

6-man Chess and Zugzwangs

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Abstract. With 6-man Chess essentially solved, the available 6-man Endgame Tables (EGTs) have been scanned for zugzwang positions where, unusually, having the move is a disadvantage. Review statistics together with some highlights and positions are provided here: the complete information is available on the ICGA website. An outcome of the review is the observation that the definition of *zugzwang* should be revisited, if only because the presence of *en passant capture* moves gives rise to three new, asymmetric types of zugzwang.

Keywords: chess, endgame tables, maximal, Nalimov, zugzwang

1 Introduction

Six-man Chess is essentially solved and the Nalimov [1] Endgame Tables (EGTs) to DTM, Depth to Mate, have been widely promulgated for some time [2]¹. The corpus of perfect information is a challenge to datamine for both helpful guidelines and for the pathological positions – the deep, exceptional, bizarre and amusing.

A *zugzwang* position is defined here as one where the side to move would prefer that it were the other side's turn to move. They are remarkable in themselves and the 'zug', much sought after by composers, is a running theme in the Study community [3-9]. The zug is also the counterexample to the assumption of the Null Move Heuristic that having the move is an advantage. When 5-man chess was solved, the two sets of respectively DTM and DTC² [10] EGTs were searched [11] for zugs³. A later review of Thompson's 6-man pawnless DTC EGTs also included a zug search [12].

The zugzwang search reported here is almost entirely of the highly available Nalimov DTM EGTs [2], [13]. The search was carried out via the web using Bourzutschky's GTBGEN and the first author's EGTs and JAVA code. Unpublished FEG DTM EGTs [14] for 5-1(p) chess and DTC EGTs for 6-man chess [15-16] exist and Bourzutschky has supplied [16] a zug review of 5-1(p) chess from the latter.

Section 2 considers *zugzwang* definitions and identifies three new types of zug; section 3 covers logistics. Section 4 is a summary of the main findings. Tables 1-5 list various illustrative statistics and positions; the full details are available via the ICGA website [17] which will host the evolving story of the zugzwang.

¹ *Essentially* because positions with castling rights are not yet included in EGTs.

² DTC ≡ Depth to Conversion, i.e. to force-change and/or Mate.

³ Incidentally providing a partial cross-check of agreement between the two sets of EGTs.

2 Definitions of *The Zugzwang*

Zugzwang is defined [18] as *pressure to take action* and a *zugzwang position* is defined to be one where this pressure is unwelcome – where the first player would rather ‘pass the position across’.⁴ However, in [19] a *zugzwang position* is defined as ‘a position in which whoever has the move would obtain a worse result than if it were the opponent’s turn to play’. Note that this now involves the 2nd player’s perspective and focuses only on the outcome without considering its achievement or likelihood. Other authorities refer to zugzwangs as *reciprocal* or *mutual zugzwangs*. The words *whoever*, *reciprocal* and *mutual* suggest a symmetry, perhaps assuming incorrectly that the 2nd player can always pass back the 1st position to the 1st player.

Consider the *en passant zone EPZ* of Chess, i.e., those positions where there is an en passant capture option. Let $p1 \in EPZ$: what now are positions $p2$ and $p3$? The proposal here⁵ is to clarify this situation by formalising the notions of *passing over* or *losing the move* as one of *playing a null move* or *nulling*. Now the rules of chess define $p2$ and $p3$: the e.p.-capture option in $p1$ disappears if not played immediately, $p2$ does not feature any e.p.-capture option as the 1st player has not moved a Pawn two squares, and similarly, $p3$ is $p1$ without e.p.-capture option. Positions in EPZ are 0.62% of those in their endgame and zugs in EPZ comprise 0.22% of the total.

Let the *Level A zugzwang*, our focus here, be described in these terms:

- a) positions are valued from 1st player’s perspective: loss (0), draw (1), win (2),
- b) if there is force-symmetry and/or no *e.p.*, 1st player is assumed to be White
Black plays first in 11 Table 1 positions (Zs 04-5, 07-9, 12, 22-4, 26-27),
- c) 1st player in position $p1$ (value $v1$) nulls to $p2$ (value $v2$) iff $v2 > v1$,⁶
- d) 2nd player may *or may not*, c.f. Table 1’s Z08-9, null to $p3$, value $v3 \leq v2$,
- e) if $p3 \equiv p1$, $v3 \equiv v1$. If $p3 \neq p1$, $p3$ is ‘stalemate’ or $v3 \leq v1$.

Fig. 1 is formatted so that the 1st player nulls ‘to the right’ and the 2nd player nulls to the left to increase value to themselves. Clearly, in addition to the three familiar zug types A1-3,⁷ we now have, exclusively in the EPZ, just three more types A4-6:⁸

- 1) type A1 \equiv ‘121’ \equiv ‘draw-win-draw’, q.v. Z01 in Table 1
- 2) type A2 \equiv ‘010’ \equiv ‘loss-draw-loss’, q.v. Z02: A1-3 are *no net gain* for player 1
- 3) type A3 \equiv ‘020’ \equiv ‘loss-win-loss’, the *full-point zug*, q.v. Z03
- 4) type A4 \equiv ‘120’ \equiv ‘draw-win-loss’, a *net loss* for player 1, q.v. Z05-6
- 5) type A5 \equiv ‘021’ \equiv ‘loss-win-draw’, a *net gain* for player 1, q.v. Z07
- 6) type A6 \equiv ‘01(1)’ \equiv ‘loss-draw(-draw)’, a *net gain* for player 1⁹, q.v. Z08-9

Clearly, type A4-6 zugs are asymmetric. Considered only in terms of the first two positions, A4 becomes A1, A5 becomes A3 and A6 becomes A2.

⁴ Not strictly possible, as in ‘passing the position across’, the side to move changes.

⁵ Our project log notes that Bourzutschky proposed the *Null Move* concept on 2005-05-31.

⁶ *Value* is calculated as normally, assuming that the option of a null move is not available.

⁷ Types are distinguished by the sequence $v_1-v_2-v_3$ rather than just by the sequence v_1-v_2 .

⁸ Unless position $p3$ is *stalemate*, its value $v3 \leq v1$ as the 1st player has one less move in $p3$.

⁹ The 2nd player may even prefer to play on rather than stalemate their opponent.

3 Enumeration: Endgames and Zugzwang Occurrences

These notes explain the lexical ordering of men and of endgames, and the principles used for counting the occurrences of zugzwangs.

The men are listed in the *strength order* K-Q-R-B-N-P. White has at least as many men as Black. In *m-m* endgames, White's *lead men* are at least as strong as Black's.

No attempt is made to eliminate unreachable positions in EGT statistics: this is usual as there is no general algorithm. With this limitation, the count is of FEN-distinct and functionally unique zugzwangs. Thus, no zugzwang can be physically transformed into any other. The following subtleties should be noted:

- a) For force-symmetric zugs $z \notin \text{EPZ}$, type A1 and A2 zugs are equivalent: the count of A2 zugs is shown in brackets,
- b) Given force symmetry, A3 zugs usually appears in two physical forms:
 - the two physical versions were identified¹² and counted as one here,
- c) When both Kings are on a1-h8 or a8-h1 in pawnless endgames: Nalimov has both physical versions of the position if there are two; the two physical versions were identified and counted as one here,
- d) When e.p.-capture has been enabled but is actually illegal, q.v. Z12: the position is counted here as different from that without the *e.p.*, 1st player would have to realize that the e.p.-option is illusory, FIDE's recently reworded Article 9.2 now seeks to ignore the *e.p.* [20-22],
- e) An example of an unreachable zug:
 - position Z12 also implies the prior 1. b2-b4, impossible on two grounds:
 - before 1. b2-b4, the side not to move, Black, is in (double) check,
 - check from a Pawn on its home square is itself impossible,
 - therefore, the position prior to Z12 is also unreachable.

We note one small, historical and now resolved hiatus with respect to these results. The identification of zugs of types A4-6 was a serendipitous accident¹³ and was initially regarded as a bug by GTBGEN's author Marc Bourzutschky.

When the 'last 16' 3-3p endgames to KPPKPP were published, both by MB converting his FEG EGTs to Nalimov's format¹⁴ [23] and by Nalimov returning a disc to Hernandez [24], MB discovered that although he had anticipated a 2-byte format for the KQPK(B/R)P EGTs in GTBGEN, Nalimov had in fact discovered that the 1-byte format would suffice [25]. MB realigned with Nalimov and removed the ability to detect type A4-6 zugs before a GTBGEN was provided that could address these two EGTs. MB now advises [16] that no type A4-6 KQPKBP or KQPKRP zug exists.

Results for Level A zugs have been published as a set of files [17]. Table 2 provides summary statistics for the six blocks of 6-man zugs, 3-3(p), 4-2(p) and 5-1(p). There are 293 5-1p zugs, 261 created by a touring Knight being on the wrong foot.

Table 3 details the occurrences of e.p. zugs including the new types of zug. Table 4 lists in lexical order all endgames with A3 zugs, together with an example of their deepest such zug. Table 5 is a miscellany of exemplar positions: an A2 e.p. zug with

¹² The identification of symmetries and equivalences was done by the first author's code.

¹³ The serendipitous accident has its place of honour in the history of discovery.

¹⁴ After reverse-engineering the unpublished format of the FEG EGT data format.

p3's depth less than p1's (P03), an A3 e.p. zug (N01), A2 and A3 zugs dezugged by the addition of an e.p. opportunity (N02-3) or castling option (P01), zugs unaffected (P02) or created (N04-8) by giving castling rights to the 1st or 2nd player, 5-1p zugs (P06-13), 7-man zugs (B01-6) and *Zugzwang Studies* (S01-12) featuring zugs. Elkie's [26] composed N01-10 and Bourzutschky [15-16] found B01-6.

Table 2. Level A zugs: summary statistics for each 6-man endgame group.¹⁵⁻¹⁶

Item \ Group	3-3	4-2	5-1	3-3p	4-2p	5-1p	Total
01 Endgames	55	80	35	65	95	35	365
02 <i>No zugs</i>	12	50	35	12	29	28	166
03 No A1 zugs	12	50	35	12	30	28	167
04 No A2 zugs	31 (+ 9)	71	—	22 (+ 3)	51	—	175 (+ 12)
05 No A3 zugs	55	80	—	50	67	—	252
06 <i>A unique zug</i>	4	1	0	1	2	0	8
07 One A1 zug	4	1	0	1	4	0	10
08 One A2 zug	1	2	—	1	6	—	10
09 One A3 zug	0	0	—	6	3	—	9
10 A1-A6 zugs	27,597	20,017	0	380,363	478,682	293	906,952
11 A1 zugs	27,470	8,434	0	361,725	373,479	293	771,401
12 A2 zugs	127	11,583	—	15,543	105,069	—	132,322
13 A3 zugs	0	0	—	2,700	133	—	2,833
14 A4 zugs	—	—	—	394	0	—	394
15 A5 zugs	—	—	—	0	0	—	0
16 A6 zugs	—	—	—	1	1	—	2

Table 3. Statistics for the fourteen endgames with *e.p.-zugs*.

Endgame	e.p.-zugs		A1		A2		A3		A4		A5		A6	
	wtm	btm	wtm	btm	wtm	btm	wtm	btm	wtm	btm	wtm	btm	wtm	btm
KPPKP	35	20	35	0	0	20	0	0	0	0	0	0	0	0
KBPKBP	10	—	10	—	0	—	0	—	0	—	0	—	0	—
KBPKNP	130	5	130	2	0	3	0	0	0	0	0	0	0	0
KBPKPP	18	18	18	4	0	13	0	0	0	1	0	0	0	0
KNPKNP	156	—	156	—	0	—	0	—	0	—	0	—	0	—
KNPKPP	250	19	250	17	0	2	0	0	0	0	0	0	0	0
KPPKPP	1,301	—	869	—	39	—	0	—	393	—	0	—	0	—
KQPKQP	75	—	72	—	3	—	0	—	0	—	0	—	0	—
KRPKBP	20	7	20	0	0	7	0	0	0	0	0	0	0	0
KRPKNP	27	14	27	0	0	14	0	0	0	0	0	0	0	0
KRPKPP	0	2	0	0	0	1	0	0	0	0	0	0	0	1
KBPPKP	1	0	1	0	0	0	0	0	0	0	0	0	0	0
KNPPKP	0	1	0	0	0	1	0	0	0	0	0	0	0	0
KPPPKP	8	2	8	0	0	1	0	0	0	0	0	0	0	1
Totals	2,031	88	1,596	23	42	62	0	0	393	1	0	0	0	2

¹⁵ (+n) indicates that there are n endgames whose A2 zugs merely mirror their A1 zugs.

¹⁶ The eight zugzwangs which are unique across their endgames are Z12-Z19 in Table 1.

Table 4. The 43 endgames with *full-point* type A3 zugs: maxDTM examples.^{17,18,19}

id	Endgame	Position	val / DTM			Total Depth	Flag
			p1	p2	p3		
F01	KBPKNP	k7/Bp4n1/1K1P4/8/8/8/8 w	-25	+16	-25	41	
F02	KBPKPP	8/8/8/3K1k1p/3P2p1/6B1/8 w	-57	+28	-57	85	
F03	KNPKNP	8/8/8/8/K1k5/P1p5/n1N5 w	-13	+24	-13	37	u
F04	KNPKPP	8/8/8/4kp1p/3N4/2KP4/8 w	-71	+26	-71	97	
F05	KPPKPP	8/8/8/p4p2/k1P5/2K1P3/8 w	-17	+103	-17	120	c.f. '18'
F06	KQNKQP	8/8/8/3N4/8/k2p4/3q4/KQ6 w	-14	+6	-14	20	u
F07	KQNKRP	QN6/Kpk5/1r6/8/8/8/8 w	-8	+32	-8	40	u
F08	KQPKQP	8/8/8/1Pq5/8/1K1Q4/5p2/2k5 w	-24	+95	-24	119	
F09	KQPKRB	2K1k3/2P5/8/8/8/1r2b3/4Q3 w	-20	+15	-20	35	u
F10	KQPKRP	8/8/8/1Q6/8/1pP5/2k2r2/K7 w	-12	+42	-12	54	
F11	KRNKNP	8/8/8/8/p7/N2k4/RK1n4 w	-1	+28	-1	29	u
F12	KRNKPP	8/8/8/2N5/8/7p/k5pR/2K5 w	-33	+17	-33	50	
F13	KRPKBP	8/8/8/8/2p5/2Pk4/1KR1b3 w	-15	+30	-15	45	
F14	KRPKNP	8/8/8/8/3n4/k7/p1P5/K1R5 w	-1	+33	-1	34	u
F15	KRPKPP	8/8/6R1/k1P5/2K5/7p/6p1/8 w	-71	+9	-71	80	
F16	KBBPKQ	8/8/8/1K6/BBP5/8/qk6 w	-102	+13	-102	115	
F17	KBNPKB	K7/P1k5/8/8/8/6N1/5B1b w	-2	+14	-2	16	
F18	KBNPKN	BK1n4/NP1k4/8/8/8/8/8 w	-1	+32	-1	33	
F19	KBPPKB	K7/P1k5/8/8/8/6P1/5B1b w	-2	+14	-2	16	
F20	KBPPKP	8/8/8/3k4/1K1p4/1P3P2/B7 w	-18	+16	-18	34	
F21	KBPPKQ	3K1kq1/8/4PB2/3P4/8/8/8 w	-34	+40	-34	74	c.f. '19'
F22	KBPPKR	8/8/8/8/2k5/P1P5/rBK5 w	-18	+22	-18	40	
F23	KNNPKN	K1k5/P2N4/4N3/3n4/8/8/8 w	-1	+19	-1	20	
F24	KNPPKN	K7/P1kN4/8/3P4/n7/8/8 w	-2	+20	-2	22	
F25	KNPPKP	8/8/8/K7/P1k5/1p6/3P4/4N3 w	-20	+23	-20	43	
F26	KNPPKQ	1K1k2q1/8/2P5/3N4/8/2P5/8/8 w	-20	+45	-20	65	
F27	KNPPKR	N1k5/2P5/rPK5/8/8/8/8 w	-22	+12	-22	34	
F28	KPPPKN	n7/P1k5/K7/PP6/8/8/8 w	-2	+13	-2	15	
F29	KPPPKP	8/8/8/5k2/3K1p2/3P3P/3P4/8 w	-20	+19	-20	39	
F30	KPPPKQ	k7/q1PK4/P7/8/8/2P5/8/8 w	-15	+19	-15	34	
F31	KPPPKR	1K6/1P1k4/1r6/1P6/2P5/8/8/8 w	-21	+36	-21	57	
F32	KQNPKN	QN6/KP6/8/nk6/8/8/8 w	-1	+8	-1	9	
F33	KQNPKQ	8/8/8/5N2/1q6/8/Q2P4/K1k5 w	-3	+35	-3	38	
F34	KQPPKQ	8/8/8/1q6/5P2/P7/Q7/K1k5 w	-4	+33	-4	37	
F35	KRBPKB	K7/P1k5/8/8/8/6R1/5B1b w	-2	+14	-2	16	
F36	KRBPKN	RK6/B3n3/1Pk5/8/8/8/8 w	-2	+14	-2	16	u
F37	KRBPKP	8/8/8/8/1k6/pP6/BRK5 w	-11	+21	-11	32	
F38	KRBPKQ	1qk5/8/RBP5/8/8/8/1K6 w	-41	+13	-41	54	u
F39	KRNPKN	8/8/8/n7/P7/K1k5/RN6 w	-1	+24	-1	25	
F40	KRNPKQ	1K3N2/3R1P2/1kq5/8/8/8/8 w	-11	+16	-11	27	
F41	KRPPKN	K7/P1k5/R1P5/8/2n5/8/8/8 w	-1	+20	-1	21	
F42	KRPPKQ	3R4/q7/k1P5/P7/K7/8/8/8 w	-36	+12	-36	48	u
F43	KRPPKR	8/8/8/8/rPK5/1RP5/2k5 w	-26	+29	-26	55	

¹⁷ The depth of a type A3 or A5 zug is defined as the sum of the depths of *p1* and *p2*.

¹⁸ KPPKPP zug F05 is the deepest A3 zug with *dtm* = 120.

¹⁹ KBPPKQ zug F21 has the maximal DTM-depth 'shallower side' loss (here) with *dtm* = 34.

Table 5. More didactic positions including 5-1p zugs and e.p. and/or castling effects.²⁰

id	Endgame	Position	DTx					
			Type	p1	p2	p3	x	Flag
P01	KQNKRR	1KQNK2r/7r/8/8/8/8/8 w - -	A1	=	+54	=	Z	c
P02	KRRKRb	r3kb2/1RK4R/8/8/8/8/8 w - -	A1	=	+7	=	Z	c
P03	KPPKPP	8/6p1/4k1P1/4Pp2/3K4/8/8/8 w - f6	A2	-24	=	-19	M	e
P04	KNNKP	7k/8/5NK1/7p/8/8/N7/8 b - -	B2	-0	1	-0	Z	B2
P05	KPPKPP	8/8/p2k4/6p1/3K2P1/P7/8/8 b - -	C	=	=	=	Z	C
P06	KBPPPK	5kBK/5P1P/7P/8/8/8/8 w - -	A1	=	+2	=	Z	5
P07	KBPPPK	7K/5kBP/5P1P/8/8/8/8 w - -	A1	=	+2	=	Z	5
P08	KBPPPK	8/B1k5/K7/P7/P7/8/8 w - -	A1	=	+3	=	Z	5
P09	KBPPPK	1k6/8/KP6/BP6/1P6/8/8/8 w - -	A1	=	+1	=	Z	5
P10	KNPPPK	7K/5k1P/4N2P/7P/8/8/8 w - -	A1	=	+2	=	Z	5
P11	KPPPKP	5k2/5P2/4K3/7P/7P/8/8 w - -	A1	=	+2	=	Z	5
P12	KQNPPK	4k1KQ/5NPP/8/8/8/8/8 w - -	A1	=	+2	=	Z	5
P13	KRPPPK	1k6/1P6/K7/RP6/P7/8/8/8 w - -	A1	=	+2	=	Z	5
N01	KPPPKPPP	8/1p6/8/pP4pK/5K1/P7/8/8 w - a6	A3	-1	+1	-1	Z	e
N02	KPPKPP	3K4/8/3k4/8/3Pp3/4P3/8/8 b - -	A2	-4	=	-4	Z	e
N03	KPPKPP	8/8/5pK1/4kPp1/8/7P/8 b - -	A3	-1	+1	-1	Z	e
N04	KQP(6)KRRBP(3)	Q1K1k2r/PPP1p2p/b1r1P2P/2p5/2P5/8/8/8 w k -	A2	-1	=	-1	Z	c
N05	KQP(6)KRRBP(5)	Q1K1k2r/PPP1p2p/bprpP2P/2p5/2P5/8/8/8 w k -	A3	-1	+2	-1	Z	c
N06	KQP(8)KRRBP(7)	Q1K1k2r/PPP1p2p/bprpP1pP/2p5/2P2pP1/8/5P2/8 b k g3	A3	-2	+2	-2	Z	c e
N07	KRBPN(3)KRBP	r3k1KR/3p2PB/3P2N1/3P3b/8/8/8 w q -	A3	-1	?	-1	Z	c
N08	KRBPN(4)KRBP(3)	r3k1KR/3p2PB/2pP2N1/7b/1pP5/8/1P6/8 b q c3	A3	-?	+1	-?	Z	c e
N09	KPPKPP	8/8/3k2P1/4pKP1/8/8/8 w - -	B1	+84	+25	+84	M	B1
N10	KPPKPP	8/8/3k4/1K1p4/1P6/1P6/8/8 b - -	C	=	=	=	Z	C
B01	KNNNKNN	7k/8/4N3/4NN2/n2K4/8/8/3n4 w - -	A1	=	+17	=	Z	s
B02	KRBBKQB	8/8/8/2b2q2/B7/1R3B2/2k1K3 w - -	A3	-96	+2	-96	Z	u
B03	KRBBKQN	8/5B1q/6R1/3n4/8/8/2KB4/k7 w - -	A3	-6	+2	-6	Z	
B04	KRRRKR	8/8/8/8/3Rr3/kr6/2KRR3 w - -	A1	=	+2	=	Z	U
B05	KBBBBKQ	6B1/1B4qB/5k2/8/3K4/8/6B1/8 w - -	A2	-35	=	-35	Z	s
B06	KBBNNKQ	8/8/8/8/4q3/2k4N/5B2/N1K2B2 w - -	A3	-7	+41	-7	Z	u
S01	KRKN	8/8/2k1K3/8/3R4/4n3/8 w - -	[9] #2, ar-Razi (~850)					
S05	KQKRP	1rk5/8/8/3Q4/8/1p6/1K6/8 w - -	[9] #6					
S03	KRKBN	k1K5/2n5/8/8/b7/1R6/8/8 w - -	[4] #457, [9] #5, Nunn					
S05	KRKBN	k3b3/n1K5/R7/8/8/8/8 w - -	[9] #5a					
S06	KNNKP	8/8/1p6/1K6/2N5/3N4/8/k7 w - -	[9] #9					
S07	KNPKP	8/8/8/6Pk/4K3/4N2p/8/8 w - -	[9] #7					
S08	KNPKN	8/8/8/5KPk/8/8/5N1n b - -	[9] #7a: A2 zug					
S09	KNPKP	8/8/8/4k3/7p/P3K1N1/8 w - -	[9] #8					
S10	KNPKN	8/8/8/4k3/8/P3K3/5N1n w - -	[9] #8a					
S11	KNPKPP	8/8/8/8/5p2/4k1p1/4N1P1/5K2 w - -	[39] #5.1, Mandler					
S12	KPPKPP	8/5pk1/8/2p1PK2/2P5/8/8/8 w - -	[9] #4					

Positions S01-S12 are from studies where White wins in an essentially unique way. They all feature a level A zug in both a *try* and the mainline solution and are mainly taken from Beasley [9]. The appendix can accommodate only a few of the solutions so there are plenty of exercises here for the reader.

²⁰ c ≡ zug-significant castling rights given to 1st or 2nd player, B/C ≡ Level B/C zug, e ≡ e.p. significant, and 5 ≡ 5-1p zug.

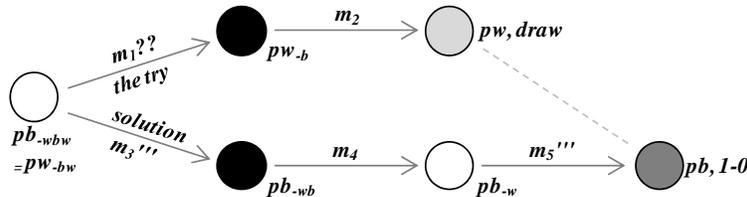


Fig. 2. The Zugzwang Study scenario.

4 Commentary: Statistics, Gems and Studies

Our EGT search has identified a corpus of over 900,000 zugs which may be reviewed in statistical terms, datamined for gems of various sorts, and put in the context of the Chess Study, the chess-engine and the game itself.

In addition to the Level A zug defined above, two further levels of zug are notable, q.v. Table 5. At *Level B*, a B1 (B2) zug is merely inconvenient, requiring the winner (loser) to make more (less) moves in some metric before some defined goal. Positions P04 (B2), N09 (B1) and S01-S12 feature examples. Note that the number of *moves to goal* may not be affected in all metrics.²¹ At *Level C*, zugs do not impact value or depth in any metric but the side to move would rather play a null move: Regan [27] identified P05 and Elkies [26] identified N10, both draws in which a null move eases the task of the defender. The likelihood of a win in a theoretically drawn position can be modelled using the concept of a Reference Fallible Player [28-32].

4.1 The Statistics

Pawnless zugs are just 5.26% of the total; pawnful zugs account for the vast majority. There are more pawnful than pawnless positions but the presence of at least one pawn increases the density of zugs by a factor of four. Zugs are also more frequent when Knights, parity-bound and unable to *lose the move*²², are present.

Type A3 zugs are clearly rarer than A1-2 zugs and have been the focus to date: it had been established that no pawnless A3 zug exists with 5 men or less [11].

The presence of an *en passant* feature in a zug has not attracted attention so far, perhaps for three reasons. Type A1-3 zugs in EPZ remain zugs of the same type if the en passant opportunity is removed. Secondly, e.p. constrains two pawns to 14 of the 2,256 positions available and, at 0.22% of the total as in Table 3, e.p. zugs are few indeed. Finally, the very few type A4-6 zugs had not been discovered.

The next challenges are to trawl for zugs in small EGTs for positions with castling rights, and in large 7-man EGTs. Under the DTZ_{50} metric which recognizes the 50-move Rule, some zugs, q.v. Table 4, lose their status or change type.

²¹ KNNKP position P04: $dtz = dtc = 4m$ if a null move is available; $dtm = 4m$ regardless.

²² Alternating as they do between white and black squares.

4.2 Datamining for Gems

The deepest zugs come closest to not being zugs at all: the shallowest zugs feature the greatest change of advantage achievable by the null move. Relative depths may vary with the depth-metric chosen.

Rare gems have intrinsic value; zugs may be absolutely or type-unique within their endgame and/or have some rare feature. There are only 51 type A2 e.p. zugs in which position $p3$ is shallower than position $p1$: 20 in KPPKP and 31 in KPPKPP.

The absence of a pawnless 5-man A3 zug naturally led to a search for one in 6-man chess. In response to the so-called *Pawnless Trébuchet Challenge* [33], Elkies had conjectured that position Z10, identified three years earlier [8], might just be one such: he did not claim [8] that it was as incorrectly announced by Roycroft [34]. The search turned to 6-man endgames [12], [33], [35] but the authors confirmed earlier in the work reported here that there were none [29]. Evidence of Elkies' remarkable prescience [36] is that Z10 perhaps comes closest of all 6-man pawnless positions to being an A3 zug. This is the only position found in which the 2nd player has to avoid the loss by first playing four unique draw-saving moves.²³

With KRBKNN Z11 derived from KRKNN Z10 [8], the *A3 challenge* became one of reducing the number of Knights in such a zug. The zugs B06 [29], [37], B03 [16] and B02 [38] feature two, one and no Knights respectively though B02 leaves a residual challenge by requiring *obtrusive*, i.e. obviously-promoted, force.

Surprisingly, there are no A3 e.p. zugs in 6-man chess but Elkies recalled an 8-man example (N01) derived from an actual game [39]. Other than the 393 A4 KPPKPP zugs, there are just three A4-6 zugs: one A4 (Z05), no A5 zugs²⁴ and two A6 zugs (Z08-9). The A4 zug is unique in that the value of position $p3$ is worse than the value of position $p1$, but the 1st player is a net winner in A5-6 zugs: the 2nd player's perspective is irrelevant in A6 zugs. Elkies has provided the first known A5 zug (Z07) and examples N02-3 of an added e.p. opportunity dezugging a zug.

Castling rights have not been included in EGTs. However P01 and P02 are the first known zugs where added castling rights dezug or not. Elkies [26] provided exemplar zugs (N04-8) where the provision of 0-0(-0) castling rights to the 1st or 2nd player creates a zug: some also feature a significant e.p. opportunity.

4.3 Zugzwangs and Studies

In the Chess Study, White is by convention challenged to draw or win. The appearance of a zugzwang position in a study is notable in itself and, if it is Black to move, suggests that White is just one ply from missing its objective. Mandler's study S11 [40] requires White to revisit a previous physical position 11 plies later but with Black to move: that position is therefore a Level B zug.

A *Zugzwang Study* is defined to be one in which the zugzwang not only appears in the main line of a study in White's favour but also appears as the refutation of a plausible sideline *try* [9]. Fig.2 illustrates the requirements for such a study: a position p

²³ 1. ... Nc5' 2. Nd4 Ne3+' 3. Kd2 Kf2' 4. Ne6 {other moves pressure more} Nxe6'.

²⁴ And MB [16] reports that KPPKPP* (assuming only P=Q allowed) has no A5 zugs.

must appear in its wtm form pw in the try and in its btm form pb in the main line; White's moves should be essentially unique and Black should play its 'best defence'. Beasley gives some remarkable examples of the genre and his article on the theme, from which most of the study positions S01-S12 are taken, is recommended. The zugzwang study demonstrates that the aesthetic contribution [41] of a zugzwang position to a study must be judged in the context of that study and not in isolation.

5 Summary

The authors have searched the available Nalimov DTM EGTs for 6-man chess to identify all the Level A zugzwangs. Somewhat accidentally, we have discovered three new types of zugzwang to make six types in all: there are no other types.

Work will turn to zugs in the more recently arrived 6-man 'DTC' results [16] which will be compared with those of Nalimov²⁵ and Thompson [12], to Level B and C zugs, and to an examination of the occurrence of zugs in studies [42-43].

Complementing this review, the full results, including statistics, highlights and lists of all the zugs with their DTM depths, are published on the ICGA website [17]. The zugs may be studied using EGT query services on the web [13], [44] and we look forward to them being mined for gems by the Chess Studies community and others.

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References

1. Nalimov, E., Haworth, G.M^cC., Heinz, E.A.: Space-efficient Indexing of Endgame Tables for Chess. ICGA J. 23-3, 148--162 (2000)
2. Kryukov, K.: EGTs Online. <http://kirill-kryukov.com/chess/tablebases-online/> (2006)
3. Roycroft, A.J.: Test Tube Chess: A Comprehensive Introduction to the Chess Endgame Study. Faber and Faber Ltd. (1972)
4. Nunn, J.: Secrets of Pawnless Endings. B.T. Batsford, London. 2nd edition (2004)
5. Nunn, J.: Secrets of Rook Endings. B.T. Batsford, London. 2nd edition. Gambit (1999)
6. Nunn, J.: Secrets of Minor-Piece Endings. B.T. Batsford, London (1995)
7. Beasley, J., Whitworth, T.: Endgame Magic. B.T. Batsford, London (1996)
8. Elkies, N.D.: No. 10965: mutual full-point zugzwang? EG 8-128, 320 (1998)
9. Beasley J.: Creating reciprocal zugzwang studies. EBUR 12-2, 8--12 (2000)

²⁵ First indications [31] are that the 'DTC' and 'DTM' statistics are in full agreement.

10. Wirth, C., Nievergelt, J.: Exhaustive and Heuristic Retrograde Analysis of the KPPKP Endgame. *ICCA J.* 22-2, 67--80 (1999)
11. Haworth, G.M^cC., Karrer, P., Tamplin, J.A., Wirth, C.: 3-5-Man Chess: Maximals and Mzugs. *ICGA J.* 24-4, 225--230 (2001)
12. Tamplin, J., Haworth, G.M^cC.: Ken Thompson's 6-man Tables. *ICGA J.* 24-2, 83--85 (2001)
13. Bleicher, E.: Endgame Service based on Nalimov's EGTs <http://www.k4it.de/index.php?topic=egtb&lang=en> (2009)
14. Tay, A.: A Guide to Endgame Tablebases. <http://www.horizonchess.com/FAQ/Winboard/egtb.html> (2009)
15. Konoval, Y., Bourzutschky, M.S.: Private Communications (2007-8)
16. Bourzutschky, M.S.: Private Communications (2009)
17. ICGA: The ICGA website. Menu: Game-specific information – Western Chess - Endgames. www.icga.org (2009)
18. Thyen, O., Clark, M., Scholze-Stubenrecht, W., Sykes, J.B.: *The Oxford-Duden German Dictionary*. OUP (2005)
19. Hooper, D., Whyld, K.: *The Oxford Companion to Chess*. OUP (1992)
20. Gijssen, G.: An Arbiter's Notebook: Monroi and Other Matters. Chesscafe.com <http://www.chesscafe.com/text/geurt105.pdf> (2006)
21. Gijssen, G.: An Arbiter's Notebook: Interpreting the Rules. Chesscafe.com <http://www.chesscafe.com/text/geurt110.pdf> (2007)
22. Gijssen, G.: An Arbiter's Notebook: Interpreting the Rules. Chesscafe.com <http://www.chesscafe.com/text/geurt129.pdf> (2009)
23. Bourzutschky, M.S.: The 16 "missing" Nalimov files. <http://preview.tinyurl.com/aeqvp2> (2006-07-18)
24. Hernandez, N.: 'Missing 16 received!'. Private Communication (2006-08-08)
25. Bourzutschky, M.S.: Tablebase comparison. <http://www.tinyurl.com/d3wny4> (2006-08-10)
26. Elkies, N.D.: Private Communications (2009)
27. Regan, K.W.: Private Communications (2009-04)
28. Haworth, G.M^cC.: Reference Fallible Endgame Play. *ICGA J.* 26-2, 81--91 (2003)
29. Haworth, G.M^cC.: Chess Endgame News. *ICGA J.* 28-4, 243 (2005)
30. Haworth, G.M^cC.: Gentlemen, Stop Your Engines! *ICGA J.* 30-3, 150--156 (2007)
31. Di Fatta, G., Haworth, G.M^cC., Regan, K.W.: Skill Rating by Bayesian Inference. In: Proc. IEEE Conf. on Computational Intelligence and Data Mining, Nashville, USA, 89--94 (2009)
32. Haworth, G.M^cC., Di Fatta, G., Regan, K.W.: Performance and Prediction: Bayesian Modelling of Fallible Choice in Chess. In: Proc. Advances in Computer Games 12. Springer LNCS (2009)
33. Roycroft, A.J.: Announcement of the Pawnless Trébuchet task. *EG* 7-116, 633 (1995)
34. Roycroft, A.J.: Report 1 on the Pawnless Trebuchet task. *EG* 7-117, 645 (1995)
35. Costeff, G.: EG 1-152 online. <http://www.gadycosteff.com/eg/eg.html> (2009)
36. Beasley, J.: Gems Discovered by Computer. *BESN Special Number* 46, 6--7 (2005)
37. Bourzutschky, M.S., Konoval, Y.: 7-man Endgame Databases. *EG* 11-162, 493--510 (2006)
38. Haworth, G.M^cC.: Chess Endgame News. *ICGA J.*, 29-2, 79 (2006)
39. Elkies, N.D.: On Numbers and Endgames: Combinatorial Game Theory in Chess Endgames. In: *Games of No Chance* (ed. R.J. Nowakowski), MSRI 29 (1996)
40. Beasley, J.: *Depth and Beauty: The chess endgame studies of Artur Mandler* (2003)
41. Iqbal, A.: A Discrete Computational Aesthetics Model for a Zero-Sum Perfect Information Game. [http://metalab.uniten.edu.my/~azlan/Misc/phd_thesis_azlan_\(final\).pdf](http://metalab.uniten.edu.my/~azlan/Misc/phd_thesis_azlan_(final).pdf) Ph.D. thesis. University of Malaya, Kuala Lumpur, Malaysia (2008)
42. Costeff, G., Stiller, L.B.: Chess Query Language CQL. <http://rbnn.com/cql/> (2003)
43. Van der Heijden, H.: Endgame Study Database III, 67,691 Studies (2005)
44. Tamplin, J.A.: Multimetric endgame service. <http://chess.jaet.org/endings/> (2009)

Appendix: Some Zugzwang Lines

The 7-man lines are from Bourzutschky [16], [37]. All moves are at least optimal moves given the move-selecting strategy nominated, and beyond that, the key is:

- ''' ≡ only value-saving move (independent of move-choosing strategy),
- '' ≡ the only optimal move, given the strategy nominated, e.g., SC^M, and
- ° ≡ only move.

KPPKP Z04: positions *p1* and *p3* have different depths to mate.

p1, btm: {*dtm* = 25} SM/SM⁺ 1. ... cxd3'' 2. Kxd3''' Kc5'' 3. Kc3''' Kb5'' 4. Kb3''' Ka5 5. Kc4'' Kb6'' 6. Kb4''' Kc6 7. Ka5'' Kb7'' 8. Kb5'' Ka7'' 9. Kc6'' Ka8'' 10. b4 Kb8'' 11. Kb6'' Ka8'' 12. b5 Kb8° 13. Ka6'' Kc7'' 14. b6 Kc8'' 15. Ka7''' 1-0

p2, wtm: 1. K~ Kxd4 = or 1. b~ cxb3''' =

p3, btm: {*dtm* = 15} SM/SM⁺ 1. ... K(c6/d6/e6) 2. Kxc4'' {*dte* = 7} 1-0

KBPKPP Z05: an A4, draw-win-loss, zug:

p1, btm: 1. ... cxb3''' 2. Bxb3''' =

p2, wtm: SM⁺Z⁺/SM⁻Z⁻ 1. Ke1'' c3'' 2. Bb3 c2'' 3. K~ K(b1/d1/d2) 4. Bxc2'' Kxc2''' 5. Ke1 K(b3/c3) 6. Kd1 Kxb4''' 0-1

p3, btm: SM⁻Z⁻/SM⁺Z⁺ 1. ... Kb1'' 2. Ke3'' Kc1'' 3. Be2 Kc2'' 4. Kd4''' c3'' 5. Bd3+'' Kd2'' 6. Be4 c2'' 7. Bxc2''' Kxc2'' 8. Kc5''' Kb3 9. Kxb5''' Ka3 10. Kc5 Kb3 11. b5'' 1-0

KRBKNKN Z11: an A3 zug adaption of Z10 which 'just failed' to be A3.

p1 ≡ *p3*, wtm: 1. N~ N(x)d3#''''; 1. R~ Nxb2#'''' 0-1

p2, btm: {"Black cannot maintain the bind" [8]} 1. ... Nc5 2. Nd4 Ne3+ 3. Kc1 Kg1 4. Bf5 Ng2 5. Nf3+ Kf1' 6. Bh3 Nd3+' 7. Kb1 Nb2 8. Kxb2 {*dte* = 1m, *dtm* = 8m}

KNNNKNN B01: the bK is boxed in but White must avoid a KNNK endgame.

p2, btm: {*dtz* = 17} SZ/SZ⁺ 1... Ndc3 2. Nd7''' Kh7 3. Nf4''' Kg8 4. Nd6''' Kg7 5. Ke5''' Nd1 6. Kf5 Kh6 7. Kf6 Ne3 8. Nf7+''' Kh7° 9. Ng5+''' Kg8 10. Ne4''' Kh7 11. Ne5 Nb6 12. Kf7''' Nbc4 13. Nf6+''' Kh6 14. Nf3''' Nd6+ 15. Kf8 Nef5 16. Ng8+ Kh7° 17. Ng5+ Kh8° 18. Ng6# {10 of White's 17 moves were unique winning moves} 1-0

KRBBKQN B03: an A3 zug with only one Knight.

p1, wtm: SZ⁺/SZ⁻ 1. Be8 Ka2 2. Bf7 Ka3 3. Bc1+ Kb4 4. Bd2+ Kc4 5. Be8 6. Ne7 0-1

{1. Bxd5 Qxg6+''' {KBBKQ, *dtz* = 62} "is however probably *best defence*" [16]}.

p2, btm: SM⁻Z⁻/SM⁺Z⁺ 1. ... Qh3 2. Ra6+'' Qa3° 3. Rxa3#'' 1-0

KRKN S01: 1.Re3''' Ng1' 2.Kf5' (2.Kf4?? Kd4 pw =) Kd4' 3.Kf4''' pb 1-0

KRKBN S03: 1.Rb6''' Nb5' 2.Ra6+''' Na7+° 3.Kc7''' Be8' 4.Ra3''' (4.Ra2?? Ba4''' pw =) Ba4' 5.Ra2' pb 1-0

KPPKPP S12: 1.Kf4''' (1.Ke4?? Kg6'' pw) Kg6' 2.Ke4''' pb Kg5' 3.e6''' fxe6' 4.Ke5''' Kg4 5.Kxe6''' 1-0