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A Pneumatic On-off Controller with Feedback Compensation

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We attempt to apply the feedback compensation technics of on-off controller to the pneumatic systems.

In this paper, we describe the structure and the action of a new developed pneumatic on-off controller with feedback compensation, and the principle of the special pilot valve which is used in the controller as the on-off element.

Next, it is shown analytically and experimentally that the dynamic characteristics of the controller are nearly equal to those of the continuous one which has an infinite proportional gain instead of the on-off element, if the time average of its discontinuous output is treated as the output of the controller.

§ 1. Introduction

One of the methods to decrease the amplitude of cycling in the on-off control system is the so-called feedback compensation. This is the method by which the output of the on-off controller is fed back to its input side through a suitable compensating element. It is essential for the compensating element to make its lag smaller than in the controlled system. Then, the cycling in the compensating loop has a period smaller than that in the main control loop. Consequently, the switching period of the on-off element becomes smaller and the controlled variable is smoothed. Of course, it is desirable that the signal in the compensating loop shoud act not only to shorten the switching period but also to make satisfactory the transient and steady-state response of the main loop.

In connection with the feedback compensation of on-off control system, various studies have been made theoretically and experimentally. C. Kessler[1] analyzed approximately an on-off controller with a sort of feedback compensating element. Izawa and Yamaguchi[2] also analyzed a similar controller by the frequency response method. W. Böttcher[3] studied the optimum adjustment of controller by using the analog computer. Yamaguchi[4] synthesized the controller which gives the time optimal response to a step input to the control system. Moreover, S. Paul[5], and Iwanaga[6] applied the technics to the temperature control of electric furnace.

This paper describes an attempt to apply the above-mentioned technics to the pneumatic controller. First, the structure and the action of a pneumatic on-off controller with the feedback compensating element, which manufactured for trial, are explained. Secondly, the dynamic and the static characteristics of the controller are analyzed, and these are in satisfactory agreement with the experimental results. Finally, the analysis and the design of the special pilot valve which acts as the on-off element with hysteresis are mentioned.

§ 2. Pneumatic on-off controller

The on-off element in pneumatic control system may usually be a pilot valve or an on-off type control valve. When we attempt to improve the system behaviour by the feedback compensation technics, if we regard the control valve as the on-off element, it is necessary to detect and feed back the controlled variable. Then it will become difficult to satisfy the basic requirement : the lag in the compensation loop must be smaller than in the main control loop. Therefore, we regard the pilot valve as the on-off element.

The compensating element used in usual studies is an integrating or a first order lag element, or the connection of these two in an opposite sign. But, in the former two cases the mean value of the output of compensating element is generally not equal to zero even in the steady state. Then, as the compensating element not only raises the switching frequency of on-off element but also affects the input of the on-off element and consequently the controlled deviation, a considerable off-set arises in the system. On the other hand, in the last case the mean values of two elements are set off against each other, and the off-set becomes comparatively small. Therefore, we use the connection of two first order lag elements in opposite sign as the compensating element.

The controlling unit is constructed of a nozzle-flapper mechanism and a pilot valve. It is necessary for this unit to have the exact on-off characteristics and the proper hysteresis because of the continuation of stable cycling in compensating loop.

Based on the considerations mentioned above, a pneumatic on-off controller with the feedback compensation is manufactured for trial. Its scheme is shown in Fig. 1. Four bellows are attached on the error link supported by a movable fulcrum, and each of these is subjected to the pressure correspondent to the reference input, the primary feedback signal, and the minor feedback signals for compensation. The difference of these pressures gives the pressure change on the diaphragm of the pilot valve by means of the error link and the nozzle-flapper mechanism.

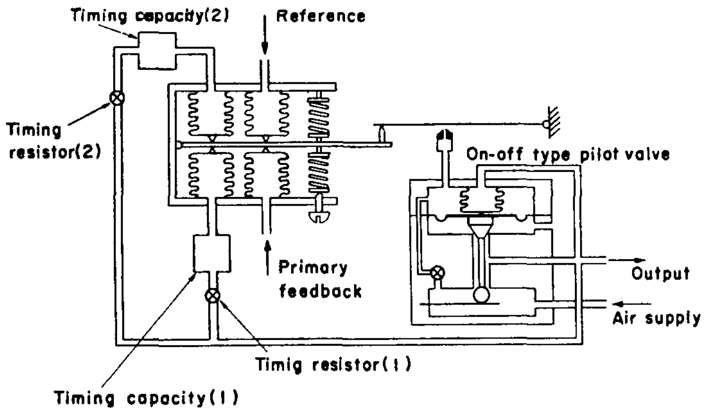

Fig. 1 Pneumatic on-off controller with feedback compensation.

Then, in addition to the nozzle back pressure, the output pressure itself of the pilot valve acts on the diaphragm through a bellows. Accordingly, if the input change to the pilot valve causes a slight increase of the output, the output has the same effect as a greater change of the input, so that it increases rapidly and this pilot valve has a characteristic of the on-off type. After the output pressure once reaches its maximum value, a decrease of the input does not cause a decrease of the output, until the force due to the input pressure overcomes the force due to the pressure in the positive feedback bellows. But, if the input decreases below the above critical value, the output decreases rapidly to its minimum value. As the result, it is known that the characteristic of this special pilot valve is the on-off type with hysteresis.

Each of the first order lag elements, which compose the feedback compensating loop, is made up by a pneumatic resistance of a needle valve type and a pneumatic capacitance of a tank, and the time constant of the lag element is changed by the pneumatic resistance.

§ 3. Linear approximation of dynamic characteristics of controller

Nomenclatures are as follows and as shown in Fig. 2:

A, $-A$ = output of controller

A_b = effective cross-sectional area of bellows

C_1, C_2 = pneumatic capacitance of compensating element

$2h'$ = hysteresis of on-off pilot valve

k_s = effective spring constant of error link

K_n = gain of nozzle-flapper

p_1, p_2 = output pressure of compensating element

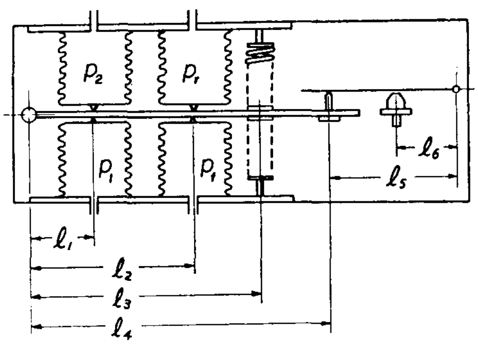

Fig. 2 Dimensions of the controller.

p_f = primary feedback pressure
p_r = reference pressure
R_1, R_2 = pneumatic resistance of compensating element.

The block diagram can be described as shown in Fig. 3 (a) and Fig. 3 (b) in a simplified form, where $\varepsilon = p_r - p_f$, $h = k_8 l_3^2 l_5 h'/K_n A_b l_2 l_4 l_6$, $K_f = l_1/l_2$, $T_1 = C_1 R_1$, and $T_2 = C_2 R_2$.

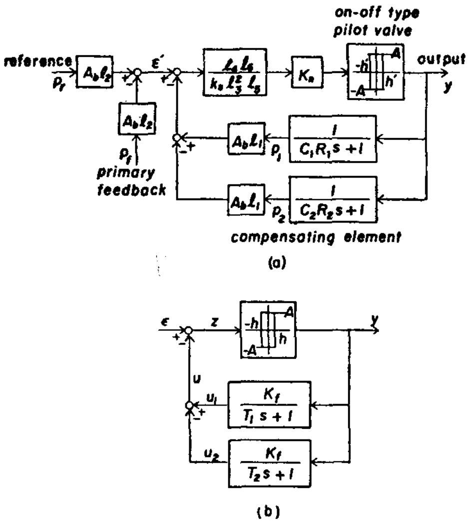

Fig. 3 Block diagram of the controller.

Assuming the cycling is continuing in the compensating loop, the amplitude of the input and the output of the on-off element is respectively equal to h and A, thus the equivalent gain of the on-off element is considered as $4A/\pi h$. As the value of A/h is very large, the on-off element can be approximately regarded as a proportional element of large gain. Thus the transfer function of the system in Fig. 3 (b) is described as follows:

$$\frac{L\{y(t)\}}{L\{\varepsilon(t)\}} \fallingdotseq \frac{T_1+T_2}{K_f(T_2-T_1)} \left\{1+\frac{1}{(T_1+T_2)s}+\frac{T_1T_2}{T_1+T_2}s\right\} \tag{1}$$

As the output $y(t)$ is in fact a rectangular wave of the height $2A$, it is necessary to express $y(t)$ in a time average defined properly. Because it is necessary that the work done by the output defined as a time average should be equal to the work done by the on-off controller, the expression

$$\tilde{y}(t) = \frac{1}{P}\int_t^{t+P} y(t)dt = A\frac{t_n - t_f}{t_n + t_f} \tag{2}$$

may be used reasonablly instead of $y(t)$, where t_n and t_f are respectively the time duration in which the output of the on-off element is the value of A or $-A$, and $P = t_n + t_f$.

Therefore, substituting $y(t)$ in Eq. (1) by $\tilde{y}(t)$ in Eq. (2), the next expression is obtained as a linear approximation of the characteristics of the on-off controller as shown in Fig. 3 (b):

$$\frac{L\{\tilde{y}(t)\}}{L\{\varepsilon(t)\}} \fallingdotseq K_P\left(1+\frac{1}{T_I s}+T_D s\right) \tag{3}$$

where

$$K_P = \frac{T_1+T_2}{K_f(T_2-T_1)}, \quad T_I = T_1+T_2,$$

$$T_D = \frac{T_1T_2}{T_1+T_2}. \tag{4}$$

§ 4. Investigation of linear approximation by analog computer

Though the linear approximation in the last paragraph is devoid of mathematical strictness, it allows to apply the linear control theory in further analysis and synthesis, and it confers much benefit upon the practical use. Therefore, the property of the Eq. (3) must be verified by any other method, and this is executed by a series of experiments using an analog computer.

The flow chart of the analog computer shown in Fig. 4 is analogous to the system shown in Fig. 3 (b), where NL indicates a limiter with the gain of unity and the limiting value of A and $-A$. When the output of NL is fed back

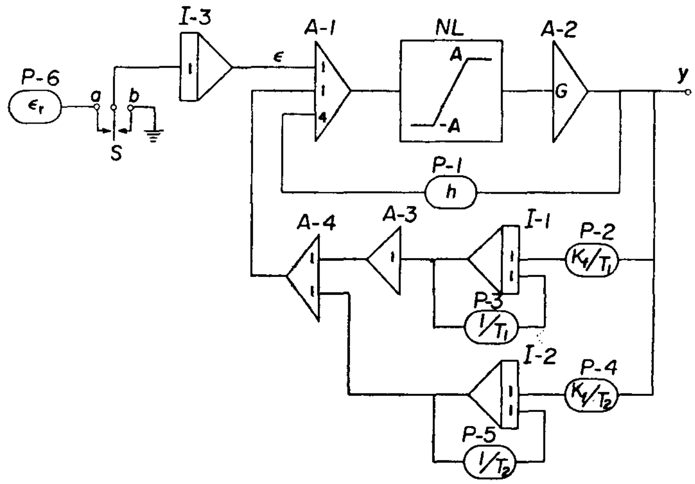

Fig. 4 Simulation of the controller by analog computer.

in a positive sign to its input side through the potentiometer $P-1$ of larger gain than unity, the on-off element with hysteresis can be obtained. The integrator $I-3$ is used to make the ramp input $\varepsilon_r t$, and the time derivative of ε_r is changed by $P-6$. S indicates a switch to give the input.

According to the last paragraph, the output $\tilde{y}(t)$ of the controller to a ramp input $\varepsilon(t)$ is given by

$$\tilde{y}(t) \fallingdotseq \varepsilon_r K_P\left(T_D + t + \frac{1}{2T_I}t^2\right) \tag{5}$$

where

$$\varepsilon(t) = \varepsilon_r t. \tag{6}$$

In other words, $\tilde{y}(t)$ to a ramp input $\varepsilon_r t$ is expressed by a quadratic equation of t, and its constant, first order, and second order coefficients that are described by α, β, and γ, are respectively given as follows:

$$\left.\begin{aligned} \alpha &= \varepsilon_r K_P T_D = \varepsilon_r \frac{T_1 T_2}{K_f(T_2 - T_1)} \\ \beta &= \varepsilon_r K_P = \varepsilon_r \frac{T_1 + T_2}{K_f(T_2 - T_1)} \\ \gamma &= \varepsilon_r \frac{K_P}{2T_I} = \varepsilon_r \frac{1}{2K_f(T_2 - T_1)} \end{aligned}\right\} \tag{7}$$

Now, the experiments on the ramp response are made by the circuit in Fig. 4, and the response is represented by the form:

$$\tilde{y}(t) = \alpha + \beta t + \gamma t^2 \tag{8}$$

where the coefficients α, β, and γ satisfy the following equations:

$$\left.\begin{aligned} \alpha N + \beta \sum t_i + \gamma \sum t_i^2 &= \sum \tilde{y}_i \\ \alpha \sum t_i + \beta \sum t_i^2 + \gamma \sum t_i^3 &= \sum t_i \tilde{y}_i \\ \alpha \sum t_i^2 + \beta \sum t_i^3 + \gamma \sum t_i^4 &= \sum t_i^2 \tilde{y}_i \end{aligned}\right\} \tag{9}$$

The time t_i's are decided in such a manner as is shown in Fig. 5: for instance, if $y = A$ at the moment when the input is given, the time when y changes from A to $-A$ is chosen as $t=0$, and since then each time when y changes from A to $-A$ is taken as t_i for the cycle of y_i.

If the coefficients α_{exp}, β_{exp}, and γ_{exp} which are obtained by experiments through the Eq. (9) are respectively equal to the coefficients α_{cal}, β_{cal}, and γ_{cal} which are calculated by the Eq. (7), it can be stated that the linear approximaton mentioned above is reasonable.

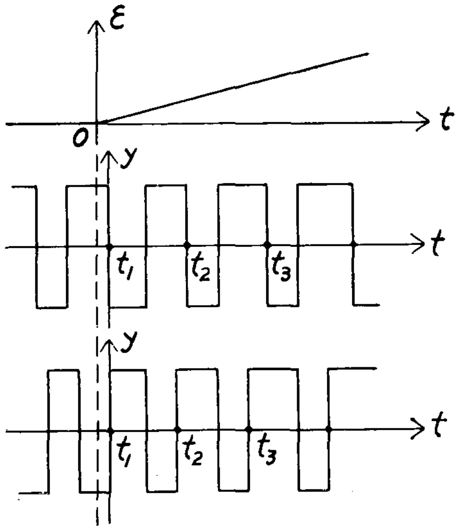

Fig. 5 Representation of times.

Fig. 6 shows an example of the ramp response y by the circuit shown in Fig. 4. Small circles in Fig. 7 indicate y_i's which are the time averages in a cycle of the rectangular wave y, and the solid line is the most plausible curve of the quadratic form.

Parameters of a series of experiments are shown in Table 1. Experimental results are arranged in Fig. 8 where α_{cal} - α_{exp}, β_{cal} - β_{exp}, and γ_{cal} - γ_{exp} are respectively plotted on a rectangular axis. Naturally the results in Fig. 8 contain errors which may be caused by the analog computer itself and by the reading of the recorded graphs, so that any statistical technics would be used to conclude the relation between the linear approximation and the original non-linear system.

The correlation coefficient r between the two variables ξ and η is given by the formula[7]:

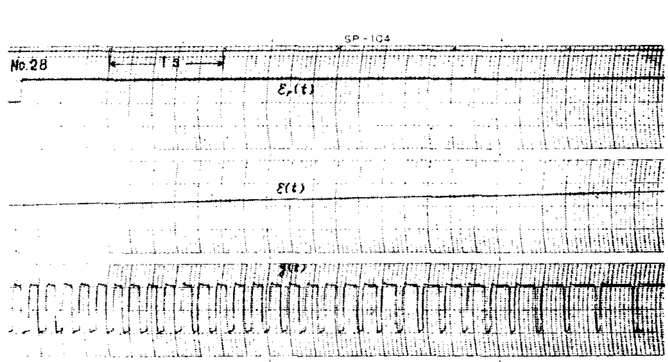

Fig. 6 Transient response of the system shown in Fig. 4 to a ramp input.

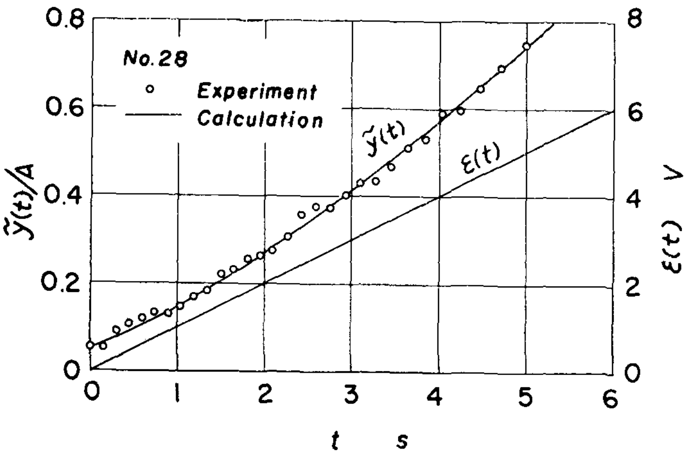

Fig. 7 Quadratic expression of $y(t)$ by r. m. s. method.

Table 1 System parameters used in experiments.

No.	A V	h V	K_f	T_1 s	T_2 s	ε_r V/s
1	50	1	1	1	2	0.5
2						1
3					3	0.5
4						1
5				2	4	0.5
6						1
7					6	0.5
8						1
9			0.5	1	2	0.5
10						1
11					3	0.5
12						1
13				2	4	0.5
14						0.2
15					6	0.5
16						0.2
17		0.5	1	1	2	0.5
18						1
19					3	0.5
20						1
21				2	4	0.5
22						1
23					6	0.5
24						1
25			0.5	1	2	0.5
26						0.25
27					3	0.5
28						1
29				1	4	0.5
30						0.25
31					6	0.5
32						1
33		1	1	1	4	0.5
34						1

$$r=\frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{\xi_i-\bar{\xi}}{S_\xi}\right)\left(\frac{\eta_i-\bar{\eta}}{S_\eta}\right) \tag{10}$$

where

$$\left.\begin{aligned} S_\xi &= \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(\xi-\bar{\xi})^2} \\ S_\eta &= \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(\eta_i-\bar{\eta})^2} \\ \bar{\xi} &= \frac{1}{N}\sum_{i=1}^{N}\xi_i, \\ \bar{\eta} &= \frac{1}{N}\sum_{i=1}^{N}\eta_i. \end{aligned}\right\} \tag{11}$$

Regression line of η to ξ is given by the formula[8]:

$$\eta-\bar{\eta}=r\frac{S_\eta}{S_\xi}(\xi-\bar{\xi}) \tag{12}$$

or

$$\eta=a+b\xi \tag{13}$$

where

$$a=\bar{\eta}-r\frac{S_\eta}{S_\xi}\bar{\xi}, \qquad b=r\frac{S_\eta}{S_\xi}. \tag{14}$$

Using Eqs. (10) and (14), the values of r, a, and b of the experimental results shown in Fig. 8 are calculated as listed in Table 2. From the values of r, it is easily proved that the null hypothesis, namely the correlation coefficient of population $\hat{r}=0$, is rejected about α, β, and γ. Though it is not able to test the hypothesis $\hat{r}=1$, the values of r's indidate that there is a considerably significant correlation between the experimented and the calculated results.

Table 2 Correlation and regression of the experiments with the linear approximations.

	Correlation coefficient r	Regression line a	Regression line b
α	0.909	0.0131	0.860
β	0.981	−0.0012	1.033
γ	0.965	0.0005	0.812

Next, the analyses of regression are carried out as follows. 1) The regression is linear: by the F-test on the ratio of the variance about regression and the variance within clusters of the experimental values, it can be proved that the hypothesis of the linearity cannot be rejected under the 95% confidence. 2) Intercept of the regression line of population $\hat{a}=0$: the

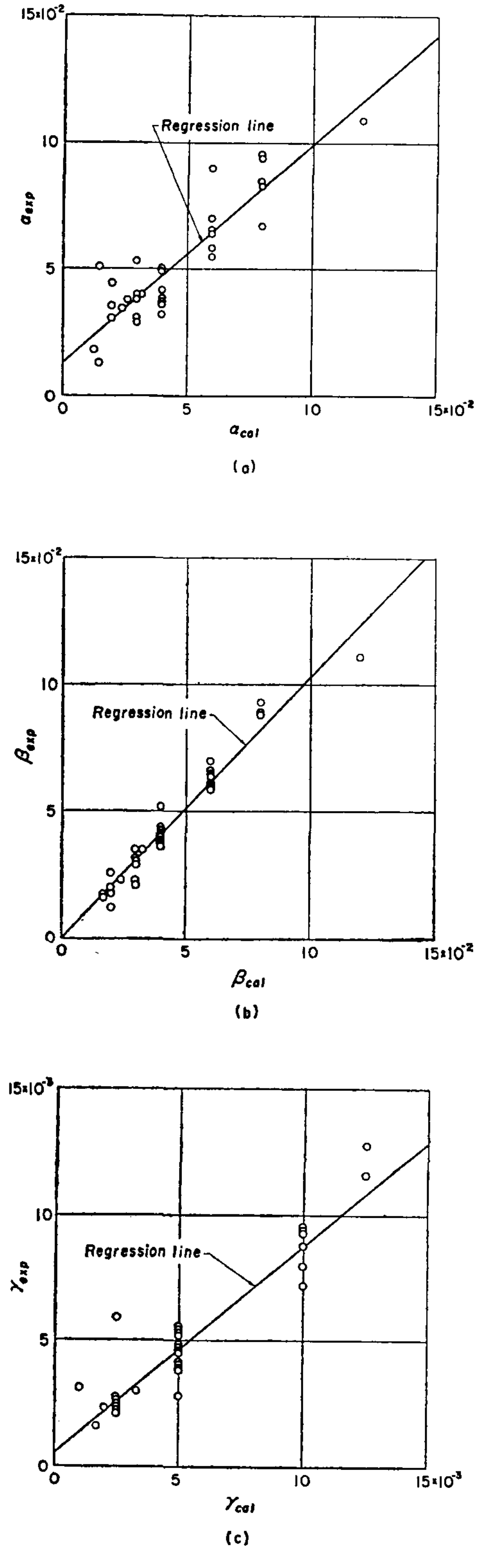

Fig. 8 Coefficients of the quadratic expressions.

statistic on a-$\hat{a}$ has the t-distribution, so that it is shown that the hypothesis cannot be rejected under the 95% confidence by using the t-test.
3) Gradient of the regression line of population $\hat{b}=1$: by the same test as the one used in a, the hypothesis also cannot be rejected.

On the basis of the considerations mentioned above it may be given as a conclusion that the characteristics of the controller shown in Fig. 3 are equivalent to that of the linear system which is obtained by substituting the on-off element by a proportional one with gain of infinity, the output being evaluated by Eq. (2) as a time mean.

§ 5. Statical characteristics

Assume that the loop in Fig. 3 (b) is opened at the part of z and the on-off element continues the steady on-off action with time intervals t_n and t_f by an appropriate input z. Then, the output u_1 of a first lag element of the compensating loop is given by

$$\left.\begin{aligned} u_{1n}(t) &= u_{1n}(0)e^{-\frac{t}{T_1}} + AK_f(1-e^{-\frac{t}{T_1}}) \\ u_{1f}(t) &= u_{1f}(0)e^{-\frac{t}{T_1}} - AK_f(1-e^{-\frac{t}{T_1}}) \end{aligned}\right\} \quad (15)$$

where the subscripts n and f indicate respectively the states $y=A$ and $y=-A$, and the time t is measured from the instant of the last switching of y.

In the steady state the formulas are obtained:

$$\left.\begin{aligned} u_{1n}(t_n) &= u_{1f}(0) = u_{1n}(0)e^{-\frac{t_n}{T_1}} + AK_f(1-e^{-\frac{t_n}{T_1}}) \\ u_{1f}(t_f) &= u_{1n}(0) = u_{1f}(0)e^{-\frac{t_f}{T_1}} - AK_f(1-e^{-\frac{t_f}{T_1}}) \end{aligned}\right\} \quad (16)$$

Thus, we obtain

$$\left.\begin{aligned} u_{1n}(0) &= -AK_f(1+e^{-\frac{t_0}{T_1}} - 2e^{-\frac{t_f}{T_1}}) \Big/ (1-e^{-\frac{t_0}{T_1}}) \\ u_{1f}(0) &= AK_f(1+e^{-\frac{t_0}{T_1}} - 2e^{-\frac{t_n}{T_1}}) \Big/ (1-e^{-\frac{t_0}{T_1}}) \end{aligned}\right\} \quad (17)$$

where $t_0 = t_n + t_f$. And similarly

$$\left.\begin{aligned} u_{2n}(0) &= -AK_f(1+e^{-\frac{t_0}{T_2}} - 2e^{-\frac{t_f}{T_2}}) \Big/ (1-e^{-\frac{t_0}{T_2}}) \\ u_{2f}(0) &= AK_f(1+e^{-\frac{t_0}{T_2}} - 2e^{-\frac{t_n}{T_2}}) \Big/ (1-e^{-\frac{t_0}{T_2}}). \end{aligned}\right\} \quad (18)$$

From Eqs. (17) and (18), we obtain

$$\left.\begin{aligned}
u_n(0) &\equiv u_{1n}(0)-u_{2n}(0)\\
&= -AK_f\left\{\frac{1+e^{-\frac{t_0}{T_1}}-2e^{-\frac{t_f}{T_1}}}{1-e^{-\frac{t_0}{T_1}}}-\frac{1+e^{-\frac{t_0}{T_2}}-2e^{-\frac{t_f}{T_2}}}{1-e^{-\frac{t_0}{T_2}}}\right\}\\
u_f(0) &\equiv u_{1f}(0)-n_{2f}(0)\\
&= AK_f\left\{\frac{1+e^{-\frac{t_0}{T_1}}-2e^{-\frac{t_n}{T_1}}}{1-e^{-\frac{t_0}{T_1}}}-\frac{1+e^{-\frac{t_0}{T_2}}-2e^{-\frac{t_n}{T_2}}}{1-e^{-\frac{t_0}{T_2}}}\right\}.
\end{aligned}\right\} \tag{19}$$

Using the expressions:

$$\tau\equiv t/T_1,\ \lambda\equiv T_1/T_2 \tag{20}$$

then the output and its derivatives are

$$\left.\begin{aligned}
u_n(\tau) &= -2AK_f\left\{\frac{1-e^{-\tau_f}}{1-e^{-\tau_0}}\cdot e^{-\tau}-\frac{1-e^{-\lambda\tau_f}}{1-e^{-\lambda\tau_0}}\cdot e^{-\lambda\tau}\right\}\\
u_f(\tau) &= 2AK_f\left\{\frac{1-e^{-\tau_n}}{1-e^{-\tau_0}}\cdot e^{-\tau}-\frac{1-e^{-\lambda\tau_n}}{1-e^{-\lambda\tau_0}}\cdot e^{-\lambda\tau}\right\}
\end{aligned}\right\} \tag{21}$$

$$\left.\begin{aligned}
\frac{du_n(\tau)}{d\tau} &= 2AK_f\left\{\frac{1-e^{-\tau_f}}{1-e^{-\tau_0}}\cdot e^{-\tau}-\frac{1-e^{-\lambda\tau_f}}{1-e^{-\lambda\tau_0}}\cdot \lambda e^{-\lambda\tau}\right\}\\
\frac{du_f(\tau)}{d\tau} &= -2AK_f\left\{\frac{1-e^{-\tau_n}}{1-e^{-\tau_0}}\cdot e^{-\tau}-\frac{1-e^{-\lambda\tau_n}}{1-e^{-\lambda\tau_0}}\cdot \lambda^{-\lambda\tau}\right\}
\end{aligned}\right\}. \tag{22}$$

When the loop is closed again, the conditions for the occurrence of cycling are written as follows:

$$\tau=\tau_f:\ \varepsilon(\tau)-u_f(\tau)=h,\ \dot{\varepsilon}(\tau)-\dot{u}_f(\tau)>0 \tag{23}$$

$$\tau=\tau_n:\ \varepsilon(\tau)-u_n(\tau)=-h,\ \dot{\varepsilon}(\tau)-\dot{u}_n(\tau)<0 \tag{24}$$

When ε is constant, these conditions are rewritten by Eqs. (21) and (22)

$$\left.\begin{aligned}
&\frac{1-e^{-\tau_f}}{1-e^{-\tau_0}}-\frac{1-e^{-\lambda\tau_f}}{1-e^{-\lambda\tau_0}}=\frac{\varepsilon-h}{2AK_f}\\
&\frac{1-e^{-\tau_f}}{1-e^{-\tau_0}}-\lambda\frac{1-e^{-\lambda\tau_f}}{1-e^{-\lambda\tau_0}}<1-\lambda
\end{aligned}\right\} \tag{25}$$

$$\left.\begin{aligned}
&\frac{1-e^{-\tau_n}}{1-e^{-\tau_0}}-\frac{1-e^{-\lambda\tau_n}}{1-e^{-\lambda\tau_0}}=\frac{\varepsilon+h}{2AK_f}\\
&\frac{1-e^{-\tau_n}}{1-e^{-\tau_0}}-\lambda\frac{1-e^{-\lambda\tau_n}}{1-e^{-\lambda\tau_0}}<1-\lambda
\end{aligned}\right\}. \tag{26}$$

From these equations the steady state response y to the constant input ε can be determined by τ_n and τ_f. However, as the equations cannot be solved in an explicit form, a numerical solution is shown in Fig. 9. The solution indicates that the controller has the proportional characteristics in the vicinity of $\varepsilon=0$. Therefore, if the controlled system has no integral characteristics, the controller may leave the off-set in the control system, which depends on λ and h/AK_f.

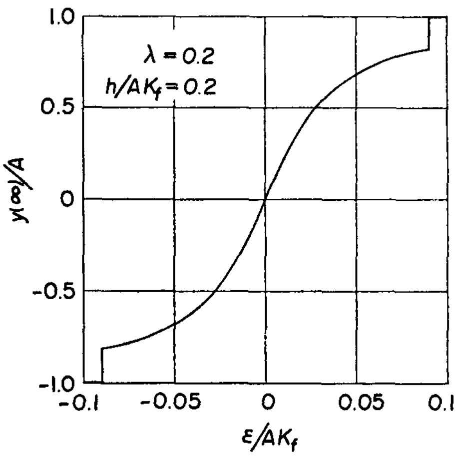

Fig. 9 Statical characteristics of the controller.

§ 6. Pilot valve as the on-off element

The pilot valve which is used in the controller is a special one with positive feedback. The characteristics and the designing method on it are described in the following.

Referring to the scheme in Fig. 1, the total gain of the pilot valve k is given by

$$k\equiv\frac{dp_0}{dp_n}=\frac{k_p}{1-k_f\cdot k_p} \tag{27}$$

where

A_b, A_a = effective sectional area of feedback bellows and diaphragm of pilot valve

$k_f = A_b/A_a$

k_p = gain of pilot valve without feedback

p_n, p_0 = input and output pressure of pilot valve.

As is generally known, k_p is a function of the input p_n, so that Eq. (27) is rewritten

$$k(p_n)=\frac{k_p(p_n)}{1-k_f\cdot k_p(p_n)}. \tag{28}$$

Thus, to the input p_{n1} satisfying the relation

$$1-k_f\cdot k_p(p_n)=0, \tag{29}$$

the total gain k tends to infinity. That is, if the curve A in Fig. 10 indicates k_p (p_n) and the

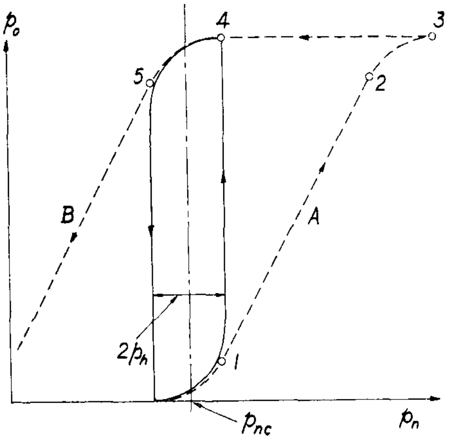

Fig. 10 Action of the pilot valve.

point p_{n1} satisfies Eq. (29), the output of the pilot valve reaches its maximum value p_s at the state where p_n reaches p_{n1}.

After that, the output does not change, even if the input should increase or decrease until it reaches p_{n4} which satisfies the relation:

$$A_a\cdot p_{n4}+A_b\cdot p_s=k_s d_0 \tag{30}$$

where

d_0 = initial compression of effective spring

k_s = effective spring constant on valve stem.

On the other hand, if the pilot valve has no feedback, the input pressure corresponding to p_{n4} is p_{n3} which satisfies the following relation:

$$A_a p_{n3}=k_s d_0. \tag{31}$$

And, the output pressure decreases rapidly at $p_n=p_{n5}$ for the same reason as at p_{n1}. Therefore, if p_{n1} and p_{n5} are chosen appropriately, the pilot valve has the on-off characteristics with desired hysteresis.

Referring to Fig. 10, the center of action p_{nc} and the hysteresis $2p_h$ of the pilot valve are given by the equations:

$$\left.\begin{aligned} p_{nc}+p_h&=p_{n1}\\ p_{nc}-p_h&=p_{n5}.\end{aligned}\right\} \tag{32}$$

By Eqs. (30) and (31)

$$\begin{aligned} p_{n5}&=p_{n2}-(p_{n2}-p_{n5})=p_{n2}-(p_{n3}-p_{n4})\\ &=p_{n2}-k_f p_s.\end{aligned}$$

As the result, when p_{nc} and p_h are given, the pilot valve without feedback and the feedback element must be so designed as to satisfy the conditions:

$$\left.\begin{aligned} p_{n1}&=p_{nc}+p_h\\ p_{n2}&=p_{nc}-p_h+k_f p_s.\end{aligned}\right\} \tag{33}$$

§ 7. Experiments

Statical characteristics of the pilot valve are shown in Fig. 11. It is evident that the usual pilot valve is changed into an on-off type one by

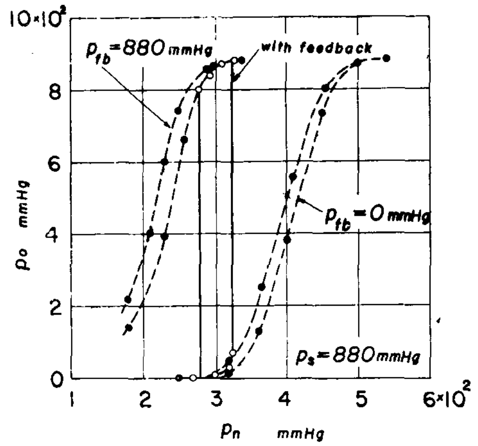

Fig. 11 Statical characteristics of the pilot valve, p_{fb} = pressure in feedback bellows.

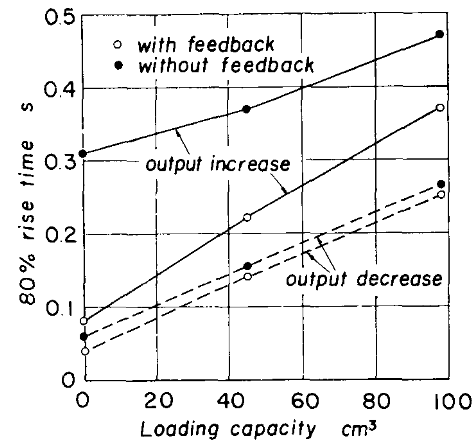

Fig. 12 Dynamic characteristics of the pilot valve.

the positive feedback, and the characteristics are in considerable agreement with those estimated on the theory in the last paragraph.

Fig. 12 shows the dynamic characteristics of the pilot valve with and without feedback, which give the 80% rise time of the output pressure in various loading capacitances. Thus it is known that the change in characteristics is more remarkable in the dynamic one than in the statical one.

Fig. 13 indicates the statical characteristics of the controller, which agree qualitatively with the theoretical result shown in Fig. 9. A ramp response of the controller is shown in Fig. 14, where the ramp change of the input pressure is substituted by a step change of pressure in the pneumatic resistance-capacitance element. And the pressure change is recorded by a pen recording oscillograph with pressure head of the electric strain meter type. Fig. 15 indicates the time average $\tilde{y}(t)$ of the response $y(t)$ in Fig. 14, in which the small circles and the solid line describe respectively the experimental results and the result calculated by Eq. (6), and these are in good agreement.

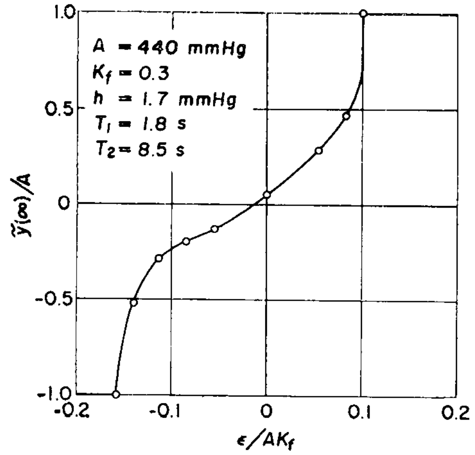

Fig. 13 Statical characteristics of the controller.

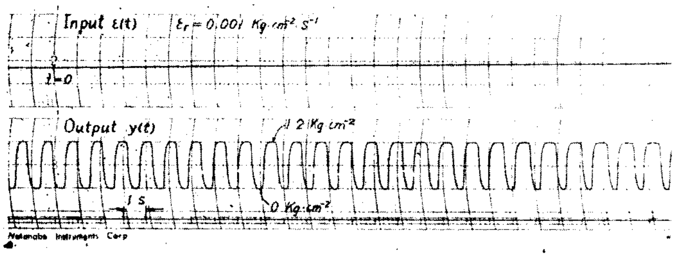

Fig. 14 Transient response of the controller to a ramp input.

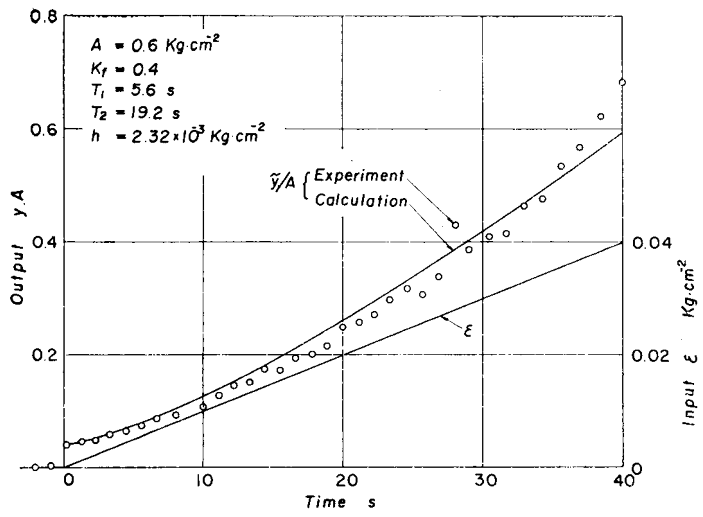

Fig. 15 Representation of the transient response in the linearized form.

§ 8. Conclusion

A pneumatic on-off controller with feedback compensation was manufactured for trial to apply the feedback compensation technics of on-off control system to the pneumatic systems. For the stable on-off action of the controller, the developement of a pneumatic on-off element with hysteresis was required, and the requirement was satisfied by a pilot valve with positive feedback.

Exact characteristics of the controller are too complex to formulate for its non-linearity. In this paper a linear approximation technics was used for expressing the dynamic behaviour of the controller, and the appropriateness of the expression was investigated statistically, based on the experimental results by the analog computer. Consequently, it was explained that the linear approximation could express well the dynamic characteristics of the on-off controller.

Further, the transient and the steady state characteristics were experimented on the special pilot valve and the controller, and these results were investigated theoretically.

In conclusion the authers wish

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