UNIVERSITY

# Monitoring of Passive Optical Networks Utilising an Optical Coding Technique 

A thesis submitted in fulfillment of the requirements for the degree of Doctor of Philosophy by

Huda Saleh Abbas<br>MEng, BCS

School of Engineering<br>College of Science, Engineering and Health RMIT University

September 2017

## Statement

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

Huda Saleh Abbas
12 September 2017

## Acknowledgements

This thesis is dedicated to my parents, brothers and sisters, my husband, and my children for their support, patience and love.

I wish to acknowledge the important contributions of Associate Professor Mark A Gregory, my primary supervisor, who advised and supported me. I thank him for trusting in my capabilities. Also, I thank my second supervisor Professor Michael Austin for his guidance. I acknowledge the opportunity given to me by the government of Saudi Arabia and The Custodian of the Two Holy Mosques Scholarship for financial support, which provided the opportunity to further my education. By undertaking this PhD , I have an opportunity to contribute to my country, the education sector and society.


#### Abstract

Passive Optical Networks (PONs) have become the most popular fibre based access networks over the last decade. They are widely deployed for use in Fibre-to-the-Premises (FTTP) scenarios. PON is a point-to-multipoint connection (P2MP) between an optical line terminal (OLT) located at the central office (CO) and multiple optical network units (ONU) at the customer premises. The next generation of PONs (NG-PON) are likely to deploy a ring-andspur long reach PON (LR-PON). NG-PON aims to accommodate more ONUs, extend the network coverage out to 100 km , minimize complexity and improve operational outcomes. An all fibre access network, operating over extended distances, presents a reliability risk, thereby increasing the need for a reliable and cost-effective monitoring system to enhance protection and reduce restoration time. Among existing monitoring techniques, attention is focused on approaches that use optical code division multiplexing (OCDM), also known as optical coding (OC). The OC is applied to a signal that is sent from the network management system (NMS) to the ONUs. The monitoring signal is transmitted onto a fibre and split into a number of sub-signals that are equal in number to the ONUs. Each one of the ONUs receives a sub-signal, encodes it, and then reflects it back to the NMS. The NMS has the capability to identify faulty ONUs by examining the code received from the ONUs. A review of the literature has shown that the use of OCs does improve system performance, especially in the timely detection of faults. Many of the studies, found in the literature, focus on how to implement optical spreading codes that are used in OCDM Access (OCDMA) systems and currently the optical orthogonal code (OOC) is the dominant code implemented for timedomain coding. Although the OOC code performs well, its construction is relatively complex. The available code-words (cardinality) that are offered by OOC are proportional to the code length. Implementing OOC in a high capacity PON requires a long code length causes an inevitable degradation of system performance. Therefore, an improved optical coding


technique for PONs should provide code-words that conform to PON split ratios.

The main objective of the research was to develop an optical spreading code, based on a prime code family for OCDMA systems, that has the capability to accommodate different PON split ratios and with characteristics that improve transmission system performance when compared to existing prime code families. The novel code presented in this thesis is identified as the extended grouped new modified prime code (EG-nMPC). The number of code-words generated by the proposed codes are substantially higher than those generated by the existing code families and more compatible to the different PON splitting ratios. In addition, with a low code weight, both power consumption and hardware complexity decreases.

The code performance was evaluated using mathematical models for two transmission formats - pulse position modulation (PPM) and on-off keying (OOK) modulation. The performance of EG-nMPC was compared to other prime codes, and the results show that the proposed code improves the performance of OCDMA in terms of bit-error rate (BER).

As PON is a point-to-multipoint connection oriented access network, downstream traffic is encrypted and broadcast to all ONUs, while the unencrypted upstream traffic from each ONU terminal occurs in a burst mode. The OLT carries out a ranging process to determine transmission delays between ONUs, to prevent collisions between the burst mode traffic from each of the ONUs. In this research, the burst mode traffic ranging process has been replicated in the monitoring system, with this replication providing a fixed equalization delay time for the monitoring transmissions.

To investigate the ring-and-spur LR-PON reliability several protection architectures were evaluated, in term of cost and availability, to determine the optimal protection architecture. In this thesis, the reliability parameter Failure Impact Robustness (FIR), has been used to calculate the failure impact of the different components in ring-and-spur LR-PON, hence
selecting the optimal protection scheme.

A PON-based optical communication system model was developed and the proposed EGnMPC code was incorporated. Fibre split ratios of 32, 64 and 128, were considered in this study. The simulation results show that the EG-nMPC code improves the performance, efficiency and accuracy of the PON transmission monitoring system.

To conclude, this research aims to enhance the PON performance by a fast detection of the fault and quick restoration. This research has contributed to knowledge by identifying a new and novel spreading code that is compatible with the different PON splitting ratios for OC monitoring techniques. By using the ranging process, a fixed equalization delay time has been assigned to each ONU to manage the upstream burst traffic. The spreading code has been implemented in a real-time simulation to show the status of each fibre link. The implementation was carried out based on 1-D tree topology system. However, the proposed EG-nMPC can be exploited to enable network monitoring that is based on hybrid 1D/2D coding. This coding is complementary with the structure of LR-PON as explained in section 8.2.3. In addition, with the use of the FIR parameter for the different components in the ring-and-spur architecture, an optimal protection scheme for both OLT and the ring (feeder fibre), has been nominated. This protection scheme ensures that protection, availability and cost are at their optimal values.

## Table of Contents

Statement ..... ii
Acknowledgements ..... iii
Abstract ..... iv
Table of Contents ..... vii
List of Figures .....
List of Tables ..... xiii
List of Acronyms and Abbreviations ..... xiv
Chapter 1 INTRODUCTION ..... 1
1.1 Introduction ..... 2
1.2 Research Problem ..... 8
1.3 Research Objectives ..... 11
1.4 Research Contribution ..... 12
1.5 Research Methodology ..... 13
1.6 Publications ..... 14
1.7 Structure of Thesis ..... 15
Chapter 2 LITERATURE REVIEW ..... 17
2.1 Next Generation-PON ..... 18
2.2 Multiplexing Techniques ..... 19
2.2.1 Pure Techniques ..... 19
2.2.1.1 Time Division Multiplexing- Passive Optical Network ..... 19
2.2.1.2 Wavelength Division Multiplexing-Passive Optical Networks ..... 20
2.2.1.3 Optical Frequency Multiplexing PON ..... 22
2.2.1.4 Optical Code Division Multiplexing PON ..... 23
2.2.2 Hybrid TDM and WDM-PON ..... 24
2.3 Long-reach PON ..... 26
2.4 PON Reliability ..... 28
2.4.1 PON Protection Schemes ..... 28
2.4.2 Protection for ring-and-spur LR-PON ..... 30
2.5 PON Monitoring ..... 32
2.6 GPON ..... 36
2.7 OCDMA Systems ..... 39
2.7.1 Encoding Principle ..... 39
2.7.2 Encoder/Decoder Devices ..... 42
2.7.2.1 Optical Delay Line ..... 42
2.7.2.2 Fibre Bragg Grating ..... 44
2.7.2.3 Arrayed Waveguide Grating ..... 46
2.7.3 OCDMA System Challenges ..... 47
2.7.4 Spreading Code Design Issues ..... 48
2.7.5 Optical Spreading Codes ..... 49
2.7.6 OCDMA Modulation Schemes ..... 51
2.8 PON Monitoring based OC Technique Research ..... 52
2.9 Summary ..... 57
Chapter 3 OPTICAL SPREADING CODE ..... 59
3.1 Optical Orthogonal Code ..... 60
3.2 Quadratic Congruence Code ..... 61
3.4 Prime Code ..... 63
3.4.1 Basic Prime Code ..... 63
3.4.2 Extended Prime Code ..... 65
3.4.3 Modified Prime Code ..... 65
3.4.4 New Modified Prime Code ..... 66
3.4.5 Padded Modified Prime Code ..... 67
3.4.6 Group padded Modified Prime Code ..... 68
3.4.7 Double padded Modified Prime Code ..... 69
3.4.8 Transposed Modified Prime Code ..... 71
3.4.9 Transposed Sparse-padded Modified Prime Code ..... 73
3.5 Summary ..... 77
This chapter provides a broad background of the most common codes applied into OCDMA system including OOC, QCC and PC families. It gives details about the construction of the codes, the parameters and an example of the resulted code-words ..... 77
Chapter 4 EXTENDED GROUPED NEW MODIFIED PRIME CODE ..... 79
4.1 Proposed Code ..... 80
4.1.1 EG-nMPC Construction ..... 80
4.1.2 Code Parameters ..... 83
4.2 Performance Analysis ..... 88
4.2.1 Performance Analysis of OOK-OCDMA System ..... 89
4.2.2 Performance Analysis of PPM-OCDMA System ..... 89
4.3 Discussion ..... 92
4.4 Code Comparison for GPON Splitting Ratios ..... 96
4.5 Summary ..... 99
Chapter 5 NG-PON PROTECTION ..... 100
5.1 NG-PON Architecture ..... 101
5.2 FIR for Network Components ..... 102
5.3 Protection Schemes for LR-PON ..... 104
5.3.1 Availability ..... 105
5.3.2 Cost ..... 108
5.3.3 Discussion ..... 109
5.4 Summary ..... 111
Chapter 6 NG-PON MONITORING ..... 113
6.1 GPON Ranging Process for the Monitoring Layer ..... 114
6.2 Principle of Monitoring System ..... 115
6.3 Monitoring pulse width ..... 116
6.4 Numerical results ..... 119
6.5 Summary ..... 123
Chapter 7 IMPLEMENTATION ..... 125
7.1 VPI TransmissionMAker overview ..... 126
7.2 Network Simulation with Four ONUs ..... 127
7.2.1 Monitoring Signal Generator ..... 128
7.2.2 Remote Node Splitter ..... 129
7.2.3 Encoding ..... 130
7.2.4 Remote Node Combiner ..... 135
7.2.5 Fibre Link ..... 136
7.2.6 Decoding and Fault Identification ..... 137
7.2.8 Fibre degradation ..... 143
7.3 A Splitting Ratio of 32 ..... 145
7.4 A Splitting Ratio of 64 ..... 150
7.5 A Splitting Ratio of 128 ..... 156
7.6 Discussion ..... 162
7.7 Summary ..... 163
Chapter 8 CONCLUSION AND FUTURE WORK ..... 164
8.1 Conclusion ..... 165
8.2 Future work ..... 167
8.2.1 Hybridization of OTDR and OC for LR-PON ..... 167
8.2.2 Constructing a 2-D coding using EG-nMPC as one of its dimensions ..... 167
8.2.3 Implementing 1D/2D coding in LR-PON ..... 167
BIBLIOGRAPHY ..... 169
Appendix A1. Simulation of Four ONUs ..... 178
A1.1. Simulation parameters of four ONUs ..... 178
Appendix A2. A Splitting Ratio of 32 ..... 179
A2.1. Simulation parameters of 32 ONUs ..... 179
A2.2. Sub-pulses times before and after delay ..... 182
Appendix A3. A Splitting Ratio of 64 ..... 184
A3.1. Simulation parameters of 64 ONUs ..... 184
A3.2. Binary codes for EG-nMPC, $P=5$ ..... 191
A3.3. Sub-pulses times before and after delay ..... 194
Appendix A4. A Splitting Ratio of 128 ..... 198
A4.1. Simulation parameters of 128 ONUs ..... 198
A4.2. Binary codes for EG-nMPC, $P=7$ ..... 217
A4.3. Sub-pulses times after delay ..... 221

## List of Figures

Figure 1-1 PON architecture [4] ..... 3
Figure 1-2 OCDMA system [25] ..... 5
Figure 2-1 Design of TDM-PON [13] ..... 20
Figure 2-2 WDM-PON based legacy TDM-PON [12] ..... 21
Figure 2-3 Standard WDM-PON [44] ..... 21
Figure 2-4 Design of OCDM-PON [54] ..... 24
Figure 2-5 Design of TWDM-PON [57] ..... 24
Figure 2-6 TWDM-PON, utilizes a combination of AWG and power splitters [58] ..... 25
Figure 2-7 Long reach PON [74] ..... 27
Figure 2-8 Ring-and-spur long reach PON design [79] ..... 31
Figure 2-9 Protection systems based on [80] ..... 32
Figure 2-10 OTDR fault trace [81] ..... 34
Figure 2-11 Optical coding monitoring technique [81] ..... 36
Figure 2-12 Downstream frame [90] ..... 37
Figure 2-13 Upstream frame [90] ..... 37
Figure 2-14 Upstream and downstream [91] ..... 38
Figure 2-15 Coding dimensions [96] ..... 41
Figure 2-16 1-D ODL-based encoder/decoder [27] ..... 42
Figure 2-17 2-D ODL-based encoder/decoder [27] ..... 43
Figure 2-18 2-D FBG-based serial encoder/decoder [101] ..... 46
Figure 2-19 2-D FBG-based parallel encoder/decoder [101] ..... 46
Figure 2-20 AWG with ODL feedback [101] ..... 47
Figure 2-21 AWG with mirrored ODL [101] ..... 47
Figure 2-22 Signalling format, (a) OOK-OCDMA, (b) PPM-OCDMA [106] ..... 52
Figure 4-1 Auto-correlation of EG-nMPC of $C_{010}$, at synchronization time, $T$ ..... 86
Figure 4-2 Cross-correlation of EG-nMPC of $C_{014}$ and $C_{112}$, at synchronization time, $T$ ..... 86
Figure 4-3 Cross-correlation of EG-nMPC of $C_{022}$ and $C_{125}$, at synchronization time, $T$ ..... 87
Figure 4-4 Cross-correlation of EG-nMPC of $C_{110}$ and $C_{112}$, at synchronization time, $T$ ..... 87
Figure 4-5 Cross-correlation of EG-nMPC of $C_{024}$ and $C_{013}$, at synchronization time, $T$ ..... 88
Figure 4-6 Cross-correlation of EG-nMPC of $C_{020}$ and $C_{210}$, at synchronization time, $T$ ..... 88
Figure 4-7 Cross-correlation expectations of MPC, T-SPMPC and EG-nMPC ..... 94
Figure 4-8 BER versus number of communication channels, for MPC, T-SPMPC, and EG-nMPC, using OOK system for $P=11$ and $P=13$ ..... 94
Figure 4-9 BER versus number of communication channels, for MPC, T-SPMPC, and EG-nMPC using OOK system, for $P=23$ and $P=37$ ..... 95
Figure 4-10 BER versus number of communication channels, for MPC, and EG-nMPC using PPM- OCDMA system, for $P=11$, and $P=13$ ..... 95
Figure 4-11 BER versus number of communication channels, for MPC, and EG-nMPC using PPM- OCDMA system, for $P=11, M=8$ and $M=16$ ..... 96
Figure 4-12 Cardinality of MPC, T-SPMPC and EG-nMPC ..... 96
Figure 5-1 LR-PON ..... 102
Figure 5-2 FIR for different network components in ring-and-spur ..... 103
Figure 5-3 ABD for the proposed protection schemes, (a) OLT-Ring protection, (b) OLT-Ring-DF protection, (c) Ring-DF protection ..... 104
Figure 5-4 Availability of different protection schemes of LR-PON ..... 111
Figure 5-5 Cost of different protection schemes of LR-PON ..... 111
Figure 6-1 Principle of upstream transmission using equalization delay ..... 115
Figure 6-2 Monitoring system (1-D) ..... 116
Figure 6-3 SNR versus pulse width for dark and thermal noises for all splitting ratios of 32, 64, and 128 ..... 121
Figure 6-4 SNR versus pulse width for shot noise for splitting ratios of 32, 64, and 128 ..... 121
Figure 6-5 SNR versus pulse width for beat noise for splitting ratios of 32,64 , and 128 ..... 122
Figure 6-6 Beat and Shot noises for different splitting ratios ..... 122
Figure 6-7 SIR versus $T_{c}$, for $N=32,64$ and 128 ..... 123
Figure 6-1 VPI Hierarchical organization ..... 127
Figure 7-1 VPI model of four ONUs ..... 128
Figure 7-2 VPI OOK transmitter design ..... 129
Figure 7-3 Monitoring pulse generator output ..... 129
Figure 7-4 VPI splitter and combiner ..... 130
Figure 7-5 VPI encoder design ..... 130
Figure 7-6 Encoder 1 output ..... 131
Figure 7-7 Encoder 2 output ..... 131
Figure 7-8 Encoder 3 output ..... 131
Figure 7-9 Encoder 4 output ..... 132
Figure 7-10 Start and end times. ..... 133
Figure 7-11 Encoder 1 output with delay ..... 133
Figure 7-12 Encoder 2 output with delay ..... 133
Figure 7-13 Encoder 3 output with delay ..... 134
Figure 7-14 Encoder 4 output with delay ..... 134
Figure 7-15 Encoder 1 output with delay closeup ..... 134
Figure 7-16 Encoder 2 output with delay closeup ..... 134
Figure 7-17 Encoder 3 output with delay closeup ..... 135
Figure 7-18 Encoder 4 output with delay closeup ..... 135
Figure 7-19 Analyser before combiner output ..... 136
Figure 7-20 Combined signal ..... 136
Figure 7-21 Fibre and reflection ..... 137
Figure 7-22 Reference signal ..... 137
Figure 7-23 Reference signal output ..... 137
Figure 7-24 Sampler output ..... 138
Figure 7-25 Thresholder output ..... 138
Figure 7-26 VPI analyser data example for thresholder and reference signal and the exported data in Excel ..... 140
Figure 7-27 Filtered data for reference signal ..... 141
Figure 7-28 Filtered data for the thresholder ..... 142
Figure 7-29 Fibre brake module in VPI. ..... 143
Figure 7-30 Encoder 1 output for a Fault ..... 143
Figure 7-31 Encoded signals for a fibre fault to ONU1 before combiner ..... 144
Figure 7-32 Encoded signals for a fibre fault to ONU1 after combiner ..... 144
Figure 7-33 Sampler output for a fibre fault to ONU1 ..... 144
Figure 7-34 Thresholder output for a fibre fault to ONU1 ..... 145
Figure 7-35 VPI model of 32 ONUs ..... 145
Figure 7-36 Encoded combined signals ..... 146
Figure 7-37 Sampler output ..... 146
Figure 7-38 Thresholder output ..... 147
Figure 7-39 Reference signal outputs ..... 147
Figure 7-40 Encoded combined signals, (faulty case) ..... 148
Figure 7-41 Sampler output, (faulty case) ..... 149
Figure 7-42 Thresholder output, (faulty case) ..... 149
Figure 7-43 VPI model for 64 ONUs ..... 150
Figure 7-44 Combined signal output for 64 ONUs, (healthy case) ..... 151
Figure 7-45 Sampler output for 64 ONUs, (healthy case) ..... 151
Figure 7-46 Thresholder output for 64 ONUs, (healthy case) ..... 151
Figure 7-47 Reference signal output for 64 ONUs, (healthy case) ..... 151
Figure 7-48 Closeup output of the Combiner, thresholder and reference signal of encoder 1, 2, and 3 with delay ..... 153
Figure 7-49 Sub-pulses times before and after delay ..... 153
Figure 7-50 Combined signal output for 64 ONUs, (faulty case) ..... 154
Figure 7-51 Sampler output for 64 ONUs, (faulty case) ..... 154
Figure 7-52 Thresholder output for 64 ONUs, (faulty case) ..... 154
Figure 7-53 VPI model for 128 ONUs ..... 156
Figure 7-54 Combined signal output for 128 ONUs (healthy case) ..... 157
Figure 7-55 Combined signal output for 128 ONUs (faulty case) ..... 157
Figure 7-56 Reference signal output for 128 ONUs ..... 157
Figure 7-57 Combined signal output for 64 ONUs out of 128, (healthy case) ..... 158
Figure 7-58 Sampler output for 64 ONUs out of 128, (healthy case) ..... 158
Figure 7-59 Thresholder output for 64 ONUs out of 128, (healthy case) ..... 159
Figure 7-60 Reference signal output for 64 ONUs out of 128 ..... 159
Figure 7-61 Combined signal output for 64 ONUs out of 128, (faulty case). ..... 159
Figure 7-62 Sampler output for 64 ONUs out of 128, (healthy case) ..... 160
Figure 7-63 Thresholder output for 64 ONUs out of 128, (healthy case) ..... 160
Figure 7-64 Closeup output of the Combiner, thresholder and reference signal of encoder 1, 2, and 3 with delay ..... 161
Figure 7-65 Sub-pulse delay times after and before delay for encoder1, 2, and 3 ..... 161

## List of Tables

Table 1-1 The characteristics of different Prime code families ..... 10
Table 2-1 ITU PON maintenance recommendations [32] ..... 33
Table 2-2 Monitoring techniques [84] ..... 35
Table 2-3 Coding domain and relevant devices [96] ..... 41
Table 2-4 Encoder/decoder delay lines [100] ..... 44
Table 2-5 OCDM codes and protocols [27] ..... 51
Table 3-1 OOC ( $N, 3,1$ ) sequence indexes of various lengths [30] ..... 61
Table 3-2 Sequence codes of OOC $(31,3,1)$ [30] ..... 61
Table 3-3 QCC code sequences for $P=7$ [27] ..... 62
Table 3-4 Basic prime code sequences for $P=5$ [27] ..... 63
Table 3-5 Binary sequence of basic PC, $P=5$ [27] ..... 64
Table 3-6 Extended prime code sequences, $P=5$ [123] ..... 65
Table 3-7 Modified prime code sequences, $P=5$ [124] ..... 66
Table 3-8 New modified prime code sequences, $P=5$ [105] ..... 67
Table 3-9 Padded modified prime code sequences, $P=5$ [30] ..... 68
Table 3-10 Double padded modified prime code sequences, $P=5$ [30] ..... 69
Table 3-11 Double padded modified prime code, $P=5$ [129] ..... 70
Table 3-12 Full padded modified prime code, $P=5$ [132] ..... 71
Table 3-13 Transposed modified prime code sequences, $P=5$ [132] ..... 72
Table 3-14 Sparse padded sequence, SP, $P=5$ [133] ..... 74
Table 3-15 Intermediate sparse padded MPC [133] ..... 74
Table 3-16 Sparse padded MPC, $P=5$ [133] ..... 75
Table 3-17 Transposed sparse-padded modified prime code, $P=5$ [133] ..... 76
Table 4-1 Time shifting of extended prime code, $P=3$ ..... 81
Table 4-2 Binary sequence of extended new modified prime code ..... 82
Table 4-3 Binary sequence of extended grouped new modified prime code ..... 82
Table 4-4 Examples of optimal OOC code [27] ..... 97
Table 4-5 Code Parameter Comparison ..... 99
Table 5-1 FIR parameters ..... 102
Table 5-2 Parameters used in the simulation of Cost and availability ..... 110
Table 6-1 Simulation parameters ..... 123
Table 7-1 Sub-pulse times for the four encoders ..... 132
Table 7-2 Expected start/end times and sub-pulses times for the ONUs ..... 133
Table 7-3 Fibre ID and status for four ONUs (healthy case) ..... 142
Table 7-4 Fibre ID and status for four ONUs (faulty case) ..... 145
Table 7-5 Fibre ID and status (healthy case) ..... 147
Table 7-6 Fibre ID and status, (Faulty case) ..... 149
Table 7-7 Fibre ID and status for 64 ONUs, (healthy case) ..... 152
Table 7-8 Fibre ID and status for 64 ONUs, (faulty case) ..... 155
Table 7-9 Fibre ID and status for 64 ONUs out of 128 (healthy case) ..... 162
Table 7-10 Fibre ID and status for 64 ONUs out of 128 (faulty case) ..... 162

## List of Acronyms and Abbreviations

| ABD | availability block diagrams |
| :---: | :---: |
| Alloc-ID | Allocation Identifier |
| APON | Asynchronous transfer mode passive optical network |
| ATM | Asynchronous transfer mode |
| AWG | Array waveguide grating |
| BER | Bit-error rate |
| BFSA | Brillouin frequency shift assignment |
| BPON | Broadband passive optical network |
| BW map | Bandwidth map |
| CO | Central Office |
| CoS | Classes of Service |
| CWDM | Coarse wavelength division multiplexing |
| DBA | Dynamic bandwidth allocation |
| DBWA | Dense bandwidth and wavelength allocation |
| DPMPC | Double-padded modified prime code |
| DPSK | Differential phase-shift keying |
| DRA | Distributed Raman amplifier |
| DS-OCDM | Direct Sequence OCDM |
| DWDM | Dense wavelength division multiplexing |
| EDFA | Erbium doped fibre amplifier |
| EG-nMPC | extended grouped new modified prime code |
| EPC | Extended prime code |


| EPON | Ethernet PON |
| :---: | :---: |
| FBG | Fibre Bragg grating |
| FIR | Failure Impact Robustnes |
| FSAN | Full service access network group |
| FTTP | Fibre-to-the-Premises |
| Gbps | Gigabyte per second |
| GEM | GPON Encapsulation Method |
| GF | Galois field |
| GPMPC | Group padded modified prime code |
| GPON | Gigabit passive optical network |
| GTC | the GPON Transmission Convergence |
| IEEE | Institute of Electrical and Electronic Engineers |
| ITU | International Telecommunications Union and its standards (ITU-T) |
| IP | Internet Protocol |
| IPPC | Incrementally Pulse Positioned Code |
| LAG | Link Aggregation |
| LAN | Local area network |
| LR-PON | Long reach PON |
| MAI | Multiple access interference |
| MCIP | Multiple-Customers Interference Probability |
| ML-OOC | Multilevel-Optical Orthogonal Code |
| MPC | Modified prime code |
| NG-PON1 | Next generation PON stage 1 |
| NG-PON2 | Next generation PON stage 2 |
| n -MPC | New modified prime code |


| NMS | Network Management System |
| :---: | :---: |
| OBS | Optical burst switching |
| OC | Optical coding |
| OCDM | Optical code division multiplexing |
| OCDMA | Optical code division multiple access system |
| ODN | Optical distribution network |
| OFDM | Optical frequency division multiplexing |
| OLT | Optical line terminal |
| ONU | Optical network unit |
| OOC | Optical orthogonal codes |
| OOK | On-off keying |
| ODL | Optical tapped delay line |
| OTDR | Optical time domain reflectometer |
| P | Prime number |
| P2MP | Point to multi-point |
| PC | Prime code |
| PCBd | Physical Control Block downstream |
| PLOu | Physical Layer Overhead |
| PMPC | Padded modified prime code |
| PON | Passive optical networks |
| PPM | Pulse position modulation |
| PSC | Power Splitter/Combiner |
| QCC | Quadratic congruence code |
| QoS | Quality of service |
| RN | Remote node |


| SIR | Signal to Interference Ratio |
| :---: | :---: |
| SLA | Service Level Agreement |
| SL-RSOA | Self-injection locked reflective semiconductor optical amplifier |
| SMAC | Slotted Medium Access Control |
| SNI | Server Node Interface |
| SNR | Signal-to-Noise Ratio |
| SNIR | Signal-to-Noise-and-Interference Ratio |
| SOA | Semiconductor optical amplifier |
| SPON | Slotted-PON |
| T-CONT | traffic containers |
| TDM | Time division multiplexing |
| TDMA | Time division multiple access |
| T-MPC | Transposed modified prime code |
| T-SPMPC | Transposed sparse-padded modified prime code |
| TWDM | Time/wavelength division multiplexing |
| UNI | User Network Interface |
| WDM | Wavelength division multiplexing |
| XG-PON | 10 Gigabit-capable passive optical network |
| XGS-PON | 10-gigabit symmetrical passive optical network |
| X-TDM | Hybrid time division multiplexing |
| X-WDM | Hybrid wavelength division multiplexing |

## Chapter 1 INTRODUCTION

The research carried out relates to the monitoring of Passive Optical Networks (PON), utilizing the Optical Coding (OC) technique. In the introductory chapter, an overview of PON monitoring is provided followed by the research problem, research objectives, contribution, and methodology. The introductory chapter ends with a thesis summary.

### 1.1 Introduction

PON is defined as a broadband access network technology that offer advantages, when employed to provide Fibre-to-the-Premises (FTTP), including cost-effectiveness; a point to multi-point architecture; high quality triple-play service capabilities for data, voice and video; reduced maintenance and operational costs; and high speed internet access [1]. The International Telecommunications Union (ITU) and the Institute of Electrical and Electronic Engineers (IEEE) developed the Asynchronous Transfer Mode (ATM) technology in the 1980s, and this was utilised as the basis for early PON versions including ATM PON (APON), Broadband PON (BPON) and the more recent variation Gigabit PON (GPON); and a second group named Ethernet PON (EPON) [2, 3]. Of these, EPON and GPON are now ubiquitous. A conventional PON architecture is presented in Figure 1-1 [4].

Figure 1-1 shows the PON architecture which consists of an Optical Line Terminal (OLT), Optical Distribution Network (ODN), and Optical Network Units (ONU). The PON Point to Multi-Point (P2MP) architecture includes an OLT that is located in the Central Office (CO), ONUs that are located in premises within a region around the CO and fibre distribution that utilises optical splitters to connect the feeder fibre from the OLT to multiple ONUs using drop fibres [5].


Figure 1-1 PON architecture [4]
The first PON generation was based on Time Division Multiple Access (TDMA), and provided an EPON downstream rate of one gigabit per second (Gbps) and a GPON downstream rate of 2.4 Gbps. The next generation PON stage 1 (NG-PON1) increased the data rate to up to 10 Gbps for both variants. With the rapid expansion of high bandwidth applications and internet services, however, the NG-PON1 was found to be insufficient to meet the demand for more bandwidth and an improved Quality of Service (QoS). To determine an acceptable upgrade pathway, research was conducted to investigate options for the next generation PON stage 2 and one implementation has now occurred (NG-PON2) [4].

PON technologies are classified as either pure or hybrid technologies [6] [7]. Pure technologies include high speed Time Division Multiplexing PON (TDM-PON), Wavelength Division Multiplexing PON (WDM-PON), Optical Code Division Multiplexing PON (OCDM-PON), and Orthogonal Frequency Division Multiplexing PON (OFDM-PON) [8]. Hybrid technologies include hybrid TDM and WDM (TWDM), X-WDM and X-TDM [4, 9, 10].

The predominant methods for implementing PON use either WDM-PON or time TDM-PON, or a combination of the two techniques [11]. Although TDM-PON is a cost-effective technology, it was found to be restrictive in terms of security and bandwidth. WDM-PON provides high bandwidth to multiple ONUs. It is, however, more expensive due to the need for additional equipment [12]. OFDM-PON is scalable and the current implementation has a higher capacity ( 40 Gbps ). It is also efficient and flexible, with dynamic bandwidth allocations for downstream and upstream transmissions [13]. Its flexibility and optical transmission complexity, however, requires more power, setup and may be less secure [1416]. OCDM-PON makes efficient use of bandwidth and is more secure. However, with an increasing number of ONUs, issues such as multiple access and noise interference could occur [17].

Hybridised TDM-PON and WDM-PON combine the advantages of both technologies [12, 18]. Over time, a TWDM-PON design emerged as standard for NG-PON2 [19], based on performance ( $40 \mathrm{Gbps} / 10 \mathrm{Gbps}$ ), power and cost [20].

In terms of topology, ring-and-tree, identified in some of the literature as ring-and-spur, is the dominant topology for NG-PON [21-23]. The ring, feeder fibre, is based on WDM and connects the OLT to a number of Remote Nodes (RN), each of which serves a spur that is based on TDM. A ring-and-spur topology meets the requirements of the NG-PON as it aims to support more than 1000 ONUs over distances up to 100 km and to optimize the fibre infrastructure utilization, enhance scalability, and increase distribution flexibility as additional RNs could be added [24].

The general structure of an OCDM Access (OCDMA) system is based on a star topology as shown in Figure 1-2 [25]. Each communication channel in the system is composed of both transmission and receiver sides connected by a star coupler. The main components of the
transmission side are 'information source' and 'encoder'. The main components of the receiver side are 'decoder' and 'receiver' [26]. Each transmitter on the OCDMA system has been allocated a unique code-word. Transmitted data is based on a binary number to represent the presence or absence of the light ( $1=$ presence, $0=$ absence ) [27]. If the data is a 1 , the encoding process will be executed and if it is a 0 , the encoded process will not be executed [25]. Each transmitter encodes the data with its corresponding code-word. The decoding process is a reverse operation of the encoding process.


Figure 1-2 OCDMA system [25]
Of the two OCDMA systems, one utilises a coherent approach while the other has an incoherent approach. Incoherent OCDMA is more popular than coherent OCDMA because of the simplicity and robustness of its encoding and decoding processes [28]. Encoding and decoding systems that are based on incoherent OCDMA utilise unipolar codes, due to improved performance compared to the bipolar codes that have been used in radio frequency CDMA [27]. As a result, research has focused on developing new codes. Common unipolar codes include optical orthogonal/pseudo-orthogonal codes (OOC), prime codes (PC), and
quadratic congruence codes ( QCC ). The OOC is quite common due to its performance when compared to other codes utilising measures such as the correlation functions (auto-and cross correlation). However, the construction of an OOC is complex. Several studies have been proposed to improve the performance of PC and QCC. The construction of a PC that is based on congruence techniques is less complex than the construction of OOC [27].

Code-words can be formed by utilizing a one-dimensional (1-D) encoding scheme in either the time or frequency domains. In the case of 1-D coding, the code cardinality (number of available code-words) will be limited. It requires a longer code length to provide more codewords. A two-dimensional (2-D) encoding scheme is a combination of both domains. 2-D encoding has been proposed to solve the limitation associated with code cardinality in 1-D encoding. A three-dimensional (3-D) encoding scheme that uses space or polarization as the third domain, provides a further increase in the code cardinality [29].

The aims of an OCDMA system are, firstly, to accommodate a large number of communication channels in the system and, secondly, to distinguish the signal of the intended transmitter from other transmitters' signals. Increasing the number of communication channels leads to an increase in the Multiple Access Interference (MAI), an undesirable outcome as it leads to system performance degradation. Therefore, implementing an orthogonal code with high auto correlation and low cross-correlation is highly recommended to make the required distinction between signals [30].

Ensuring service reliability is one of the most important requirements for end-users [31]. With the NG-PON architecture, its high capacity and large coverage area, fault management has gained in importance. Network providers aim to reduce failure impact, which refers to minimizing the number of ONUs that are affected by a single failure in the network [31]. Thus, a reliability parameter named Failure Impact Robustness (FIR) has been introduced by

Four types of protection schemes, known as Type A to D, have been standardized in the ITU, Series G, Supplement 51 "Passive optical network protection considerations" to protect sections of the network.

Besides network protection, monitoring the network is another important factor that ensures service continuity. PON monitoring was standardised for surveillance, testing and control by the ITU as either pre- or post-fault [32]. Monitoring techniques are classified as either based on Optical Time Domain Reflectometry (OTDR), or based on non-OTDR. OTDR is based on Fresnel reflection and Rayleigh backscattering, therefore it is a promising technique for monitoring point-to-point connections [33]. Examples of non-OTDR based monitoring techniques are Brillouin Frequency Shift Assignment (BFSA), reference reflector-broadband source, reflected signal, Self-Injection Locked Reflective Semiconductor Optical Amplifier (SL-RSOA), and monitoring based on OCDM, which is referred to as an OC technique [34]. Amongst these techniques, OC has received considerable attention due to its simplicity and cost-effectiveness. This technique overcomes the limitations associated with OTDR and provides a favourable solution to monitor PON with its potentially large number of ONUs connected to each feeder fibre [33]. Several studies have been proposed for PON monitoring based on OC and are discussed in more detail in the literature review.

As discussed in above, the standard OCDMA system requires a code with a high correlation to recognize the intended signal [25]. However, in PON, the OLT performs a ranging process to assign an equalization delay for each ONU to avoid burst collision. Monitoring based OC can apply this ranging process into the monitoring layer; hence the condition of high autocorrelation and low cross-correlation has become less important. The main objective is to provide a number of code-words that are compatible with the PON splitting ratio with smaller

Prime number $(P)$. There is no mention in the literature of a monitoring system that uses codes based on OCDMA codes other than OOC. The use of a PC in this research provides a new and innovative approach that is shown to have several benefits over alternate approaches.

### 1.2 Research Problem

The aim of PON technologies is to increase data throughput to 40 Gbps and beyond, extend the access network reach, and accommodate more ONUs. This architecture leads to challenges in network monitoring and thus a cost-effective management system is required. The challenges have led to a substantial amount of research being carried out in this area [35]. The ITU L66 recommendation 'Optical fibre cable maintenance criteria for in-service fibre testing in access networks' recommended the standardization of PON maintenance and monitoring functions. According to this recommendation, OTDR has been picked up as the optimal technique for in-service monitoring, testing and measurement. OTDR is a reliable technique that detects and locates the exact location of a fault. However, standard OTDR cannot be implemented in a PON, as PON utilises a P2MP architecture; therefore, research has been carried out to modifying standard OTDR in order to cope with the PON P2MP architecture. The modifications led to limitations in network capacity and an increase in the system complexity [33].

The cost of design, installation and establishment, or the capital cost, for monitoring each fibre link, and the total ongoing system maintenance costs are calculated and provide a life cycle cost for a monitoring system. Thus, monitoring systems should be simplified to reduce costs. Constant monitoring of the ONU links enhances reliability. Utilizing OTDR, however, is not economically feasible for a PON network with a large number of ONUs. The monitoring system should be designed for a large and increasing number of ONUs so there is scope for further research [4]. Monitoring utilising an OC technique is preferred over other
techniques due to its simplicity and low cost. This technique utilizes the OOC and PC codes that have been used in OCDMA systems [36].

PON accommodates a potentially large and increasing number of ONUs as new PON versions are developed. The address code chosen must be compatible with the maximum number of ONUs supported by the PON version. The existing families of PC include: basic prime code (PC), extended PC (EPC), modified PC (MPC), new-MPC (n-MPC), padded MPC (PMPC), group PMPC (GPMPC), double PMPC (DPMPC), Transposed-MPC (TMPC), and transposed-sparse padded MPC (T-SPMPC). Characteristics such as cardinality $(|C|)$, length $(L)$, and weight $(w)$ (number of the digit 1 in the code-word) are a function of the prime number ( P ), hence this number plays an important role in determining those characteristics. Basic PC provides $P$ code-word with length of $P^{2}$ and weight of $P$. MPC, n MPC, PMPC, DPMPC, and GPMPC provide $P^{2}$ code-words with lengths of $P^{2}, P^{2}+P, P^{2}+P$, $P^{2}+2 P$, and $P^{2}+2 P$, respectively, and weights of $P, P+1, P+1, P+2$, and $P+2$, respectively. TMPC and T-SPMPC provide $2 P^{2}$ code-words with length of $P^{2}$ and weight of $P$ and $(\mathrm{P}+1) / 2$, respectively.

Considering the existing PC families that support the PON architecture, Table 1-1 below shows the characteristics of different PC families that support the different PON splitting ratios.

It can be observed that there are issues with the current codes to accommodate the different PON splitting ratios. Firstly, the code-words are far more than what is required in many PON networks, which results in a large number of unused codes, leading to an inefficient system. Secondly, increasing the prime number leads to an increase in the weight, which leads to an increase in the hardware (encoder/decoder) complexity, hence power consumption. In addition, increasing the prime number will increase length, resulting in, understandably, an
increase in processing time and a drop in efficiency [37].

Table 1-1 The characteristics of different Prime code families

|  | 32 |  |  |  | 64 |  |  |  | 128 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PC <br> family | P | $\begin{gathered} \|C\| \\ \text { (unused } \\ \text { codes) } \\ \hline \end{gathered}$ | $L$ | W | P | $\begin{gathered} \|C\| \\ \text { (unused } \\ \text { codes) } \\ \hline \end{gathered}$ | $L$ | W | P | $\begin{gathered} \|C\| \\ \text { (unused } \\ \text { codes) } \end{gathered}$ | $L$ | $w$ |
| Basic PC | 33 | $33$ <br> (1) | 1089 | 33 | 67 | 67 <br> (1) | 4486 | 67 | 131 | $131$ <br> (3) | 17161 | 131 |
| MPC | 7 | $\begin{gathered} \hline 49 \\ (17) \end{gathered}$ | 49 | 7 | 11 | $\begin{aligned} & 121 \\ & (57) \end{aligned}$ | 121 | 11 | 13 | $\begin{aligned} & 169 \\ & (41) \end{aligned}$ | 169 | 13 |
| n-MPC | 7 | $\begin{gathered} 49 \\ (17) \end{gathered}$ | 56 | 8 | 11 | $\begin{aligned} & 121 \\ & (57) \end{aligned}$ | 132 | 12 | 13 | $\begin{aligned} & 169 \\ & (41) \end{aligned}$ | 182 | 14 |
| PMPC | 7 | $\begin{gathered} \hline 49 \\ (17) \end{gathered}$ | 56 | 8 | 11 | $\begin{aligned} & 121 \\ & (57) \end{aligned}$ | 132 | 12 | 13 | $\begin{aligned} & 169 \\ & (41) \\ & \hline \end{aligned}$ | 182 | 14 |
| DPMPC | 7 | $\begin{gathered} \hline 49 \\ (17) \end{gathered}$ | 63 | 9 | 11 | $\begin{aligned} & 121 \\ & (57) \end{aligned}$ | 143 | 13 | 13 | $\begin{aligned} & 169 \\ & (41) \end{aligned}$ | 195 | 15 |
| GPMPC | 7 | $\begin{gathered} \hline 49 \\ (17) \end{gathered}$ | 63 | 9 | 11 | $\begin{aligned} & 121 \\ & (57) \end{aligned}$ | 143 | 13 | 13 | $\begin{aligned} & 169 \\ & (41) \end{aligned}$ | 195 | 15 |
| T-MPC | 5 | $\begin{gathered} \hline 50 \\ (18) \end{gathered}$ | 25 | 5 | 7 | $\begin{gathered} \hline 98 \\ (34) \end{gathered}$ | 49 | 7 | 11 | $\begin{gathered} 242 \\ (114) \end{gathered}$ | 121 | 11 |
| TSPMPC | 5 | $\begin{gathered} 50 \\ (18) \end{gathered}$ | 25 | 3 | 7 | $\begin{gathered} \hline 98 \\ (34) \end{gathered}$ | 49 | 4 | 11 | $\begin{gathered} 242 \\ (114) \end{gathered}$ | 121 | 6 |

The objective of this research is to develop a spreading code that is suitable for the different PON splitting ratios, and that also possesses improved characteristics in terms of cardinality, length, weight, correlation functions, and BER performance [5].

In terms of protection, the protection types demonstrated in the ITU-T standard are for a PON with a typical tree topology [38]. The literature shows the performance of three protection schemes for a ring-and-spur topology in terms of cost and availability. However, according to ring-and-spur architecture that supports a large number of ONUs, it is important to examine the FIR parameter for every component in the network and, hence, select the appropriate protection schemes.

The research questions are:

1. Can the existing codes implemented in OCDMA systems be modified to
accommodate, with better characteristics, the number of ONUs supported in PON?
2. How would the new application of the FIR parameter improve system performance?
3. What advantages would be achieved when applying PON ranging into the monitoring layer?

These research questions are formulated in such a way so that, when investigated and answered, PON system performance will be enhanced, which leads to faster detection and traffic restoration. Enhancing the monitoring system can be realized by utilizing a proper spreading code with improved characteristics coupled with valuable information gathered from the ranging process that helps to distinguish between codes coming from multiple ONUs. Quick restoration is dependant on selecting the optimal protection scheme, selected by evaluating the FIR parameter. Responses to those questions, when put together, will result in an improvement in the overall performance of PON and a reduction in the outage time.

### 1.3 Research Objectives

The objective of this research was to improve the design of OCDMA system codes by reducing the prime number used. The smaller prime number should accommodate the number of ONUs in the PON with enhanced characteristics. In addition, the FIR parameter will be used to determine the protection scheme, in terms of cost and availability, for ring-and-spur PON.

Additional research objectives include:

- Modifying the existing code in OCDMA systems, based on the PC family, to achieve a cardinality that has a lower number of unused codes and supports the different PON splitting ratios (32, 64, and 128) with improved characteristics.
- Simplifying encoder and decoder hardware design, based on an optical tapped delay
line by reducing the code weight, hence the number of optical tapped delay lines.
- Gaining an energy-efficient code with a lower code weight where the code sequences are patched with blocks of zeros.
- Using the concept of the PON upstream transmission ranging process found at the data layer, in the monitoring layer as well thereby taking advantage of the fixed upstream transmission interval at the monitoring layer to provide an equalization delay for each ONU.
- Calculating the FIR of the different components in the ring-and-spur system to select the optimal protection scheme, and evaluate its performance.


### 1.4 Research Contribution

Although there is no shortage of literature on research conducted on OC for PON monitoring, the research in the literature focuses on OOC and related enhanced coding schemes. By focusing on the use of a PC, rather than an OOC, the research carried out, and presented in this thesis, provides a new and innovative approach to improve PON monitoring outcomes. Although PC has been used in OCDMA schemes found in the literature, the outcomes were non-optimal. The proposed PC and monitoring approach is suitable for the different PON splitting ratios and provides the basis for an improvement to the overall monitoring outcomes for PON networks. The modified PC is identified by variations to the code-word numbers, length, and weight. Improved performance was obtained for the three splitting ratios studied ( 32,64 and 128 ) using a PC with a smaller prime number, reduced number of unused codes, shorter length and reduced weight. In addition, FIR was used to determine the components with a high probability of failure thereby potentially affecting a large number of ONUs in a ring-and-spur architecture. The resulting monitoring technique was found to be cost efficient, reliable and suitable for typical PON implementations. The research was modelled and
simulations of typical configurations demonstrated the effectiveness of the proposed approach in providing the Network Management System (NMS) with accurate information about the fibre links.

### 1.5 Research Methodology

The research methodology was designed to permit a logical consideration of the research questions and to build upon the research carried out for each of the research questions. The research outcome was a new approach in the design of spreading codes for PON monitoring based OC techniques.

The methodology adopted for Research Question 1 was to identify code families that might be suitable candidates that could modified to improve monitoring outcomes. The investigation identified that by combining the two coding techniques, padding and extension, that were used in the n-MPC and EPC, respectively, it would be possible to design a new code that could satisfy the research objectives. The resulting code-words were doubled to provide the number of code-words needed and the next step was to build a network model and to carry out simulations to gather results for analysis. The performance of the proposed code was compared to that of the most recent code reported in the literature (T-SPMPC) and to MPC which is the basis of most of the codes identified in the literature.

MATLAB was utilized to evaluate the performance of the codes in an on-off keying model and a pulse position modulation model.

Research Question 2 was approached with the objective to add to what had been found so far by improving the measurement of the reliability of the protection scheme. A variety of protection schemes for ring-and-spur have been utilized to find the optimal design in relation to both cost and reliability. By calculating the FIR of the different components in the ring-and-spur system to select the optimal protection scheme it was possible to evaluate its
performance and the outcomes identified that the protection scheme (OLT-Ring) presented in this thesis provides an improvements.

The methodology adopted for Research Question 3 was to consider an improved fibre monitoring technique suitable for PON that could be adopted with future PON systems. A critical aspect of the design was to ensure that upstream monitoring signals would not clash with the monitoring signals from other ONUs. To achieve this objective a fixed delay time was introduced into the monitoring system by taking the advantage of both the fixed burst interval at the monitoring layer, and the concept of the PON ranging process at the data layer. The outcome of this approach was to improve the monitoring system design, efficiency and reliability.

The industry standard VPITransmissionMaker tool was used to evaluate the status of each ONU's link for the three scenarios corresponding to PON splitting ratios $(32,64,128)$.

### 1.6 Publications

Peer reviewed journal and conference publications are listed below:

## Journal

1. H. S. Abbas and M. A. Gregory, "Passive optical network survivability: protection, detection and restoration," International Journal of Information, Communication Technology and Applications, vol. 1, pp.128-142, 2014. doi: https://doi.org/10.17972/ajicta20151115.
2. H. S. Abbas and M. A. Gregory, " The Next Generation of Passive Optical Networks: A Review " Journal of Network and Computer Applications (JNCA), 2016. doi: https://doi.org/10.1016/j.jnca.2016.02.015
3. H. S. Abbas, M. A. Gregory and M. Austin, "A New Family of Prime Codes for

Synchronous Optical Code Division Multiple-Access Networks" (submitted to Journal of Computer Networks and Communications - Scopus indexed)
4. H. S. Abbas and M. A. Gregory, "Implementation of OC Monitoring System in PON Networks" (progressing will submitted to Journal of Computer Networks and Communications - Scopus indexed)

## Conference

1. H. S. Abbas and M. A. Gregory, "Feeder fibre and OLT protection for ring-and-spur long-reach passive optical network," in Telecommunication Networks and Applications Conference (ATNAC), 2013 Australasian, 2013, pp. 63-68. doi: 10.1109/ATNAC.2013.6705358
2. H. S. Abbas and M. A. Gregory, "OCDM network implementation of 1-D OOC and passive correlation receiver," in Artificial Intelligence with Applications in Engineering and Technology (ICAIET), $20144^{\text {th }}$ International Conference on, 2014, pp. 311-316.doi:10.1109/ICAIET.2014.58

### 1.7 Structure of Thesis

Chapter 1 outlines the significance of this research and briefly introduces its objectives and methodology.

The literature review presented in Chapter 2 is divided into three sections; PON network, OCDMA systems, and a summary of the current research on PON monitoring optical coding techniques.

Chapter 3 focuses on a review and provides a comparative table of different PC families. This is done in order to introduce the proposed code extended grouped new modified prime code (EG-nMPC), which is the subject of this research.

In Chapter 4, the proposed code is introduced. The construction steps and the code parameters are detailed. The performance of the code is analyzed in an incoherent system using both OOK and PPM systems.

Chapter 5 introduces three protection schemes for long-reach PON (LR-PON) based on a ring-and-spur topology. The three protection schemes were examined in terms of availability and cost and then compared to the existing protection schemes.

Chapter 6 presents a definition for the fixed equalization delay for the monitoring layer with an explanation of the overall monitoring system. In addition, it analyses the effect of the pulse width on the different noise sources and on the system performance for the different PON splitting ratios.

Chapter 7 presents the design and implementation of the monitoring system in VPItransmissionMaker. Three scenarios compatible with GPON splitting ratios $(32,64,128)$ were simulated. The outcome of implementing the monitoring system does provide the network management with crucial information about the status of each drop fibre link.

The thesis ends with a conclusion, suggestions for future work, appendix and a bibliography.

## Chapter 2 LITERATURE REVIEW

The literature review chapter is composed of three main sections; NG-PONs, OCDMA, and PON monitoring based on OC. The first section, NG-PONs, comprises four sub-sections, which are: a review of the different multiplexing techniques of NG-PON including pure, and hybrid TDM and WDM design; an overview of LR-PON; an overview of GPON physical and data link layers; and finally, a review of the reliability aspects of the systems, including protection, and monitoring.

The second section on OCDMA systems includes the encoding process, the devices of encoder and decoder, challenges associated with OCDMA system, design issues of the spreading code, code types, and modulation schemes.

The third section on PON monitoring reviews the most recent work performed on PON monitoring based on OC.

### 1.1 Next Generation-PON

NG-PON is divided into two stages; NG-PON1 and NG-PON2 [8]. The first standardized NG-PON1 was XG-PON1 (X=10 G=Gigabit PON) that delivers 10 Gbps downstream and 2.5 Gbps upstream (10/2.5G). It uses a single fixed wavelength in each direction [39]. Details about the XG-PON physical layer have been described in ITU-T G.987.2 [40]. The second standardized NG-PON1was XG-PON2, which can be referred to as XGS-PON (X=10, G = Gigabit, $\mathrm{S}=$ symmetrical PON) [40]. The latest XGS-PON delivers 10 Gbps in both directions and has a dual rate transmission capability [41].

The most advanced NG-PON2, that has been introduced by the Full Service Access Network Group (FSAN), delivers a minimum of 40 Gbps in the downstream and 10 Gbps in the upstream direction [8]. It aims to extend the coverage, increase both bandwidth and transmission speed, support high capacity and ensures efficiency of both energy and cost [42]. Several enabling technologies have been proposed for NG-PON2. However, TWDM-

PON has been selected as the primary technology for NG-PON2 as it meets the requirements [8].

### 2.2 Multiplexing Techniques

### 2.2.1Pure Techniques

This section reviews the pure techniques in terms operations, advantages and disadvantages.

### 2.2.1.1 Time Division Multiplexing- Passive Optical Network

Using a single wavelength, TDM-PON allows multiple ONUs to share the same bandwidth [43]. TDM-PON structure is shown in Figure 2-1 [13].

The downstream traffic is broadcast from the OLT to all ONUs and a specific time is assigned by the OLT to each ONU to control upstream transmissions. These time slots are allocated in downstream and upstream frames where an algorithm assigns the bandwidth timing to avoid collision [4, 11]. Although TDM-PON is simple and cost effective, the number of ONUs in the system is limited due to the splitter attenuation as well as the available bitrate of the transmitter and receiver at the CO and ONUs [44]. Standard TDM operates over a maximum of 20 km from the OLT with typically 32 ONUs, and a maximum distance of 10 km from OLT typically permits 64 ONUs [44]. In addition, shared bandwidth compromises security through attack or eavesdropping [45].


Figure 2-1 Design of TDM-PON [13]
To meet the NG-PON2 standards of 40 Gbps for TDM-PON, system architecture variations were proposed. The first of these variations is a conventional on-off key (OOK). However, it requires a 40 Gbps burst-mode receiver, high-cost 40 GHz electronics and photonics, and highly sensitive receivers [46]. The other system to achieve a bitrate of 40 Gbps is duo-binary modulation [43, 47]. It is similar to a deployed PON system, but uses one wavelength for downstream and another for upstream. In duo-binary modulation, phase and amplitude modulations permit ONUs to have 20 GHz bandwidth and reduce delivery issues such as dispersion [48].

In summary, time division multiplexing was not found to be conducive to upgrading to meet the ITU standards for future technologies.

### 2.2.1.2 Wavelength Division Multiplexing-Passive Optical Networks

WDM-PON has been considered as an alternative technology to TDM-PON. It provides a virtual point-to-point connection between the OLT and several ONUs; where, each ONU is assigned a different wavelength for upstream and downstream transmission [44].

Several architectures have been introduced to enable WDM-PON implementation. One of the architectures is based on legacy TDM-PON with the use of a band pass splitter at each ONU in order to distinguish between the wavelengths of upstream and downstream transmission. This architecture is shown in Figure 2-2. It does, however, exhibit the power loss drawback that is due to the use of the splitter, reduced security, and difficulties in implementing colorless ONUs [12]. The standard structure of WDM-PON is shown in Figure 2-3 [44]. This design differs from a TDM-PON system by using devices such as an Array Waveguide Grating (AWG) in place of the TDM-PON power splitter [43].


Figure 2-2 WDM-PON based legacy TDM-PON [12]


Figure 2-3 Standard WDM-PON [44]
In the downstream transmission, the wavelength channels are routed using a specified port at the AWG from the OLT to the ONUs. ONUs are based on the colored component and each ONU employs a transmitter and receiver to transmit and receive on its specified channel. For upstream transmission, a WDM de-multiplexer and receiver array are equipped at the OLT to process the upstream transmission [44].

WDM-PON is classified into two classes based on the number of wavelengths supported and the wavelength spacing between the individual wavelengths transmitted over a single fibre.

The first class is Dense WDM (DWDM) and its wavelength plan is defined by ITU-T G.694.1, and the second class is Coarse WDM (CWDM) that has its wavelength plan defined by ITU-T G.694.2. The main objective of DWDM is to increase the network capacity by minimizing the wavelength spacing; CWDM aims to reduce the cost where the wavelength spacing is sufficiently high to permit the transmitters to be more accurately controlled [4] [12].

The multiple-wavelength characteristic in WDM-PON offers several unique features. WDMPON allows every ONU to transmit at the peak speed as the OLT bandwidth is not shared [49, 50]. Additionally, wavelengths are able to support different bitrates and, consequently, different services can be supported for each ONU [44]. Moreover, security is improved and the potential eavesdropping issue is eliminated [50], [51]. Despite these features, a few restrictions make WDM-PON an inappropriate technology to apply to NG-PON2. As a result of the limitation of the number of wavelengths allowed in the system and the large bandwidth requirement, the utilization of the bandwidth is rendered inefficient [10]. Moreover, WDMPON is cost-ineffective because of the need for extra equipment such as colored ONUs and a transceiver for every wavelength at the OLT $[46,51]$.

### 2.2.1.3 Optical Frequency Multiplexing PON

OFDM-PON is an enabling technology that is based on digital signal processing offering high speed and flexible system [52].

The OFDM-PON architecture is similar to the standard PON. It uses a wavelength for downstream transmission and another wavelength for upstream transmission [13]. The OLT generates multiple orthogonal subcarriers that are assigned to different ONUs and each subcarrier is timed. The OLT partitions and distributes the total bandwidth over subcarriers, timeslots, or both, according to ONU demand $[15,16]$.

OFDM-PON improves the bandwidth flexibility and it is an applicable technology for future PON [8]. Drawbacks of OFDM-PON are its complex receivers reliant on high speed digital signal processors and field programmable gate arrays. The architecture also has a high peak average power ratio from sinusoidal signals for subcarriers in the time domain, generating a higher than average amplitude value [14, 15]. Frequency offset from mismatch of carrier frequencies may also occur [53].

### 2.2.1.4 Optical Code Division Multiplexing PON

OCDM-PON is an efficient means of offering high data rate, asynchronous transmission, flexibility of ONU allocation, low signal processing latency, symmetric bandwidth and improved network security over other designs [54] [17]. Each ONU in OCDM-PON implements an encoder and a decoder with a unique fixed optical code. The OLT must have all encoders and decoders information to enable its communication with each ONU [8].

Figure 2-4 shows an example of the OCDM-PON network structure using multi-port encoder/decoder at the OLT. The main function of the multi-port encoder/decoder is to generate and recognize the different optical code in a device, this has been developed to support 40-G OCDMA system [54].

The two main limitations of OCDM-PON are the use of encoder and decoder for each ONU as well as at the OLT which increase the overall system cost, and the increase of interference as a result of increasing the number of ONUs [55].


Figure 2-4 Design of OCDM-PON [54]

### 2.2.2Hybrid TDM and WDM-PON

In 2012, an influential and dispersed international lobby group that operates under the pseudonym of FSAN, selected a TWDM-PON design as the multiplexing technique for NGPON2 [19]. Their decision was based on the maturity of design's technology, system performance, power consumption and cost [20]. The design was confirmed in 2013 by ITU-T under its G. 989 series [56].


Figure 2-5 Design of TWDM-PON [57]

TWDM-PON combines the simplicity and cost efficiency offered by TDM and the high number of wavelength provided by WDM into one architecture, by transmitting TDM frames to several ONUs over several wavelengths [10, 12]. Standard TWDM-PON consists of four sets of XG-PON1 stacked as four pairs of wavelengths [48].

Figure 2-5 shows TWDM-PON and the wavelength pairs " $\{\lambda 1, \lambda 5\},\{\lambda 2, \lambda 6\},\{\lambda 3, \lambda 7\}$ and $\{\lambda 4, \lambda 8\} "[48]$. Each XG-PON1 provides 10 Gbps and 2.5 Gbps of data in downstream and upstream transmissions, respectively. Thus, TWDM-PON increases the bit rate up to 40 Gbps for downstream transmission and 10 Gbps for upstream transmission [19]. In a basic network, programmable transmitters and receivers can be tuned to any wavelength. The system remains passive when the OLT has an amplifier, a multiplexor, and the de-multiplexor [48].

Another design that combines power splitters AWG is shown in Figure 2-6. This configuration makes use of identical colourless ONUs and supports a higher number of wavelengths than stacked PON [58].


Figure 2-6 TWDM-PON, utilizes a combination of AWG and power splitters [58]

A major limitation of the TWDM-PON is the cross-talk, which occurs at the OLT receiver due to the multiple wavelength channels and dynamic power ranging at the upstream transmission [59]. This issue engenders significant debate in the literature, as researchers identify sources of crosstalk in the upstream transmission and establish mitigation devices at the OLT receiver. To mitigate cross-talk between upstream channels in TWDM-PON, low cost options are offered, such as transmitter bias current and/or modulation current, or placing semiconductor optical amplifiers or variable optical attenuators in the transmitter [60].

Efficient dynamic bandwidth and wavelength allocation maximises performance in TWDMPON systems. Of the algorithms discussed in the literature, earliest finish time, earliest finish time with void filling [61, 62], off-DBWA and on-DBWA (described as DBWA) [63] were investigated. Other DBWA algorithms, such as Optical Burst Switching Dynamic Bandwidth Allocation (OBS-DBA), were designed for particular network architecture. This algorithm was designed for SARDANA network [64], Slotted Medium Access Control (SMAC) for Slotted PON (SPON) [65], and STARGATE EPON [66].

TWDM-PON has been receiving significant attention and many proposals have been submitted to evaluate its performance [24, 57, 67-72].

### 2.3 Long-reach PON

Extended reach is an important requirement for NG-PON2 [73]. LR-PON refers to studies of the consolidation of metro and regional access networks [74] (see Figure 2-7).

Figure 2-7 shows the aims of LR-PON. It aims to extend PON's domain up to 100 km . It reduces the number of active elements in the network, minimizes network planning efforts, and reduces the capital expenditure (CAPEX) and OPEX. Several techniques have been studied to realize this objective.[74].


Figure 2-7 Long reach PON [74]

The most common practices to extend the reach of PON are discussed below [9].

Optical amplification. The use of optical amplifiers is widespread. It enables LR-PONs to achieve the target power distribution quotas. There are several types of optical amplifiers including, Erbium Doped Fibre Amplifier (EDFA), Semiconductor Optical Amplifiers (SOA), and Distributed Raman Amplification (DRA). Whilst EDFA is power saving and noise attenuation efficient, it is confined to C and L transmission bands; which renders it unsuitable to cope with the burst of an upstream transmission [9]. Compared to EDFA, SOA has better gain dynamics, simplified wavelength conversion and all-optical regeneration. In addition, it can operate in the O band ( 1310 nm ), C band (1550 nm), and S band (1490 nm) ranges. However, SOA's wavelengths limit simultaneous amplification across multiple wavelengths [9]. DRA supports bidirectional amplification on flat optical gain bandwidth including channels that override those of common optical amplifiers [9].

Electrical Repeater-based networks. Using an Electrical Repeater at a local exchange is
another option for optical amplification. Electrical repeaters are useful in retransmission (1R) or re-time and re-transmit (2R) of downstream and upstream signals [9]. Electronic repeaters however, require bit-rate specific burst mode receivers of a wide dynamic range. They also require power, which is not indicative in a passive optical network.

Developed modulation formats and digital coherent detection. With the optical amplification and the electronic repeater it is necessary to provide power in the local exchange, this eliminates one aspect of the passive nature of the network. An alternative option is to keep the distribution network purely passive by using developed modulation formats as well as digital coherent detection [75].

Extending the reach creates issues of medium access control as the round-trip time increases and the DBA control loop is impacted, reducing performance [76]. Although LR-PON is of value amongst PON designs, large propagation delays occur on upstream channels [77].

### 2.4 PON Reliability

As part of ITU standard requirements, NG-PON2 aims to include high component reliability [48]. As this includes LR-PON, there is a higher probability of fibre damage. This section discusses the reliability aspects of PON including protection schemes and monitoring techniques.

### 2.4.1PON Protection Schemes

The ITU-T G series recommendations supplement 51, states that there are cases where special PON redundancy and protection switching are required: mobile backhaul, business services and high-density residential complexes [38].

The ITU-T in its recommendation G.984.1 has identified four types of protection systems known as Type A, Type B, Type C, and Type D. Since that time, several studies have
considered PON redundancy and protection switching. The variance between the four protection types depends on which part is being protected. Each part in the system has different effects on the system availability and total cost.

The ITU also pointed out that protection of the access network is expensive because the cost is shared only amongst a few ONUs, as compared to the larger number of ONUs serviced by transport networks. Hence, there is a general lack of attention to protection in the access networks.

As the bulk of a PON, the ODN itself constitutes the highest risk factor often due to a requirement for a Service Level Agreement (SLA) meeting 0.99999 availability. However, extending the SLA past 0.99999 means that risk reduction becomes cost prohibitive and arguably unachievable. Operators and providers offer a range of services at and below this capability [38]. An ODN constitutes above-ground cabling susceptible to tree, animal, weather and adverse event damage; underground cabling may be damaged during infrastructure installation (digging) or perhaps by burrowing animals.

Type A duplicates the feeder fibre and a fault requires operator intervention or the use of voltage-controlled optical switch to bypass the fault by connecting the spare fibre between the splitter and the OLT. Type A does not need an additional PON port at the OLT.

Type B protection provides redundancy to the OLT and the feeder fibre. With an additional PON port on the OLT, type B provides automated switching capability with the use of $2 \times \mathrm{N}$ optical power splitter. In standard Type B protection, both OLTs are located at the same location. However, to provide protection against catastrophic failures, the two OLTs are located at different locations. Each ONU is attached to both OLTs (primary and secondary) through a 2 x N optical splitter. In normal operation, the ONUs are connected to the primary OLT. In the case of OLT failure, the secondary OLT takes charge of controlling the PON.

This type of protection is referred to as dual-parented protection. Type B represents a simple and inexpensive, as it cost is shared among multiple ONUs.

A Type C redundancy is delivered in the OLT, ODN and ONUs. It combines two fully redundant services connected to the customers' premises as linear $1+1$ or linear 1:1. In protection based linear $1+1$, signals are copied and fed to both PONs. Each ONU can select one of the two signals based on some determined conditions (e.g. server defect indication). In the linear 1:1 option, the signal is delivered on one or the other PON, with the OLTs automated in case of failure of one or the other. The duplication, besides offering redundancy, can be used as extra bandwidth by the customer, but extra traffic will not be protected. There may be separate ONUs; with the option that one ONU will have two optical interfaces. Using Link Aggregation (LAG), the PONs are duplicated throughout, including OLTs and separate ONUs. In this type, protection is performed using LAG, a User Network Interface (UNI) and a Server Node Interface (SNI) interfaces. PON in this protection system, presents simple Ethernet links. Extra traffic available on the system is not protected.

Type D redundancy allowed a combination of both protected and non-protected ONUs. In another word, it allowed a combination between type B and Type C redundancy [78]. Type D redundancy, however, has been abandoned (deprecated) [38].

### 2.4.2Protection for ring-and-spur LR-PON

NG-PON2 is based on ring-and-spur LR-PON [80]. A LR-PON system is described below [81].

Figure 2-8 [81] shows a ring that is based on WDM connecting the OLT to a number of RNs. A Power Splitter/Combiner (PSC) is implemented to each RN to serve the tree (ONUs). Data coming downstream in a waveband $\Lambda_{D}$, is transmitted in a counter-clockwise direction from the CO. At $\mathrm{RN}_{i}, \Lambda_{i}$ sub-waveband of $n$ wavelengths is de-multiplexed and dropped to the

PSC, and then to the ONUs. The upstream data flow is multiplexed in $\mathrm{RN}_{i}$ and transmitted counter-clockwise to CO.


Figure 2-8 Ring-and-spur long reach PON design [81]
As shown in Figure 2-8, the downstream data flow is transmitted from the OLT in a counterclockwise direction. The upstream data flow is transmitted in both directions. In this scenario, there are 31 RNs, each with a mean 19.5 km link to one splitter. A splitter supports 32 ONUs at a mean 0.5 km distance. Thus, there is a total of 992 ONUs. If a fault occurred along the fibre or at an OLT, all 992 ONUs would be affected. A fault in an RN, distribution fibre or a splitter would affect 32 ONUs, while a fault in ONU or its drop fibre will have an effect on only that ONU. In [82], three protection systems are discussed for such a scenario: access only protection; ring only protection, and full protection. These protection schemes are shown in Figure 2-9 (b), (c), and (d) respectively, where (a) represents a system without protection. Esmail and Fathallah in [82] have evaluated the performance of these systems taking into account the availability and the cost. The analysis has shown that protection that is based on ring duplication is the most optimal scheme when it comes to both reliability and cost.

Some implementations of protection schemes for PON are given in literature. In [79] a
protection scheme for PON, that aims to protect the feeder fiber, is presented. In this demonstration, two optical switches are used to switch the traffic between the faulty and working fiber paths. The system implemented obtained a BER of $10^{-15}$ with feeder fiber length of 20 km . A surviving PON with simple ONUs and switchable OLTs are reported. The survivability was achieved by utilizing a protected-path switching scheme that is based on employing bidirectional colorless $N x 2$ and $2 x N$ AWGs. The affected traffic can be restored promptly under fiber feeder and distribution link failures, as well as the failure at the remote node. The system achieved a BER of $10^{-9}$ and a restoration time of 3 ms . The implementation of this work can be found in VPI transmission maker.


Figure 2-9 Protection systems based on [82]

### 2.5 PON Monitoring

The ITU has recommendations for maintenance of cable fibre networks [32]. Maintenance in ITU-T categorised into: preventative maintenance and post-fault maintenance. Each maintenance category comprises of three main functions which are: surveillance, testing and control. For post-fault, the functions are usually required. Whereas for the preventative maintenance category, the functions are optional and they are selected based on maintenance
rules set by the telecommunication providers. [32]. The recommendations are shown at Table 2-1.

In a recent discussion of optical distribution networks monitoring/checking by the ITU [32], the importance of a distinction between network and system failures within the context of the ONU being connected or not was made. The Union recommended OTDR for diagnosing faults in the network, using a power meter and a light source. OTDR results in a functional relationship between fibre length and power, that contains information about the fibre (see Figure 2-10) [84]. The Union hypothesized that XG-PON2 systems would autonomously detect and locate ODN faults especially between the CO and the first-stage splitter. Monitoring end-to-end performance enables operators to identify dropouts or throttling, and for this higher layer, technology like Ethernet performance monitoring can be used for monitoring and verifying data flows through PON network elements [40].

Standard OTDR cannot be used to monitor all of the PON branches, as PON is a point to multi-point architecture, where the reflected signal would comprise all backward signals coming from multiple branches. Other issues include short range connection points on the fibres, and lack of reliability due to complexity through many ONUs, and the need for high dynamic range for the reflectometer [11, 84].

Table 2-1 ITU PON maintenance recommendations [32]

| Maintenance category | Maintenance <br> Activity | Functions | Status |
| :---: | :---: | :---: | :---: |
| Preventative maintenance | Surveillance <br> (e.g, periodic testing) | Detection of fibre loss increase <br> Detection of fibre deterioration <br> Detection of water penetration | Optional Optional Optional |
|  | Testing <br> (e.g, fibre degradation testing) | Measurement of fibre fault location <br> Measurement of fibre strain distribution <br> Measurement of water location | Optional <br> Optional <br> Optional |
|  | Control (e.g, network element | Fibre identification | Optional |


|  | control) | Fibre transfer | Optional |
| :---: | :---: | :---: | :---: |
| Post-fault maintenance | Surveillance <br> (e.g, reception of transmission system alarm or customer trouble report) | Interface with transmission operation system <br> Interface with customer service operation | Required <br> Required |
|  | Testing (e.g, fibre fault testing) | Fault distribution between transmission equipment and fibre network <br> Measurement of fibre fault location <br> Confirmation of fibre condition | Required <br> Required <br> Optional |
|  | Control <br> (e.g, cable repair, removel) | Restoration/permanent repair <br> Fibre identification <br> Fibre transfer | Required <br> Required <br> Required |



Figure 2-10 OTDR fault trace [84]

Table 2-2 Monitoring techniques shows other optional monitoring techniques that are different from the standard OTDR. One of the techniques that is of interest to this thesis is the OC monitoring technique. This system is scalable, less complex and, consequently, has lower construction costs. However, a reliable fault locating feature has yet to be established, although there are solutions being sought in current research [84, 85].

In OC, monitoring signals are transmitted over the U-band while data signals are transmitted
over the L band. At the NMS, located at the CO, an optical downstream monitoring signal is generated. The signal splits at the RN to sub-pulses that are equal in number to the number of branches. A passive encoder, at the extremity of each drop fibre, encodes its sub-pulse by a unique code and then reflects the encoded sub-pulses back to the NMS at the CO. The NMS can detect the faulty branch by checking its corresponding code. The presence of the code denotes a healthy link and the absence of the code denotes broken link [86], (see Figure 2-11).

Table 2-2 Monitoring techniques [87]

| Monitoring technique | تָ |  |  |  | 0 0 0 0 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Single wavelength OTDR | High | Low | No | Low | Low | No | No | No |
|  | Low | High | No | Low | Low | Yes | Yes | Yes |
|  | Low | High | No | Low | Low | Yes | Yes | Yes |
|  | Low | High | No | Low | Low | Yes | Yes | Yes |
|  | High | High | Yes | Medium | Low | Yes | Yes | Yes |
| Tunable <br> OTDR | High | Low | No | High | High | Yes | Yes | Yes |
|  | High | Low | No | Medium | High | Yes | Yes | Yes |
| Brillouin OTDR | High | High | No | Medium | Medium | Yes | Yes | Yes |
| Embedded OTDR | Medium | High | Yes | High | Low | No | No | Yes |
| OFDR+IF units | Low | High | Yes | Low | Medium | Yes | Yes | Yes |
| Optical coding | Low | High | Yes | Low | Medium | Yes | Yes | Yes |
| SL-RSOA | Low | High | Yes | Medium | Low | Yes | No | Yes |
| Reflected signal | High | High | Yes | Low | Medium | Yes | Yes | Yes |



Figure 2-11 Optical coding monitoring technique [84]

### 2.6 GPON

Gigabit PON supports different downstream and upstream transmission rates. For downstream transmission, GPON defines rates of 1.2 Gbps or 2.4 Gbps , whereas bitrate of 1.5 Gbps, 6.2 Gbps, 1.2 Gbps or 2.4 Gbps have been identified for upstream transmission. Typically, GPON operates using 1.2 Gbps for upstream transmission and 2.4 Gbps for downstream transmission [88]. Wavelength bands that are allocated to downstream transmission are in the range 1480-1500 nm. For the upstream wavelength bands, GPON uses a wavelength band of 1290-1330 nm [89]. GPON provides a 1550 nm wavelength band for the video signal. GPON supports spilt ratios of 32, 64, and 128 ONUs [90].

GPON, an ITU standard defined in recommendation series G.984.1 - G.984.4, is characterized by dynamic bandwidth allocation, reliable timing and high quality service through its traffic containers (T-CONT). Internet Protocol (IP) or Ethernet elements are configured by GPON Encapsulation Method (GEM). The GEM comprises of the GEM header and GEM payload. GEM header includes information about ONU address. The GEMs are encapsulated into the GPON Transmission Convergence (GTC) payload [91].

GPON downstream frame is $125 \mu \mathrm{n}$ long and it uses TDM to divide the available bandwidth among the ONUs [92]. The downstream frame is showing in Figure 2-12. It consists of Physical Control Block downstream (PCBd), and a payload that includes ATM section and GEM section [93]. The data is broadcast to all ONUs, whereupon individual units extract the appropriate data using information in the PCBd. In addition, the Bandwidth map (BW map) within the downstream data flow allocates upstream time for each ONU to transmit its upstream traffic [93], (see Figure 2-14).


Figure 2-12 Downstream frame [93]


Figure 2-13 Upstream frame [93]


Figure 2-14 Upstream and downstream [94]
The upstream frame is $125 \mu \mathrm{n}$ based on TDMA. It contains multiple transmission bursts arriving from the ONUs [92]. The upstream frame is shown in Figure 2-13. Each of the upstream burst frames consists of a Physical Layer Overhead (PLOu) and a payload [92]. When traffic reaches the OLT, ONU traffic is queued based on Classes of Service (CoS) with a diverse QoS dependent on the type of the T-CONTs that is specified in the Allocation Identifier (Alloc-ID) in the PCBd. The OLT classifies and ranks the data flow needs of each ONU according to ITU-T Recommendation G. 983.4 for T-CONT types:

- T-CONT 1: Supports time-sensitive fixed bandwidth
- T-CONT 2: Supports assured and shared bandwidth with less time sensitive and quality data
- T-CONT 3: Supports bandwidths providing a guaranteed minimum Committed Information
- T-CONT 4: Supports 'best-effort', average need services.

The OLT controls the assignment of the time frame for each ONU based on its identified load. When each ONU transmits its upstream traffic during the assigned time slot, there is a possibility that frames from different ONUs will collide at some point due to the difference in
propagation delay [95, 96].

In order to guarantee that the upstream transmissions do not collide, a ranging process is performed by the OLT during the activation and registration of the ONUs. The ranging process is based on calculating a specific delay time for each ONU according to its logical distance from the OLT to equalize its transmission delay with other ONUs [96]. Each ONU will store and apply its delay to all the upstream transmissions. The delay values are broadcast to other ONUs using PLOAM messages and each ONU resumes its transmission based on the delay [97] [95].

The accuracy of the ranging process to control the upstream bursts is limited, thus a Guard Time $\left(T_{g}\right)$ is a very significant concept in the upstream transmission. $T_{g}$ is the time that is assigned to the period between two consecutive bursts to avoid packet collision [98]. The number of bits assigned to the guard time depends on the data rate, and increases with it. For example, a data rate of 1.244 Gbps consumes 32 bits, while that of 2.48 Gbps consumes 64 bits [98].

### 2.7 OCDMA Systems

The OCDM-PON configuration was discussed in Section 2.2.1.4. This section focuses on OCDMA system encoding principles, encoder and decoder devices, OCDMA system challenges, spreading code design issues, and the different optical spreading codes.

### 2.7.1Encoding Principle

In OCDMA systems, each communication channel is assigned a unique code. The transmitter encodes each data bit by multiplying it by its unique code before transmission [99]. The coding processes are performed using one-dimensional coding (1-D), two-dimensional coding (2-D), or three-dimensional coding (3-D).

1-D spreads codes on either the time or frequency domains. For encoding in the time domain, a light source produces short optical pulses. Then by using a fibre delay time, and an electrical modulator, each data bit generated by the laser source is encoded [100]. In the frequency domain, an ultra-short broadband light pulse passes through a diffraction grating scattering in multiple wavelengths [100]. In time based encoding, the bit time is divided into smaller chips that are equal to the code length, whereas in wavelength based encoding, the code is formed by dividing the transmitted bits into a number of unique subset of wavelengths [99]. The principle of encoding using time and wavelength domains is shown in Figure 2-15. The decoding operation is based on using a reverse operation to encoding (reverse time/wavelength) [99].

2-D spreads the code using both time and wavelength domains, simultaneously by placing pulses in different chips over a bit interval, each with a different wavelength, (see Figure 2-15) [99]. Consequently, 2-D coding possess better flexibility and security characteristics than 1-D coding [100].

3-D is executed in space, time, and wavelength; or in polarisation, time, and wavelength domains. Such codes support more cardinality than 2-D, improve the code performance; as well as minimizing the code length [101]. The Local Area Network (LAN) of an OCDMA system is based on a broadcast method. Multiple encoders at the transmission side encode the data with unique codes. All the encoded signals are coupled at the star coupler and broadcast to the receiver side. Each receiver receives the sum of the encoded signals. The decoding operation at the receiver side is based on the correlation function [30]. If an intended encoder transmits a signal, only the associated decoder (that is applying the revers encoding process) is able to recover the signal by representing an auto-correlation peak of the code.


Figure 2-15 Coding dimensions [99]
Table 2-3 Coding domain and relevant devices [99]

| Coding domain | Encoding device |
| :---: | :---: |
| Time | Optical delay line |
|  | Mach-Zehnder interferometer |
|  | External phase modulator |
|  | Phase modulator and local oscillator |
|  | Senar lightwave circuits with delay and phase modulators |
| Segmented fibre grating |  |
| Wavelength | Liquid crystal modulator Bragg grating |
|  | Broadband source |
|  | AWG and phase-plates |
|  | AWG and holograms |
| 2-D | Superstructured Bragg grating |

Other signals detected as interference or noise and are represented by cross-correlation [30, 99]. The correlation function can be implemented using several schemes. One scheme is based on multiplying the optical pulse sequence by a reference signal that is stored at the receiver. This process is performed in real time. Another approach is to perform the correlation by using passive optical means. An example of the encoding and decoding of this approach is the use of ODL (explained in Section 0).

### 2.7.2Encoder/Decoder Devices

Devices used for encoding and decoding and their corresponding coding scheme are presented in Table 2-3 [99]. The most common devices are reviewed in this section.

### 2.7.2.1 Optical Delay Line

The Optical Delay Line (ODL) is used as an encoder and a decoder for both 1-D and 2-D domains as shown in Figure 2-16 and Figure 2-17, respectively.


Figure 2-16 1-D ODL-based encoder/decoder [27]


Figure 2-17 2-D ODL-based encoder/decoder [27]
In Figure 2-16, the encoder consists of an optical splitter, several ODLs, equal in number to the code weight, and a power combiner. The light source sends an optical pulse for time $T$ composed of chips, each represented by $\mathcal{T}$, and in total equal to the code length. This signal is then split into $w$ sub-signals, each of which is assigned a delay line according to the codeword. The delay assigned is $j i \mathcal{T}$, where $0 \leq j . i \geq L$. The sub-signals are combined by the power combiner. The decoder design is the same as the encoder except for the values assigned to the delay line. The delay lines for the decoder are changed to ( $L-1-j$ ) $\mathcal{T}$. The output of the decoder represents an auto-correlation function of the code-word [29].

Example: Consider OOC, represented with length of 32 and weight of 4, this code can generate two code-words $\{10$ (9) $10(2) 10(14) 10(3)\}$ and $\{10$ (4) 10 (6) 10 (18) 1$\}$, where the internal bracketed figures denote the number of 0 s for each series. The code-words can be represented by $\{(0,10,13,28) \bmod 32\}$ and $\{(0,5,12,31) \bmod 32\}$ [102]. The delay lines for both encoder and decoder are shown in Table 2-4.

For 2-D encoding, see Figure 2-17, the light source is replaced with a multi-wavelength light
source or with a broadband light source that produces a train of short pulses [27]. In addition, the power splitter/combiner are replaced by a wavelength division de-multiplexer/multiplexer respectively.

Table 2-4 Encoder/decoder delay lines [103]

| Encoder/Decoder | Parameter | Value |
| :---: | :---: | :---: |
| Encoder 1 | Delay line 1 | 0*TimeChip |
|  | Delay line 2 | 10*TimeChip |
|  | Delay line 3 | 13*TimeChip |
|  | Delay line 4 | 28*TimeChip |
| Encoder 2 | Delay line 1 | 0*TimeChip |
|  | Delay line 2 | 5*TimeChip |
|  | Delay line 3 | 12*TimeChip |
|  | Delay line 4 | 31*TimeChip |
| Decoder 1 | Delay line 1 | 31*TimeChip |
|  | Delay line 2 | 21*TimeChip |
|  | Delay line 3 | 18*TimeChip |
|  | Delay line 4 | 3*TimeChip |
| Decoder 2 | Delay line 1 | 31*TimeChip |
|  | Delay line 2 | 26*TimeChip |
|  | Delay line 3 | 19*TimeChip |
|  | Delay line 4 | 0*TimeChip |

The de-multiplexer splits the optical pulse into $w$ sub-pulses, each of which represents a wavelength. These sub-pulses are assigned a specified delay, similar to 1-D encoding. The delay lines for $\left(\lambda_{1}, \lambda_{2}, \ldots, \lambda_{w}\right)$ are $(j 1 \mathcal{T}, j 2 \mathcal{T}, \ldots, j w \mathcal{T})$. The decoder for 2-D is the same as the encoder except for the delay line values which are $(\mathrm{n}-1-j 1),(L-1-j 2) \mathcal{T}$, ..., $(L-1-j w)$ $\mathcal{T}$. The sub-signals are combined at the multiplexer and they produce an auto-correlation corresponding to the assigned code-word [27].

### 2.7.2.2 Fibre Bragg Grating

Encoder/decoders based on Fibre Bragg Grating (FBG) have a wavelength selective filter to reflect the optical signal [29]. These encoder/decoders can be designed either in series or parallel forms. A 2-D serial FBG architecture is shown at Figure 2-18 [104].

The encoder shown in Figure 2-18 consists of a three-port based circulator, FBG and ODL both qual to $w$. The broadband light source generates a short optical pulse which is modulated by data. The output pulse is fed into the optical circulator through port 1 . The output signals from port 2 are reflected through FBGs in different wavelengths. Each two FBGs are joined together using ODL with differing delay times. The reflected optical pulse signals for port 2 of the circulator (including different wavelengths and delay times) comprise output from port 3. The decoder structure based on serial FBGs is the same as the encoder except that the FBGs are in reverse order and the values of the ODL are changed to be complementary to the delay assigned to the encoder [27].

The parallel 2-D form of the FBG structures is presented in Figure 2-19. The encoder/decoder in Figure 2-19 have a three-port based circulator, AWG, FBGs and ODL, with the latter two being, each, equal to $w$ [104]. The broadband light source generates a short optical pulse which is modulated by data and fed to port 1 of the circulator. The AWG splits the pulse into $w$ wavelength sub-pulses each with a pre-defined delay time, and reflected to the AWG through FBG. The reflected pulses are multiplexed and fed into the circulator port 2, and output is obtained from port 3. The decoder for parallel FBGs follows the design of the encoder except its delay time values, that should complement those of the encoder ODLs [27].


Figure 2-18 2-D FBG-based serial encoder/decoder [104]


Figure 2-19 2-D FBG-based parallel encoder/decoder [104]

### 2.7.2.3 Arrayed Waveguide Grating

Arrayed Waveguide Grating (AWG) based feedback ODL is shown in Figure 2-20.

In this design, the input optical pulses are divided by AWG into small sub-pulses at different wavelengths, each of which delayed by a pre-assigned delay time and fed back into the

AWG. The sub-pulses are combined at the AWG output port, providing an encoded signal. The decoder structure follows the encoder with delay time values in a reverse order, so that the encoder wavelength with the highest delay value forms the shortest for the decoder feedback [104].

An AWG with mirrored ODL is shown in Figure 2-21. The design of an encoder based on AWG with mirrored ODL is similar to the previous encoder. The difference is that the ODLs are mirrored to reduce the feedback structure [27] [104].


Figure 2-20 AWG with ODL feedback [104]


Figure 2-21 AWG with mirrored ODL [104]

### 2.7.30CDMA System Challenges

There are several challenges for OCDMA systems. The most critical challenges are discussed in this section [25].

- Coding Schemes. Codes in OCDMA systems have parameters such as auto and
cross-correlation functions, weight, length, and cardinality. These parameters are the main factors affecting the performance of OCDMA, as estimated by the probability of MAI. MAI increases as the number of transmitted communication channels increases. Hence the importance of knowing the number of the active communication channels that are able to transmit asynchronously. Researchers have proposed codes that generate the required code-words.
- Network Architecture. Network architecture can refer to the network topology that based on the scalability of the network, the integrated device, the cost of the system, and the robustness of the environment. For star based networks, the number of connection channels can exceed the number of input/output ports. One suggestion to increase the number of connection channels by employing the optical splitter and combiner, beside the coupler. Another suggestion is to use multiplexers and demultiplexers. The star topology is limited to local area networks with limited coverage. A solution to increase the coverage is the use of ring topology. The ring makes use of add/drop multiplexer which is the main component. The multipoint-tomultipoint connection provided by the star topology is not efficient to deploy in access networks because of the nature of traffic. Therefore, another alternative of optimal topology is tree topology.
- Hardware Design. Each channel in OCDMA systems has to implement an encoder and a decoder to encode/decode its data. Several devices can be used for the coding process, and there may be a need for advanced optical processing components, which will result in an increase in the total cost.


### 2.7.4 Spreading Code Design Issues

As discussed in the Section 2.6.3, identifying a coding scheme is one of the most important
challenges affecting the performance of OCDMA. Thus, the following issues have to be considered when designing an effective spreading code [25].

The length of spreading codes is constrained by increasing code cardinality (the number of available communication channels). This is particularly relevant to 1-D codes, to support a large cardinality, the length of the code increases exponentially, resulting in a code that is inefficient to implement. As stated in coding principles, 1-D time encoding is based on dividing the bit time into smaller chips (equal to code length). The chip width is, therefore, an important aspect to consider. It has been stated in [105] and in [106] that the state of the art photodiodes at the receiver are able to detect a chip time of the order of 100 ps . In addition, the authors predicted that chip times may drop to the order of 10 ps , and may even reach the order of femtosecond with the use of the fast-nonlinear optical element, e.g Kerr effect.

Code cardinality can be, assisted by 2-D and 3-D coding [27]. The code weight refers to the pulses represented by 1 in the code-word. Although, increasing the weight improves the correlation properties, it increases the power consumption as well as the hardware complexity (e.g. increasing the number of ODL in time encoding scheme). Construction mechanism is also of concern in designing the address codes. Simplifying the construction mechanism of the code renders it more popular for implementation.

### 2.7.5Optical Spreading Codes

Spreading code sets attributed to OCDMA systems can be classified into bipolar codes, those that have sequences of $(-1,1)$, and unipolar codes, those that have sequences of $(1,0)$. The optical spreading codes, their characteristics in relation to coding dimension, polarisation, and light types are summarised in Table 2-5. The following are the most common optical spreading codes.

## Bipolar codes

- maximum length sequence ( $m$-sequence) codes have lengths of $2^{L}-1$, where $L$ is the number of shift registers from a Linear Feedback Shift Register (LFSR) generator necessary to initiate the sequence
- Gold code sequences [107] are constructed from two m-sequences of the same length and rate with modulo-2 addition; for a Gold sequence of length $m=2^{L}-1$, two LFSR each of length $2^{L}-1$ are required
- Hadamard-Walsh codes are bipolar, perfectly orthogonal used to separate transceivers on the downlink channel [30].


## Unipolar code

- Optical orthogonal code.
- Prime code sets.
- Quadratic Congruence Codes.

Table 2-5 OCDM codes and protocols [27]

| Code- <br> dimensions | Title | Polarisation | Light type | Attributed codes |
| :---: | :---: | :---: | :---: | :---: |
| 1-D code | Pulse amplitude | Unipolar | Incoherent | OOC <br> PC <br> QCC |
|  | Pulse phase | Bipolar | Coherent | M-sequence <br> Gold code |
|  | Spectral amplitude | Unipolar | Incoherent | Walsh-Hadamard <br> codes |
|  | Spectral phase | Bipolar | Coherent | Wher |
| 2-D code | Wavelength-hopping <br> time spreading | Unipolar | Incoherent | 2D WH/TS OOC |
|  | Space encoding | Unipolar | Incoherent | 2D space codes |
|  | 3-dimension encoding | Unipolar codes | Incoherent | Space/time/ <br> wavelength or <br> polarisation/time/ <br> wavelength codes |

### 2.7.6OCDMA Modulation Schemes

On-off keying (OOK) and pulse position modulation (PPM) are the most popular modulation schemes used in OCDMA systems [108]. In OOK modulation, a data bit is denoted by two symbols based on binary numbers. If the data bit is equal to 1 , the bit time is divided into $L$ chips of width $\left(T_{c}\right)$. Thus the frame duration can be expressed as $T_{s}=L . T_{c}$. (see Figure 2-22, (a)) [109].

In the case of M-array PPM modulation, a data bit is denoted by $M$ symbols, thus, every symbol signifies $\log _{2}^{m}$ data bits. As shown in Figure 2-22 (b), a frame of duration $T_{s}$ is divided into $M$ symbols ( $M=4$ symbols), each symbol represents two data bits. Each symbol is divided into $L$ chips of width $T_{c}$, thus the symbol duration can be expressed as $\tau=L . T_{c}$, the frame duration can, therefore, be expressed as $T_{s}=M . \tau[109]$.


Figure 2-22 Signalling format, (a) OOK-OCDMA, (b) PPM-OCDMA [109]
The PPM-OCDMA approach has been shown to be an energy efficient scheme. This is because, for a given bit-error-rate, PPM-OCDMA can increase the number of communication channels by increasing the pulse-position multiplicity $(M)$ for the same average power, whereas OOK-OCDMA cannot [110]. On the other hand, however, the OOK-OCDMA system has a simpler signaling format than the PPM-OCDMA, as its chip width is larger than that in the PPM format (see Figure 2-22). In addition, the hardware used for the PPM modulation format is complex [109].

### 2.8 PON Monitoring based OC Technique Research

In [84] and [111], a number of the required features that must be fulfilled in the monitoring system were discussed. The first feature is for the system to be centralized; the NMS at the CO must be able to obtain monitoring information from the system without intervention by the ONU. In addition, the monitoring system must be able to automatically detect the fault
without any intervention from the technicians. This feature leads to minimizing the measurement time and reducing the cost of operational expenditure. Transparency is another must feature in monitoring systems. The monitoring process should not interrupt the transmission/receiving of the data. To achieve this, a monitoring pulse must be transmitted in the U-band and a wavelength selector should ensure data flows [111]. Simplification and cost-effectiveness of the monitoring system are highly desired. NG-PON accommodates high capacity and monitoring based TDM (utilizing one wavelength) is a simple and cost-effective way to monitor high network capacity $(64,128+)$, which is suitable for NG-PON.

Although the efficiency of the monitoring PON based OC technique is largely dependant on the spreading code, investigation of ways to enhance that code has been overlooked by researchers. The importance of the current work stems from the fact that this thesis places a heavy weight on ways to enhance the spreading code, rather than the coding scheme.

A proposed monitoring system based on Direct Sequence OCDM (DS-OCDM) can be found in [112]. In this system, each ONU implements a fixed encoder based on OOC to encode its signal. The encoded signals are combined at the RN and sent to the NMS. The decoder at the NMS is based on a switch and fixed decoders. In the case of a healthy branch, the decoder represents an auto-correlation peak; otherwise, a cross-correlation will be represented. The monitoring system is able to monitor TDM-PON, WDM-PON or TWDM-PON systems with around 1000 ONUs using a single wavelength in the U-band.

One approach to detect a faulty TDM-PON branch was described in [113] where the code implemented is an OOC, and the encoders used to distinguish between branches are based on five FBGs, that are, implemented outside the ONUs. The simulation monitoring signal was transmitted over the C-band (data band) and was trialled on a $1 \times 16$ TDM-PON with 20 km of optical fibre. The system was able to support up to 120 m branch length with 3 m spatial
resolution.

In [111], a modified monitoring system that meets most of the features required for monitoring PON has been proposed. In this system, the encoder is simple and cost-effective and is based on FBG with $100 \%$ reflectivity, a coupler, and a patch cord. The code implemented is Multilevel-Optical Orthogonal Code (ML-OOC).

A mathematical model for PON monitoring based on a 2-D OOC has been developed in [33]. The mathematical model includes three modules: monitoring pulse generator modules, encoding and transmission modules, and decoding and identification modules. The system performance was evaluated in terms of Signal-to-Noise Ratio (SNR), Signal to Interference Ratio (SIR), and optimal decision probability.

Periodic coding was introduced in [114] with two FBG on the same waveband. The main function of the first FBG is to perform partial reflectivity (38\%). The second FBG functions as a frequency selective mirror. The length of the patch cord between the two FBG has been increased to form the optical cavity. However, partial reflectivity of the first FBG has an effect on the impulse response sensitivity. In addition, increasing the number of ONUs has led to an inefficient increase in the length of the patch cords. The system was able to support up to 64 ONUs and its performance was tested using SNR. The limitations were resolved by the introduction of an Incrementally Pulse Positioned Code (IPPC) coding scheme [115]. In this scheme, each grating works to reflect the different wavelengths with $100 \%$ reflectivity, thus resulting in the construction of a 2-D code. The monitoring system performance was examined using SIR.

A different approach for PON monitoring with a periodic optical encoder was proposed in [116] where each periodic optical encoder produces a unique periodic code with the use of a FBG, with $100 \%$ reflectivity and a fibre ring with a different length for each distribution or
drop fibre. The proposed encoder improved system performance by reducing the monitoring cost, and was able to accommodate a large number of ONUs (256). The performance of the encoder was evaluated mathematically using SNR.

Another two encoders that implemented 1-D unipolar codes were suggested in [117]. The encoders were implemented using a couple of FBGs to form a cavity with the use of multi-level periodic codes (similar to [116]). The encoders produce a code with a weight of 2 and 3 that enhance the correlation properties of multi-level periodic codes. The performance has been evaluated in terms of Signal-to-Noise-and-Interference Ratio (SNIR).

A centralized monitoring system using a 2-D optical frequency time-hopping/periodic code was investigated in [118]. A low-cost encoder was used for each ONU based on two FBGs with $59 \%$ reflectivity. The system was simulated for 4 ONUs to check the status of the fibre (healthy/broken). In addition, the system was evaluated in terms of Multiple-Customers Interference Probability (MCIP). The result showed an improvement in the overall interference.

A remote coding scheme for PON monitoring was presented in [119] with the system based on a cascaded encoder that uses a $1 \times 2$ optical power splitter and combiner, and an FBG at the RN. The monitoring pulse consists of $P$ wavelengths transmitted via U-band into the RN. The encoder reflects the wavelengths and produces a specific combination of wavelength for every ONU. The monitoring pulses are reflected with the use of an identical reflector and transmitted back to the receiver. The received signal is demultiplexed into $P$ wavelengths, and then passed to a detector which checks the power received against a set value.

Another 2-D time/wavelength OCDMA physical layer PON monitoring system uses reflective encoders at the ONU based on FBG serial arrays [87]. This system makes use of a broadband source with a M sub-bands in the C band. An encoder that is based on incoherent

2-D coding is set to each ONU to reflect M pulses, a pulse for each sub-band, in different time slots. At the receiver side, a demultiplexer works to demultiplex the $M$ sub-bands. The concept was verified in a PON with five sub-bands and four closely spaced ONUs with a very high user density. Parameters including beat noise, false alarm and misdetection were tested [87].

A set of coding schemes for monitoring PON have been analysed by Fathallah and Esmail in [86] where they show the different coding scheme designs and evaluate their performance in relation to SNR, probabilities of false alarm and misdetection. The coding schemes are based on either ODL, that uses a power splitter/combiner, FBG, or ring coding. PON monitoring based on OC for a tree topology was reported in [86]. Fathallah and Esmail discussed an approach for monitoring a ring using an OC technique. Their approach was based on dividing the ring into a number of segments and then installing an encoder based on 1-D for each segment. In this approach, the ring is bidirectional fibre, where the monitoring pulse is transmitted from the NMS in the counter-clockwise direction, and received in the clockwise direction. In this way, if a code is missing, the NMS will detect the fault location. This, however, will not be the case if another simultaneous fault occurs behind the faulty segment. The strategy proposed to overcome this problem is based on transmitting another monitoring signal in the opposite direction to enable the NMS to monitor the segment located after the first fault. However, this approach requires a large number of advanced encoders.

A hybrid 1-D/2-D OCDMA coding scheme was introduced for the first time by Rad et al. in [120]. The main purpose of 1-D/2-D was to eliminate the beat noise that resulted from 1-D coding. The system combined both the simplicity and cost effectiveness offered by 1-D coding and the beat noise cleaning provided by 2-D coding. In this system, the optical transmitter is a multi-wavelength laser, the encoders for distribution fibres are based on 1-D coding, and the decoding is based on 2-D coding using standard multi-FBG with $95 \%$
reflectivity at the NMS. The performance of the system has been analysed using receiver operating characteristics. The proposed coding reduced the cost and achieved SNR higher than 16 dB for 256 ONUs.

### 2.9 Summary

This chapter provides a literature survey of the systems and structures that form current and future designs for PON. PON multiplexing technologies were investigated, and the restrictions on numbers of ONUs, security, bandwidth, interference, and costs discussed. The ITU offers guidelines for developing effective systems; however, researchers take the liberty to address issues by building hybrid schemes. TWDM-PON, selected as the optimal scheme, is the subject of this study.

In addition, OCDMA systems are thoroughly investigated to identify coding principles, and the design of the devices used for encoding and decoding. An analysis and investigation of OCDMA have proven to be challenging. The issues associated with spreading codes, including increasing the cardinality to accommodate the number of active communication channels, has led to the conclusion that there is a need for a code that is suitable for PON.

Moreover, studies focusing on PON monitoring based upon an OC technique were analysed. It is important to note that, most of these studies are based on developing a coding scheme and evaluating the system performance based on mathematical models in terms of SNR, SIR, MCIP, probability of false alarm, or probability of misdetection. This thesis takes another direction for PON monitoring; it aims to design a code that provides code-words compatible with the different splitting ratios commonly found in PONs. The proposed code is designed in such a way as to increase the code-words, reduce the prime number and, hence, reducing the hardware complexity and energy consumption. This is followed by the implementation of the code in different PON scenarios using VPItransmissionMaker. The purpose of the
implementation is to provide the NMS with useful information about the status of each fibre (either healthy or faulty).

## Chapter 3 OPTICAL SPREADING CODE

Chapter 3 analyses and investigates the most common address codes for OCDMA designs, OOC, QCC, and PCs with a focus on PCs. The importance of this chapter stems from the fact that the code designed in this thesis has its background and origins imbedded in the existing OCDMA codes.

### 3.1 Optical Orthogonal Code

OOC, or time-based coding [121], is one of the most important temporal codes [122]. An OOC has a low weight resulting in low encoding and decoding efficiency. The number of possible code-words is low in comparison to the same code length in radio communication techniques, such as Walsh-Hadamard codes. Code length is also constrained by an OOC characteristic to have a short pulse with a pulse-width smaller than the bit duration [123]. OOC is a family of binary sequences represented by $(N, w, \lambda a, \lambda c)$, where $N$ is the length of the code and $w$ is the Hamming-weight (number of single digits (1) in each code-word) that satisfies the following auto and cross correlation properties [30]:

$$
\begin{gather*}
R_{C_{i} C_{j}}(m)=\sum_{n=0}^{N-1} C_{i}(n) C_{j}(n+m) \leq \lambda_{c}, \quad \text { for, } \forall m  \tag{3-1}\\
R_{C_{i} C_{j}}(m)=\sum_{n=0}^{N-1} C_{i}(n) C_{j}(n+m) \leq \lambda_{a}, \quad \text { for, } \quad[m]_{n} \neq 0 \tag{3-2}
\end{gather*}
$$

An $(N, w, \lambda a, \lambda c)$ OOC, is a constant-weight symmetric OOC, when $\lambda a=\lambda c$, represented as $(N, w, \lambda)$ OOC. When $\lambda a \neq \lambda c$, an $(N, w, \lambda a, \lambda c)$ OOC is a constant-weight asymmetric OOC $[25,27][124]$. Otherwise, the OOC code is a variable-weight OOC represented by ( $N, w, \lambda a$, $\lambda c, Q)$, where $w$ denotes a set of code weight, defined as $w=\left\{w_{1}, w_{2}, \ldots w_{P}\right\}$, and $Q$ denotes a set of code-words defined as $Q=\left\{q_{1}, q_{2}, . ., q_{P}\right\}$ [125].

A special case of OOC, referred to as optimal OOC, is when $\lambda a=\lambda c=\lambda$, which is represented by $(N, w, \lambda)$. The maximum number of code-words of $(n, w, \lambda)$ is represented by $\Phi(N, w, \lambda)$ and is defined by [30]:

$$
\begin{equation*}
\Phi(N, w, \lambda) \leq \frac{(N-1)(N-2) \ldots(N-\lambda)}{w(w-1)(w-2) \ldots(w-\lambda)} \tag{3-3}
\end{equation*}
$$

Another case of OOC is referred to as strict OOC when $\lambda a=\lambda c=1$, where the cardinality is given by [30]:

$$
\begin{equation*}
|C| \leq\left\lfloor\frac{(N-1)}{w(w-1)}\right\rfloor \tag{3-4}
\end{equation*}
$$

OOC code construction is complex, and methods to construct it include combinational construction, algebraic construction, and construction of finite projective geometrics [27]. Table 3-1 shows sequence indexes of OOC code with different $N$ [30].

Table 3-1 OOC ( $N, 3,1$ ) sequence indexes of various lengths [30]

| $\boldsymbol{N}$ | Sequence index, $\boldsymbol{N} \leq \mathbf{4 9}$ |
| :---: | :---: |
| 7 | $\{1,2,4\}$ |
| 13 | $\{1,2,5\}\{1,3,8\}$ |
| 19 | $\{1,2,6\}\{1,3,9\}\{1,4,11\}$ |
| 25 | $\{1,2,7\}\{1,3,10\}\{1,4,12\}\{1,5,14\}$ |
| 31 | $\{1,2,8\}\{1,3,12\}\{1,4,16\}\{1,5,15\}\{1,6,14\}$ |
| 37 | $\{1,2,12\}\{1,3,10\}\{1,4,18\}\{1,5,13\}\{1,6,19\}\{1,7,13\}$ |
| 43 | $\{1,2,20\}\{1,3,23\}\{1,4,16\}\{1,5,14\}\{1,6,17\}\{1,7,15\}\{1,8,19\}$ |

Table 3-2 presents the OOC sequence codes $(31,3,1)$.

Table 3-2 Sequence codes of OOC $(31,3,1)[30]$

| Index | Sequence Codes |
| :---: | :---: |
| \{1,2, 8\} | 110000010000000000000000000000 |
| \{1, 3, 12\} | 101000000001000000000000000000 |
| $\{1,4,16\}$ | 100100000000000100000000000000 |
| $\{1,5,15\}$ | 100010000000001000000000000000 |
| $\{1,6,14\}$ | 100001000000010000000000000000 |

### 3.2 Quadratic Congruence Code

QCC is constructed in two steps.

## Step 1.

Select a prime number, $P$, and construct a quadratic congruence sequence based on the Galois
field (GF) [27].

$$
\begin{equation*}
S_{i}=\left(S_{i, 0}, S_{i, 1}, \ldots, S_{i, j}, \ldots, S_{i,(P-1)}\right), \quad i=1,2, \ldots, P-1 \tag{3-5}
\end{equation*}
$$

where, the elements of $S_{i}$ are:

$$
\begin{equation*}
S_{i, j}=\left\{i . \frac{j(j+1)}{2}\right\}(\bmod P), \quad 1 \leq i \leq P-1, \quad 0 \leq i \leq P-1 \tag{3-6}
\end{equation*}
$$

where,

$$
\begin{equation*}
S_{i, j} \in G F(P)=\{0,1, \ldots, P-1\} \tag{3-7}
\end{equation*}
$$

## Step 2.

Construct a binary sequence from the quadratic congruence sequence.

$$
\begin{equation*}
C_{i}=\left(C_{i, 0}, C_{i, 1}, \ldots, C_{i, k}, \ldots, C_{i,(n-1)}\right), \quad i=1,2, \ldots, P-1 \tag{3-8}
\end{equation*}
$$

where,

$$
\begin{align*}
& C_{i, k}  \tag{3-9}\\
& = \begin{cases}1, & \text { if } k=s_{i, j}+j . P, \text { for } j=\{0,1, \ldots, P-1\} \text { and } j=\left|\frac{k}{P}\right|, \text { for } k=0,1, \ldots, P^{2}-1 \\
0, & \text { elsewhere }\end{cases}
\end{align*}
$$

where, $[\mathrm{x}]$ is an integer $\leq \mathrm{x}$. QCC has a length of $n=P^{2}$, a weight of $w=P$ and a cardinality of $|C|=P-1$. The maximum auto and cross-correlation functions are $\lambda a=2, \lambda a=4$, respectively. Table 3-3 is an example of QCC code sequences for $P=7$ [27].

Table 3-3 QCC code sequences for $P=7$ [27]

| $i$ | $S_{i}$ | $C_{i}$ |
| :---: | :---: | :---: |
| 1 | $S_{1}=0136310$ | 1000000010000000010000000001000100001000001000000 |
| 2 | $S_{2}=0265620$ | 1000000001000000000010000010000000100100001000000 |
| 3 | $S_{3}=0324530$ | 1000000000100000100000000100001000000010001000000 |
| 4 | $S_{4}=0453540$ | 1000000000010000000100001000000001000001001000000 |
| 5 | $S_{5}=0512150$ | 1000000000001001000000010000010000000000101000000 |
| 6 | $S_{6}=0641460$ | 1000000000000100001000100000000010000000011000000 |

### 3.4 Prime Code

PCs are simpler to construct than other code types, and are able to be modified to increase the cardinality [30]. This section considers prime codes families.

### 3.4.1 Basic Prime Code

Basic PC was introduced in $[126,127]$ and is constructed sequentially;

First select a prime number and build a set of prime sequences based on a Galois field $G F(P)$.

$$
\begin{equation*}
S_{i}=\left(S_{i, 0}, S_{i, 1}, \ldots, S_{i, j}, \ldots, S_{i,(P-1)}\right), \quad i=0,1 \ldots, P-1 \tag{3-10}
\end{equation*}
$$

where,

$$
\begin{equation*}
S_{i, j}=\{i . j\}(\bmod P) \tag{3-11}
\end{equation*}
$$

$i$ and $j$ are elements over $G F(P)=\{0,1,2, \ldots, P-1\}$. The prime sequences $S_{i}$ for 5 , as a prime number, are shown as an example in Table 3-4.

Table 3-4 Basic prime code sequences for $P=5$ [27]

| Index | $\boldsymbol{S}_{\boldsymbol{i}}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\boldsymbol{j}=\boldsymbol{0}$ | $\boldsymbol{j}=\boldsymbol{1}$ | $\boldsymbol{j}=\mathbf{2}$ | $\boldsymbol{j}=\mathbf{3}$ | $\boldsymbol{j}=\mathbf{4}$ |
| $S_{0}$ | 0 | 0 | 0 | 0 | 0 |
| $S_{1}$ | 0 | 1 | 2 | 3 | 4 |
| $S_{2}$ | 0 | 2 | 4 | 1 | 3 |
| $S_{3}$ | 0 | 3 | 1 | 4 | 2 |
| $S_{4}$ | 0 | 4 | 3 | 2 | 1 |

Secondly, map $S_{i}$ into a binary sequence $C_{i}$ as follows:

$$
\begin{equation*}
C_{i}=\left(C_{i, 0}, C_{i, 1}, \ldots, C_{i, k}, \ldots, C_{i,(N-1)}\right), \quad i=0,1 \ldots, P-1, N=P^{2} \tag{3-12}
\end{equation*}
$$

where,

$$
C_{i, k}= \begin{cases}1, & \text { if } k=s_{i, j}+j . P, \text { for } j=\{0,1, \ldots, P-1\}  \tag{3-13}\\ 0, & \text { elsewhere }\end{cases}
$$

A binary prime sequence is generated as in For $n_{1}, n_{2} \in\{1,2, \ldots, P\}$. The cross-correlation value of the basic PC could reach 2 and the auto-correlation value could be reduced to $(P-1)$
in the case of non-zero shift [128]. However, these changes in PC correlation properties complicate the receiver detection process, therefore, the maximum cross-correlation cannot exceed 1 and the auto-correlation must be equal to the weight [128].

Table 3-5 [27]. As shown in For $n_{1}, n_{2} \in\{1,2, \ldots, P\}$. The cross-correlation value of the basic PC could reach 2 and the auto-correlation value could be reduced to $(P-1)$ in the case of non-zero shift [128]. However, these changes in PC correlation properties complicate the receiver detection process, therefore, the maximum cross-correlation cannot exceed 1 and the auto-correlation must be equal to the weight [128].

Table 3-5 Binary sequence of basic $\mathrm{PC}, P=5$ the basic PC support cardinality of $P$ with weight (w) of $P$ and length $(L)$ of $P^{2}$. The correlation function of any two code sequences $C_{n}$ and $C_{m}$ can be defined as:

$$
C_{n 1} \cdot C_{n 2}= \begin{cases}P, & m=n ; \text { Auto }- \text { correlation }  \tag{3-14}\\ 1, & m \neq n ; \text { Cross }- \text { correlation }\end{cases}
$$

For $n_{1}, n_{2} \in\{1,2, \ldots, P\}$. The cross-correlation value of the basic PC could reach 2 and the auto-correlation value could be reduced to ( $P-1$ ) in the case of non-zero shift [128]. However, these changes in PC correlation properties complicate the receiver detection process, therefore, the maximum cross-correlation cannot exceed 1 and the auto-correlation must be equal to the weight [128].

Table 3-5 Binary sequence of basic PC, $P=5$ [27]

| Index | $\boldsymbol{C}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\boldsymbol{j}=\mathbf{0}$ | $\boldsymbol{j}=\mathbf{1}$ | $\boldsymbol{j}=\mathbf{2}$ | $\boldsymbol{j}=\mathbf{3}$ | $\boldsymbol{j}=\mathbf{4}$ |
| $C_{0}$ | 10000 | 10000 | 10000 | 10000 | 10000 |
| $C_{1}$ | 10000 | 01000 | 00100 | 00010 | 00001 |
| $C_{2}$ | 10000 | 00100 | 00001 | 01000 | 00010 |
| $C_{3}$ | 10000 | 00010 | 01000 | 00001 | 00100 |
| $C_{4}$ | 10000 | 00001 | 00010 | 00100 | 01000 |

### 3.4.2Extended Prime Code

Extended prime code (EPC) was proposed by Yang and Kwong in [126] and is used to overcome changes in correlation properties associated with basic PC [126, 129]. EPC is constructed in a similar way to the basic $P C$, but with each subsequence padded with $(P-1)$ or more zeros. Mapping $S_{i}$ into binary sequence $C_{i}$ can be modified:

$$
C_{i, k}= \begin{cases}1, & \text { if } k=s_{i, j}+j(2 P-1), \text { for } j=\{0,1, \ldots, P-1\}  \tag{3-15}\\ 0, & \text { elsewhere }\end{cases}
$$

The EPC increases the length up to $2 P-1$, thus improving the correlation functions. It supports a weight of $P$ and cardinality of $P^{2}$. Table 3-6, shows sequences of EPC when $P=5$.

Table 3-6 Extended prime code sequences, $P=5$ [126]

|  | $\boldsymbol{C}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Index | $\boldsymbol{j}=\mathbf{0}$ | $\boldsymbol{j}=\mathbf{1}$ | $\boldsymbol{j}=\mathbf{2}$ | $\boldsymbol{j}=\mathbf{3}$ | $\boldsymbol{j}=\mathbf{4}$ |
| $\boldsymbol{C}_{\mathbf{0}}$ | 100000000 | 100000000 | 100000000 | 100000000 | 100000000 |
| $\boldsymbol{C}_{\mathbf{1}}$ | 100000000 | 010000000 | 001000000 | 000100000 | 000010000 |
| $\boldsymbol{C}_{2}$ | 100000000 | 001000000 | 000010000 | 010000000 | 000100000 |
| $\boldsymbol{C}_{\mathbf{3}}$ | 100000000 | 000100000 | 010000000 | 000010000 | 001000000 |

### 3.4.3Modified Prime Code

A modified prime code (MPC) is applicable with increasing numbers of network transceivers, as it is an address code with low weight and large cardinality. MPC has been proposed in [27]. It can increase cardinality up to $P^{2}$ with weight and length of $P$ and $P^{2}$, respectively [127, 130]. Code construction is performed on the basic PC, and then every code sequence is time shifted $P$ times. The auto- and cross-correlation of MPC are:

$$
C_{n 1} \cdot C_{n 2}= \begin{cases}1, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in different groups }  \tag{3-16}\\ 0, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in same group } \\ P, & n 1=n 2\end{cases}
$$

where, $n_{1}, n_{2} \in\{1,2, \ldots, P\}$. An example of MPC is shown in Table 3-7, when $P=5$.

An example of MPC is shown in Table 3-7, when $P=5$.

Table 3-7 Modified prime code sequences, $P=5$ [127]

| Index |  | MPC sequences |
| :---: | :---: | :---: |
| $C_{0}$ | $C_{00}$ | 1000010000100001000010000 |
|  | $\mathrm{C}_{01}$ | 0100001000010000100001000 |
|  | $\mathrm{C}_{02}$ | 0010000100001000010000100 |
|  | $\mathrm{C}_{03}$ | 0001000010000100001000010 |
|  | $\mathrm{C}_{04}$ | 0000100001000010000100001 |
| $C_{1}$ | $C_{10}$ | 1000001000001000001000001 |
|  | $C_{11}$ | 0100000100000100000110000 |
|  | $C_{12}$ | 0010000010000011000001000 |
|  | $\mathrm{C}_{13}$ | 0001000001100000100000100 |
|  | $\mathrm{C}_{14}$ | 0000110000010000010000010 |
| $C_{2}$ | $\mathrm{C}_{20}$ | 1000000100000010100000010 |
|  | $\mathrm{C}_{21}$ | 0100000010100000010000001 |
|  | $\mathrm{C}_{22}$ | 0010000001010000001010000 |
|  | $\mathrm{C}_{23}$ | 0001010000001000000101000 |
|  | $\mathrm{C}_{24}$ | 0000101000000101000000100 |
| $C_{3}$ | $\mathrm{C}_{30}$ | 1000000010010000000100100 |
|  | $\mathrm{C}_{31}$ | 0100000001001001000000010 |
|  | $\mathrm{C}_{32}$ | 0010010000000100100000001 |
|  | $\mathrm{C}_{33}$ | 0001001000000010010010000 |
|  | $\mathrm{C}_{34}$ | 0000100100100000001001000 |
| $C_{4}$ | $\mathrm{C}_{40}$ | 1000000001000100010001000 |
|  | $C_{41}$ | 0100010000000010001000100 |
|  | $\mathrm{C}_{42}$ | 0010001000100000000100010 |
|  | $\mathrm{C}_{43}$ | 0001000100010001000000001 |
|  | $\mathrm{C}_{44}$ | 0000100010001000100010000 |

### 3.4.4New Modified Prime Code

A new modified prime code (n-MPC) has been introduced by Liu et al. in [108] and it aims to improve BER by increasing the weight and enhancing system security by increasing the length. n-MPC is constructed as MPC, with the last subsequence repeated at the end the following code sequence in the same group and rotating it in the same group [108] (presented in bold in Table 3-8). In n-MPC, the cardinality is $P^{2}$, the weight is $P+1$ and the length is $P^{2}$ $+P$. The cross- and auto-correlation functions are:

$$
C_{n 1} \cdot C_{n 2}=\left\{\begin{align*}
\leq 2, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in different groups }  \tag{3-17}\\
0, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in the same group } \\
P+1, & n 1=n 2
\end{align*}\right.
$$

where, $n_{1}, n_{2} \in\{1,2, \ldots, P\}$. An example of n-MPC is shown in Table 3-8, where $P=5$.

Table 3-8 New modified prime code sequences, $P=5$ [108]

| Index | MPC sequence | Padded sequence |  |
| :---: | :---: | :---: | :---: |
|  | $C_{00}$ | 1000010000100001000010000 | 01000 |
|  | $C_{01}$ | 0000100001000010000100001 | 10000 |
| $C_{0}$ | $C_{02}$ | 0001000010000100001000010 | 00001 |
|  | $C_{03}$ | 0010000100001000010000100 | 00010 |
|  | $C_{04}$ | 0100001000010000100001000 | 00100 |
|  | $C_{10}$ | 1000001000001000001000001 | 00010 |
|  | $C_{11}$ | 0100000100000100000110000 | 00001 |
| $C_{1}$ | $C_{12}$ | 0010000010000011000001000 | 10000 |
|  | $C_{13}$ | 0001000001100000100000100 | 01000 |
|  | $C_{14}$ | 0000110000010000010000010 | 00100 |
|  | $C_{20}$ | 1000000100000010100000010 | 01000 |
|  | $C_{21}$ | 0010000001010000001010000 | 00010 |
| $C_{2}$ | $C_{22}$ | 0000101000000101000000100 | 10000 |
|  | $C_{23}$ | 0100000010100000010000001 | 00100 |
|  | $C_{24}$ | 0001010000001000000101000 | 00001 |
|  | $C_{30}$ | 1000000010010000000100100 | 00001 |
|  | $C_{31}$ | 0100001000000010010010000 | 00100 |
| $C_{3}$ | $C_{32}$ | 0010000001001001000000010 | 10000 |
|  | $C_{33}$ | 0001000100100000001001000 | 00010 |
|  | $C_{34}$ | 0000100100000100100000001 | 01000 |
|  | $C_{40}$ | 1000000001000100010001000 | 10000 |
|  | $C_{41}$ | 0000100010001000100010000 | 01000 |
| $C_{4}$ | $C_{42}$ | 0001000100010001000000001 | 10000 |
|  | $C_{43}$ | 0010001000100000000100010 | 00001 |
|  | $C_{44}$ | 0100010000000010001000100 | 00010 |

### 3.4.5Padded Modified Prime Code

A padded modified prime code (PMPC) has been proposed by Liu and Tsao in [131]. It increases the auto-correlation peak to identify a single signal from the flow. Codes are constructed from MPC and then each group is padded with a sub-sequence stream of $P$ length. The padded sub-sequence is constant within sequences in the same group and time shifted for other groups. PMPC cardinality is $P^{2}$ and the length and weight increased to $\left(P^{2}+\right.$ $P)$ and $(P+1)$ respectively.

The auto-correlation and cross-correlation functions are [131]:

$$
C_{n 1} \cdot C_{n 2}=\left\{\begin{align*}
P+1, & n 1=n 2 ; \text { auto }- \text { correlation }  \tag{3-18}\\
1, & n 1 \neq n 2 ; \text { cross }- \text { correlation }
\end{align*}\right.
$$

where, $n_{1}, n_{2} \in\{1,2, \ldots, P\}$. The code-words of PMPC where $P=5$ are listed in .

Table 3-9.

Table 3-9 Padded modified prime code sequences, $P=5$ [30]

| Index |  | MPC sequence | Padded sequence |
| :---: | :---: | :---: | :---: |
| $C_{0}$ | $C_{00}$ | 1000010000100001000010000 | 10000 |
|  | $\mathrm{C}_{01}$ | 0000100001000010000100001 | 10000 |
|  | $\mathrm{C}_{02}$ | 0001000010000100001000010 | 10000 |
|  | $\mathrm{C}_{03}$ | 0010000100001000010000100 | 10000 |
|  | $\mathrm{C}_{04}$ | 0100001000010000100001000 | 10000 |
| $C_{1}$ | $C_{10}$ | 1000001000001000001000001 | 01000 |
|  | $C_{11}$ | 0100000100000100000110000 | 01000 |
|  | $C_{12}$ | 0010000010000011000001000 | 01000 |
|  | $C_{13}$ | 0001000001100000100000100 | 01000 |
|  | $\mathrm{C}_{14}$ | 0000110000010000010000010 | 01000 |
| $C_{2}$ | $\mathrm{C}_{20}$ | 1000000100000010100000010 | 00100 |
|  | $\mathrm{C}_{21}$ | 0010000001010000001010000 | 00100 |
|  | $C_{22}$ | 0000101000000101000000100 | 00100 |
|  | $\mathrm{C}_{23}$ | 0100000010100000010000001 | 00100 |
|  | $\mathrm{C}_{24}$ | 0001010000001000000101000 | 00100 |
| $C_{3}$ | $\mathrm{C}_{30}$ | 1000000010010000000100100 | 00010 |
|  | $C_{31}$ | 0100001000000010010010000 | 00010 |
|  | $\mathrm{C}_{32}$ | 0010000001001001000000010 | 00010 |
|  | $\mathrm{C}_{33}$ | 0001000100100000001001000 | 00010 |
|  | $\mathrm{C}_{34}$ | 0000100100000100100000001 | 00010 |
| $C_{4}$ | $\mathrm{C}_{40}$ | 1000000001000100010001000 | 00001 |
|  | $\mathrm{C}_{41}$ | 0000100010001000100010000 | 00001 |
|  | $\mathrm{C}_{42}$ | 0001000100010001000000001 | 00001 |
|  | $\mathrm{C}_{43}$ | 0010001000100000000100010 | 00001 |
|  | $\mathrm{C}_{44}$ | 0100010000000010001000100 | 00001 |

### 3.4.6Group padded Modified Prime Code

The group padded modified prime code (GPMPC) expands the process used in n-MPC, where the last two sub-sequences are repeated and rotated at the end of the code sequence in the same group. Available codes are $P^{2}$ and the length and weight are increased up to $(P+2)$ and $\left(P^{2}+2 P\right)$, respectively. The auto- and cross-correlations functions follow [30]:

$$
C_{n 1} \cdot C_{n 2}=\left\{\begin{align*}
\leq 2, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in different groups }  \tag{3-19}\\
0, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in the same group } \\
P+2, & n 1=n 2
\end{align*}\right.
$$

where, $n_{1}, n_{2} \in\{1,2, \ldots, P\}$. The code-words of GPMPC with $P=5$ are shown in Table 3-10.

Table 3-10 Double padded modified prime code sequences, $P=5$ [30]

| Index | MPC sequences | Double padded sequence |  |
| :---: | :---: | :---: | :---: |
|  | $C_{00}$ | 1000010000100001000010000 | 0000100001 |
| $C_{0}$ | $C_{01}$ | 0100001000010000100001000 | 1000010000 |
|  | $C_{02}$ | 0010000100001000010000100 | 0100001000 |
|  | $C_{03}$ | 0001000010000100001000010 | 0010000100 |
|  | $C_{04}$ | 0000100001000010000100001 | 0001000010 |
| $C_{10}$ | 1000001000001000001000001 | 0000110000 |  |
|  | $C_{11}$ | 0100000100000100000110000 | 1000001000 |
|  | $C_{12}$ | 0010000010000011000001000 | 0100000100 |
|  | $C_{13}$ | 0001000001100000100000100 | 0010000010 |
|  | $C_{14}$ | 0000110000010000010000010 | 0001000001 |
|  | $C_{20}$ | 1000000100000010100000010 | 0000101000 |
|  | $C_{21}$ | 0100000010100000010000001 | 1000000100 |
| $C_{2}$ | $C_{22}$ | 0010000001010000001010000 | 0100000010 |
|  | $C_{23}$ | 0001010000001000000101000 | 0010000001 |
|  | $C_{24}$ | 0000101000000101000000100 | 0001010000 |
|  | $C_{30}$ | 1000000010010000000100100 | 0000100100 |
|  | $C_{31}$ | 0100000001001001000000010 | 1000000010 |
| $C_{3}$ | $C_{32}$ | 0010010000000100100000001 | 0100000001 |
|  | $C_{33}$ | 0001001000000010010010000 | 0010010000 |
|  | $C_{34}$ | 0000100100100000001001000 | 0001001000 |
|  | $C_{40}$ | 1000000001000100010001000 | 0000100010 |
|  | $C_{41}$ | 0100010000000010001000100 | 1000000001 |
| $C_{4}$ | $C_{42}$ | 0010001000100000000100010 | 0100010000 |
|  | $C_{43}$ | 0001000100010001000000001 | 0010001000 |
|  | $C_{44}$ | 0000100010001000100010000 | 0001000100 |

### 3.4.7Double padded Modified Prime Code

The double padded modified prime code (DPMPC) has been proposed by Karbassian and Ghafouri-Shiraz in [132]. The generation of DPMPC is based on the following three steps:

Step 1. Generating MPC sequence.

Step 2. The first padded sub-sequence is based on padding the last sub-sequence of MPC in the same group (similarly to n-MPC), (presented in underline in Table 3-11).

Step 3. The second padded sub-sequence is based on routing the final sub-sequence of the last MPC in same group, (presented in bold in Table 3-11).

Table 3-11 Double padded modified prime code, $P=5$ [132]

|  | ndex | MPC sequences | Double padded sequence |
| :---: | :---: | :---: | :---: |
| $C_{0}$ | $\mathrm{Co}_{0}$ | 1000010000100001000010000 | 1000001000 |
|  | $\mathrm{C}_{01}$ | 0100001000010000100001000 | 0100000100 |
|  | $\mathrm{C}_{02}$ | 0010000100001000010000100 | 0010000010 |
|  | $C_{03}$ | 0001000010000100001000010 | 0001000001 |
|  | $\mathrm{C}_{04}$ | 0000100001000010000100001 | 0000101000 |
| $C_{1}$ | $C_{10}$ | $10000010000010000010 \underline{00001}$ | $\underline{00001} \mathbf{0 0 0 1 0}$ |
|  | $C_{11}$ | 0100000100000100000110000 | 1000000001 |
|  | $C_{12}$ | 0010000010000011000001000 | 0100010000 |
|  | $C_{13}$ | 0001000001100000100000100 | 0010001000 |
|  | $\mathrm{C}_{14}$ | 0000110000010000010000010 | 0001000100 |
| $C_{2}$ | $\mathrm{C}_{20}$ | 1000000100000010100000010 | 0001000100 |
|  | $\mathrm{C}_{21}$ | 0100000010100000010000001 | 0000100010 |
|  |  | 0010000001010000001010000 | 1000000001 |
|  | $\mathrm{C}_{23}$ | 0001010000001000000101000 | 0100010000 |
|  | $\mathrm{C}_{24}$ | 0000101000000101000000100 | 0010001000 |
| $\mathrm{C}_{3}$ | $\mathrm{C}_{30}$ | 1000000010010000000100100 | 0010001000 |
|  | $\mathrm{C}_{31}$ | 0100000001001001000000010 | 0001000100 |
|  |  | 0010010000000100100000001 | 0000100010 |
|  | $\mathrm{C}_{33}$ | 0001001000000010010010000 | 1000000001 |
|  | $\mathrm{C}_{34}$ | 0000100100100000001001000 | 0100010000 |
| $C_{4}$ | $\mathrm{C}_{40}$ | 1000000001000100010001000 | 0100010000 |
|  | $\mathrm{C}_{41}$ | 0100010000000010001000100 | 0010001000 |
|  | $C_{4} C_{42}$ | 0010001000100000000100010 | 0001000100 |
|  | $\mathrm{C}_{43}$ | 0001000100010001000000001 | 0000100010 |
|  | $\mathrm{C}_{44}$ | 0000100010001000100010000 | 1000000001 |

Available codes are $P^{2}$ and the length and weight are increased up to $(P+2)$ and $\left(P^{2}+2 P\right)$, respectively compared to MPC. The auto- and cross-correlations functions follow:

$$
C_{n 1} \cdot C_{n 2}=\left\{\begin{align*}
1, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in different groups }  \tag{3-20}\\
0, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in the same group } \\
P+2, & n 1=n 2
\end{align*}\right.
$$

where, $n_{1}, n_{2} \in\{1,2, \ldots, P\}$.

### 3.4.8Transposed Modified Prime Code

The OCDMA system aims for a large number of transceivers and efficient receiver recognition. To increase cardinality, $P$ must grow, creating an inefficient code due to increasing length [133]. Transposed-MPC (T-MPC) doubles cardinality and improves the spectral efficiency [134]. The code construction aims at constructing an MPC, then using a full padded technique similar to $\mathrm{n}-\mathrm{MPC}$ and DPMPC, where the last code sequence is repeated at the end the following code sequence and rotated in the same group. (See Table 3-12). T-MPC has cardinality of $P^{2}$, length of $2 P^{2}$ and weight of $2 P$. The auto-correlation and cross-correlation functions are:

$$
C_{n 1} \cdot C_{n 2}=\left\{\begin{align*}
\leq 2, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in different groups }  \tag{3-21}\\
0, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in the same group } \\
2 P, & n 1=n 2
\end{align*}\right.
$$

Table 3-12 Full padded modified prime code, $P=5$ [135]

| Index |  | MPC sequences | Padded sequences |
| :---: | :---: | :---: | :---: |
| $C_{0}$ | $C_{00}$ | 1000010000100001000010000 | 0000100001000010000100001 |
|  | $C_{01}$ | 0100001000010000100001000 | 1000010000100001000010000 |
|  | $\mathrm{C}_{02}$ | 0010000100001000010000100 | 0100001000010000100001000 |
|  | $\mathrm{C}_{03}$ | 01000010000100001000010 | 0010000100001000010000100 |
|  | $\mathrm{C}_{04}$ | 0000100001000010000100001 | 0001000010000100001000010 |
| $C_{1}$ | $\mathrm{C}_{10}$ | 1000001000001000001000001 | 0000110000010000010000010 |
|  | $C_{11}$ | 00000100000100000110000 | 1000001000001000001000001 |
|  | $C_{12}$ | 0000010000011000001000 | 0100000100000100000110000 |
|  | $C_{13}$ | 0001000001100000100000100 | 0010000010000011000001000 |
|  | $\mathrm{C}_{14}$ | 0000110000010000010000010 | 0001000001100000100000100 |
| $C_{2}$ | $C_{20}$ | 1000000100000010100000010 | 0000101000000101000000100 |
|  | $C_{21}$ | 0100000010100000010000001 | 1000000100000010100000010 |
|  | $\mathrm{C}_{22}$ | 10000001010000001010000 | 0100000010100000010000001 |
|  | $\mathrm{C}_{23}$ | 0001010000001000000101000 | 0010000001010000001010000 |
|  | $\mathrm{C}_{24}$ | 0000101000000101000000100 | 0001010000001000000101000 |
| $C_{3}$ | $\mathrm{C}_{30}$ | 1000000010010000000100100 | 0000100100100000001001000 |
|  | $\mathrm{C}_{31}$ | 0100000001001001000000010 | 1000000010010000000100100 |
|  | $\mathrm{C}_{32}$ | 0010010000000100100000001 | 0100000001001001000000010 |
|  | $\mathrm{C}_{33}$ | 0001001000000010010010000 | 0010010000000100100000001 |
|  | $\mathrm{C}_{34}$ | 0000100100100000001001000 | 0001001000000010010010000 |
| $C_{4}$ | $\mathrm{C}_{40}$ | 0000001000100010001000 | 0000100010001000100010000 |
|  | $\mathrm{C}_{41}$ | 0100010000000010001000100 | 1000000001000100010001 |

$C_{42} \quad 0010001000100000000100010$
$C_{43} 0001000100010001000000001$
$C_{44} 0000100010001000100010000$

0100010000000010001000100 0010001000100000000100010 0001000100010001000000001

The third step is to consider Table 3-12, as a matrix and apply a 'transpose function' to produce a new matrix. The new matrix comprises the T-MPC sequences in Table 3-13 which shows that the cardinality of T-MPC is doubled with the same length and weight of MPC ( $w$ $=P, L=P^{2}$ ), using the following correlation functions:

$$
C_{n 1} \cdot C_{n 2}=\left\{\begin{align*}
\leq 2, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in diff erent groups }  \tag{3-22}\\
0, & n 1 \neq n 2, n 1 \text { and } n 2 \text { in the same group } \\
P, & n 1=n 2
\end{align*}\right.
$$

Table 3-13 Transposed modified prime code sequences, $P=5$ [135]

| Index | TMPC sequences | Index | TMPC sequences |
| :---: | :---: | :---: | :---: |
| $C_{00}$ | 1000010000100001000010000 | $\mathrm{C}_{50}$ | 0100000100001000010000100 |
| $C_{01}$ | 0000101000000100010000001 | $C_{51}$ | 1000000010100000000101000 |
| $0 C_{02}$ | 0001000100010000000100010 | $5 \quad C_{52}$ | 0000100001000100100010000 |
| $\mathrm{C}_{03}$ | 0010000010000010100000100 | $\mathrm{C}_{53}$ | 0001010000010000001000001 |
| $\mathrm{C}_{04}$ | 0100000001001000001001000 | $\mathrm{C}_{54}$ | 0010001000000011000000010 |
| $C_{10}$ | 1000000001000010000100001 | $\mathrm{C}_{60}$ | 0010000010000100001000010 |
| $C_{11}$ | 0000110000001000100000010 | $C_{61}$ | 0100000001010001000000100 |
| $1 C_{12}$ | 0001001000100000001000100 | $6 C_{62}$ | 1000010000000010010001000 |
| $C_{13}$ | 0010000100000101000001000 | $C_{63}$ | 0000101000001000000110000 |
| $C_{14}$ | 0100000010010000010010000 | $C_{64}$ | 0001000100100000100000001 |
| $\mathrm{C}_{20}$ | 1000000010000100001000010 | $C_{70}$ | 0001000001000010000100001 |
| $\mathrm{C}_{21}$ | 0000100001010001000000100 | $\mathrm{C}_{71}$ | 0010010000001000100000010 |
| $2 C_{22}$ | 0001010000000010010001000 | $7 \quad C_{72}$ | 0100001000100000001000100 |
| $\mathrm{C}_{23}$ | 0010001000001000000110000 | $\mathrm{C}_{73}$ | 1000000100000101000001000 |
| $\mathrm{C}_{24}$ | 0100000100100000100000001 | $\mathrm{C}_{74}$ | 0000100010010000010010000 |
| $\mathrm{C}_{30}$ | 1000000100001000010000100 | $\mathrm{C}_{80}$ | 0000110000100001000010000 |
| $C_{31}$ | 0000100010100000000101000 | $\mathrm{C}_{81}$ | 0001001000000100010000001 |
| $3 C_{32}$ | 0001000001000100100010000 | $8 C_{82}$ | 0010000100010000000100010 |
| $\mathrm{C}_{33}$ | 0010010000010000001000001 | $\mathrm{C}_{83}$ | 0100000010000010100000100 |
| $\mathrm{C}_{34}$ | 0100001000000011000000010 | $\mathrm{C}_{84}$ | 1000000001001000001001000 |
| $\mathrm{C}_{40}$ | 1000001000010000100001000 | $\mathrm{C}_{90}$ | 1111100000000000000000000 |
| $C_{41}$ | 0000100100000010001010000 | $\mathrm{C}_{91}$ | 0000011111000000000000000 |
| $4 C_{42}$ | 0001000010001001000000001 | $9 \quad C_{92}$ | 0000000000111110000000000 |
| $\mathrm{C}_{43}$ | 0010000001100000010000010 | $\mathrm{C}_{93}$ | 0000000000000001111100000 |
| $\mathrm{C}_{44}$ | 0100010000000100000100100 | $\mathrm{C}_{94}$ | $00 \quad 0000000000011111$ |

### 3.4.9Transposed Sparse-padded Modified Prime Code

Transposed sparse-padded MPC (T-SPMPC) is constructed in four steps.

Step 1. Generates a MPC sequence.

Step 2. Generates a sparse padded sequence $\left(S P_{x y}\right)$ as follows:

$$
\begin{equation*}
S P_{x y}=\left\{S P_{x y i} \mid i=0,1, \ldots, P-1\right\} \tag{3-23}
\end{equation*}
$$

where, each $S P_{x y i}$ sequence consists of $M_{x y i}$ (MPC sequences), and $L_{x y i}$, defined as follows:

$$
M_{x y i}=\left\{\begin{align*}
S_{x i}, & y=0  \tag{3-24}\\
M_{x(y \ominus 1)(i \oplus 1)}, & \text { elsewhere }
\end{align*}\right.
$$

where, $\ominus$ denotes subtraction $\bmod P$, and $\oplus$ denotes addition $\bmod P$.

$$
L_{x y i}=\left\{\begin{array}{cc}
x, & \text { if } M_{x y i}=x  \tag{3-25}\\
N, & \text { elsewhere }
\end{array}\right.
$$

where, x denotes the number of the group and $N$ refers to "Null" sequence. The resulting code-words are shown in Table 3-14 [136].

Step 3. Exchange between $M_{x y i}$ and $L_{x y i}$ sequences. Cross-correlations in Table 3-14 occur through $M_{x y i}$; however, $L_{x y i}$ sequences are sparse and avoid cross-correlation. The exchange process follows:

$$
\begin{align*}
& M_{x y i}(\text { new })=L_{x y i}(\text { old }) \\
& L_{x y i}(\text { new })=M_{x y i}(\text { old }) \tag{3-26}
\end{align*}
$$

By applying the following condition:

$$
\begin{equation*}
x \ominus M_{x y i}<\frac{P+1}{2} \tag{3-27}
\end{equation*}
$$

Table 3-15, shows the intermediate table for SPMPC. Table 3-16 shows the binary sequences.

Table 3-14 Sparse padded sequence, SP, P = 5 [136]

| Group | Sequence | $S P_{x y 0}$ |  | $\mathbf{S P}_{\mathrm{xy} 1}$ |  | $\boldsymbol{S} \boldsymbol{P}_{x y 2}$ |  | $\boldsymbol{S} \boldsymbol{P}_{x y 3}$ |  | $S \boldsymbol{P}_{x y 4}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $M_{x y i}$ | $L_{x y i}$ | $M_{x y i}$ | $L_{x y i}$ | $M_{x y i}$ | $L_{x y i}$ | $M_{x y i}$ | $L_{x y i}$ | $M_{x y i}$ | $L_{x y i}$ |
| 0 | $S P_{00}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | $S P_{01}$ | 4 | N | 4 | N | 4 | N | 4 | N | 4 | N |
|  | $S P_{02}$ | 3 | N | 3 | N | 3 | N | 3 | N | 3 | N |
|  | $S P_{03}$ | 2 | N | 2 | N | 2 | N | 2 | N | 2 | N |
|  | $S P_{04}$ | 1 | N | 1 | N | 1 | N | 1 | N | 1 | N |
| 1 | $S P_{10}$ | 0 | N | 1 | 1 | 2 | N | 3 | N | 4 | N |
|  | $S P_{11}$ | 1 | 1 | 2 | N | 3 | N | 4 | N | 0 | N |
|  | $S P_{12}$ | 2 | N | 3 | N | 4 | N | 0 | N | 1 | 1 |
|  | $S P_{13}$ | 3 | N | 4 | N | 0 | N | 1 | 1 | 2 | N |
|  | $S P_{14}$ | 4 | N | 0 | N | 1 | 1 | 2 | N | 3 | N |
| 2 | $S P_{20}$ | 0 | N | 2 | 2 | 4 | N | 1 | N | 3 | N |
|  | $S P_{21}$ | 2 | 2 | 4 | N | 1 | N | 3 | N | 0 | N |
|  | $S P_{22}$ | 4 | N | 1 | N | 3 | N | 0 | N | 2 | 2 |
|  | $S P_{23}$ | 1 | N | 3 | N | 0 | N | 2 | 2 | 4 | N |
|  | $\mathrm{SP}_{24}$ | 3 | N | 0 | N | 2 | 2 | 4 | N | 1 | N |
| 3 | $\mathrm{SP}_{30}$ | 0 | N | 3 | 3 | 1 | N | 4 | N | 2 | N |
|  | $S P_{31}$ | 3 | 3 | 1 | N | 4 | N | 2 | N | 0 | N |
|  | $S P_{32}$ | 1 | N | 4 | N | 2 | N | 0 | N | 3 | 3 |
|  | $S P_{33}$ | 4 | N | 2 | N | 0 | N | 3 | 3 | 1 | N |
|  | $S P_{34}$ | 2 | N | 0 | N | 3 | 3 | 1 | N | 4 | N |
| 4 | $S P_{40}$ | 0 | N | 4 | 4 | 3 | N | 2 | N | 1 | N |
|  | $S P_{41}$ | 4 | 4 | 3 | N | 2 | N | 1 | N | 0 | N |
|  | $S P_{42}$ | 3 | N | 2 | N | 1 | N | 0 | N | 4 | 4 |
|  | $S P_{43}$ | 2 | N | 1 | N | 0 | N | 4 | 4 | 3 | N |
|  | $S P_{44}$ | 1 | N | 0 | N | 4 | 4 | 3 | N | 2 | N |

Table 3-15 Intermediate sparse padded MPC [136]

| Group | Sequence | $\boldsymbol{S} \boldsymbol{P}_{x y 0}$ |  | $\boldsymbol{S} \boldsymbol{P}_{x y 1}$ |  | $\boldsymbol{S} \boldsymbol{P}_{x y 2}$ |  | $\boldsymbol{S} \boldsymbol{P}_{x y 3}$ |  | $\boldsymbol{S P} \mathrm{P}_{\text {x } 4}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $M_{x y i}$ | $L_{x y i}$ | $M_{x y i}$ | $L_{x y i}$ | $\boldsymbol{M}_{x y i}$ | $L_{x y i}$ | $M_{x y i}$ | $L_{x y i}$ | $M_{x y i}$ | $L_{x y i}$ |
| 0 | SP00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | SP01 | N | 4 | N | 4 | N | 4 | N | 4 | N | 4 |
|  | SP02 | N | 3 | N | 3 | N | 3 | N | 3 | N | 3 |
|  | SP03 | 2 | N | 2 | N | 2 | N | 2 | N | 2 | N |
|  | SP04 | 1 | N | 1 | N | 1 | N | 1 | N | 1 | N |
| 1 | SP10 | N | 0 | 1 | 1 | 2 | N | 3 | N | N | 4 |
|  | SP11 | 1 | 1 | 2 | N | 3 | N | N | 4 | N | 0 |
|  | SP12 | 2 | N | 3 | N | N | 4 | N | 0 | 1 | 1 |
|  | SP13 | 3 | N | N | 4 | N | 0 | 1 | 1 | 2 | N |
|  | SP14 | N | 4 | N | 0 | 1 | 1 | 2 | N | 3 | N |
| 2 | SP20 | N | 0 | 2 | 2 | 4 | N | N | 1 | 3 | N |
|  | SP21 | 2 | 2 | 4 | N | N | 1 | 3 | N | N | 0 |


|  | SP22 | 4 | N | N | 1 | 3 | N | N | 0 | 2 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | SP23 | N | 1 | 3 | N | N | 0 | 2 | 2 | 4 | N |
|  | SP24 | 3 | N | N | 0 | 2 | 2 | 4 | N | N | 1 |
| 3 | SP30 | 0 | N | 3 | 3 | N | 1 | 4 | N | N | 2 |
|  | SP31 | 3 | 3 | N | 1 | 4 | N | N | 2 | 0 | N |
|  | SP32 | N | 1 | 4 | N | N | 2 | 0 | N | 3 | 3 |
|  | SP33 | 4 | N | N | 2 | 0 | N | 3 | 3 | N | 1 |
|  | SP34 | N | 2 | 0 | N | 3 | 3 | N | 1 | 4 | N |
| 4 | SP40 | 0 | N | 4 | 4 | N | 3 | N | 2 | 1 | N |
|  | SP41 | 4 | 4 | N | 3 | N | 2 | 1 | N | 0 | N |
|  | SP42 | N | 3 | N | 2 | 1 | N | 0 | N | 4 | 4 |
|  | SP43 | N | 2 | 1 | N | 0 | N | 4 | 4 | N | 3 |
|  | SP44 | 1 | N | 0 | N | N | 4 | N | 3 | N | 2 |

Table 3-16 Sparse padded MPC, $P=5$ [136]

| Group | Sequence | SP $\boldsymbol{x y y}^{0}$ |  | $\boldsymbol{S P} \boldsymbol{P}_{\text {y } 1}$ |  | $\boldsymbol{S P} \boldsymbol{P}_{\text {y } 2}$ |  | $\boldsymbol{S P} \boldsymbol{P}_{\text {y } 3}$ |  | $S P_{\text {xy }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $M_{\text {xyi }}$ | $L_{x y i}$ | $M_{x y i}$ | $L_{x y i}$ | $M_{\text {xyi }}$ | $M_{\text {xyi }}$ | $L_{x y i}$ | $M_{\text {xyi }}$ | $L_{x y i}$ | $M_{\text {xyi }}$ |
| 0 | $S P_{00}$ | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 |
|  | $S P_{01}$ | 00000 | 00001 | 00000 | 00001 | 00000 | 00001 | 00000 | 00001 | 00000 | 00001 |
|  | $S P_{02}$ | 00000 | 00010 | 00000 | 00010 | 00000 | 00010 | 00000 | 00010 | 00000 | 00010 |
|  | $S P_{03}$ | 00100 | 00000 | 00100 | 00000 | 00100 | 00000 | 00100 | 00000 | 00100 | 00000 |
|  | $S P_{04}$ | 01000 | 00000 | 01000 | 00000 | 01000 | 00000 | 01000 | 00000 | 01000 | 00000 |
| 1 | $S P_{10}$ | 00000 | 10000 | 01000 | 01000 | 00100 | 00000 | 00010 | 00000 | 00000 | 00001 |
|  | $S P_{11}$ | 01000 | 01000 | 00100 | 00000 | 00010 | 00000 | 00000 | 00001 | 00000 | 10000 |
|  | SP ${ }_{12}$ | 00100 | 00000 | 00010 | 00000 | 00000 | 00001 | 00000 | 10000 | 01000 | 01000 |
|  | $S P_{13}$ | 00010 | 00000 | 00000 | 00001 | 00000 | 10000 | 01000 | 01000 | 00100 | 00000 |
|  | $S P_{14}$ | 00000 | 00001 | 00000 | 10000 | 01000 | 01000 | 00100 | 00000 | 00010 | 00000 |
| 2 | $S P_{20}$ | 00000 | 10000 | 00100 | 00100 | 00001 | 00000 | 00000 | 01000 | 00010 | 00000 |
|  | SP ${ }_{21}$ | 00100 | 00100 | 00001 | 00000 | 00000 | 01000 | 00010 | 00000 | 00000 | 10000 |
|  | $S P_{22}$ | 00001 | 00000 | 00000 | 01000 | 00010 | 00000 | 00000 | 10000 | 00100 | 00100 |
|  | $S P_{23}$ | 00000 | 01000 | 00010 | 00000 | 00000 | 10000 | 10000 | 00100 | 00001 | 00000 |
|  | $S_{24}$ | 00010 | 00000 | 00000 | 10000 | 00100 | 00100 | 00001 | 00000 | 00000 | 01000 |
| 3 | $S P_{30}$ | 10000 | 00000 | 00010 | 00010 | 00000 | 01000 | 00001 | 00000 | 00000 | 00100 |
|  | $S P_{31}$ | 00010 | 00010 | 00000 | 01000 | 00001 | 00000 | 00000 | 00100 | 10000 | 00000 |
|  | $S P_{32}$ | 00000 | 01000 | 00001 | 00000 | 00000 | 00100 | 10000 | 00000 | 00010 | 00010 |
|  | $S P_{33}$ | 00001 | 00000 | 00000 | 00100 | 10000 | 00000 | 00010 | 00010 | 00000 | 01000 |
|  | $S P_{34}$ | 00000 | 00100 | 10000 | 00000 | 00010 | 00010 | 00000 | 01000 | 00001 | 00000 |
| 4 | $S P_{40}$ | 10000 | 00000 | 00001 | 00001 | 00000 | 00010 | 00000 | 00100 | 01000 | 00000 |
|  | $S P_{41}$ | 00001 | 00001 | 00000 | 00010 | 00000 | 00100 | 01000 | 00000 | 10000 | 00000 |
|  | $S P_{42}$ | 00000 | 00010 | 00000 | 00100 | 01000 | 00000 | 10000 | 00000 | 00001 | 00001 |
|  | $S P_{43}$ | 00000 | 00100 | 01000 | 00000 | 10000 | 00000 | 00001 | 00001 | 00000 | 00010 |
|  | $S P_{44}$ | 01000 | 00000 | 10000 | 00000 | 00001 | 00001 | 00000 | 00010 | 00000 | 00100 |

Step 4. Apply the transpose function for T-MPC in Table 3-16 to generate the T-SPMPC code sequences as shown in Table 3-17.

The cardinality, length and weight are $2 P^{2}, P^{2}$ and $(P+1) / 2$, respectively. The auto-correlation and cross-correlation functions of T-SPMPC are as follows:

$$
\mathrm{Cn} . \mathrm{C} m=\left\{\begin{array}{lc}
0, & \frac{3 p^{3}-p^{2}-3 p+1}{4 p^{3}-2 p} \text { probability }  \tag{3-28}\\
1, & \frac{p^{3}+p^{2}+p-1}{4 p^{3}-2 p} \text { probability }
\end{array}\right.
$$

Table 3-17 Transposed sparse-padded modified prime code, $P=5$ [136]

| Index | TMPC sequences | Index | TMPC sequences |
| :---: | :---: | :---: | :---: |
| $C_{00}$ | 1000000000000001000010000 | $\mathrm{C}_{50}$ | 1000000010000100000000000 |
| $C_{01}$ | 0000101000000000000000001 | $\mathrm{C}_{51}$ | 0000000001010001000000000 |
| $0 C_{02}$ | 0001000100010000000000000 | 5 | 0000000000000010010001000 |
| $\mathrm{C}_{03}$ | 0000000010000010100000000 | $\mathrm{C}_{53}$ | 0010000000000000000110000 |
| $\mathrm{C}_{04}$ | 0000000000001000001001000 | $\mathrm{C}_{54}$ | 0100000100000000000000001 |
| $\mathrm{C}_{10}$ | 1000010000100000000000000 | \% | 1000000000000000010000100 |
| $C_{11}$ | 0000001000000100010000000 | $\mathrm{C}_{61}$ | 0000100010000000000001000 |
| $1 C_{12}$ | 0000000000010000000100010 | $6 C_{62}$ | 0001000001000100000000000 |
| $C_{13}$ | 0010000000000000100000100 | $C_{6}$ | 0000010000010000001000000 |
| $\mathrm{C}_{14}$ | 0100000001000000000001000 | $C_{64}$ | 0000000000000011000000010 |
| $\mathrm{C}_{20}$ | 1000000000000000000100001 | $\mathrm{C}_{70}$ | 1000000100001000000000000 |
| $\mathrm{C}_{21}$ | 0000110000000000000000010 | $\mathrm{C}_{71}$ | 0000000010100000000100000 |
| $2 C_{22}$ | 0001001000100000000000000 | $7 \quad C_{72}$ | 0000000000000100100010000 |
| $\mathrm{C}_{23}$ | 0000000100000101000000000 | C | 0010000000000000001000001 |
| $\mathrm{C}_{24}$ | 0000000000010000010010000 | C | 0100001000000000000000010 |
| $\mathrm{C}_{30}$ | 1000000001000010000000000 | $\mathrm{C}_{80}$ | 1000000000000000100001000 |
| $C_{31}$ | 0000010000001000100000000 | $\mathrm{C}_{81}$ | 0000100100000000000010000 |
| $3 \quad C_{32}$ | 0000000000100000001000100 | 8 | 0001000010001000000000000 |
| $\mathrm{C}_{33}$ | 0010000000000001000001000 | 83 | 0000000001100000010000000 |
| $\mathrm{C}_{34}$ | 0100000010000000000010000 | $\mathrm{C}_{84}$ | 0000000000000100000100100 |
| $\mathrm{C}_{40}$ | 1000000000000000001000010 | 90 | 1000001000010000000000000 |
| $\mathrm{C}_{41}$ | 0000100001000000000000100 | $C_{91}$ | 0000000100000010001000000 |
| $4 C_{42}$ | 0001010000000010000000000 | $9 \quad C_{92}$ | 0000000000001001000000001 |
| $\mathrm{C}_{43}$ | 0000001000001000000100000 | $\mathrm{C}_{93}$ | 0010000000000000010000010 |
| $C_{44}$ | 0000000000100000100000001 | $\mathrm{C}_{94}$ | 0100010000000000000000100 |

### 3.5 Summary

This chapter provides a broad background of the most common codes applied into OCDMA system including OOC, QCC and PC families. It gives details about the construction of the codes, the parameters and an example of the resulted code-words

Chapter 4 EXTENDED GROUPED NEW MODIFIED PRIME CODE

This chapter answers Research Question 1" Can the existing codes implemented in OCDMA systems be modified to accommodate, with better characteristics, the number of ONUs supported in PON?. This chapter introduces the proposed code "EG-nMPC", with steps of code construction detailed in section 4.1.1, and the parameters of the code are given in section 4.1.2. In addition, the code performance is evaluated in term of BER in an incoherent system using OOK and PPM modulations. Furthermore, the performance of the proposed code is compared to other prime codes and a discussion of the improvements achieved by the proposed code are highlighted in section 4.3

### 4.1 Proposed Code

This section contains a description of the code construction and parameters.

### 4.1.1EG-nMPC Construction

The EG-nMPC construction is described in the following seven steps:

Step 1. Choose the prime number, for example $P=3$

Step 2. Generate the prime sequence based on a Galois Field GF (P)

$$
\begin{equation*}
S_{\mathrm{i}}=\left(S_{i, 0}, S_{i, 1}, \ldots, S_{i, j}, \ldots, S_{i,(P-1)}\right), i=0,1, \ldots, P-1 \tag{4-1}
\end{equation*}
$$

where the element of the sequence is

$$
\begin{equation*}
S_{i, j}=\{i . j\}(\bmod P) \tag{4-2}
\end{equation*}
$$

where $i$ and $j$ are elements over $G F(P)=\{0,1,2, \ldots, P-1\}$.

Step 3. Map $S_{i}$ into a binary sequence $C_{i}$ as follows:

$$
\begin{equation*}
C_{i}=\left(C_{i, 0}, C_{i, 1}, \ldots, C_{i, k}, \ldots, C_{i,(n-1)}\right), i=0,1, \ldots, P-1, n=P^{2} \tag{4-3}
\end{equation*}
$$

where

$$
C_{i, k}= \begin{cases}1, & \text { if } k=S_{i, j}+j . P, \text { for } j=\{0,1, \ldots, P-1\}  \tag{4-4}\\ 0, & \text { elsewhere }\end{cases}
$$

Step 4. Extend the prime code by patching each sub-sequence by $P$ of 0 s.

Step 5. For each code sequence, apply the time-shifting method described in MPC. Table 4-1 shows the resulting code-words with $P=3$.

Table 4-1 Time shifting of extended prime code, $P=3$

| Group | Binary Code $C_{i}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $C_{00}$ | 100000 | 100000 | 100000 |
|  | $C_{01}$ | 010000 | 010000 | 010000 |
| $\mathbf{0}$ | $C_{02}$ | 001000 | 001000 | 001000 |
|  | $C_{03}$ | 000100 | 000100 | 000100 |
|  | $C_{04}$ | 000010 | 000010 | 000010 |
|  | $C_{05}$ | 000001 | 000001 | 000001 |
|  | $C_{10}$ | 100000 | 010000 | 001000 |
|  | $C_{11}$ | 010000 | 001000 | 000100 |
| $\mathbf{1}$ | $C_{12}$ | 001000 | 000100 | 000010 |
|  | $C_{13}$ | 000100 | 000010 | 000001 |
|  | $C_{14}$ | 000010 | 000001 | 100000 |
|  | $C_{15}$ | 000001 | 100000 | 010000 |
|  | $C_{20}$ | 100000 | 001000 | 010000 |
|  | $C_{21}$ | 010000 | 000100 | 001000 |
|  | $C_{22}$ | 001000 | 000010 | 000100 |
| $\mathbf{2}$ | $C_{23}$ | 000100 | 000001 | 000010 |
|  | $C_{24}$ | 000010 | 100000 | 000001 |
|  | $C_{25}$ | 000001 | 010000 | 100000 |

Step 6. Apply the method used in n-MPC which is based on repeating the last sequence stream of the earlier code-word and rotating within the same group [30]. Table 4-2 shows the result of this step. In the table, the padded sequence stream of the first code and the last sequence stream of the earlier code are presented in bold.

Step 7. From each group, generate two groups where:

- Group 1: Consists of the $S_{i, j}$ where $i=0, \ldots, P-1$ and $j$ can take the values of zero or any even number. Other sub-sequences are patched with blocks of zeros.
- Group 2: Consists of the $S_{i, j}$ where $i=0, \ldots, P-1$ and $j$ is an odd number. Other subsequences are patched with blocks of zeros.

The final code-words are presented in

Table 4-3. The code is represented by $C_{x y z}$, where $x$ denotes the group ( 0 to $P-1$ ), $y$ denotes the sub-group ( 1 or 2 ) and $z$ denotes the code sequence ( 0 to $2 P-1$ ).

Table 4-2 Binary sequence of extended new modified prime code

| Group | Binary Code $C_{i}$ |  |  |  | Padded sequence |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | $C_{00}$ | 100000 | 100000 | 100000 | $\mathbf{0 0 0 0 0 1}$ |
|  | $C_{01}$ | 010000 | 010000 | 010000 | 100000 |
|  | $C_{02}$ | 001000 | 001000 | 001000 | 010000 |
|  | $C_{03}$ | 000100 | 000100 | 000100 | 001000 |
|  | $C_{04}$ | 000010 | 000010 | 000010 | 000100 |
|  | $C_{05}$ | 000001 | 000001 | $\mathbf{0 0 0 0 0 1}$ | 000010 |
| $\mathbf{1}$ | $C_{10}$ | 100000 | 010000 | 001000 | 010000 |
|  | $C_{11}$ | 010000 | 001000 | 000100 | 001000 |
|  | $C_{12}$ | 001000 | 000100 | 000010 | 000100 |
|  | $C_{13}$ | 000100 | 000010 | 000001 | 000010 |
|  | $C_{14}$ | 000010 | 000001 | 100000 | 000001 |
|  | $C_{15}$ | 000001 | 100000 | 010000 | 100000 |
|  | $C_{20}$ | 100000 | 001000 | 010000 | 100000 |
|  | $C_{21}$ | 010000 | 000100 | 001000 | 010000 |
| 2 | $C_{22}$ | 001000 | 000010 | 000100 | 001000 |
|  | $C_{23}$ | 000100 | 000001 | 000010 | 000100 |
|  | $C_{24}$ | 000010 | 100000 | 000001 | 000010 |
|  | $C_{25}$ | 000001 | 010000 | 100000 | 000001 |

Table 4-3 Binary sequence of extended grouped new modified prime code

| Group | Binary Code $C_{i}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | $C_{010}$ | 000000 | 100000 | 000000 | 000001 |
|  |  | $C_{011}$ | 000000 | 010000 | 000000 | 100000 |
|  |  | $C_{012}$ | 000000 | 001000 | 000000 | 010000 |
|  |  | $C_{013}$ | 000000 | 000100 | 000000 | 001000 |
|  |  | $C_{014}$ | 000000 | 000010 | 000000 | 000100 |
|  |  | $C_{025}$ | 000000 | 000001 | 000000 | 000010 |
|  | 2 | $C_{020}$ | 100000 | 000000 | 100000 | 000000 |
|  |  | $C_{021}$ | 010000 | 000000 | 010000 | 000000 |
|  |  | $C_{022}$ | 001000 | 000000 | 001000 | 000000 |
|  |  | $\mathrm{C}_{023}$ | 000100 | 000000 | 000100 | 000000 |


|  |  | $\begin{aligned} & C_{024} \\ & C_{025} \end{aligned}$ | $\begin{aligned} & 000010 \\ & 000001 \end{aligned}$ | $\begin{aligned} & 000000 \\ & 000000 \end{aligned}$ | $\begin{aligned} & 000010 \\ & 000001 \end{aligned}$ | $\begin{aligned} & 000000 \\ & 000000 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | $C_{110}$ | 000000 | 010000 | 000000 | 010000 |
|  |  | $C_{111}$ | 000000 | 001000 | 000000 | 001000 |
|  |  | $C_{112}$ | 000000 | 000100 | 000000 | 000100 |
|  |  | $C_{113}$ | 000000 | 000010 | 000000 | 000010 |
|  |  | $C_{114}$ | 000000 | 000001 | 000000 | 000001 |
|  |  | $C_{115}$ | 000000 | 100000 | 000000 | 100000 |
|  | 2 | $C_{120}$ | 100000 | 000000 | 001000 | 000000 |
|  |  | $C_{121}$ | 010000 | 000000 | 000100 | 000000 |
|  |  | $C_{122}$ | 001000 | 000000 | 000010 | 000000 |
|  |  | $C_{123}$ | 000100 | 000000 | 000001 | 000000 |
|  |  | $C_{124}$ | 000010 | 000000 | 100000 | 000000 |
|  |  | $\mathrm{C}_{125}$ | 000001 | 000000 | 010000 | 000000 |
| 2 | 1 | $\mathrm{C}_{210}$ | 000000 | 001000 | 000000 | 100000 |
|  |  | $C_{211}$ | 000000 | 000100 | 000000 | 010000 |
|  |  | $\mathrm{C}_{212}$ | 000000 | 000010 | 000000 | 001000 |
|  |  | $\mathrm{C}_{213}$ | 000000 | 000001 | 000000 | 000100 |
|  |  | $C_{214}$ | 000000 | 100000 | 000000 | 000010 |
|  |  | $\mathrm{C}_{215}$ | 000000 | 010000 | 000000 | 000001 |
|  | 2 | $\mathrm{C}_{220}$ | 100000 | 000000 | 010000 | 000000 |
|  |  | $C_{221}$ | 010000 | 000000 | 001000 | 000000 |
|  |  | $\mathrm{C}_{222}$ | 001000 | 000000 | 000100 | 000000 |
|  |  | $\mathrm{C}_{223}$ | 000100 | 000000 | 000010 | 000000 |
|  |  | $\mathrm{C}_{224}$ | 000010 | 000000 | 000001 | 000000 |
|  |  | $\mathrm{C}_{225}$ | 000001 | 000000 | 100000 | 000000 |

### 4.1.2Code Parameters

The length $L$, weight $w$, cardinality $|C|$, autocorrelation $\lambda_{x y z}$, and cross-correlation $\lambda_{x 1 y 1 z 1, \times 2 y 2 z 2}$ of EG-nMPC are given by:

$$
\begin{gather*}
L=2 P^{2}+2 P  \tag{4-5}\\
w=\frac{(P+1)}{2}  \tag{4-6}\\
|C|=4 P^{2}  \tag{4-7}\\
\lambda_{x y z}=w  \tag{4-8}\\
\lambda_{x_{1} y_{1} z_{1}, x_{2} y_{2} z_{2}}= \begin{cases}0, & \frac{P^{2}-1}{8 P^{2}-2} \text { probability } \\
1, & \frac{7 P^{2}-1}{8 P^{2}-2} \text { probability }\end{cases} \tag{4-9}
\end{gather*}
$$

## Equation (4-9) is obtained as follows:

Each code-word causes $w(P-1)$ pairs of cross-correlations. Thus the cross-correlation value of 1 for a one communication channel and for all communication channels can be formed respectively as follow:

$$
\begin{gather*}
\lambda_{1(\text { one } C-\text { channel })}=\frac{P^{2}-1}{2}  \tag{4-10}\\
\lambda_{1(\text { all } C \text {-channels })}=2 P^{4}-2 P^{2} \tag{4-11}
\end{gather*}
$$

The total number of correlation pairs of EG-nMPC is:

$$
\begin{equation*}
\lambda_{\text {Corr-pairs }}=4 P^{2} \cdot\left(4 P^{2}-1\right)=16 P^{4}-4 P^{2} \tag{4-12}
\end{equation*}
$$

The probability of a cross-correlation value of 1 is given by:

$$
\begin{equation*}
P b_{\lambda 1}=\frac{\lambda_{1(\text { allusers })}}{\lambda_{\text {corr-pairs }}}=\frac{P^{2}-1}{8 P^{2}-2} \tag{4-13}
\end{equation*}
$$

The probability of a cross-correlation value of 0 is given by:

$$
\begin{equation*}
P b_{\lambda 0}=1-P b_{\lambda 1}=\frac{7 P^{2}-1}{8 P^{2}-2} \tag{4-14}
\end{equation*}
$$

Taking the code $C_{010}$ as an example, the code will not cause any cross-correlation with any code in the same group, either in the same or different subgroups. In addition, the code will not cause any cross correlation with any code in a different group and in different sub-group, as a consequence of step 7 in section 2.1. Each code- weight ( $w$ ) in the code causes a crosscorrelation with only one code from other groups ( $P-1$ group). Thus the code $C_{010}$ will cause a cross correlation with the codes $C_{114}, C_{115}, C_{214}$, and $C_{215}$.

A system that uses an incoherent OCDMA is based on the binary 1 and 0 , where, 1 represents the presence of light and hence the encoding process executes and generates a waveform $s(n)$. The code sequence represents the destination address $f(n)$. The 0 on the other hand, represents
the absence of light which means the encoding process will not be executed. After the encoding process and transmission to the communication channel, the receiver correlates the received signal with the destination address. This correlation function is denoted by $r(n)$ and expressed as [136]:

$$
\begin{equation*}
r(n)=\sum_{i=1}^{N} s(i) f(i-n) \tag{4-15}
\end{equation*}
$$

where, $N$ is the number of communication channels. If the signal arrives correctly, then the received signal is equal to the destination address with $s(n)=f(n)$ and 1 signifies an autocorrelation function. Otherwise, 1 signifies a cross-correlation function [136].

In a synchronous OCDMA system, time synchronization $T$ is performed at the end of each bit duration, where $T=L$ chip width [136]. For a "1 1001 " data stream, and synchronization time $T$, the correlation values of alternate codes are presented in Figure 4-1 and Figure 4-6. Figure 4-1 shows the auto-correlation value of $C_{010}$ at each time interval $T$ and it is equal to $w$ (2 in Fig. 1). Figure 4-2 and Figure 4-3 show the cross-correlation value between two codes in a different group and same sub-group respectively. Figure 4-2 shows the case when the crosscorrelation value is 1 and Figure 4-3 shows the case when the cross-correlation value is 0 . Figure 4-4 to Figure 4-6 present the cross-correlation values of two codes from the same group and same sub-group, same group different sub-group and different group different subgroup respectively. As shown in these Figures, the value of the cross-correlation at time $T$ is 0.


Figure 4-1 Auto-correlation of EG-nMPC of $C_{010}$, at synchronization time, $T$


Figure 4-2 Cross-correlation of EG-nMPC of $C_{014}$ and $C_{112}$, at synchronization time, $T$


Figure 4-3 Cross-correlation of EG-nMPC of $C_{022}$ and $C_{125}$, at synchronization time, $T$


Figure 4-4 Cross-correlation of EG-nMPC of $C_{110}$ and $C_{112}$, at synchronization time, $T$


Figure 4-5 Cross-correlation of EG-nMPC of $C_{024}$ and $C_{013}$, at synchronization time, $T$


Figure 4-6 Cross-correlation of EG-nMPC of $C_{020}$ and $C_{210}$, at synchronization time, $T$

### 4.2 Performance Analysis

In this section, the code performance analysis is provided.

### 4.2.1Performance Analysis of OOK-OCDMA System

The BER of the one dimensional time spreading code using an incoherent OOK modulation system relies on three main factors: the binary stream, the threshold range and the probability of 'hits' between ' 1 s ' within different code-words [134]. The BER can be formed as:

$$
\begin{equation*}
B E R=\frac{1}{2} \sum_{i=0}^{w}(-1)^{i}\binom{w}{i}\left(1-\frac{i q}{w}\right)^{N-1} \tag{4-16}
\end{equation*}
$$

In (4-16), $w$ is the weight, $N$ is the number of active communication channels $s$ and $q$ is the hit possibility of 1 between a particular communication channels and the intended communication channel which given by:

$$
\begin{equation*}
q=\frac{w^{2}}{2 L} \tag{4-17}
\end{equation*}
$$

However, $q$ in (4-17) is for asynchronous OCDMA applying the OOC. For synchronous OCDMA system, $q$ depends on the correlation expectation $\left(E_{\lambda}\right)$ between two codes which can be expressed as [136]:

$$
\begin{equation*}
E_{\lambda}=\frac{\sum \lambda_{x 1 y 1, x 2 y 2}}{|C| \cdot(|C|-1)} \tag{4-18}
\end{equation*}
$$

The BER probability of MPC and T-SPMPC can be obtained from [136]. The BER of EGnMPC can be formulated as:

$$
\begin{equation*}
B E R=\frac{1}{2} \sum_{i=0}^{w}(-1)^{i}\binom{w}{i}\left(1-\frac{i}{2 w} \cdot \frac{P^{2}-1}{8 P^{2}-2}\right)^{N-1} \tag{4-19}
\end{equation*}
$$

### 4.2.2Performance Analysis of PPM-OCDMA System

EG-nMPC provides $4 P^{2}$ code sequences, thus the number of communication channels is $4 P^{2}$. By assuming $N$ is the number of active communication channels, $4 P^{2}-N$ is the number of idle communication channels. $Y_{n}$ is defined as a random variable where:

$$
\begin{gather*}
Y_{n}, n \in\left\{1,2, \ldots, 4 P^{2}\right\}  \tag{4-20}\\
Y_{n}= \begin{cases}1, & \text { if communication channel } n \text { is active } \\
0, & \text { if communication channel } n \text { is idle }\end{cases} \tag{4-21}
\end{gather*}
$$

thus,

$$
\begin{equation*}
N=\sum_{n=1}^{4 P^{2}} \mathrm{Y}_{n} \tag{4-22}
\end{equation*}
$$

By assuming communication channel 1 is the desired communication channel, the random variable $T$ is defined to represent the number of active communication channels that are not causing cross correlation with communication channel 1 and $t$ is the realization of $T$. According to (4-10), $T$ can be defined as:

$$
\begin{equation*}
T \stackrel{\text { def }}{=} \sum_{n=1}^{\left(7 \mathrm{P}^{2}+1\right) / 2} Y_{n} \tag{4-23}
\end{equation*}
$$

The probability distribution of the random variable $T$ can be obtained as:
where $t \in\left\{t_{\min }, \ldots, t_{\max }\right\}$

$$
\begin{gathered}
\left.\left.t_{\min }=\max \left(2 N-P^{2}+1\right) / 2\right), 1\right) \\
t_{\max }=\min \left(N,\left(7 P^{2}+1\right) / 2\right)
\end{gathered}
$$

Let the photon count collection be denoted by the vector $Y_{n}$ for communication channel $n$, where $Y_{n}=\left(Y_{n, 0}, Y_{n, 1}, \ldots, Y_{n, M-1}\right)$. Let the interference denoted by the random vector $k$, where $k$ $=\left(k_{0}, k_{1}, \ldots, k_{m-1}\right)^{T}$. Let the vector $u$ denote the realization of $k$, where $u=\left(u_{0}, u_{1}, \ldots, u_{M-1}\right)^{T}$. By assuming $T=t, k$ can be defined as a multinomial random vector with the following probability:

$$
\begin{equation*}
P_{k \mid T}\left(u_{0}, u_{1}, \ldots, u_{M-1} \mid t\right)=\frac{1}{M^{N-t}} \cdot \frac{(N-t)!}{u_{0}!, u_{1}!, \ldots, u_{M-1}!} \tag{4-25}
\end{equation*}
$$

where,

$$
\begin{equation*}
N-t=\sum_{i=0}^{M-1} u_{i} \tag{4-26}
\end{equation*}
$$

Then the BER of can be given by [132]:

$$
\begin{equation*}
P_{b}=\frac{M}{2(M-1)} \sum_{t=t_{\text {min }}}^{t_{\text {max }}} P_{E} \cdot P_{t}(t) \tag{4-27}
\end{equation*}
$$

where $M$ denotes the multiplicity and $P_{E}$ denotes the error probability [132].

Similarly to (9) in [132], the lower bounded BER can be obtained in (4-30) by modifying (5) in [110] based on EG-nMPC properties. By taking $Q \rightarrow \infty$, where $Q$ defined as:

$$
\begin{equation*}
\mathrm{Q}=\frac{\mu \cdot(\ln M)}{w} \tag{4-28}
\end{equation*}
$$

$\mu$ in (4-28) is the average photons count per pulse (photons/nat), nat can be expressed as follow [137] [138]:

$$
\begin{align*}
& 1 \text { nat }=\log _{2} e \text { bits }  \tag{4-29}\\
& P_{E} \geq \sum_{u 1=\left(\frac{P+1}{2}+1\right)}^{N-t}\binom{N-t}{u_{1}} \frac{1}{M^{u 1}} \cdot(1 \\
&\left.-\frac{1}{M}\right)^{N-t-u_{1}} \cdot \sum_{u_{0}=0}^{\min \left(u_{1}-\left(\frac{P+1}{2}+1\right), N-t-u_{1}\right)}\binom{N-t-u_{1}}{u_{0}} \cdot \frac{1}{(M-1)^{u_{0}}} \cdot(1 \\
&\left.-\frac{1}{M-1}\right)^{N-t-u_{0}-u_{1}} \\
&\left.+0.5 \sum_{\frac{N-t+\frac{P+1}{2}}{\sum^{2}}}^{u_{1=\frac{P+1}{2}}^{N-t}} \begin{array}{c}
N \\
u_{1}
\end{array}\right) \frac{1}{M^{u_{1}}} \cdot(1 \\
&\left.-\frac{1}{M}\right)^{N-t-u_{1}} \cdot\binom{N-t-u_{1}}{u_{1}-\left(\frac{P+1}{2}\right)} \cdot \frac{1}{(M-1)^{u_{1}-\left(\frac{P+1}{2}\right)} \cdot\left(1-\frac{1}{M-1}\right)^{N-t-2 u_{0}+\frac{P+1}{2}}}
\end{align*}
$$

### 4.3 Discussion

A comparison of the results for EG-nMPC and MPC, the basis for many of the standard codes found today, highlights the performance advantages of EG-nMPC. When EG-nMPC is compared with T-SPMPC, a recently developed code that supports a large number of communication channels with a low weight, the result shows that EG-nMPC provides an improvement.

In Figure 4-7, the correlation expectation of the MPC, T-SPMPC, and EG-nMPC are presented and as shown in the figure, the correlation expectation of MPC is the highest. With $P=5$ the correlation expectation is 0.83 and it increases to 0.96 as $P$ increases to 23. TSPMPC shows a lower correlation expectation. With $P=5$ the correlation expectation is 0.32 and decreases to 0.29 as $P$ increases to 23 . EG-nMPC shows the lowest correlation expectation compared to the other codes. With $P=5$, the correlation expectation is 0.1212 and increases slightly to 0.1248 with $P$ increasing to 23 .

The low value of correlation expectation is a result of Step 7 in the code construction. This step aims to duplicate the number of available code-words at the same time it minimizes the correlation between the codes by replacing some of the sub-sequences with block of zeros.

Figure 4-8 presents the BER performance versus the number of communication channels of an OOK-OCDMA system. It compares the performance of the three codes with $P=11$, and $P$ $=13$. The figure shows that at the BER of $10^{-9}$, the performance of the three codes are similar with a slight improvement for MPC, followed by EG-nMPC and then T-SPMPC. However, with an increasing number of the communication channels, the performance of the EG-nMPC has improved and its performance with $P=11$ and $P=13$ is superior. In addition, this improvement can be seen clearly in Figure $4-9$ with $P=23$ and $P=37$. The figure shows that EG-nMPC was achieved the required BER with more communication channels.

Figure 4-10 illustrates the BER performance versus the number of communication channels of a PPM-OCDMA system. The figure compares the performance of MPC and EG-nMPC, with, $\mu=150, M=16, P=11$, and $P=13$. It is important to note that the comparison is made between EG-nMPC and MPC only. This is due to the absence of mathematical expressions to evaluate the BER of T-SPMPC, in the case of a PPM-OCDMA system without interference cancellation. The figure shows that at the BER value of $10^{-9}$, MPC accommodates a slightly higher number of communication channels when compared to EG-nMPC. However, as the number of communication channels increases, the BER of EG-nMPC reduces, pointing to an improved performance. The smaller BER values are attributed to the EG-nMPC capability to accommodate a larger number of communication channels. The capability of EG-nMPC to accommodate more communication channels with the same average photons/nat, is also evident from Figure 4-10, an outcome that points to the code being energy-efficient.

It is important to note the significant role played by the multiplicity, $M$, in improving the system performance, as shown in Figure 4-11. The figure shows the performance of BER versus the number of communication channels, with $M=16$ and $M=8, P=13$, and $\mu=150$. As shown in the figure, an increase in the value of $M$ results in an improvement in system performance, however, larger values of $M$ lead to an increase in system cost and complexity.

EG-nMPC increases the cardinality of the code with a smaller $P$, as shown in Figure 4-12. For a large number of communication channels, EG-nMPC has a lower BER overall compared to the other code families with both modulation systems as shown in Figure 4-8, Figure 4-9, and Figure 4-10. Therefore, it is effective to implement EG-nMPC over GPON thereby supporting a high number of ONUs.


Figure 4-7 Cross-correlation expectations of MPC, T-SPMPC and EG-nMPC


Figure 4-8 BER versus number of communication channels, for MPC, T-SPMPC, and EG-nMPC, using
OOK system for $P=11$ and $P=13$


Figure 4-9 BER versus number of communication channels, for MPC, T-SPMPC, and EG-nMPC using OOK system, for $P=23$ and $P=37$


Figure 4-10 BER versus number of communication channels, for MPC, and EG-nMPC using PPM-
OCDMA system, for $P=11$, and $P=13$


Figure 4-11 BER versus number of communication channels, for MPC, and EG-nMPC using PPMOCDMA system, for $P=11, M=8$ and $M=16$


Figure 4-12 Cardinality of MPC, T-SPMPC and EG-nMPC

### 4.4 Code Comparison for GPON Splitting Ratios

As stated in Chapter 1, the spreading code of an OCDMA system can be used in a monitoring
system for PON networks. For 1-D coding, a 1-D OOC has improved performance over prime codes in terms of correlation. Its construction mechanism, however, is relatively complex, in addition to its cardinality being bound by its length and weight [30]. Therefore, an OOC can only support a limited number of code-words. To increase the number of codewords, the length needs to be increased. Table 4-4 shows examples of optimal OOC codes [27]. On the other hand, 1-D PC is characterized by a simple construction mechanism, in addition to its cardinality not being subject to an upper bound and there are no mathematical limitations on the way the code is used, other than matching the code used with the optical transmission system capabilities [27, 30]. The correlation probabilities of OOC are higher than those of PCs, however, this feature has become less important in PON due to the use of TDMA and the ranging process performed by the OLT and ONU to avoid upstream burst collision. The selection of the spreading code implemented in OC techniques is determined by the cardinality of the spreading code selected [36]. Hence, the main objective is to provide the number of code-words that are compatible with PON splitting ratios (32, 64, and 128).

Table 4-4 Examples of optimal OOC code [27]

| Weight, $\boldsymbol{w}$ | Length, $\boldsymbol{n}$ | Cardinality, $\|\boldsymbol{C}\|$ |
| :---: | :---: | :---: |
| 3 | 31 | 5 |
| 3 | 255 | 42 |
| 3 | 511 | 85 |
| 3 | 1023 | 170 |
| 4 | 40 | 3 |
| 4 | 364 | 30 |
| 4 | 1093 | 91 |
| 5 | 85 | 4 |
| 5 | 341 | 17 |
| 5 | 1365 | 68 |
| 6 | 156 | 5 |
| 6 | 631 | 21 |
| 6 | 3156 | 105 |

The proposed code can be implemented in OC techniques as it provides compatible cardinality with lower unused code-words when compared to other PCs, thereby leading to an
increase in the code utilization factor that is defined as the number of active communication channels at a given BER [133].

EG-nMPC increases the cardinality of the code with a smaller value of $P$, as shown in Figure 4-12. Implementing EG-nMPC as a spreading code for an OC technique to monitor PON networks that support splitting ratios of 32,64 , and 128 could be seen to be an improved solution. Table 4-5 shows a comparison of the code parameters for the different PON splitting ratios. For a splitting ratio of 32, EG-nMPC has been shown to have the lowest weight, length and unused codes, i.e. 2, 24, 4, respectively. This renders EG-nMPC a superior code for supporting a splitting ratio of 32 , as compared to other codes. T-SPMPC ranks second in superiority, with MPC being the least desirable code among the three. The very slight increase in the number of unused codes exhibited by T-SPMPC as compared to MPC, does not carry any significant weight on this superiority ranking. The inferiority of MPC, for a splitting ratio of 32 , is evidenced by the values of its weight, its length, and the number of unused codes, i.e. $7,49,17$ respectively.

For a splitting ratio of 64, MPC shows the least desirable option with largest values for weight, length, and unused codes, i.e. 11, 121 and 57 respectively. When it comes to code length and the number of unused codes, T-SPMPC is shown to be a better option when compared to EG-nMPC, with a length of 49 , as compared to 60 , and the number of unused codes being 34 as compared to 36 . However, EG-nMPC possesses a lower weight when compared to T-SPMPC, i.e. 3 and 4, respectively. For a splitting ratio of 128, although MPC has the lowest number of unused codes, its weight is far larger and the length is far longer than other codes. EG-nMPC exhibits lower weight, length, and a smaller number of unused codes compared to T-SPMPC. Consequently, it could be concluded that EG-nMPC possesses improved numerical values for weight, length and number of unused codes.

Table 4-5 Code Parameter Comparison

| 32 |  |  |  |  | 64 |  |  |  | 128 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PC | $P$ | Cardinality (unused codes) | $L$ | w | $P$ | Cardinality (unused codes) | $L$ | w | $P$ | Cardinality (unused codes) | $L$ | w |
| MPC | 7 | $\begin{gathered} 49 \\ (17) \end{gathered}$ | 49 | 7 | 11 | $\begin{aligned} & 121 \\ & (57) \end{aligned}$ | 121 | 11 | 13 | $\begin{aligned} & 169 \\ & (41) \end{aligned}$ | 169 | 13 |
| T-SPMPC | 5 | $\begin{gathered} \hline 50 \\ (18) \end{gathered}$ | 25 | 3 | 7 | $\begin{gathered} \hline 98 \\ (34) \end{gathered}$ | 49 | 4 | 11 | $\begin{gathered} 242 \\ (114) \end{gathered}$ | 121 | 6 |
| EG-nMPC | 3 | $36$ <br> (4) | 24 | 2 | 5 | $\begin{aligned} & 100 \\ & (36) \end{aligned}$ | 60 | 3 | 7 | $\begin{aligned} & 196 \\ & (68) \end{aligned}$ | 112 | 4 |

### 4.5 Summary

In this chapter, the construction of the proposed code for a PON monitoring based OC technique has been described and its parameters have been defined. In addition, the performance of the proposed code has been evaluated using an incoherent OOK-OCDMA system, and then compared to both MPC and T-SPMPC. In incoherent PPM-OCDMA systems, however, performance of the proposed code was evaluated and compared to MPC only. In addition, it has been shown that, the use of EG-nMPC as a spreading code for a monitoring-based OC technique is valid as it possesses overall superior characteristics, when compared to the other two codes, and that is in regard to weight, length, and the number of unused codes.

## Chapter 5 NG-PON PROTECTION

This chapter answers Research Question 2 "How would the new application of the FIR parameter improve system performance? ". To answer this question, the chapter starts with an overview of NG-PON architecture that is based on LR-PON in section 5.1. It then followed by evaluation of the FIR parameter for the different network components in section 5.2 . Based on this parameter, three protection schemes for LR-PON were proposed as shown in section 5.3. The performance of the proposed protection schemes were then evaluated in term of cost and availability, and compared to the existing protection schemes.

### 5.1 NG-PON Architecture

The LR-PON architecture is an optimal solution that meets the requirements of NG-PON [12]. It aims to extend coverage and to accommodate 1024 ONUs [8]. This can be achieved by implementing optical amplifiers in the field [139]. Depending on GPON splitting ratios and the architecture of LR-PON, that is based on ring-and spur, NG-PON that support 1024 ONUs, at least, can be realized by implementing one of the following scenarios:

- Scenario A: 32 RNs each with a splitting ratio of 32
- Scenario B: 16 RNs each with a splitting ratio of 64
- Scenario C: 8 RNs each with a splitting ratio of 128

The calculations in this chapter are based on Figure 5-1 and the follwing asumptions.

The feeder fibre (FF), 100 km long, is connected to a number of $K$ RNs, where $K \in\{32,16$, $8\}$. Hence, there are a $(K+1)$ fibre segments $\left(L_{F S}\right)$. Each RN connects to a splitter using a distribution fibre (DF) with a length of 20 km each. The splitter is connected to $N$ ONUs, where, $N \in\{32,64,128\}$. In the calculations, presented in this chapter, the ONU is assumed to be at a distance of 1 km from the splitter, which is connected to (K/2) RN.


Figure 5-1 LR-PON

### 5.2 FIR for Network Components

FIR is an important parameter that is used to quantify the resistance of a component or a connection to a single failure [140]. There is an inverse relationship between FIR and the number of affected ONUs. So, as the value of FIR decreases, the number of affected ONUs increases [31]. The FIR for a network component can be calculated from the following equation [31]:

$$
\begin{equation*}
F I R_{\text {component }}=\frac{1}{(C A F * \text { Un component })} \tag{5-1}
\end{equation*}
$$

where CAF is the number of ONUs affected by a failure in the component, and $U n_{\text {component }}$ is the unavailability of that component [31]. The parameters used to find the FIR for the different compnents are presented in Table 5-1.

Table 5-1 FIR parameters

| Components | Unavailablity | Availablity | CAF |
| :---: | :---: | :---: | :---: |
| OLT | $5.12 \mathrm{E}-7$ | 0.999999488 | KxN |
| FF | $3.42 \mathrm{E}-4$ | 0.999658 | KxN |
| DF | $6.4 \mathrm{E}-5$ | 0.999936 | N |
| DDF | $3.42 \mathrm{E}-6$ | 0.99999658 | 1 |
| RN | $4.00 \mathrm{E}-7$ | 0.9999996 | N |
| Splitter | $1.00 \mathrm{E}-7$ | 0.9999999 | N |
| ONU | $5.12 \mathrm{E}-7$ | 0.999999488 | 1 |

Figure 5-2 shows the FIR of different network components for a ring-and-spur LR-PON. It should be noted that the FIR of the OLT, FF, ONU, and DDF have the same numerical value for the different scenarios. This is due to the fact that a fault in the OLT or FF will affect 1024 ONUs, while a fault in an ONU or DDF will affect only one ONU. FIR values for DF, splitter, and RN, however, affect different numbers of ONUs based on the splitting ratios. It is evident from Figure 5-2 that the FIR values of the ONU and DDF are the highest. This is due to the fact that only one ONU is affected by that fault. The ONU acquires a higher FIR value, compared to the DDF, due to its higher availability. The FIR values of the splitter and the RN are lower than those of ONU and DDF, as the number of users affected by a single fault in these components is $N \epsilon(32,64$, or 128$)$ for scenarios $\mathrm{A}, \mathrm{B}$, and C respectively. The ranking of the FIR of those components is based on their availability, where the splitter has higher availability than the RN. Although a fault in the OLT would affect 1024 ONUs and a fault in DF would affect $N$ ONUs, the FIR of the OLT is larger than that of DF because of its higher availability. It is evident also that the FIR of the FF is the lowest because of its low availability as well as the fact that a faulty FF would affect 1024 ONUs.


Figure 5-2 FIR for different network components in ring-and-spur

### 5.3 Protection Schemes for LR-PON

Based on the numerical FIR values, three protection schemes are proposed for LR-PON, i.e. Ring-DF protection, OLT-Ring-DF protection and OLT-Ring protection schemes.

Figure 5-3 presents the availability block diagrams (ABD) based on the ring-and-spur LRPON architecture, a sketch diagram that is similar in design to that reported in the literature, see Section 2.4.2. In the ABD above, blocks are used to represent components, and lines are used to represent connections between components.

Figure 5-3 (a) shows an OLT-Ring protection scheme. In this scheme, the protection of the ring is carried out by duplicating the RNs and using a spare FF fibre, where, two OSs are used to re-route the traffic from the faulty FF fibre to the spare one. The protection of the OLT is performed by duplicating the OLT. In case of a faulty primary OLT, the secondary OLT is activated to take control of the PON. The switching process is entirely within the OLT, hence, no need for a switching protocol [141].


Figure 5-3 ABD for the proposed protection schemes, (a) OLT-Ring protection, (b) OLT-Ring-DF protection, (c) Ring-DF protection

Figure 5-3 (b) shows an OLT-Ring-DF protection scheme. In this scheme, OSs are used to reroute traffic in case of a fault detected in the operational $\mathrm{FF}, \mathrm{DF}$ or both.

Figure 5-3 (c) shows a Ring-DF protection scheme. This type of protection is similar to Type D protection, an independent duplication of feeder and branch fibre that was discussed in ITU-T Recommendation G.983.1 [142]. This protection was proposed for a typical tree topology, where each distribution fibre was protected by a spare fibre, as well as each ONU being connected to an OS to switch between the faulty distribution fibre and the spare one [142, 143]. However, for Ring-DF protection based LR-PON, the DF connects RN and splitter together; hence, the duplication of the DF requires only two OSs.

### 5.3.1 Availability

The network availability is the collective availability of all the network components, and is calculated based on the assumption that the components are able to provide services and perform functions in a predetermined period of time without any failure [82]. The availability can be defined as [81]:

$$
\begin{equation*}
A=1-\left(\frac{M T T R}{M T B F}\right) \tag{5-2}
\end{equation*}
$$

Where, MTTR denotes the mean time to repair and MTBF denotes the mean time between failures. The unavailability is the "complement of availability" that can be derived as follows:

$$
\begin{equation*}
U n=M T T R \cdot F I T_{s} / 10^{9} \tag{5-3}
\end{equation*}
$$

Where, FIT denotes failure in time, with 1 FIT signifying a failure in $10^{9} \mathrm{~h}$. The values of MTTR and FIT for network components are found in [82].

The unavailability of non-protection, access protection, ring protection and fully protection schemes have been defined in [82]. Non-protection scheme provides a single path between the OLT and the ONUs. Access protection scheme duplicates the path between the RN and
the ONUs. Ring protection is based on duplicating the ring which includes the RNs. Full protection aims to duplicate each part in the LR-PON. The proposed protection schemes that include OLT-ring protection, ring-DF protection and OLT-ring-DF aim to duplicate the ring and the OLT, the ring and the distribution fibers, and the OLT, ring, and the distribution fibers, respectively. In the following equations, the duplicated parts of each protection scheme are squared, that is raised to power 2 . The abbreviation of the formulas (5-4) - (5-10) are given in Table 5-2.

The unavailability of non-protection, access protection, ring protection, fully protection, OLT-Ring protection, Ring-DF protection, and OLT-Ring-DF protection schemes, respectively, are defined as follow:

$$
\begin{align*}
& U n_{\text {nonProt. }}=U n_{O L T}+\frac{K}{2} \cdot L_{F S} \cdot U n_{F}+\frac{K}{2} \cdot U n_{R N}+L_{D F} \cdot U n_{F}+U n_{S}+L_{D D F} \cdot U n_{F}+U n_{O N U}  \tag{5-4}\\
& U n_{\text {Access Prot. }}=U n_{O L T}+\frac{K}{2} \cdot L_{F S} \cdot U n_{F}+\frac{K}{2} \cdot U n_{R N}+U n_{O S}  \tag{5-5}\\
& +\left[L_{D F} \cdot U n_{F}+U n_{S}+L_{D D F} \cdot U n_{F}+U n_{O N U}\right]^{2} \\
& \begin{aligned}
U n_{\text {Ring Prot. }}= & U n_{O L T}+U n_{O S}+\left[\frac{K}{2} \cdot L_{F S} \cdot U n_{F}+\frac{K}{2} \cdot U n_{R N}\right]^{2}+U n_{O S}+L_{D F} \cdot U n_{F}+U n_{S}+L_{D D F} \cdot U n_{F} \\
& +U n_{O N U}
\end{aligned}  \tag{5-6}\\
& U n_{\text {Fully Prot. }}=\left[U n_{O L T}\right]^{2}+U n_{O S}+\left[\frac{K}{2} \cdot L_{F S} \cdot U n_{F}+\frac{K}{2} \cdot U n_{R N}\right]^{2}+U n_{O S}+\left[L_{D F} \cdot U n_{F}+U n_{S}\right.  \tag{5-7}\\
& \left.+L_{D D F} \cdot U n_{F}+U n_{O N U}\right]^{2} \\
& U n_{O L T-\text { RingProt. }}=\left[U n_{O L T}\right]^{2}+U n_{O S}+\left[\frac{K}{2} \cdot L_{F S} \cdot U n_{F}+\frac{K}{2} \cdot U n_{R N}\right]^{2}+U n_{O S}+L_{D F} \cdot U n_{F}+U n_{S}  \tag{5-8}\\
& +L_{D D F} \cdot U n_{F}+U n_{O N U} \\
& \begin{aligned}
U n_{\text {Ring }-D F \text { Prot. }}= & U n_{O L T}+U n_{O S}+\left[\frac{K}{2} \cdot L_{F S} \cdot U n_{F}+\frac{K}{2} \cdot U n_{R N}\right]^{2}+U n_{O S}+\left[L_{D F} \cdot U n_{F}\right]^{2}+U n_{O S}+U n_{S} \\
& +L_{D D F} \cdot U n_{F}+U n_{O N U}
\end{aligned} \tag{5-9}
\end{align*}
$$

$U n_{\text {OLT-Ring-DF Prot. }}$

$$
\begin{align*}
& =\left[U n_{O L T}\right]^{2}+U n_{O S}+\left[\frac{K}{2} \cdot L_{F S} \cdot U n_{F}+\frac{K}{2} \cdot U n_{R N}\right]^{2}+U n_{O S}+\left[L_{D F} \cdot U n_{F}\right]^{2}+U n_{O S}  \tag{5-10}\\
& +U n_{S}+L_{D D F} \cdot U n_{F}+U n_{O N U}
\end{align*}
$$

Table 5-2 Abbrevation of unavailablity evaluation

| Abbreviations |  |
| :---: | :---: |
| $U n_{\text {non prot }}$. | Unavailability of non-protected scheme |
| $U n_{\text {Access prot. }}$ | Unavailability of access protected scheme |
| Un $n_{\text {Ring prot. }}$ | Unavailability of ring protected scheme |
| $U n_{\text {Fully Prot. }}$ | Unavailability of fully protected scheme |
| Un ${ }_{\text {OLT-Ring Prot }}$ | Unavailability of OLT and ring protected scheme |
| $U n_{\text {Ring-DF Prot. }}$ | Unavailability of ring and distribution fibers protected scheme |
| $U n_{\text {OLT-Ring-DF Prot. }}$ | Unavailability of OLT, ring and distribution protected scheme |
| $U n_{\text {OLt }}$. | Unavailability of OLT |
| $U n_{\text {F }}$. | Unavailability of the fiber |
| $U n_{R N}$ | Unavailability of the RN |
| $U n_{s}$ | Unavailability of the splitter |
| $U n_{\text {ONU }}$ | Unavailability of the ONU |
| $U n_{O S}$ | Unavailability of the optical switch |
| $\mathrm{L}_{\text {FS }}$ | Length of the fibre segments |
| $L_{\text {DF }}$ | Length of the distribution fiber |


| $L_{D D F}$ | Length of the ddrop fiber |
| :--- | :--- |

### 5.3.2 Cost

Similar to the availability, and in each equation of the protection scheme, the costs of the duplicated parts are doubled. The cost of non-protection, access protection, ring protection, fully protection, OLT-Ring protection, Ring-DF protection, and OLT-Ring-DF protection schemes, respectively, are defined similar to [82] as follows:

$$
\begin{align*}
& C_{\text {non Prot. }}=C_{O L T}+L_{F F}\left(C_{F}+C_{B}\right)+K . C_{R N}+K . C_{S}+K . L_{D F}\left(C_{F}+C_{B}\right)  \tag{5-11}\\
& +K . N\left[\left(L_{D D F}\left(C_{F}+C_{B}\right)+C_{O N U}\right)\right] \\
& \begin{aligned}
& C_{\text {Access Prot. }}=C_{O L T}+L_{F F}\left(C_{F}+C_{B}\right)+K . C_{R N}+K . C_{O S}+2 K . L_{D F}\left(C_{F}+C_{B}\right)+2 K . C_{S} \\
&+2 K . N\left[\left(L_{D D F}\left(C_{F}+C_{B}\right)+C_{O N U}\right)\right]
\end{aligned}  \tag{5-12}\\
& C_{\text {Ring Prot. }}=C_{O L T}+C_{O S}+2 L_{F F}\left(C_{F}+C_{B}\right)+2 K . C_{R N}+K . C_{O S}+K . L_{D F}\left(C_{F}+C_{B}\right)+K . C_{S}  \tag{5-13}\\
& +K . N\left[\left(L_{D D F}\left(C_{F}+C_{B}\right)+C_{O N U}\right)\right] \\
& C_{\text {Fully Prot. }}=2 C_{O L T}+C_{O S}+2 L_{F F}\left(C_{F}+C_{B}\right)+2 K . C_{R N}+2 K . C_{O S}+2 K .\left[L_{D F}\left(C_{F}+C_{B}\right)\right]  \tag{5-14}\\
& +2 K . C_{S}+2 K . N\left[\left(L_{D D F}\left(C_{F}+C_{B}\right)+C_{O N U}\right)\right] \\
& C_{\text {OLT-Ring Prot. }}=2 C_{O L T}+C_{O S}+2 L_{F F}\left(C_{F}+C_{B}\right)+2 K . C_{R N}+K . C_{O S}+K . L_{D F}\left(C_{F}+C_{B}\right)  \tag{5-15}\\
& +K . C_{S}+K . N\left[\left(L_{D D F}\left(C_{F}+C_{B}\right)+C_{O N U}\right)\right] \\
& C_{\text {Ring-DF Prot. }}=C_{O L T}+C_{O S}+2 L_{F F}\left(C_{F}+C_{B}\right)+2 K . C_{R N}+K . C_{O S}+2 K . L_{D F}\left(C_{F}+C_{B}\right)  \tag{5-16}\\
& +K . C_{O S}+K . C_{S}+K . N\left[\left(L_{D D F}\left(C_{F}+C_{B}\right)+C_{O N U}\right)\right] \\
& C_{\text {OLT-Ring-DF Prot. }}  \tag{5-17}\\
& \begin{array}{l}
=2 C_{O L T}+C_{O S}+2 L_{F F}\left(C_{F}+C_{B}\right)+2 K . C_{R N}+K . C_{O S}+2 K . L_{D F}\left(C_{F}+C_{B}\right) \\
+K . C_{O S}+K . C_{S}+K . N\left[\left(L_{D D F}\left(C_{F}+C_{B}\right)+C_{O N U}\right)\right]
\end{array}
\end{align*}
$$

The cost of the components can be found in [82, 144]. The abbreviation of the formulas $(5-11)-(5-17)$ are given in Table 5-3.

Table 5-3 Abbrevation of cost evaluation

| Abbreviations |  |
| :--- | :--- |
| $C_{\text {non prot. }}$ | Cost of non-protected scheme |


| $C_{\text {Access prot. }}$ | Cost of access protected scheme |
| :--- | :--- |
| $C_{\text {Ring prot. }}$ | Cost of ring protected scheme |
| $C_{\text {Fully Prot. }}$ | Cost of fully protected scheme |
| $C_{\text {OLT-Ring Prot. }}$ | Cost of OLT and ring protected scheme |
| $C_{\text {Ring-DF Prot. }}$ | Cost of ring and distribution fibers protected scheme |
| $C_{\text {OLT-Ring-DF Prot. }}$ | Cost of OLT, ring and distribution protected scheme |
| $C_{\text {OLT. }}$ | Cost of OLT |
| $C_{\text {F. }}$ | Cost of the fiber (/km) |
| $C_{B}$ | Cost of burying fiber (/km) |
| $C_{R N}$ | Cost of the RN |
| $C_{S}$ | Cost of the splitter |
| $C_{\text {ONU }}$ | Cost of the ONU |
| $C_{\text {OS }}$ | Cost of the optical switch |
| $L_{\text {PF }}$ | Length of the feeder fiber |

### 5.3.3Discussion

Figure 5-4 and Figure 5-5 represent statistical analysis and comparison of all protection schemes in terms of both cost and availability, respectively. The parameters used are presented in Table 5-4. It can be seen that the fully protected architecture is the most reliable scheme because of its high availability, as compared to other protection schemes. It is,
however, the most expensive one. The access protection scheme is an inefficient solution as it is very costly and with low availability, when compared to its counterparts. The cost of each of OLT-Ring-DF and Ring-DF protection schemes, which is very comparable as can be seen in Figure 5-5, is lower than that of the full and access protection schemes. The availability of OLT-Ring-DF and Ring-DF schemes is comparable, and is higher than that of access protection. The cost of ring protection and OLT-Ring protection are slightly higher than the corresponding techniques with no protection. Likewise, the availability of the schemes are higher than the corresponding techniques with no protection. Therefore, those schemes are the more satisfactory schemes.

The main contribution of this chapter is the selection of the optimal protection scheme. While [82] recommends ring only protection scheme as it achieves high availability (99.992), this research recommends the use of an OLT-Ring protection scheme that achieves a similar availability (99.993) at a similar cost becoming an improved protection scheme. This recommendation is based on the results of the FIR analysis and CAF values for the different components. In addition, the cost per user of the OLT-Ring protection scheme is minimal.

Table 5-4 Parameters used in the simulation of Cost and availability

| Components | FIT | MTTR | Unavaliablity | Cost <br> US \$ |
| :---: | :---: | :---: | :---: | :---: |
| OLT | 256 | 2 | $5.12 \mathrm{E}-7$ | 500 |
| Fibre (km) | 570 | 6 | $3.42 \mathrm{E}-6$ | 1 |
| Remote node | 200 | 2 | $4 \mathrm{E}-7$ | 600 |
| Splitter 1x32 | 50 | 2 | $1 \mathrm{E}-7$ | 13 |
| Splitter 1x64 | 50 | 2 | $1 \mathrm{E}-7$ | 37 |
| Splitter 1 x 128 | 50 | 2 | $1 \mathrm{E}-7$ | 190 |
| ONU | 256 | 2 | $5.12 \mathrm{E}-7$ | 150 |
| Burying Fibres (km) | - | - | - | 250 |



Figure 5-4 Availability of different protection schemes of LR-PON


Figure 5-5 Cost of different protection schemes of LR-PON

### 5.4 Summary

This chapter introduces three protection schemes for LR-PON. The performance of the protection schemes has been investigated in terms of both availability and cost, and then compared to the existing protection schemes. It has been shown that an OLT-Ring protection
scheme is a superior scheme for LR-PON.

## Chapter 6 NG-PON MONITORING

This chapter answers Research Question 3, "What advantages would be achieved when applying PON ranging into the monitoring layer". This chapter focuses on GPON network monitoring system. To answer this research question, the chapter starts with an explanation of the use of the GPON ranging process in the monitoring layer and the benefits gained in eliminating interference between upstream bursts from different ONUs. This is followed by an explanation of the proposed GPON monitoring system in a 1-D scenario. In addition, the chapter mathematically analyses the effect of the pulse width on the different noise sources in terms of SNR. Furthermore, there is an analysis of the effect of reducing the pulse width on reduced interference in terms of SIR.

### 6.1 GPON Ranging Process for the Monitoring Layer

The OLT measures the distance to each ONU and then calculates the differences in distance between two successive ONUs in order to assign an equalization delay for each ONU. The process is as follows.

The OLT measures the distance to $\mathrm{ONU}_{i}$ and $\mathrm{ONU}_{i+1}$ as follows: (see Figure 6-1)

$$
\begin{align*}
D_{O N U_{i}} & =2 D_{s}+2 D_{O N U_{i}}  \tag{6-1}\\
D_{O N U_{i+1}} & =2 D_{s}+2 D_{O N U_{i+1}} \tag{6-2}
\end{align*}
$$

where $D_{s}$ is the distance between the OLT and the splitter. $D_{\text {ONUi }}$ and $D_{\text {ONUi+1 }}$ are the distances between the splitter and $\mathrm{ONU}_{i}$, and $\mathrm{ONU}_{i+1}$, respectively. The OLT calculates the distance, $\left(D_{\Delta}\right)$, between $\mathrm{ONU}_{i+1}$ and $\mathrm{ONU}_{i}$ as follows:

$$
\begin{equation*}
D_{\Delta}=2\left(D_{O N U_{i+1}}-D_{O N U_{i}}\right) \tag{6-3}
\end{equation*}
$$

Based on this distance, the equalization time $\left(T_{\Delta}\right)$ can be calculated as follows:

$$
\begin{equation*}
T_{\Delta}=D_{\Delta} / v \tag{6-4}
\end{equation*}
$$

where $v$ is the velocity defined as:

$$
\begin{equation*}
\mathrm{v}=C / \eta \tag{6-5}
\end{equation*}
$$

where $C$ is the speed of the light and $\eta$ is the refractive index.

The OLT sends the $T_{\Delta}$ to each ONU to equalize its traffic with the other ONUs. In the case where each ONU suspends its traffic by a period of time that is equal to $T_{\Delta}$, the probability of interference at the optical combiner between upstream bursts will be eliminated as shown in Figure 6-1 (A: No interference). However, there is a probability of interference between bursts if an ONU suspends its upstream burst by a period of time that is less than $T_{\Delta}$, as shown in Figure 6-1 (B: interference).


Figure 6-1 Principle of upstream transmission using equalization delay

### 6.2 Principle of Monitoring System

The principle of the GPON monitoring system based 1-D scenario is shown in Figure 6-2. 1D system is based on tree topology where the OLT is located at the CO and connected to multiple ONUs over different distances. In this scenario, a pulse using one wavelength in the U-band is generated from the NMS and launched into the feeder fibre. The pulse is split into
$N$ sub-pulses, where $N \in(32,64$, or 128$)$, each of which travels through a unique fibre length. Encoders are placed outside the ONUs. Each encoder encodes the monitoring sub-signal by its unique code using EG-nMPC. Encoders are based on 1-D ODLs. Each ONU suspends the transmission of its upstream burst for a time that is equal to its assigned $T_{\Delta}$. All encoded subpulses from ONUs are combined at the RN by the combiner and then transmitted to the NMS. This monitoring system does not require implementing a decoder at the receiving side at NMS; instead, the NMS stores a reference signal that generates a time reference of the expected time of receiving each chip of each code. This time reference can be easily stored by knowing the code-word and the $T_{\Delta}$ of each encoder. The NMS works to compare the received chips of each code to the time reference. If a code for a specific ONU has been received correctly at the assigned time, the fiber status for that fiber will be healthy (case 1 in the Figure); otherwise the fiber will be detected as faulty (case 2 in the Figure).


Figure 6-2 Monitoring system (1-D)

### 6.3 Monitoring pulse width

The pulse width has a major impact on the performance of the monitoring system. In this
section, the effect of the different noise sources, including shot, beat, dark and thermal noises, on the pulse width in terms of SNR is evaluated. In addition, the effects of the pulse width on the performance of SNR and SIR for the different GPON splitting ratios are analysed.

The SNR and SIR are expressed as follows [33]:

$$
\begin{gather*}
S N R=\frac{E\left(\mu_{\text {sig }}^{2}\right)}{E\left(\alpha_{\text {Noises }}^{2}\right)}  \tag{6-6}\\
S I R=\frac{E\left(\mu_{\text {sig }}^{2}\right)}{E\left(\mu_{\text {int }}^{2}\right)} \tag{6-7}
\end{gather*}
$$

where, $E\left(\mu_{\text {sig }}^{2}\right)$ is the expectation of the desired signal power, $E\left(\alpha_{\text {Noises }}^{2}\right)$ is the expectation of all noises power, and $E\left(\mu_{\text {int }}^{2}\right)$ is the expectation of the interference signal power. $\mu_{\text {sig }}$ is derived as follows [33]:

$$
\begin{equation*}
\mu_{\text {sig }}=G \alpha_{T} \xi_{i} e^{-2 \alpha_{a} l_{i}} \sum_{n=1}^{w} \frac{1}{T_{c}} \int_{t \in T c}\left|Q_{i n}\left(t_{i}, \lambda\right)\right|^{2} d t \tag{6-8}
\end{equation*}
$$

where $G$ is the photodiode gain, $\alpha_{T}$ is the total loss budget, $\zeta_{i}$ is the fibre status, which is equal to " 1 " for a healthy fibre, and " 0 " otherwise. $\alpha_{a}$ is the fibre attenuation coefficient, $l i$ is the distance between OLT and $\mathrm{ONU}_{i}, w$ is the weight, $T_{c}$ is the pulse width, $Q_{i n}(t, \lambda)$ is the generated monitoring pulse with a width of $T_{c}$ and a power of $P_{s}$. Details of mathematical modelling of the encoding, decoding and the detection process can be found in [33]. This chapter focuses on selecting the optimal pulse width for the different GPON scenarios. Equation (6-8) can now be written as [33]:

$$
\begin{equation*}
\mu_{s i g}=G \alpha_{T} w P_{s} \xi_{i} e^{-2 \alpha_{\alpha_{i}}} \tag{6-9}
\end{equation*}
$$

$\mu_{\text {int }}$ is defined as:

$$
\begin{equation*}
\mu_{\mathrm{int}}=G \alpha_{T} \sum_{n=2}^{N} \rho P_{s} n e^{-2 \alpha_{a} l_{n}} \tag{6-10}
\end{equation*}
$$

where $N$ is the number of ONUs, $\rho$ is the interference probability given in (4-13), and $l_{n}$ is the distance between OLT and $\mathrm{ONU}_{N}$. The probability of interference between two transmission bursts can be eliminated by the use of a ranging process. The interference probability can be redefined as:

$$
\begin{equation*}
\rho=\lambda_{C} \cdot T_{\Delta} \tag{6-11}
\end{equation*}
$$

The noise sources $i_{\text {Noises }}$ in the system can be defined as:

$$
\begin{equation*}
i_{\text {Noises }}(t)=i_{\text {ShotN }}(t)+i_{\text {DarkN }}(t)+i_{\text {ThermalN }}(t)+i_{\text {BeatN }}(t) \tag{6-12}
\end{equation*}
$$

where $i_{\text {ShotN }}, i_{\text {DarkN }}, i_{\text {ThermalN }}$, and $i_{\text {BeatN, }}$, are shot noise, dark noise, thermal noise, and beat noise, respectively. The power of the noises $\left(\alpha_{\text {Noises }}^{2}\right)$ can be calculated by adding the power of all noise components. It is given as follows [33]:

$$
\begin{equation*}
\alpha_{\text {Noises }}^{2}=\alpha_{\text {ShotN }}^{2}+\alpha_{\text {DarkN }}^{2}+\alpha_{\text {ThermalN }}^{2}+\alpha_{\text {BeatN }}^{2} \tag{6-13}
\end{equation*}
$$

where the power of shot noise $\left(\alpha_{S h o t N}^{2}\right)$ is given by:

$$
\begin{equation*}
\alpha_{\text {ShotN }}^{2}=q G(1+\zeta)\left(\mu_{\text {sig }}+\mu_{\text {int }}\right) B_{e} \tag{6-14}
\end{equation*}
$$

where $q$ is the electron charge, $1+\varsigma$ is photodiode excess noise factor, and $B_{\mathrm{e}}$ is the electrical bandwidth. $B_{e}$ is defined as follows:

$$
\begin{equation*}
B_{e}=\frac{1}{T_{C}} \tag{6-15}
\end{equation*}
$$

The power of dark noise $\left(\alpha_{\text {Dark }}^{2}\right)$ is given by:

$$
\begin{equation*}
\alpha_{\text {DarkN }}^{2}=q G I_{\text {DarkN }} B_{e} \tag{6-16}
\end{equation*}
$$

where, $I_{\text {DarkN }}$ is the dark noise current.

The power of thermal noise $\left(\alpha_{\text {Thermaln }}^{2}\right)$ is given by:

$$
\begin{equation*}
\alpha_{\text {ThermalN }}^{2}=I_{\text {thermalN }} B_{e} \tag{6-17}
\end{equation*}
$$

where $I_{\text {ThermalN }}$ is the thermal noise current. The power of beat noise $\left(\alpha_{\text {BeatN }}^{2}\right)$ is given by:

$$
\begin{equation*}
\alpha_{\text {BeatN }}^{2}=2 \xi_{i} \cdot \beta(1+\varsigma)\left(\alpha_{T} G P_{s}\right)^{2} \cdot w e^{-2 \alpha_{a} l_{i}} \sum_{n=2}^{N} 2 \rho k e^{-2 \alpha_{a} l_{n}^{2}} \tag{6-18}
\end{equation*}
$$

where $\beta$ is defined as:

$$
\begin{equation*}
\beta=\frac{B_{e}}{B_{o}} \tag{6-19}
\end{equation*}
$$

where $B_{o}$ is the optical bandwidth.

### 6.4 Numerical results

For the different noise sources, plots of the SNR versus the pulse width for PON splitting ratios, 32, 64, and 128, are presented in Figure 6-3, Figure 6-4, and Figure 6-5 respectively. The parameters used for the MATLAB are presented in Table 6-1.

Figure 6-3 shows the effect of dark noise and thermal noise on the SNR for different pulse widths. The values of SNR are inversely proportional to pulse widths; as shown in (6-16) and (6-17). In addition, according to those equations, the code parameters chosen, i.e. weight, length, and available cardinality, are irrelevant, hence one figure to represent the three different splitting ratios.

Figure 6-4 presents the effect of the pulse width on the SNR for shot noise for splitting ratios
of 32,64 , and 128 . As can be seen, there is an inverse relationship between the shot noise and the pulse width as stated in (6-14) and (6-15). In this figure, the effect of the splitting ratios on the results can be seen. The shot noise is affected by $\mu_{\text {sig, }}$ which, in turn, is affected by the weight as well as $\mu_{\mathrm{int}}$, with the latter acquiring different values of interference probabilities.

Figure 6-5, illustrates the performance of the SNR versus different pulse widths for beat noise for splitting ratios of 32,64 , and 128 . There is a proportional relationship between pulse width and probability of interference, which leads to higher values of SNR, see (6-18) and (6-11). However, the SNR reaches a steady state value as the pulse width reaches $10^{-6}$ for 32 users, and $10^{-7}$ for 64 and 128 users. This is due to the inverse relationship between the beat noise and the pulse width as shown in (6-18), (6-19) and (6-15)

From the discussion, it can be seen that only the shot and beat noises have an impact on the SNR and SIR. Thus, the selected pulse width should consider these noises. From Figure 6-6, it is noted that the optimal pulse width for a splitting ratio of 32 , (a) in the figure, is $10^{-10}$ while the optimal pulse for splitting ratios of 64 and 128, (b) and (c) in the Figure, respectively, is $10^{-11}$.

Figure 6-7, shows the impact of different pulse widths on the SIR. Once again, the inverse relationship could be noted.


Figure 6-3 SNR versus pulse width for dark and thermal noises for all splitting ratios of 32, 64, and 128


Figure 6-4 SNR versus pulse width for shot noise for splitting ratios of 32, 64, and 128


Figure 6-5 SNR versus pulse width for beat noise for splitting ratios of 32, 64, and 128


Figure 6-6 Beat and Shot noises for different splitting ratios


Figure 6-7 SIR versus $T_{c}$, for $N=32$, 64 and 128
Table 6-1 Simulation parameters

| Parameter | Value |  | Parameter | Value |
| :---: | :---: | :---: | :---: | :---: |
| $N$ | Case 1 | 32 | $1+\varsigma$ | 2.97 |
|  | Case 2 | 64 | G | 100 |
|  | Case 3 | 128 | $\alpha_{a}$ | $0.3 \mathrm{~dB} / \mathrm{km}$ |
| W | Case 1 | 2 | $\alpha_{L}$ | 5 dB |
|  | Case 2 | 3 | $I_{\text {Therral } N}$ | 160 nA |
|  | Case 3 | 4 | $I_{\text {Dark N }}$ | $10^{26} \mathrm{~A}^{2} / \mathrm{Hz}$ |
| $L$ | Case 1 | 24 | $P_{s}$ | 4 dBm |
|  | Case 2 | 60 | $T_{C}$ | $T_{c}\left[10^{-12}: 10^{-6}\right]$ |
|  | Case 3 | 112 | $\lambda$ | 1650 nm |
| $B_{e}$ | $\begin{gathered} 1 / T_{C} \\ 1 \mathrm{THz} \end{gathered}$ |  | $L_{\text {FF }}$ | 20 km |
| $B_{o}$ |  |  | c | 299,792,458 m\s |

### 6.5 Summary

This chapter exhibits the significance of utilizing the information gathered from the GPON ranging process at the data layer in eliminating the interference between upstream bursts in the monitoring layer. In addition, it explains the overall system principle for a 1-D scenario. In addition, this chapter has analysed the effect of the pulse width in the system performance in terms of SNR and SIR using a mathematical model.

## Chapter 7 IMPLEMENTATION

This chapter provides an overview of VPI TransmissionMaker simulation tool including its hierarchy and parameters. It followed by implementation of 1-D EG-nMPC using VPI TransmissionMaker. For simplicity and readability, the chapter provides a description of a network simulation with four ONUs. The results for networks with capacities of 32, 64, and 128 splitting ratios are then presented based on the results obtained for 4 ONUs.

### 7.1 VPI TransmissionMAker overview

VPI transmissionMaker is one of the most robust tools to model and simulate photonic systems and networks such as microwave photonics application and optical fiber networks. It combines the advantages of a powerful graphical interface and a reliable simulation scheduler with flexibility in representing the optical signal. In addition, VPI provides advanced tools such as interactive simulation, data import with automatic file, and co-simulation utilizing some of the standard programming languages including Python and MATLAB [145].

VPI is hierarchically organized in a such way to permits the user to manage the modules easily. The hierarchy is divided into three levels and every level can be processed independently. These levels can be referred to as: universe, galaxy and star [146]. An example of the hierarchy is shown in Figure 7-1. As shown in the Figure, the star level is the lowest level of the simulation interface. It represents a single module with a certain function. The galaxy level represents the second level and it constructed of a combination of interconnected stars or other galaxies. This level requires at least one input and/or output port. The universe level is the highest level in the hierarchy and it may include stars and/or galaxies. The universe level shows the simulation scheme where the scheme can be executed [146].

The parameters of the modules of the VPI is divided into global and specific parameters. The global parameters impact all the modules while the specific parameter impact a specific
module [146].


Figure 7-1 VPI Hierarchy

### 7.2 Network Simulation with Four ONUs

The monitoring system simulation model consists of four main parts; monitoring signal generator, RN , encoding, and decoding and fault identification. The simulation model with four ONUs is shown in

Figure 7-2. The module parameters used in this simulation are presented in the Appendix, while the parmeters for the undefined modules are set to their default values.


Figure 7-2 VPI model of four ONUs

### 7.2.1Monitoring Signal Generator

The monitoring signal generator is based on an OOK transmitter, that is available in the VPItransmissionMaker simulator. The interior design of the OOK transmitter is shown in Figure 7-3. The modules used in this OOK transmitter include:
(1) Laser source (LaserCW). This is used to generate a continuous wave (CW) optical signal.
(2) Pseudo Random Binary Sequence generator (PRBS).: This module is used to produce a pseudo random data sequence.
(3) Coder driver OOK. This module is used to produce an electrical signal.
(4) $1 \times 2$ Fork. The function of this module is to divide the input into two equal output paths.
(5) Modulator Differential Mach-Zehnder (DiffMZ_DSM).
(6) Add logical channel (LogicAddChannel). This module is used to assign a logical channel to the signal.
(7) DC-Source. This module produces a constant-amplitude electrical signal at a defined value.
(8) Attenuator. This module attenuates the optical signal.
(9) Null sources. This model is used to terminate unused input.

The output of the monitroing signal generator is shown in Figure 7-4. As shown in the Figure, the monitoring signal has a duration of $T_{c}=1 \mathrm{~ns}$ and a power of $P_{s}=0.00251 \mathrm{~W}$, (4 dBm).

## Monitoring Signal Generator



Figure 7-3 VPI OOK transmitter design


Figure 7-4 Monitoring pulse generator output

### 7.2.2Remote Node Splitter

The design of the RN is presented in Figure 7-5 and consists of:
(1) $1 \mathrm{x} N$ power splitter. Splits the input power into $N$ equal output ports.
(2) $N \mathrm{x} 1$ power combiner. Combines the power coming from $N$ input ports into one output port.


Figure 7-5 VPI splitter and combiner

### 7.2.3Encoding

The encoder design is presented in Figure 7-6 and consists of:
(1) $1 \times N$ power splitter.
(2) $N$ ODLs, each with a specific delay time corresponding to the encoder's code-word.
(3) $N x 1$ power combiner.


Figure 7-6 VPI encoder design
The outputs of the four encoder showing the $w$ sub-pulses locations, in the interval $T=L . T_{c}$, are presented in Figure 7-7 to Figure 7-10. The time for each sub-pulse is presented in Table 7-1.


Figure 7-7 Encoder 1 output


Figure 7-8 Encoder 2 output


Figure 7-9 Encoder 3 output


Figure 7-10 Encoder 4 output
Table 7-1 Sub-pulse times for the four encoders

| Encoder | Sub-pulse (1) *Tc | Sub-pulse (2) *Tc |
| :---: | :---: | :---: |
| Encoder 1 | 10 | 21 |
| Encoder 2 | 7 | 18 |
| Encoder 3 | 8 | 19 |
| Encoder 4 | 9 | 20 |

In this simulation, the distance to the first ONU is set to be 500 m , it then increases by 5 m for each successive ONU. Therefore, according to (6-4), the $T_{\Delta}$ between any two successive ONUs is set to be equal to 50 ns . Since $T_{\Delta}$ is not assigned to the first ONU , a time that is equal to $T_{\Delta}$ has been assigned to $\mathrm{ONU}_{1}$. Thus, the expected start and end times for the four encoders and their corresponding sub-pulses times are presented in Table 7-2 (see Figure 7-11).

The encoders outputs with their assigned $T_{\Delta}$ are shown in Figure 7-12 to Figure 7-15. In addition, a scaling up of the time zones of concern has been conducted in order to obtain more precise values of the sub-pulse times. Results of this scaling up are shown in Figure 7-16 to Figure 7-19.


Figure 7-11 Start and end times

Table 7-2 Expected start/end times and sub-pulses times for the ONUs

| ONU | Start Time (ns) | End Time (ns) | Sub-pulse (1) | Sub-pulse (2) |
| :---: | :---: | :---: | :---: | :---: |
| ONU1 | 51 | 74 | 60 | 71 |
| ONU2 | 124 | 148 | 131 | 142 |
| ONU3 | 198 | 222 | 206 | 217 |
| ONU4 | 272 | 296 | 281 | 292 |



Figure 7-12 Encoder 1 output with delay


Figure 7-13 Encoder 2 output with delay


Figure 7-14 Encoder 3 output with delay


Figure 7-15 Encoder 4 output with delay


Figure 7-16 Encoder 1 output with delay closeup


Figure 7-17 Encoder 2 output with delay closeup


Figure 7-18 Encoder 3 output with delay closeup


Figure 7-19 Encoder 4 output with delay closeup

### 7.2.4Remote Node Combiner

Two signal analyzers are used to analyse the combined signal. The first is located before the combiner, where it collects all of the encoded signals and assigns a unique color to each of the signals for ease of identification. The second analyser is located after the combiner to show the encoded signal. The outputs of both analysers are shown in Figure 7-20 and Figure 7-21, respectively. It should be noted that, for the analyser before the combiner, the output of each encoder should be connected to the analyser in the order of their numbers, and in an ascending manner.


Figure 7-20 Analyser before combiner output


Figure 7-21 Combined signal

### 7.2.5Fibre Link

After the encoding process, the encoded signal is reflected back to the RN and then transmitted to the NMS. In VPItransmissionMaker, the fibre module available has one input port and one output port. To simulate this structure, a reflector and two fibres (with the same parameters) are used. The reflector is placed between the delay and the second fibre as shown Figure 7-22.


Figure 7-22 Fibre and reflection

### 7.2.6Decoding and Fault Identification

Fault identification is based on checking the presence/absence of the encoded signal corresponding to each ONU by comparing it to a reference signal that is stored in the NMS. The model of the reference signal and its output for four ONUs are shown in Figure 7-23 and Figure 7-24, respectively.


Figure 7-23 Reference signal


Figure 7-24 Reference signal output
When the encoded signal reaches the NMS, it passes through a photo-diode to convert the optical signal to an electrical signal, and then to a sampler and a thresholder. The NMS will
then compare the thresholder output to the reference signal output to determine the status of each fibre link. The fibre to an ONU is healthy if the output of the thresholder matches the output of the reference signal at the time assigned for that ONU, otherwise, it is unhealthy.

The output of the sampler and the thresholder for four ONUs are shown in Figure 7-25 and Figure 7-26, respectively. By comparing the reference signal to the thresholder, it can be noted that all signals are received correctly, indicating a healthy fibre link to the four ONUs.

To increase readability of the results, especially when number of ONUs is 32 or higher, the outputs of each of thresholder and the reference signal are exported to an Excel file, and then processed to produce a table that consists of the ID of each fibre and its corresponding status.


Figure 7-25 Sampler output


Figure 7-26 Thresholder output

A screenshot of VPI output in a text view for thresholder and reference signal and the exported data to Excel for thresholder and reference signal are presented in Figure 7-27 (a, b, and c , respectively). This figure shows power values corresponding to successive increments of 0.04 ns and 0.0017 ns in time for the reference signal and thresholder respectively. To show results at increments of 1 ns , the data has been filtered. The output of the filtered data for the reference signal and thresholder that shows the time assigned to sub-pulses corresponding to $\mathrm{ONU}_{1}$ is shown in Figure 7-28 and Figure 7-29, respectively. For more information see Table 7-2. The results of both sets of filtered data are compared to check the status for each fibre link. Comparison results are shown in Table 7-3.

| A thresholder |  | －回 | $\mathscr{C}$ | A reference |  | $\square$ 回 | x |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bands |  |  |  | Bands |  |  |  |
| Thresholder |  |  | ， | Input 1 |  |  | A |
|  | （ns） | Electrical Sign．．． |  | Time（ns） |  | Power（W） | 三 |
| 1 | 0 | 0 |  | 1 | 0 | 0 |  |
| 2 | 0.00173611111111111 | 0 |  | 2 | 0.0416666666666667 | 0 |  |
| 3 | 0.00347222222222222 | 0 |  | 3 | 0.0833333333333333 | 0 |  |
| 4 | 0.00520833333333333 | 0 |  | 4 | 0.125 | 0 |  |
| 5 | 0.00694444444444444 | 0 |  | 5 | 0.166666666666667 | 0 |  |
| 6 | 0.00868055555555556 | 0 |  | 6 | 0.208333333333333 | 0 |  |
| 7 | 0.0104166666666667 | 0 |  | 7 | 0.25 | 0 |  |
| 8 | 0.0121527777777778 | 0 |  | 8 | 0.291666666666667 | 0 |  |
| 9 | 0.0138888888888889 | 0 |  | 9 | 0.333333333333333 | 0 |  |
| 10 | 0.015625 | 0 |  | 10 | 0.375 | 0 |  |
| 11 | 0.0173611111111111 | 0 |  | 11 | 0.416666666666667 | 0 |  |
| 12 | 0.0190972222222222 | 0 |  | 12 | 0.458333333333333 | 0 |  |
| 13 | 0.0208333333333333 | 0 |  | 13 | 0.5 | 0 |  |
| 14 | 0.0225694444444444 | 0 |  | 14 | 0.541666666666667 | 0 |  |
| 15 | 0.0243055555555556 | 0 |  | 15 | 0.583333333333333 | 0 |  |
| 16 | 0.0260416666666667 | 0 |  | 16 | 0.625 | 0 |  |
| 17 | 0.0277777777777778 | 0 |  | 17 | 0.666666666666667 | 0 |  |
| 18 | 0.0295138888888889 | 0 |  | 18 | 0.708333333333333 | 0 |  |
| 19 | 0.03125 | 0 |  | 19 | 0.75 | 0 |  |
| 20 | 0.0329861111111111 | 0 |  | 20 | 0.791666666666667 | 0 |  |
| 21 | 0.0347222222222222 | 0 |  | 21 | 0.833333333333333 | 0 |  |
| 22 | 0.0364583333333333 | 0 |  | 22 | 0.875 | 0 |  |
| 23 | 0.0381944444444444 | 0 |  | 23 | 0.916666666666667 | 0 |  |
| 24 | 0.0399305555555556 | 0 |  | 24 | 0.958333333333333 | 0 |  |
| 25 | 0.0416666666666667 | 0 | － | 25 | 1 | 0 | － |

（a）Screenshot of VPI output（text view）for thresholder and reference signal

| 4 | ／／TRACE |  | Thresholder |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 | ／／SampledBand |  |  |  |
| 6 | \＃Time | Electrical Signal |  |  |
| 7 | \＃（ns） | （a．u．） |  |  |
| 8 | 0 | 0 |  |  |
| 9 | 0.001736 | 0 |  |  |
| 10 | 0.003472 | 0 |  |  |
| 11 | 0.005208 | 0 |  |  |
| 12 | 0.006944 | 0 |  |  |
| 13 | 0.008681 | 0 |  |  |
| 14 | 0.010417 | 0 |  |  |
| 15 | 0.012153 | 0 |  |  |
| 16 | 0.013889 | 0 |  |  |
| 17 | 0.015625 | 0 |  |  |
| 18 | 0.017361 | 0 |  |  |
| 19 | 0.019097 | 0 |  |  |
| 20 | 0.020833 | 0 |  |  |
| 21 | 0.022569 | 0 |  |  |
| 22 | 0.024306 | 0 |  |  |
| 23 | 0.026042 | 0 |  |  |
| 24 | 0.027778 | 0 |  |  |
| 25 | 0.029514 | 0 |  |  |
| 26 | 0.03125 | 0 |  |  |
| 27 | 0.032986 | 0 |  |  |
| 28 | 0.034722 | 0 |  |  |
| 29 | 0.036458 | 0 |  |  |
| 30 | 0.038194 | 0 |  |  |
| 31 | 0.039931 | 0 |  |  |
| 32 | 0.041667 | 0 |  |  |

（b）Thresholder exported data

| 4 | ／／TRACE |  |  |  |  |
| ---: | :--- | :--- | :--- | :--- | :--- |
| 5 | ／／SampledBand |  |  |  |  |
| 6 | \＃Time | Power | Reference Signal |  |  |
| 7 | \＃（ns） | （W） |  |  |  |
| 8 | 0 |  | 0 |  |  |
| 9 | 0.041667 | 0 |  |  |  |
| 10 | 0.083333 | 0 |  |  |  |
| 11 | 0.125 | 0 |  |  |  |
| 12 | 0.166667 | 0 |  |  |  |
| 13 | 0.208333 | 0 |  |  |  |
| 14 | 0.25 | 0 |  |  |  |
| 15 | 0.291667 | 0 |  |  |  |
| 16 | 0.333333 | 0 |  |  |  |
| 17 | 0.375 | 0 |  |  |  |
| 18 | 0.416667 | 0 |  |  |  |
| 19 | 0.458333 | 0 |  |  |  |
| 20 | 0.5 | 0 |  |  |  |
| 21 | 0.541667 | 0 |  |  |  |
| 22 | 0.583333 | 0 |  |  |  |
| 23 | 0.625 | 0 |  |  |  |
| 24 | 0.666667 | 0 |  |  |  |
| 25 | 0.708333 | 0 |  |  |  |
| 26 | 0.75 | 0 |  |  |  |
| 27 | 0.791667 | 0 |  |  |  |
| 28 | 0.833333 | 0 |  |  |  |
| 29 | 0.875 | 0 |  |  |  |
| 30 | 0.916667 | 0 |  |  |  |
| 31 | 0.958333 | 0 |  |  |  |
| 32 |  | 1 | 0 |  |  |
|  |  | 0 |  |  |  |

（c）Reference signal exported data

Figure 7－27 VPI analyser data example for thresholder and reference signal and the exported data in Excel


Figure 7-28 Filtered data for reference signal


Figure 7-29 Filtered data for the thresholder
Table 7-3 Fibre ID and status for four ONUs (healthy case)

| Fibre ID | Status |
| :---: | :---: |
| 1 | Healthy |
| 2 | Healthy |
| 3 | Healthy |
| 4 | Healthy |

### 7.2.8Fibre degradation

Fibre degradation can be simulated by assigning a high attenuation value to the fibre. The model of the fibre with degradation (a break) is presented in Figure 7-30. Each attenuator is assigned an attenuation value of 50 dB .

This section presents results for a break in the fibre to $\mathrm{ONU}_{1}$. Figure 7-31 shows the output of the encoder, the figure shows no sub-pulses. Figure 7-32 and Figure 7-33 show the output of the analyser before and after the combiner, respectively. It can be noted that, the signal of the encoder corresponding to $\mathrm{ONU}_{1}$ (red colour) is missing since there are no sub-pulses. Figure 7-34 and Figure 7-35 show the sampler and thresholder outputs, respectively, where the signal corresponding to $\mathrm{ONU}_{1}$ is missing. Table 7-4 presents the ID of each fibre and its corresponding status.

Fiber break


Figure 7-30 Fibre brake module in VPI


Figure 7-31 Encoder 1 output for a Fault


Figure 7-32 Encoded signals for a fibre fault to ONU1 before combiner


Figure 7-33 Encoded signals for a fibre fault to ONU1 after combiner


Figure 7-34 Sampler output for a fibre fault to ONU1


Figure 7-35 Thresholder output for a fibre fault to ONU1
Table 7-4 Fibre ID and status for four ONUs (faulty case)

| Fibre ID | Status |
| :---: | :---: |
| 1 | Faulty |
| 2 | Healthy |
| 3 | Healthy |
| 4 | Healthy |

### 7.3 A Splitting Ratio of 32

The general structure of a system with 32 ONUs is shown in Figure 7-36. The parameters used are presented in the Appendix.


Figure 7-36 VPI model of 32 ONUs

The output of the combiner, sampler, thresholder and the reference signal are shown in Figure $7-37$ to Figure $7-40$, respectively. From these figures, it can be noted that all signals are received correctly, indicating healthy fibre links to all ONUs as shown in Table 7-5. For a network with 32 ONUs and more, an amplifier has been used to reduce the effect of the noise.


Figure 7-37 Encoded combined signals


Figure 7-38 Sampler output


Figure 7-39 Thresholder output


Figure 7-40 Reference signal outputs
Table 7-5 Fibre ID and status (healthy case)

| Fibre ID | Status |
| :---: | :---: |
| 1 | Healthy |
| 2 | Healthy |
| 3 | Healthy |
| 4 | Healthy |
| 5 | Healthy |
| 6 | Healthy |
| 7 | Healthy |
| 8 | Healthy |
| 9 | Healthy |
| 10 | Healthy |
| 11 | Healthy |
| 12 | Healthy |
| 13 | Healthy |
| 14 | Healthy |
| 15 | Healthy |


| 16 | Healthy |
| :--- | :--- |
| 17 | Healthy |
| 18 | Healthy |
| 19 | Healthy |
| 20 | Healthy |
| 21 | Healthy |
| 22 | Healthy |
| 23 | Healthy |
| 24 | Healthy |
| 25 | Healthy |
| 26 | Healthy |
| 27 | Healthy |
| 28 | Healthy |
| 29 | Healthy |
| 30 | Healthy |
| 31 | Healthy |
| 32 | Healthy |

Figure 7-41 to Figure 7-43 show the output of the combiner, sampler and the thresholder, respectively, in the case of a fibre fault to $\mathrm{ONU}_{5}, \mathrm{ONU}_{16}$ and $\mathrm{ONU}_{25}$. From these figures, it can be noted that the signals corresponding to the ONUs are missing, indicating a fault in their fibre links as stated in Table 7-6.


Figure 7-41 Encoded combined signals, (faulty case)


Figure 7-42 Sampler output, (faulty case)


Figure 7-43 Thresholder output, (faulty case)

Table 7-6 Fibre ID and status, (Faulty case)

| Fibre ID | Status |
| :---: | :---: |
| 1 | Healthy |
| 2 | Healthy |
| 3 | Healthy |
| 4 | Healthy |
| 5 | Faulty |
| 6 | Healthy |
| 7 | Healthy |
| 8 | Healthy |
| 9 | Healthy |
| 10 | Healthy |
| 11 | Healthy |
| 12 | Healthy |
| 13 | Healthy |
| 14 | Healthy |


| 15 | Healthy |
| :---: | :---: |
| 16 | Faulty |
| 17 | Healthy |
| 18 | Healthy |
| 19 | Healthy |
| 20 | Healthy |
| 21 | Healthy |
| 22 | Healthy |
| 23 | Healthy |
| 24 | Healthy |
| 24 | Healthy |
| 25 | Faulty |
| 26 | Healthy |
| 27 | Healthy |
| 28 | Healthy |
| 29 | Healthy |
| 30 | Healthy |
| 31 | Healthy |
| 32 | Healthy |

### 7.4 A Splitting Ratio of 64

The general structure of a system with 64 ONUs is shown in Figure 7-44. The parameters used are presented in Appendix.


Figure 7-44 VPI model for 64 ONUs

In this scenario, the encoder structure is the same as described in (7.2.3) with an additional ODL. The output of the combiner, sampler, thresholder, and the reference signal are shown in Figure 7-45 to Figure 7-48, respectively. From those figures, it can be noted that all signals are received correctly, indicating a healthy fibre links to all ONUs as shown in Table 7-7


Figure 7-45 Combined signal output for 64 ONUs, (healthy case)


Figure 7-46 Sampler output for 64 ONUs, (healthy case)


Figure 7-47 Thresholder output for 64 ONUs, (healthy case)


Figure 7-48 Reference signal output for 64 ONUs, (healthy case)

Table 7-7 Fibre ID and status for 64 ONUs, (healthy case)

| Fibre ID | Status | Fibre ID | Status |
| :---: | :---: | :---: | :---: |
| 1 | Healthy | 33 | Healthy |
| 2 | Healthy | 34 | Healthy |
| 3 | Healthy | 35 | Healthy |
| 4 | Healthy | 36 | Healthy |
| 5 | Healthy | 37 | Healthy |
| 6 | Healthy | 38 | Healthy |
| 7 | Healthy | 39 | Healthy |
| 8 | Healthy | 40 | Healthy |
| 9 | Healthy | 41 | Healthy |
| 10 | Healthy | 42 | Healthy |
| 11 | Healthy | 43 | Healthy |
| 12 | Healthy | 44 | Healthy |
| 13 | Healthy | 45 | Healthy |
| 14 | Healthy | 46 | Healthy |
| 15 | Healthy | 47 | Healthy |
| 16 | Healthy | 48 | Healthy |
| 17 | Healthy | 49 | Healthy |
| 18 | Healthy | 50 | Healthy |
| 19 | Healthy | 51 | Healthy |
| 20 | Healthy | 52 | Healthy |
| 21 | Healthy | 53 | Healthy |
| 22 | Healthy | 54 | Healthy |
| 23 | Healthy | 55 | Healthy |
| 24 | Healthy | 56 | Healthy |
| 25 | Healthy | 57 | Healthy |
| 26 | Healthy | 58 | Healthy |
| 27 | Healthy | 59 | Healthy |
| 28 | Healthy | 60 | Healthy |
| 29 | Healthy | 61 | Healthy |
| 30 | Healthy | 62 | Healthy |
| 31 | Healthy | 63 | Healthy |
| 32 | Healthy | 64 | Healthy |

A scaling up of the time zones for the first three encoders is shown in Figure 7-49. The time corresponding to those encoder before and after delay is shown in Figure 7-50.


Figure 7-49 Closeup output of the Combiner, thresholder and reference signal of encoder 1, 2, and 3 with delay

| Encoder | Sub-pulses delay | Before Delay | After delay |
| :---: | :---: | :---: | :---: |
| 1 | Delay 1 | 11 | 61 |
|  | Delay 2 | 31 | 81 |
|  | Delay 3 | 50 | 100 |
| 2 | Delay 1 | 12 | 172 |
|  | Delay 2 | 32 | 192 |
|  | Delay 3 | 51 | 211 |
| 3 | Delay 1 | 13 | 283 |
|  | Delay 2 | 33 | 303 |
|  | Delay 3 | 52 | 322 |

Figure 7-50 Sub-pulses times before and after delay

Figure 7-51, Figure 7-52, and Figure 7-53 show the output of the combiner, sampler, and the thresholder, respectively, in case of a fault in the fibre to $\mathrm{ONU}_{16}, \mathrm{ONU}_{48}$ and $\mathrm{ONU}_{60}$. From these figures, it can be noted that the signals corresponding to the ONUs are missing, indicating a fault in their fibre links as stated in Table 7-8.


Figure 7-51 Combined signal output for 64 ONUs, (faulty case)


Figure 7-52 Sampler output for 64 ONUs, (faulty case)


Figure 7-53 Thresholder output for 64 ONUs, (faulty case)

Table 7-8 Fibre ID and status for 64 ONUs, (faulty case)

| Fibre ID | Status | Fibre ID | Status |
| :---: | :---: | :---: | :---: |
| 1 | Healthy | 33 | Healthy |
| 2 | Healthy | 34 | Healthy |
| 3 | Healthy | 35 | Healthy |
| 4 | Healthy | 36 | Healthy |
| 5 | Healthy | 37 | Healthy |
| 6 | Healthy | 38 | Healthy |
| 7 | Healthy | 39 | Healthy |
| 8 | Healthy | 40 | Healthy |
| 9 | Healthy | 41 | Healthy |
| 10 | Healthy | 42 | Healthy |
| 11 | Healthy | 43 | Healthy |
| 12 | Healthy | 44 | Healthy |
| 13 | Healthy | 45 | Healthy |
| 14 | Healthy | 46 | Healthy |
| 15 | Healthy | 47 | Healthy |
| 16 | Faulty | 48 | Faulty |
| 17 | Healthy | 49 | Healthy |
| 18 | Healthy | 50 | Healthy |
| 19 | Healthy | 51 | Healthy |
| 20 | Healthy | 52 | Healthy |
| 21 | Healthy | 53 | Healthy |
| 22 | Healthy | 54 | Healthy |
| 23 | Healthy | 55 | Healthy |
| 24 | Healthy | 56 | Healthy |
| 25 | Healthy | 57 | Healthy |
| 26 | Healthy | 58 | Healthy |
| 27 | Healthy | 59 | Healthy |
| 28 | Healthy | 60 | Faulty |
| 29 | Healthy | 61 | Healthy |
| 30 | Healthy | 62 | Healthy |
| 31 | Healthy | 63 | Healthy |
| 32 | Healthy | 64 | Healthy |

### 7.5 A Splitting Ratio of 128

The general structure of a system with 128 ONUs is shown in Figure 7-54. The parameters used are presented in the Appendix.

Due to the large amount of data exported from the analyser, and the limited number of rows permitted in Microsoft Excel, the output of the comparison for the healthy and fault cases in this scenario is displayed for 64 ONUs only. However, Figure 7-55 and Figure 7-56, show the output of the combined signal for 128 for healthy and fault cases, where faults detected in links to $\mathrm{ONU}_{12}, \mathrm{ONU}_{48}, \mathrm{ONU}_{96}$, and $\mathrm{ONU}_{128}$, respectively. Figure 7-57 shows the output of the reference signal for 128.


Figure 7-54 VPI model for 128 ONUs


Figure 7-55 Combined signal output for 128 ONUs (healthy case)


Figure 7-56 Combined signal output for 128 ONUs (faulty case)


Figure 7-57 Reference signal output for 128 ONUs
The output of the combiner, sampler, thresholder, and the reference signal for 64 ONUs out of 128 are shown in Figure 7-58 to Figure 7-61, respectively. Figure 7-62, Figure 7-63, and Figure 7-64, show the output of the combiner, sampler, and the thresholder, respectively, in case of a fault in $\mathrm{ONU}_{12}$ and $\mathrm{ONU}_{48}$. In addition, a scaling up of the time zones for the first
three encoders is shown in Figure 7-65, where their corresponding times before and after delay is shown in Figure 7-66. The outputs of the comparison for the healthy and fault cases for 64 ONUs out of 128 are presented in Table 7-9 and

Table 7-10, respectively.


Figure 7-58 Combined signal output for 64 ONUs out of 128, (healthy case)


Figure 7-59 Sampler output for 64 ONUs out of 128, (healthy case)


Figure 7-60 Thresholder output for 64 ONUs out of 128, (healthy case)


Figure 7-61 Reference signal output for 64 ONUs out of 128


Figure 7-62 Combined signal output for 64 ONUs out of 128, (faulty case)


Figure 7-63 Sampler output for 64 ONUs out of 128, (healthy case)


Figure 7-64 Thresholder output for 64 ONUs out of 128, (healthy case)


Figure 7-65 Closeup output of the Combiner, thresholder and reference signal of encoder 1, 2, and 3 with delay

| Encoder | Sub-pulses delay | Before delay | After delay |
| :---: | :---: | :---: | :---: |
| 1 | Delay1 | 16 | 66 |
|  | Delay2 | 43 | 93 |
|  | Delay3 | 72 | 122 |
|  | Delay4 | 99 | 149 |
| 2 | Delay1 | 17 | 229 |
|  | Delay2 | 44 | 256 |
|  | Delay3 | 73 | 447 |
|  | Delay4 | 100 | 312 |
|  | Delay1 | 18 | 392 |
|  | Delay2 | 45 | 419 |
|  | Delay3 | 74 | 448 |
|  | Delay4 | 101 | 475 |

Figure 7-66 Sub-pulse delay times after and before delay for encoder1, 2, and 3

Table 7-9 Fibre ID and status for 64 ONUs out of $\mathbf{1 2 8}$ (healthy case)

| Fibre ID | Status | Fibre ID | Status | Fibre ID | Status | Fibre ID | Status |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Healthy | 17 | Healthy | 33 | Healthy | 49 | Healthy |
| 2 | Healthy | 18 | Healthy | 34 | Healthy | 50 | Healthy |
| 3 | Healthy | 19 | Healthy | 35 | Healthy | 51 | Healthy |
| 4 | Healthy | 20 | Healthy | 36 | Healthy | 52 | Healthy |
| 5 | Healthy | 21 | Healthy | 37 | Healthy | 53 | Healthy |
| 6 | Healthy | 22 | Healthy | 38 | Healthy | 54 | Healthy |
| 7 | Healthy | 23 | Healthy | 39 | Healthy | 55 | Healthy |
| 8 | Healthy | 24 | Healthy | 40 | Healthy | 56 | Healthy |
| 9 | Healthy | 25 | Healthy | 41 | Healthy | 57 | Healthy |
| 10 | Healthy | 26 | Healthy | 42 | Healthy | 58 | Healthy |
| 11 | Healthy | 27 | Healthy | 43 | Healthy | 59 | Healthy |
| 12 | Healthy | 28 | Healthy | 44 | Healthy | 60 | Healthy |
| 13 | Healthy | 29 | Healthy | 45 | Healthy | 61 | Healthy |
| 14 | Healthy | 30 | Healthy | 46 | Healthy | 62 | Healthy |
| 15 | Healthy | 31 | Healthy | 47 | Healthy | 63 | Healthy |
| 16 | Healthy | 32 | Healthy | 48 | Healthy | 64 | Healthy |

Table 7-10 Fibre ID and status for $\mathbf{6 4}$ ONUs out of $\mathbf{1 2 8}$ (faulty case)

| Fibre ID | Status | Fibre ID | Status | Fibre ID | Status | Fibre ID | Status |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Healthy | 17 | Healthy | 33 | Healthy | 49 | Healthy |
| 2 | Healthy | 18 | Healthy | 34 | Healthy | 50 | Healthy |
| 3 | Healthy | 19 | Healthy | 35 | Healthy | 51 | Healthy |
| 4 | Healthy | 20 | Healthy | 36 | Healthy | 52 | Healthy |
| 5 | Healthy | 21 | Healthy | 37 | Healthy | 53 | Healthy |
| 6 | Healthy | 22 | Healthy | 38 | Healthy | 54 | Healthy |
| 7 | Healthy | 23 | Healthy | 39 | Healthy | 55 | Healthy |
| 8 | Healthy | 24 | Healthy | 40 | Healthy | 56 | Healthy |
| 9 | Healthy | 25 | Healthy | 41 | Healthy | 57 | Healthy |
| 10 | Healthy | 26 | Healthy | 42 | Healthy | 58 | Healthy |
| 11 | Healthy | 27 | Healthy | 43 | Healthy | 59 | Healthy |
| 12 | Faulty | 28 | Healthy | 44 | Healthy | 60 | Healthy |
| 13 | Healthy | 29 | Healthy | 45 | Healthy | 61 | Healthy |
| 14 | Healthy | 30 | Healthy | 46 | Healthy | 62 | Healthy |
| 15 | Healthy | 31 | Healthy | 47 | Healthy | 63 | Healthy |
| 16 | Healthy | 32 | Healthy | 48 | Faulty | 64 | Healthy |

### 7.6 Discussion

The demonstration of the monitoring system using 1-D time encoding has shown the
capability of the EG-nMPC to accommodate the maximum number of the different splitting ratios of GPON. In addition, the demonstration showed that the use of information gathered from the ranging process can lead to improvement in the performance by easily distinguishing between the codes coming from the multiple ONUs. For fault identification, the received signal of a particular code is compared to a reference signal in order to make a decision about the status of the link. If the signal for a practicular code and the reference signal simultaneously match, then the thresholder returns a peak at the time assigned for that particular code, indicating a healthy link. Otherwise, the link is faulty. In addition, and by utilizing the option of exporting the data into the Excel file, and processing the same using VBA, this chapter provides a readable information about each link, where the healthy link is marked in green and a faulty one in red.

### 7.7 Summary

This chapter has provided a demonstration of the monitoring system implementation of 1-D EG-nMPC using VPItransmissionMaker. For simplicity and readability, the chapter provided a description of a network simulation with four ONUs. The simulation results for networks with capacities of 32,64 , and 128 splitting ratios were presented based on the results obtained for 4 ONUs.

## Chapter 8 CONCLUSION AND FUTURE WORK

### 8.1 Conclusion

The research has been successfully completed and the research questions identified in Chapter 1 have been answered. The research identified a new and novel monitoring approach for NG-PON and demonstrated the viability of this approach rigorously. The research has resulted in a number of peer reviewed publications.

With the high capacity and wide coverage of NG-PON there is a need to ensure that high network reliability and improved timely fault management can be achieved. Research into monitoring systems and technologies is a key aspect of current research. Applying protection schemes can help in fast traffic restoration when a fault occurs, however, there is a need to identify the location of the fault and eliminate its causes. Therefore, several studies have been proposed in literature for monitoring NG-PONs. It is evident that the features provided by an OC based monitoring technique, such as improved performance, cost-effectiveness, and simplicity, would make it a promising candidate for a future monitoring system. The key requirement of monitoring based OC is the ability of the spreading code to provide a cardinality that is compatible with the number of ONUs under consideration. The main aim of this research was, therefore, to design a spreading code that is able to accommodate the typical PON splitting ratios.

To achieve the research objectives defined in Chapter 1, a new prime code for OCDMA systems has been proposed. In addition, the FIR parameter was used as a criterion to determine the optimal protection scheme. The key achievements of this research are summarised as:

- The code proposed in Chapter 4, has the capability to accommodate the different PON splitting ratios with a smaller prime number when compared to other prime code approaches. In addition, the proposed code requires a lower code weight and shorter code
length, leading to a reduction in energy consumption and processing time. In Chapter 4, the construction of the code and its parameters are detailed, as well as the evaluation of its performance using OOK-OCDMA and PPM-OCDMA modulation formats. The proposed code exhibited an improved performance, compared to other codes, when used with an increasing number of channels for both modulation techniques.
- The three protection schemes for ring-and-spur LR-PON presented in Chapter 5 are proposed based on evaluating the FIR parameter for the different network components. The performance of the proposed protection schemes, in terms of availability and cost, are evaluated and compared to the existing protection schemes. It has been shown that the OLT-ring protection scheme, suggested by this research, provides higher availability with a minimal increase in the cost, when compared to ring only protection.
- The significance of utilizing the PON ranging process information, carried out by the OLT at the data layer, to eliminate the interference between the upstream burst at the monitoring layer, is shown in Chapter 6. In addition, Chapter 6 presents a mathematical evaluation of the influence of the different pulse widths on the system SNR and SIR.
- The monitoring system implementation, using the code proposed in Chapter 4, and the $T_{\Delta}$ introduced in Chapter 6, are shown in Chapter 7. Chapter 7 presents the details for the design of the system components used in VPItransmissionMaker.

To conclude, the implementation of the proposed code, EG-nMPC, offers better performance than other prime codes when it comes to monitoring PON using an OC based technique. The code reduces the hardware complexity by reducing the code weight required, and consequently the number of ODLs used in encoders and decoders, leading to a reduction in cost and power consumption. In addition, to support the GPON splitting ratios, the proposed code requires shorter code lengths, leading to a reduction in processing time.

### 8.2 Future work

Research presented in this thesis can be built upon to further improve the performance of the monitoring system. Potential future work is discussed in the following sections.

### 8.2.1 Hybridization of OTDR and OC for LR-PON

As has been shown in Chapter 5, the FF is the most critical part in the network due to its very low availability and high CAF. Therefore, it requires a highly reliable monitoring system. In addition, OTDR has been recommended by the ITU-T for monitoring PON due to its ability to immediately locate faults and analyse the cause. Thus, hybridization of OTDR and OC can be investigated, where OTDR could be used to monitor FF , while an OC technique can be implemented to monitor the drop fibre links.

### 8.2.2Constructing a 2-D coding using EG-nMPC as one of its dimensions

Although, the proposed 1-D code resulted in a dramatic increase in code cardinality, this code still has the limitation that is needs to be adapted for NG-PON where number of ONUs exceeds 1024. It is important to explore the possibility of constructing a 2-D code, that is based on EG-nMPC as one of the dimensions, if a larger number of ONUs is to be accommodated. Similar to any 2-D coding scheme, this approach requires ONUs and OLT to implement a 2-D encoder and decoder, respectively. It should be noted that, the cardinality of 2-D codes is upper bounded by its parameters. Hence, an additional RN could require changing the coding scheme if the total number of ONUs, after the addition of more ONUs, exceeds that of the available cardinality of the chosen code.

### 8.2.3Implementing 1D/2D coding in LR-PON

1D/2D coding is a valuable area for further research to provide flexibility when one or more remote nodes are added to the ring. If it is required to add another monitoring wavelength to support the RN then it is appropriate that a broader and more flexible approach be
investigated. However, the potential variability in ring-and-spur PON means that additional components such as add/drop filters at the RNs may be needed. In addition, the number of available monitoring wavelengths is dependent on the monitoring band.

## BIBLIOGRAPHY

[1] S.-J. Park, C.-H. Lee, K.-T. Jeong, H.-J. Park, J.-G. Ahn, and K.-H. Song, "Fiber-to-the-home services based on wavelength-division-multiplexing passive optical network," Journal of Lightwave Technology, vol. 22, p. 2582, 2004.
[2] G. Kramer, B. Mukherjee, and G. Pesavento, "Ethernet PON (ePON): Design and analysis of an optical access network," Photonic Network Communications, vol. 3, pp. 307-319, 2001.
[3] S. E. Minzer, "Broadband ISDN and asynchronous transfer mode (ATM)," IEEE Communications Magazine, vol. 27, pp. 17-24, 1989.
[4] T. Muciaccia, F. Gargano, and V. Passaro, "Passive Optical Access Networks: State of the Art and Future Evolution," in Photonics, 2014, pp. 323-346.
[5] T. Pfeiffer, "An Introduction to PON Technologies," IEEE Communications Magazine, p. S18, 2007.
[6] S.-L. Lee, C.-H. Sun, and K.-C. Feng, "Hybrid passive optical networking architecture and techniques for constructing green broadband access networks," 2014.
[7] R. Alvizu, A. Arcia, M. Hernández, M. Huerta, and I. Tafur Monroy, "Hybrid WDMXDM PON Architectures for Future Proof Access Networks," International Journal On Advances in Systems and Measurements, vol. 5, pp. 139-153, 2012.
[8] S. Bindhaiq, A. S. M. Supa, N. Zulkifli, A. B. Mohammad, R. Q. Shaddad, M. A. Elmagzoub, et al., "Recent development on time and wavelength-division multiplexed passive optical network (TWDM-PON) for next-generation passive optical network stage 2 (NG-PON2)," Optical Switching and Networking, vol. 15, pp. 53-66, 2015.
[9] E. Wong, "Next-generation broadband access networks and technologies," Lightwave Technology, Journal of, vol. 30, pp. 597-608, 2012.
[10] M. Hernandez, A. Arcia, R. Alvizu, and M. Huerta, "A review of XDMA-WDM-PON for Next Generation Optical Access Networks," in Global Information Infrastructure and Networking Symposium (GIIS), 2012, 2012, pp. 1-6.
[11] M. A. Esmail and H. Fathallah, "Physical Layer Monitoring Techniques for TDMPassive Optical Networks: A Survey," Communications Surveys \& Tutorials, IEEE, vol. 15, pp. 943-958, 2013.
[12] A. M. Ragheb and H. Fathallah, "Performance analysis of next generation-PON (NGPON) architectures," in High Capacity Optical Networks and Enabling Technologies (HONET), 2011, 2011, pp. 339-345.
[13] R. Shaddad, A. Mohammad, S. Al-Gailani, A. Al-hetar, and M. Elmagzoub, "A survey on access technologies for broadband optical and wireless networks," Journal of Network and Computer Applications, vol. 41, pp. 459-472, 2014.
[14] I. N. Cano, X. Escayola, A. Peralta, V. Polo, M. C. Santos, and J. Prat, "A study of flexible bandwidth allocation in statistical OFDM-based PON," in Transparent Optical Networks (ICTON), 2013 15th International Conference on, 2013, pp. 1-4.
[15] N. Cvijetic, "OFDM for next-generation optical access networks," Lightwave Technology, Journal of, vol. 30, pp. 384-398, 2012.
[16] N. Cvijetic, Q. Dayou, and H. Junqiang, "100 Gb/s optical access based on optical orthogonal frequency-division multiplexing," Communications Magazine, IEEE, vol. 48, pp. 70-77, 2010.
[17] N. Kataoka, N. Wada, W. Xu, G. Cincotti, and K. Kitayama, "10Gbps-Class, bandwidth-symmetric, OCDM-PON system using hybrid multi-port and SSFBG en/decoder," in Optical Network Design and Modeling (ONDM), 2010 14th Conference on, 2010, pp. 1-4.
[18] D. Gutierrez, K. S. Kim, S. Rotolo, F.-T. An, and L. G. Kazovsky, "FTTH standards, deployments and research issues," Photonics and Networking Research Lab., Proceedings of JCIS ‘05, pp. 1358-1361, 2005.
[19] Y. Luo, X. Zhou, F. Effenberger, X. Yan, G. Peng, Y. Qian, et al., "Time-and wavelength-division multiplexed passive optical network (TWDM-PON) for nextgeneration PON stage 2 (NG-PON2)," Journal of Lightwave Technology, vol. 31, pp. 587-593, 2013.
[20] Y. Luo, X. Yan, and F. Effenberger, "Next generation passive optical network offering $40 \mathrm{~Gb} / \mathrm{s}$ or more bandwidth," in Communications and Photonics Conference (ACP), 2012 Asia, 2012, pp. 1-3.
[21] J. Prat, J. Lázaro, P. Chanclou, R. Soila, P. Velanas, A. Teixeira, et al., "Passive optical network for long-reach scalable and resilient access," in Telecommunications, 2009. ConTEL 2009. 10th International Conference on, 2009, pp. 271-275.
[22] J. Prat, J. A. Lázaro, K. Kanonakis, and I. Tomkos, "New FTTH Architectures for NG-PON-2," in Access Networks and In-house Communications, 2010, p. ATuA4.
[23] M. Cen, J. Chen, V. Moeyaert, P. Mégret, and M. Wuilpart, "Full monitoring for long-reach TWDM passive optical networks," Optics express, vol. 24, pp. 1578215797, 2016.
[24] J. Prat, V. Polo, B. Schrenk, J. A. Lazaro, F. Bonada, E. T. Lopez, et al., "Demonstration and field trial of a resilient hybrid NG-PON test-bed," Optical Fiber Technology, vol. 20, pp. 537-546, 2014.
[25] S. Jindal and N. Gupta, "OCDMA: Study and Future Aspects," in Recent Development in Wireless Sensor and Ad-hoc Networks, ed: Springer, 2015, pp. 125167.
[26] V. Baby, D. Rand, C.-S. Bres, L. Xu, I. Glesk, and P. R. Prucnal, "Incoherent optical CDMA systems," 2005.
[27] H. Yin and D. J. Richardson, Optical Code Division Multiple Access Communication Networks:Theory and Applications, 1 ed., 2009.
[28] C.-S. Bres, I. Glesk, and P. R. Prucnal, "Demonstration of an eight-user 115-Gchip/s incoherent OCDMA system using supercontinuum generation and optical time gating," IEEE photonics technology letters, vol. 18, pp. 889-891, 2006.
[29] N. F. Naim, M. S. Ab-Rahman, H. A. Bakarman, and A. A. A. Bakar, "Real-time monitoring in passive optical networks using a superluminescent LED with uniform and phase-shifted fiber Bragg gratings," Journal of Optical Communications and Networking, vol. 5, pp. 1425-1430, 2013.
[30] H. Ghafouri-Shiraz and M. M. Karbassian, Optical CDMA networks: principles, analysis and applications vol. 38: John Wiley \& Sons, 2012.
[31] A. Dixit, B. Lannoo, D. Colle, M. Pickavet, C. Jiajia, and M. Mahloo, "Efficient protection schemes for hybrid WDM/TDM Passive Optical Networks," in Communications (ICC), 2012 IEEE International Conference on, 2012, pp. 62206224.
[32] "Optical fiber cable network maintenance", ITU-T L.25, ed.
[33] M. Zhu, J. Zhang, D. Wang, and X. Sun, "Optimal Fiber Link Fault Decision for Optical 2D Coding-Monitoring Scheme in Passive Optical Networks," Journal of Optical Communications and Networking, vol. 8, pp. 137-147, 2016.
[34] "Optical fibre cable maintenance criteria for in-service fibre testing in access networks", ITU-T Recommendation L.66, ed.
[35] R. F. Moghaddam, Y. Lemieux, and M. Cheriet, "40 Gbps Access for Metro networks: Implications in terms of Sustainability and Innovation from an LCA Perspective," arXiv preprint arXiv:1504.06262, 2015.
[36] M. M. Rad, H. A. Fathallah, and L. A. Rusch, "Fiber fault PON monitoring using optical coding: effects of customer geographic distribution," IEEE transactions on communications, vol. 58, 2010.
[37] R. Yadav and G. Kaur, "Design and performance analysis of 1D, 2D and 3D prime sequence code family for optical CDMA network," Journal of Optics, pp. 1-14, 2016.
[38] "Passive Optical Network Protection Considerations", ITU-T G. Sup 51, ed.
[39] P. Begovic, N. Hadziahmetovic, and D. Raca, "10G EPON vs. XG-PON1 efficiency," in Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), 2011 3rd International Congress on, 2011, pp. 1-9.
[40] "10-Gigabit-capable passive optical networks (XG-PON): Physical media dependent (PMD) layer specification", ITU-T G.987.2.
[41] D. Van Veen, D. Suvakovic, L. Man Fai, H. Krimmel, A. J. de Lind van Wijngaarden, J. Galaro, et al., "Demonstration of a symmetrical $10 / 10 \mathrm{Gbit} / \mathrm{s}$ XG-PON2 system," in Optical Fiber Communication Conference and Exposition (OFC/NFOEC), 2011 and the National Fiber Optic Engineers Conference, 2011, pp. 1-3.
[42] J.-i. Kani, "Enabling technologies for future scalable and flexible WDM-PON and WDM/TDM-PON systems," IEEE Journal of Selected Topics in Quantum Electronics, vol. 16, pp. 1290-1297, 2010.
[43] D. Nesset, "NG-PON2 technology and standards," Journal of Lightwave Technology, vol. 33, pp. 1136-1143, 2015.
[44] A. Banerjee, Y. Park, F. Clarke, H. Song, S. Yang, G. Kramer, et al., "Wavelength-division-multiplexed passive optical network (WDM-PON) technologies for broadband access: a review [Invited]," Journal of optical networking, vol. 4, pp. 737758, 2005.
[45] D. Gutierrez, J. Cho, and L. G. Kazovsky, "TDM-PON security issues: Upstream encryption is needed," in Optical Fiber Communication and the National Fiber Optic Engineers Conference, 2007. OFC/NFOEC 2007. Conference on, 2007, pp. 1-3.
[46] N. Sotiropoulos, T. Koonen, and H. de Waardt, "Advanced Differential Modulation Formats for Optical Access Networks," Lightwave Technology, Journal of, vol. 31, pp. 2829-2843, 2013.
[47] D. van Veen, D. Suvakovic, H. Chow, V. Houtsma, E. Harstead, P. J. Winzer, et al., "Options for TDM PON beyond 10G," in Access Networks and In-house Communications, 2012, p. AW2A. 1.
[48] "40-Gigabit-capable passive optical networks 2 (NG-PON2): Physical media dependent (PMD) layer specification", ITU-T G989.2, 2015., ed.
[49] S. Yoshima, M. Noda, E. Igawa, S. Shirai, K. Ishii, M. Nogami, et al., "Recent progress of high-speed burst-mode transceiver technologies for TDM-PON systems," in Wireless and Optical Communications Conference (WOCC), 2012 21st Annual, 2012, pp. 59-62.
[50] A. Srivastava, "Next generation PON evolution," in SPIE OPTO, 2013, pp. 864509-864509-15.
[51] R. Urata, C. Lam, H. Liu, and C. Johnson, "High performance, low cost, colorless ONU for WDM-PON," in Optical Fiber Communication Conference and Exposition (OFC/NFOEC), 2012 and the National Fiber Optic Engineers Conference, 2012, pp. 1-3.
[52] S. Biswas and S. Adak, "OFDMA-PON: High Speed PON Access System," International Journal of Soft Computing, vol. 1.
[53] J. Armstrong, "OFDM for optical communications," Journal of Lightwave Technology, vol. 27, pp. 189-204, 2009.
[54] R. Matsumoto, T. Kodama, S. Shimizu, R. Nomura, K. Omichi, N. Wada, et al.,
"40G-OCDMA-PON System With an Asymmetric Structure Using a Single MultiPort and Sampled SSFBG Encoder/Decoders," Journal of Lightwave Technology, vol. 32, pp. 1132-1143, 2014.
[55] L. G. Kazovsky, N. Cheng, W.-T. Shaw, D. Gutierrez, and S.-W. Wong, Broadband optical access networks: John Wiley \& Sons, 2011.
[56] R. Murano, "Optical component technology options for NGPON2 systems," in Optical Fiber Communication Conference, 2014, p. M3I. 1.
[57] N. Cheng, J. Gao, C. Xu, B. Gao, D. Liu, L. Wang, et al., "Flexible TWDM PON system with pluggable optical transceiver modules," Optics express, vol. 22, pp. 2078-2091, 2014.
[58] G. Kramer, M. De Andrade, R. Roy, and P. Chowdhury, "Evolution of Optical Access Networks: Architectures and Capacity Upgrades," Proceedings of the IEEE, vol. 100, pp. 1188-1196, 2012.
[59] R. Bonk, W. Poehlmann, H. Schmuck, and T. Pfeiffer, "Cross-talk in TWDM-PON beyond NG-PON2," in Optical Fiber Communication Conference, 2015, p. Tu3E. 2.
[60] L. Han Hyub, L. Jong Hyun, L. Sang Soo, R. Hee Yeal, Y. Hark, and K. YoonKoo, "Investigation of ONU power leveling method for mitigating inter-channel crosstalk in TWDM-PONs," in Optical Internet 2014 (COIN), 2014 12th International Conference on, 2014, pp. 1-2.
[61] A. Buttaboni, M. De Andrade, and M. Tornatore, "New and Improved approaches for Dynamic Bandwidth and Wavelength allocation in LR WDM/TDM PON."
[62] M. I. Dias, D. P. Van, L. Valcarenghi, and E. Wong, "Energy-efficient dynamic wavelength and bandwidth allocation algorithm for TWDM-PONs with tunable VCSEL ONUs," 2014.
[63] M. Dias, D. P. Van, L. Valcarenghi, and E. Wong, "Energy-Efficient Framework for Time and Wavelength Division Multiplexed Passive Optical Networks," Journal of Optical Communications and Networking, vol. 7, pp. 496-504, 2015.
[64] J. Segarra, V. Sales, and J. Prat, "OLT design approach for resilient extended PON with OBS dynamic bandwidth allocation sharing the OLT optical resources," in Transparent Optical Networks, 2008. ICTON 2008. 10th Anniversary International Conference on, 2008, pp. 139-144.
[65] L. Hui-Tang, H. Zhong-Huan, C. Hung-Chen, and C. Wang-Rong, "SPON: A slotted long-reach PON architecture for supporting internetworking capability," in Military Communications Conference, 2009. MILCOM 2009. IEEE, 2009, pp. 1-8.
[66] L. Meng, C. M. Assi, M. Maier, and A. R. Dhaini, "Resource management in STARGATE-based Ethernet passive optical networks (SG-EPONs)," Journal of Optical Communications and Networking, vol. 1, pp. 279-293, 2009.
[67] L. Yi, Z. Li, M. Bi, W. Wei, and W. Hu, "Symmetric 40-Gb/s TWDM-PON With 39dB Power Budget," IEEE Photonics Technology Letters, vol. 25, pp. 644-647, 2013.
[68] Z. Li, L. Yi, and W. Hu, "Symmetric 40-Gb/s TWDM-PON with $51-\mathrm{dB}$ loss budget by using a single SOA as preamplifier, booster and format converter in ONU," Optics express, vol. 22, pp. 24398-24404, 2014.
[69] M. Bi, S. Xiao, L. Yi, H. He, J. Li, X. Yang, et al., "Power budget improvement of symmetric $40-\mathrm{Gb} / \mathrm{s}$ DML-based TWDM-PON system," Optics express, vol. 22, pp. 6925-6933, 2014.
[70] Y. Guo, S. Zhu, G. Kuang, Y. Yin, Y. Gao, D. Zhang, et al., "Demonstration of 10G burst-mode DML and EDC in symmetric 40Gbit/s TWDM-PON over 40km passive reach," in Optical Fiber Communications Conference and Exhibition (OFC), 2014, 2014, pp. 1-3.
[71] Z. Zhou, M. Bi, S. Xiao, Y. Zhang, and W. Hu, "Experimental Demonstration of

Symmetric 100-Gb/s DML-Based TWDM-PON System," Photonics Technology Letters, IEEE, vol. 27, pp. 470-473, 2015.
[72] L. Chengjun, G. Wei, W. Wei, and H. Weisheng, "A novel TWDM-PON architecture with control channel," in Optical Internet 2014 (COIN), 2014 12th International Conference on, 2014, pp. 1-2.
[73] I. M. Mohamed and M. S. B. Ab-Rahman, "Options and challenges in next-generation optical access networks (NG-OANs)," Optik-International Journal for Light and Electron Optics, vol. 126, pp. 131-138, 2015.
[74] H. Song, B.-W. Kim, and B. Mukherjee, "Long-reach optical access networks: A survey of research challenges, demonstrations, and bandwidth assignment mechanisms," Communications Surveys \& Tutorials, IEEE, vol. 12, pp. 112-123, 2010.
[75] D. Lavery and S. J. Savory, "Digital coherent technology for long-reach optical access," in Optical Fiber Communications Conference and Exhibition (OFC), 2014, 2014, pp. 1-3.
[76] B. Skubic, J. Chen, J. Ahmed, B. Chen, L. Wosinska, and B. Mukherjee, "Dynamic bandwidth allocation for long-reach PON: overcoming performance degradation," Communications Magazine, IEEE, vol. 48, pp. 100-108, 2010.
[77] M. De Andrade, M. Maier, M. P. McGarry, and M. Reisslein, "Passive optical network (PON) supported networking," Optical Switching and Networking, 2014.
[78] "Gigabit-capable passive optical networks (G-PON):) General characteristics", ITUT G.984.1, 2008, ed.
[79] M. S. Ab-Rahman, S. A. C. Aziz, and K. Jumari, "Protection route Mechanism for Survivability in FTTH-PON
Network " International Journal of Computer Science and Network Security (IJCSNS), vol. 8, 2008.
[80] C.-Y. Chuang, C.-C. Wei, J.-J. Liu, H.-Y. Wu, H.-M. Nguyen, C.-W. Wang, et al., "A High Loss Budget 400-Gbps WDM-OFDM Long-Reach PON over 60 km Transmission by 10G-class EAM and PIN without In-line or Pre-Amplifier," in Optical Fiber Communication Conference, 2017, p. W1K. 3.
[81] M. A. Esmail and H. Fathallah, "Fiber fault management and protection solution for ring-and-spur WDM/TDM long-reach PON," in Global Telecommunications Conference (GLOBECOM 2011), 2011 IEEE, 2011, pp. 1-5.
[82] M. A. Esmail and H. Fathallah, "Optical coding for next-generation survivable longreach passive optical networks," Optical Communications and Networking, IEEE/OSA Journal of, vol. 4, pp. 1062-1074, 2012.
[83] S. McGettrick, F. Slyne, N. Kitsuwan, D. B. Payne, and M. Ruffini, "Experimental End-to-End Demonstration of Shared N:1 Dual Homed Protection in Long Reach PON and SDN-Controlled Core," in Optical Fiber Communication Conference, Los Angeles, California, 2015, p. Tu2E.5.
[84] M. M. Rad, K. Fouli, H. A. Fathallah, L. A. Rusch, and M. Maier, "Passive optical network monitoring: challenges and requirements," Communications Magazine, IEEE, vol. 49, pp. s45-S52, 2011.
[85] T. Zhao, H. Han, J. Zhang, X. Liu, X. Chang, A. Wang, et al., "Precise Fault Location in TDM-PON by Utilizing Chaotic Laser Subject to Optical Feedback," IEEE Photonics Journal, vol. 7, pp. 1-9, 2015.
[86] H. Fathallah and M. Esmail, "Performance evaluation of special optical coding techniques appropriate for physical layer monitoring of access and metro optical networks," Photonic Network Communications, vol. 30, pp. 223-233, 2015.
[87] D. Pastor, K. Jamshidi, and C.-A. Bunge, "Physical layer monitoring based on 2D-

OCDMA concepts and electronic decoding for high density PON networks," in 2015 17th International Conference on Transparent Optical Networks (ICTON), 2015, pp. 1-4.
[88] F. Selmanovic and E. Skaljo, "GPON in Telecommunication Network," in Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), 2010 International Congress on, 2010, pp. 1012-1016.
[89] V. Eržen and B. Batagelj, "NG-PON1: technology presentation, implementation in practice and coexistence with the GPON system."
[90] S. Dahlfort, "Comparison of 10 Gbit/s PON vs WDM-PON," in Optical Communication, 2009. ECOC'09. 35th European Conference on, 2009, pp. 1-2.
[91] J. Segarra, V. Sales, and J. Prat, "GPON scheduling disciplines under multi-service bursty traffic and long-reach approach," in 2010 12th International Conference on Transparent Optical Networks, 2010, pp. 1-6.
[92] I. Cale, A. Salihovic, and M. Ivekovic, "Gigabit passive optical network-GPON," in Information Technology Interfaces, 2007. ITI 2007. 29th International Conference on, 2007, pp. 679-684.
[93] O. Marmur and E. Shraga, "GPON: the next big thing in optical access networks," in Asia-Pacific Optical and Wireless Communications, 2004, pp. 199-209.
[94] "Gigabit-capable passive optical networks (G-PON): Transmission convergence layer specification", ITU-T G.984.3, 2014, ed.
[95] G. Kramer, "The Problem of Upstream Traffic Synchronization in Passive Optical Networks"," 1999.
[96] M. Radivojević and P. Matavulj, "PON Evolution," in The Emerging WDM EPON, ed: Springer, 2017, pp. 67-99.
[97] X.-Z. Qiu, P. Ossieur, J. Bauwelinck, Y. Yi, D. Verhulst, J. Vandewege, et al., "Development of GPON upstream physical-media-dependent prototypes," Journal of Lightwave Technology, vol. 22, p. 2498, 2004.
[98] G. Keiser, FTTX concepts and applications vol. 91: John Wiley \& Sons, 2006.
[99] K. Fouli and M. Maier, "OCDMA and Optical Coding: Principles, Applications, and Challenges [Topics in Optical Communications]," Communications Magazine, IEEE, vol. 45, pp. 27-34, 2007.
[100] A. Sayed, L. Jolly, and P. Khot, "Analysis of effect of MAI on an OCDMA system," in International Conference and Workshop on Emerging Trends in Technology, 2014, p. 39.
[101] R. C. S. CHAUHAN, "DESIGN OF UNIPOLAR (OPTICAL) ORTHOGONAL CODES AND THEIR MAXIMAL CLIQUE SETS," UTTAR PRADESH TECHNICAL UNIVERSITY, 2015.
[102] S. Han, "Optical CDMA with Optical Orthogonal Code," Multiuser Wireless Communication (EE381K) Class Project, Fall, 2002.
[103] H. Abbas and M. Gregory, "OCDM network implementation of 1-D OOC and passive correlation receiver," in ICAIET 2014, 2014, pp. 311-318.
[104] L. R. Chen, "Technologies for hybrid wavelength/time optical CDMA transmission," in Electrical and Computer Engineering, 2001. Canadian Conference on, 2001, pp. 435-440.
[105] M. E. Marhic, "Trends in optical CDMA," Proc. Multigigabit Fiber Communication (SPIE), vol. 1787, pp. 80-98, 1992.
[106] I. Andonovic, L. Tancevski, M. Shabeer, and L. Bazgaloski, "Incoherent all-optical code recognition with balanced detection," Journal of lightwave technology, vol. 12, pp. 1073-1080, 1994.
[107] P. Prucnal, M. Santoro, and T. Fan, "Spread spectrum fiber-optic local area network
using optical processing," Journal of Lightwave Technology, vol. 4, pp. 547-554, 1986.
[108] F. Liu, M. Karbassian, and H. Ghafouri-Shiraz, "Novel family of prime codes for synchronous optical CDMA," Optical and quantum electronics, vol. 39, pp. 79-90, 2007.
[109] J. F. A. Rida, A. Bhardwaj, and A. Jaiswal, "Design optimization of optical communication systems using carbon nanotubes (CNTs) based on optical code division multiple access (OCDMA)," International Journal of Computer Science and Network Security (IJCSNS), vol. 14, p. 102, 2014.
[110] H. M. Shalaby, "Cochannel interference reduction in optical PPM-CDMA systems," IEEE transactions on communications, vol. 46, pp. 799-805, 1998.
[111] M. A. Esmail and H. Fathallah, "Current and next-generation passive optical networks monitoring solution," in 8th International Conference on High-capacity Optical Networks and Emerging Technologies, 2011, pp. 334-338.
[112] H. Fathallah and L. A. Rusch, "Code-division multiplexing for in-service out-of-band monitoring of live FTTH-PONs," Journal of Optical Networking, vol. 6, pp. 819-829, 2007.
[113] M. Thollabandi, X. Cheng, and Y.-K. Yeo, "Encoded probing technique for detection of the faulty branch in TDM-PON," IEEE Photonics Technology Letters, vol. 24, pp. 1610-1613, 2012.
[114] H. Fathallah, M. M. Rad, and L. A. Rusch, "PON monitoring: periodic encoders with low capital and operational cost," IEEE Photon. Technol. Lett, vol. 20, pp. 2039-2041, 2008.
[115] M. M. Rad, H. A. Fathallah, M. Maier, L. A. Rusch, and M. Uysal, "A novel pulsepositioned coding scheme for fiber fault monitoring of a PON," IEEE communications letters, vol. 15, pp. 1007-1009, 2011.
[116] M. A. Esmail and H. Fathallah, "Novel coding for PON fault identification," IEEE communications letters, vol. 15, pp. 677-679, 2011.
[117] M. P. Fernández, P. A. C. Caso, and L. A. B. Rossini, "Design and performance evalauation of an optical coding scheme for PON monitoring," in Information Processing and Control (RPIC), 2015 XVI Workshop on, 2015, pp. 1-6.
[118] X. Zhou, F. Zhang, and X. Sun, "Centralized PON monitoring scheme based on optical coding," IEEE Photonics Technology Letters, vol. 25, pp. 795-797, 2013.
[119] X. Zhang, S. Chen, F. Lu, X. Zhao, M. Zhu, and X. Sun, "Remote Coding Scheme Using Cascaded Encoder for PON Monitoring," IEEE Photonics Technology Letters, vol. 28, pp. 2183-2186, 2016.
[120] M. M. Rad, H. Fathallah, and L. A. Rusch, "Beat Noise Mitigation via Hybrid 1-D/2-D-OCDM: Application to Monitoring of High Capacity PONs," in Optical Fiber Communication Conference, 2008, p. OMR7.
[121] T. Tsujioka and H. Yamamoto, "Design of optical orthogonal codes with variable chip rate for flexible optical CDMA systems," in Advanced Communication Technology (ICACT), 2012 14th International Conference on, 2012, pp. 1156-1161.
[122] J. A. Salehi, "Code division multiple-access techniques in optical fiber networks. I. Fundamental principles," IEEE transactions on communications, vol. 37, pp. 824-833, 1989.
[123] B. Skubic, D. Betou, E. In, T. Ayhan, and S. Dahlfort, "Energy-efficient nextgeneration optical access networks," Communications Magazine, IEEE, vol. 50, pp. 122-127, 2012.
[124] S. De Lausnay, L. De Strycker, J.-P. Goemaere, N. Stevens, and B. Nauwelaers, "Optical CDMA codes for an indoor localization system using VLC," in Optical

Wireless Communications (IWOW), 2014 3rd International Workshop in, 2014, pp. 50-54.
[125] J. O. Anaman and S. Prince, "Correlation properties and performance evaluation of 1dimensional OOC's for OCDMA," in Devices, Circuits and Systems (ICDCS), 2012 International Conference on, 2012, pp. 167-171.
[126] G.-C. Yang and W. C. Kwong, "Performance analysis of optical CDMA with prime codes," Electronics Letters, vol. 31, pp. 569-570, 1995.
[127] W. C. Kwong, P. A. Perrier, and P. R. Prucnal, "Performance comparison of asynchronous and synchronous code-division multiple-access techniques for fiberoptic local area networks," IEEE transactions on communications, vol. 39, pp. 16251634, 1991.
[128] M. H. Zoualfaghari, "Co-channel Interference Reduction in
Optical Code Division Multiple Access Systems," School of electronic, electrical and computer engineering, University of Birmingham, 2015.
[129] "Prime Codes with Applications to CDMA Wireless and Optical Networks.(Book Review)(Brief Article)," vol. 26, ed, 2002, p. 188.
[130] M. M. Karbassian and H. Ghafouri-Shiraz, "Capacity enhancement in synchronous optical overlapping PPM-CDMA network by a novel spreading code," in Global Telecommunications Conference, 2007. GLOBECOM'07. IEEE, 2007, pp. 2407-2411.
[131] M. Y. Liu and H. W. Tsao, "Cochannel interference cancellation via employing a reference correlator for synchronous optical CDMA systems," Microwave and Optical Technology Letters, vol. 25, pp. 390-392, 2000.
[132] M. M. Karbassian and H. Ghafouri-Shiraz, "Fresh prime codes evaluation for synchronous PPM and OPPM signaling for optical CDMA networks," Journal of Lightwave Technology, vol. 25, pp. 1422-1430, 2007.
[133] M. M. Karbassian and F. Kueppers, "OCDMA code utilization increase: capacity and spectral efficiency enrichment," in Global Telecommunications Conference (GLOBECOM 2010), 2010 IEEE, 2010, pp. 1-5.
[134] M. M. Karbassian and F. Küppers, "Enhancing spectral efficiency and capacity in synchronous OCDMA by transposed-MPC," Optical Switching and Networking, vol. 9, pp. 130-137, 2012.
[135] M. M. Karbassian and F. Kueppers, "Synchronous optical CDMA networks capacity increase using transposed modified prime codes," Journal of Lightwave Technology, vol. 28, pp. 2603-2610, 2010.
[136] Q. Jin, M. M. Karbassian, and H. Ghafouri-Shiraz, "Energy-Efficient High-Capacity Optical CDMA Networks by Low-Weight Large Code-Set MPC," Lightwave Technology, Journal of, vol. 30, pp. 2876-2883, 2012.
[137] I. Garrett, "Towards the fundamental limits of optical-fiber communications," Journal of Lightwave Technology, vol. 1, pp. 131-138, 1983.
[138] I. Kaminow and T. Koch, Optical Fiber Telecommunications III, 1997.
[139] E. Wong and K.-L. Lee, "Automatic protection, restoration, and survivability of longreach passive optical networks," in Communications (ICC), 2011 IEEE International Conference on, 2011, pp. 1-6.
[140] J. Chen, "Reducing the impact of failures in Next-Generation Optical Access Networks," in Asia Communications and Photonics Conference, 2012.
[141] F. J. Effenberger, H. Ichibangase, and H. Yamashita, "Advances in broadband passive optical networking technologies," IEEE Communications Magazine, vol. 39, pp. 118124, 2001.
[142] T. Koonen, "Fiber to the Home/Fiber to the Premises: What, Where, and When?," Proceedings of the IEEE, vol. 94, pp. 911-934, 2006.
[143] N. Ghazisaidi, M. Scheutzow, and M. Maier, "Survivability Analysis of NextGeneration Passive Optical Networks and Fiber-Wireless Access Networks," IEEE Transactions on Reliability, vol. 60, pp. 479-492, 2011.
[144] (11/08/2017). Available: https://www.cozlink.com
[145] Available: http://www.vpiphotonics.com/Tools/OpticalSystems
[146] E. H. Miguel, "Fiber-based Orthogonal Frequency Division Multiplexing Transmission Systems," 2010.

## Appendix A1. Simulation of Four ONUs

## A1.1. Simulation parameters of four ONUs



|  |  | Length | 515 m |
| :---: | :---: | :---: | :---: |
|  |  | GroupRefractiveIndex | 1.47 |
| Monitoring Pulse generator | General | EmissionFrequency | 184.5 e 12 |
|  |  | ModulationType | NRZ |
|  | PRBS | PRBS_Type | CodeWord |
|  |  | CodeWord | 10 [295] |
| Encoding | Encoder 1 | SignalDelay 1 | $10 \times T_{c}$ |
|  |  | SignalDelay 2 | $21 \times T_{c}$ |
|  | $T_{\Delta 1}$ | SignalDelay | $1 \mathrm{x} T_{\Delta}$ |
|  | Encoder 2 | SignalDelay 1 | $7 \times T_{c}$ |
|  |  | SignalDelay 2 | $18 \times T_{c}$ |
|  | $T_{\Delta 2}$ | SignalDelay | $2 \mathrm{x} T_{\Delta}+T_{\text {TDM }}$ |
|  | Encoder 3 | SignalDelay 1 | $8 \times T_{c}$ |
|  |  | SignalDelay 2 | $19 \times T_{c}$ |
|  | $T_{\Delta 3}$ | SignalDelay | $3 \mathrm{x} T_{\Delta}+2 \mathrm{x} T_{\text {TDM }}$ |
|  | Encoder 4 | SignalDelay 1 | $9 \times T_{c}$ |
|  |  | SignalDelay 2 | $20 \times T_{c}$ |
|  | $T_{\Delta 4}$ | SignalDelay | $4 \mathrm{x} T_{\Delta}+3 \mathrm{x} T_{\text {TDM }}$ |
| Reference signal | PRBS | CodeWord | Td c1 Td c2 Td c3 Td c4 |

## Appendix A2. A Splitting Ratio of 32

## A2.1. Simulation parameters of $\mathbf{3 2}$ ONUs

| Category | Module | Parameter | Value |
| :---: | :---: | :---: | :---: |
| Global |  | Time window | $32 \mathrm{x}(50+L) /$ BitRateDefault |
|  |  | BitRateDefault | 1 e 9 |
|  |  | SampleModeCentreFrequency | 184.5 e 12 |
|  |  | Code-length ( $L$ ) | 24 |
|  |  | Time-Chip ( $T_{\text {c }}$ ) | 1 ns |
|  |  | $T_{\Delta}$ | 50 ns |
|  |  | $T_{\text {TDM }}$ | 24 ns |


|  |  | Burst-Duration ( $T_{b}$ ) | L. $T_{c} \mathrm{~ns}$ |
| :---: | :---: | :---: | :---: |
|  |  | w | 0 [50] |
|  |  | C1 | 0 [6] 10 [16] 1 |
|  |  | C2 | 0 [7] 10 [10] 10 [5] |
|  |  | C3 | 0 [8] 10 [10] 10 [4] |
|  |  | C4 | 0 [9] 10 [10] 10 [3] |
|  |  | C5 | 0 [10] 10 [10] 10 [2] |
|  |  | C6 | 0 [11] 10 [10] 10 [1] |
|  |  | C7 | 10 [11] 10 [11] |
|  |  | C8 | 0 [1] 10 [11] 10 [10] |
|  |  | C9 | 0 [2] 10 [11] 10 [9] |
|  |  | C10 | 0 [3] 10 [11] 10 [8] |
|  |  | C11 | 0 [4] 10 [11] 10 [7] |
|  |  | C12 | 0 [5] 10 [11] 10 [6] |
|  |  | C13 | 0 [7] 10 [11] 10 [4] |
|  |  | C14 | 0 [8] 10 [11] 10 [3] |
|  |  | C15 | 0 [9] 10 [11] 10 [2] |
|  |  | C16 | 0 [10] 10 [11] 10 |
|  |  | C17 | 0 [11] 10 [11] 1 |
|  |  | C18 | 0 [6] 10 [11] 10 [5] |
|  |  | C19 | 10 [13] 10 [9] |
|  |  | C20 | 0 [1] 10 [13] 10 [8] |
|  |  | C21 | 0 [2] 10 [13] 10 [7] |
|  |  | C22 | 0 [3] 10 [13] 10 [6] |
|  |  | C23 | 0 [4] 10 [7] 10 [11] |
|  |  | C24 | 0 [5] 10 [7] 10 [10] |
|  |  | C25 | 0 [8] 10 [9] 10 [5] |
|  |  | C26 | 0 [9] 10 [9] 10 [4] |
|  |  | C27 | 0 [10] 10 [9] 10 [3] |
|  |  | C28 | 0 [11] 10 [9] 10 [2] |
|  |  | C29 | 0 [6] 10 [15] 10 [1] |
|  |  | C30 | 0 [7] 10 [15] 1 |
|  |  | C31 | 10 [12] 10 [10] |
|  |  | C32 | 010 [12] 10 [9] |
| Fibers | Feeder Fiber | ReferenceFrequency | 184.5 e 12 |
|  |  | Length | 10 km |
|  |  | GroupRefractiveIndex | 1.47 |
|  | Drop fibers | ReferenceFrequency | 184.5 e 12 |
|  |  | Length | $\mathrm{L} 1=500 \mathrm{~m}$ <br> Increases by $5 \mathrm{~m}-$ $\mathrm{L} 32=655 \mathrm{~m}$ |



|  |  | Signal Delay 2 | 15 |
| :---: | :---: | :---: | :---: |
|  | Encoder 19 | Signal Delay1 | 2 |
|  |  | Signal Delay 2 | 16 |
|  | Encoder 20 | Signal Delay1 | 3 |
|  |  | Signal Delay 2 | 17 |
|  | Encoder 21 | Signal Delay1 | 4 |
|  |  | Signal Delay 2 | 12 |
|  | Encoder 22 | Signal Delay1 | 5 |
|  |  | Signal Delay 2 | 13 |
|  | Encoder 23 | Signal Delay1 | 8 |
|  |  | Signal Delay 2 | 18 |
|  | Encoder 24 | Signal Delay1 | 9 |
|  |  | Signal Delay 2 | 19 |
|  | Encoder 25 | Signal Delay1 | 10 |
|  |  | Signal Delay 2 | 20 |
|  | Encoder 26 | Signal Delay1 | 11 |
|  |  | Signal Delay 2 | 21 |
|  | Encoder 27 | Signal Delay1 | 6 |
|  |  | Signal Delay 2 | 22 |
|  | Encoder 28 | Signal Delay1 | 7 |
|  |  | Signal Delay 2 | 23 |
|  | Encoder 29 | Signal Delay1 | 0 |
|  |  | Signal Delay 2 | 13 |
|  | Encoder 30 | Signal Delay1 | 1 |
|  |  | Signal Delay 2 | 14 |
|  | Encoder 31 | Signal Delay1 | 2 |
|  |  | Signal Delay 2 | 15 |
|  | Encoder 32 | Signal Delay1 | 3 |
|  |  | Signal Delay 2 | 16 |
| Reference signal | PRBS | CodeWord | wc1 wc2 wc3 wc4 wc5 w c6 w c7 w c8 w c9 w c10 wc11 wc1 wcl3 w c14 w c15 w c16 w c17 w c18 w c19 w c20 w c21 w c22 w c23 w c24 w c25 w c26 w c27 w c28 w c29 w c30 w c31 w c32 |

## A2.2. Sub-pulses times before and after delay

| Encoder | Signal Delay | Time (ns) |
| :---: | :---: | :---: |
| Encoder 1 | Signal Delay1 | 56 |


|  | Signal Delay 2 | 73 |
| :---: | :---: | :---: |
| Encoder 2 | Signal Delay1 | 131 |
|  | Signal Delay 2 | 142 |
| Encoder 3 | Signal Delay1 | 206 |
|  | Signal Delay 2 | 217 |
| Encoder 4 | Signal Delay1 | 281 |
|  | Signal Delay 2 | 292 |
| Encoder 5 | Signal Delay1 | 356 |
|  | Signal Delay 2 | 367 |
| Encoder 6 | Signal Delay1 | 421 |
|  | Signal Delay 2 | 433 |
| Encoder 7 | Signal Delay1 | 496 |
|  | Signal Delay 2 | 508 |
| Encoder 8 | Signal Delay1 | 571 |
|  | Signal Delay 2 | 583 |
| Encoder 9 | Signal Delay1 | 646 |
|  | Signal Delay 2 | 658 |
| Encoder 10 | Signal Delay1 | 721 |
|  | Signal Delay 2 | 733 |
| Encoder 11 | Signal Delay1 | 797 |
|  | Signal Delay 2 | 809 |
| Encoder 12 | Signal Delay1 | 872 |
|  | Signal Delay 2 | 884 |
| Encoder 13 | Signal Delay1 | 947 |
|  | Signal Delay 2 | 959 |
| Encoder 14 | Signal Delay1 | 1022 |
|  | Signal Delay 2 | 1034 |
| Encoder 15 | Signal Delay1 | 1097 |
|  | Signal Delay 2 | 1109 |
| Encoder 16 | Signal Delay1 | 1166 |
|  | Signal Delay 2 | 1178 |
| Encoder 17 | Signal Delay1 | 1234 |
|  | Signal Delay 2 | 1248 |
| Encoder 18 | Signal Delay1 | 1309 |
|  | Signal Delay 2 | 1323 |
| Encoder 19 | Signal Delay1 | 1384 |
|  | Signal Delay 2 | 1398 |
| Encoder 20 | Signal Delay1 | 1459 |
|  | Signal Delay 2 | 1473 |


| Encoder 21 | Signal Delay1 | 1534 |
| :---: | :---: | :---: |
|  | Signal Delay 2 | 1542 |
| Encoder 22 | Signal Delay1 | 1609 |
|  | Signal Delay 2 | 1617 |
| Encoder 23 | Signal Delay1 | 1686 |
|  | Signal Delay 2 | 1696 |
| Encoder 24 | Signal Delay1 | 1761 |
|  | Signal Delay 2 | 1771 |
| Encoder 25 | Signal Delay1 | 1836 |
|  | Signal Delay 2 | 1846 |
| Encoder 26 | Signal Delay1 | 1911 |
|  | Signal Delay 2 | 1921 |
| Encoder 27 | Signal Delay1 | 1980 |
|  | Signal Delay 2 | 1996 |
| Encoder 28 | Signal Delay1 | 2055 |
|  | Signal Delay 2 | 2071 |
| Encoder 29 | Signal Delay1 | 2122 |
|  | Signal Delay 2 | 2135 |
| Encoder 30 | Signal Delay1 | 2197 |
|  | Signal Delay 2 | 2210 |
| Encoder 31 | Signal Delay1 | 2272 |
|  | Signal Delay 2 | 2285 |
| Encoder 32 | Signal Delay1 | 2347 |
|  | Signal Delay 2 | 2360 |

## Appendix A3. A Splitting Ratio of 64

## A3.1. Simulation parameters of 64 ONUs

| Category | Module | Parameter | Value |
| :---: | :---: | :---: | :---: |
| Global | Time window | $64 \times(50+L) /$ BitRateDefault |  |
|  | BitRateDefault | 1 e 9 |  |
|  |  | SampleModeCentreFrequency | 184.5 e 12 |
|  | Code-length $(L)$ | 60 |  |
|  | Time-Chip $(T c)$ | 1 ns |  |
|  | $T_{\Delta}$ | 50 ns |  |
|  | $T_{\text {TDM }}$ | 60 ns |  |
|  | Burst-Duration $(T b)$ | $L . T c \mathrm{~ns}$ |  |
| Code-words | $\boldsymbol{w}$ | $0[50]$ |  |


|  | C2 | 0[12] 10 [ 19] 10 [18] 10 [ 8] |
| :---: | :---: | :---: |
|  | C3 | 0[13] 10 [ 19] 10 [ 18] 10 [ 7] |
|  | C4 | 0[14] 10 [ 19] 10 [ 18] 10 [ 6] |
|  | C5 | 0 [ 15] 10 [ 19] 10 [18] 10 [5] |
|  | C6 | 0[16] 10 [ 19] 10 [ 18] 10 [4] |
|  | C7 | 0[17] 10 [ 19] 10 [ 18] 10 [3] |
|  | C8 | 0[18] 10 [ 19] 10 [ 18] 10 [2] |
|  | C9 | 0[19] 10 [ 19] 10 [ 18] 10 [ 1] |
|  | C10 | 0 [1] 10 [ 19] 10 [ 19] 10 [ 18] |
|  | C11 | 0[2] 10 [ 19] 10 [ 19] 10 [ 17] |
|  | C12 | 0 [3] 10 [19] 10 [ 19] 10 [ 16] |
|  | C13 | 0 [4] 10 [ 19] 10 [ 19] 10 [ 15] |
|  | C14 | 0 [5] 10 [ 19] 10 [ 19] 10 [ 14] |
|  | C15 | 0 [6] 10 [ 19] 10 [ 19] 10 [ 13] |
|  | C16 | 0 [7] 10 [ 19] 10 [ 19] 10 [ 12] |
|  | C17 | 0 [8] 10 [ 19] 10 [ 19] 10 [ 11] |
|  | C18 | 0 [9] 10 [ 19] 10 [ 19] 10 [ 10] |
|  | C19 | 0[11] 10 [21] 10 [ 19] 10 [6] |
|  | C20 | 0 [12] 10 [21] 10 [19] 10 [ 5] |
|  | C21 | 0[13] 10 [21] 10 [19] 10 [4] |
|  | C22 | 0 [14] 10 [21] 10 [ 19] 10 [3] |
|  | C23 | 0 [ 15] 10 [21] 10 [19] 10 [2] |
|  | C24 | 0[16] 10 [21] 10 [ 19] 10 [ 1] |
|  | C25 | 0[18] 10 [11] 10 [ 19] 10 [ 9 ] |
|  | C26 | 0[19] 10 [ 11] 10 [ 19] 10 [ 8] |
|  | C27 | 0[10] 10 [21] 10 [ 19] 10 [ 7] |
|  | C28 | 0 [1] 10 [21] 10 [21] 10 [14] |
|  | C29 | 0 [2] 10 [21] 10 [21] 10 [13] |
|  | C30 | 0[3] 10 [21] 10 [21] 10 [ 12 ] |
|  | C31 | 0 [4] 10 [21] 10 [21] 10 [11] |
|  | C32 | 0 [5] 10 [21] 10 [21] 10 [ 10$]$ |
|  | C33 | 0 [6] 10 [21] 10 [ 11] 10 [ 19] |
|  | C34 | 0 [7] 10 [21] 10 [ 11$] 10$ [ 18] |
|  | C35 | 0 [8] 10 [11] 10 [21] 10 [17] |
|  | C36 | 0 [9] 10 [11] 10 [21] 10 [ 16] |
|  | C37 | 0 [12] 10 [ 18] 10 [20] 10 [ 7] |
|  | C38 | 0[13] 10 [ 18] 10 [20] 10 [ 6] |
|  | C39 | 0 [ 14] 10 [ 18] 10 [20] 10 [ 5] |
|  | C40 | 0[15] 10 [ 18] 10 [20] 10 [4] |
|  | C41 | 0[16] 10 [ 18] 10 [20] 10 [3] |
|  | C42 | 0[17] 10 [ 18] 10 [20] 10 [2] |



|  |  | Signal Delay 3 | 52 |
| :---: | :---: | :---: | :---: |
|  | Encoder 4 | Signal Delay1 | 14 |
|  |  | Signal Delay 2 | 34 |
|  |  | Signal Delay 3 | 53 |
|  | Encoder 5 | Signal Delay1 | 15 |
|  |  | Signal Delay 2 | 35 |
|  |  | Signal Delay 3 | 54 |
|  | Encoder 6 | Signal Delay1 | 16 |
|  |  | Signal Delay 2 | 36 |
|  |  | Signal Delay 3 | 55 |
|  | Encoder 7 | Signal Delay1 | 17 |
|  |  | Signal Delay 2 | 37 |
|  |  | Signal Delay 3 | 56 |
|  | Encoder 8 | Signal Delay1 | 18 |
|  |  | Signal Delay 2 | 38 |
|  |  | Signal Delay 3 | 57 |
|  | Encoder 9 | Signal Delay1 | 19 |
|  |  | Signal Delay 2 | 39 |
|  |  | Signal Delay 3 | 58 |
|  | Encoder 10 | Signal Delay1 | 1 |
|  |  | Signal Delay 2 | 22 |
|  |  | Signal Delay 3 | 41 |
|  | Encoder 11 | Signal Delay1 | 2 |
|  |  | Signal Delay 2 | 23 |
|  |  | Signal Delay 3 | 42 |
|  | Encoder 12 | Signal Delay1 | 3 |
|  |  | Signal Delay 2 | 24 |
|  |  | Signal Delay 3 | 43 |
|  | Encoder 13 | Signal Delay1 | 4 |
|  |  | Signal Delay 2 | 25 |
|  |  | Signal Delay 3 | 44 |
|  | Encoder 14 | Signal Delay1 | 5 |
|  |  | Signal Delay 2 | 26 |
|  |  | Signal Delay 3 | 45 |
|  | Encoder 15 | Signal Delay1 | 6 |
|  |  | Signal Delay 2 | 27 |
|  |  | Signal Delay 3 | 46 |
|  | Encoder 16 | Signal Delay1 | 7 |
|  |  | Signal Delay 2 | 28 |
|  |  | Signal Delay 3 | 47 |
|  | Encoder 17 | Signal Delay1 | 8 |


|  |  | Signal Delay 2 | 29 |
| :---: | :---: | :---: | :---: |
|  |  | Signal Delay 3 | 348 |
|  |  | Signal Delay1 | 9 |
|  | Encoder 18 | Signal Delay 2 | 30 |
|  |  | Signal Delay 3 | 49 |
|  |  | Signal Delay1 | 11 |
|  | Encoder 19 | Signal Delay 2 | 34 |
|  |  | Signal Delay 3 | 53 |
|  |  | Signal Delay1 | 12 |
|  | Encoder 20 | Signal Delay 2 | 35 |
|  |  | Signal Delay 3 | 54 |
|  |  | Signal Delay1 | 13 |
|  | Encoder 21 | Signal Delay 2 | 36 |
|  |  | Signal Delay 3 | 55 |
|  |  | Signal Delay1 | 14 |
|  | Encoder 22 | Signal Delay 2 | 37 |
|  |  | Signal Delay 3 | 56 |
|  |  | Signal Delay1 | 15 |
|  | Encoder 23 | Signal Delay 2 | 38 |
|  |  | Signal Delay 3 | 57 |
|  |  | Signal Delay1 | 16 |
|  | Encoder 24 | Signal Delay 2 | 39 |
|  |  | Signal Delay 3 | 58 |
|  |  | Signal Delay1 | 18 |
|  | Encoder 25 | Signal Delay 2 | 30 |
|  |  | Signal Delay 3 | 50 |
|  |  | Signal Delay1 | 19 |
|  | Encoder 26 | Signal Delay 2 | 31 |
|  |  | Signal Delay 3 | 51 |
|  |  | Signal Delay1 | 10 |
|  | Encoder 27 | Signal Delay 2 | 32 |
|  |  | Signal Delay 3 | 52 |
|  |  | Signal Delay1 | 1 |
|  | Encoder 28 | Signal Delay 2 | 23 |
|  |  | Signal Delay 3 | 45 |
|  |  | Signal Delay1 | 2 |
|  | Encoder 29 | Signal Delay 2 | 24 |
|  |  | Signal Delay 3 | 46 |
|  |  | Signal Delay1 | 3 |
|  | Encoder 30 | Signal Delay 2 | 25 |
|  |  | Signal Delay 3 | 47 |



|  |  | Signal Delay 3 | 50 |
| :---: | :---: | :---: | :---: |
|  | Encoder 45 | Signal Delay1 | 11 |
|  |  | Signal Delay 2 | 30 |
|  |  | Signal Delay 3 | 51 |
|  | Encoder 46 | Signal Delay1 | 1 |
|  |  | Signal Delay 2 | 25 |
|  |  | Signal Delay 3 | 44 |
|  | Encoder 47 | Signal Delay1 | 2 |
|  |  | Signal Delay 2 | 26 |
|  |  | Signal Delay 3 | 45 |
|  | Encoder 48 | Signal Delay1 | 3 |
|  |  | Signal Delay 2 | 27 |
|  |  | Signal Delay 3 | 46 |
|  | Encoder 49 | Signal Delay1 | 4 |
|  |  | Signal Delay 2 | 28 |
|  |  | Signal Delay 3 | 47 |
|  | Encoder 50 | Signal Delay1 | 5 |
|  |  | Signal Delay 2 | 29 |
|  |  | Signal Delay 3 | 48 |
|  | Encoder 51 | Signal Delay1 | 6 |
|  |  | Signal Delay 2 | 20 |
|  |  | Signal Delay 3 | 49 |
|  | Encoder 52 | Signal Delay1 | 7 |
|  |  | Signal Delay 2 | 21 |
|  |  | Signal Delay 3 | 40 |
|  | Encoder 53 | Signal Delay1 | 8 |
|  |  | Signal Delay 2 | 22 |
|  |  | Signal Delay 3 | 41 |
|  | Encoder 54 | Signal Delay1 | 9 |
|  |  | Signal Delay 2 | 23 |
|  |  | Signal Delay 3 | 42 |
|  | Encoder 55 | Signal Delay1 | 13 |
|  |  | Signal Delay 2 | 34 |
|  |  | Signal Delay 3 | 51 |
|  | Encoder 56 | Signal Delay1 | 14 |
|  |  | Signal Delay 2 | 35 |
|  |  | Signal Delay 3 | 52 |
|  | Encoder 57 | Signal Delay1 | 15 |
|  |  | Signal Delay 2 | 36 |
|  |  | Signal Delay 3 | 53 |
|  | Encoder 58 | Signal Delay1 | 16 |



A3.2. Binary codes for EG-nMPC, $P=5$

| Binary code-words |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0000000000 | 1000000000 | 0000000000 | 1000000000 | 0000000000 | 0000000001 |
| 000000000 | 0100000000 | 0000000000 | 0100000000 | 0000000000 | 100000000 |
| 000000000 | 0010000000 | 0000000000 | 0010000000 | 000000000 | 010000000 |
| 000000000 | 0001000000 | 000000000 | 0001000000 | 000000000 | 001000000 |
| 0000000000 | 0000100000 | 0000000000 | 0000100000 | 0000000000 | 0001000000 |
| 0000000000 | 0000010000 | 0000000000 | 0000010000 | 0000000000 | 0000100000 |


| 0000000000 | 0000001000 | 0000000000 | 0000001000 | 0000000000 | 0000010000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 000000000 | 0000000100 | 0000000000 | 0000000100 | 000000000 | 0000001000 |
| 000000000 | 0000000010 | 0000000000 | 0000000010 | 000000000 | 0000000100 |
| 0000000000 | 0000000001 | 0000000000 | 0000000001 | 0000000000 | 0000000010 |
| 100000000 | 000000000 | 100000000 | 000000000 | 1000000000 | 000000000 |
| 0100000000 | 0000000000 | 0100000000 | 0000000000 | 0100000000 | 0000000000 |
| 0010000000 | 000000000 | 0010000000 | 000000000 | 0010000000 | 000000000 |
| 0001000000 | 0000000000 | 0001000000 | 000000000 | 0001000000 | 000000000 |
| 0000100000 | 000000000 | 0000100000 | 000000000 | 0000100000 | 000000000 |
| 0000010000 | 000000000 | 0000010000 | 000000000 | 0000010000 | 000000000 |
| 0000001000 | 000000000 | 0000001000 | 000000000 | 0000001000 | 000000000 |
| 0000000100 | 000000000 | 0000000100 | 000000000 | 0000000100 | 000000000 |
| 0000000010 | 0000000000 | 0000000010 | 0000000000 | 0000000010 | 0000000000 |
| 0000000001 | 000000000 | 0000000001 | 000000000 | 0000000001 | 000000000 |
| 000000000 | 0100000000 | 0000000000 | 0001000000 | 0000000000 | 0001000000 |
| 000000000 | 0010000000 | 0000000000 | 0000100000 | 000000000 | 0000100000 |
| 000000000 | 0001000000 | 0000000000 | 0000010000 | 0000000000 | 0000010000 |
| 000000000 | 0000100000 | 0000000000 | 0000001000 | 0000000000 | 0000001000 |
| 000000000 | 0000010000 | 0000000000 | 0000000100 | 0000000000 | 0000000100 |
| 000000000 | 0000001000 | 0000000000 | 0000000010 | 0000000000 | 0000000010 |
| 000000000 | 0000000100 | 0000000000 | 0000000001 | 0000000000 | 0000000001 |
| 000000000 | 0000000010 | 0000000000 | 1000000000 | 0000000000 | 100000000 |
| 000000000 | 0000000001 | 0000000000 | 0100000000 | 0000000000 | 010000000 |
| 0000000000 | 1000000000 | 0000000000 | 0010000000 | 0000000000 | 0010000000 |
| 100000000 | 0000000000 | 0010000000 | 000000000 | 0000100000 | 000000000 |
| 0100000000 | 0000000000 | 0001000000 | 000000000 | 0000010000 | 000000000 |
| 001000000 | 000000000 | 0000100000 | 000000000 | 0000001000 | 000000000 |
| 0001000000 | 0000000000 | 0000010000 | 000000000 | 0000000100 | 000000000 |
| 0000100000 | 000000000 | 0000001000 | 000000000 | 0000000010 | 000000000 |
| 0000010000 | 0000000000 | 0000000100 | 000000000 | 0000000001 | 000000000 |
| 0000001000 | 000000000 | 0000000010 | 000000000 | 1000000000 | 000000000 |
| 0000000100 | 000000000 | 0000000001 | 000000000 | 0100000000 | 000000000 |
| 0000000010 | 000000000 | 1000000000 | 000000000 | 0010000000 | 000000000 |
| 0000000001 | 000000000 | 0100000000 | 000000000 | 0001000000 | 000000000 |
| 000000000 | 0010000000 | 0000000000 | 0100000000 | 0000000000 | 0010000000 |
| 000000000 | 0001000000 | 0000000000 | 0010000000 | 000000000 | 0001000000 |
| 000000000 | 0000100000 | 0000000000 | 0001000000 | 0000000000 | 0000100000 |
| 0000000000 | 0000010000 | 0000000000 | 0000100000 | 0000000000 | 0000010000 |
| 000000000 | 0000001000 | 0000000000 | 0000010000 | 0000000000 | 0000001000 |
| 000000000 | 0000000100 | 0000000000 | 0000001000 | 0000000000 | 0000000100 |
| 000000000 | 0000000010 | 0000000000 | 0000000100 | 0000000000 | 0000000010 |
| 000000000 | 0000000001 | 0000000000 | 0000000010 | 0000000000 | 0000000001 |
| 000000000 | 1000000000 | 0000000000 | 0000000001 | 0000000000 | 100000000 |


| 000000000 | 0100000000 | 0000000000 | 1000000000 | 0000000000 | 0100000000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 100000000 | 000000000 | 0000100000 | 000000000 | 0001000000 | 000000000 |
| 010000000 | 0000000000 | 0000010000 | 000000000 | 0000100000 | 000000000 |
| 001000000 | 000000000 | 0000001000 | 000000000 | 0000010000 | 000000000 |
| 0001000000 | 0000000000 | 0000000100 | 0000000000 | 0000001000 | 000000000 |
| 0000100000 | 000000000 | 0000000010 | 000000000 | 0000000100 | 000000000 |
| 0000010000 | 0000000000 | 0000000001 | 000000000 | 0000000010 | 000000000 |
| 0000001000 | 0000000000 | 1000000000 | 0000000000 | 0000000001 | 000000000 |
| 0000000100 | 000000000 | 0100000000 | 000000000 | 1000000000 | 000000000 |
| 0000000010 | 000000000 | 0010000000 | 000000000 | 0100000000 | 000000000 |
| $0000000001$ | 000000000 | 0001000000 | 000000000 | 0010000000 | 000000000 |
| 0000000000 | 0001000000 | 0000000000 | 0000100000 | 0000000000 | 0100000000 |
| 000000000 | 0000100000 | 0000000000 | 0000010000 | 0000000000 | 0010000000 |
| 000000000 | 0000010000 | 0000000000 | 0000001000 | 000000000 | 0001000000 |
| 000000000 | 0000001000 | 0000000000 | 0000000100 | 0000000000 | 0000100000 |
| 0000000000 | 0000000100 | 0000000000 | 0000000010 | 0000000000 | 0000010000 |
| 000000000 | 0000000010 | 0000000000 | 0000000001 | 000000000 | 0000001000 |
| 000000000 | 0000000001 | 0000000000 | 1000000000 | 0000000000 | 0000000100 |
| 000000000 | 1000000000 | 0000000000 | 0100000000 | 0000000000 | 0000000010 |
| 0000000000 | 0100000000 | 0000000000 | 0010000000 | 0000000000 | 0000000001 |
| $0000000000$ | $0010000000$ | $0000000000$ | 0001000000 | 0000000000 | $1000000000$ |
| 1000000000 | 0000000000 | 0100000000 | 0000000000 | 0010000000 | 000000000 |
| 0100000000 | 0000000000 | 0010000000 | 0000000000 | 0001000000 | 000000000 |
| 0010000000 | 0000000000 | 0001000000 | 0000000000 | 0000100000 | 0000000000 |
| $0001000000$ | $0000000000$ | $0000100000$ | 0000000000 | 0000010000 | 0000000000 |
| 0000100000 | 0000000000 | 0000010000 | 0000000000 | 0000001000 | 000000000 |
| 0000010000 | 0000000000 | 0000001000 | 0000000000 | 0000000100 | 0000000000 |
| $0000001000$ | $0000000000$ | $0000000100$ | $0000000000$ | $0000000010$ | $0000000000$ |
| 0000000100 | 0000000000 | 0000000010 | 0000000000 | 0000000001 | 0000000000 |
| 0000000010 | 0000000000 | 0000000001 | 0000000000 | 1000000000 | 000000000 |
| 0000000001 | 0000000000 | 1000000000 | 0000000000 | 0100000000 | 0000000000 |
| 0000000000 | 0000100000 | 0000000000 | 0010000000 | 0000000000 | 1000000000 |
| 0000000000 | 0000010000 | 0000000000 | 0001000000 | 0000000000 | 010000000 |
| 0000000000 | 0000001000 | 0000000000 | 0000100000 | 0000000000 | 0010000000 |
| $0000000000$ | 0000000100 | 0000000000 | 0000010000 | 0000000000 | 0001000000 |
| 0000000000 | 0000000010 | 0000000000 | 0000001000 | 0000000000 | 0000100000 |
| 0000000000 | 0000000001 | 0000000000 | 0000000100 | 0000000000 | 0000010000 |
| 0000000000 | 1000000000 | 0000000000 | 0000000010 | 0000000000 | 0000001000 |
| 000000000 | 0100000000 | 000000000 | 0000000001 | 0000000000 | 0000000100 |
| 0000000000 | 0010000000 | 0000000000 | 1000000000 | 0000000000 | 0000000010 |
| 0000000000 | 0001000000 | 0000000000 | 0100000000 | 0000000000 | 0000000001 |
| 100000000 | 0000000000 | 0001000000 | 0000000000 | 0100000000 | 000000000 |
| 0100000000 | 0000000000 | 0000100000 | 0000000000 | 0010000000 | 000000000 |


| 0010000000 | 0000000000 | 0000010000 | 0000000000 | 0001000000 | 0000000000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0001000000 | 0000000000 | 0000001000 | 0000000000 | 0000100000 | 0000000000 |
| 0000100000 | 0000000000 | 0000000100 | 0000000000 | 0000010000 | 0000000000 |
| 0000010000 | 0000000000 | 0000000010 | 0000000000 | 0000001000 | 0000000000 |
| 0000001000 | 0000000000 | 0000000001 | 0000000000 | 0000000100 | 0000000000 |
| 0000000100 | 0000000000 | 1000000000 | 0000000000 | 0000000010 | 0000000000 |
| 0000000010 | 0000000000 | 0100000000 | 0000000000 | 0000000001 | 0000000000 |
| 0000000001 | 0000000000 | 0010000000 | 0000000000 | 1000000000 | 0000000000 |

## A3.3. Sub-pulses times before and after delay

| Encoder | Signal Delay | Time (ns) |
| :---: | :---: | :---: |
| Encoder 1 | Signal Delay1 | 61 |
|  | Signal Delay 2 | 81 |
|  | Signal Delay 3 | 100 |
| Encoder 2 | Signal Delay1 | 172 |
|  | Signal Delay 2 | 192 |
|  | Signal Delay 3 | 211 |
| Encoder 3 | Signal Delay1 | 283 |
|  | Signal Delay 2 | 303 |
|  | Signal Delay 3 | 322 |
| Encoder 4 | Signal Delay1 | 394 |
|  | Signal Delay 2 | 414 |
|  | Signal Delay 3 | 433 |
| Encoder 5 | Signal Delay1 | 505 |
|  | Signal Delay 2 | 525 |
|  | Signal Delay 3 | 544 |
| Encoder 6 | Signal Delay1 | 616 |
|  | Signal Delay 2 | 636 |
|  | Signal Delay 3 | 655 |
| Encoder 7 | Signal Delay1 | 727 |
|  | Signal Delay 2 | 747 |
|  | Signal Delay 3 | 766 |
| Encoder 8 | Signal Delay1 | 838 |
|  | Signal Delay 2 | 858 |
|  | Signal Delay 3 | 877 |
| Encoder 9 | Signal Delay1 | 949 |
|  | Signal Delay 2 | 969 |
|  | Signal Delay 3 | 988 |
| Encoder 10 | Signal Delay1 | 1041 |
|  | Signal Delay 2 | 1062 |
|  | Signal Delay 3 | 1081 |
| Encoder 11 | Signal Delay1 | 1152 |


|  | Signal Delay 2 | 1173 |
| :---: | :---: | :---: |
|  | Signal Delay 3 | 1192 |
|  | Signal Delay1 | 1263 |
| Encoder 12 | Signal Delay 2 | 1284 |
|  | Signal Delay 3 | 1303 |
|  | Signal Delay1 | 1374 |
| Encoder 13 | Signal Delay 2 | 1395 |
|  | Signal Delay 3 | 1414 |
|  | Signal Delay1 | 1485 |
| Encoder 14 | Signal Delay 2 | 1506 |
|  | Signal Delay 3 | 1525 |
|  | Signal Delay1 | 1596 |
| Encoder 15 | Signal Delay 2 | 1617 |
|  | Signal Delay 3 | 1636 |
|  | Signal Delay1 | 1707 |
| Encoder 16 | Signal Delay 2 | 1728 |
|  | Signal Delay 3 | 1747 |
|  | Signal Delay1 | 1818 |
| Encoder 17 | Signal Delay 2 | 1839 |
|  | Signal Delay 3 | 1858 |
|  | Signal Delay1 | 1929 |
| Encoder 18 | Signal Delay 2 | 1950 |
|  | Signal Delay 3 | 1969 |
|  | Signal Delay1 | 2041 |
| Encoder 19 | Signal Delay 2 | 2064 |
|  | Signal Delay 3 | 2083 |
|  | Signal Delay1 | 2152 |
| Encoder 20 | Signal Delay 2 | 2175 |
|  | Signal Delay 3 | 2194 |
|  | Signal Delay1 | 2263 |
| Encoder 21 | Signal Delay 2 | 2286 |
|  | Signal Delay 3 | 2305 |
|  | Signal Delay1 | 2374 |
| Encoder 22 | Signal Delay 2 | 2397 |
|  | Signal Delay 3 | 2416 |
|  | Signal Delay1 | 2485 |
| Encoder 23 | Signal Delay 2 | 2508 |
|  | Signal Delay 3 | 2527 |
|  | Signal Delay1 | 2596 |
| Encoder 24 | Signal Delay 2 | 2619 |
|  | Signal Delay 3 | 2638 |
| Encoder 25 | Signal Delay1 | 2708 |
| Encoder 25 | Signal Delay 2 | 2720 |


|  | Signal Delay 3 | 2740 |
| :---: | :---: | :---: |
| Encoder 26 | Signal Delay1 | 2819 |
|  | Signal Delay 2 | 2831 |
|  | Signal Delay 3 | 2851 |
| Encoder 27 | Signal Delay1 | 2920 |
|  | Signal Delay 2 | 2942 |
|  | Signal Delay 3 | 2962 |
| Encoder 28 | Signal Delay1 | 3021 |
|  | Signal Delay 2 | 3043 |
|  | Signal Delay 3 | 3065 |
| Encoder 29 | Signal Delay1 | 3132 |
|  | Signal Delay 2 | 3154 |
|  | Signal Delay 3 | 3176 |
| Encoder 30 | Signal Delay1 | 3243 |
|  | Signal Delay 2 | 3265 |
|  | Signal Delay 3 | 3287 |
| Encoder 31 | Signal Delay1 | 3354 |
|  | Signal Delay 2 | 3376 |
|  | Signal Delay 3 | 3398 |
| Encoder 32 | Signal Delay1 | 3465 |
|  | Signal Delay 2 | 3487 |
|  | Signal Delay 3 | 3509 |
| Encoder 33 | Signal Delay1 | 3576 |
|  | Signal Delay 2 | 3598 |
|  | Signal Delay 3 | 3610 |
| Encoder 34 | Signal Delay1 | 3687 |
|  | Signal Delay 2 | 3709 |
|  | Signal Delay 3 | 3721 |
| Encoder 35 | Signal Delay1 | 3798 |
|  | Signal Delay 2 | 3810 |
|  | Signal Delay 3 | 3832 |
| Encoder 36 | Signal Delay1 | 3909 |
|  | Signal Delay 2 | 3921 |
|  | Signal Delay 3 | 3943 |
| Encoder 37 | Signal Delay1 | 4022 |
|  | Signal Delay 2 | 4041 |
|  | Signal Delay 3 | 4062 |
| Encoder 38 | Signal Delay1 | 4133 |
|  | Signal Delay 2 | 4152 |
|  | Signal Delay 3 | 4173 |
| Encoder 39 | Signal Delay1 | 4244 |
|  | Signal Delay 2 | 4263 |
|  | Signal Delay 3 | 4284 |


| Encoder 40 | Signal Delay1 | 4355 |
| :---: | :---: | :---: |
|  | Signal Delay 2 | 4374 |
|  | Signal Delay 3 | 4395 |
| Encoder 41 | Signal Delay1 | 4466 |
|  | Signal Delay 2 | 4485 |
|  | Signal Delay 3 | 4506 |
| Encoder 42 | Signal Delay1 | 4577 |
|  | Signal Delay 2 | 4596 |
|  | Signal Delay 3 | 4617 |
| Encoder 43 | Signal Delay1 | 4688 |
|  | Signal Delay 2 | 4707 |
|  | Signal Delay 3 | 4728 |
| Encoder 44 | Signal Delay1 | 4790 |
|  | Signal Delay 2 | 4819 |
|  | Signal Delay 3 | 4830 |
| Encoder 45 | Signal Delay1 | 4901 |
|  | Signal Delay 2 | 4920 |
|  | Signal Delay 3 | 4941 |
| Encoder 46 | Signal Delay1 | 5001 |
|  | Signal Delay 2 | 5025 |
|  | Signal Delay 3 | 5044 |
| Encoder 47 | Signal Delay1 | 5112 |
|  | Signal Delay 2 | 5136 |
|  | Signal Delay 3 | 5155 |
| Encoder 48 | Signal Delay1 | 5223 |
|  | Signal Delay 2 | 5247 |
|  | Signal Delay 3 | 5266 |
| Encoder 49 | Signal Delay1 | 5334 |
|  | Signal Delay 2 | 5358 |
|  | Signal Delay 3 | 5377 |
| Encoder 50 | Signal Delay1 | 5445 |
|  | Signal Delay 2 | 5469 |
|  | Signal Delay 3 | 5488 |
| Encoder 51 | Signal Delay1 | 5556 |
|  | Signal Delay 2 | 5570 |
|  | Signal Delay 3 | 5599 |
| Encoder 52 | Signal Delay1 | 5667 |
|  | Signal Delay 2 | 5681 |
|  | Signal Delay 3 | 5700 |
| Encoder 53 | Signal Delay1 | 5778 |
|  | Signal Delay 2 | 5792 |
|  | Signal Delay 3 | 5811 |
| Encoder 54 | Signal Delay1 | 5889 |


|  | Signal Delay 2 | 5903 |
| :---: | :---: | :---: |
|  | Signal Delay 3 | 5922 |
|  | Signal Delay1 | 6003 |
| Encoder 55 | Signal Delay 2 | 6024 |
|  | Signal Delay 3 | 6041 |
|  | Signal Delay1 | 6114 |
| Encoder 56 | Signal Delay 2 | 6135 |
|  | Signal Delay 3 | 6152 |
|  | Signal Delay1 | 6225 |
| Encoder 57 | Signal Delay 2 | 6246 |
|  | Signal Delay 3 | 6263 |
|  | Signal Delayl | 6336 |
| Encoder 58 | Signal Delay 2 | 6357 |
|  | Signal Delay 3 | 6374 |
|  | Signal Delay1 | 6447 |
| Encoder 59 | Signal Delay 2 | 6468 |
|  | Signal Delay 3 | 6485 |
|  | Signal Delay1 | 6558 |
| Encoder 60 | Signal Delay 2 | 6579 |
|  | Signal Delay 3 | 6596 |
|  | Signal Delay1 | 6669 |
| Encoder 61 | Signal Delay 2 | 6680 |
|  | Signal Delay 3 | 6707 |
|  | Signal Delay1 | 6770 |
| Encoder 62 | Signal Delay 2 | 6791 |
|  | Signal Delay 3 | 6818 |
|  | Signal Delay1 | 6882 |
| Encoder 63 | Signal Delay 2 | 6903 |
|  | Signal Delay 3 | 6920 |
|  | Signal Delay1 | 6981 |
| Encoder 64 | Signal Delay 2 | 7002 |
|  | Signal Delay 3 | 7023 |

## Appendix A4. A Splitting Ratio of 128

## A4.1. Simulation parameters of $\mathbf{1 2 8}$ ONUs

| Category | Module | Parameter | Value |
| :---: | :---: | :---: | :---: |
| Global | Time window | $128 \times(50+L) /$ |  |
|  |  |  |  |$|$



|  |  | [27] 10 [ 17] |
| :---: | :---: | :---: |
|  | C24 | $\begin{gathered} 0[11] 10[27] 10[27] 10 \\ {[27] 10[16]} \end{gathered}$ |
|  | C25 | $\begin{gathered} 0[12] 10[27] 10[27] 10 \\ {[27] 10[15]} \\ \hline \end{gathered}$ |
|  | C26 | $\begin{gathered} 0 \text { [13] } 10 \text { [27] } 10 \text { [27] } 10 \\ {[27] 10[14]} \\ \hline \end{gathered}$ |
|  | C27 | $\begin{gathered} 0 \text { [ 15] } 10 \text { [29] } 10 \text { [29] } 10 \\ {[27] 10 \text { [ 8] }} \end{gathered}$ |
|  | C28 | $\begin{gathered} 0 \text { [ 16] } 10 \text { [29] } 10 \text { [29] } 10 \\ {[27] 10[7]} \\ \hline \end{gathered}$ |
|  | C29 | $\begin{gathered} 0[17] 10[29] 10[29] 10 \\ {[27] 10[6]} \\ \hline \end{gathered}$ |
|  | C30 | $\begin{gathered} 0 \text { [ 18] } 10 \text { [ 29] } 10 \text { [ 29] } 10 \\ {[27] 10[5]} \end{gathered}$ |
|  | C31 | $\begin{gathered} 0 \text { [19] } 10 \text { [29] } 10 \text { [ 29] } 10 \\ {[27] 10 \text { [4] }} \\ \hline \end{gathered}$ |
|  | C32 | $\begin{gathered} 0 \text { [20] } 10 \text { [29] } 10 \text { [29] } 10 \\ {[27] 10[3]} \\ \hline \end{gathered}$ |
|  | C33 | $\begin{gathered} 0[21] 10[29] 10[29] 10 \\ {[27] 10[2]} \\ \hline \end{gathered}$ |
|  | C34 | $\begin{gathered} 0[22] 10[29] 10[29] 10 \\ {[27] 10[1]} \end{gathered}$ |
|  | C35 | $\begin{gathered} 0[24] 10[29] 10[15] 10 \\ {[27] 10[13]} \\ \hline \end{gathered}$ |
|  | C36 | $\begin{gathered} 0[25] 10[29] 10[15] 10 \\ {[27] 10[12]} \end{gathered}$ |
|  | C37 | $\begin{gathered} 0 \text { [26] } 10 \text { [ 15] } 10 \text { [ 29] } 10 \\ {[27] 10[11]} \end{gathered}$ |
|  | C38 | $\begin{gathered} 0 \text { [ 27] } 10 \text { [ 15] } 10 \text { [29] } 10 \\ {[27] 10[10]} \\ \hline \end{gathered}$ |
|  | C39 | $\begin{gathered} 0[14] 10[27] 10[31] 10 \\ {[27] 10[9]} \end{gathered}$ |
|  | C40 | $\begin{gathered} 0[1] 10[29] 10[29] 10 \\ {[29] 10[20]} \end{gathered}$ |
|  | C41 | $\begin{gathered} 0[2] 10[29] 10[29] 10 \\ {[29] 10[19]} \end{gathered}$ |
|  | C42 | $\begin{gathered} 0[3] 10[29] 10[29] 10 \\ {[29] 10[18]} \end{gathered}$ |
|  | C43 | $\begin{gathered} 0[4] 10[29] 10[29] 10 \\ {[29] 10[17]} \\ \hline \end{gathered}$ |
|  | C44 | $\begin{gathered} 0[5] 10[29] 10[29] 10 \\ {[29] 10[16]} \end{gathered}$ |
|  | C45 | $\begin{gathered} 0[6] 10[29] 10[29] 10 \\ {[29] 10[15]} \end{gathered}$ |
|  | C46 | $\begin{gathered} 0[7] 10[29] 10[29] 10 \\ {[29] 10[14]} \end{gathered}$ |
|  | C47 | $\begin{gathered} 0[8] 10[29] 10[29] 10 \\ {[15] 10[27]} \end{gathered}$ |
|  | C48 | 0 [9] 10 [29] 10 [29] 10 |


|  |  | [ 15] 10 [ 26] |
| :---: | :---: | :---: |
|  | C49 | $\begin{gathered} 0[10] 10[29] 10[15] 10 \\ {[29] 10[25]} \end{gathered}$ |
|  | C50 | $\begin{gathered} 0[11] 10[29] 10[15] 10 \\ {[29] 10[24]} \end{gathered}$ |
|  | C51 | $\begin{gathered} 0[12] 10[15] 10[29] 10 \\ {[29] 10[23]} \end{gathered}$ |
|  | C52 | $\begin{gathered} 0 \text { [13] } 10 \text { [ 15] } 10 \text { [ 29] } 10 \\ {[29] 10 \text { [ 22] }} \end{gathered}$ |
|  | C53 | $\begin{gathered} 0[16] 10[31] 10[24] 10 \\ {[28] 10[9]} \end{gathered}$ |
|  | C54 | $\begin{gathered} 0[17] 10[31] 110[24] 10 \\ {[28] 10[8]} \\ \hline \end{gathered}$ |
|  | C55 | $\begin{gathered} 0[18] 10[31] 10[24] 10 \\ {[28] 10[7]} \end{gathered}$ |
|  | C56 | $\begin{gathered} 0 \text { [ 19] } 10 \text { [ 31] } 10 \text { [ 24] } 10 \\ {[28] 10[6]} \end{gathered}$ |
|  | C57 | $\begin{gathered} 0[20] 10[31] 110[24] 10 \\ {[28] 10[5]} \\ \hline \end{gathered}$ |
|  | C58 | $\begin{gathered} 0[21] 10[31] 10[24] 10 \\ {[28] 10[4]} \end{gathered}$ |
|  | C59 | $\begin{gathered} 0[22] 10[31] 10[24] 10 \\ {[28] 10[3]} \end{gathered}$ |
|  | C60 | $\begin{gathered} 0[23] 10[31] 110[24] 10 \\ {[28] 10[2]} \\ \hline \end{gathered}$ |
|  | C61 | $\begin{gathered} 0[24] 10[17] 10[38] 10 \\ {[28] 10[1]} \end{gathered}$ |
|  | C62 | $\begin{gathered} 0[26] 10[17] 10[38] 10 \\ {[14] 10[13]} \end{gathered}$ |
|  | C63 | $\begin{gathered} 0[27] 10[17] 10[24] 10 \\ {[28] 110[12]} \\ \hline \end{gathered}$ |
|  | C64 | $\begin{gathered} 0[14] 10[31] 10[24] 10 \\ {[28] 10[11]} \end{gathered}$ |
|  | C65 | $\begin{gathered} 0[15] 10[31] 10[22] 10 \\ {[30] 10[10]} \end{gathered}$ |
|  | C66 | $\begin{gathered} 0[1] 10[31] 10[24] 10 \\ {[31] 10[21]} \\ \hline \end{gathered}$ |
|  | C67 | $\begin{gathered} 0[2] 10[31] 10[24] 10 \\ {[31] 10[20]} \end{gathered}$ |
|  | c68 | $\begin{gathered} 0[3] 10[31] 10[24] 10 \\ {[31] 10[19]} \end{gathered}$ |
|  | C69 | $\begin{gathered} 0[4] 10[31] 10[24] 10 \\ {[31] 10[18]} \end{gathered}$ |
|  | C70 | $\begin{gathered} 0[5] 10[31] 10[24] 10 \\ {[31] 10[17]} \end{gathered}$ |
|  | C71 | $\begin{gathered} 0[6] 10[31] 10[24] 10 \\ {[31] 10[16]} \\ \hline \end{gathered}$ |
|  | C72 | $\begin{gathered} 0[7] 10[31] 10[24] 10 \\ {[31] 10[15]} \end{gathered}$ |
|  | C73 | 0 [8] 10 [31] 10 [24] 10 |


|  |  | [31] 10 [ 14] |
| :---: | :---: | :---: |
|  | C74 | $\begin{gathered} 0[9] 10[31] 10[24] 10 \\ {[17] 10[27]} \end{gathered}$ |
|  | C75 | $\begin{gathered} 0[10] 10[17] 10[38] 10 \\ {[17] 10[26]} \\ \hline \end{gathered}$ |
|  | C76 | $\begin{gathered} 0[11] 10[17] 110[38] 10 \\ {[17] 10[25]} \\ \hline \end{gathered}$ |
|  | C77 | $\begin{gathered} 0[12] 10[17] 10[38] 10 \\ {[17] 10[24]} \end{gathered}$ |
|  | C78 | $\begin{gathered} 0 \text { [ 13] } 10[17] 10[24] 10 \\ {[31] 100[23]} \\ \hline \end{gathered}$ |
|  | C79 | $\begin{gathered} 0[17] 10[26] 10[26] 10 \\ {[29] 110[10]} \\ \hline \end{gathered}$ |
|  | C80 | $\begin{gathered} 0[18] 10[26] 10[26] 10 \\ {[29] 10[9]} \\ \hline \end{gathered}$ |
|  | C81 | $\begin{gathered} 0[19] 10[26] 110[26] 10 \\ {[29] 10[8]} \end{gathered}$ |
|  | C82 | $\begin{gathered} 0[20] 10[26] 10[26] 10 \\ {[29] 10[7]} \\ \hline \end{gathered}$ |
|  | C83 | $\begin{gathered} 0[21] 10[26] 10[26] 10 \\ {[29] 10[6]} \end{gathered}$ |
|  | C84 | $\begin{gathered} 0[22] 10[26] 10[26] 10 \\ {[29] 10[5]} \end{gathered}$ |
|  | C85 | $\begin{gathered} 0 \text { [23] } 10 \text { [26] } 10 \text { [26] } 10 \\ {[29] 10 \text { [4] }} \end{gathered}$ |
|  | C86 | $\begin{gathered} 0[24] 10[26] 10[26] 10 \\ {[29] 10[3]} \\ \hline \end{gathered}$ |
|  | C87 | $\begin{gathered} 0[25] 10[26] 110[26] 10 \\ {[29] 10[2]} \end{gathered}$ |
|  | C88 | $\begin{gathered} 0[26] 10[26] 10[26] 10 \\ {[29] 10[1]} \end{gathered}$ |
|  | C89 | $\begin{gathered} 0[14] 10[40] 10[26] 10 \\ {[15] 10[13]} \end{gathered}$ |
|  | C90 | $\begin{gathered} 0[15] 10[26] 10[40] 10 \\ {[15] 10[12]} \end{gathered}$ |
|  | C91 | $\begin{gathered} 0[14] 10[28] 110[26] 10 \\ {[29] 110[11]} \\ \hline \end{gathered}$ |
|  | C92 | $\begin{gathered} 0[1] 10[33] 10[26] 10 \\ {[26] 10[22]} \end{gathered}$ |
|  | C93 | $\begin{gathered} 0[2] 10[33] 10[26] 10 \\ {[26] 10[21]} \end{gathered}$ |
|  | C94 | $\begin{gathered} 0[3] 10[33] 10[26] 10 \\ {[26] 10[20]} \end{gathered}$ |
|  | C95 | $\begin{gathered} 0 \text { [4] } 10[33] 10[26] 10 \\ {[26] 10[19]} \\ \hline \end{gathered}$ |
|  | C96 | $\begin{gathered} 0[5] 10[33] 10[26] 10 \\ {[26] 10[18]} \\ \hline \end{gathered}$ |
|  | C97 | $\begin{gathered} 0[6] 10[33] 10[26] 10 \\ {[26] 10[17]} \end{gathered}$ |
|  | C98 | 0 [ 7] 10 [33] 10 [ 26] 10 |


|  |  | [26] 10 [ 16] |
| :---: | :---: | :---: |
|  | C99 | $\begin{gathered} 0[8] 10[19] 10[40] 10 \\ {[26] 10[15]} \end{gathered}$ |
|  | C100 | $\begin{gathered} 0[9] 10[19] 10[26] 10 \\ {[40] 10[14]} \end{gathered}$ |
|  | C101 | $\begin{gathered} 0[10] 10[19] 110[26] 10 \\ {[26] 10[27]} \\ \hline \end{gathered}$ |
|  | C102 | $\begin{gathered} 0[11] 10[19] 110[26] 10 \\ {[26] 10[26]} \end{gathered}$ |
|  | C103 | $\begin{gathered} 0[12] 10[19] 10[26] 10 \\ {[26] 10[25]} \end{gathered}$ |
|  | C104 | $\begin{gathered} 0[13] 10[19] 10[26] 10 \\ {[26] 10[24]} \end{gathered}$ |
|  | C105 | $\begin{gathered} 0[18] 10[28] 10[28] 10 \\ {[21] 10[13]} \end{gathered}$ |
|  | C106 | $\begin{gathered} 0[19] 10[28] 10[28] 10 \\ {[23] 10[10]} \end{gathered}$ |
|  | C107 | $\begin{gathered} 0[20] 10[28] 10[28] 10 \\ {[23] 10[9]} \\ \hline \end{gathered}$ |
|  | C108 | $\begin{gathered} 0[21] 10[28] 10[28] 10 \\ {[23] 10[8]} \end{gathered}$ |
|  | C109 | $\begin{gathered} 0[22] 10[28] 10[28] 10 \\ {[23] 10[7]} \\ \hline \end{gathered}$ |
|  | C110 | $\begin{gathered} 0[23] 10[28] 10[28] 10 \\ {[23] 10[6]} \end{gathered}$ |
|  | C111 | $\begin{gathered} 0[24] 10[28] 10[28] 10 \\ {[23] 10[5]} \end{gathered}$ |
|  | C112 | $\begin{gathered} 0[25] 10[28] 10[28] 10 \\ {[23] 10[4]} \end{gathered}$ |
|  | C113 | $\begin{gathered} 0[26] 10[28] 10[14] 10 \\ {[37] 10[3]} \\ \hline \end{gathered}$ |
|  | C114 | $\begin{gathered} 0[27] 10[14] 10[28] 10 \\ {[37] 10[2]} \\ \hline \end{gathered}$ |
|  | C115 | $\begin{gathered} 0[14] 10[28] 10[28] 10 \\ {[37] 10[1]} \end{gathered}$ |
|  | C116 | $\begin{gathered} 0[16] 10[28] 110[28] 10 \\ {[23] 110[13]} \\ \hline \end{gathered}$ |
|  | C117 | $\begin{gathered} 0[17] 10[28] 10[28] 10 \\ {[23] 10[12]} \end{gathered}$ |
|  | C118 | $\begin{gathered} 0[1] 10[28] 10[28] 10 \\ {[28] 10[23]} \end{gathered}$ |
|  | C119 | $\begin{gathered} 0[2] 10[28] 10[28] 10 \\ {[28] 10[22]} \end{gathered}$ |
|  | C120 | $\begin{gathered} 0 \text { [3] } 10[28] 10[28] 10 \\ {[28] 10[21]} \\ \hline \end{gathered}$ |
|  | C121 | $\begin{gathered} 0[4] 10[28] 10[28] 10 \\ {[28] 10[20]} \end{gathered}$ |
|  | C122 | $\begin{gathered} 0[5] 10[28] 10[28] 10 \\ {[28] 10[19]} \end{gathered}$ |
|  | C123 | 0 [6] 10 [ 28] 10 [ 28] 10 |


|  |  |  | [28] 10 [18] |
| :---: | :---: | :---: | :---: |
|  |  | C124 | $\begin{gathered} 0[7] 10[28] 10[28] 10 \\ {[28] 10[17]} \end{gathered}$ |
|  |  | C125 | $\begin{gathered} 0[8] 10[28] 10[28] 10 \\ {[28] 10[16]} \end{gathered}$ |
|  |  | C126 | $\begin{gathered} 0[9] 10[28] 10[28] 10 \\ {[28] 10[15]} \end{gathered}$ |
|  |  | C127 | $\begin{gathered} 0[10] 10[28] 10[28] 10 \\ {[28] 10[14]} \end{gathered}$ |
|  |  | C128 | $\begin{gathered} 0[11] 10[28] 10[28] 10 \\ {[14] 10[27]} \end{gathered}$ |
|  |  | $w$ | 0 [50] |
| Fibers | Feeder Fiber | ReferenceFrequency | 184.5 e 12 |
|  |  | Length | 10 km |
|  |  | GroupRefractiveIndex | 1.47 |
|  | Drop fibers | ReferenceFrequency | 184.5 e 12 |
|  |  | Length | $\begin{gathered} L 1=500 \mathrm{~m} \\ \text { Increases by } 5 \mathrm{~m}- \\ L 128=1135 \mathrm{~m} \end{gathered}$ |
| Monitoring Pulse generator | General | BitRate | BitRateDefault |
|  |  | EmissionFrequency | 184.5e12 |
|  |  | ModulationType | NRZ |
|  | PRBS | PRBS_Type | CodeWord |
|  |  | CodeWord | 10 [20735] |
| Encoding x Tc | Encoder 1 | Signal Delay1 | 16 |
|  |  | Signal Delay 2 | 43 |
|  |  | Signal Delay 3 | 72 |
|  |  | Signal Delay 4 | 99 |
|  | Encoder2 | Signal Delay1 | 17 |
|  |  | Signal Delay 2 | 44 |
|  |  | Signal Delay 3 | 7573 |
|  |  | Signal Delay 4 | 100 |
|  | Encoder 3 | Signal Delay1 | 18 |
|  |  | Signal Delay 2 | 45 |
|  |  | Signal Delay 3 | 74 |
|  |  | Signal Delay 4 | 101 |
|  | Encoder 4 | Signal Delay1 | 19 |
|  |  | Signal Delay 2 | 46 |
|  |  | Signal Delay 3 | 75 |
|  |  | Signal Delay 4 | 102 |
|  | Encoder 5 | Signal Delay1 | 20 |
|  |  | Signal Delay 2 | 47 |
|  |  | Signal Delay 3 | 76 |
|  |  | Signal Delay 4 | 103 |


|  |  | Signal Delay1 | 21 |
| :---: | :---: | :---: | :---: |
|  | Encoder 6 | Signal Delay 2 | 48 |
|  |  | Signal Delay 3 | 77 |
|  |  | Signal Delay 4 | 104 |
|  |  | Signal Delay1 | 22 |
|  | E | Signal Delay 2 | 49 |
|  |  | Signal Delay 3 | 78 |
|  |  | Signal Delay 4 | 105 |
|  |  | Signal Delay1 | 23 |
|  | Encoder 8 | Signal Delay 2 | 50 |
|  |  | Signal Delay 3 | 79 |
|  |  | Signal Delay 4 | 106 |
|  |  | Signal Delay1 | 24 |
|  | Encoder 9 | Signal Delay 2 | 51 |
|  | Encoder 9 | Signal Delay 3 | 80 |
|  |  | Signal Delay 4 | 107 |
|  |  | Signal Delay1 | 25 |
|  | Encoder 10 | Signal Delay 2 | 52 |
|  | Encoder 10 | Signal Delay 3 | 81 |
|  |  | Signal Delay 4 | 108 |
|  |  | Signal Delay1 | 26 |
|  | Encor 11 | Signal Delay 2 | 53 |
|  | Encoder 11 | Signal Delay 3 | 82 |
|  |  | Signal Delay 4 | 109 |
|  |  | Signal Delay1 | 27 |
|  | Enc | Signal Delay 2 | 54 |
|  |  | Signal Delay 3 | 83 |
|  |  | Signal Delay 4 | 110 |
|  |  | Signal Delay1 | 28 |
|  | Encoder 13 | Signal Delay 2 | 55 |
|  |  | Signal Delay 3 | 84 |
|  |  | Signal Delay 4 | 111 |
|  |  | Signal Delay1 | 2 |
|  | E | Signal Delay 2 | 30 |
|  | Encoder 14 | Signal Delay 3 | 58 |
|  |  | Signal Delay 4 | 86 |
|  |  | Signal Delay1 | 3 |
|  | Encoder 15 | Signal Delay 2 | 31 |
|  | Encoder 15 | Signal Delay 3 | 59 |
|  |  | Signal Delay 4 | 87 |
|  |  | Signal Delay1 | 4 |
|  | Encoder 16 | Signal Delay 2 | 32 |
|  |  | Signal Delay 3 | 60 |


|  |  | Signal Delay 4 | 88 |
| :---: | :---: | :---: | :---: |
|  | Encoder 17 | Signal Delay1 | 5 |
|  |  | Signal Delay 2 | 33 |
|  |  | Signal Delay 3 | 61 |
|  |  | Signal Delay 4 | 89 |
|  | Encoder 18 | Signal Delay1 | 6 |
|  |  | Signal Delay 2 | 34 |
|  |  | Signal Delay 3 | 62 |
|  |  | Signal Delay 4 | 90 |
|  | Encoder 19 | Signal Delay1 | 7 |
|  |  | Signal Delay 2 | 35 |
|  |  | Signal Delay 3 | 63 |
|  |  | Signal Delay 4 | 91 |
|  | Encoder 20 | Signal Delay1 | 8 |
|  |  | Signal Delay 2 | 36 |
|  |  | Signal Delay 3 | 64 |
|  |  | Signal Delay 4 | 92 |
|  | Encoder 21 | Signal Delay1 | 9 |
|  |  | Signal Delay 2 | 37 |
|  |  | Signal Delay 3 | 65 |
|  |  | Signal Delay 4 | 93 |
|  | Encoder 22 | Signal Delay1 | 10 |
|  |  | Signal Delay 2 | 38 |
|  |  | Signal Delay 3 | 66 |
|  |  | Signal Delay 4 | 94 |
|  | Encoder 23 | Signal Delay1 | 11 |
|  |  | Signal Delay 2 | 39 |
|  |  | Signal Delay 3 | 67 |
|  |  | Signal Delay 4 | 95 |
|  | Encoder 24 | Signal Delay1 | 12 |
|  |  | Signal Delay 2 | 40 |
|  |  | Signal Delay 3 | 68 |
|  |  | Signal Delay 4 | 96 |
|  | Encoder 25 | Signal Delay1 | 13 |
|  |  | Signal Delay 2 | 41 |
|  |  | Signal Delay 3 | 69 |
|  |  | Signal Delay 4 | 97 |
|  | Encoder 26 | Signal Delay1 | 14 |
|  |  | Signal Delay 2 | 42 |
|  |  | Signal Delay 3 | 70 |
|  |  | Signal Delay 4 | 98 |
|  | Encoder 27 | Signal Delay1 | 16 |
|  |  | Signal Delay 2 | 46 |


|  |  | Signal Delay 3 | 76 |
| :---: | :---: | :---: | :---: |
|  |  | Signal Delay 4 | 104 |
|  |  | Signal Delay1 | 17 |
|  |  | Signal Delay 2 | 47 |
|  |  | Signal Delay 3 | 77 |
|  |  | Signal Delay 4 | 105 |
|  |  | Signal Delay1 | 18 |
|  |  | Signal Delay 2 | 48 |
|  |  | Signal Delay 3 | 78 |
|  |  | Signal Delay 4 | 106 |
|  |  | Signal Delay1 | 19 |
|  | Encoder 30 | Signal Delay 2 | 49 |
|  |  | Signal Delay 3 | 79 |
|  |  | Signal Delay 4 | 107 |
|  |  | Signal Delay1 | 20 |
|  | Encoder 31 | Signal Delay 2 | 50 |
|  | Encoder 31 | Signal Delay 3 | 80 |
|  |  | Signal Delay 4 | 108 |
|  |  | Signal Delay1 | 21 |
|  | Encorer 32 | Signal Delay 2 | 51 |
|  |  | Signal Delay 3 | 81 |
|  |  | Signal Delay 4 | 109 |
|  |  | Signal Delay1 | 22 |
|  | Encod | Signal Delay 2 | 52 |
|  | Encod | Signal Delay 3 | 82 |
|  |  | Signal Delay 4 | 110 |
|  |  | Signal Delay1 | 23 |
|  |  | Signal Delay 2 | 53 |
|  |  | Signal Delay 3 | 83 |
|  |  | Signal Delay 4 | 111 |
|  |  | Signal Delay1 | 25 |
|  |  | Signal Delay 2 | 55 |
|  | Encoder 35 | Signal Delay 3 | 71 |
|  |  | Signal Delay 4 | 90 |
|  |  | Signal Delay1 | 26 |
|  | Encoder 36 | Signal Delay 2 | 56 |
|  | Encoder 36 | Signal Delay 3 | 72 |
|  |  | Signal Delay 4 | 100 |
|  |  | Signal Delay1 | 27 |
|  | Encoder 37 | Signal Delay 2 | 43 |
|  | Encoder 37 | Signal Delay 3 | 73 |
|  |  | Signal Delay 4 | 101 |
|  | Encoder 38 | Signal Delay1 | 28 |


|  |  | Signal Delay 2 | 44 |
| :---: | :---: | :---: | :---: |
|  |  | Signal Delay 3 | 74 |
|  |  | Signal Delay 4 | 102 |
|  |  | Signal Delay1 | 15 |
|  | Encoder 39 | Signal Delay 2 | 45 |
|  | Encoder 39 | Signal Delay 3 | 75 |
|  |  | Signal Delay 4 | 103 |
|  |  | Signal Delay1 | 2 |
|  | Encoder 40 | Signal Delay 2 | 32 |
|  | Encoder 40 | Signal Delay 3 | 62 |
|  |  | Signal Delay 4 | 92 |
|  |  | Signal Delay1 | 3 |
|  | Encoder 41 | Signal Delay 2 | 33 |
|  | Encoder 41 | Signal Delay 3 | 63 |
|  |  | Signal Delay 4 | 93 |
|  |  | Signal Delay1 | 4 |
|  | Encoder 42 | Signal Delay 2 | 34 |
|  | Encoder 42 | Signal Delay 3 | 64 |
|  |  | Signal Delay 4 | 94 |
|  |  | Signal Delay1 | 5 |
|  | Encoder 43 | Signal Delay 2 | 35 |
|  | Encoder 43 | Signal Delay 3 | 65 |
|  |  | Signal Delay 4 | 95 |
|  |  | Signal Delay1 | 6 |
|  | Encoder 44 | Signal Delay 2 | 36 |
|  | Encoder 44 | Signal Delay 3 | 66 |
|  |  | Signal Delay 4 | 96 |
|  |  | Signal Delay1 | 7 |
|  | Encoder 45 | Signal Delay 2 | 37 |
|  | Encoder 45 | Signal Delay 3 | 67 |
|  |  | Signal Delay 4 | 97 |
|  |  | Signal Delay1 | 8 |
|  | Encoder 46 | Signal Delay 2 | 38 |
|  | Encoder 46 | Signal Delay 3 | 68 |
|  |  | Signal Delay 4 | 98 |
|  |  | Signal Delay1 | 9 |
|  | Encoder 47 | Signal Delay 2 | 39 |
|  | Encoder 47 | Signal Delay 3 | 69 |
|  |  | Signal Delay 4 | 85 |
|  |  | Signal Delay1 | 10 |
|  | Encoder 48 | Signal Delay 2 | 40 |
|  | Encoder 48 | Signal Delay 3 | 70 |
|  |  | Signal Delay 4 | 86 |


|  |  | Signal Delay1 | 11 |
| :---: | :---: | :---: | :---: |
|  | Encoder 49 | Signal Delay 2 | 41 |
|  | Encoder 49 | Signal Delay 3 | 57 |
|  |  | Signal Delay 4 | 87 |
|  |  | Signal Delay1 | 12 |
|  | Encoder 50 | Signal Delay 2 | 42 |
|  | Encoder 50 | Signal Delay 3 | 58 |
|  |  | Signal Delay 4 | 88 |
|  |  | Signal Delay1 | 13 |
|  | Encoder 51 | Signal Delay 2 | 29 |
|  | Encoder 51 | Signal Delay 3 | 58 |
|  |  | Signal Delay 4 | 89 |
|  |  | Signal Delay1 | 14 |
|  | Encoder 52 | Signal Delay 2 | 30 |
|  | Encoder 52 | Signal Delay 3 | 60 |
|  |  | Signal Delay 4 | 90 |
|  |  | Signal Delay1 | 17 |
|  | Encoder 53 | Signal Delay 2 | 49 |
|  | Encoder 53 | Signal Delay 3 | 74 |
|  |  | Signal Delay 4 | 103 |
|  |  | Signal Delay1 | 18 |
|  | Encoder 54 | Signal Delay 2 | 50 |
|  | Encoder 54 | Signal Delay 3 | 75 |
|  |  | Signal Delay 4 | 104 |
|  |  | Signal Delay1 | 19 |
|  | Encoder 55 | Signal Delay 2 | 51 |
|  | Encoder 55 | Signal Delay 3 | 76 |
|  |  | Signal Delay 4 | 105 |
|  |  | Signal Delay1 | 20 |
|  | Encoder 56 | Signal Delay 2 | 52 |
|  | Encoder 56 | Signal Delay 3 | 77 |
|  |  | Signal Delay 4 | 106 |
|  |  | Signal Delay1 | 21 |
|  | Encoder 57 | Signal Delay 2 | 53 |
|  | Encoder 57 | Signal Delay 3 | 78 |
|  |  | Signal Delay 4 | 107 |
|  |  | Signal Delay1 | 22 |
|  | Encoder 58 | Signal Delay 2 | 54 |
|  | Encoder 58 | Signal Delay 3 | 79 |
|  |  | Signal Delay 4 | 108 |
|  |  | Signal Delay1 | 23 |
|  | Encoder 59 | Signal Delay 2 | 55 |
|  |  | Signal Delay 3 | 80 |


|  |  | Signal Delay 4 | 109 |
| :---: | :---: | :---: | :---: |
|  |  | Signal Delay1 | 24 |
|  | Encoder 60 | Signal Delay 2 | 56 |
|  | 析 60 | Signal Delay 3 | 81 |
|  |  | Signal Delay 4 | 110 |
|  |  | Signal Delay1 | 25 |
|  | Encoder 61 | Signal Delay 2 | 43 |
|  | Encoder 61 | Signal Delay 3 | 82 |
|  |  | Signal Delay 4 | 111 |
|  |  | Signal Delay1 | 27 |
|  | Encoder 62 | Signal Delay 2 | 45 |
|  | Encoder 62 | Signal Delay 3 | 84 |
|  |  | Signal Delay 4 | 99 |
|  |  | Signal Delay1 | 28 |
|  | Encoder 63 | Signal Delay 2 | 46 |
|  | oder | Signal Delay 3 | 71 |
|  |  | Signal Delay 4 | 100 |
|  |  | Signal Delay1 | 15 |
|  | Encoder 64 | Signal Delay 2 | 47 |
|  | Encoder 64 | Signal Delay 3 | 72 |
|  |  | Signal Delay 4 | 101 |
|  |  | Signal Delay1 | 16 |
|  | Encoder 65 | Signal Delay 2 | 48 |
|  | Encoder 65 | Signal Delay 3 | 73 |
|  |  | Signal Delay 4 | 102 |
|  |  | Signal Delay1 | 2 |
|  | Encoder 66 | Signal Delay 2 | 34 |
|  | Encoder 66 | Signal Delay 3 | 59 |
|  |  | Signal Delay 4 | 91 |
|  |  | Signal Delay1 | 3 |
|  | Encoder 67 | Signal Delay 2 | 35 |
|  | Encoder 67 | Signal Delay 3 | 60 |
|  |  | Signal Delay 4 | 92 |
|  |  | Signal Delay1 | 4 |
|  | Encoder 68 | Signal Delay 2 | 36 |
|  | Encoder 6 | Signal Delay 3 | 61 |
|  |  | Signal Delay 4 | 93 |
|  |  | Signal Delay1 | 5 |
|  | E | Signal Delay 2 | 37 |
|  | Encoder 69 | Signal Delay 3 | 62 |
|  |  | Signal Delay 4 | 94 |
|  | Encoder 70 | Signal Delay1 | 6 |
|  | Encoder 70 | Signal Delay 2 | 38 |


|  |  | Signal Delay 3 | 63 |
| :---: | :---: | :---: | :---: |
|  |  | Signal Delay 4 | 95 |
|  |  | Signal Delay1 | 7 |
|  |  | Signal Delay 2 | 39 |
|  |  | Signal Delay 3 | 64 |
|  |  | Signal Delay 4 | 96 |
|  |  | Signal Delay1 | 8 |
|  | Encoder 72 | Signal Delay 2 | 40 |
|  |  | Signal Delay 3 | 65 |
|  |  | Signal Delay 4 | 97 |
|  |  | Signal Delay1 | 9 |
|  | Encoder 73 | Signal Delay 2 | 41 |
|  |  | Signal Delay 3 | 66 |
|  |  | Signal Delay 4 | 98 |
|  |  | Signal Delay1 | 10 |
|  | Encoder 74 | Signal Delay 2 | 42 |
|  | Encoder | Signal Delay 3 | 67 |
|  |  | Signal Delay 4 | 85 |
|  |  | Signal Delay1 | 11 |
|  | Encoder 75 | Signal Delay 2 | 29 |
|  |  | Signal Delay 3 | 68 |
|  |  | Signal Delay 4 | 86 |
|  |  | Signal Delay1 | 12 |
|  | Enc | Signal Delay 2 | 30 |
|  |  | Signal Delay 3 | 69 |
|  |  | Signal Delay 4 | 87 |
|  |  | Signal Delay1 | 13 |
|  |  | Signal Delay 2 | 31 |
|  |  | Signal Delay 3 | 70 |
|  |  | Signal Delay 4 | 88 |
|  |  | Signal Delay1 | 14 |
|  |  | Signal Delay 2 | 32 |
|  | Encoder 78 | Signal Delay 3 | 57 |
|  |  | Signal Delay 4 | 89 |
|  |  | Signal Delay1 | 18 |
|  | Encoder 79 | Signal Delay 2 | 45 |
|  | Encoder 79 | Signal Delay 3 | 72 |
|  |  | Signal Delay 4 | 102 |
|  |  | Signal Delay1 | 19 |
|  | Encoder 80 | Signal Delay 2 | 46 |
|  | Encoder 80 | Signal Delay 3 | 73 |
|  |  | Signal Delay 4 | 103 |
|  | Encoder 81 | Signal Delay1 | 20 |


|  |  | Signal Delay 2 | 47 |
| :---: | :---: | :---: | :---: |
|  |  | Signal Delay 3 | 74 |
|  |  | Signal Delay 4 | 104 |
|  |  | Signal Delay1 | 21 |
|  | Encoder 82 | Signal Delay 2 | 48 |
|  | Encoder 82 | Signal Delay 3 | 75 |
|  |  | Signal Delay 4 | 105 |
|  |  | Signal Delay1 | 22 |
|  | Encoder 83 | Signal Delay 2 | 49 |
|  | Encoder 83 | Signal Delay 3 | 76 |
|  |  | Signal Delay 4 | 106 |
|  |  | Signal Delay1 | 23 |
|  | Encoder 84 | Signal Delay 2 | 50 |
|  | Encoder 84 | Signal Delay 3 | 77 |
|  |  | Signal Delay 4 | 107 |
|  |  | Signal Delay1 | 24 |
|  | Encoder 85 | Signal Delay 2 | 51 |
|  | Encoder 85 | Signal Delay 3 | 78 |
|  |  | Signal Delay 4 | 108 |
|  |  | Signal Delay1 | 25 |
|  | Encoder 86 | Signal Delay 2 | 52 |
|  | Encoder 86 | Signal Delay 3 | 79 |
|  |  | Signal Delay 4 | 109 |
|  |  | Signal Delay1 | 26 |
|  | Encoder 87 | Signal Delay 2 | 53 |
|  | Encoder 87 | Signal Delay 3 | 80 |
|  |  | Signal Delay 4 | 110 |
|  |  | Signal Delay1 | 27 |
|  | Encoder 88 | Signal Delay 2 | 54 |
|  | Encoder 88 | Signal Delay 3 | 81 |
|  |  | Signal Delay 4 | 111 |
|  |  | Signal Delay1 | 15 |
|  | Encoder 89 | Signal Delay 2 | 56 |
|  | Encoder 89 | Signal Delay 3 | 83 |
|  |  | Signal Delay 4 | 99 |
|  |  | Signal Delay1 | 16 |
|  | Encoder 90 | Signal Delay 2 | 43 |
|  | Encoder 90 | Signal Delay 3 | 84 |
|  |  | Signal Delay 4 | 100 |
|  |  | Signal Delay1 | 17 |
|  | Encoder 91 | Signal Delay 2 | 44 |
|  | Encoder 91 | Signal Delay 3 | 71 |
|  |  | Signal Delay 4 | 101 |


|  |  | Signal Delay1 | 2 |
| :---: | :---: | :---: | :---: |
|  | Encoder 92 | Signal Delay 2 | 36 |
|  | Encoder 92 | Signal Delay 3 | 63 |
|  |  | Signal Delay 4 | 90 |
|  |  | Signal Delay1 | 3 |
|  | Encoder 93 | Signal Delay 2 | 37 |
|  |  | Signal Delay 3 | 64 |
|  |  | Signal Delay 4 | 91 |
|  |  | Signal Delay1 | 4 |
|  | Encoder 94 | Signal Delay 2 | 38 |
|  |  | Signal Delay 3 | 65 |
|  |  | Signal Delay 4 | 92 |
|  |  | Signal Delay1 | 5 |
|  | Encoder 95 | Signal Delay 2 | 39 |
|  |  | Signal Delay 3 | 66 |
|  |  | Signal Delay 4 | 93 |
|  |  | Signal Delay1 | 6 |
|  |  | Signal Delay 2 | 40 |
|  | Encoder 96 | Signal Delay 3 | 67 |
|  |  | Signal Delay 4 | 94 |
|  |  | Signal Delay1 | 7 |
|  | Encoder 97 | Signal Delay 2 | 41 |
|  |  | Signal Delay 3 | 68 |
|  |  | Signal Delay 4 | 95 |
|  |  | Signal Delay1 | 8 |
|  | Encoder 98 | Signal Delay 2 | 42 |
|  | E | Signal Delay 3 | 69 |
|  |  | Signal Delay 4 | 96 |
|  |  | Signal Delay1 | 9 |
|  | Encoder | Signal Delay 2 | 29 |
|  | Encoder 9 | Signal Delay 3 | 70 |
|  |  | Signal Delay 4 | 97 |
|  |  | Signal Delay1 | 10 |
|  | Encoder 100 | Signal Delay 2 | 30 |
|  | Encoder 100 | Signal Delay 3 | 57 |
|  |  | Signal Delay 4 | 98 |
|  |  | Signal Delay1 | 11 |
|  |  | Signal Delay 2 | 31 |
|  | Encoder 101 | Signal Delay 3 | 58 |
|  |  | Signal Delay 4 | 85 |
|  |  | Signal Delay1 | 12 |
|  | Encoder 102 | Signal Delay 2 | 32 |
|  |  | Signal Delay 3 | 59 |


|  |  | Signal Delay 4 | 86 |
| :---: | :---: | :---: | :---: |
|  | Encoder 103 | Signal Delay1 | 13 |
|  |  | Signal Delay 2 | 33 |
|  |  | Signal Delay 3 | 59 |
|  |  | Signal Delay 4 | 87 |
|  | Encoder 104 | Signal Delay1 | 14 |
|  |  | Signal Delay 2 | 34 |
|  |  | Signal Delay 3 | 61 |
|  |  | Signal Delay 4 | 88 |
|  | Encoder 105 | Signal Delay1 | 19 |
|  |  | Signal Delay 2 | 48 |
|  |  | Signal Delay 3 | 77 |
|  |  | Signal Delay 4 | 99 |
|  | Encoder 106 | Signal Delay1 | 20 |
|  |  | Signal Delay 2 | 49 |
|  |  | Signal Delay 3 | 78 |
|  |  | Signal Delay 4 | 102 |
|  | Encoder 107 | Signal Delay1 | 21 |
|  |  | Signal Delay 2 | 50 |
|  |  | Signal Delay 3 | 79 |
|  |  | Signal Delay 4 | 103 |
|  | Encoder 108 | Signal Delay1 | 22 |
|  |  | Signal Delay 2 | 51 |
|  |  | Signal Delay 3 | 80 |
|  |  | Signal Delay 4 | 104 |
|  | Encoder 109 | Signal Delay1 | 23 |
|  |  | Signal Delay 2 | 52 |
|  |  | Signal Delay 3 | 81 |
|  |  | Signal Delay 4 | 105 |
|  | Encoder 110 | Signal Delay1 | 24 |
|  |  | Signal Delay 2 | 53 |
|  |  | Signal Delay 3 | 82 |
|  |  | Signal Delay 4 | 106 |
|  | Encoder 111 | Signal Delay1 | 25 |
|  |  | Signal Delay 2 | 54 |
|  |  | Signal Delay 3 | 83 |
|  |  | Signal Delay 4 | 107 |
|  | Encoder 112 | Signal Delay1 | 26 |
|  |  | Signal Delay 2 | 55 |
|  |  | Signal Delay 3 | 84 |
|  |  | Signal Delay 4 | 108 |
|  | Encoder 113 | Signal Delay1 | 27 |
|  |  | Signal Delay 2 | 56 |


|  |  | Signal Delay 3 | 71 |
| :---: | :---: | :---: | :---: |
|  |  | Signal Delay 4 | 109 |
|  |  | Signal Delay1 | 28 |
|  | Encoder 114 | Signal Delay 2 | 43 |
|  | Encoder 114 | Signal Delay 3 | 72 |
|  |  | Signal Delay 4 | 110 |
|  |  | Signal Delay1 | 15 |
|  | Encoder 115 | Signal Delay 2 | 44 |
|  | Encoder 115 | Signal Delay 3 | 73 |
|  |  | Signal Delay 4 | 111 |
|  |  | Signal Delay1 | 17 |
|  | Encoder 116 | Signal Delay 2 | 46 |
|  | Encoder 116 | Signal Delay 3 | 75 |
|  |  | Signal Delay 4 | 99 |
|  |  | Signal Delay1 | 18 |
|  | Encoder 117 | Signal Delay 2 | 47 |
|  | Encoder 117 | Signal Delay 3 | 76 |
|  |  | Signal Delay 4 | 100 |
|  |  | Signal Delay1 | 2 |
|  | Encoder 118 | Signal Delay 2 | 31 |
|  | Encoder 118 | Signal Delay 3 | 60 |
|  |  | Signal Delay 4 | 89 |
|  |  | Signal Delay1 | 3 |
|  | Encoder 119 | Signal Delay 2 | 32 |
|  | Encoder 119 | Signal Delay 3 | 61 |
|  |  | Signal Delay 4 | 90 |
|  |  | Signal Delay1 | 4 |
|  | Encoder 120 | Signal Delay 2 | 33 |
|  | Encoder 120 | Signal Delay 3 | 62 |
|  |  | Signal Delay 4 | 91 |
|  |  | Signal Delay1 | 5 |
|  | Encoder 121 | Signal Delay 2 | 34 |
|  | Encoder 121 | Signal Delay 3 | 63 |
|  |  | Signal Delay 4 | 92 |
|  |  | Signal Delay1 | 6 |
|  | Encoder 122 | Signal Delay 2 | 35 |
|  | Encoder 122 | Signal Delay 3 | 64 |
|  |  | Signal Delay 4 | 93 |
|  |  | Signal Delay1 | 7 |
|  | Encoder 123 | Signal Delay 2 | 36 |
|  | Encoder 123 | Signal Delay 3 | 65 |
|  |  | Signal Delay 4 | 94 |
|  | Encoder 124 | Signal Delay1 | 8 |



A4.2. Binary codes for $E G-n M P C, P=7$

| Binary code-words for EG-nMPC, P=7 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | \%o, ${ }^{\text {O2000 }}$ | 100000000000 | 00000 0000000 | 100008 |  |  |
| \%oo | ${ }^{01000000080}$ | 0 | ${ }^{010000000}$ | 000000000 | ${ }_{0}^{000}$ |  | Ooa00000 |
| Oood | 0 |  | \%o | Oonoooso $0^{0}$ | 0 | 0 | ${ }^{1100009000000}$ |
| ${ }^{\text {aocooosoo }}$ | 0 | 00000a00000 | ${ }^{000100000}$ | . | $\bigcirc$ | 0000000000000 | 0 |
| 00000000 | ${ }^{0000108000}$ | 00000000000 | 0 | 0000080 | ${ }^{00001000}$ | 000000000000 | 0010000 |
| ${ }^{\text {aonoosoos }}$ | ${ }^{000000} 100$ | 000 | doi | 000 | ${ }^{\text {Oouoo } 100}$ |  | \%o |
| \% | 0 | ${ }^{000000000000}$ | 000000 $0_{0} 1000$ | \%o | ${ }^{00000910} 80$ | 0000 | (00 1000000 |
| 00000000 | ${ }^{0000009} 8$ |  | ${ }^{\text {acooosor }}$ | ${ }^{000000080} 80$ | . |  |  |
| O000000 |  | \%oos | 0 | 0 | 0 | 0 | 0 |
| Oo | 0 | 000000000000 | Sorooon | Oonoonoo | 00000000 | 0 | , |
| \%oos | \%oos | \%oi | (oos | Ooosoor | ${ }^{00000000}$ | 9000000 | \%oid |
| 0000000 $0^{0}$ | 9000000\% | 000000000000 | Ooos | 000000000 | ${ }^{0000000} 0$ | Oood | , |
| 00000000 ${ }_{0}$ | ${ }^{00000} 0_{10}$ | 000000000000 | ${ }^{\text {00000a00000 }} 10$ | ${ }^{\text {a0000a000000 }}$ | 10 | 9000000000 | Ooosoosoon |
| 00000000 | ${ }^{0000000}$ |  | ${ }^{\text {Oonoono }} 1$ |  | ${ }^{\text {aroonoor }}$ | \%ooonoo |  |
| Ooos | ${ }^{00000000}$ | \%oob | \%oo | \%oos | ${ }^{00000080}$ | \%oos | \%oonoon |
| 0100aooob | ${ }^{\text {aoaososo }}$ | ${ }^{010000 a 000000}$ | \%ood | ${ }^{\text {ooos }}$ | ${ }^{000000808}$ | ooob | Oood |
| 0010aooso | ${ }^{\text {a000000 }} 0$ | ${ }^{001000000000}$ | 00000000 80 | 001000000 | ${ }^{0000000080}$ | (10ooooono | Sooono |
| 00010000 $0_{0}$ | ${ }^{\text {arooosoos }}$ |  | 000000000 | ${ }^{000100000}$ | . | \%oovoo |  |
| Oo, | 0 | \%os | Oosoor | 001000 $0_{00}$ | ${ }^{00000080} 8$ | O\% | Ooposomod |
| ${ }^{\circ}$ | 0 | 0 | (ooso | ${ }^{000000} 100$ | O000000\% | 9100 | 0 |
| ${ }^{\text {a00000 } 100}$ | ${ }^{\text {couoson }}$ | 0 | Ooo | ${ }^{000000} 10$ | ${ }^{00000080}$ | ${ }_{0}^{10}$ | Ooosoon |
| 000000, $0_{0}$ | \%oo | ${ }^{00000000000}$ | 000000000 ${ }_{0}$ | ${ }^{\text {aoaoosor }}$ | ${ }^{\text {aoaoosoob }}$ | 50090010000 | ${ }^{\text {anoonogoono }}$ |
| Ooid | 9oos | 0 | Oood | ${ }^{\text {aocoosoob }}$ | 0 | 0 | 0 |
| Oo, | \%oo | Ooo | O00 | Oonoosoon | ${ }^{\text {Onoonoso }}$ | Ooot 00 | 0 |
| \%oon | ooos |  | oood | ooos | onoooso | aooool0 | , |
| 00000096 | 9000000 | 0 | $\bigcirc$ | ${ }^{\text {aocoosoo }}$ | 0 | \%oonool | 900000 |
| 1 | 0 | , | $\bigcirc$ | , | ${ }^{\circ}$ | 0000000 |  |
| 000009000000 | 0000000000 |  |  | ${ }^{\text {000000000 }}$ | ${ }^{00000000} 80$ | \%o, | Oo, |
| 0 | 0 | 0 | 0 | 0 | 0 | \%oio | 0 |
| 0 | 0 |  | 0 | 0 | ${ }^{\text {acooos } 1_{0} 0^{\circ}}$ | \%oo | \%o, |
| Oo, | Oo, | oos | Ooino | \%osoo | ${ }^{00000001}$ | 900 | oo, |
| Ooi | ${ }^{1000}$ | Oo, | ${ }^{0} 1000$ | ${ }^{\text {onoob }}$ | ${ }^{00000080} 80$ | (000 | Ooo |
| 0 | ${ }_{0}^{0100}$ | 00000000000 | \%os |  | O |  | Oosoo |
|  | 0 | 0 | 0 | . | . | 0 | Oooosoono |
| \%oo | Oo, ${ }_{0} 10$ | ooo | Oosoof | 0 | ${ }^{\text {Onoonoso }}$ | \%oo | , |
| 0 | Ooo, | \%oonoo | \%ooon | \%osoo | ${ }^{\text {ono }} 10$ | Oo, |  |
| \%oio | oooo | - | 崖 | - | ${ }^{\text {00000a000000 }}$ | Ooooooo |  |
|  | aocosoos |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |



| 000000000000 <br> 10 |
| :---: |
| $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ |
| $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ |



| $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| :---: | :---: |
| $\begin{gathered} 0100000000000 \\ 0 \end{gathered}$ | $0000000000000000000$ |
| $\begin{gathered} 10000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 00001000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000100 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 00000000010 \\ 000 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000001 \\ 000 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 01 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 10000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0010000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 000 \end{gathered}$ |


| $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ |
| :---: |
| $\begin{gathered} 000100000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |

000000000000
00
000000000000
00
000000000000
00
000000100000
00
000000010000
00
000000001000
00
0000000000100
00

001000000000
00
000100000000 00
00010000000 00 000000000000 00
000000000 00 00000000000
00 000000000000 000000000000
000000000010
000000000001
000
000000000000
000000000000
01
100000000000 00000000000 00
000000000000
010000000000

000000000000

$$
001000000000
$$

$$
\begin{gathered}
00 \\
000100000000 \\
00
\end{gathered}
$$

000010000000
00
000001000000
000000000000
000010000000
000000000000
00

| $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{aligned} & 100000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000010000000000 \\ 0 \end{gathered}$ | $0000000000000$ | $000100000000$ |  | $0010000000000$ | $0000000000000$ | $\begin{gathered} 0000100000000 \\ 00 \end{gathered}$ |
| $000000000000000000$ | $\begin{gathered} 0000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $0001000000000000$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ |
| $000000000000$ | $000000010000$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $000000100000$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{array}{cc} 0000010000000 \\ 0 \end{array}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000100 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000100 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000001 \\ 000000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000010 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ |
| $000000000000000000$ | $\begin{gathered} 000000000000 \\ 10 \\ \hline \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $00000000000000000$ | $\begin{array}{ll} 0 & 000000000000 \\ 00 \end{array}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{aligned} & 100000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0100000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0010000000000 \\ 00 \end{gathered}$ |
| $\begin{aligned} & 100000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 0 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{aligned} & 000000000010 \\ & 00 \end{aligned}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000010000000 \\ 00 \end{gathered}$ | $000000000000000000$ | $0000000000001$ | $000000000000$ | $00000000000000000$ | $000000000000000000$ | $00000000000000000$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $000000010000$ | $000000000000$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $0000000000000$ | $\begin{array}{ll} 0 & 000000000000 \\ 10 \end{array} 0$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $0000000000000000$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000001000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 10000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{aligned} & 1000000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $0000000000000$ |  | $0000000000000$ | $001000000000000000$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $010000000000$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 00000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00001000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \\ \hline \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{aligned} & 000000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000100000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0001000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000100000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000100000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ |  | $\begin{gathered} 000000000100 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000001000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000001 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000100000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 000 \end{gathered}$ | $000000000000000000$ | $\begin{gathered} 00000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000000001 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $0000000000000$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000001000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 10000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{aligned} & 10000000000000 \\ & 000 \end{aligned}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 010000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 10000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{aligned} & 000000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 01000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0100000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $000100000000$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000100000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 1000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ |
| $\begin{aligned} & 100000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0001000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |


| $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 0 \\ 0 \end{gathered} 10000000000$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{array}{cc} 0000010000000 \\ 0 & 0 \end{array}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 0000100000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000001000000 \\ 000 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{array}{cc} 0 & 000000000000 \\ 0 \end{array}$ | $\begin{gathered} 00000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 0 \\ 0 \end{gathered} 000000000000000$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ | $\begin{array}{rl} 0 & 00000000 \\ 00 \end{array}$ |
| $\begin{gathered} 0000000100000 \\ 00 \end{gathered}$ | $\begin{array}{ll} 000000000000000 \\ 0 \end{array}$ | $000000001000$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $000000000100$ | $\begin{array}{cc} 000000000000000 \\ 0 \end{array}$ | $000000000010$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ | $\begin{aligned} & 000000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 00000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000000001 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 00000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 00000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 00000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 00000000 \\ 00 \end{gathered}$ | $\begin{aligned} & 1000000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 010000000000000 \\ 000 \end{gathered}$ | $\begin{aligned} & 00000000000 \\ & 00 \end{aligned}$ |
| $\begin{gathered} 0000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 0000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 10000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 01000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{aligned} & 0000000000 \\ & 00 \end{aligned}$ | $\begin{aligned} & 0000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 10000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 000 \end{gathered}$ | $\begin{aligned} & 001000000000000 \\ & 00 \end{aligned}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000001000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{array}{rl} 0 & 0 \\ 0 & 000000000 \\ 0 \end{array}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 00000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $000000000000$ | $\begin{gathered} 0000010000000 \\ 00 \end{gathered}$ | $\begin{aligned} & 00000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00001000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000100 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000001000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 00000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |  |
| $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000001 \\ 000000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $000000000100$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{aligned} & 10000000000000 \\ & 000 \end{aligned}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{aligned} & 000000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 001000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{aligned} & 100000000000 \\ & 00 \end{aligned}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0010000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 010000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ | $000100000000$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 0010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $000010000000$ | $\begin{gathered} 0000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $00000000000000000$ | $0000000000000000000$ | $000000100000$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ |  | $\begin{gathered} 000000000100 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000000 \\ 000 \end{gathered}$ |  | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $0000000000000000000$ | $000000000010$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000100 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 10000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 000 \end{gathered}$ | $000000000000$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 0000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 00000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $0000000000000$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000100000 \\ 000 \end{gathered}$ | $\begin{array}{cc} 0000000000000 \\ 0 & 0 \end{array}$ | $\begin{gathered} 0000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 00100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ |


| $00000000000000000$ | $\begin{gathered} 000000000100 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0001000000000000 \\ 0 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000010000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000100 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000100000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{array}{cc} 000000000000 \\ 0 \end{array}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000010000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 10000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{array}{cc} 000000000000 \\ 10 \end{array}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ |
| $\begin{array}{cc} 0000000000000 \\ 0 \end{array}$ | $\begin{gathered} 0100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $0000000000000$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0 \\ 0 \\ 0 \end{gathered} 0000000000$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ |
| $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0 \\ 0 \end{gathered} 10000000000$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ |
| $\begin{gathered} 1000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0100000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ |
| $\begin{gathered} 0 \\ 0 \end{gathered} 00000000000$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 0010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 000 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000100000000 \\ 0 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 0000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000100000000 \\ 0 \end{gathered}$ | $\begin{array}{cc} 0000000000000 \\ 0 & 0 \end{array}$ |
| $\begin{gathered} 00000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000001000000 \\ 00 \end{gathered}$ | $\begin{array}{cc} 000000000000 \\ 0 & 0 \end{array}$ |
| $\begin{gathered} 000000100000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000100 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000010000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $000000000000$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 000 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{array}{cc} 00000000000000 \\ 0 \end{array}$ |
| $\begin{gathered} 000000001000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000100 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 0000000001000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000010 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 0 \end{gathered}$ | $\begin{gathered} 00000000000000 \\ 0 \end{gathered}$ |
| $\begin{gathered} 000000000001 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 001000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 0000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{array}{cc} 000000000000 \\ 0 & 0 \end{array}$ |
| $\begin{gathered} 000000000000 \\ 10 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000100000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 010000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |
| $\begin{gathered} 000000000000 \\ 01 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000010000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 100000000000 \\ 00 \end{gathered}$ | $\begin{gathered} 000000000000 \\ 00 \end{gathered}$ |

## A4.3. Sub-pulses times after delay

| Encoder | Signal Delay | Time (ns) |
| :---: | :---: | :---: |
| Encoder 1 | Signal Delay1 | 66 |
|  | Signal Delay 2 | 93 |
|  | Signal Delay 3 | 122 |
| Encoder2 | Signal Delay 4 | 149 |
|  | Signal Delay1 | 229 |
|  | Signal Delay 2 | 256 |
|  | Encoder 3 | Signal Delay 3 |
|  | Signal Delay 4 | 447 |
|  | Signal Delay1 | 312 |
|  | Signal Delay 2 | 392 |
|  | Signal Delay 3 | 419 |
| Encoder 4 | Signal Delay 4 | 448 |
|  | Signal Delay1 | 475 |
|  | Signal Delay 2 | 555 |


|  | Signal Delay 3 | 611 |
| :---: | :---: | :---: |
|  | Signal Delay 4 | 638 |
| Encoder 5 | Signal Delay1 | 718 |
|  | Signal Delay 2 | 745 |
|  | Signal Delay 3 | 774 |
|  | Signal Delay 4 | 801 |
| Encoder 6 | Signal Delay1 | 881 |
|  | Signal Delay 2 | 908 |
|  | Signal Delay 3 | 937 |
|  | Signal Delay 4 | 964 |
| Encoder 7 | Signal Delay1 | 1044 |
|  | Signal Delay 2 | 1071 |
|  | Signal Delay 3 | 1100 |
|  | Signal Delay 4 | 1127 |
| Encoder 8 | Signal Delay1 | 1207 |
|  | Signal Delay 2 | 1234 |
|  | Signal Delay 3 | 1263 |
|  | Signal Delay 4 | 1290 |
| Encoder 9 | Signal Delay1 | 1370 |
|  | Signal Delay 2 | 1397 |
|  | Signal Delay 3 | 1426 |
|  | Signal Delay 4 | 1453 |
| Encoder 10 | Signal Delay1 | 1533 |
|  | Signal Delay 2 | 1560 |
|  | Signal Delay 3 | 1589 |
|  | Signal Delay 4 | 1616 |
| Encoder 11 | Signal Delay1 | 1696 |
|  | Signal Delay 2 | 1723 |
|  | Signal Delay 3 | 1752 |
|  | Signal Delay 4 | 1779 |
| Encoder 12 | Signal Delay1 | 1859 |
|  | Signal Delay 2 | 1886 |
|  | Signal Delay 3 | 1915 |
|  | Signal Delay 4 | 1942 |
| Encoder 13 | Signal Delay1 | 2022 |
|  | Signal Delay 2 | 2049 |
|  | Signal Delay 3 | 2078 |
|  | Signal Delay 4 | 2105 |
| Encoder 14 | Signal Delay1 | 2158 |
|  | Signal Delay 2 | 2186 |
|  | Signal Delay 3 | 2214 |
|  | Signal Delay 4 | 2242 |
| Encoder 15 | Signal Delay1 | 2321 |


|  | Signal Delay 2 | 2349 |
| :---: | :---: | :---: |
|  | Signal Delay 3 | 2377 |
|  | Signal Delay 4 | 2405 |
| Encoder 16 | Signal Delay1 | 2484 |
|  | Signal Delay 2 | 2512 |
|  | Signal Delay 3 | 2540 |
|  | Signal Delay 4 | 2568 |
| Encoder 17 | Signal Delay1 | 2647 |
|  | Signal Delay 2 | 2675 |
|  | Signal Delay 3 | 2703 |
|  | Signal Delay 4 | 2731 |
| Encoder 18 | Signal Delay1 | 2810 |
|  | Signal Delay 2 | 2838 |
|  | Signal Delay 3 | 2866 |
|  | Signal Delay 4 | 2894 |
| Encoder 19 | Signal Delay1 | 2973 |
|  | Signal Delay 2 | 3001 |
|  | Signal Delay 3 | 3029 |
|  | Signal Delay 4 | 3057 |
| Encoder 20 | Signal Delay1 | 3136 |
|  | Signal Delay 2 | 3164 |
|  | Signal Delay 3 | 3192 |
|  | Signal Delay 4 | 3220 |
| Encoder 21 | Signal Delay1 | 3299 |
|  | Signal Delay 2 | 3327 |
|  | Signal Delay 3 | 3355 |
|  | Signal Delay 4 | 3383 |
| Encoder 22 | Signal Delay1 | 3462 |
|  | Signal Delay 2 | 3490 |
|  | Signal Delay 3 | 3518 |
|  | Signal Delay 4 | 3546 |
| Encoder 23 | Signal Delay1 | 3625 |
|  | Signal Delay 2 | 3653 |
|  | Signal Delay 3 | 3681 |
|  | Signal Delay 4 | 3709 |
| Encoder 24 | Signal Delay1 | 3788 |
|  | Signal Delay 2 | 3816 |
|  | Signal Delay 3 | 3844 |
|  | Signal Delay 4 | 3872 |
| Encoder 25 | Signal Delay1 | 3951 |
|  | Signal Delay 2 | 3979 |
|  | Signal Delay 3 | 4007 |
|  | Signal Delay 4 | 4035 |


| Encoder 26 | Signal Delay1 | 4114 |
| :---: | :---: | :---: |
|  | Signal Delay 2 | 4142 |
|  | Signal Delay 3 | 4170 |
|  | Signal Delay 4 | 4198 |
| Encoder 27 | Signal Delay1 | 4278 |
|  | Signal Delay 2 | 4308 |
|  | Signal Delay 3 | 4338 |
|  | Signal Delay 4 | 4366 |
| Encoder 28 | Signal Delay1 | 4441 |
|  | Signal Delay 2 | 4471 |
|  | Signal Delay 3 | 4501 |
|  | Signal Delay 4 | 4529 |
| Encoder 29 | Signal Delay1 | 4604 |
|  | Signal Delay 2 | 4634 |
|  | Signal Delay 3 | 4664 |
|  | Signal Delay 4 | 4692 |
| Encoder 30 | Signal Delay1 | 4767 |
|  | Signal Delay 2 | 4797 |
|  | Signal Delay 3 | 4827 |
|  | Signal Delay 4 | 4855 |
| Encoder 31 | Signal Delay1 | 4930 |
|  | Signal Delay 2 | 4960 |
|  | Signal Delay 3 | 4990 |
|  | Signal Delay 4 | 5018 |
| Encoder 32 | Signal Delay1 | 5093 |
|  | Signal Delay 2 | 5123 |
|  | Signal Delay 3 | 5153 |
|  | Signal Delay 4 | 5181 |
| Encoder 33 | Signal Delay1 | 5256 |
|  | Signal Delay 2 | 5286 |
|  | Signal Delay 3 | 5316 |
|  | Signal Delay 4 | 5344 |
| Encoder 34 | Signal Delay1 | 5419 |
|  | Signal Delay 2 | 5449 |
|  | Signal Delay 3 | 5479 |
|  | Signal Delay 4 | 5507 |
| Encoder 35 | Signal Delay1 | 5583 |
|  | Signal Delay 2 | 5613 |
|  | Signal Delay 3 | 5629 |
|  | Signal Delay 4 | 5648 |
| Encoder 36 | Signal Delay1 | 5746 |
|  | Signal Delay 2 | 5776 |
|  | Signal Delay 3 | 5792 |


|  | Signal Delay 4 | 5820 |
| :---: | :---: | :---: |
| Encoder 37 | Signal Delay1 | 5909 |
|  | Signal Delay 2 | 5925 |
|  | Signal Delay 3 | 5955 |
|  | Signal Delay 4 | 5983 |
| Encoder 38 | Signal Delay1 | 6072 |
|  | Signal Delay 2 | 6088 |
|  | Signal Delay 3 | 6118 |
|  | Signal Delay 4 | 6146 |
| Encoder 39 | Signal Delay1 | 6221 |
|  | Signal Delay 2 | 6251 |
|  | Signal Delay 3 | 6281 |
|  | Signal Delay 4 | 6309 |
| Encoder 40 | Signal Delay1 | 6370 |
|  | Signal Delay 2 | 6400 |
|  | Signal Delay 3 | 6430 |
|  | Signal Delay 4 | 6460 |
| Encoder 41 | Signal Delay1 | 6533 |
|  | Signal Delay 2 | 6563 |
|  | Signal Delay 3 | 6593 |
|  | Signal Delay 4 | 6623 |
| Encoder 42 | Signal Delay1 | 6696 |
|  | Signal Delay 2 | 6726 |
|  | Signal Delay 3 | 6756 |
|  | Signal Delay 4 | 6786 |
| Encoder 43 | Signal Delay1 | 6859 |
|  | Signal Delay 2 | 6889 |
|  | Signal Delay 3 | 6919 |
|  | Signal Delay 4 | 6949 |
| Encoder 44 | Signal Delay1 | 7022 |
|  | Signal Delay 2 | 7052 |
|  | Signal Delay 3 | 7082 |
|  | Signal Delay 4 | 7112 |
| Encoder 45 | Signal Delay1 | 7185 |
|  | Signal Delay 2 | 7215 |
|  | Signal Delay 3 | 7245 |
|  | Signal Delay 4 | 7275 |
| Encoder 46 | Signal Delay1 | 7348 |
|  | Signal Delay 2 | 7378 |
|  | Signal Delay 3 | 7408 |
|  | Signal Delay 4 | 7438 |
| Encoder 47 | Signal Delay1 | 7511 |
|  | Signal Delay 2 | 7541 |


|  | Signal Delay 3 | 7571 |
| :---: | :---: | :---: |
|  | Signal Delay 4 | 7587 |
| Encoder 48 | Signal Delay1 | 7674 |
|  | Signal Delay 2 | 7704 |
|  | Signal Delay 3 | 7734 |
|  | Signal Delay 4 | 7750 |
| Encoder 49 | Signal Delay1 | 7837 |
|  | Signal Delay 2 | 7867 |
|  | Signal Delay 3 | 7883 |
|  | Signal Delay 4 | 7913 |
| Encoder 50 | Signal Delay1 | 8000 |
|  | Signal Delay 2 | 8030 |
|  | Signal Delay 3 | 8046 |
|  | Signal Delay 4 | 8076 |
| Encoder 51 | Signal Delay1 | 8163 |
|  | Signal Delay 2 | 8179 |
|  | Signal Delay 3 | 8208 |
|  | Signal Delay 4 | 8239 |
| Encoder 52 | Signal Delay1 | 8326 |
|  | Signal Delay 2 | 8342 |
|  | Signal Delay 3 | 8372 |
|  | Signal Delay 4 | 8402 |
| Encoder 53 | Signal Delay1 | 8491 |
|  | Signal Delay 2 | 8523 |
|  | Signal Delay 3 | 8548 |
|  | Signal Delay 4 | 8577 |
| Encoder 54 | Signal Delay1 | 8654 |
|  | Signal Delay 2 | 8686 |
|  | Signal Delay 3 | 8711 |
|  | Signal Delay 4 | 8740 |
| Encoder 55 | Signal Delay1 | 8817 |
|  | Signal Delay 2 | 8849 |
|  | Signal Delay 3 | 8874 |
|  | Signal Delay 4 | 8903 |
| Encoder 56 | Signal Delay1 | 8980 |
|  | Signal Delay 2 | 9012 |
|  | Signal Delay 3 | 9037 |
|  | Signal Delay 4 | 9066 |
| Encoder 57 | Signal Delay1 | 9143 |
|  | Signal Delay 2 | 9175 |
|  | Signal Delay 3 | 9200 |
|  | Signal Delay 4 | 9229 |
| Encoder 58 | Signal Delay1 | 9306 |


|  | Signal Delay 2 | 9338 |
| :---: | :---: | :---: |
|  | Signal Delay 3 | 9363 |
|  | Signal Delay 4 | 9392 |
| Encoder 59 | Signal Delay1 | 9469 |
|  | Signal Delay 2 | 9501 |
|  | Signal Delay 3 | 9526 |
|  | Signal Delay 4 | 9555 |
| Encoder 60 | Signal Delay1 | 9632 |
|  | Signal Delay 2 | 9664 |
|  | Signal Delay 3 | 9689 |
|  | Signal Delay 4 | 9718 |
| Encoder 61 | Signal Delay1 | 9795 |
|  | Signal Delay 2 | 9813 |
|  | Signal Delay 3 | 9852 |
|  | Signal Delay 4 | 9881 |
| Encoder 62 | Signal Delay1 | 9959 |
|  | Signal Delay 2 | 9977 |
|  | Signal Delay 3 | 10016 |
|  | Signal Delay 4 | 10031 |
| Encoder 63 | Signal Delay1 | 10122 |
|  | Signal Delay 2 | 10140 |
|  | Signal Delay 3 | 10165 |
|  | Signal Delay 4 | 10194 |
| Encoder 64 | Signal Delay1 | 10271 |
|  | Signal Delay 2 | 10303 |
|  | Signal Delay 3 | 10328 |
|  | Signal Delay 4 | 10357 |
| Encoder 65 | Signal Delay1 | 10434 |
|  | Signal Delay 2 | 10466 |
|  | Signal Delay 3 | 10491 |
|  | Signal Delay 4 | 10520 |
| Encoder 66 | Signal Delay1 | 10582 |
|  | Signal Delay 2 | 10614 |
|  | Signal Delay 3 | 10639 |
|  | Signal Delay 4 | 10671 |
| Encoder 67 | Signal Delay1 | 10745 |
|  | Signal Delay 2 | 10777 |
|  | Signal Delay 3 | 10802 |
|  | Signal Delay 4 | 10834 |
| Encoder 68 | Signal Delay1 | 10908 |
|  | Signal Delay 2 | 10940 |
|  | Signal Delay 3 | 10965 |
|  | Signal Delay 4 | 10997 |


| Encoder 69 | Signal Delay1 | 11071 |
| :---: | :---: | :---: |
|  | Signal Delay 2 | 11103 |
|  | Signal Delay 3 | 11128 |
|  | Signal Delay 4 | 11160 |
| Encoder 70 | Signal Delay1 | 11234 |
|  | Signal Delay 2 | 11266 |
|  | Signal Delay 3 | 11291 |
|  | Signal Delay 4 | 11323 |
| Encoder 71 | Signal Delay1 | 11397 |
|  | Signal Delay 2 | 11429 |
|  | Signal Delay 3 | 11454 |
|  | Signal Delay 4 | 11486 |
| Encoder 72 | Signal Delay1 | 11560 |
|  | Signal Delay 2 | 11592 |
|  | Signal Delay 3 | 11617 |
|  | Signal Delay 4 | 11649 |
| Encoder 73 | Signal Delay1 | 11723 |
|  | Signal Delay 2 | 11755 |
|  | Signal Delay 3 | 11780 |
|  | Signal Delay 4 | 11812 |
| Encoder 74 | Signal Delay1 | 11886 |
|  | Signal Delay 2 | 11918 |
|  | Signal Delay 3 | 11943 |
|  | Signal Delay 4 | 11961 |
| Encoder 75 | Signal Delay1 | 12049 |
|  | Signal Delay 2 | 12067 |
|  | Signal Delay 3 | 12106 |
|  | Signal Delay 4 | 12124 |
| Encoder 76 | Signal Delay1 | 12212 |
|  | Signal Delay 2 | 12230 |
|  | Signal Delay 3 | 12269 |
|  | Signal Delay 4 | 12287 |
| Encoder 77 | Signal Delay1 | 12375 |
|  | Signal Delay 2 | 12393 |
|  | Signal Delay 3 | 12432 |
|  | Signal Delay 4 | 12450 |
| Encoder 78 | Signal Delay1 | 12538 |
|  | Signal Delay 2 | 12556 |
|  | Signal Delay 3 | 12581 |
|  | Signal Delay 4 | 12613 |
| Encoder 79 | Signal Delay1 | 12704 |
|  | Signal Delay 2 | 12731 |
|  | Signal Delay 3 | 12758 |


|  | Signal Delay 4 | 12788 |
| :---: | :---: | :---: |
| Encoder 80 | Signal Delay1 | 12867 |
|  | Signal Delay 2 | 12894 |
|  | Signal Delay 3 | 12921 |
|  | Signal Delay 4 | 12951 |
| Encoder 81 | Signal Delay1 | 13030 |
|  | Signal Delay 2 | 13057 |
|  | Signal Delay 3 | 13084 |
|  | Signal Delay 4 | 13114 |
| Encoder 82 | Signal Delay1 | 13193 |
|  | Signal Delay 2 | 13220 |
|  | Signal Delay 3 | 13247 |
|  | Signal Delay 4 | 13277 |
| Encoder 83 | Signal Delay1 | 13356 |
|  | Signal Delay 2 | 13383 |
|  | Signal Delay 3 | 13410 |
|  | Signal Delay 4 | 13440 |
| Encoder 84 | Signal Delay1 | 13519 |
|  | Signal Delay 2 | 13546 |
|  | Signal Delay 3 | 13573 |
|  | Signal Delay 4 | 13603 |
| Encoder 85 | Signal Delay1 | 13682 |
|  | Signal Delay 2 | 13709 |
|  | Signal Delay 3 | 13736 |
|  | Signal Delay 4 | 13604 |
| Encoder 86 | Signal Delay1 | 13845 |
|  | Signal Delay 2 | 13872 |
|  | Signal Delay 3 | 13899 |
|  | Signal Delay 4 | 13929 |
| Encoder 87 | Signal Delay1 | 14008 |
|  | Signal Delay 2 | 14035 |
|  | Signal Delay 3 | 14062 |
|  | Signal Delay 4 | 14092 |
| Encoder 88 | Signal Delay1 | 14171 |
|  | Signal Delay 2 | 14198 |
|  | Signal Delay 3 | 14225 |
|  | Signal Delay 4 | 14255 |
| Encoder 89 | Signal Delay1 | 14321 |
|  | Signal Delay 2 | 14362 |
|  | Signal Delay 3 | 14389 |
|  | Signal Delay 4 | 14405 |
| Encoder 90 | Signal Delay1 | 14484 |
|  | Signal Delay 2 | 14511 |


|  | Signal Delay 3 | 14552 |
| :---: | :---: | :---: |
|  | Signal Delay 4 | 14568 |
| Encoder 91 | Signal Delay1 | 14647 |
|  | Signal Delay 2 | 14674 |
|  | Signal Delay 3 | 14701 |
|  | Signal Delay 4 | 14731 |
| Encoder 92 | Signal Delay1 | 14794 |
|  | Signal Delay 2 | 14828 |
|  | Signal Delay 3 | 14855 |
|  | Signal Delay 4 | 14882 |
| Encoder 93 | Signal Delay1 | 14957 |
|  | Signal Delay 2 | 14991 |
|  | Signal Delay 3 | 15018 |
|  | Signal Delay 4 | 15045 |
| Encoder 94 | Signal Delay1 | 15120 |
|  | Signal Delay 2 | 15154 |
|  | Signal Delay 3 | 15181 |
|  | Signal Delay 4 | 15208 |
| Encoder 95 | Signal Delay1 | 15283 |
|  | Signal Delay 2 | 15317 |
|  | Signal Delay 3 | 15344 |
|  | Signal Delay 4 | 15371 |
| Encoder 96 | Signal Delay1 | 15446 |
|  | Signal Delay 2 | 15480 |
|  | Signal Delay 3 | 15507 |
|  | Signal Delay 4 | 15534 |
| Encoder 97 | Signal Delay1 | 15609 |
|  | Signal Delay 2 | 15643 |
|  | Signal Delay 3 | 15670 |
|  | Signal Delay 4 | 15697 |
| Encoder 98 | Signal Delay1 | 15772 |
|  | Signal Delay 2 | 15806 |
|  | Signal Delay 3 | 15833 |
|  | Signal Delay 4 | 15860 |
| Encoder 99 | Signal Delay1 | 15935 |
|  | Signal Delay 2 | 15955 |
|  | Signal Delay 3 | 15996 |
|  | Signal Delay 4 | 16023 |
| Encoder 100 | Signal Delay1 | 16098 |
|  | Signal Delay 2 | 16118 |
|  | Signal Delay 3 | 16145 |
|  | Signal Delay 4 | 16186 |
| Encoder 101 | Signal Delay1 | 16261 |


|  | Signal Delay 2 | 16281 |
| :---: | :---: | :---: |
|  | Signal Delay 3 | 16308 |
|  | Signal Delay 4 | 16335 |
| Encoder 102 | Signal Delay1 | 16424 |
|  | Signal Delay 2 | 16444 |
|  | Signal Delay 3 | 16471 |
|  | Signal Delay 4 | 16498 |
| Encoder 103 | Signal Delay1 | 16587 |
|  | Signal Delay 2 | 16607 |
|  | Signal Delay 3 | 16633 |
|  | Signal Delay 4 | 16661 |
| Encoder 104 | Signal Delay1 | 16750 |
|  | Signal Delay 2 | 16770 |
|  | Signal Delay 3 | 16797 |
|  | Signal Delay 4 | 16824 |
| Encoder 105 | Signal Delay1 | 16917 |
|  | Signal Delay 2 | 16946 |
|  | Signal Delay 3 | 16975 |
|  | Signal Delay 4 | 16997 |
| Encoder 106 | Signal Delay1 | 17080 |
|  | Signal Delay 2 | 17109 |
|  | Signal Delay 3 | 17138 |
|  | Signal Delay 4 | 17162 |
| Encoder 107 | Signal Delay1 | 17243 |
|  | Signal Delay 2 | 17272 |
|  | Signal Delay 3 | 17301 |
|  | Signal Delay 4 | 17325 |
| Encoder 108 | Signal Delay1 | 17406 |
|  | Signal Delay 2 | 17435 |
|  | Signal Delay 3 | 17464 |
|  | Signal Delay 4 | 17488 |
| Encoder 109 | Signal Delay1 | 17569 |
|  | Signal Delay 2 | 17598 |
|  | Signal Delay 3 | 17627 |
|  | Signal Delay 4 | 17651 |
| Encoder 110 | Signal Delay1 | 17732 |
|  | Signal Delay 2 | 17761 |
|  | Signal Delay 3 | 17790 |
|  | Signal Delay 4 | 17814 |
| Encoder 111 | Signal Delay1 | 17895 |
|  | Signal Delay 2 | 17924 |
|  | Signal Delay 3 | 17953 |
|  | Signal Delay 4 | 17977 |


| Encoder 112 | Signal Delay1 | 18058 |
| :---: | :---: | :---: |
|  | Signal Delay 2 | 18087 |
|  | Signal Delay 3 | 18116 |
|  | Signal Delay 4 | 18140 |
| Encoder 113 | Signal Delay1 | 18221 |
|  | Signal Delay 2 | 18250 |
|  | Signal Delay 3 | 18265 |
|  | Signal Delay 4 | 18303 |
| Encoder 114 | Signal Delay1 | 18384 |
|  | Signal Delay 2 | 18399 |
|  | Signal Delay 3 | 18428 |
|  | Signal Delay 4 | 18466 |
| Encoder 115 | Signal Delay1 | 18533 |
|  | Signal Delay 2 | 18562 |
|  | Signal Delay 3 | 18591 |
|  | Signal Delay 4 | 18629 |
| Encoder 116 | Signal Delay1 | 18697 |
|  | Signal Delay 2 | 18726 |
|  | Signal Delay 3 | 18755 |
|  | Signal Delay 4 | 18779 |
| Encoder 117 | Signal Delay1 | 18860 |
|  | Signal Delay 2 | 18889 |
|  | Signal Delay 3 | 18918 |
|  | Signal Delay 4 | 18942 |
| Encoder 118 | Signal Delay1 | 19006 |
|  | Signal Delay 2 | 19035 |
|  | Signal Delay 3 | 19064 |
|  | Signal Delay 4 | 19093 |
| Encoder 119 | Signal Delay1 | 19169 |
|  | Signal Delay 2 | 19198 |
|  | Signal Delay 3 | 19227 |
|  | Signal Delay 4 | 19256 |
| Encoder 120 | Signal Delay1 | 19332 |
|  | Signal Delay 2 | 19361 |
|  | Signal Delay 3 | 19390 |
|  | Signal Delay 4 | 19419 |
| Encoder 121 | Signal Delay1 | 19495 |
|  | Signal Delay 2 | 19524 |
|  | Signal Delay 3 | 19553 |
|  | Signal Delay 4 | 19582 |
| Encoder 122 | Signal Delay1 | 19658 |
|  | Signal Delay 2 | 19687 |
|  | Signal Delay 3 | 19716 |


|  | Signal Delay 4 | 19745 |
| :---: | :---: | :---: |
| Encoder 123 | Signal Delay1 | 19821 |
|  | Signal Delay 2 | 19850 |
|  | Signal Delay 3 | 19879 |
|  | Signal Delay 4 | 19908 |
| Encoder 124 | Signal Delay1 | 19984 |
|  | Signal Delay 2 | 20013 |
|  | Signal Delay 3 | 20042 |
|  | Signal Delay 4 | 20071 |
| Encoder 125 | Signal Delay1 | 20147 |
|  | Signal Delay 2 | 20176 |
|  | Signal Delay 3 | 20205 |
|  | Signal Delay 4 | 20234 |
| Encoder 126 | Signal Delay1 | 20310 |
|  | Signal Delay 2 | 20339 |
|  | Signal Delay 3 | 20368 |
|  | Signal Delay 4 | 20397 |
| Encoder 127 | Signal Delay1 | 20473 |
|  | Signal Delay 2 | 20502 |
|  | Signal Delay 3 | 20531 |
|  | Signal Delay 4 | 20560 |
| Encoder128 | Signal Delay1 | 20636 |
|  | Signal Delay 2 | 20665 |
|  | Signal Delay 3 | 20694 |
|  | Signal Delay 4 | 20709 |

