Survey Expectations

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August 2005

CWPE 0536

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29th July, 2005

Abstract

This paper focusses on survey expectations and discusses their uses for testing and modeling of expectations. Alternative models of expectations formation are reviewed and the importance of allowing for heterogeneity of expectations is emphasized. A weak form of the rational expectations hypothesis which focusses on average expectations rather than individual expectations is advanced. Other models of expectations formation, such as the adaptive expectations hypothesis, are briefly discussed. Testable implications of rational and extrapolative models of expectations are reviewed and the importance of the loss function for the interpretation of the test results is discussed. The paper then provides an account of the various surveys of expectations, reviews alternative methods of quantifying the qualitative surveys, and discusses the use of aggregate and individual survey responses in the analysis of expectations and for forecasting.

JEL Classifications: C40, C50, C53, C80

Key Words: Models of Expectations Formation, Survey Data, Heterogeneity, Tests of Rational Expectations.

*Prepared for inclusion in the Handbook of Economic Forecasting, G. Elliott, C.W.J. Granger, and A. Timmermann (eds.), North-Holland (forthcoming 2006). Helpful comments by two anonymous referees, Kajal Lahiri and Ron Smith are gratefully acknowledged.

Contents

1 Introduction

Expectations formation is an integral part of the decision making process by households, firms, as well as the private and public institutions. At the theoretical level the rational expectations hypothesis as advanced by Muth (1961) has gained general acceptance as the dominant model of expectations formation. It provides a fully theory-consistent framework where subjective expectations of individual decision makers are set to their objective counterparts, assuming a known true underlying economic model. Expectations can be in the form of point expectations, or could concern the whole conditional probability distribution of the future values of the variables that influence individual decisions, namely probability or density expectations. Point expectations would be sufficient in the case of linear-quadratic decision problems where the utility (or cost) functions are quadratic and the constraints linear. For more general decision problems density expectations might be required.

From an empirical viewpoint, expectations formation is closely linked to point and density forecasting and as such is subject to data and model uncertainty. Assuming that individual decision makers know the true model of the economy is no more credible than claiming that economic forecasts made using econometric models will be free of systematic bias and informational inefficiencies. This has led many investigators to explore the development of a weaker form of the rational expectations hypothesis that allows for model uncertainty and learning.[1] In this process experimental and survey data on expectations play an important role in providing better insights into how expectations are formed. There is now a substantial literature on survey expectations. Experimental data on expectations are also becoming increasingly available and are particularly important for development of a better understanding of how learning takes place in the expectations formation process.

As with many areas of applied econometrics, initial studies of survey data on expectations tended to focus on the properties of aggregate summaries of survey findings, and their role in aggregate time-series models. The first study of individual responses was published in 1983 and much of the more recent work has been focused on this. Obviously, when a survey covers expectations of individual experience, such as firm's sales or a consumer's income, it is desirable to assess the survey data in the light of the subsequent outcome for the individual. This allows an assessment of the reported expectations in a manner which is not possible using only time-series aggregates but it requires some form of panel data set. Even where

[1] Evans & Honkapohja (2001) provide an excellent account of recent developments of expectations formation models subject to learning.

a survey collects information on expectations of some macro-economic aggregate, such as the rate of inflation, it is likely that analysis of individual responses will provide richer and more convincing conclusions than would be found from the time-series analysis of aggregated responses alone.

This paper focusses on the analysis of survey expectations at the individual and at the aggregate levels and discusses their uses in forecasting and for testing and modelling of expectations. Most expectations data are concerned with point expectations, although some attempts have been made to elicit density expectations, in particular expectations of second order moments. Survey data are often compiled in the form of qualitative responses and their conversion into quantitative measures might be needed. The elicitation process embodied in the survey techniques also presents further problems for the use of survey expectations. Since respondents tend to lack proper economic incentives when answering survey questions about their expectations, the responses might not be sufficiently accurate or reliable. Finally, survey expectations tend to cover relatively short horizons, typically 1 to 12 months, and their use in long-term forecasting or impulse response analysis will be limited, and would require augmenting the survey data with a formal expectations formation model for generation of longer term expectations, beyond the horizon of the survey data. The literature on these and on a number of other related issues will be covered. In particular, we consider the evidence on the use of survey expectations in forecasting. The question of interest would be to see if expectations data when used as supplementary variables in forecasting models would lead to better forecasting performance. We note that many expectational surveys also collect information about the recent past. Such data are potentially useful for "nowcasting" because they are typically made available earlier than the "hard" official data to which they are supposed to relate. While their study falls outside a synthesis of work on survey measures of expectations, and is not discussed here, it is worth noting that many of the methods used to analyse and test survey data about expectations of the future also apply with little or no modification, to analysis of these retrospective data. In some circumstances, as we discuss in section 3.3 , they may be required to assess the performance of surveys about expectations of the future.

While we focus on survey expectations rather than the forecasting properties of particular statistical or econometric models, it is worth emphasizing that the distinction is more apparent than real. Some surveys collate the forecasts of professional forecasters, and it is likely that at least some of these are generated by formal forecasting models and forecasting processes of various types. Even where information on such expectations is collected from the public at large, such expectations may be closely informed by the published forecasts of professional forecasters. There are some circumstances where it is, however, unlikely that formal models are implied. If consumers are asked about their expectations of their own incomes, while these may be influenced by forecasts for the macro-economy, they are unlikely to be solely the outcome of formal forecasting procedures. When businesses are asked about how they expect their own sales or prices to change, the same is likely to be true. The ambiguity of the distinction and the fact that some important issues are raised by surveys which collect information from professional analysts and forecasters does mean, however, that we give some consideration to such surveys as well as to those which are likely to reflect expectations rather than forecasts.

Our review covers four separate but closely related topics and is therefore organized in four distinct parts. In part one we address the question of concepts and models of expectations formation. Part two looks at the development of measures of expectations including issues arising in the quantification of qualitative measures of expectations. Part three considers the use of survey expectations in forecasting, and part four considers how survey data are used in testing theories with particular emphasis on models of expectations formation. Conclusions follow.

We begin part one by introducing some of the basic concepts and the various models of expectations formation advanced in the literature. In section 2.1 we introduce the rational expectations hypothesis and discuss the importance of allowing for heterogeneity of expectations in relating theory to survey expectations. To this end a weak form of the rational expectations hypothesis which focusses on average expectations rather individual expectations is advanced. Other models of expectations formation, such as the adaptive expectations hypothesis, are briefly reviewed in section 2.2. In section 2.3 we discuss some of the issues involved in testing models of expectations. Section 2.4 further considers the optimality of survey forecasts in the case where loss functions are asymmetric.

The introductory section to part two provides a historical account of the development of surveys of expectations. As noted above, many of these surveys collect qualitative data; section 3.1 considers ways of quantifying these qualitative expectations paying attention to both the use of aggregated data from these surveys and to the use of individual responses. In section 3.2 we discuss different ways of providing and interpreting information on uncertainty to complement qualitative or quantitative information on expectations. In section 3.3 we discuss the analysis of individual rather than aggregated responses to surveys about expectations.

The focus of part three is on the uses of survey data in producing economic forecasts. In section 4.1 we discuss the use of survey data in the context of forecast combination as a means of using disparate forecasts to produce a more accurate compromise forecast. Section 4.2 considers how they can be used to indicate the uncertainty of forecasts and in section 4.3 we discuss the use of qualitative surveys to produce forecasts of quantitative macro-economic aggregates.

Methods of testing models of expectation formation, discussed in part four are split between analysis based on the results of quantitative surveys of expectations in section 5.1 and the analysis of qualitative disaggregated data in section 5.2. A substantial range of econometric issues arises in both cases. Perhaps not surprisingly more attention has been paid to the former than to the latter although, since many of the high-frequency expectations surveys are qualitative in form, the second area is likely to develop in importance.

2 Part I: Concepts and Models of Expectations Formation

Expectations are subjectively held beliefs by individuals about uncertain future outcomes or the beliefs of other individuals in the market place.[2] How expectations are formed, and whether they lend themselves to mathematical representations have been the subject of considerable debate and discussions. The answers vary and depend on the nature and the source of uncertainty that surrounds a particular decision. Knight (1921) distinguishes between 'true uncertainty' and 'risk' and argues that under the former it is not possible to reduce the uncertainty and expectations to 'an objective quantitatively determined probability' (p. 321). Pesaran (1987) makes a distinction between exogenous and behavioural uncertainty and argues that the former is more likely to lend itself to formal probabilistic analysis. In this review we focus on situations where individual expectations can be formalized.

Denote individual i's point expectations of a k dimensional vector of future variables, say $\mathbf{x}_{t+1}$, formed with respect to the information set, Ω_{it}, by $E_i(\mathbf{x}_{t+1}|\Omega_{it})$. Similarly, let $f_i(\mathbf{x}_{t+1}|\Omega_{it})$ be individual i's density expectations, so that

$$E_i(\mathbf{x}_{t+1}|\Omega_{it}) = \int \mathbf{x}_{t+1} f_i\left(\mathbf{x}_{t+1}|\Omega_{it}\right) d\mathbf{x}_t.$$

[2] It is also possible for individuals to form expectations of present or past events about which they are not fully informed. This is related to "nowcasting" or "backcasting" in the forecasting literature mentioned above.

Individual i's belief about individual j's expectations of $\mathbf{x}_{t+1}$ may also be written as

$$E_i[E_j\left(\mathbf{x}_{t+1}|\Omega_{jt}\right)|\Omega_{it}].$$

Clearly, higher order expectations can be similarly defined but will not be pursued here.

In general, point expectations of the same variable could differ considerably across individuals, due to differences in Ω_{it} (information disparity), and differences in the subjective probability densities, $f_i(.)$ (belief disparity). The two sources of expectations heterogeneity are closely related and could be re-inforcing. Information disparities could initiate and maintain disparities in beliefs, whilst differences in beliefs could lead to information disparities when information processing is costly.[3]

Alternative models of expectations formation provide different characterizations of the way subjective beliefs and the objective reality are related. At one extreme lies the rational expectations hypothesis of Muth (1961) that postulates the coincidence of the two concepts, with Knightian view that denies any specific links between expectations and reality. In what follows we provide an overview of the alternative models, confining ourselves to expectations formation models that lend themselves to statistical formalizations.

2.1 The Rational Expectations Hypothesis

For a formal representation of the rational expectations hypothesis (REH), as set out by Muth, we first decompose the individual specific information sets, Ω_{it}, into a *public information* set Ψ_t, and an individual-specific *private information* set Φ_{it} such that

$$\Omega_{it} = \Psi_t \cup \Phi_{it},$$

for $i = 1, 2, ..., N$. Further, we assume that the 'objective' probability density function of $\mathbf{x}_{t+1}$ is given by $f(\mathbf{x}_{t+1}|\Psi_t)$. Then the REH postulates that

$$H_{REH} : f_i(\mathbf{x}_{t+1}|\Omega_{it}) = f(\mathbf{x}_{t+1}|\Psi_t), \text{ for all } i. \tag{1}$$

Under the Muthian notion of the REH, private information plays no role in the expectations formation process, and expectations are fully efficient with respect to the public information, Ψ_t. In the case of point expectations, the optimality of the REH is captured by the "orthogonality" condition

$$E(\boldsymbol{\xi}_{t+1}|S_t) = \mathbf{0}, \tag{2}$$

[3]Models of rationally heterogeneous expectations are discussed, for example, in Evans & Ramey (1992), Brock & Hommes (1997) and Branch (2002). See also section 5.1.5 for discussion of evidence on expectations heterogeneity.

where $\boldsymbol{\xi}_{t+1}$ is the error of expectations defined by

$$\boldsymbol{\xi}_{t+1} = \mathbf{x}_{t+1} - E(\mathbf{x}_{t+1}|\Psi_t), \tag{3}$$

and $S_t \subseteq \Psi_t$, is a subset of Ψ_t. The orthogonality condition (2) in turn implies that, under the REH, expectations errors have zero means and are serially uncorrelated. It does not, for example, require the expectations errors to be conditionally or unconditionally homoskedastic. From a formal mathematical perspective, it states that under the REH (in the sense of Muth) expectations errors form a martingale difference process with respect to the non-decreasing information set available to the agent at the time expectations are formed. In what follows we shall use the term 'orthogonality condition' and the 'martingale property' of the expectations errors interchangeably. The orthogonality condition is often used to test the informational efficiency of survey expectations. But as we shall see it is neither necessary nor sufficient for rationality of expectations if individual expectations are formed as optimal forecasts with respect to general cost functions under incomplete learning.

Also, the common knowledge assumptions that underlie the rationality of individual expectations in the Muthian sense is rather restrictive, and has been relaxed in the literature where different notions of the rational expectations equilibria are defined and implemented under asymmetric and heterogeneous information. See, for example, Radner (1979), Grossman & Stiglitz (1980), Hellwig (1980) and Milgrom (1981), just to mention some of the early important contributions.

In advancing the REH, Muth (1961) was in fact fully aware of the importance of allowing for cross section heterogeneity of expectations.[4] One of his aims in proposing the REH was to explain the following stylized facts observed using expectations data

> 1. Averages of expectations in an industry are more accurate than naive models and as accurate as elaborate equation systems, although there are considerable cross-sectional differences of opinion.
>
> 2. Reported expectations generally underestimate the extent of changes that actually take place.Muth (1961)[p. 316]

One of the main reasons for the prevalence of the homogeneous version of the rational expectations hypothesis given by (1) has been the conceptual and technical difficulties of

[4]Pigou (1927) and Keynes (1936) had already emphasized the role of heterogeneity of information and beliefs across agents for the analysis of financial markets.

dealing with rational expectations models under heterogeneous information.[5] Early attempts to allow for heterogeneous information in rational expectations models include Lucas (1973), Townsend (1978) and Townsend (1983). More recent developments are surveyed by Hommes (forthcoming) who argues that an important paradigm shift is occurring in economics and finance from a representative rational agent model towards heterogeneous agent models. Analysis of heterogeneous rational expectations models invariably involve the "infinite regress in expectations" problem that arise as agents need to forecast the forecasts of others. A number of different solution strategies have been proposed in the literature which in different ways limit the scope of possible solutions. For example, Binder & Pesaran (1998) establish that a unique solution results if it is assumed that each agent bases his/her forecasts of others only on the information set that is common knowledge, Ψ_t.

When the heterogeneous rational expectations model has a unique solution, expectations errors of individual agents continue to satisfy the usual orthogonality conditions. However, unlike in models under homogeneous information, the average expectations error across decision makers, defined as $\boldsymbol{\xi}_{t+1} = \mathbf{x}_{t+1} - \sum_{i=1}^{N} w_{it} E(\mathbf{x}_{t+1}|\Omega_{it})$ is generally not orthogonal with respect to the individual decision makers' information sets, where w_{it} is the weight attached to the i^{th} individual in forming the the average expectations measure. Seen from this perspective a weaker form of the REH that focusses on 'average' expectations might be more desirable. Consider the average density expectations computed over N individuals

$$\bar{f}_w(\mathbf{x}_{t+1}|\Omega_t) = \sum_{i\equiv 1}^{N} w_{it} f_i(\mathbf{x}_{t+1}|\Omega_{it}). \tag{4}$$

The average form of the REH can then be postulated as

$$\overline{H}_{REH} : \bar{f}_w(\mathbf{x}_{t+1}|\Omega_t) = f(\mathbf{x}_{t+1}|\Psi_t), \tag{5}$$

where $\Omega_t = U_{i=1}^{N} \Omega_{it}$, and w_{it} are non-negative weights that satisfy the conditions:

$$\sum_{i\equiv 1}^{N} w_{it} = 1, \;\; \sum_{i\equiv 1}^{N} w_{it}^2 = O\left(\frac{1}{N}\right). \tag{6}$$

In terms of point expectations, the average form of the REH holds if

$$\bar{E}_w(\mathbf{x}_{t+1}|\Omega_t) = \sum_{i\equiv 1}^{N} w_{it} E_i(\mathbf{x}_{t+1}|\Omega_{it}) = E(\mathbf{x}_{t+1}|\Psi_t), \tag{7}$$

[5]For example, as recently acknowledged by Mankiw, Reis & Wolfers (2004), the fact that expectations are not the same across individuals is routinely ignored in the macroeconomic literature.

which is much weaker than the REH and allows for a considerable degree of heterogeneity of individual expectations.

This version of the REH is, for example, compatible with systematic errors of expectations being present at the individual level. Suppose that individual expectations can be decomposed as

$$E_i(\mathbf{x}_{t+1}|\Omega_{it}) = \mathbf{H}_i E(\mathbf{x}_{t+1}|\Psi_t) + \mathbf{u}_{it}, \tag{8}$$

where $\mathbf{u}_{it}$, $i = 1, 2, ..., N$, are the individual-specific components. The individual expectations errors are now given by

$$\boldsymbol{\xi}_{i,t+1} = \mathbf{x}_{t+1} - E_i(\mathbf{x}_{t+1}|\Omega_{it}) = \boldsymbol{\xi}_{t+1} + (\mathbf{I}_k - \mathbf{H}_i)\, E(\mathbf{x}_{t+1}|\Psi_t) - \mathbf{u}_{it},$$

and clearly do not satisfy the REH if $\mathbf{H}_i \neq \mathbf{I}_k$, and/or $\mathbf{u}_{it}$ are, for example, serially correlated. Using the weights w_{it}, the average expectations errors are now given by

$$\bar{\boldsymbol{\xi}}_{w,t+1} = \boldsymbol{\xi}_{t+1} + \left(\mathbf{I}_k - \bar{\mathbf{H}}_w\right) E(\mathbf{x}_{t+1}|\Psi_t) - \bar{\mathbf{u}}_{wt},$$

where

$$\bar{\boldsymbol{\xi}}_{w,t+1} = \sum_{i=1}^{N} w_{it}\boldsymbol{\xi}_{i,t+1},\ \bar{\mathbf{H}}_{wt} = \sum_{i=1}^{N} w_{it}\mathbf{H}_i,\ \bar{\mathbf{u}}_{wt} = \sum_{i=1}^{N} w_{it}\mathbf{u}_{it}.$$

The conditions under which average expectations are 'rational' are much less restrictive as compared to the conditions required for the rationality of individual expectations. A set of sufficient conditions for the rationality of average expectations is given by

1. N is sufficiently large.
2. $\mathbf{u}_{it}$ are distributed independently across i, and for each i they are covariance stationary.
3. the weights, w_{it}, satisfy the conditions in (6) and are distributed independently of $\mathbf{u}_{jt}$, for all i and j.
4. $\mathbf{H}_i$ are distributed independently of w_{it} and across i with mean $\mathbf{I}_k$ and finite second order moments.

Under these conditions (for each t) we have[6]

$$\bar{\boldsymbol{\xi}}_{w,t+1} \overset{q.m.}{\to} \boldsymbol{\xi}_{t+1},\ \text{as } N \to \infty,$$

where $\overset{q.m.}{\to}$ denotes convergence in quadratic means. Therefore, average, 'consensus' or market rationality can hold even if the underlying individual expectations are non-rational in the

[6]For a proof, see Pesaran (2004)[Appendix A].

sense of Muth.[7] The above conditions allow for a high degree of heterogeneity of expectations, and are compatible with individual expectations errors being biased and serially correlated As we shall see this result is particularly relevant to tests of the REH that are based on survey responses.

2.2 Extrapolative Models of Expectations Formation

In addition to the REH, a wide variety of expectations formation models has been advanced in the literature with differing degrees of informational requirements. Most of these models fall under the "extrapolative" category, where point expectations are determined by weighted averages of past realizations. A general extrapolative formula is given by

$$E_i(\mathbf{x}_{t+1}|\Omega_{it}) = \sum_{s=0}^{\infty} \mathbf{\Phi}_{is}\mathbf{x}_{t-s}, \tag{9}$$

where the coefficient matrices, $\mathbf{\Phi}_{is}$, are assumed to be absolute summable subject to the adding up condition

$$\sum_{s=0}^{\infty} \mathbf{\Phi}_{is} = \mathbf{I}_k. \tag{10}$$

This condition ensures that *unconditionally* expectations and observations have the same means. For example, suppose that $\mathbf{x}_t$ follows the first-order stationary autoregressive process (unknown to the individuals)

$$\mathbf{x}_t = \boldsymbol{\mu} + \mathbf{\Psi}\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t,$$

where all eigenvalues of $\mathbf{\Psi}$ lie inside the unit circle. It is then easily seen that

$$E\left[E_i(\mathbf{x}_{t+1}|\Omega_{it})\right] = \left(\sum_{s=0}^{\infty} \mathbf{\Phi}_{is}\right)\left(\mathbf{I}_k - \mathbf{\Psi}\right)^{-1}\boldsymbol{\mu},$$

and under the adding up condition, (10), yields, $E\left[E_i(\mathbf{x}_{t+1}|\Omega_{it})\right] = E\left(\mathbf{x}_t\right) = \left(\mathbf{I}_k - \mathbf{\Psi}\right)^{-1}\boldsymbol{\mu}$. Under (10), time averages of extrapolative expectations will be the same as the sample mean of the underlying processes, an implication that can be tested using quantitative survey expectations, if available.

The average (or consensus) version of the extrapolative hypothesis derived using the weights, w_{it} defined by (6), has also the extrapolative form

$$\bar{E}(\mathbf{x}_{t+1}|S_t) = \sum_{s=0}^{\infty} \mathbf{\Phi}_{st}\mathbf{x}_{t-s}, \tag{11}$$

[7] The term consensus forecasts or expectations was popularized by Joseph Livingston, the founder of the Livingston Survey in the U.S. See Section 3 for further details and references.

where S_t contains $\mathbf{x}_t, \mathbf{x}_{t-1}, ...; w_{1t}, w_{2t}, ...$and

$$\mathbf{\Phi}_{st} = \sum_{i=1}^{N} w_{it}\mathbf{\Phi}_{is}.$$

It is clear that under extrapolative expectations individual expectations need not be homogeneous and could follow a number of different processes all of which are special cases of the general extrapolative scheme. Once again, under the adding up condition, (10), $E\left[\bar{E}(\mathbf{x}_{t+1}|S_t)\right] = E(\mathbf{x}_t)$, so long as $\sum_{i=1}^{N} w_{it} = 1$.

2.2.1 Static Models of Expectations

The simplest form of an extrapolative model is the static expectations model considered by Keynes (1936). In its basic form it is defined by

$$E_i\left(\mathbf{x}_{t+1}|\Omega_{it}\right) = \bar{E}(\mathbf{x}_{t+1}|S_t) = \mathbf{x}_t,$$

and is optimal (in the mean squared error sense) if $\mathbf{x}_t$ follows a pure random walk model. A more recent version of this model, used in the case of integrated processes is given by

$$\bar{E}(\mathbf{x}_{t+1}|S_t) = \mathbf{x}_t + \Delta\mathbf{x}_{t-1},$$

which is applicable when $\Delta\mathbf{x}_{t+1}$ follows a random walk. This latter specification has the advantage of being robust to shifts in the unconditional mean of the $\mathbf{x}_t$ processes. Neither of these specifications, however, allows for any form of adaptation to the changing nature of the underlying time series.

2.2.2 Return to Normality Models

A simple generalisation of the static model that takes account of the evolution of the underlying processes is the 'mean-reverting' or the 'return to normality' model defined by

$$\bar{E}\left(\mathbf{x}_{t+1} \mid S_t\right) = \left(\mathbf{I}_k - \mathbf{\Lambda}\right)\mathbf{x}_t + \mathbf{\Lambda}\mathbf{x}_t^*, \tag{12}$$

where $\mathbf{\Lambda}$ is a non-negative definite matrix, and $\mathbf{x}_t^*$ represents the 'normal' or 'the long-run equilibrium' level of $\mathbf{x}_t$. In this formulation, expectations are adjusted downward if $\mathbf{x}_t$ is above its normal level and *vice versa* if $\mathbf{x}_t$ is below its normal level. Different specifications of $\mathbf{x}_t^*$ can be considered. For example, assuming

$$\mathbf{x}_t^* = \left(\mathbf{I}_k - \mathbf{W}\right)\mathbf{x}_t + \mathbf{W}\mathbf{x}_{t-1},$$

yields the regressive expectations model

$$\bar{E}\left(\mathbf{x}_{t+1} \mid S_t\right) = \left(\mathbf{I}_k - \mathbf{\Lambda W}\right)\mathbf{x}_t + \mathbf{\Lambda W x}_{t-1},$$

where $\mathbf{W}$ is a weight matrix.

2.2.3 Adaptive Expectations Model

This is the most prominent form of extrapolative expectations, and can be obtained from the general extrapolative formula, (11), by setting

$$\mathbf{\Phi}_s = \mathbf{\Gamma}\left(\mathbf{I}_k - \mathbf{\Gamma}\right)^s, \; s = 0, 1, ..$$

and assuming that all eigenvalues of $\mathbf{I}_k - \mathbf{\Gamma}$ line inside the unit circle. Alternatively, the adaptive expectations model can be obtained from the return to normality model (12), by setting

$$\mathbf{x}_t^* = \left(\mathbf{I} - \mathbf{W}\right)\mathbf{x}_t + \mathbf{W}\bar{E}\left(\mathbf{x}_t \mid S_{t-1}\right),$$

which yields the familiar representation

$$\bar{E}(\mathbf{x}_{t+1}|S_t) - \bar{E}(\mathbf{x}_t|S_{t-1}) = \mathbf{\Gamma}\left[\mathbf{x}_t - \bar{E}(\mathbf{x}_t|S_{t-1})\right]. \tag{13}$$

Higher order versions of the adaptive expectations model have also been employed in the analysis of expectations data. A general r^{th} order vector adaptive model is given by

$$\bar{E}(\mathbf{x}_{t+1}|S_t) - \bar{E}(\mathbf{x}_t|S_{t-1}) = \sum_{j=0}^{r-1} \mathbf{\Psi}_j \left[\mathbf{x}_{t-j} - \bar{E}(\mathbf{x}_{t-j}|S_{t-j-1})\right]. \tag{14}$$

Under this model expectations are revised in line with past errors of expectations. In the present multivariate setting, past expectations errors of all variables can potentially affect the extent to which expectations of a single variable are revised. Univariate adaptive expectations models can be derived by restricting $\mathbf{\Psi}_j$ to be diagonal for all j.

The univariate version of the adaptive expectations model was introduced into economics by Koyck (1954) in a study of investment, by Cagan (1956) in a study of money demand in conditions of hyper-inflation and by Nerlove (1958) in a study of the cobweb cycle. Adaptive expectations were also used extensively in empirical studies of consumption and the Phillips curve prior to the ascendancy of the REH in early 1970s.

In general, adaptive expectations need not be informationally efficient, and expectations errors generated by adaptive schemes could be serially correlated. Originally, the adaptive expectations hypothesis was advanced as a plausible 'rule of thumb' for updating and revising expectations, without claiming that it will be optimal. Muth (1960) was the first to show that the adaptive expectations hypothesis is optimal (in the sense of yielding minimum mean squared forecast errors) only if the process generating $\mathbf{x}_{t+1}$ has the following integrated, first-order moving average representation (IMA(1)):

$$\Delta \mathbf{x}_{t+1} = \boldsymbol{\varepsilon}_{t+1} - (\mathbf{I}_k - \boldsymbol{\Gamma})\, \boldsymbol{\varepsilon}_t, \;\; \boldsymbol{\varepsilon}_{t+1} \,|S_t \sim IID(\mathbf{0}, \boldsymbol{\Sigma}_{\varepsilon}).$$

In general, adaptive expectations need not be optimal and could perform particularly poorly when the underlying processes are subject to structural breaks.

2.2.4 Error-Learning Models

The adaptive expectations hypothesis is concerned with one-step ahead expectations, and how they are updated, but it can be readily generalised to deal with expectations formed over longer horizons. Denoting the h-step ahead expectations by $\bar{\mathrm{E}}(\mathbf{x}_{t+h} \mid S_t)$, the error-learning model is given by

$$\bar{E}\left(\mathbf{x}_{t+h} \mid S_t\right) - \bar{E}\left(\mathbf{x}_{t+h} \mid S_{t-1}\right) = \boldsymbol{\Gamma}_h \left[\mathbf{x}_t - \bar{E}\left(\mathbf{x}_t \mid S_{t-1}\right)\right], \tag{15}$$

which for $h = 1$ reduces to the simple adaptive expectations scheme. The error-learning model states that revision in expectations of $\mathbf{x}_{t+h}$ over the period t-1 to t is proportional to the current error of expectations. Different expectations formation models can be obtained assuming different patterns for the revision coefficients $\boldsymbol{\Gamma}_h$. The error-learning models have been proposed in the literature by Meiselman (1962), Mincer & Zarnowitz (1969) and Frenkel (1975) and reduce to the adaptive expectations model if the revision coefficients, $\boldsymbol{\Gamma}_h$, are restricted to be the same across different horizons. Mincer & Zarnowitz (1969) show that the revision coefficients are related to the weights $\boldsymbol{\Phi}_j$ in the general extrapolations formula via the following recursive relations:

$$\boldsymbol{\Gamma}_h = \sum_{j=0}^{h-1} \boldsymbol{\Phi}_j \boldsymbol{\Gamma}_{h-1-j}, \;\; h = 1, 2, ..., \tag{16}$$

when $\boldsymbol{\Gamma}_0 = \mathbf{I}_k$. They demonstrate that the revision coefficients will be falling (rising) when the weights $\boldsymbol{\Phi}_j$ decline (rise) more than exponentially. The error-correction and the general extrapolation model are algebraically equivalent, but the former is particularly convenient when survey data is available on expectations over different horizons.

2.3 Testable Implications of Expectations Formation Models

Broadly speaking there are two general approaches to testing expectations formation models. 'Direct' tests that make use of survey data on expectations, and the 'indirect' tests that focus on cross equation parametric restrictions of the expectations formation models when combined with a particular parametric economic model. The direct approach is applicable to testing the REH as well as the extrapolative models, whilst the indirect approach has been used primarily in testing of the REH. Given the focus of this paper we shall confine our discussion to the direct tests.

2.3.1 Testing the REH

Suppose that quantitative expectations of $\mathbf{x}_{t+h}$ are available on individuals, $i = 1, 2, ..., N$, formed at time $t = 1, 2, ..., T$, over different horizons, $h = 1, 2, ..., H$, and denote these by ${}_t\mathbf{x}^e_{i,t+h}$. In the case of many surveys only qualitative responses are available and they need to be converted into quantitative measures, a topic that we return to in section 3.1. The realizations, $\mathbf{x}_{t+h}$, are often subject to data revisions that might not have been known to the individuals when forming their expectations. The agent's loss function might not be quadratic. These issues will be addressed in subsequent sections. For the time being, we abstract from data revisions and conversion errors and suppose that ${}_t\mathbf{x}^e_{i,t+h}$ and the associated expectations errors

$$\boldsymbol{\xi}_{i,t+h} = \mathbf{x}_{t+h} - {}_t\mathbf{x}^e_{i,t+h}, \tag{17}$$

are observed free of measurement errors. Under this idealized set up the test of the REH can proceed by testing the orthogonality condition, (2), applied to the individual expectations errors, $\boldsymbol{\xi}_{i,t+h}$, assuming that

$${}_t\mathbf{x}^e_{i,t+h} = E_i(\mathbf{x}_{t+h}|\Omega_{it}) = \int \mathbf{x}_{t+h} f_i\left(\mathbf{x}_{t+h}|\Omega_{it}\right) d\mathbf{x}_t, \tag{18}$$

namely that survey responses and mathematical expectations of individual's density expectations are identical. The orthogonality condition applied to the individual expectations errors may now be written as

$$E_i\left(\mathbf{x}_{t+h} - {}_t\mathbf{x}^e_{i,t+h}|S_{it}\right) = \mathbf{0}, \text{ for } i = 1, 2, .., N \text{ and } h = 1, 2, ..., H, \tag{19}$$

namely expectations errors (at all horizons) form martingale difference processes with respect to the information set S_{it}, where S_{it} could contain any sub-set of the public information set, Ψ_t, specifically $\mathbf{x}_t$, $\mathbf{x}_{t-1}, \mathbf{x}_{t-2}, ..$, and the past values of individual-specific expectations,

${}_{t-\ell}\mathbf{x}^e_{i,t+h-\ell}$, $\ell = 1, 2, ...$ Information on other individuals' expectations, ${}_{t-\ell}\mathbf{x}^e_{j,t+h-\ell}$ for $j \neq i$ should not be included in S_{it} unless they are specifically supplied to the individual respondents being surveyed. In such a case the test encompasses the concept that the explanatory power of a rational forecast cannot be enhanced by the use of information provided by any other forecast (Fair & Shiller 1990, Bonham & Dacy 1991). A test of unbiasedness of the rational expectations can be carried out by including a vector of unity, $\boldsymbol{\tau} = (1, 1, ..., 1)'$ amongst the elements of S_{it}. As noted earlier, the REH does not impose any restrictions on conditional or unconditional volatilities of the expectations errors, so long as the underlying losses are quadratic in those errors.

The REH can also be tested using the time consistency property of mathematical expectations, so long as at least two survey expectations are available for the same target dates (i.e. $H \geq 2$). The subjective expectations, $E_i(\mathbf{x}_{t+h}|S_{i,t+\ell})$ formed at time $t+\ell$ for period $t+h$ ($h > \ell$) is said to be consistent if expectations of $E_i(\mathbf{x}_{t+h}|S_{i,t+\ell})$ formed at time t are equal to $E_i(\mathbf{x}_{t+h}|S_{it})$ for all ℓ. See Pesaran (1989) and Froot & Ito (1990). Clearly, expectations formed rationally also satisfy the consistency property, and in particular

$$E_i\left[E_i(\mathbf{x}_{t+h}|S_{i,t+1})|S_{it}\right] = E_i(\mathbf{x}_{t+h}|S_{it}).$$

Therefore, under (18)

$$E_i\left[\left({}_{t+1}\mathbf{x}^e_{i,t+h} - {}_{t}\mathbf{x}^e_{i,t+h}\right)|S_{it}\right] = \mathbf{0},$$

which for the same target date, t, can be written as

$$E_i\left[\left({}_{t-h+1}\mathbf{x}^e_{it} - {}_{t-h}\mathbf{x}^e_{it}\right)|S_{i,t-h}\right] = \mathbf{0}, \text{ for } h = 2, 3, .., H. \tag{20}$$

Namely revisions in expectations of $\mathbf{x}_t$ over the period $t-h$ to $t-h+1$ must be informationally efficient. As compared to the standard orthogonality conditions (19), the orthogonality conditions in (20) have the added advantage that they do not necessarily require data on realizations, and are therefore likely to be more robust to data revisions. Davies & Lahiri (1995) utilize these conditions in their analysis of Blue Chip Survey of Professional Forecasts and in a later paper (Davies & Lahiri 1999) they study the Survey of Professional Forecasters.

Average versions of (19) and (20) can also be considered, namely

$$\bar{E}\left(\mathbf{x}_{t+h} - {}_{t}\bar{\mathbf{x}}^e_{t+h}|S_t\right) = \mathbf{0}, \text{ for } h = 1, 2, .., H, \tag{21}$$

where

$${}_{t}\bar{\mathbf{x}}^e_{t+h} = \sum_{i=1}^{N} w_i \; {}_{t}\mathbf{x}^e_{i,t+h}, \tag{22}$$

and $S_t \subseteq \Psi_t$. Similarly,

$$E_i\left[\left({}_{t-h+1}\bar{\mathbf{x}}_t^e - {}_{t-h}\bar{\mathbf{x}}_t^e\right)|S_{t-h}\right] = \mathbf{0}, \text{ for } h = 2, 3, .., H. \tag{23}$$

In using these conditions special care need be exercised in the choice of S_{t-h}. For example, inclusion of past average expectations, ${}_{t-h}\bar{\mathbf{x}}_t^e$, ${}_{t-h-1}\bar{\mathbf{x}}_t^e$, .. in S_{t-h} might not be valid if information on average expectations were not publicly released.[8] But in testing the rationality of individual expectations it would be valid to include past expectations of the individual under consideration in his/her information set, S_{it}.

2.3.2 Testing Extrapolative Models

In their most general formulation, as set out in (9), the extrapolative models have only a limited number of testable implications; the most important of which is the linearity of the relationship postulated between expectations, $\bar{E}\left(\mathbf{x}_{t+1} \mid S_t\right)$, and $\mathbf{x}_t, \mathbf{x}_{t-1}, ...$ Important testable implications, however, follow if it is further assumed that extrapolative expectations must also satisfy the time consistency property discussed above. The time consistency of expectations requires that

$$\bar{E}\left\{\bar{E}\left(\mathbf{x}_{t+1} \mid S_t\right) \mid S_{t-1}\right\} = \bar{E}\left(\mathbf{x}_{t+1} \mid S_{t-1}\right),$$

and is much less restrictive than the orthogonality condition applied to the forecast errors. Under time consistency and using (11) we have

$$\bar{E}\left(\mathbf{x}_{t+1} \mid S_{t-1}\right) = \mathbf{\Phi}_0 \bar{E}\left(\mathbf{x}_t \mid S_{t-1}\right) + \sum_{s=1}^{\infty} \Phi_s \mathbf{x}_{t-s},$$

and hence

$$\bar{E}\left(\mathbf{x}_{t+1} \mid S_t\right) - \bar{E}\left(\mathbf{x}_{t+1} \mid S_{t-1}\right) = \mathbf{\Phi}_0\left[\mathbf{x}_t - \bar{E}\left(\mathbf{x}_t \mid S_{t-1}\right)\right].$$

When losses are quadratic in expectations errors, under time consistency the survey expectations would then satisfy the relationships

$${}_t\bar{\mathbf{x}}_{t+1}^e - {}_{t-1}\bar{\mathbf{x}}_{t+1}^e = \mathbf{\Phi}_0\left(\mathbf{x}_t - {}_{t-1}\bar{\mathbf{x}}_t^e\right), \tag{24}$$

which states that revisions in expectations of $\mathbf{x}_{t+1}$ over the period $t-1$ to t should depend only on the expectations errors and not on $\mathbf{x}_t$ or its lagged values. Under asymmetrical losses expectations revisions would also depend on revisions in expected volatilities, and the

[8]The same issue also arises in panel tests of the REH where past average expectations are included as regressors in a panel of individual expectations. For a related critique see Bonham & Cohen (2001).

time consistency of the extrapolative expectations can be tested only if direct observations on expected volatilities are available. The new testable implications discussed in Patton & Timmermann (2004) are also relevant here.

Relation (24) also shows that extrapolative expectations could still suffer from systematic errors, even if they satisfy the time consistency property. Finally, using the results (15) and (16) obtained for the error learning models, time consistency implications of the extrapolation models can be readily extended to expectations formed at time t and time $t-1$ for higher order horizons, $h > 1$.

As noted earlier, direct tests of time consistency of expectations require survey data on expectations of the same target date formed at two different previous dates at the minimum. In cases where such multiple observations are not available, it seems meaningful to test only particular formulations of the extrapolation models such as the mean-reverting or the adaptive hypothesis. Testable implications of the finite-order adaptive models are discussed further in Pesaran (1985) and Pesaran (1987, Chapter 9) where an empirical analysis of the formation of inflation expectations in British manufacturing industries is provided.

2.4 Testing the Optimality of Survey Forecasts under Asymmetric Losses

The two orthogonality conditions, (19) and (20), are based on the assumption that individual forecast responses are the same as conditional mathematical expectations. See (18). This assumption is, however, valid if forecasts are made with respect to loss functions that are quadratic in forecast errors and does not hold in more general settings where the loss function is non-quadratic or asymmetric. Properties of optimal forecasts under general loss functions are discussed in Patton & Timmermann (2004) where new testable implications are also established. Asymmetric losses can arise in practice for a number of different reasons, such as institutional constraints, or non-linear effects in economic decisions. In a recent paper Elliott, Komunjer & Timmermann (2003) even argue that 'on economic grounds one would, if anything, typically expect asymmetric losses.'[9] Once the symmetric loss function is

[9] In a related paper, Elliot, Komunjer & Timmermann (forthcoming) consider the reverse of the rationality testing problem and derive conditions under which the parameters of an assumed loss function can be estimated from the forecast responses and the associated realizations assuming that the forecasters are rational.

abandoned, as shown by Zellner (1986), optimal forecasts need not be unbiased [10]. This point is easily illustrated with respect to the LINEX function introduced by Varian (1975), and used by Zellner (1986) in a Bayesian context. The LINEX function has the following simple form

$$\varphi_i\left(\xi_{i,t+1}\right) = \frac{2}{\alpha_i^2}\left[\exp\left(\alpha_i \xi_{i,t+1}\right) - \alpha_i \xi_{i,t+1} - 1\right],$$

where $\xi_{i,t+1}$ is the forecast error defined by (17). To simplify the exposition we assume here that $\xi_{i,t+1}$ is a scalar. For this loss function the optimal forecast is given by [11]

$${}_t x_{i,t+h}^e = \alpha_i^{-1} \log\left\{E_i\left(\exp\left(\alpha_i x_{t+h}\right) \mid \mathbf{\Omega}_{it}\right)\right\}.$$

In the case where individual i^{th} conditional expected density of x_{t+h} is normal we have

$${}_t x_{i,t+h}^e = E_i\left(\mathbf{x}_{t+h} \mid \mathbf{\Omega}_{it}\right) + \left(\frac{\alpha_i}{2}\right) V_i\left(\mathbf{x}_{t+h} \mid \mathbf{\Omega}_{it}\right),$$

where $V_i\left(\mathbf{x}_{t+h} \mid \mathbf{\Omega}_{it}\right)$ is the conditional variance of individual i^{th} expected density. The degree of asymmetry of the cost function is measured by α_i. When $\alpha_i > 0$, under-predicting is more costly than over-predicting, and the reverse is true when $\alpha_i < 0$. This is reflected in the optimal forecasts ${}_t\mathbf{x}_{i,t+h}^e$, that exceeds $E_i\left(\mathbf{x}_{t+h} \mid \mathbf{\Omega}_{it}\right)$ when $\alpha_i < 0$ and falls below it when $\alpha_i > 0$.

It is interesting that qualitatively similar results can be obtained for other seemingly different loss functions. A simple example is the so-called "Lin-Lin" function:

$$C_i\left(\xi_{i,t+1}\right) = a_i \xi_{i,t+1} I\left(\xi_{i,t+1}\right) - b_i \xi_{i,t+1} I\left(-\xi_{i.t+1}\right), \tag{25}$$

where $a_i, b_i > 0$, and $I(A)$ is an indicator variable that takes the value of unity if $A > 0$ and zero otherwise. The relative cost of over and under-prediction is determined by a_i and b_i. For example, under-predicting is more costly if $a_i > b_i$. The optimal forecast for this loss function is given by

$${}_t x_{i,t+h}^e = \arg\min_{x^*}\left\{E_i\left[C_i\left(x_{t+h} - x^*\right) \mid \mathbf{\Omega}_{it}\right]\right\}.$$

Since the Lin-Lin function is not differentiable a general closed form solution does not seem possible. But, assuming that $x_{t+h} \mid \mathbf{\Omega}_{it}$ is normally distributed the following simple solution can be obtained[12]

$${}_t x_{i,t+h}^e = E_i\left(x_{t+h} \mid \mathbf{\Omega}_{it}\right) + \kappa_i \sigma_i\left(x_{t+h} \mid \mathbf{\Omega}_{it}\right),$$

[10] For further discussion, see Batchelor & Zarkesh (2000), Granger & Pesaran (2000) and Elliott et al. (2003).

[11] For a derivation, see Granger & Pesaran (2000).

[12] See Christoffersen & Diebold (1997). An alternative derivation is provided in Appendix A.

where

$$\sigma_i\left(x_{t+h} \mid \mathbf{\Omega}_{it}\right) = \sqrt{V_i\left(x_{t+h} \mid \mathbf{\Omega}_{it}\right)},\ \kappa_i = \Phi^{-1}\left(\frac{a_i}{a_i + b_i}\right),$$

and $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function of a standard normal variate. The similarity of the solutions under the LINEX and the Lin-Lin cost functions is striking, although the quantitative nature of the adjustments for the asymmetries differ. Not surprisingly, under symmetrical losses, $a_i = b_i$ and $\kappa_i = \Phi^{-1}(1/2) = 0$, otherwise, $\kappa_i > 0$ if $a_i > b_i$ and *vice versa*. Namely, it is optimal to over-predict if cost of over-prediction (b_i) is low relative to the cost of under-prediction (a_i). The size of the forecast bias, $\kappa_i\sigma_i\left(x_{t+h} \mid \mathbf{\Omega}_{it}\right)$, depends on $a_i/\left(a_i + b_i\right)$ as well as the expected volatility. Therefore, under asymmetric cost functions, the standard orthogonality condition (19) is not satisfied, and in general we might expect $E_i\left(\xi_{i,t+h} \mid \mathbf{\Omega}_{it}\right)$ to vary with $\sigma_i\left(x_{t+h} \mid \Omega_{it}\right)$. The exact nature of this relationship depends on the assumed loss function, and tests of rationality need to be conducted in relation to suitable restrictions on the expected density functions and not just its first moments. At the individual level, valid tests of the 'rationality' hypothesis require survey observations on forecast volatilities as well as on mean forecasts. Only in the special case where forecast volatilities are not time varying, a test of informational efficiency of individual forecasts can be carried out without such additional observations. In the homoskedastic case where $\sigma_i\left(x_{t+h} \mid \Omega_{it}\right) = \sigma_{ih}$, the relevant orthogonality condition to be tested is given by

$$E_i\left(x_{t+h} - \ {}_t x^e_{i,t+h} | S_{it}\right) = d_{ih},$$

where d_{ih} is given by $-\left(\alpha_i/2\right)\sigma^2_{ih}$ in the case of the LINEX loss function and by $-\kappa_i\sigma_{ih}$ in the case of the Lin-Lin function. In this case, although biased survey expectations no longer constitute evidence against rationality, statistical significance of time varying elements of S_{it} as regressors do provide evidence against rationality.

The orthogonality conditions, (20), based on the time consistency property can also be used under asymmetrical losses. For example, for the Lin-Lin loss function we have

$$E_i\left[\left(x_t - {}_{t-h+1}\, x^e_{it}\right) \mid \mathbf{\Omega}_{i,t-h+1}\right] = -\kappa_i\sigma_i\left(x_t \mid \mathbf{\Omega}_{i,t-h+1}\right),$$

$$E_i\left[\left(x_t - {}_{t-h+1}\, x^e_{it}\right) \mid \mathbf{\Omega}_{i,t-h}\right] = -\kappa_i\sigma_i(x_t \mid \mathbf{\Omega}_{i,t-h}),$$

and hence

$$\begin{aligned} & E_i\left({}_{t-h+1}x^e_{it} - {}_{t-h}\, x^e_{it} \mid S_{i,t-h}\right) \\ = \ & -\kappa_i\left\{E\left[\sigma_i\left(x_t \mid \mathbf{\Omega}_{i,t-h+1}\right) \mid S_{i,t-h}\right] - E\left[\sigma_i\left(x_t \mid \mathbf{\Omega}_{i,t-h}\right) \mid S_{i,t-h}\right]\right\}. \end{aligned}$$

Once again, if $\sigma_i(x_{t+h} \mid \Omega_{it}) = \sigma_{ih}$ we have

$$E_i\left({}_{t-h+1}x^e_{it} - {}_{t-h}\,x^e_{it} \mid S_{i,t-h}\right) = -\kappa_i\left(\sigma_{i,h-1} - \sigma_{ih}\right),$$

and the rationality of expectations can be conducted with respect to the time-varying components of $S_{i,t-h}$.

Similarly modified orthogonality conditions can also be obtained for the consensus forecasts, when $\sigma_i(x_{t+h} \mid \Omega_{it}) = \sigma_{ih}$. Specifically, we have

$$E_i\left(x_{t+h} - \ {}_t\bar{x}^e_{t+h}|S_{it}\right) = \bar{d}_h,$$

and

$$E_i\left({}_{t-h+1}\bar{x}^e_t - {}_{t-h}\,\bar{x}^e_t \mid S_{t-h}\right) = \bar{d}_{h-1} - \bar{d}_h,$$

where $\bar{d}_h = \sum_{i=1}^N w_i d_{ih}$.

In the more general case where expected volatilities are time varying, tests of rationality based on survey expectations also require information on individual or average expected volatilities, $\sigma_i(x_{t+h} \mid \Omega_{it})$. Direct measurement of $\sigma_i(x_{t+h} \mid \Omega_{it})$ based on survey expectations have been considered in the literature by Demetriades (1989), Batchelor & Jonung (1989), Dasgupta & Lahiri (1993) and Batchelor & Zarkesh (2000). But with the exception of Batchelor & Zarkesh (2000), these studies are primarily concerned with the cross section variance of expectations over different respondents, rather than $\sigma_i(x_{t+h} \mid \Omega_{it})$, an issue which we discuss further in section 4.2 in the context of the forecasts of event probabilities collated by the Survey of Professional Forecasters. An empirical analysis of the relationship between expectations errors and expected volatilities could be of interest both for shedding lights on the importance of asymmetries in the loss functions, as well as for providing a more robust framework for orthogonality testing. With direct observations on $\sigma_i(x_{t+h} \mid \Omega_{it})$, say ${}_t\sigma^e_{i,t+1}$, one could run regressions of $x_{t+h} - \ {}_tx^e_{i,t+h}$ on ${}_t\sigma^e_{i,t+1}$ and other variables in Ω_{it}, for example $\mathbf{x}_t, \mathbf{x}_{t-1}$,.... Under rational expectations with asymmetric losses, only the coefficient of ${}_t\sigma^e_{i,t+1}$ should be statistically significant in this regression. Similar tests based on the time consistency conditions can also be developed.

3 Part II: Measurement of Expectations: History and Developments

The collection of data on future expectations of individuals has its roots in the development of survey methodology as a means of compiling data in the years before the Second World

War. Use of sample surveys made it possible to collect information on a range of topics which could not be covered by administrative sources and full enumeration censuses; it was natural that these began to extend themselves to covering questions about the future as well as the past. It also has to be said that interest in measuring expectations was likely only after economists had started to understand the importance expectations of future events as determinants of the current decisions. This was a process which began in the 1920s with discussions on the nature of risk and uncertainty (Knight 1921), expanded in the 1930s through Keynes' contributions and has continued to develop ever since.

The earliest systematic attempt to collect information on expectations which we have been able to trace was a study carried out in 1944 by the United States Department of Agriculture. This was a survey of consumer expectations attempting to measure consumer sentiment (Katona 1975) with the latter calculated by aggregating the categorical answers provided to a variety of questions. Dominitz & Manski (2005) present a statistical analysis of the way in which the sentiment indicator is produced. Currently the survey is run by the University of Michigan and is known as the Michigan survey, with many other similar surveys conducted across OECD countries so as to provide up to date information on consumer expectations. Questions on expectations are also sometimes included in panel surveys. The British Household Panel Survey is one such example which asks questions such as whether households expect their financial positions to improve or worsen over the coming year. Such surveys, as well as offering an insight into how such expectations may be formed, do of course make it possible to assess whether, or how far, such expectations are well-founded by comparing the experiences of individual households with their prior expectations.

A key aspect of the Michigan survey, and of many other more recent surveys, is that some of its questions ask for qualitative responses. Consumers are not asked to say what they think their income next week or next year will be, by what percentage they expect it to change from their current income or even to provide a range in which they expect the change in their income to lie. Instead they are simply asked to provide a qualitative indication of whether they expect to be better off or worse off. That this structure has been widely copied, in surveys of both consumers and businesses is perhaps an indication that it is easier to obtain reliable responses to qualitative questions of this sort than to more precise questions. In other words there is believed to be some sort of trade-off between the loss of information consequent on qualitative questions of this sort and the costs in terms of response rate and therefore possible bias from asking more precise questions. It may also be that the answers to more precise questions yield more precise but not necessarily more accurate answers. (the truth elicitation problem). For either reason the consequence is that a key research issue in the use of expectational data is handling the link between the qualitative data and the quantitative variables which indicate the outcomes of business and consumer decisions and experiences. It is also the case that some surveys which collect qualitative information on the future also collect qualitative information on the past; the question of linking these latter data to quantitative variables also arises and many of the questions are the same as those arising in the interpretation of prospective qualitative data.

Household surveys were later complemented with business surveys on the state of economic activity. In the period before the Second World War a number of countries produced reports on the state of business. These do not, however, appear to have collected any formal indicators of sentiment. The United States enquiry into the state of business developed into the Institute of Purchasing Managers Survey. This asks firms a range of questions about the state of business including the level of order books and capacity utilisation. It does not ask either about what is expected to happen in the future or about firms' experiences of the very recent past. However, the Institut für Wirtschaftsforschung in Munich in 1948 started to ask manufacturing firms questions about their expectations of output growth and price increase in the near future as well as questions about recent movement of these variables. They also included a question about firms' expectations of the evolution of the business environment. The sort of questions covered in the Purchasing Managers' Survey were also covered.

This survey structure has since been adopted by other countries. For example, the Confederation of British Industry began to ask similar questions of the UK manufacturing sector in 1958 and has continued to do so ever since. The Tankan surveys cover similar grounds in Japan. Policy-makers and financial economists often rely on the results of these surveys as indicators of both past and future short-term movements of the economic variables. There has, moreover, gradually been a recognition that similar methods can be used to cover other aspects of the economy; in the European Union, EC-sponsored surveys now cover industrial production, construction, retail sales and other services.

Another type of survey expectations has also been initiated in the United States. In 1946 a journalist, Joseph Livingston started to ask a number of economists their expectations of inflation over the coming year and the coming five years. Quantitative rather than qualitative survey data were collected, relating not to expectations of individual experiences but regarding the macro-economy as a whole. Although people are being asked to produce forecasts in both cases, the performance of forecasts about individual experiences can be verified satisfactorily only if data are collected on how the circumstances of the individuals actually evolve over time. The performance of the second type of forecast can, by contrast, be verified by direct comparisons with realized macroeconomic data.

The exercise gave rise to what has become known as the Livingston Survey (Croushore 1997, Thomas 1999) and has broadened in scope to collect information on expectations about a range of economic variables including consumer and wholesale prices, the Standard and Poor's industrial share price index, real and nominal GNP (now GDP), corporate profits and the unemployment rate from a number of economists. It is the oldest continuous survey of economists' expectations and is now conducted by the Federal Reserve Bank of Philadelphia.

In contrast to the consumer expectations questions, these respondents were expected to provide point estimates of their expectations. No doubt this was more practical than with the consumer expectations survey because the respondents were practising economists and therefore might be assumed to be more capable of and more comfortable with providing quantitative answers to the questions. After a survey of this type it is possible to calculate not only the mean but also the standard deviation of the responses. The mean, though appealing as a representation of the consensus, is unlikely to be the best prediction generated from the individual forecasts.

Other surveys of macroeconomic forecasts include the Philadelphia Fed's Survey of Professional Forecasters[13], the Blue Chip Survey of Professional Forecasters, and the National Association of Business Economists (NABE) surveys that are produced quarterly and consists of annual forecasts for many macroeconomic variables.[14] The Goldsmith-Nagan Bond and Money Market Letter, provides an aggregation of forecasts of the yield on 3-month US Treasury Bills and other key interest rates from 30-40 analysts. Interest rates, unlike many of the variables considered in the Livingston Survey are typically inputs to rather than outputs of macro-economic models and forecasts. In that sense the survey is probably reporting judgements as to how individual expectations might differ from the pattern implied by the yield curve rather than the outcome of a more formal forecasting process.

To the extent that there is a difference between opinions and formal forecasts produced by some sort of forecasting model, this not made clear in the information collected in the Livingston Survey. The Survey of Blue Chip Forecasters, on the other hand focuses on organisations making use of formal forecasting models. As always it is unclear how far the forecasts generated by the latter are the products of the models rather than the judgements

[13]This was formerly conducted by the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER). It was known as the ASA/NBER survey.

[14]The variables included in the Survey of Profesional Forecasters and other details are described in Croushore (1993).

of the forecasters producing them. But this survey, too, indicates the range of opinion of forecasters and means and standard deviations can be computed from it.

The Survey of Professional Forecasters asks respondents to provide probabilities that key variables will fall into particular ranges, instead of simply asking forecasters to provide their forecasts. This does, therefore, make available serious information on the degree of uncertainty as perceived by individual forecasters. The production and assessment of these forecasts is discussed elsewhere in this volume. A range of other surveys (Manski 2004) also asks questions about event probabilities from households and individuals about their perceptions of the probabilities of events which affect them, such as job loss[15], life expectancy[16] and future income[17]. We discuss the importance of these subsequently in section 4.2.

Surveys similar to these exist for other countries although few collect information on individual perceptions of uncertainty. Consensus Forecasts collates forecasts produced for a number of different countries and Isiklar, Lahiri & Loungani (2005) use the average of the forecasts for each country as a basis for an analysis of forecast revision. In the UK, HM Treasury publishes its own compilation of independent economic forecasts. The Zentrum für Europäische Wirtschaftsforschung (ZEW) collects data on the views of "financial experts" about the German economy's prospects. We provide a summary of key surveys in table 1. A list of key references is presented in appendix B.

To the extent that the surveys report forecasts produced as a result of some forecasting process, it is questionable how far such forecasts actually represent anyone's expectations, at least in a formal sense. Sometimes they are constructed to show what will happen if a policy which is not expected to be followed is actually followed. Thus the forecasts produced by the UK's Monetary Policy Committee are usually based on two interest rate assumptions. The first is that interest rates are held constant for two years and the second that they follow the pattern implied by the yield curve. Both of these assumptions may be inconsistent with the Monetary Policy Committee's view of the economic situation. There is the separate question of whether such forecasts contain predictive power over and above that of the direct quantitative and qualitative information mentioned above; and the weaker question of whether the predictive power of such forecasts can be enhanced by combining them with

[15]U.S. Survey of Economic Expectations (Dominitz & Manski 1997*a*, Dominitz & Manski 1997*b*)

[16]U.S. Health and Retirement Survey (Juster & Suzman 1995, Hurd & McGarry 2002)

[17]Italy's Survey of Household Income and Wealth (Guiso, Japelli & Terlizzese 1992, Guiso, Japelli & Pistaferri 2002), the Netherlands' VSB Panel Survey(Das & Donkers 1999), the US Survey of Economic Expectations (Dominitz and Manski, op.cit) and the U.S. Survey of Consumers (Dominitz & Manski 2003, Dominitz & Manski 2004).

official and other data sets based on past realizations. Obviously the answer to the latter depends in part on whether and how forecasters use such information in the production of their forecasts.

A third category of information on expectations is implied by prices of financial assets. Subject to concerns over risk premia which are widely discussed (and never completely resolved) long-term interest rates are an average of expected future short-term interest rates, so that financial institutions are able to publish the future short-term rates implied by them. Forward exchange rates and commodity prices have to be regarded as expectations of future spot prices. In the case of the foreign exchange markets arbitrage, which should reinforce the role of futures prices as market expectations, is possible at very low cost. In the case of commodities which are perishable or expensive to store there is less reason to expect arbitrage to ensure that the future price is a clearly defined market expectation. Such markets have existed in the past, but since 1981 we have started to see the introduction of index-linked government debt. With the assumption that the difference between the yield on nominal and indexed debt represents expected inflation, it becomes possible to deduce a market series for expectations of inflation in each period for which future values can be estimated for both nominal and real interest rates. When using such estimates it must be remembered that the markets for indexed debt are often rather thin and that, at least historically, the range of available securities has been small, reducing the accuracy with which future real interest rates can be predicted. The development of options markets has meant that it is possible to infer estimates of interest rate uncertainty from options prices. The markets for options in indexed debt have, however, not yet developed to the point at which it is possible to infer a measure of the uncertainty of expected inflation.

We now proceed to a discussion of the quantification of qualitative survey data. This then allows us to discuss the empirical issues concerning alternative models of expectations formation.

Table 1: A Selected List of Sources for Survey Data

Institution	Country/Region	Web link	Availability Free?	Type	Notes
European Commission Business and Consumer Surveys	European Union	http://www.europa.eu.int/comm/economy_finance/indicators/businessandconsumersurveys_en.htm	Yes	Qualitative	Business and consumer data on expectations and experience
IFO Business Survey (now CESifo)	Germany	http://www.cesifo-group.de/portal/page?_pageid=36,34759&_dad=portal&_schema=PORTAL		Both	Provides data on business expectations and experience
Tankan	Japan	http://www.boj.or.jp/en/stat/tk/tk.htm	Yes	Both	Qualitative business data. Quantitiative forecasts of profit and loss accounts
Consensus Economics	Most of the World excluding Africa and parts of Asia	http://www.consensuseconomics.com		Quantitative	Collates economics forecasts
Confederation of British Industry	UK	http://www.cbi.org.uk/ndbs/content.nsf/b80e12d0cd1cd37c802567bb00491cbf/2f172e85d0508cea80256e20003e95c6?OpenDocument		Qualitative	Provides data on business expectations and experience
HM Treasury Survey of UK Forecasters	UK	http://www.hm-treasury.gov.uk/economic_data_and_tools/forecast_for_the_uk_economy/data_forecasts_index.cfm	Yes	Quantitative	Collates economics forecasts
Blue Chip Economic Indicators	US. Limited information on other countries	http://www.aspenpublishers.com/bluechip.asp	Yes	Quantitative	Collates economic forecasts
Institute of Supply Management (formerly National Association Purchasing Managers)	USA	http://www.ism.ws/ISMReport/index.cfm		Qualitative	Does not collect data on expectations or forecasts
Livingston Survey	USA	http://www.phil.frb.org/econ/liv/index.html	Yes	Quantitative	Covers inflation expectations
Survey of Consumers University of Michigan	USA	http://www.sca.isr.umich.edu/main.php		Both	Data on consumer expectations and experience
Survey of Professional Forecasters	USA	http://www.phil.frb.org/econ/spf/index.html	Yes	Quantitative	Collates economic forecasts. Includes indicators of forecast density functions

3.1 Quantification and Analysis of Qualitative Survey Data

Consider a survey that asks a sample of N_t respondents (firms or households) whether they expect a variable, $x_{i,t+1}$(if it is specific to respondent i), or x_{t+1} (if it is a macro-economic variable) to "go up" $\left(u^e_{i,t+1}\right)$, "stay the same" $\left(s^e_{i,t+1}\right)$, or "go down" $\left(d^e_{i,t+1}\right)$ relative to the previous period.[18] The number of respondents could vary over time and tends to differ markedly across sample surveys. The individual responses, $u^e_{i,t+1}$, $s^e_{i,t+1}$ and $d^e_{i,t+1}$ formed at time t, are often aggregated (with appropriate weights) into proportions of respondents expecting a rise, no change or a fall, typically denoted by U^e_{t+1}, S^e_{t+1} and D^e_{t+1}, respectively. A number of procedures have been suggested in the literature for converting these proportions into aggregate measures of expectations[19]. We shall consider two of these methods in some detail and briefly discuss their extensions and further developments. The conversion techniques can be applied to aggregation of responses that concern an individual-specific variable such as the output growth or price change of a particular firm. They can also be applied when respondents are asked questions regarding the same common variable, typically macro-economic variables such as the inflation, GDP growth or exchange rates. The main conversion techniques are:

1. the probability approach of Carlson & Parkin (1975).

2. the regression approach of Pesaran (1984) and Pesaran (1987).[20]

Although motivated in different ways, the two approaches are shown to share a common foundation. Consider each approach in turn.[21]

3.1.1 The probability approach

This approach was first employed by Theil (1952) to motivate the use by Anderson (1952) of the so-called "balance statistic" $(U^e_{t+1} - D^e_{t+1})$ as a method of quantification of qualitative survey observations. The balance statistic, up to a scalar factor, provides an accurate measure of the average expected, or actual, change in the variable in question if the percentage changes of falls and rises reported by the respondents remain constant and of the same order of magnitudes for rises and falls over time.

[18] To simplify the notations we have suppressed the left-side t subscript of ${}_tu^e_{i,t+1}$, ${}_ts^e_{i,t+1}$, and etc.

[19] Nardo (2003) provides a useful survey of the issues surrounding quantification of qualitative expectations.

[20] A related procedure is the reverse-regression approach advanced by Cunningham, Smith & Weale (1998) and Mitchell, Smith & Weale (2002), which we shall also be discussed briefly later.

[21] The exposition draws on Pesaran (1987) and Mitchell et al. (2002). For alternative reviews and extensions of the probability and regression approaches see Wren-Lewis (1985) and Smith & McAleer (1995).

The probability approach relaxes this restrictive assumption, and instead assumes that responses by the i^{th} respondent about the future values of x_{it} (say the i^{th} firm's output) are based on her/his subjective probability density function conditional on the available information. Denote this subjective probability density function by $f_i(x_{i,t+1} \mid \Omega_{it})$. It is assumed that the responses are constructed in the following manner:

- if ${}_tx^e_{i,t+1} \geq b_{it}$ respondent i expects a "rise" in output; $u^e_{i,t+1} = 1$, $d^e_{i,t+1} = s^e_{i,t+1} = 0$,
- if ${}_tx^e_{i,t+1} \leq -a_{it}$ respondent i expects a "fall" in output; $d^e_{i,t+1} = 1$, $u^e_{i,t+1} = s^e_{i,t+1} = 0$,
- if $-a_{it} < {}_tx^e_{i,t+1} < b_{it}$ respondent i expects "no change" in output; $s^e_{i,t+1} = 1$, $u^e_{i,t+1} = d^e_{i,t+1} = 0$,

where as before ${}_tx^e_{i,t+1} = E\left(x_{i,t+1} \mid \Omega_{it}\right)$ and $(-a_{it}, b_{it})$ is known as the indifference interval for given positive values, a_{it} and b_{it}, that define perceptions of falls and rises in output.

It is clear that, in general, it will not be possible to derive the individual expectations, ${}_tx^e_{i,t+1}$, from the qualitative observations, $u^e_{i,t+1}$ and $d^e_{i,t+1}$.[22] The best that can be hoped for is to obtain an average expectations measure. Suppose that individual expectations, ${}_tx^e_{i,t+1}$, can be viewed as independent draws from a common distribution, represented by the density function, $g({}_tx^e_{i,t+1})$, with mean ${}_tx^e_{t+1}$ and the standard deviation, ${}_t\sigma^e_{t+1}$. Further assume that the perception thresholds a_{it} and b_{it} are symmetric and fixed both across respondents and over time, $a_{it} = b_{it} = \lambda$, $\forall i, t$. Then the percentage of respondents reporting rises and falls by U^e_{t+1} and D^e_{t+1}, respectively, converge to the associated population values (for a sufficiently large N_t),

$$U^e_{t+1} \xrightarrow{p} \Pr({}_tx^e_{i,t+1} \geq \lambda) = 1 - G_{t+1}(\lambda), \text{ as } N_t \to \infty, \tag{26}$$

$$D^e_{t+1} \xrightarrow{p} \Pr({}_tx^e_{i,t+1} \leq -\lambda) = G_{t+1}(-\lambda), \text{as } N_t \to \infty, \tag{27}$$

where $G_{t+1}(\cdot)$ is the cumulative density function of $g({}_tx^e_{i,t+1})$, assumed common across i. Then, conditional on a particular value for λ and a specific form for the aggregate density function, ${}_tx^e_{t+1} = E_c\left({}_tx^e_{i,t+1}\right)$ can be derived in terms of U^e_{t+1} and D^e_{t+1}. Notice that expectations are taken with respect to the cross section distribution of individual expectations. It is also important to note that $\left({}_t\sigma^e_{t+1}\right)^2 = E_c\left[{}_tx^e_{i,t+1} - E_c\left({}_tx^e_{i,t+1}\right)\right]^2$ is a cross section variance and measures the cross section dispersion of individual expectations and should not be confused with the volatility of individual expectations that could be denoted by

[22] Note that $s^e_{i,t+1} = 1 - u^e_{i,t+1} - d^e_{i,t+1}$.

$V\left(x_{i,t+1} \mid \Omega_{it}\right)$. ${}_t\sigma^e_{t+1}$ is best viewed as a measure of discord or disagreement across agents, whilst $V\left(x_{i,t+1} \mid \Omega_{it}\right)$ represents the extent to which the i^{th} individual is uncertain about his/her future point expectations.

The accuracy of the probability approach clearly depends on its underlying assumptions and the value of N_t. As the Monte Carlo experiments carried out by Löffler (1999) show the sampling error of the probability approach can be considerable when N_t is not sufficiently large, even if distributional assumptions are satisfied. It is, therefore, important that estimates of ${}_tx^e_{t+1}$ based U^e_{t+1} and D^e_{t+1} are used with care, and with due allowance for possible measurement errors involved.[23] Jeong & Maddala (1991) use the generalised method of moments to deal with the question of measurement error. Cunningham et al. (1998) and Mitchell et al. (2002) apply the method of reverse regression to address the same problem. This is discussed further in Section 3.1.3. Ivaldi (1992) considers the question of measurement error when analysing responses of individual firms.

The traditional approach of Carlson & Parkin (1975) assumes the cross section density of ${}_tx^e_{i,t+1}$ to be normal. From (26) and (27)

$$1 - U^e_{t+1} = \Phi\left(\frac{\lambda -_t x^e_{t+1}}{\sigma^e_{t+1}}\right), \tag{28}$$

$$D^e_{t+1} = \Phi\left(\frac{-\lambda -_t x^e_{t+1}}{{}_t\sigma^e_{t+1}}\right), \tag{29}$$

where $\Phi(.)$ is the cumulative distribution function of a standard normal variate. Using (28) and (29) we note that

$$r^e_{t+1} = \Phi^{-1}\left(1 - U^e_{t+1}\right) = \frac{\lambda -_t x^e_{t+1}}{{}_t\sigma^e_{t+1}}, \tag{30}$$

$$f^e_{t+1} = \Phi^{-1}\left(D^e_{t+1}\right) = \frac{-\lambda -_t x^e_{t+1}}{{}_t\sigma^e_{t+1}}, \tag{31}$$

where r^e_{t+1} can be calculated as the abscissa of the standard normal distribution corresponding to the cumulative probability of $(1 - U^e_{t+1})$, and f^e_{t+1} is the abscissa corresponding to D^e_{t+1}. Other distributions such as logistic and the Student t distribution have also been used in the literature. See, for example, Batchelor (1981).

We can solve for ${}_tx^e_{t+1}$ and ${}_t\sigma^e_{t+1}$

$${}_tx^e_{t+1} = \lambda\left(\frac{f^e_{t+1} + r^e_{t+1}}{f^e_{t+1} - r^e_{t+1}}\right), \tag{32}$$

[23] Measurement errors in survey expectations their implications for testing of the expectations formation models are discussed, for example, in Pesaran (1987), Jeong & Maddala (1991), Jeong & Maddala (1996) and Rich, Raymond & Butler (1993).

and

$$ {}_t\sigma^e_{t+1} = \frac{2\lambda}{r^e_{t+1} - f^e_{t+1}}. \tag{33} $$

This leaves only λ unknown . Alternative approaches to the estimation of λ have been proposed in the literature. Carlson and Parkin assume unbiasedness of generated expectations over the sample period, $t = 1, ..., T$ and estimate λ by

$$ \hat{\lambda} = \left(\sum\nolimits_{t=1}^{T} x_t\right) / \left(\sum\nolimits_{t=1}^{T} \left(\frac{f^e_t + r^e_t}{f^e_t - r^e_t}\right)\right), $$

where x_t is the realizations of the variable under consideration. For alternatives see *inter alia* Batchelor (1981), Batchelor (1982), Pesaran (1984) and Wren-Lewis (1985). Since λ is a constant its role is merely to scale ${}_tx^e_{t+1}$.

Further discussions of the Carlson and Parkin estimator of ${}_tx^e_{t+1}$ can be found in Fishe & Lahiri (1981), Batchelor & Orr (1988) and Dasgupta & Lahiri (1992). There is, however, one key aspect of it which has not received much attention. The method essentially exploits the fact that when data are presented in the trichotomous classification, there are two independent proportions which result from this. The normal distribution is fully specified by two parameters, its mean and its variance. Thus Carlson and Parkin use the two degrees of freedom present in the reported proportions to determine the two parameters of the normal distribution. If the survey were dichotomous- reporting only people who expected rises and those who expected falls, then it would be possible to deduce only one of the parameters, typically the mean by assuming that the variance is constant at some known value.

A problem also arises if the survey covers more than three categories- for example if it asks firms to classify their expectations or experiences into one of five categories, a sharp rise, a modest rise, no change, a modest fall or a sharp fall. Taking a time series of such a survey it is impossible to assume that the thresholds are constant over time; the most that can be done is to set out some minimand, for example making the thresholds in each individual period as close as possible to the sample mean. The regression approach which follows is unaffected by this problem.

3.1.2 The regression approach

Consider the aggregate variable x_t as a weighted average of the variables associated with the individual respondents (c.f. (22))

$$ {}_t\bar{x}^e_{t+1} = \sum\nolimits_{i=1}^{N_t} w_{it}\ {}_tx^e_{i,t+1}, \tag{34} $$

where w_{it} is the weight of the i^{th} respondent which is typically set to $1/N_t$. Suppose now that at time t respondents are grouped according to whether they reported an expectation of a rise or a fall. Denote the two groups by $\mathcal{U}_{t+1}$ and $\mathcal{D}_{t+1}$ and rewrite (34) equivalently as

$$_t\bar{x}^e_{t+1} = \sum_{i \in \mathcal{U}_{t+1}} w^+_{it} \ {}_tx^{e+}_{i,t+1} + \sum_{i \in \mathcal{D}_{t+1}} w^-_{it} \ {}_tx^{e-}_{i,t+1}, \tag{35}$$

where the superscripts $+$ and $-$ denote the respondent expecting an increase and a decrease, respectively. From the survey we do not have exact quantitative information on $x^{e+}_{i,t+1}$ and $_tx^{e-}_{i,t+1}$. Following Pesaran (1984), suppose that

$$x^{e+}_{i,t+1} = \alpha + v_{i\alpha}, \text{ and } {}_tx^{e-}_{i,t+1} = -\beta + v_{i\beta}, \tag{36}$$

where $\alpha, \beta > 0$ and $v_{i\alpha}$ and $v_{i\beta}$ are independently distributed across i with zero means and variances σ^2_α and σ^2_β. Assume that these variances are sufficiently small and the distributions of $v_{i\alpha}$ and $v_{i\beta}$ are appropriately truncated so that $x^{e+}_{i,t+1} > 0$ and $_tx^{e-}_{i,t+1} < 0$ for all i and t. Using these in (35) we have (for sufficiently large elements in $\mathcal{U}_{t+1}$ and $\mathcal{D}_{t+1}$)[24]

$$_t\bar{x}^e_{t+1} \approx \alpha \sum_{i \in} w^+_{it} - \beta \sum_{i \in \mathcal{D}_{t+1}} w^-_{it}, \tag{37}$$

or simply

$$_t\bar{x}^e_{t+1} \approx \alpha U^e_{t+1} - \beta D^e_{t+1}, \tag{38}$$

where U^e_{t+1} and D^e_{t+1} are the (appropriately weighted) proportion of firms that reported an expected rise and fall, respectively, and α and β are unknown positive parameters. The balance statistic, $U^e_{t+1} - D^e_{t+1}$ advocated by Anderson (1952) and Theil (1952) is a special case of (38) where $a = \beta = 1$. Pesaran (1984) allows for possible asymmetries and non-linearities in the relationship that relates $_t\bar{x}^e_{t+1}$ to $U^e_{t+1}$and D^e_{t+1}. The unknown parameters are estimated by linear or non-linear regressions (as deemed appropriate) of the realized values of x_t (the average underlying variable) on past realizations U_t and D_t, corresponding to the expected proportions U^e_{t+1} and D^e_{t+1}, respectively. As noted above, this approach can be straigthtforwardly extended if the survey provides information on more than two categories.

[24]Recent evidence on price changes in European economies suggest that on average out of every 100 price changes 60 relate to price rises and the remaining 40 to price falls. There is also a remarkable symmetry in the average sizes of price rises and price falls. These and other important findings of the Inflation Persistence Network (sponsored by the European Central Bank) are summarized in Gadzinski & Orlandi (2004).

3.1.3 Other conversion techniques - further developments and extensions

There have been a number of related contributions that construct models in which parameters can vary over time. For example Kanoh & Li (1990) use a logistic model to explain the proportions giving each of three categorical responses to a question about expected inflation in Japan. They assume that expected inflation is a linear function of current and past inflation. A model in which the parameters are time-varying is preferred to one in which they are not. Smith & McAleer (1995) suggested that the thresholds in Carlson and Parkin's model might be varying over time, assuming that they were subject to both permanent shocks and short-term shocks which were uncorrelated over time. The model was then estimated using a Kalman filter technique finding that the time-series model is preferred to the standard model.

Cunningham et al. (1998) and Mitchell et al. (2002) relate survey responses to official data by regressing the proportions of firms reporting rises and falls on the official data. Cunningham et al. (1998), however, take the view that the survey data represent some transformation of the underlying latent variable with an additional error term added on arising for perception and measurement reasons. For this reason it may seem more appropriate to estimate regression equations which explain observed proportions, U^e_{t+1} and D^e_{t+1} (or U_t and D_t) rather than explaining output by the survey aggregates. This means that estimates of the variable represented by the survey have to be derived by inverting each regression equation. Since the number of independent regression equations is equal to the number of categories less one, there are this number of separate estimates of the variable of interest produced. Since, however, the covariance of the vector of these distinct estimates can be estimated from the standard properties of regression equations, it is possible to produce a variance-weighted mean of the different estimates to give a best estimate of the variable of interest (Stone, Champernowne & Meade 1942).

Mitchell et al. (2002) extend this technique using the CBI survey data. Instead of explaining the two survey proportions (the proportion reporting or expecting a rise in output and the proportion reporting or expecting a fall) they look at the proportions reporting/expecting rises or falls as proportions of those who had reported/expected rises, no change or falls in the previous period[25]. This creates a system of six equations which can be estimated in the same way. Mitchell, Smith and Weale describe this as a semi-disaggregate approach. They found evidence suggesting that the thresholds which underpin both the regression and the

[25] These variables are not published but can, of course, be constructed from access to the firms' individual responses.

reverse regression models, are functions of the responses in the previous period. While there is some evidence of serial correlation in the relevant aggregate regressions, the evidence for this is much weaker in the six semi-disaggregate regressions suggesting that the apparent serial correlation may result from a failure to take account of the dependence of thresholds on previous responses. The semi-disaggregate approach gave a better within-sample fit than did the aggregate approach which Cunningham et al. found outperformed the usual regression approach.

3.2 Measurement of Expectations Uncertainty

In Section 3.1.1 we discussed alternative methods of obtaining an estimate of cross section mean and dispersion of individual expectations, and it was noted that the dispersion measures of the type defined by (33), do not necessarily measure the uncertainty faced by individual respondents when forming their expectations. To measure expectations uncertainty one needs further survey measurements where respondents are explicitly asked about the degree of confidence they attach to their point expectations. There are only a few surveys that address this issue of expectations uncertainty.

Surveys sometimes collect qualitative data on uncertainty. For example the Confederation of British Industry's survey asks respondents to indicate whether their investment plans are constrained by demand uncertainty. Here respondents are being asked to report if they are influenced by the second moment of their own sales growth. The impact of this could be substantial even if there were very little difference between both the experience and the point expectations of the individual respondents. With some modifications the approach set out in Section 3.1.1 can be used to analyse these data.

There the analysis relied on the assumption that the underlying latent variable was normally distributed, which is clearly not appropriate for quantification of higher order moments of expected probability distributions. One possibility would be to assume that the distribution of the logarithm of the variance is normally distributed. For example, suppose that a firm reports being constrained by uncertainty if its own subjective variance of future output growth, ${}_t\sigma^2_{i,t+1}$, is greater than a threshold, $\bar{\sigma}^2$. In addition assume that $\ln\left({}_t\sigma^2_{i,t+1}\right) \sim N\left(\ln\left(\bar{\sigma}^2_t\right), \omega^2_t\right)$, where ω^2_t is the cross section variance of $\ln\left({}_t\sigma^2_{i,t+1}\right)$ which we take to be fixed across i. Under these assumptions we have

$$P({}_t\sigma^2_{i,t+1} > \bar{\sigma}^2) = 1 - P({}_t\sigma^2_{i,t+1} \leq \bar{\sigma}^2) = 1 - \Phi\left(\frac{\ln \bar{\sigma}^2 - \ln\left(\bar{\sigma}^2_t\right)}{\omega_t}\right),$$

where $P_t = P({}_t\sigma^2_{i,t+1} > \bar{\sigma}^2)$ and can be estimated by the proportion of respondents reporting their investment as being constrained by demand. This set up is analogous to the Carlson and Parkin approach discussed above for quantification of point expectations and yields

$$\bar{\sigma}_t^2 = \bar{\sigma}^2 e^{-\omega_t \ \Phi^{-1}(1-P_t)}. \tag{39}$$

Without some independent observation on subjective uncertainty it is not possible to go further than this. Carlson and Parkin relied on actual measures of inflation to estimate their threshold parameter, λ. Here in addition to $\bar{\sigma}^2$, which performs a role analogous to λ, we also need to restrict ω_t to be time invariant. Under these rather restrictive assumptions it is possible to estimate $\ln\left(\bar{\sigma}_t^2\right)$ consistently up to a linear transformation.

In other cases, as we have noted above, surveys ask respondents to provide probabilities that variables will lie in particular ranges. In this case the variance of the expectation can be estimated by fitting an appropriate density function to the event probabilities provided by the respondents (Dominitz & Manski 1997*b*, Dominitz 1998).

3.3 Analysis of Individual Responses

The previous sections discuss ways of quantifying aggregated responses, such as the proportion of respondents expecting a rise or a fall in the variable in question so as to be able to make use of them either in interpreting the results of the surveys in real time or in the more general use of such surveys in applied macro analysis. As we noted in the introduction, however, analysis of individual responses, particularly in a panel context, is generally more satisfactory.

A number of surveys, such as the surveys conducted by the Confederation of British Industries, ask respondents to provide categorical information about some variable of interest, both *ex ante* and *ex post.* Where these surveys are conducted from samples drawn afresh on each occasion there is little that can be done beyond exploring the link between the *ex ante* data and future income growth or the *ex post* data and past income growth using one of the methods discussed in Section 3.1. But where the data are collected from a panel so that it is possible to keep track of the expectations and outcomes as reported by individual respondents, then it becomes possible to explore whether respondents' expectations are consistent with rationality according to a number of different definitions.

Nerlove (1983) was one of the first to discuss the problem of exploring the relationship between individuals' expectations and the associated realizations using two-way tables of categorical data. Obviously this can be used to explore association between any pairs of variables. The most obvious comparison is that between reports of expectations for period

t made in period $t-1$ and the subsequent out-turns reported *ex post* for period t. One may also explore the relationship between expected or reported price rises and expected or reported output growth, or the way in which expectations are linked to past realisations. Gourieroux & Pradel (1986), Ivaldi (1992) and Das & Donkers (1999) discuss ways of testing the rationality of expectations in such data.

In order to explore these issues further we first recap and extend our notation. Suppose that there are m (taken to be an odd number) possible categories and respondent i is asked to report *ex ante* which category is relevant to his/her expectation, ${}_tx^e_{i,t+1}$, formed at the end of period t of the variable whose outcome, $x_{i,t+1}$ is realized in period $t+1$. The mid-category, $(m+1)/2$ is taken as the "no-change" category.

1. The prediction is denoted by the discrete random variables $y^e_{i,t+1,j}$, $j = 1, 2, .., m$ where

$$ {}_ty^e_{i,t+1,j} = 1 \text{ if } c^e_{j-1} < {}_tx^e_{i,t+1} \leq c^e_j; \text{ and 0 otherwise.} \tag{40} $$

2. The outcome is denoted by the discrete random variable $y_{i,t+1,j}$, $j = 1, 2, .., m$ defined similarly as

$$ y_{i,t+1,j} = 1 \text{ if } c_{j-1} < x_{i,t+1} \leq c_j; \text{ and 0 otherwise.} $$

We follow convention and assume $\{c^e_0, c_0\} = -\infty$ and $\{c^e_m, c_m\} = \infty$. Let

$$ p^e_j = \Pr\left({}_ty^e_{i,t+1,j} = 1\right);\ p_j = \Pr\left(y_{i,t+1,j} = 1\right), $$

and

$$ p_{jk} = \Pr\left({}_ty^e_{i,t+1,j} = 1 \mid y_{i,t+1,k} = 1\right), $$

and assume that p^e_j, p_j and p_{jk} are invariant across i and t, and denote the estimates of these probabilities by $\hat{p}^e_j, \hat{p}_j$ and $\hat{p}_{jk}$, respectively. Under this set up $\hat{p}^e_j$, $\hat{p}_j$ and $\hat{p}_{jk}$ can be computed consistently from the *ex ante* and *ex post* responses, assuming that individual responses are independent draws from a common multivariate distribution.

Nerlove notes the distinction between an expectation and a forecast. If positive and negative surprises are equally likely, then it would not be surprising to find a substantial number of respondents expecting no change. On the other hand if everyone subsequently experiences a sizeable shock, very few people will report an out-turn of no change. The French and German surveys of past and expected future output growth (Germany) or past and expected future demand (France) certainly exhibit this feature with more expectations than subsequent responses falling in the no change category. Thus we generally observe $p^e_{(m+1)/2} > p_{(m+1)/2}$.

Suppose now that the aim is to derive forecasts of the proportion of observations that fall in a given category, j, which we denote by p_j^*. By Bayes theorem and using the above notations

$$p_j^* = \sum_{k=1}^{m} p_k p_{jk}.$$

In general, the conditional probabilities, p_{jk}, are not known and need to be estimated. Nerlove suggests estimating p_{jk} using past observations of the relationship between forecasts and out-turns. This is, however, subject to a number of problems. The most important of which is probably that past relationships between expectations and out-turns have been affected by macro-economic shocks. If the effects of these can be removed or averaged out, then the relationship is more likely to be satisfactory. Not surprisingly, the move from p_j^e to p_j^* disperses the responses from the centre to the extremes of the distribution. Nerlove then uses measures of association suggested by Goodman & Kruskal (1979) to identify patterns in the two-way tables looking at links between expectations and previous out-turns and errors in previous out-turns in order to explore how quickly expectations are influenced by the past, as well as the link between expectations and the out-turns described by them. While a number of interactions are identified, the exercise suffers from the problem that it does not actually offer a means of exploring the performance of different expectational models, except in the most general of terms.

Gourieroux & Pradel (1986) prove that, for expectations to be rational it must be that $p_{kk} > \max_{j \neq k} p_{jk}$, for $k = 1, 2, ..., m$. Ivaldi (1992) notes that a test of rationality based on Gourieroux and Pradel criterion is likely to lack power and instead proposes a two-step estimation method based on a latent variable model where in the first step, using the theory of polychoric correlations, the correlation matrix of the latent variables are estimated, and in the second step, under certain exact identifying assumptions, the estimated correlation coefficients are used to obtain consistent estimates of the underlying parameters of the latent variable model. This estimation approach is applied to business surveys of French manufacturing industry conducted by INSÉE that asks firms about their expectations of output growth and the subsequent out-turns over four periods of three successive surveys during 1984-1985 (giving two estimates of the relationship between expectation and out-turn in each period). The hypothesis of rational expectations is rejected in five out of eight cases. However, Ivaldi argues that the test tends to over-reject when samples are large and therefore the case against rationality is weaker than these findings suggest. The data, however, pass an efficiency test in that he can accept the hypothesis that the influence of out-turns up to period t on the expectation for $t+1$ is the same as that on the out-turn for period $t+1$.

Das & Donkers (1999) point out that the respondents are not given precise instructions about how to respond to the questions. There are a number of possible answers that individuals might give to a question about what they expect to happen. For example, they might report the category in which lies the mean, the median or the mode and with a skewed probability distribution these will differ. Using the multivariate normal distribution as the limiting case of the polynomial distribution Das & Donkers (1999) show that, if the expectations reported are those of the mode and if the *ex post* responses are drawn from the same distribution

$$\sqrt{\frac{N_k}{2\hat{p}_{kk}}}\,(\hat{p}_{kk}-\hat{p}_{jk}) \longrightarrow N(0,1),$$

where N_k is the number of realizations that fall in the k^{th} category. We note that the modal assumption is awkward in the situation where the density function is symmetric and the central category has low probability because the range $[c_{(m+1)/2}, c_{(m-1)/2})$ is small.

Where the reported category k is that in which the median value of the underlying variable lies, then the most one can say is that

$$\sum_{j=k+1}^{m} p_{jk} \leqq 0.5, \text{and } \sum_{j=1}^{k-1} p_{jk} \leqq 0.5,$$

which can again be tested using the normal distribution. If, however respondents report their mean values, then Das & Donkers (1999) point out that without information on the underlying variable and the thresholds, it is impossible to test that the initial expectations are consistent with the underlying distribution.

4 Part III: Uses of Survey Data in Forecasting

Both qualitative and quantitative survey data on expectations could be potentially important in forecasting, either on their own or in conjunction with other variables. Concern about the future is widespread and the demand for forecasts is obvious. Where expectational data are quantitative, as with the Livingston survey, their interpretation seems equally obvious. Users nevertheless, are likely to be interested in whether they have any predictive power. With qualitative data the same question arises but with the additional complication that the performance of any indicator is bound to depend on the econometric methods used to carry out the conversions.

Obviously in most circumstances survey data are not the only means of forecasting available. Unless other methods (such as time series methods) have no predictive power, it is likely that good forecasts will involve either the addition of survey data to time series models or the use of forecast combination techniques to combine forecasts arising from surveys with those generated by time-series techniques.

4.1 Forecast Combination

It is generally, and not surprisingly, found that combinations of forecasts produced by different bodies tend to be more accurate than forecasts produced by any individual. Granger & Ramanathan (1984) show that, when the covariance structure of the forecasting errors is stable, then the regression coefficients of an equation explaining the out-turn in terms of the disparate forecasts provides the optimal combination of the different forecasts. Clearly, the forecasts thus combined can be of different types and from different sources. Thus it is perfectly possible to combine forecasts such as those presented in the Survey of Professional Forecasters with those generated using similar approaches by public bodies such as the Federal Reserve Board and those which are the expectations of 'experts' rather than properly worked out forecasts as such. A recent development of work of this type is provided by Elliott & Timmermann (forthcoming). They show that the framework provided by a switching model offers a means of forecast combination better than the traditional approach. They also compare their results with other methods using time-varying weights (Zellner, Hong & C-K Min 1991, Deutsch, Granger & Terasvirta 1994).

4.2 Indicating Uncertainty

Economists and policy-makers need to take an interest not only in expected future values but also in the uncertainty that surrounds such expectations. As we noted above, there is no reason to believe that the cross dispersion of point estimates is a good indication of the uncertainty perceived by individual respondents. Survey data can, in principle, provide information about subjective uncertainty as well as about some form of point expectations. The topic is ignored in many surveys and not given much emphasis in others. As we have noted above, the CBI survey does, however, ask respondents whether their investment plans are limited by uncertainty about demand. This is plainly a question about second moments which provides information about subjective views of uncertainty and, given an appropriate quantification method, can be used to represent the latter. We have already discussed means of doing this in section 3.2; it remains to be implemented.

There have, however, been a small number of attempts to infer income uncertainty in surveys of consumers. Thus Guiso et al. (1992) asked respondents to provide probabilities for inflation in twelve months time and "your opinion about labour earnings or pensions [growth] twelve months from now" falling into particular ranges, with the probabilities being designed to sum to one.(p.332) These questions were included in the 1989 Survey of Household Income and Wealth run by the Bank of Italy.

Dominitz & Manski (1997*b*) designed a survey specifically to elucidate information on income uncertainty, as part of the University of Wisconsin's Survey of Economic Expectations and thereby produced an indication of subjective income uncertainty of households. They concluded that the best way of collecting data on the subjective distribution was to ask people the probabilities that their incomes over the next twelve months would be below each of four thresholds, with the thresholds chosen in the light of reported realised income. Respondents were also asked to report the lowest and highest possible amounts their household incomes might be. Subsequent analysis of these data (Dominitz 1998) suggests that the measures of expectation which can be reconstituted from these density functions are a reasonably good guide to out-turns (Dominitz 2001), and that the estimates of uncertainty thus derived correlate reasonably well with measures deduced from the Panel Survey of Income Dynamics on the basis of the forecast performance of time-series models of incomes. On the basis mainly of these findings Manski (2004) is optimistic about the ability of surveys of this type to collect information on expectations and plans of individuals.

Rather more work has been done on the data collected in surveys of economists expectations/forecasts of macro-economic variables, with particular use being made of the event probabilities collated by the Survey of Professional Forecasters. Zarnowitz & Lambros (1987) and Lahiri, Teigland & Zaporowski (1988) use this survey to compare the dispersion of point forecasts of inflation and real GNP growth produced by economic forecasters in the United States with the uncertainty that individual forecasters report for their forecasts. Zarnowitz & Lambros (1987) find that the dispersion of point forecasts understates the uncertainty of the predictive probability distribution, with some evidence that high inflation is associated with uncertainty about inflation. Confirmation of the importance of this distinction is provided by the observation that, while the average variance of forecasters' individual distributions has little influence in an equation for the real rate of interest, the average measures of skewness and kurtosis seemed to have a significant depressing influence on interest rates.

Bomberger (1996) suggested, comparing the dispersion of forecasts in the Livingston survey with variance estimates of inflation generated by ARCH/GARCH processes, that there was a close relationship between the two, although the latter was around four times the former. This work was criticised by Rich & Butler (1998) with a defence by Bomberger (1999). The reader is, however left with the feeling that there is something unsatisfactory in using the dispersion if an arbitrary scaling factor is required to relate it to a suitable statistical measure. This malaise persists even if as Bomberger claims, the scaling factor is stable over the period he considered. Giordani & Söderlind (2003), looking again at the results of the Survey of Professional Forecasters, extend Bomberger's analysis. They derive three measures of uncertainty, (i) disagreement or dispersion of point forecasts, (ii) the average of the estimated standard deviations of the individual forecasts (calculated from the event probabilities presented in the Survey) and (iii) a measure of the aggregate variance derived by aggregating the individual event probabilities to produce an aggregate probability density function. They report a correlation between measures (i) and (ii) of 0.60 when looking at inflation with a correlation of 0.44 when they consider output. The correlations between (i) and (iii) are 0.75 in both cases. From these results they argue that "disagreement is a fairly good proxy for other measures" despite the fact that it accounts for at most just over half of the variation in the other measures. However, they found that estimates of uncertainty generated by time-series GARCH methods did not match those generated from the survey data. Lahiri & Liu (forthcoming) explore the changes in the pattern of the individual density forecasts presented in the survey. They find less persistence in the uncertainty associated with each individual forecast than studies based on aggregate data suggest and also that past forecast error has little influence on reported forecast uncertainty. This is clearly an important area for further research.

4.3 Aggregated Data from Qualitative Surveys

Despite the apparent advantages in using quantified surveys, qualitative surveys are widespread and considerable attention is paid to them in the real-time economic debate. It is therefore important also to consider at their performance as measures of the state of the macro-economy.

4.3.1 Forecasting: Output Growth

As we have noted, qualitative surveys typically include questions about output prospects. As is clear from section 3.1, the method of analysis is essentially the same as that used to analyse responses to questions about past output movements. However, the relevant survey response in period t is aligned to the actual out-turn in period $t+1$ rather than anything which is known in period t. Obviously when applying the reverse regression approach due attention has to be made of the fact that future output is unknown at the time the survey is carried out, and an appropriate form of GMM such as instrumental variables must be used to deal with this.

There is a wide range of studies addressing the capacity of prospective survey questions to anticipate output movements. We discuss these before considering work on anticipating inflationary expectations and predicting future price movements.

Öller (1990) finds balance statistics useful as a means of identifying cyclical turning points in economic data. Entorf (1993) finds, however, looking at the question in the IFO survey about expected future business conditions (rather than the expected output of the respondent itself) that the proportion of respondents expecting business conditions to worsen is a better predictor of future output changes than is the balance statistic. Cunningham et al. (1998) examining surveys from the United Kingdom also find that use of the balance statistic results in loss of information. Smith & McAleer (1995) use the survey collected by the Confederation of Australian Industries to explore the capacity of six questions to predict future movements in five variables, output, employment, prices, stocks of finished goods and overtime. Here we focus on the results obtained on output, discussing price movements in the next section. The survey is of form similar to those discussed above, with categorical responses for “up”, “same”, “down” and a small “not available” category. The authors explore the performance of different methods of quantifying the surveys and also test whether expectations are rational, by exploring whether expectational errors are orthogonal to expectations themselves (see Section 2.3). The performance of the models is assessed only in-sample over the period, 1966Q2-1989Q2. In-sample the best-performing model is the time-varying parameters model with a root mean square error lower than that of the probability model and an ARIMA(2,1,0) model. Obviously the time-varying parameters model has fewer degrees of freedom left than the probability model. Driver & Urga (2004) by contrast, looking at out of sample performance for the UK find that a regression model based on the balance statistic offers the best out of sample performance for interpreting retrospective data about output, investment and employment. The best-performing model was therefore different from that found for the retrospective analysis of the UK by Cunningham et al. (1998). Comparison of these studies indicates that generalization about which method of quantification works best is not possible. Although both Smith & McAleer (1995) and Driver & Urga (2004) compare various approaches over long periods, they do not consider whether for the series they investigate the performance ranking of the conversion procedures remain stable across different sub-periods or variables.

There have been a number of other studies looking at the performance of these prospective measures of economic performance, often published by the bodies which produce the indicators themselves. But in most cases they do not go beyond the general question of whether the indicators have some ability to fit the data. Rahiala & Teräsvirta (1993) consider the role of business surveys in predicting output growth in metal and engineering industries in Finland and Sweden. Bergström (1995) explores the link between manufacturing output and a range of business surveys in Sweden, and Madsen (1993) studies the predictive power of production expectations in eight OECD countries. Klein & Moore (1991) look at the capacity of diffusion indices[26] constructed from the National Association of Purchasing Managers' Surveys in the United States to predict turning points of the United States economy. Hild (2002) uses the method of principal components to explore the inter-relationships between variables in the French survey, but does not concern himself with the fact that polychoric correlations should be used to evaluate the principal components while Bouton & Erkel-Rousse (2002) look at the information contained in qualitative data on the service sector for France. Gregoir & Lenglart (2000) use the survey to derive a coincident indicator based on a two-state Markov process. Parigi & Schlitzer (1995) consider forecasts of the Italian business cycle.

4.3.2 Forecasting: Inflation

As we have noted above, the question of the link between expectational data and inflation has received more attention than that between expectational data and output movements, partly because of the importance attached to inflationary expectations in a number of macroeconomic models such as the expectations-augmented Philips curve and the assumption that a real interest rate can be derived by deducting inflationary expectations from the nominal interest rate. Thus both Carlson & Parkin (1975) and Pesaran (1984) developed their models with specific reference to inflation expectations. We address the performance of qualitative and quantitative expectations data in the context of models and theories in the next section. Here we focus on the capacity of both types of expectations data to anticipate inflation, at least to some extent.

Looking first at qualitative data Lee (1994) uses the probability method to explore the link between firms' expectations of price and cost increases and the response to the ret-

[26] Diffusion indices offer a means of combining a number of different but related indicators. They show the proportion of indicators registering a rise rather than a fall in whatever variable each indicator happens to report.

rospective questions about the same variables. He studied the period 1972Q2 to 1989Q4 which covered the very rapid price increases of the 1970s and the much lower rate of price increase, particularly from 1983 onwards. He carried out his analysis for the nine subsectors of manufacturing identified by the CBI survey as well as for the manufacturing sector as a whole. He was able to reject the hypothesis that there was a unit root in unanticipated inflation of output prices for all sectors except electrical engineering on the basis of an ADF(4) test statistic. Even for electrical engineering the test statistic of -2.35 makes it likely that the process is I(0) rather than I(1). He found that expectations were conservative in that changes in the actual rate of price increase are only partially reflected in changes in the expected rate of price increase. Moreover a test for rationality of expectations (see section 2.1) suggested that the expectational error could be explained by information available at the time the expectations were formed; in other words expectations were not rational. The variables used to explain the errors were manufacturing output price increases, manufacturing materials cost increases, manufacturing wage costs, the change in the exchange rate, the growth of total output, the growth of the money stock and the aggregate unemployment rate all lagged one quarter. Only for the chemical industry could the hypothesis of rationality be accepted at a 5% significance level. He also found that the "conversion errors" the difference between actual price increases and those deduced from the qualitative survey were explained to some extent by the variables used to explain the expectational errors. This raised the possibility that the rejection of rationality of expectations was a consequence of some flaw in the conversion process rather than a defect with the expectations themselves. If the expectational errors are corrected for the conversion errors, then the position is more mixed, with the rational expectations hypothesis rejected for five out of nine sectors. Compared to the retrospective and prospective studies of output growth mentioned above, this takes us further because it actually points to a failing of a particular conversion method- that the conversion errors are predictable in terms of known variables- rather than simply offering a comment on the performance of different methods.

4.3.3 Forecasting: Consumer Sentiment and Consumer Spending

As we noted in section 3, the first surveys to collect information on expectations were the studies of consumer sentiment. Dominitz & Manski (1997*b*) provide a brief account of early attempts to assess their value. They explain how the surveys acquired a poor reputation because they seemed to have little capacity to explain future consumption. Early econometric studies (Tobin 1959, Adams 1964) use methods which would now be regarded as unsuitable- such as estimation of relationships between variables which are probably I(1)- without exploring issues of co-integration and dynamic adjustment.

The value of these surveys was questioned by Federal Reserve Consultant Committee on Consumer Survey Statistics (1955) leading to a response from Katona (1957). Juster (1964) also thought their value was limited and Dominitz & Manski (1997*b*) concluded that this interchange left most economists with the feeling that qualitative expectational survey data were of limited use. Nevertheless, the Michigan survey has continued and the European Union supports the collection of similar data in its member states, perhaps because Praet & Vuchelen (1984) found that they had some power to predict future movements in aggregate consumption. We save our discussion of more recent work on disaggregated data for section 5.2.2 below.

5 Part IV: Uses of Survey Data in Testing Theories: Evidence on Rationality of Expectations

An obvious role for expectational data is in the testing of models of the way in which expectations are formed. Market mechanisms which might penalise people who form their expectations 'inefficiently' are likely to be weak or could take a long time to work. Thus given a number of competing models of the way in which people might actually form expectations, such as those discussed in Part I, it is possible to use actual measures of expected future out-turns to try to distinguish between different expectations formation models.

In many cases economic theories refer to the influence of expected future events on current behaviour. Where there is no independent measure of expectations, then it is impossible to test the theory independently of the assumption made about the way in which people form their expectations. It is not possible to test this assumption independently of the model of behaviour consequent on that assumption. Independent measures of expected future values mean that it is possible to test theories contingent only on the assumption that the expectational data do in fact represent people's or firms' expectations of the future.

Two examples can make this clear. The life-cycle model of consumer behaviour leads to the conclusion that, at any age, people who have an expectation of a rapidly rising income are likely to have lower asset levels than those who do not. If one makes an assumption that people's expectations of future income growth are based on some particular model (such as reversion to the mean for their cohort appropriately adjusted for individual characteristics such as education level), then it is possible to explore this question. But if expectations are in fact different, then the model may be rejected for the wrong reasons. Information from individual households on their own expectations of how their financial situations are likely to develop allows a cleaner assessment of the model in question.

Another obvious example where survey data on expectations can be used for testing a theory concerns the role of uncertainty in limiting investment. Because firms always have the choice of delaying irreversible investment until the future becomes clearer, high uncertainty is likely to reduce investment. But, unless there is a direct measure of uncertainty available, it is almost impossible to test the theory independently of the assumption made about the determinants of uncertainty.

Manski (2004) discusses many other examples and similarly concludes

> "Economists have long been hostile to subjective data. Caution is prudent, but hostility is not warranted. The empirical evidence cited in this article shows that, by and large, persons respond informatively to questions eliciting probabilistic expectations for personally significant events. We have learned enough for me to recommend, with some confidence, that economists should abandon their antipathy to measurement of expectations. The unattractive alternative to measurement is to make unsubstantiated assumptions." p. 1370

In the remainder of this part we shall focus on the use of survey expectations for testing the expectations formation process in economics and finance. We begin with an overview of the studies that use quantitative (or quantified) survey responses, before turning to studies that base their analysis directly on qualitative responses.

5.1 Analysis of Quantified Surveys, Econometric Issues and Findings

On the face of it exploration of the results of quantified surveys is straightforward. Numerical forecasts or expectations can be compared *ex post* with numerical out-turns and tests of the orthogonality conditions, as discussed in Section 2.3, can be explored as a test for rationality. One is also in a position to explore questions of non-linearity. There are, nevertheless, a number of important econometric considerations which need to be taken into account in carrying out such tests.

5.1.1 Simple Tests of Expectations Formation: Rationality in the Financial Markets

As we have noted above, some surveys cover the expectations of people involved in financial markets. Dominguez (1986), looking at a survey run by Money Market Services Inc. of thirty people involved in the foreign exchange markets, tested the hypothesis that expectations were rational. She had weekly data for the period 1983-1985 for the exchange rates of the US$ against sterling, the Deutsche Mark, the Swiss Franc and the Yen and looked at the subperiods 1983-1984 and 1984-1985, using one-week and two-week non-overlapping observations. She rejected the hypothesis of rationality at at least a 5% significance level in all the cases she examined. Over longer horizons she rejected rationality at three months but not at one month. Frankel & Froot (1987*b*) continued with the same theme, looking at the exchange rate expectations of people involved in the foreign exchange markets and comparing them with out-turns over the period 1976-1985. They found that expectations were relatively inelastic and that expectational errors could often be explained statistically by past forecasting errors. Thus the people they surveyed could be described as slow to learn. Nevertheless, the nature of the departure of expectations from the pattern implied by rationality depended on the period under consideration. Elliott & Ito (1999) found that, although survey data for the Yen/US$ rate were worse than random-walk predictions in terms of mean-square error, they could identify a profitable trading rule based on the subjective forecasts compared to the forward rate; the profits were, however, very variable. Takagi (1991) presents a survey of literature on survey measures of foreign exchange expectations.

The studies by Dominguez (1986) and Frankel & Froot (1987*b*) were time-series analyses applied to the median response in each period of the relevant sample. Elliott & Ito (1999) looked at the mean, minimum and maximum of the reported responses in each period. However, we consider the issue of heterogeneity in more detail in section 5.1.5.

There is also the question whether and how far the departure from rationality can be explained in terms of a risk premium, either constant or varying over time, rather than systematic errors in expectations. We explore this in section 5.1.4.

5.1.2 Testing Rationality with Aggregates and in Panels

Tests of rationality and analysis of expectations formation have been carried out using the mean of the forecasts produced by a number of different forecasters e.g. Pesando (1975), Friedman (1980), Brown & Maital (1981) and Caskey (1985). While these can report on the rationality of the mean they cannot imply anything about the rationality of individual forecasts (Keane & Runkle 1990, Bonham & Cohen 2001). It is perfectly possible that the different forecasts have offsetting biases with the mean of these biases being zero or some value not significantly different from zero. Thus the conclusion that the mean is unbiased (or more generally orthogonal to the information set) does not make it possible to draw any similar conclusion about the individual expectations/forecasts.

But it is also possible that the hypothesis of rationality might be rejected for the aggregate when it is in fact true of all of the individuals, at least if the individual forecasts are produced using both private and public information as Figlewski & Wachtel (1983) make clear. We have, in section 2.1 distinguished the public information set, Ψ_t from the private information set available to agent i, Φ_{it}. Suppose that $\mathbf{y}_t \in \Psi_t$ and $\mathbf{z}_{it} \in \Phi_{it}$, for $i = 1, 2, ..., N$ such that

$$\begin{aligned} E\left(\mathbf{z}_{it} \left| \Phi_{jt} \right.\right) &= \mathbf{z}_{it} \text{ if } i = j \\ &= 0 \text{ if } i \neq j, \end{aligned}$$

and assume that each individual forms his/her expectations based on the same data generating process given by

$$x_{t+1} = \boldsymbol{\gamma}' \mathbf{y}_t + N^{-1} \sum_{j=1}^{N} \boldsymbol{\delta}_j' \mathbf{z}_{jt} + \varepsilon_{t+1},$$

where ε_{t+1} are martingale processes with respect to the individual information sets, $\Omega_{it} = \Psi_t \cup \Phi_{it}$. Under this set up individual i's expectations are given by

$${}_t x_{i,t+1}^e = \boldsymbol{\gamma}' \mathbf{y}_t + N^{-1} \boldsymbol{\delta}_i' \mathbf{z}_{it},$$

and by construction the individual expectations errors

$$x_{t+1} - {}_t x_{i,t+1}^e = N^{-1} \sum_{j=1,\, j \neq i}^{N} \boldsymbol{\delta}_j' \mathbf{z}_{jt} + \varepsilon_{t+1},$$

form martingale processes with respect to Ω_{it}, namely $E\left(x_{t+1} - {}_t x_{i,t+1}^e \left| \Omega_{it} \right.\right) = 0$.

Consider now the expectations errors associated with mean or consensus forecasts, ${}_t \bar{x}_{t+1}^e = N^{-1} \sum_{i=1}^{N} {}_t x_{i,t+1}^e$, and note that

$$\eta_{t+1} = x_{t+1} - {}_t \bar{x}_{t+1}^e = \left(1 - \frac{1}{N}\right) \bar{z}_t + \varepsilon_{t+1},$$

where $\bar{z}_t = N^{-1} \sum_{i=1}^{N} \boldsymbol{\delta}_i' \mathbf{z}_{it}$. Therefore, since ${}_t \bar{x}_{t+1}^e = \boldsymbol{\gamma}' \mathbf{y}_t + N^{-1} \bar{z}_t$, the orthogonality regression often carried out using the consensus forecasts:

$$x_{t+1} - {}_t \bar{x}_{t+1}^e = \alpha + \beta \, {}_t \bar{x}_{t+1}^e + u_{t+1}, \tag{41}$$

is likely to yield a biased inference for a given $N > 1$. In other words the hypothesis of rationality, requiring $\alpha = \beta = 0$ may be rejected even when true. Figlewski & Wachtel (1983) refer to this as the private information bias.

If the mean forecast is unsuitable as a variable with which to explore rationality, use of panel regression for this problem might not be satisfactory either. Consider the panel version of (41),

$$x_{t+1} - {}_t\bar{x}^e_{i,t+1} = \alpha_i + \beta_i \; {}_t x^e_{i,t+1} + u_{i,t+1}. \tag{42}$$

If the regression equation errors are correlated across forecasters, so that $Cov(u_{i,t+1}, u_{j,t+1}) \neq 0$ when $i \neq j$, then estimating the equations jointly for all forecasters as a seemingly unrelated set of regression equations will deliver estimates of the parameters more efficient than those found by Ordinary Least Squares. But, as authors such as Pesaran & Smith (1995) have pointed out in other contexts, the restrictions $\alpha_i = \alpha$, $\beta_i = \beta$ for all i should not be imposed without being tested. If the restrictions (described as micro-homogeneity) can be accepted then regression (41) produces consistent estimates of α and β. If these restrictions do not hold, then all of the forecasters cannot be producing rational forecasts, so the consensus equation cannot be given any meaningful interpretation.

Having made these observations Bonham & Cohen (2001) develop a GMM extension of the seemingly unrelated regression approach of Zellner (1962) in order to explore rationality in the forecasts reported in the Survey of Professional Forecasters. They find that they reject micro-homogeneity in most cases with the implication that the REH needs to be tested at the level of individual forecasters, albeit taking account of the increased efficiency offered by system methods.

5.1.3 Three-dimensional Panels

The work discussed above looks at the analysis of a panel of forecasts in which each forecaster predicts a variable or variables of interest over the same given horizon. But Davies & Lahiri (1995) point out that in many cases forecasters produce forecasts for a number of different future target dates (horizons). At any date they are likely to forecast GDP growth in the current year, the next year and possibly even in the year after that. Thus any panel of forecasts has a third dimension given by the horizon of the forecasts. Davies and Lahiri develop a GMM method for exploiting this third dimension; obviously its importance lies in the fact that there is likely to be a correlation in the forecast errors of forecasts produced by any particular forecaster for the same variable at two different horizons. People who are optimistic about GDP growth in the near future are likely to be optimistic also in the more distant future. The three-dimensional panel analysis takes account of this.

5.1.4 Asymmetries or Bias

Froot & Frankel (1989) used survey data as measures of expectations to the explore whether the apparent inefficiency in the foreign exchange market which they observed, could be attributed to expectations not being rational or to the presence of a risk premium. They rejected the hypothesis that none of the bias was due to systematic expectational errors, and could not reject the hypothesis that it was entirely due to this cause. They also could not reject the hypothesis that the risk premium was constant. MacDonald (2000) surveyed more recent work in the same vein and discussed work on bond and equity markets. A general finding in bond markets was that term premia were non-zero and tended to rise with time to maturity. They also appeared to be time-varying and related to the level of interest rates. There was also evidence of systematic bias in the US stock market (Abou & Prat 1995). Macdonald drew attention to the heterogeneity of expectations across market participants, evidence for the latter being the scale of trading in financial markets.

As we noted in section 2.4, in the presence of asymmetric loss functions, the optimal forecast is different from the expectation. Since the loss function has to be assumed invariant over time if it is to be of any analytical use, the offset arising from an asymmetric loss function could be distinguished from bias only if the second moment of the process driving the variable of interest changes over time. If the variance of the variable forecast is constant it is not possible to distinguish bias from the effect of asymmetry, but if it follows some time-series process, it should be possible to distinguish the two.

Batchelor & Peel (1998) exploit this to test for the effects of asymmetric loss functions in the forecasts of 3-month yields on US Treasury Bills contained in the Goldsmith-Nagan Bond and Money Market Letter. They fit a GARCH process to the variance of the interest rate around its expected value, and assume that the individuals using the forecast have a Lin-Lin loss function (section 2.4). They apply the analysis to the mean of the forecasts reported in the survey despite the criticisms of the use of the mean identified above. The Lin-Lin loss function provides a framework indicating how they should expect the offset of the interest rate forecast from its expectation to vary over time. Batchelor and Peel find that, although the GARCH process is poorly defined and does not enter into the equation testing forecast performance with a statistically significant coefficient, its presence in the regression equation means that one is able to accept the joint hypothesis that the forecast is linked to the outcome with unit coefficient and zero bias. It is, of course, not clear how much weight should be placed on this finding, but the analysis does suggest that there is some point in looking for the consequences of asymmetries for optimal forecasts when the variances of the variables forecasted follow a GARCH process.

Elliott et al. (2003) devise an alternative method of testing jointly the hypothesis that forecasts are rational and that offsets from expected values are the consequence of asymmetric loss functions. They use the forecasts of money GDP growth collated by the Survey of Professional Forecasters and assess the individual forecasts reported there instead of the mean of these. Estimating equation (42), they reject the hypothesis of rationality at the 5% level for twenty-nine participants out of the ninety-eight in the panel.

They then propose a generalised form of the Lin-Lin loss function. In their alternative a forecaster's utility is assumed to be a non-linear function of the forecast error. The function is constructed in two stages, with utility linked to a non-linear function of the absolute forecast error by means of a constant absolute risk aversion utility function, with the Lin-Lin function arising when risk-aversion is absent. It is, however, assumed that the embarrassment arising from a positive forecast error differs from that associated with a negative forecast error giving a degree of asymmetry. Appropriate choice of parameters means that the specification is flexible over whether under-forecasting is more or less embarrassing than over-forecasting. The resulting loss function has the form

$$L_i(e_{i,t+1}) = \{\alpha + (1-2\alpha)I(-e_{i,t+1})\}\, e^p_{i,t+1}, \tag{43}$$

where $e_{i,t+1} = x_{t+1} -_t x^*_{i,t+1}$ denoting the difference between the outcome and the forecast, ${}_t x^*_{it+1}$, which is of course no longer equal to the expectation, and $I()$ is the indicator function which takes the value 1 when its argument is zero or positive and 0 otherwise. $p = 1$ and $0 < \alpha < 1$ deliver the Lin-Lin function.

The authors show that OLS estimates of β_i in equation (42) are biased when the true loss function is given by (43) and that the distribution of β_i is also affected. It follows that the F-test used to explore the hypothesis of rationality is also affected, with the limiting case, as the number of observations rises without limit, being given by a non-central χ^2 distribution. If the parameters of the loss function are known it is possible to correct the standard tests, and ensure that the hypothesis of rationality can be appropriately tested. Even where these are unknown the question can be explored using GMM estimation and the J-test for over-identification.

When the joint hypothesis of symmetry and rationality is tested (setting $p = 2$), this is rejected for 34/98 forecasters at a 5% level. However once asymmetry is allowed rationality is rejected only for four forecasters at the same significance level; such a rejection rate could surely be regarded as the outcome of chance.

Patton & Timmermann (2004) develop a flexible approach designed to allow for the possibility that different forecasters have different loss functions. This leads to testable implications of optimality even if the loss functions of the forecasters are unknown. They explore the consensus (i.e. mean) forecasts published by the Survey of Professional Forecasters for inflation and output growth (GNP growth before 1992) for 1983-2003. They find evidence of suboptimality against quadratic loss functions but not against alternative loss functions for both variables. Their work supports the idea that the loss functions of inflation forecasters are asymmetric except at low levels of inflation.

5.1.5 Heterogeneity of Expectations

Many studies allow for the possibility that some individuals may be rational and others may not. But they do not look at the mechanisms by which the irrational individuals might form their expectations.

Four papers explore this issue.[27] Ito (1990) looks at expectations of foreign exchange rates, using a survey run by the Japan Centre for International Finance, which, unlike the studies mentioned above (Dominguez 1986, Frankel & Froot 1987*b*) provides individual responses. He finds clear evidence for the presence of individual effects which are invariant over time and that these are related to the trades of the relevant respondents. Thus exporters tend to anticipate a yen depreciation while importers anticipate an appreciation, a process described by Ito as 'wishful thinking'. These individual effects are due to fixed effects rather than different time-series responses to past data. As with the earlier work, rationality of expectations is generally rejected. So too is consistency of the form described in section 2.3. Frankel & Froot (1987*a*), Frankel & Froot (1987*b*), Frankel & Froot (1990*a*), Frankel & Froot (1990*b*), Allen & Taylor (1990) and Ito (1990) also show that at short horizons traders tend to use extrapolative chartist rules, whilst at longer horizons they tend to use more mean reverting rules based on fundamentals.

Dominitz & Manski (2005) present summary statistics for heterogeneity of expectations about equity prices. Respondents to the Michigan Survey were asked how much they thought a mutual fund (unit trust) investment would rise over the coming year and what they thought

[27] In an interesting paper, Kurz (2001) also provide evidence on the heterogeneity of forecasts across the private agents and the Staff of the Federal Reserve Bank in the U.S., and explores its implications for the analysis of rational beliefs, as developed earlier in Kurz (1994).

were the chances it would rise in nominal and real terms. The Survey interviews most respondents twice. The authors classify respondents into three types, those who expect the market to follow a random walk, those who expect recent rates of return to persist and those who anticipate mean reversion. The Michigan Survey suggests that where people are interviewed twice only 15% of the population can be thus categorised. It finds that young people tend to be more optimistic than old people about the stock market , that men are more optimistic than women and that optimism increases with education. The other two papers we consider explore expectations formation in more detail.

Carroll (2003) draws on an epidemiological framework to model how households form their expectations. He models the evolution of households' inflationary expectations as reported in the Michigan Survey with the assumption that households gradually form their views from news reports and that these in turn absorb the views of people whose trade is forecasting as represented in the Survey of Professional Forecasters. The diffusion process is, however, slow, because neither the journalists writing news stories nor the people reading give undivided attention to the matter of updating their inflationary expectations. Even if the expectations of professional forecasters are rational this means that expectations of households will be slow to change. Carroll finds that the Michigan Survey has a mean square error almost twice that of the Survey of Professional Forecasters and also that the former has a much lower capacity than the latter to predict inflation in equations which also allow for the effects of lagged dependent variables. The Michigan Survey adds nothing significant to an equation which includes the results of the Survey of Professional Forecasters but the opposite is not true. Indeed the professional forecasts Granger-cause household expectations but household expectations do not seem to Granger-cause professional forecasts.

Carroll assumes that there is a unit root or near unit root in the inflation rate- a proposition which is true for some countries with some policy regimes but which is unlikely to be true for monetary areas with clear public inflation targets- and finds that the pattern by which Michigan Survey expectations are updated from those of professional forecasters is consistent with a simple diffusion process similar to that by which epidemics spread. There is, however, a constant term in the regression equation which implies some sort of residual view about the inflation rate- or at least that there is an element in household expectations which may be very slow indeed to change. Carroll also finds that during periods of intense news coverage the gap between household expectations and those of professional forecasters narrows faster than when inflation is out of the news. This of course does not, in itself demonstrate that heavy news coverage leads to the faster updating; it may simply be that when inflation matters more people pay more attention to it. Nevertheless it is consistent with a view that dissemination occurs from professional forecasters through news media to households.

In a second paper, Branch (2004) explores the heterogeneity of inflation expectations as measured by the Michigan Survey that covers the expectations reported by individuals rather than by professional forecasters. He considers the period from January 1977 to December 1993 and, although the survey interviews each respondent twice with a lag of six months, he treats each monthly observation as a cross-section and does not exploit the panel structure of the data set. Unlike earlier work on testing expectations which sought to understand the determination of the mean forecast, Branch explores the dispersion of survey responses and investigates the characteristics of statistical processes which might account for that dispersion. With an average of just under seven hundred responses each month, the probability density that underlies the forecasts is well-populated and it is possible to explore structures more complicated than distributions such as the normal density.

The framework he uses is a mixture of three normal distributions. However, instead of extending the methods surveyed by Fowlkes (1979) to find the parameters of each distribution and the weight attached to each in each period, he imposes strong assumptions on the choice of the models used to generate the means of each distribution from three relatively simple specifications; first naive expectations where expected inflation of the i^{th} respondent, π^e_{it}, is set equal to π_{t-1}, the lagged realized of inflation, secondly adaptive expectations (with the adaption coefficient determined by least squares over the data as a whole), and thirdly a forecast generated from a vector autoregression. Branch assumes that the proportion of respondents using each of the three forecasting mechanism depends on the 'success', U_{jt}, associated with the choice of the j^{th} forecast for $j = 1, 2, 3$. Success is calculated as the sum of a constant term specific to each of the three methods (C_j, $j = 1, 2, 3$), and a mean square error term, $MSE_{j,t}$, calculated as an exponential decay process applied to current and past mean square errors

$$MSE_{jt} = (1-\delta)MSE_{j,t-1} + \delta(\pi^e_{it} - \pi_t)^2,$$

with

$$U_{jt} = -(MSE_{jt} + C_j), \tag{44}$$

The probability that an individual uses method j, n_{jt} is then given by a restricted logistic function as

$$n_{jt} = \frac{e^{-\beta U_{jt}}}{\sum_j e^{-\beta U_{jt}}}. \tag{45}$$

Given the series of forecasts produced by the three methods and the standard deviation of the disturbance around each point forecast added on to each point forecast by the individual who uses that forecasting method, it is then possible to calculate the cost associated with each method and thus the proportion of respondents who "should" use this means of forecasting. Branch assumes that the standard deviation of the disturbance is time invariant and is also the same for each of these three forecasting methods; these hypotheses are not tested and no justification is given for the restrictions. He then finds that, conditional on the underlying structure he has imposed, the model 'fits' the data, with the proportions of respondents using each of the three forecasting methods consistent with (44) and (45) and that one can reject the hypothesis that only one of the forecasting methods is used.

The evidence presented shows that heterogeneity of expectations in itself does not contradict the rationality hypothesis in that people choose between forecasting methods depending on their performance and their cost, and different individual could end up using different forecasting models depending on their particular circumstances. The results do not, however, provide a test of 'rationality' of the individual choices since in reality the respondents could have faced many other model choices not considered by Branch. Also there is no reason to believe that the same set of models will be considered by all respondents at all times. Testing 'rationality' in the presence of heterogeneity and information processing costs poses new problems, very much along the lines discussed in Pesaran & Timmermann (1995) and Pesaran & Timmermann (2005) on the use sequential (recursive) modelling in finance.

Nevertheless, an examination of the raw data raises a number of further questions which might be addressed. In figure 1 we show the density of inflation expectations in the United States as reported by the Michigan Survey. The densities are shown averaged for three subperiods, 1978-1981, 1981-1991 and 1991-1999, and are reproduced from Bryan & Palmqvist (2004). As they point out, there is a clear clustering of inflationary expectations, with 0% p.a., 3% p.a. and 5% p.a. being popular answers in the 1990s. Thus there is a question whether in fact many of the respondents are simply providing categorical data. This observation and its implications for the analysis of expectations remain to be investigated.

5.2 Analysis of Disaggregate Qualitative Data

The studies surveyed above all, in various forms, provide interpretations of aggregated qualitative data. One might imagine, however, that both in terms of extracting an aggregate signal from the data and in studying expectations more generally, that there would be substantial gains from the use of individual responses and especially where the latter are available in

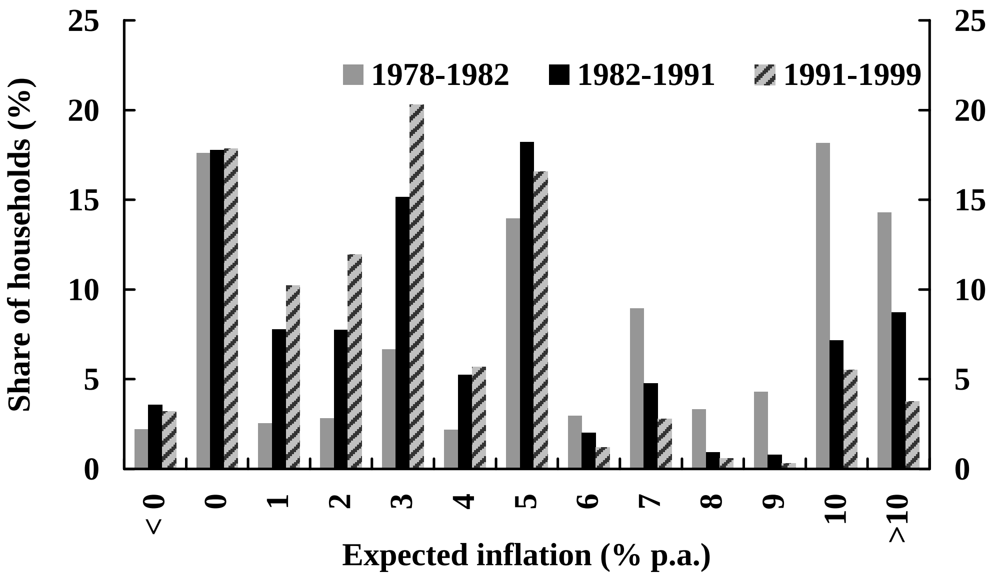

Figure 1: The Density Function of Inflation Expectations as Identified in the Michigan Survey

panel form. The main obstacle to their use is that the data are typically not collected by public sector bodies and records are often not maintained to the standards which might be expected in the public sector. We are, however, able to identify a number of studies which make use of disaggregate data collected in wide-ranging surveys.

5.2.1 Disaggregate Analysis of Expectations of Inflation and Output

Horvath, Nerlove & Wilson (1992) examine the rationality of expectations of price increases held by British firms, using the data from the Confederation of British Industry Survey. We have drawn attention in section 5.2 of what can and cannot be done using categorical data in a non-parametric framework. However, more detailed analysis is possible if one is prepared to make use of parametric models. The idea is to explore the relationship between the latent variables explaining the categorical responses to the questions about both expected future price movements and past price changes conditional on a set of exogenous variables, $\mathbf{z}_{t-1}$. For example, in the context of the following parametric model

$$x_{i,t+1} = \alpha_i + \beta_i \; {}_t x^e_{i,t+1} + \boldsymbol{\gamma}_i' \mathbf{z}_{t-1} + \varepsilon_{i,t+1},$$

since only qualitative measurements are available on $x_{i,t+1}$ and ${}_t x^e_{i,t+1}$ it is necessary to infer the regression relationship from what can be deduced about the polychoric correlations of the latent variables (Olsson 1979). In order to identify the model so as to test the hypothesis of rationality it is necessary to make two further assumptions, first that expectations are on average correct over the period and secondly that the thresholds involved in the categorisation of expectations are the same as those involved in the categorisation of the out-turn ($c_j^p = c_j^r$ for all j). Having estimated their model in this way, the authors reject the restrictions required by rationality. Kukuk (1994) used similar methods to explore the rationality of both inflation and output expectations in the IFO survey. He too rejected the hypothesis of rationality.

Mitchell, Smith & Weale (2005) addressed the question how one might produce aggregate indicators of expected output change from an analysis of the disaggregated qualitative responses to the CBI survey. They were therefore concerned with how to use the survey for forecasting purposes rather than testing any particular economic hypothesis. In essence therefore the issue they addressed was, that, while the conversion methods identified in section 3.1 may be sensible ways of extracting aggregate signals from the surveys once they have been consolidated, they may not be the best method of using the survey if one has access to the individual responses. In other words, the conventional method of reporting the results may itself be inefficient if the survey is intended to be used to provide a macro-economic signal.

The method they used is applicable only to surveys which maintain a panel of respondents. On the basis of the past relationship between each respondent's answer and actual output change, they gave each firm a score. This score can be estimated non-parametrically, as simply the mean growth in output in those periods in which the firm gave the response in question. Alternatively a probit model can be estimated to link the firm's response to output change. Given an aggregate forecasting model for output growth (such as a time-series model) Bayes' theorem can be used to derive expected output growth conditional on the response of the firm.

To produce an estimate of aggregate output growth the mean of the individual scores is taken. Experience showed that the resulting series, although strongly correlated with output growth, was much less volatile and a regression equation was needed to align it against actual output growth. Out of sample testing of the approach suggested that it performed better than the more conventional methods based on the approaches discussed in section 3.1. Nevertheless the results did not suggest that the survey was very informative as compared to a simple time-series model.

5.2.2 Consumer Expectations and Spending Decisions

Das & Donkers (1999) study the answers given by households to questions about expected income growth collected in the Netherlands' Socio-Economic Panel. Using the methods of Section 5.2 they reject the hypothesis that the respondents have rational expectations about their future income growth. Respondents to the survey are asked to give one of five categorical responses to expectations of income growth over the coming twelve months and also to report in the same way actual income growth over the past twelve months. The categorical responses are: "Strong decrease", "Decrease", "No change", "Increase", and "Strong increase".

It was found that, for people who had expected a decrease the number actually experiencing no change was larger than those reporting a decrease *ex post* in all five of the years considered and that the difference was statistically significant in four of the five years. For those reporting category "Strong decrease" *ex ante* the condition for rationality was violated in three of the five years but the violation was not statistically significant. For those reporting the last three the condition for rationality was not violated. Analysis on the assumption that the reported expectations were medians similarly led to rejection of the assumption of rationality for those expecting categories one and two. Analysis of the means was disrupted by outliers and the authors imposed 5% upper and lower trims on the sample.

They explored the idea that expectations might be based on the means by using the actual incomes reported by the households, with a weak condition being that the means of *ex post* income growth for each *ex ante* category should be increasing in the categorical ordering. Although this condition is violated sometimes for categories one and five, the violation is not statistically significant. However real income growth was positive in three of the five years for those expecting a decline in income and in two of the years the growth was significantly above zero. This leads to the conclusion that, at least as reported in the survey from the Netherlands, expectations were not rational and tended to be excessively pessimistic. Thus greater ingenuity is needed to exploit the cross-section information contained in these data.

Souleles (2004) also uses data from the Michigan Survey and explores whether the survey provides any information beyond that present in current consumption to predict future consumption. The problem he faces is that the Michigan Survey does not collect data on actual consumption and he deals with this problem by imputing information on expectations from the Michigan Survey to the United States Consumer Expenditure Survey; the latter collects consumption data from households four times in a year, providing information on spending in four quarterly periods.

Thus a discrete choice model was fitted to the Michigan Survey data to explain household responses by demographic data and income with the effects of age and income being allowed to vary over time, although no formal tests were presented for parameter stability. Given the model parameters it was possible to impute the underlying continuous variables being the responses to each of the five questions. It is then possible to explore the augmented Euler equation for consumption

$$\Delta \ln c_{i,t+1} = \mathbf{b}_0' \mathbf{d}_t + \mathbf{b}_1' \mathbf{w}_{i,t+1} + b_2 \hat{q}_{it} + \eta_{i,t+1},$$

where $\mathbf{d}_t$ is a full set of month dummies, $\mathbf{w}_{i,t+1}$ includes the age of the household head and changes in the number of adults and children and $\hat{q}_{it}$ is the fitted value of the latent expectational variable imputed to household i in period t. Note that the augmentation of the Euler equation to include demographic variables in an *ad hoc* fashion is done frequently in micro-econometric studies of household spending. In fact, although changes in household size should be expected to influence the change in household consumption, the impact of the former is specified very tightly in the population-augmented Euler equation; the restrictions implied by economic theory are rarely tested. Also the econometric specification imposes slope homogeneity which could bias the estimates.

The survey asks about past income growth and expectations of future income growth. An underlying latent variable can also be fitted to these as a function of time and demographic characteristics. It then becomes possible to work out the revision to the underlying latent variable for each household; the life-cycle model suggests that expectational errors such as these should also be expected to have an impact on consumption growth and that, too can be tested.

The study finds that non-durable consumption growth was sensitive to a number of indicators from the Michigan Survey, both the expectation and realisation of the financial position, business conditions over five years, expected income growth and expected inflation. Some of these variables may be standing in for real interest rates, omitted from the Euler equation but the study does offer *prima facie* evidence that current consumption is not a sufficient statistic for future consumption. There is also evidence that consumption growth is sensitive to expectational errors although, somewhat surprisingly, errors in expectations of future income do not seem to play a role.

This study sheds light on the link between consumer sentiment, expectations and spending growth. While its research method is innovative, it has less to say than Branch (2004) on the mechanisms by which expectations are formed. Readers are therefore unable to judge why or how far the apparent inadequacy of the Euler equation model is associated with the failure of households to make efficient predictions of the future.

6 Conclusions

The collection of data on expectations about both macro-economic variables and individual experiences provides a means of exploring mechanisms of expectations formation, linking theory to expectation and identifying the forecasting power of those expectations. A number of important issues arise. First of all there is the important question: what is the nature of expectations and how do they relate to any particular loss function? Secondly, how are expectations formed and to what extent do people learn from experience? Thirdly, what is the relationship between assumptions standard to economic theory and expectations formation in practice? Finally, how far can expectational data enhance the performance of conventional forecasting methods such as time-series models.

The studies we have discussed have identified many of these questions to some extent. However, it remains the case that the analysis of individual responses to such surveys, and in particular to those which collect only qualitative information, is underdeveloped. We expect that, as this literature develops, it will yield further valuable insights about the way people form expectations and the link between those expectations and subsequent reality. Most studies have focussed on point expectations, although studies which look at the Survey of Professional Forecasters do often also consider the information provided on the density function of expectations. By contrast there has been very little work done on qualitative information on uncertainty even though surveys such as the Confederation of British Industry's have collected such data for many years. This appears to be another vein likely to yield interesting results.

The utility of many of the data sets is limited by the fact that they are collected as cross-sections rather than panels; such surveys are likely to be more informative if they are run as well-maintained panels even if this results in a reduction in sample size. For those surveys which collect expectational information from a large number of respondents (i.e. not usually those of the forecasts of professional economists) we have not been able to find much evidence of interplay between the design of the surveys and the analysis of the information that they collect. In many countries the use made of such surveys in key decisions such as interest rate setting, has increased considerably because of the perception that they provide rapid economic information. There does not yet, however, appear to be a science of rapid data collection relating the design of these surveys to the uses made of the data that they provide. Work on this topic is also likely to be of great value.

Separately there is the question how the surveys themselves might be expected to evolve. As the tools and computing power needed to analyse panels have developed so the value of surveys maintained as panels is likely to increase. At present some are and others are not, but there appears to be no consensus developing yet about the merits of maintaining a panel, even if it is one which rotates fairly rapidly. Secondly there is the issue of collecting event probabilities rather than or in addition to quantitative or qualitative expectations. Studies carried out to date suggest that such data are useful and one might expect that increasing attention will be paid to this by data collectors.

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A Appendix A: Derivation of Optimal Forecasts under a 'Lin-Lin' Cost Function

To simplify the notations we abstract from individual subscripts, i, and write the Lin-Lin cost function, (25) for $h = 1$ as:

$$C\left(\xi_{t+1}\right) = (a+b)\left(x_{t+1} - {}_t x_{t+1}^*\right) I\left(x_{t+1} - {}_t x_{t+1}^*\right) - b\left(x_{t+1} - {}_t x_{t+1}^*\right).$$

We also assume that

$$\mathbf{x}_{t+1}\left|\Omega_t \sim N\left[E\left(x_{t+1}\left|\Omega_t\right.\right),\ \sigma^2\left(x_{t+1}\left|\Omega_t\right.\right)\right]\right..$$

Under this assumption it is easily seen that

$$E\left[\left(x_{t+1} - {}_t x_{t+1}^*\right) I\left(x_{t+1} - {}_t x_{t+1}^*\right)\left|\Omega_t\right.\right] = \sigma^2\left(x_{t+1}\left|\Omega_t\right.\right)\int_{z=\mu_{t+1}}^{\infty}\left(z + \mu_{t+1}\right)\phi\left(z\right)dz,$$

where $\phi(\cdot)$ is the probability density function of the standard normal variate, and

$$\mu_{t+1} = \frac{{}_t\mathbf{x}_{t+1}^* - E(x_{t+1} | \Omega_t)}{\sigma(x_{t+1} | \Omega_t)}.$$

Hence,

$$E\left[\left(x_{t+1} - x_{t+1}^*\right) I\left(x_{t+1} - x_{t+1}^*\right) | \Omega_t\right] = \sigma(x_{t+1} | \Omega_t) \left\{\phi\left(\mu_{t+1}\right) - \mu_{t+1}\left(1 - \Phi\left(\mu_{t+1}\right)\right)\right\},$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal variate. Therefore,

$$E\left[C\left(\xi_{t+1}\right) | \Omega_t\right] = (a+b)\,\sigma(x_{t+1} | \Omega_t)\left\{\phi\left(\mu_{t+1}\right) + \mu_{t+1}\left[\Phi\left(\mu_{t+1}\right) - \theta\right]\right\}, \tag{46}$$

where $\theta = a/(a+b)$. The first-order condition for minimization of the expected cost function is given by

$$\frac{\delta E_x\left[C\left(\xi_{t+1}\right)\right]}{\delta \mu_{t+1}} = (a+b)\,\sigma(x_{t+1} | \Omega_t)\left[\Phi\left(\mu_{t+1}\right) - \theta\right],$$

and $E_x\left[C\left(\xi_{t+1}\right)\right]$ is globally minimized for

$$\mu_{t+1}^* = \Phi^{-1}(\theta), \tag{47}$$

and hence the optimal forecast, ${}_t x_{t+1}^*$, is given by

$${}_t x_{t+1}^* = E(x_{t+1} | \Omega_t) + \sigma(x_{t+1} | \Omega_t)\,\Phi^{-1}\left(\frac{a}{a+b}\right).$$

Also, using (47) in(46), the expected loss evaluated at ${}_t x_{t+1}^*$ can be obtained as

$$E^*\left[C\left(\xi_{t+1}\right) | \Omega_t\right] = (a+b)\,\sigma(x_{t+1} | \Omega_t)\,\phi\left[\Phi^{-1}(\theta)\right],$$

which is proportional to expected volatility. The expected cost of ignoring the asymmetric nature of the loss function when forming expectations is given by

$$(a+b)\,\sigma(x_{t+1} | \Omega_t)\left\{\phi(0) - \phi\left[\Phi^{-1}(\theta)\right]\right\} \geq 0,$$

which is an increasing function of expected volatility.

B Appendix B: References to the Main Sources of Expectational Data

1. CBI: Carlson & Parkin (1975), Cunningham et al. (1998), Demetriades (1989), Driver & Urga (2004), Horvath et al. (1992), Lee (1994), Mitchell et al. (2002), Mitchell et al. (2005),Pesaran (1984),Pesaran (1985), Pesaran (1987), Wren-Lewis (1985)

2. IFO: Anderson (1952), Entorf (1993), Hüfner & Schröder (2002), Kukuk (1994), Nerlove (1983), Scholer, Schlemper & Ehlgen (1993*a*), Scholer, Schlemper & Ehlgen (1993*b*), Theil (1952)

3. INSEE: Bouton & Erkel-Rousse (2002),Gregoir & Lenglart (2000), Hild (2002), Ivaldi (1992), Nerlove (1983)

4. Livingston[28]: Bomberger (1996), Bomberger (1999), Brown & Maital (1981), Caskey (1985), Croushore (1997), Figlewski & Wachtel (1981), Figlewski & Wachtel (1983), Pesando (1975), Rich & Butler (1998), Thomas (1999)

5. Michigan: Adams (1964), Branch (2004), Bryan & Palmqvist (2004), Carroll (2003), Dominitz & Manski (1997*b*), Dominitz & Manski (2004),, Dominitz & Manski (2005),, Fishe & Lahiri (1981), Katona (1957), Katona (1975), Maddala, Fishe & Lahiri (1983), Rich et al. (1993), Souleles (2004)

6. NAPM: Klein & Moore (1991), Dasgupta & Lahiri (1993)

7. SPF[29]: Bonham & Dacy (1991), Bonham & Cohen (2001),Davies & Lahiri (1999), Elliott & Timmermann (forthcoming)Fair & Shiller (1990), Giordani & Söderlind (2003), Jeong & Maddala (1996), Keane & Runkle (1990), Lahiri et al. (1988), Zarnowitz & Lambros (1987)

8. Others: Bergström (1995), Davies & Lahiri (1995), Dominguez (1986), Frankel & Froot (1987*b*), Hüfner & Schröder (2002), Ito (1990), Kanoh & Li (1990), Kauppi, Lassila & Teräsvirta (1996), MacDonald (2000), Madsen (1993), Nerlove (1983), Öller (1990), Parigi & Schlitzer (1995), Praet & Vuchelen (1984), Praet (1985), Rahiala & Teräsvirta (1993), Smith & McAleer (1995), Tobin (1959).

[28] A full bibliography can be found at http://www.phil.frb.org/econ/liv/livbib.html

[29] A full bibliography can be found at http://www.phil.frb.org/econ/spf/spfbib.html