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Computer Program of Line Balancing under the Multiple Workers in Each Station (LBMW)

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Synopsis

An assembly line with no paralleling of work elements and work stations is called a serial line. The cycle time of the serial line must be at least equal to the maximum work element time. To lower the cycle time beyond the limit and increase the production rate, one may permit the paralleling of work elements or work stations.

So in this paper we propose the parallel assignment method for achieving a higher production rate. In this method, work elements are assigned to work stations under the multiple upper time limits which are the products of the various numbers of workers and the limiting cycle time.

Further we develop the computer program of the proposed method and provide an illustrative problem and computational results.

1. Introduction

The typical assembly line is serial with no paralleling of work elements and work stations [1]. The serial line has one worker in each work station. And the serial line balancing is to assign the work elements to the work stations so as to make the work content at each of the stations as close as possible to one limiting cycle time, i.e., an upper time limit of the stations. Then the sum of the times for work elements assigned to any one station (i.e., station time) does not exceed the upper time limit.

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Therefor the cycle time, which is the largest value among the stations times, must be at least equal to the maximum work element time. Hence the production rate depending on the cycle time is restricted by the maximum work element time. This consequently confines the wide application of line balancing methods.

An alternative way to attack the increase of the production rate (hence lowering the cycle time) is by assigning the multiple workers to a definite station, that is, paralleling the station work. The effect of this assignment is to allow the multiple workers to perform the same station work, thereby increasing the upper time limit by the number of workers at the station. In this case, the problem is how to obtain the best possible combination of the number of workers and work elements in each station so that the efficiency of line balancing may be maximized.

Then we propose the method of assigning work elements to work stations under the multiple upper time limits which are the products of the number of workers and the limiting cycle time. We call this method the line balancing method under multiple workers in each station (LBMW).

Further we develop the computer program of line balancing with the proposed method. And an illustrative problem and computational results are provided to explain the proposed method.

2. Notations

We use the following notations for the assembly line terminology.

n : number of work elements
w_k : kth work element in the element list
t_k : performance time of w_k
P_k : set of work elements preceding to w_k
M_k : performance restrictions of w_k
$k=1,...,n$: work element serial number
$T=\sum t_k, k=1,..,n$: total work content per unit product
$t_{max}=max\{t_k, k=1,..,n\}$: maximum work element time
Pu : limiting cycle time (depending on production rate)
N : number of work stations
n_i : number of work elements in the ith station
w_{ij} : jth work element in the ith station
t_{ij} : performance time of w_{ij}
$j=1,...,n_i$: work element serial number in the ith station
$i=1,...,N$: station serial number
$T_i=\sum t_{ij}, j=1,..,n_i$: station time of the ith station

$P=max\{T_i, i=1,..,N\}$: cycle time of the line

$Eb=\frac{T}{N\times P}$: efficiency of line balancing

3. Method proposed

Work elements are assigned to work stations which have one or more workers performing the same station work. So we add the other notations as follows :

m_i : number of workers in the ith station

$m=\sum m_i, i=1,..,N$: total number of workers on the line

$Mimax$: admitted maximum number of workers at the ith station

$l=1,...,Mimax$: serial number of stages

where

$m_i \in \{1,...,Mimax\}$

$P=max\{T_i/m_i, i=1,...,N\}$

$Eb=\frac{T}{m\times P}$

In this method the problem is how to obtain the combination of m_i and $\{w_{ij}, j=1,...,n_i\}$ in each station so that the ratio of $T_i/(m_i\times Pu)$ may be maximized.

Work elements, their performance time, precedence relations, and other constraints are assumed to be given. Further Pu and $Mimax$ must be given to execute the proposed method LBMW. The selection of assignable work elements is based on the preceding relations and the largest candidate rule [2].

The procedure of LBMW is as follows.

Step 1. Set the initial data.

$R = \{w_k, k=1,...,n\}$: set of still unassigned work elements

$i = 0$. Go to step 2.

Step 2. Proceeding to the next station.

$i = i + 1$

$l = 0$. Go to step 3.

Step 3. Proceeding to the next upper time limit at ith station.

$l = l + 1$

$n_i(l) = 0$, $T_i(l) = 0$. Go to step 4.

Step 4. Select the maximum work element w_a among the ones which satisfy all the following four conditions.

(1) $w_a \in R$

w_a is one of the unassigned work elements.

(2) $\{ w_a | P_a \cap R = \phi \}$

w_a is one of the workable work elements with precedence relations.

(3) $\{ w_a | t_a < l\times Pu - T_i(l) \}$

w_a is one of the assignable work elements with the bound of slack times.

(4) $\{ w_a \mid (M_a=M_{ij}(l), j=1,..,n_i(l)) \bigcup (M_a=\phi) \}$

w_a is one of the assignable work elements with the performance restrictions.

If w_a can be selected, go to step 5. If not, go to step 6.

Step 5. Assign the selected work element w_a to ith station with lth upper time limit and the following calculations are done.

$n_i(l) = n_i(l) + 1$,

$j = n_i(l)$,

$w_{ij}(l) = w_a$, $t_{ij}(l) = t_a$, $T_i(l) = T_i(l) + t_a$, $R = R - \{w_a\}$.

Return to step 4.

Step 6. Reset the assigned work elements with lth limit.

$R = R + \{w_{ij}(l), j=1,...,n_i(l)\}$.

If $l < Mimax$, return to step 3. If $l \geqq Mimax$, go to step 7.

Step 7. Select the best combination of the number of workers and work elements at ith station. First,

$l_0 = \{l \mid max \{T_i(l)/ (l \times Pu), l=1,...,Mimax\}$

and using l_0, the following calculations are done.

$m_i = l_0$, $n_i = n_i(l_0)$, $T_i= T_i(l_0)$,

$w_{ij}= w_{ij}(l_0)$, $t_{ij}= t_{ij}(l_0)$, $j=1,...,n_i(l_0)$,

$R = R - \{w_{ij}, j=1,...,n_i\}$.

If $R \neq \phi$, return to step 2. If $R = \phi$, go to step 8.

Step 8. Compute the balance and stop the procedure.

$N = i$

$P = max\{T_i/m_i, i=1,...,N\}$, $Eb = T/(\sum m_i \times P) = T/(m \times P)$.

4. Program

The work assignment method under the condition of multiple upper time limits is programmed in FORTRAN Ⅳ. The program is the form of subroutine and its name is LBMW.

```
SUBROUTINE LBMW(PU,MIMAX,NW,NWK,TWK,KINDP,NAMEP,MACHIN,NSTN,TI,MI,NI,NWIJ)
```

4.1. Usage

Argument list

ARGUMENT	I/O	TYPE	SIZE	DEFINITION
PU	I	REAL	1	Pu, the limiting cycle time
MIMAX	I	INTEGER	1	$Mimax$, admitted number of workers at a station
NW	I	INTEGER	1	n, number of work elements
NWK	I	INTEGER	NW	w_k, work element number
TWK	I	REAL	NW	t_k, time durations of work element
KINDP	I	INTEGER	NW	number of pre work elements
NAMEP	I	INTEGER	NW×KINDP	P_k, pre work element number
MACHIN	I	INTEGER	NW	M_k, performance restriction

ARGUMENT	I/O	TYPE	SIZE	DEFINITION	
NSTN	0	INTEGER	1	N,	number of stations
TI	0	REAL	NSTN	T_i,	station time
MI	0	INTEGER	NSTN	m_i,	number of workers in the station
NI	0	INTEGER	NSTN	n_i,	number of work elements in the station
NWIJ	0	INTEGER	NSTN×NI		work element number in the station

$\text{NW} \leq 50$, $\text{KINDP} \leq 10$, $\text{NSTN} \leq 50$, $\text{NI}(i) \leq 10$, $i=1,\ldots,\text{NSTN}$, $\text{NWIJ} \in \{1,2,..,\text{NW}\}$

4.2. Suggestion on using

Subroutine SWWEPR, SAWEBS, SAWMAC and MAXGRP are used in LBMW. These are used to select the maximum work element w_a at step 4 in the procedure of the method.

The program list of LBMW is shown in Table 1.

5. Illustrative problem and computational results

The assembly work of the small electric switches is used as an example to illustrate the proposed method. The total assembly work has been analyzed and divided into work elements. The list of work elements has been developed and shown in Table 2.

The line balncing consists of two procedures [3];

(1) Minimize the number of workers on the line given the limiting cycle time.

(2) Minimize the cycle time given the number of workers on the line.

LBMW can be applied to both procedures.

We assume that the production schedule needs two thousands of the switches per day (420 minutes). So the cycle time of the line must be lower than 420/2000, or 0.210 minutes, and the limiting cycle time Pu is 0.210 minutes.

But the work element w_1 (01) takes 0.323 minutes, which is longer than Pu. In this case, the establishment of the assembly line is impossible by the serial balancing methods. So the LBMW will be applied to solve this problem.

As t_{max}/Pu=0.323/0.210=1.58, $Mimax$ must be at least equal to 2, and we set $Mimax$=4.

5.1. Assignment under the given Pu=0.210

Giving Pu=0.210 and $Mimax$=4, LBMW is called. The step of obtaining the solution is shown in Table 3. Computational results in this case are shown in Table 4. The obtained line is constructed by 3 work stations(N=3) and 5 workers(m=5). The cycle time of that is 0.201 and the efficiency of line balancing is 0.909 (*i.e.*, $P=0.201$ and $Eb=0.909$) In detail the *1*st station has 3 workers($m_1=3$), 3 work elements($n_1=3$, $w_{1j}=\{01,02,04\}$), and the station time of 0.602($T_1=0.602$). The 2nd station is $m_2=1$, $n_2=2$, $w_{2j}=\{05,03\}$, and $T_2=0.184$. At the 3rd station

$m_3=1$, $n_3=1$, $w_{3j}=\{06\}$, and $T_3=0.126$.

5.2. Assignment to lower the cycle time for 5 workers

To lower the cycle time and improve Eb, we use the cycle time of the previous obtained line for the next Pu. Giving $Pu=P=0.201$ (obtained in 5.1.) and $Mimax=4$, LBMW is called. As the obtained line has 5 workers, the higher balance for $m=5$ is obtained. Computational results in this case are $N=2$, $m=5$, $P=0.196$ and $Eb=0.928$. The 1st station has 4 workers($m_1=4$), 5 work elements($n_1=5$, $w_{1j}=\{01,02,04,05,03\}$) and the station time 0.786($T_1=0.786$). The 2nd station has $m_2=1$, $n_2=1$, $w_{21}=06$ and $T_2=0.126$. They are shown in Table 5.

Further we use $P=0.196$ for Pu and perform the same procedure. As the line has 5 workers, the higher balance for $m=5$ is obtained. Computational results in this case are $N=2$, $m=5$, $P=0.184$, $Eb=0.991$, shown in Table 6. The 1st station is 4 workers($m_1=4$), 4 work elements ($n_1=4$, $w_{1j}=\{01,02,04,05\}$), and $T_1=0.728$. And the 2nd station is $m_2=1$, $n_2=2$, $w_{2j}=\{06,03\}$ and $T_2=0.184$.

Further we use $P=0.184$ for Pu and perform the same procedure. In this case the obtained line has one more workers than the previous line. This means that the cycle time can not be lower than 0.184 for 5 workers on the line.

As the results, three different work assignments which satisfy the production rate 2000 units/day were obtained. One of them may be selected and adopted by the other measurements except the efficiency.

References

[1] P. Pinto, et al., *Int. J. Prod. Res.*, Vol.13, No.2, p.183(1975)
[2] C.L.Moodie and H.H.Young, *J. Indust. Engng.*, Vol.16, No.1, p.23(1965)
[3] W.B.Helgeson and D.P.Birnie, *J.Indust. Engng.*, Vol.12, No.6, p.394(1961)

Table 2. List of work elements of a switch.

NWK	TWK	KINDP	NAMEP	MACHIN
w_k	t_k(min.)		P_k	M_k
01	0.323	0		
02	0.153	0		
03	0.058	1	02	
04	0.126	0		
05	0.126	0		
06	0.126	0		

$t_{max}=0.323$, $T=0.912$

Table 3. Steps of obtaining the solution under $Pu=0.210$ and $Mimax=4$.

```
I   L   TI(L)   TI(L)/(L*PU)   *IJ(L)
1
    1   0.153      0.729         2
    2   0.323      0.769         1
    3   0.602      0.956         1   2   4
    4   0.786      0.936         1   2   4   5   3
2
    1   0.184      0.876         5   3
    2   0.310      0.738         5   6   3
    3   0.310      0.492         5   6   3
    4   0.310      0.369         5   6   3
3
    1   0.126      0.600         6
    2   0.126      0.300         6
    3   0.126      0.200         6
    4   0.126      0.150         6
```

Table 4. Computational results.

```
N= 3   M= 5   PU=     .210
  I       TI       MI     WIJ    TWIJ
  1      .602       3
                          01    .323
                          02    .153
                          04    .126
  2      .184       1
                          05    .126
                          03    .058
  3      .126       1
                          06    .126
 P=     .201     EB= .909
```

Table 5. Computational results.

```
N= 2   M= 5   PU=     .201
  I       TI       MI     WIJ    TWIJ
  1      .786       4
                          01    .323
                          02    .153
                          04    .126
                          05    .126
                          03    .053
  2      .126       1
                          06    .126
 P=     .196     EB= .923
```

Table 6. Computational results.

```
N= 2   M= 5   PU=     .196
  I       TI       MI     WIJ    TWIJ
  1      .728       4
                          01    .323
                          02    .153
                          04    .126
                          05    .126
  2      .184       1
                          06    .126
                          03    .058
 P=     .184     EB= .991
```

Table 1. Program list.

```
C *** CPLB UNDER THE MULTIPLE WORKERS IN EACH STATION ***
      SUBROUTINE LBMW(PU,MIMAX,NW,NWK,TWK,KINDP,NAMEP,MACHIN,
     &                          NSTN,TI,MI,NI,NWIJ)
      DIMENSION NWK(50),TWK(50),KINDP(50),NAMEP(50,10),MACHIN(50)
      DIMENSION TI(50),MI(50),NI(50),NWIJ(50,10)
      DIMENSION TIL(10),NIL(10),NWIJL(10,10),ND(50)
      DIMENSION NGROUP(50),NASSGP(50)
      DO 12 I=1,NW
   12 ND(I)=NWK(I)
      IS=0
  400 IS=IS+1
      DO 14 I=1,NW
      IF(ND(I) .NE. 0) GO TO 15
   14 CONTINUE
      GO TO 500
   15 CONTINUE
      LS=0
  330 LS=LS+1
      IF(LS .GT.MIMAX) GO TO 315
      TREST=PU*FLOAT(LS)
      TIL(LS)=0.0
      NIL(LS)=0
      MACRES=0
  300 CONTINUE
      CALL SWWEPR(NW,ND,KINDP,NAMEP,NUMGRP,NGROUP)
      IF(NUMGRP .LE. 0) GO TO 310
      CALL SAWESS(NUMGRP,NGROUP,TREST,TWK,NUMASS,NASSGP)
      IF(NUMASS.LE.0) GO TO 310
      CALL SAWMAC(MACRES,NUMASS,NASSGP,MACHIN)
      IF(NUMASS .LE. 0) GO TO 310
      MAXET=MAXGRP(NUMASS,NASSGP,TWK)
      TIL(LS)=TIL(LS)+TWK(MAXET)
      NIL(LS)=NIL(LS)+1
      NWIJL(LS,NIL(LS))=MAXET
      ND(MAXET)=0
      TREST=TREST-TWK(MAXET)
      NTREST=TREST*10000.0
      TREST=NTREST
      TREST=TREST/10000.0
      IF(MACHIN(MAXET) .NE. 0) MACRES=MACHIN(MAXET)
      GO TO 300
  310 IF(NIL(LS).LE.0) GO TO 330
      EBL=TIL(LS)/(LS*PU)
```

```
      DO 35 I=1,NIL(LS)
       NET=NWIJL(LS,I)
  35   ND(NET)=NWK(NET)
      GO TO 330
 315  EBMAX=0.0
      DO 30 I=1,MIMAX
      EBL=TIL(I)/(I*PU)
      IF(EBL.LE.EBMAX) GO TO 30
      EBMAX=EBL
      LS=I
  30  CONTINUE
      TI(IS)=TIL(LS)
      MI(IS)=LS
      NI(IS)=NIL(LS)
      DO 38 J=1,NIL(LS)
      ND(NWIJL(LS,J))=0
  38  NWIJ(IS,J)=NWIJL(LS,J)
      NSTN=IS
      GO TO 400
 300  RETURN
      END
C   *** SELECT THE WORKABLE WORK ELEMENTS WITH PRECEDENCE RELATIONS ***
      SUBROUTINE SWWEPR(NW,ND,KINDP,NAMEP,NUMGRP,NGROUP)
      DIMENSION ND(50),KINDP(50),NAMEP(50,10),NGROUP(50)
      NUMGRP=0
      DO 10 I=1,NW
       IF(ND(I) .EQ. 0) GO TO 10
       KP=KINDP(I)
       DO 20 J=1,KP
        DO 20 K=1,NW
        IF(ND(K) .EQ. 0) GO TO 20
        IF(NAMEP(I,J) .EQ. ND(K)) GO TO 10
  20   CONTINUE
      NUMGRP=NUMGRP+1
      NGROUP(NUMGRP)=I
  10  CONTINUE
      RETURN
      END
C   *** SELECT THE ASSIGNABLE WORK ELEMENTS WITH THE BOUND OF SLACK TIMES ***
      SUBROUTINE SAWEBS(NUMGRP,NGROUP,TREST,TWK,NUMASS,NASSGP)
      DIMENSION NGROUP(50),TWK(50),NASSGP(50)
      NUMASS=0
      DO 10 I=1,NUMGRP
       NAS=NGROUP(I)
       IF(TWK(NAS) .GE. TREST) GO TO 10
       NUMASS=NUMASS+1
       NASSGP(NUMASS)=NGROUP(I)
  10  CONTINUE
      RETURN
      END
C   *** SELECT ASSIGNABLE WORKS WITH THE MACHINE RESTRICTIONS ***
      SUBROUTINE SAWMAC(MACRES,NUMASS,NASSGP,MACHIN)
      DIMENSION NASSGP(50),MACHIN(50)
      IF(MACRES .EQ. 0) RETURN
      N=0
      DO 10 I=1,NUMASS
       NAS=NASSGP(I)
       IF(MACHIN(NAS) .LE. 0) GO TO 15
       IF(MACRES.NE.MACHIN(NAS)) GO TO 10
  15  N=N+1
       NASSGP(N)=NASSGP(I)
  10  CONTINUE
      NUMASS=N
      RETURN
      END
C   *** LARGEST CANDIDATE RULE ***
      FUNCTION MAXGRP(NUMASS,NASSGP,TWK)
      DIMENSION NASSGP(50),TWK(50)
      MAXGRP=NASSGP(1)
      BIG=TWK(MAXGRP)
      DO 100 I=1,NUMASS
       N=NASSGP(I)
       IF(TWK(N) .LE. BIG) GO TO 100
       MAXGRP=N
       BIG=TWK(N)
 100  CONTINUE
      RETURN
      END
```