set Covering Heuristic

In the, what we term, Large Sheet Set Covering (LSSC) heuristic we use as input the order data which shows the quantity (in the number of sheets) and the dimensions of the small sheets (in *mm*). The heuristic outputs a number of large sheet types that covers the small format order set. The restrictions with respect to the rotary cut machine are taken into account when executing the heuristic.

The LSSC-heuristic uses six stages that can be described as follows. In the first stage, it determines all so-called maxcliques, where a *maxclique* is any complete subgraph that is not properly contained in another subgraph (see *e.g.* McKee and McMorris (1999)). This operation is performed in order to reduce the solution space. It may happen that the optimal solution does not correspond to a maxclique. For example, in Section 2 the maxclique set (MCS) is equal to

$$\left\{\left\{\left\{V_1^{1,1}\right\}, \left\{V_2^{1,1}\right\}, \left\{V_3^{1,1}\right\}\right\}, \left\{\left\{V_3^{1,1}\right\}, \left\{V_4^{1,2}\right\}\right\}, \left\{\left\{V_3^{2,1}\right\}, \left\{V_4^{2,2}\right\}\right\}, \left\{V_2^{1,2}\right\}, \left\{V_4^{1,1}\right\}, \left\{V_4^{2,1}\right\}\right\},$$

while the optimal solution uses a subclique of the maxclique that covers $V_1^{1,1}$.

In stage 2 we construct three types of incidence matrices to be used as input for the set covering problem from stage 4. Because the management of the company is interested in a balanced solution between a minimal stock of large formats and a minimal trim loss level, the trim loss is used as so-called cost operator in the set covering problem. In stage 5 the solution of the set covering set is reduced. In stage 6 the actual large format inventory is determined. The heuristic reads as follows.

Large Sheet-Set Covering heuristic

Input: Graph $G = (V, E)$; vertex set $\mathcal{V} = \left\{V_i^{n_{1,i},n_{2,i}}\right\}$ with M vertices; edges $\left\langle V_i^{n_{1,i},n_{2,i}}, V_h^{q_{1,h},q_{2,h}}\right\rangle$, with $h, i = 1, \ldots, I$, $n_{\alpha,i}, q_{\alpha,h} = 1, \ldots N_{\alpha,i}$, $\alpha = 1, 2$; order partition of $\mathcal{V} = \left\{P_1, \ldots, P_{|\mathcal{V}|}\right\}$.

Output: Large format stock dimensions $(g_1(j), g_2(j))$ for $j = 1, \ldots, J$, with J the number of large formats, and the number of sheets Q^j necessary to cover the demand in large sheet type $(g_1(j), g_2(j))$ of a particular period, for $j = 1, \ldots, J$.

Stage 1: *Maxcliques.* Calculate all maxcliques of G. The method from Carraghan and Pardalos (1990) can be used. The vertices are labeled in the order of nondecreasing degrees, say $v_1, \ldots, v_M$. First all maxcliques containing v_1 are determined. Then all maxcliques containing v_2, but not v_1 are determined, and so on, until we have found all maxcliques.

Stage 2: *Incidence matrices.* Calculate the maxclique-vertex incidence matrix $C = (c_{pm})$, defined as $c_{pm} = 1$ if $v_m \in \mathcal{V}^p$, and $c_{pm} = 0$ otherwise, for $p \in \mathcal{P}$, $m = 1, \ldots, M$; see Golumbic (1980) and Booth and Lueker (1975). Calculate the vertex-order incidence matrix $D = (d_{mi})$, with $d_{mi} = 1$ if v_m is a vertex of small format order i, and $d_{mi} = 0$ otherwise for $i \in I$, $m = 1, \ldots, M$. Calculate the order-maxclique incidence matrix $O = (o_{ip})$, with $o_{ip} = 1$ if maxclique $\mathcal{V}^p$ covers order i, and $o_{ip} = 0$ otherwise for $i \in I$, $p \in \mathcal{P}$. Note that $O = D^T \times C^T$.

Stage 3: *Trim loss.* Calculate the trim loss for each maxclique $\mathcal{V}^p$ $(p \in \mathcal{P})$. These values are used as cost operators in next stages. Before the trim loss can be calculated we need the optimal large sheet dimensions for each maxclique $\mathcal{V}^p$. A large sheet dimension $g_\alpha^*(\mathcal{V}^p)$, $p = 1, \ldots, P$ is called *optimal* if

$$g_\alpha^*(\mathcal{V}^p) = \max\left\{\min g_\alpha^*(v_{m_1}), \ldots, \min g_\alpha^*(v_{m_n}), \ldots, \min g_\alpha^*(v_{m_N})\right\}$$
$$\alpha \in \{1, 2\};\; g_\alpha^*(v_{m_n}) \in v_{m_n};\; v_{m_n} \in \mathcal{V}^p;\; \forall m_n \in M;\; n = 1, \ldots, N;\; N \leq M. \quad (6)$$

The trim loss is now equal to

$$T\left(\mathcal{V}^p\right) = \frac{\sum_{i=1}^{I} o_{ij} Q_i\left(g_\alpha^*\left(\mathcal{V}^p\right)\right) \times T_i\left(g_\alpha^*\left(\mathcal{V}^p\right)\right)}{\sum_{i=1}^{I} o_{ij} Q_i\left(g_\alpha^*\left(\mathcal{V}^p\right)\right)}, \tag{7}$$

with $Q_i\left(g_\alpha^*\left(\mathcal{V}^p\right)\right)$ and $T_i\left(g_\alpha^*\left(\mathcal{V}^p\right)\right)$ using the definitions of (4) and (5).

Stage 4: *Set covering.* Solve a set covering problem for the matrix O, *i.e.* determine a minimum number of rows of O such that each column has at least one 1 in one of the selected rows. In linear programming optimization then the problem reads as follows.

Let $x^p = 1$ if clique $\mathcal{V}^p$ (cost $T\left(\mathcal{V}^p\right)$) is in the solution, and $x^p = 0$ otherwise. Then

$$\min \left\{ \sum_{p=1}^{P} T\left(\mathcal{V}^p\right) x^p \;\middle|\; \begin{array}{ll} \sum_{p=1}^{P} o_{ip} x^p \geq 1 & i \in I \\ x^p \in (0,1) & p \in \mathcal{P} \end{array} \right\}. \tag{8}$$

Let $\mathcal{W}^r = \left\{v_{t_1}, \ldots, v_{t_{R_r}}\right\} \in \left\{\mathcal{V}^1, \ldots, \mathcal{V}^P\right\}$ for $r = 1, \ldots, R \leq P$ be the solution of this set covering problem. Note that the entries of the matrix O are input parameters of the set covering problem. Define $O' = (o'_{ir})$ with $o'_{ir} = 1$ if maxclique $\mathcal{W}^r$ covers order i, and 0 otherwise.

Stage 5: *Redundancy in solution.* Solve the bipartite matching problem (see e.g. Ahuja *et al* (1993))

$$\min \left\{ \sum_{i=1}^{I} \sum_{r=1}^{R} k_{ir} o''_{ir} \;\middle|\; \sum_{i=1}^{I} o''_{ir} = 1,\ o''_{ir} \in (0,1) \quad i \in I,\ r \in R \right\}$$

where $k_{ir} = -T_i\left(g_\alpha^*\left(\mathcal{W}^r\right)\right)$ if $o'_{ir} = 1$, and $k_{ir} = -\infty$ if $o'_{ir} = 0$. Note that the entries of the matrix $O'' = (o''_{ir})$ are the decision variables in this problem. These entries show which orders are covered by $\overline{\mathcal{W}}^r$. The cliques $\overline{\mathcal{W}}^1, \ldots, \overline{\mathcal{W}}^R$ are derived from $\mathcal{W}^1, \ldots, \mathcal{W}^R$ by deleting vertices v_m that are not covered anymore. The vertices v_m that are removed from $\mathcal{W}^r$ are determined by using the matrices C and D. Let O''' be the matrix obtained by deleting the zero columns from O''. Let $\overline{\mathcal{W}}^1, \ldots, \overline{\mathcal{W}}^J$ be the set of the remaining cliques, corresponding to the demanded J large format types.

Stage 6: *Large format inventory.* From the cliques that are selected in stage 5 the large sheet dimensions are determined by using (6). For each $j = 1, \ldots, J$ the net need Q^j, being the number of sheets in a particular large sheet type $\left(g_1(j), g_2(j)\right)$, is calculated by means of (5).

The size of the incidence matrices C, D, and O in stage 2 can be reduced if for a row (column) with only one entry 1 there is another row (column) that also contains this entry. This will not change the procedure (see the example below). The specification of the large sheet dimensions is nothing else than determining the lower left corner point $(g_1^*(\mathcal{V}^p), g_2^*(\mathcal{V}^p))$ of the intersection of the LFF-regions corresponding to the clique $\mathcal{V}^p$. In stage 4 a set covering formulation is used; see *e.g.* Beasley (1987). The first set of restrictions in (8) ensures that each row is covered by at least one column. The second set of restrictions are integrality constraints. For $p = 1, \ldots, P$, $T(\mathcal{V}^p)$ denotes the cost of the trim loss of the clique $\mathcal{V}^p$. The heuristic determines a feasible solution of the MCCA-problem by solving a set covering problem that minimizes the total trim loss of the maxcliques. The set $\{\mathcal{W}^1, \ldots, \mathcal{W}^R\}$ shows the solution of the set covering formulation. In stage 5, the heuristic reduces the number of vertices in the maxcliques from $\{\mathcal{W}^1, \ldots, \mathcal{W}^R\}$. After removing the empty cliques we end up with the cliques $\overline{\mathcal{W}}^1, \ldots, \overline{\mathcal{W}}^J$. The clique $\overline{\mathcal{W}}^j$ $(j = 1, \ldots, J)$ corresponds to a large sheet type and may cover several small format orders. Note that a clique $\overline{\mathcal{W}}^j$ need not be a maxclique. Finally, in stage 6 the corresponding large sheet dimensions and the net need of sheets, necessary to cover the demand of a certain period, are calculated. The working of the LSSC-heuristic is now illustrated by means of the example corresponding to Figure 3.1.

Stage 1: Start using the algorithm of Carraghan and Pardalos (1990). A boldfaced node below is one that will be expanded.

1. Initialization orders the nodes: $v_1 = V_2^{1,2}, v_2 = V_4^{1,1}, v_3 = V_4^{2,1}, v_4 = V_3^{2,1}, v_5 = V_4^{2,2}, v_6 = V_4^{1,2}, v_7 = V_1^{1,1}, v_8 = V_2^{1,1}, v_9 = V_3^{1,1}$
2. Depth 1: $\mathbf{v_1}, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9$ (cannot expand, so CBC is $\mathcal{V}_1 = \{v_1\}$, size 1)
3. Depth 1: $v_1, \mathbf{v_2}, v_3, v_4, v_5, v_6, v_7, v_8, v_9$ (cannot expand, so CBC is $\mathcal{V}_2 = \{v_2\}$, size 1)
4. Depth 1: $v_1, v_2, \mathbf{v_3}, v_4, v_5, v_6, v_7, v_8, v_9$ (cannot expand, so CBC is $\mathcal{V}_3 = \{v_3\}$, size 1)
5. Depth 1: $v_1, v_2, v_3, \mathbf{v_4}, v_5, v_6, v_7, v_8, v_9$
   Depth 2: $\mathbf{v_5}, \mathbf{v_6}, \mathbf{v_7}, \mathbf{v_8}, \mathbf{v_9}$ (cannot expand, so CBC is $\mathcal{V}_4 = \{v_4, v_5\}$, size 2)
6. Depth 1: $v_1, v_2, v_3, v_4, \mathbf{v_5}, v_6, v_7, v_8, v_9$ (cannot expand, is already in a clique, namely $\mathcal{V}^4$)
7. Depth 1: $v_1, v_2, v_3, v_4, v_5, \mathbf{v_6}, v_7, v_8, v_9$
   Depth 2: $v_7, v_8, \mathbf{v_9}$ (cannot expand, so CBC is $\mathcal{V}_5 = \{v_6, v_9\}$, size 2)
8. Depth 1: $v_1, v_2, v_3, v_4, v_5, v_6, \mathbf{v_7}, v_8, v_9$
   Depth 2: $\mathbf{v_8}, v_9$
   Depth 3: $\mathbf{v_9}$ (cannot expand, so CBC is $\mathcal{V}_6 = \{v_7, v_8, v_9\}$, size 3)

Stage 2: For each vertex in the graph of Figure 3.1, the maxcliques are used to calculate the maxclique-vertices matrix C, namely:

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
$\mathcal{V}^1$	1	0	0	0	0	0	0	0	0
$\mathcal{V}^2$	0	1	0	0	0	0	0	0	0
$\mathcal{V}^3$	0	0	1	0	0	0	0	0	0
$\mathcal{V}^4$	0	0	0	1	1	0	0	0	0
$\mathcal{V}^5$	0	0	0	0	0	1	0	0	1
$\mathcal{V}^6$	0	0	0	0	0	0	1	1	1

where the p^{th} row corresponds to clique $\mathcal{V}^p$ and the m^{th} column to the vertex v_m. The vertex-order incidence matrix D reads as follows:

	1	2	3	4
v_1	0	1	0	0
v_2	0	0	0	1
v_3	0	0	0	1
v_4	0	0	1	0
v_5	0	0	0	1
v_6	0	0	0	1
v_7	1	0	0	0
v_8	0	1	0	0
v_9	0	0	1	0

where the m^{th} row corresponds to vertex v_m and the i^{th} column to order i. The maxcliques-order incidence matrix O reads as follows:

	$\mathcal{V}^1$	$\mathcal{V}^2$	$\mathcal{V}^3$	$\mathcal{V}^4$	$\mathcal{V}^5$	$\mathcal{V}^6$
1	0	0	0	0	0	1
2	1	0	0	0	0	1
3	0	0	0	1	1	1
4	0	1	1	1	1	0

where the i^{th} row corresponds to order i and the p^{th} column to the clique $\mathcal{V}^p$.

Stage 3: In Table 4.1 the trim loss is given for each maxclique $\mathcal{V}^p$.

Tabel 4.1: Trim loss for each maxclique in the example
(The labels of the vertices refer to Figure 2.2)

Maxclique	***Large format***		***Trim loss***
	Vertex	$(g_1^*(\mathcal{V}^p), g_2^*(\mathcal{V}^p))$	$T(\mathcal{V}^p)$
$\mathcal{V}^1$	**C**	$1314 \times 1100mm$	5.0%
$\mathcal{V}^2$	**F**	$1000 \times 1050mm$	9.1%
$\mathcal{V}^3$	**H**	$1000 \times 1305mm$	26.8%
$\mathcal{V}^4$	**I**	$1200 \times 1305mm$	5.7%
$\mathcal{V}^5$	**J**	$1200 \times 1085mm$	6.9%
$\mathcal{V}^6$	**L**	$1150 \times 1150mm$	8.1%

Stage 4: Using O as input, the solution of the set covering problem is $\mathcal{W}^1 = \mathcal{V}^4, \mathcal{W}^2 = \mathcal{V}^6$. Matrix O' is equal to

	$\mathcal{W}^1$	$\mathcal{W}^2$
1	0	1
2	0	1
3	1	1
4	1	0

Stage 5: This solution contains redundancy. In order to remove it, a bipartite matching problem is solved. Figure 4.1 shows the corresponding bipartite graph.

Note that order 3 is covered by $\mathcal{W}^1$(lowest trim loss). So we remove order 3 from $\mathcal{W}^2$. The matrix $O''' = O''$ reads now as follows:

	$\overline{\mathcal{W}}^1$	$\overline{\mathcal{W}}^2$
1	0	1
2	0	1
3	1	0
4	1	0

Using the matrices C, D, and O''', we find that $\overline{\mathcal{W}}^1 = \mathcal{W}^1, \overline{\mathcal{W}}^2 = \mathcal{W}^2 \setminus \{v_9\}$.

Stage 6: The large format stock is determined from (6). Namely, $\left(g_1\left(\overline{\mathcal{W}}^1\right), g_2\left(\overline{\mathcal{W}}^1\right)\right) = (1200, 1305)$ and $\left(g_1\left(\overline{\mathcal{W}}^2\right), g_2\left(\overline{\mathcal{W}}^2\right)\right) = (1130, 1150)$. Using the data from Table 2.3, the net need satisfies $\left\{Q\left(\overline{\mathcal{W}}^1\right), Q\left(\overline{\mathcal{W}}^2\right)\right\} = \{4338, 3093\}$.

Note that the cliques $\mathcal{V}^1$, $\mathcal{V}^2$, and $\mathcal{V}^3$ are the redundant singleton cliques, because other cliques already cover the small format orders corresponding to $\mathcal{V}^1$, $\mathcal{V}^2$, or $\mathcal{V}^3$. Delete the corresponding rows and/or columns from the matrices C, D, and O. It follows from matrix O that $\mathcal{V}^4$ and $\mathcal{V}^5$ both cover all small format orders. With respect to the trim loss, $\mathcal{V}^4$ is better than $\mathcal{V}^5$. So, the rows and columns corresponding to $\mathcal{V}^5$ can be removed.

In this simple case the heuristic has found the optimal solution, namely the one corresponding to $\{\mathbf{I}, \mathbf{A}\}$ of Table 2.3.

5. Computational results

The company for which the research of this paper has been carried out produces about $40,000$ tons of rotary cut format sheets per year. The data set used in this section includes all rotary cut format orders within a specific year. We consider calipers of which the weights are at least 5%. In Table 5.1 these calipers are listed together with the number of orders and the weights of the orders (in tons). The 'share' column in this table contains the percentages of the caliper in the total number of orders. There are two order variables: 'real' and 'unique'. 'Real' is the number of orders in a specific caliper, while 'unique' is the number of order specifications with respect to the dimensions in a specific caliper.

We only use the calipers $1.7mm$, $2.0mm$, $2.4mm$, and assume that $g_1^{\min} = g_2^{\min} = g^{\min} = 1000mm$, $g_1^{\max} = g_2^{\max} = g^{\max} = 1350mm$, $a_1^{\min} = a_2^{\min} = a^{\min} = 30mm$, $a_1^{\max} = a_2^{\max} = a^{\max} = 100mm$. The rotary cut machine has 10 horizontal and 10 vertical knives. The simulations are carried out by a simplified version of the LSSC-heuristic. Table 5.2 shows the results of the calculations.

The column 'share' shows the importance of the large format for the specific caliper. If a large format specification has a weight of 40%, then from the total number of tons in

Tabel 5.1: Weights of calipers above the 5%

Caliper	***# of orders***		***# of***	***Share***
	real	***unique***	***tons***	
$1.7mm$	219	58	2992	7.2%
$1.8mm$	319	90	2580	5.7%
$1.9mm$	403	133	3156	7.6%
$2.0mm$	726	269	5982	14.4%
$2.25mm$	449	106	5247	12.7%
$2.4mm$	680	332	6900	16.7%
Total number of orders				4719
Total number of tons				41275

a specific caliper, 40% has that large format specification. Note that the total number of tons in Table 5.2 is higher than in Table 5.1. This is caused by the fact that the total number in tons denotes the released to satisfy the demand. Table 5.2 also shows the trim loss. The number of large formats denotes the number of large format dimensions necessary for cutting all rotary cut format orders under the given restrictions. Because of these restrictions, not all orders are covered in a specific caliper. Take for example the caliper of $2.0mm$. Here 254 out of 269 unique orders are covered by the set of large formats. In order to cover 254 orders, 24 different small sheet dimensions are necessary. Other orders cannot be cut, because of additional restrictions such as the number of knives.

From Table 5.2 we conclude that the proposed heuristic should perform rather well. The trim loss is in general below the 11%, where the company has as target an average of at most 13% trim loss. If the company restricts the number of large sheets in a particular caliper to three, then at least 66% of the volume is covered by these large sheets. This can be seen as a rather good result.

6. Discussion and conclusions

In this paper we analyze the problem of determining the size of the stock of large format dimensions, such that the number of rotary cut format orders that are covered is maximized and the trim loss is minimized. Until now the composition of the large format stock is determined by using the experience of the planners who prioritize the

Tabel 5.2: Simulation results for a selection of calipers

Caliper	***Large sheet***		***Trim Loss***	***Share***	***Unique orders***
	#	***Dimensions***			
$1.7mm$	13	1120×1005	9.45%	61%	11
		1334×1255	9.17%	25%	6
		1080×1110	9.06%	6%	10
		Total in tons			3294
		Covered orders / Total number of orders			57 / 58
$2.0mm$	24	1310×1290	7.41%	40%	31
		1160×1010	10.95%	14%	27
		1220×1332	10.94%	12%	35
		Total in tons			6310
		Covered orders / Total number of orders			254 / 269
$2.4mm$	27	1322×1002	9.05%	38%	35
		1344×1226	8.68%	20%	36
		1250×1002	9.43%	8%	26
		Total in tons			7441
		Covered orders / Total number of orders			313 / 332

maximization of the trim loss at the cost of minimizing stock costs. We provide a more balanced treatment of the problem. It is shown that the problem can be formulated as a generalized clique covering problem. Since the complexity of this specific problem is unknown, we are forced to use heuristics.

The heuristic proposed here uses six stages, applying general mathematical techniques for finding maxcliques, solving a set covering problem, and improving the performance by reducing the maxcliques.

The computational results show that the heuristic performs pretty well on the data set. This set covers one production year. The calculation of the large format stock is also based on this data set, although it is expected that the demand in the future may change. The results show that the heuristic may provide a significant reduction in the trim losses. Important to note is the fact that the simulations are performed on a deterministic data set. In practice, the demand fluctuates dynamically, and so do the demanded sheet dimensions. This may lead to lower reductions in the trim losses than can be expected from the simulations.

The company distinguishes a total of five market segments, each with its specific characteristics including the sizes of the demanded rotary cut format sheets in the various calipers. Some market segments are better predictable than others. This may also have an impact on the performance of the heuristic and the way it will be used in practice.

There are a number of interesting management implications of the current research. First of all, the cutting losses decreases. The advantage of a new type of inventory, the large format stock is a reduction of the additional setups leading to an increase of the available capacity on the production machines.

Acknowledgement

We would like to thank the following persons in alphabetical order: Dirk-Pieter van Donk, Diptesh Ghosh, Alain Hertz, Manuel Laguna, Marino Widmer, and the Euro Summer Winter Insitute 18 in 2000 that was participated by the first author.

References

Ausiello, G., Crescenzi, P., Gambosi, G., Kann, V., Marchetti Spaccamela, A., Protasi, M., *Complexity and Approximation. Combinatiorial Optimization Problems and their Approximability Properties*, Springer-Verlag, Berlin, 1999. Compendium also available at: http://www.nada.kth.se/theory/compendium

Abello, J., Pardalos, P.M., Resende, M.C., On maximum clique problems in very large graphs, *Dimacs Series on Discrete Mathematics and Theoretical Computer Science*, AMS, 1999

Ahuja, R.K., Magnanti, T.L., Orlin, J.B., *Network Flows*, Prentice Hall, Englewood Cliffs, 1993

Babel, L., Tinhofer, G., A branch and bound algorithm for the maximum clique problem, *ZOR-Methods and Models of Operations Research*, 34:207-217, 1990

Balas, E., Xue, J., Weighted and unweighted maximum clique algorithms with upper bounds from fractional coloring, *Algorithmica*, 15 (5):397-412, 1996

Balas, E., Xue, J., Minimum weighted coloring of triangulated graphs, with application to maximum weight vertex packing and clique finding in arbitrary graphs, *SIAM Journal on Computing*, 20 (2): 209-221, 1991

Beasley, J.E., An algorithm for set covering problem, *European Journal of Operational Research*, 31:85-93, 1987

Booth, K.S., Lueker, G.S., Linear algorithms to recognize interval graphs and test for the consecutive ones property, *Proceedings of the Seventh Annual ACM Symposium on Theory of Computing*, 255-265, 1975

Brlaz, D., New methods to color the vertices of a graph, *Communications of the ACM*, 22 (4): 251-256, 1979

Carraghan, R., Pardalos, P.M., An exact algorithm for the maximum clique problem, *Operations Research Letters*, 9:375-382, 1990

Fulkerson, D.R., Gross, O.A., Incidence Matrices and Interval Graphs, *Pacific Journal of Mathematics*, 15:835-855, 1965

Garey, M., Johnson, D.S., *Computers and Intractability, A Guide to the Theory of NP-completeness*, Freeman, San Francisco, 1979

Gibbons, L.E., Hearn, D.W., Pardalos, P.M., Ramana, M.V., Continuous characterizations of the maximum clique problem, *Mathematics of Operations Research*, 22 (3):754-768, 1997

Golumbic, M.C., *Algorithmic Graph Theory and Perfect Graphs*, Academic Press, New York, 1980

Horowitz, E., Sahni, S., Rajasekaran, S., *Computer Algorithms C++*, Computer Science Press, New York, 1996

Karger, D., Motwani, R., Sudan, M., Approximate graph coloring by semidefinite programming, *Journal of the ACM*, 45 (2):246-265, 1998

McKee, T.A., McMorris, F.R., *Topics in Intersection Graph Theory*, SIAM Monographs on Discrete Mathematics and Applications, SIAM, Philadelphia, 1999

Pardalos, P.M., Rappe, J., Resende, M.G.C., An exact parallel algorithm for the maximum clique problem,
http://www.research.att.com/˜mgcr/doc/parclq.pdf

Roberts, F.S., On the boxicity and cubicity of a graph. In: *Recent Progress in Combinatorics*, W.T. Tutte (ed.), Academic Press, New York, 301-310, 1969

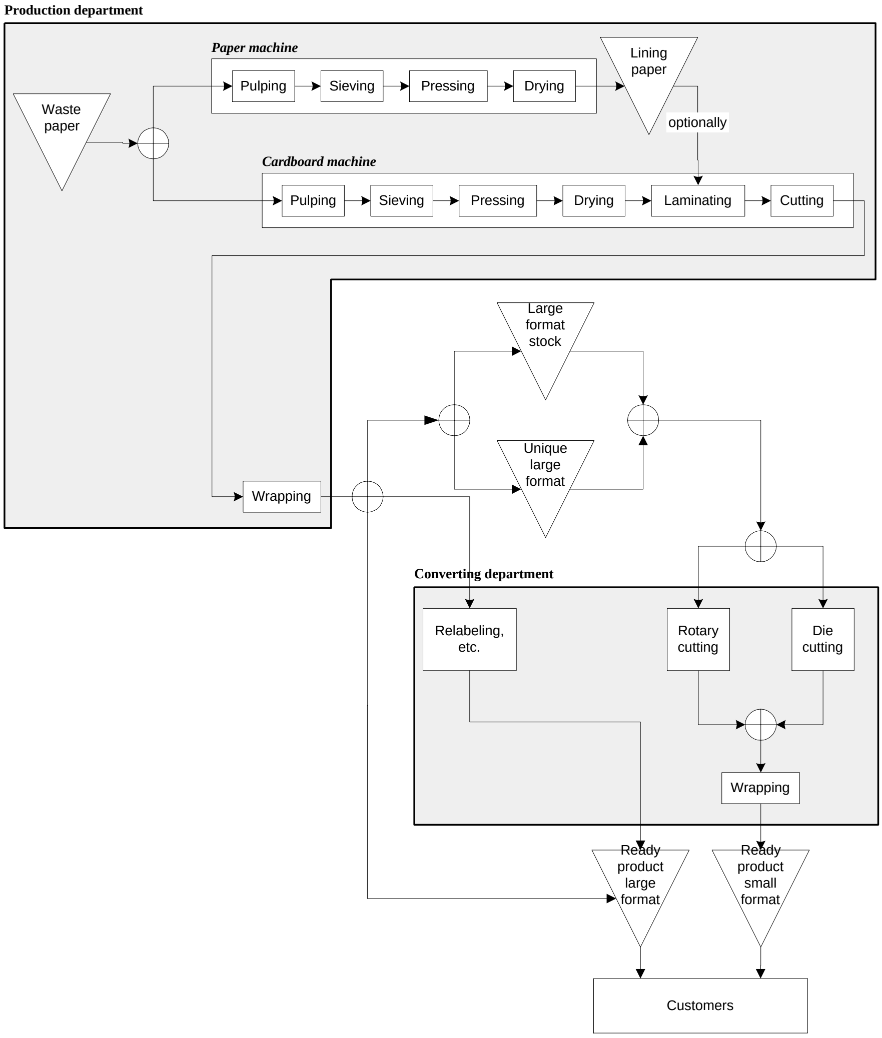
Roberts, F.S., Applications of combinatorics and graph theory to the biological and social sciencds: Seven fundamental ideas. In: *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences.* [*IMA Volumes in Mathematics* *and Its Applications* **17**], Springer, New York, 1-37, 1989

Roberts, F.S., *Graph Theory and its Applications to Problems of Society*, CBMS-NSF Regional conference series in applied mathematics, Odyssey Press, Dover, 3^{rd} edition, 1993

Sewell, E.C., A branch and bound algorithm for the stability number of a sparse graph, *INFORMS Journal on Computing*, 10 (4):438-447, 1999

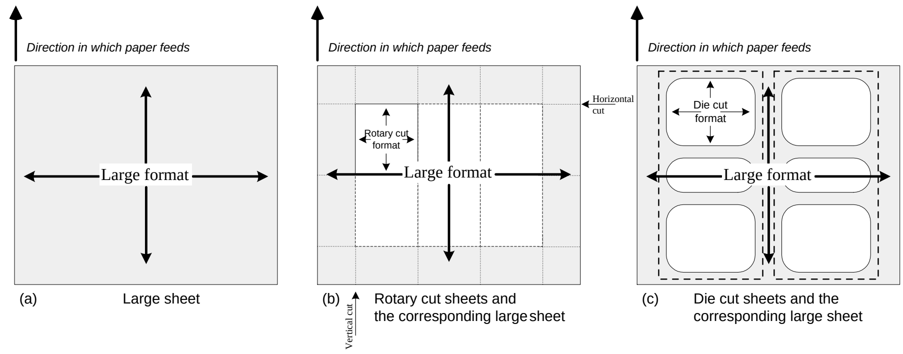
Sierksma, G., Wanders, H.L.T., The manufacturing of heavy weight cardboard, *International Journal of Production Economics*, 65:295-303, 2000

Wood, D.R., An algorithm for finding a maximum clique in a graph, *Operations Research Letters*, 21 (5):211-217, 1997

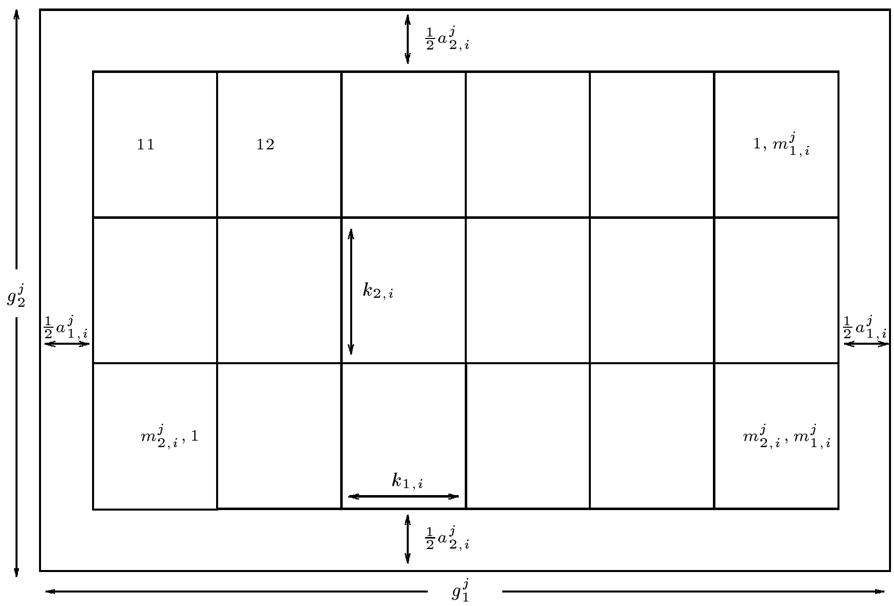


Figuur 1.1: The cardboard production process schematically

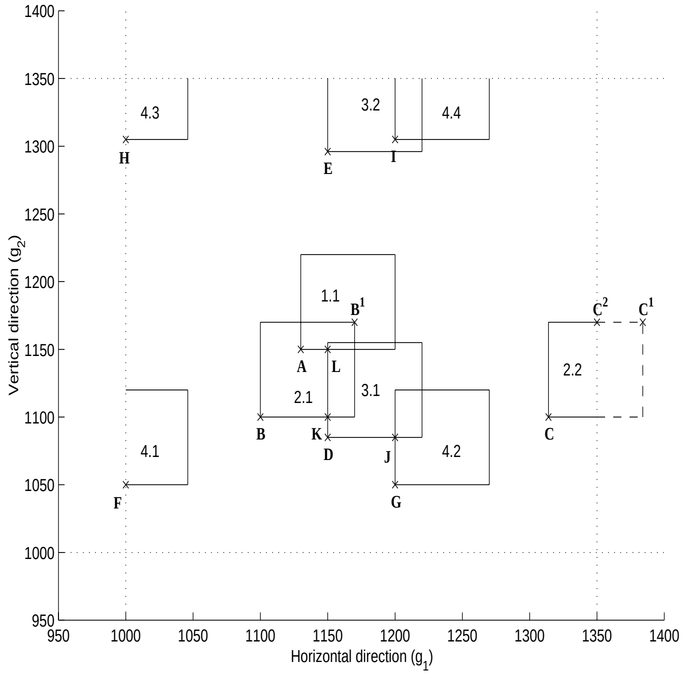
The triangles denote inventories, the rectangles processing steps, and the circles denote *either-or* structures. For example, after the wrapping, a large format is ready. It is put in the ‘ready product’ stock, or in an inventory to be processed in the final processing department (large format stock, and unique large format respectively).



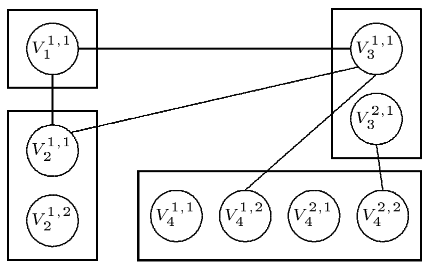
Figuur 1.2: Relation between a large and small format



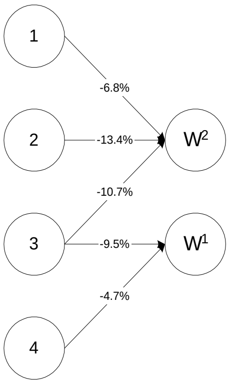
Figuur 2.1: Relation between a large sheet and rotary cut formats



Figuur 2.2: Large format feasible regions (LFF-regions)



Figuur 3.1: Graph corresponding to Figure 2.2



Figuur 4.1: Bipartite matching problem