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# Optimal Design of Efficient IPM Motor Using Finite Element Method

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**Abstract**—Techniques for the optimal design of permanent magnet motors considering rotation are investigated. The nonconforming mesh technique is used to take into account the rotation of rotor. It is shown that the technique is especially useful for the optimal design using the finite element method, because no modification of mesh is necessary during the rotation. The mesh at each angle of rotation can be obtained by only combining meshes of a rotor and a stator. By using the combined optimization technique of the experimental design method and the Rosenbrock's method, the number of FEM calculations can be fairly reduced. By applying the optimal design method, about 40% of volume of permanent magnet of IPM motor and about 15% of the torque ripple can be reduced.

**Index Terms**—Finite element method, IPM motor, nonconforming mesh, optimization.

## I. INTRODUCTION

THE INTERIOR permanent magnet (IPM) motor has many advantages, such as high efficiency etc., and this motor is becoming widely used not only in home applications like air conditioners but also electric vehicles. Although the investigation of optimal design of IPM motors is important, the analysis of IPM motors using the finite element method and the optimization method is very rare. This is, because the IPM motor should be analyzed considering the rotation of rotor in order to obtain the torque ripple, the mesh generation at each step of rotation is troublesome and the practical optimization technique with small number of iterations is not demonstrated. If the nonconforming mesh technique [1], [2] is used in the optimization problems with rotating meshes, the practical optimization considering the rotation of rotor becomes easy, because the mesh at each angle of rotation can be obtained by only combining meshes of a rotor and a stator.

In this paper, techniques for the optimal design of permanent magnet motor considering rotation are investigated. The nonconforming mesh, the combined experimental design method and Rosenbrock's method [3] are introduced for practical design. The usefulness of the optimal design method is illustrated by applying it to the design of an IPM motor with small volume of permanent magnet and with low torque ripple.

## II. MODEL OF IPM MOTOR

Fig. 1 shows the model of IPM motor. The core is made of nonoriented silicon steel, and the NdFeB magnet ( $B_r = 1.25$  T)

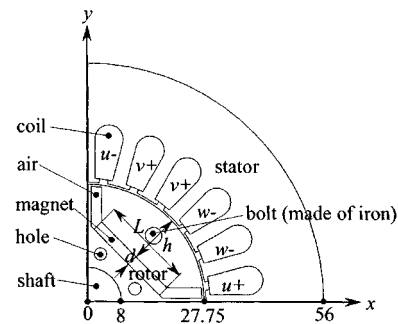


Fig. 1. Model of IPM motor.

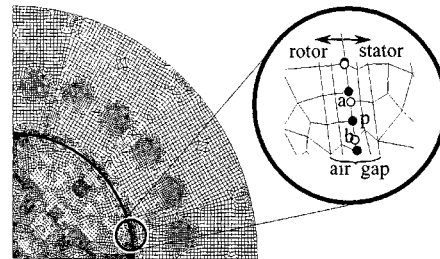


Fig. 2. Meshes (final shape).

is inserted in the rotor. The rotating speed is 1800 rpm. The thickness of the motor in the  $z$ -direction is 65 mm. The number of turns of the winding per phase is 140 and the rated current is 3 A (rms). It is assumed that the currents  $I_u, I_v, I_w$  in stator winding are three-phase sinusoidal. For example,

$$I_u = I \sin(\omega t - \theta) \quad (1)$$

where  $\theta$  is the phase difference between the no-load voltage ( $q$ -axis) of  $u$ -phase and the current  $I_u$ .  $I$  is the maximum current.

The length  $d$ , the width  $L$ , the depth  $h$  of permanent magnet and the phase angle  $\theta$  of current are chosen as design variables. These values of initial shapes are assumed  $d_0 = 2.5$  mm,  $L_0 = 20.5$  mm,  $h_0 = 12$  mm, and  $\theta_0 = 30$  deg.

## III. TECHNIQUES FOR OPTIMAL DESIGN

### A. Nonconforming Mesh Technique

The meshes of stator and rotor are generated separately, then both meshes are combined at the respective rotor position using the nonconforming mesh technique [1], [2]. Fig. 2 shows the whole mesh and enlarged mesh near the air gap. The white and black circles are the nonconforming nodes. The potentials of the nonconforming nodes on one side, where the mesh density is

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larger, are interpolated by those on the opposite side. For example, the potential at the node  $p$  is linearly interpolated by those at the nodes  $a$  and  $b$  in Fig. 2.

### B. Finite Element Analysis

Due to symmetry, 1/4 region is analyzed using the nonlinear finite element method. The stator mesh is kept constant. On the contrary, the rotor mesh is generated automatically according to the change of shape at each iteration of optimization. The analyzed region is subdivided into about 11 000 elements. The nodal force method [4] is used in the calculation of torque.

The rotor is rotated at the step angle of 1 deg. Nearly periodic torque waveform can be obtained at 30 steps of calculation. Then, in the analysis of optimization, the rotor is rotated by 30 mechanical angle (deg., 30 steps) at each iteration of optimization.

### C. Objective Functions and Constraints

The optimization is carried out so that volume  $V$  of permanent magnet or torque ripple  $Tr$  becomes minimum under the constraint that the torque is not less than 1.9 N·m. The objective functions  $W$  are chosen as follows:

*Minimum Volume of Magnet:*

$$W = V/V_0 + P \quad (2)$$

$$P = \begin{cases} 0 & (Ta \geq 1.9) \\ 10 \times (1.9 - Ta) & (Ta < 1.9) \end{cases} \quad (3)$$

where subscript (0) denotes the value at initial shape.  $P$  is the penalty function defined in (3).

*Minimum Torque Ripple:*

$$W = Td + P \quad (4)$$

$Td$  is defined by

$$Td = T \max - Ta \quad (5)$$

where  $T \max$  and  $Ta$  are the maximum torque and the average torque, respectively.

The constraints of design variables  $d$ ,  $L$ ,  $h$  and  $\theta$  are defined as follows:

$$\begin{aligned} 0.5 \leq d \leq 4.5 \\ 18 \leq L \leq 23 \\ 8 \leq H \leq 14 \\ 0 \leq \theta \leq 60. \end{aligned} \quad (6)$$

### D. Combined Method of EDM and RBM

It is already reported that the experimental design method (EDM, Taguchi's Method) is useful for the determination of initial values of design variables [2]. Therefore, first EDM is used for the determination of initial values, then Rosenbrocks method (RBM) is applied to search for the optimal value.

As an example, the procedure for applying EDM in determining initial values is explained. The constraints are divided into three levels as shown in Table I. The volume  $V$  of permanent magnet, torque ripple  $Tr$  and average torque  $Ta$ , which have nine patterns, are calculated from the orthogonal array (Nos.1-9) shown in Table II. In this table,  $V/Va + Tr$  is not calculated when  $Ta$  is less than 1.9 N·m.  $Va$  is the average volume

TABLE I  
LEVEL OF DESIGN VARIABLE

design variable	level1	level2	level3
$d$ [mm]	1.7	2.5	3.2
$L$ [mm]	19	20.5	22
$h$ [mm]	9	11.5	13
$\theta$ [deg]	20	30	40

TABLE II  
ORTHOGONAL ARRAY

model no.	design variable	$Td+P$	$V/Va$	$Tr$	$V/Va + Tr$	$Ta$			
	$d$	$L$	$h$	$\theta$					
1	1	1	1	1	3.77	0.638	0.418	-	1.86
2	1	2	2	2	0.406	0.690	0.517	1.21	1.93
3	1	3	3	3	0.437	0.739	0.579	-	1.84
4	2	1	3	2	0.441	0.937	0.575	1.51	1.90
5	2	2	1	3	0.374	1.01	0.529	1.54	2.02
6	2	3	2	1	1.07	1.09	0.511	1.60	2.29
7	3	1	2	3	0.395	1.20	0.563	1.77	1.97
8	3	2	3	1	2.35	1.30	0.586	1.88	2.06
9	3	3	1	2	0.557	1.39	0.566	1.96	2.38

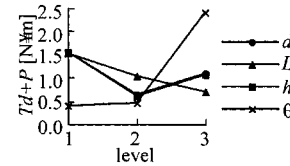


Fig. 3. Effect of design variables.

of models no. 1-9. As the models no. 1-3 have a level 1 of  $d$ , the effective value of level 1 is the average value of models Nos.1-3. The effective values of levels 2 and 3 are also the average values of models no. 4-6 and 7-9, respectively.

*Minimum Torque Ripple:* The influence of each design variable on the torque ripple  $Td + P$  can be obtained from Table II, and  $Td + P$  is plotted in Fig. 3. From the figure, the appropriate initial values for each design variable, in which  $Td + P$  becomes a minimum, can be obtained as  $d$ : level 2 (2.5 mm),  $L$ : level 3 (22 mm),  $h$ : level 2 (11.5 mm) and  $\theta$ : level 1 (20 deg).

*Minimum Volume of Magnet:* The same procedure is carried out in the determination of initial values for the case of minimum volume of magnet. In this case, the values of  $h$  and  $\theta$  in which the average torque becomes a maximum are adopted as the initial values, under the condition that the volume  $V_0$  of permanent magnet is fixed ( $V_0 = 65d_0 \cdot L_0 = 3.33 \times 10^3 \text{ mm}^3$ ).

## IV. RESULTS AND DISCUSSION

### A. Minimum Volume of Magnet

Table III shows the obtained results using only RBM (caseA) and the combined method of EDM and RBM (caseB). The volume  $V$  obtained using the combined method of EDM and RBM (caseB) is smaller than that using only RBM (caseA). Moreover, the Table denotes that the combined method of EDM and RBM is effective from the viewpoint of shorter CPU time. About 40% of the volume of permanent magnet can be reduced

TABLE III  
RESULTS OF OPTIMIZATION (MINIMUM VOLUME OF MAGNET)

	caseA (only RBM)		caseB (EDM+RBM)	
	initial	optimal	initial	optimal
$d$ [mm]	2.50	1.92	2.50	1.54
$L$ [mm]	20.5	18.6	20.5	20.2
$h$ [mm]	12.0	8.89	8.50	8.50
$\theta$ [deg]	30.0	27.7	30.0	26.5
$V$ [ $\times 10^3 \text{mm}^3$ ]	3.33	2.31	3.33	2.02
$T_a$ [N·m]	2.09	1.90	2.15	1.90
Number of iterations	93		56	
CPU time[h]	27.8		17.2	

Computer used: VT-Alpha533 (SPECfp95: 22.5)

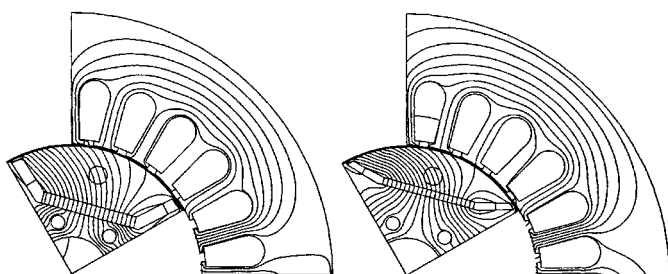


Fig. 4. Flux distributions.

TABLE IV  
RESULTS OF OPTIMIZATION (MINIMUM TORQUE RIPPLE)

	caseA (only RBM)		caseB (EDM+RBM)	
	initial	optimal	initial	optimal
$d$ [mm]	2.50	2.58	2.50	2.10
$L$ [mm]	20.5	18.3	22.0	20.3
$h$ [mm]	12.0	10.9	11.0	8.73
$\theta$ [deg]	30.0	30.3	20.0	15.5
Tr [%]	56.7	50.8	51.1	42.4
Ld [mH]	12.3	10.1	18.1	23.1
Lq [mH]	28.6	28.4	30.8	33.6
Number of iterations	93		41	
CPU time[h]	29.7		12.7	

Computer used: VT-Alpha533 (SPECfp95: 22.5)

(caseB) under the condition that the average torque is not less than 1.9 N·m. The magnet length of optimal shape is less than that of initial shape. Fig. 4 shows the flux distribution at initial and optimal shapes (caseB).

### B. Minimum Torque Ripple

Table IV shows the obtained results. The torque ripple can be reduced from 56.7% to 42.4% (caseB). Fig. 5 shows the torque waveforms.

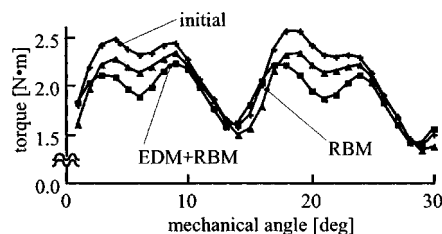


Fig. 5. Torque waveforms.

## V. CONCLUSION

The obtained results can be summarized as follows:

- 1) It is easy to optimize the motor shape taking account of rotation of rotor by using the nonconforming mesh technique.
- 2) The optimal design using the combined EDM and RBM may be applicable to the practical design, because the optimal value can be obtained with the acceptable CPU time.
- 3) It is shown that the technique discussed in this paper is effective for obtaining the shape of IPM motor having minimum volume of permanent magnet or minimum torque ripple.

The verification of the optimization of the IPM motor by experiments is the future work.

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