A theorem on semi-centralizing derivations of prime rings

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A THEOREM ON SEMI-CENTRALIZING DERIVATIONS OF PRIME RINGS

Dedicated to Professor Hisao Tominaga on his 60th birthday

ARIF KAYA

Let $R$ be an (associative) ring with center $C$, and $S$ a subset of $R$. A derivation $d : x \mapsto x'$ of $R$ is said to be centralizing (resp. skew-centralizing) on $S$ if $ss' - ss' \in C$ (resp. $ss' + ss' \in C$) for every $s \in S$. More generally, $d$ is defined to be semi-centralizing on $S$ if $ss' - ss' \in C$ or $ss' + ss' \in C$ for every $s \in S$.

The following has been proved in [1, Theorem 1 (2)] and [2, Theorem 2].

**Theorem 1.** Let $d$ be a non-zero derivation of a prime ring $R$, and $S$ a non-zero ideal of $R$.

(1) If $d$ is centralizing or skew-centralizing on $S$, then $R$ is commutative.

(2) If $d$ is semi-centralizing on $R$, then $R$ is commutative.

In this very brief note, we improve the above theorem as follows:

**Theorem 2.** Let $d$ be a non-zero derivation of a prime ring $R$, and $S$ a non-zero ideal of $R$. If $d$ is semi-centralizing on $S$, then $R$ is commutative.

**Proof.** Suppose, to the contrary, that $R$ is not commutative. In view of Theorem 1 (1), $d$ is not centralizing on $S$ and $R$ is of characteristic not 2. Then, by [1, Lemma 4], $S \cap C = 0$ and there exists $t \in S$ such that $t^4 \neq 0$ but $(t^2)' = 0$. Since $R$ is a prime ring, so is the non-zero ideal $T = Rt^2R$ of $R$. Moreover, by [1, Lemma 1 (3)], $0 \neq T' \subseteq Rt^2R + Rt^2R' \subseteq T$. Hence $d$ induces a non-zero derivation of $T$ which is semi-centralizing on $T$. Thus, $T$ is commutative by Theorem 1 (2), and therefore $R$ itself is commutative by [1, Lemma 1 (1)]. This is a contradiction.

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**References**

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