Modeling Productivity Index for Long Horizontal Well

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Horizontal wells have become a popular alternative for the development of hydrocarbon fields around the world because of their high flow efficiency caused by a larger contact area made with the reservoir. Most of the analytical work done in the past on horizontal productivity either assumed that the well is infinitely conductive or the flow is uniform along the entire well length. The infinite conductive assumption is good only when the pressure drop in the wellbore is very small compared to the drawdown in the reservoir otherwise the pressure drop in the wellbore should be taken into account. In this paper, an improved predictive model that takes into account the effect of all possible wellbore pressure losses on productivity index of long horizontal well was developed. Results show that the discrepancies in the predictions of the previous models and experimental results were not only due to effect of friction pressure losses as opined by Cho and Shah but may also be due to all prominent pressure losses such as kinetic change and fluid accumulation experienced by the flowing fluid in a conduit. The effect is most pronounced at the early production time where initial transience at the onset of flow is experienced. [DOI: 10.1115/1.4004887]

Keywords: modeling, productivity, horizontal well, pressure loss

Introduction

Horizontal well technology has become an important technique in oil and gas recovery because of the ability of horizontal well to produce with a higher flow rate at a lower reservoir pressure drawdown. Survey throughout the past years have shown that horizontal drilling can be used in almost any reservoir setting and its success rate reach up to 95%. There is convincing evidence that the implementation of horizontal well technique in any reservoir setting would increase the productivity index compared to vertical well technique. This technology has also proven to be excellent candidate for thin reservoir by its ability to create a drainage pattern that is quite different from that of vertical well. Naturally, increase in drainage area of horizontal well with increase in horizontal well length would promote the productivity index (PI) of horizontal wells. Recent experience [1–4] with horizontal wells has revealed that there are factors limiting the useful length of a long horizontal well that is in many circumstances the inflow performance of horizontal wells does not match with the expected productivity and their deliverability may be reduced by various pressure losses along the long horizontal wellbore [4]. The effect has serious implications where the horizontal well section is very long because the productivity index is no longer directly proportional to the well length [4,5].

As the length of a horizontal well is increased, its contact with the reservoir increases. But at the same time, the resistance to flow in the well also increases, which has a direct negative effect on the productivity of the well. The overall performance of a horizontal well depends on the balance of these two opposing factors. No reliable tools are currently available that account for both these factors in the evaluation of horizontal well performance.

Most of the findings [1–12] for evaluating the productivity index for horizontal wells have been developed. Most of the researches have focused on finding the analytical solution which has led to the development of different models. However, there are remarkable differences among their results which do not allow us to clearly establish which one match closely to the actual values. Almost, all these analytical predictive models assumed infinitely conductive or uniform flow along the entire long horizontal well length [6–9]. The assumption of uniform flow was made purely for mathematical convenience. It has been argued in the literatures that the infinite conductivity wellbore assumption is adequate for horizontal wells. Although, this may be a good assumption in situations where the pressure drop along the horizontal section of the wellbore is negligible compared to that in the reservoir, it is reasonable to expect the frictional pressure losses to cause noticeable pressure gradients in long horizontal wellbores which are defined as being longer than 1000 m [6]. Nonlaminar flow that may develop at reasonably high production rates further increases the wellbore pressure losses. Rigorous analysis of horizontal well responses and, therefore, requires the use of a model that takes into account the effect of frictional losses in the horizontal section of the well.

Among other authors, Dikken [4] (1990) discussed the effect of only frictional pressure losses of high flow rate in the long horizontal wellbore and analytically shows the solution for an infinite horizontal well length. Novy [2] (1995) generalised Dikken's work [4] by developing equation that lumped both single phase oil and gas flow. The results provided the criteria for the selection of reasonable horizontal well length at the point at which friction reduces productivity by 10% or more. Recently, Cho and Shah [13,14] (2000, 2001) developed a semi-analytical model which analyse quantitatively the effect of friction losses of liquid hydrocarbon flow on productivity index under inflow conditions.

In the present study, the effect of all possible well bore pressure losses on productivity index of a long horizontal well is investigated and a new model that incorporated these pressure losses as developed and compared with existing models. Robust model captures effect of different losses in wellbore. The key operational, fluid, and reservoir-wellbore parameters which influence the magnitude of productivity index have been identified through the formulation.

Horizontal Well Productivity Under Steady-State Flow

The steady-state analytical solution is the simplest solution to various horizontal well problems. The steady-state solution requires that the pressure at any point in the reservoir does not change with time. The flow rate equation in a steady-state condition is represented by [1]

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$$Q_{tp} = J_h \Delta P_T \tag{1}$$

where Q_{tp} = Horizontal well flow rate, STB/day, J_h = Productivity index of the horizontal well, STB/day/psi, ΔP_T = Pressure drop from the drainage boundary to wellbore, psi.

The productivity index of the horizontal well J_h can always be obtained by dividing the flow rate Q_{tp} by the pressure drop, ΔP or

$$J_h = \frac{Q_{tp}}{\Delta P} \tag{2}$$

There are several methods that are designed to predict the productivity index from the fluid and reservoir properties. Some of these methods include:

- Borisov's method
- The Giger-Reiss-Jourdan method
- Joshi's method
- The Renard-Dupuy method

Borisov's Method

Borisov (1984) proposed the following expression for predicting the productivity index of a horizontal well in an isotropic reservoir, i.e., $k_v = k_h$ [15]

$$J_h = \frac{0.0078hk_h}{\mu_0 B_0 \left[\ln\left(\frac{4r_{eh}}{L}\right) + \left(\frac{h}{L}\right)\ln\left(\frac{h}{2\pi r_w}\right)\right]} \tag{3}$$

where

h = thickness, ft
k_h = horizontal permeability, md
k_v = vertical permeability, md
L = length of the horizontal well, ft
r_{eh} = drainage radius of the horizontal well, ft
r_w = wellbore radius, ft
J_h = productivity index, STB/day/psi

The Giger-Reiss-Jourdan Method

For an isotropic reservoir where the vertical permeability k_v, equals the horizontal permeability k_h, Giger et al. (1984) proposed the following expression for determining J_h [5]:

$$J_h = \frac{0.0078Lk_h}{\mu_0 B_0 \left[\ln\left(\frac{L}{h}\right)\ln(X) + \ln\left(\frac{h}{2r_w}\right)\right]} \tag{4}$$

$$X = \frac{1 + \sqrt{1 + \left(\frac{L}{2r_{eh}}\right)^2}}{L/(2r_{eh})} \tag{5}$$

To account for the reservoir anisotropy, the authors proposed the following relationships:

$$J_h = \frac{0.0078k_h}{\mu_0 B_0 \left[\left(\frac{1}{h}\right)\ln(X) + \left(\frac{B^2}{L}\right)\ln\left(\frac{h}{2r_w}\right)\right]} \tag{6}$$

With the parameter B as defined by

$$B = \sqrt{\frac{K_h}{K_v}} \tag{7}$$

where K_v = vertical permeability, md
L = length of the horizontal section, ft

Joshi's Method

Joshi (1991) presented the following expression for estimating the productivity index of a horizontal well in isotropic reservoirs [6,7]

$$J_h = \frac{0.0078hk_h}{\mu_0 B_0 \left[\ln(R) + \left(\frac{h}{L}\right)\ln\left(\frac{h}{2r_w}\right)\right]} \tag{8}$$

with

$$R = \frac{a + \sqrt{a^2 - (L/2)^2}}{(L/2)} \tag{9}$$

and a is half the major axis of drainage ellipse and given by

$$a = (L/2)\left[0.5 + \sqrt{0.25 + (2r_{eh}/L)^4}\right]^{0.5} \tag{10}$$

Joshi accounted for the influence of the reservoir anisotropy by introducing the vertical permeability K_v via Equation (7)

$$J_h = \frac{0.0078hk_h}{\mu_0 B_0 \left[\ln(R) + \left(\frac{B^2 h}{L}\right)\ln\left(\frac{h}{2r_w}\right)\right]} \tag{11}$$

where the parameters B and R are defined above.

The Renard-Dupuy Method

For an isotropic reservoir, Renard and Dupuy (1990) proposed the following expression [1]:

$$J_h = \frac{0.0078hk_h}{\mu_0 B_0 \left[\cosh^{-1}\left(\frac{2a}{L}\right) + \left(\frac{h}{L}\right)\ln\left(\frac{h}{2r_w}\right)\right]} \tag{12}$$

where, a is half the major axis of drainage ellipse.

For anisotropic reservoirs, the authors proposed the following relationship:

$$J_h = \frac{0.0078hk_h}{\mu_0 B_0 \left[\cosh^{-1}\left(\frac{2a}{L}\right) + \left(\frac{Bh}{L}\right)\ln\left(\frac{h}{2r'_w}\right)\right]} \tag{13}$$

where

$$r'_w = \frac{(1+B)r_w}{2B} \tag{14}$$

Model Formulation

Considering the specific productivity index of long horizontal well without neglecting any of the pressure drop terms in the fundamental governing differential fluid flow equation for horizontal well. The equation can be simply represented as

$$J_h = \frac{Q}{\Delta P_T} = \frac{Q_{tp}}{\Delta P_{\text{fluid}} + \Delta P_{\text{dam}} + \Delta P_{\text{fric}} + \Delta P_{\text{acc}} + \Delta P_{K.E}} \tag{15}$$

where Q_{tp}: flow rate is obtained by considering total pressure difference between wellbore end and heel point due to inflow conditions.

ΔP_{fluid}: Pressure drop due to fluid flow via horizontal conduit.
ΔP_{dam}: Pressure drop due to formation damage near the horizontal well.

ΔP_{fric}: Pressure drop due to frictional losses in the horizontal portion of the well.

ΔP_{acc}: Pressure drop due to accumulation of fluid flow in the horizontal well.

$\Delta P_{K.E}$: Pressure drop due to convective acceleration or kinetic energy change.

Pressure Profile

Giger [5] and Joshi [6] presented pressure profile drainage of horizontal wells. Once the pressure distribution is known, oil production rates can be calculated by Darcy's law. The pressure distribution caused by steady-state flow to the horizontal well is approximated by subdividing the 3D flow problem into two 2D, according to Joshi's [6] simplification. This will approximate the pressure loss problem into two categories: (1) oil flow into a horizontal well in a horizontal plane and (2) oil flow into a horizontal well in a vertical plane.

$$\frac{P_e - p_H}{3D - xyz} = \frac{P_e - P_F}{2D - xy} + \frac{P_F - P_H}{2D - yz} + \frac{\Delta P_{\text{fric}}}{2D - xy} + \frac{\Delta P_{K.E}}{2D - xy} + \frac{\Delta P_{\text{acc}}}{2D - xy} \tag{16}$$

In this first zone (2D-xy), flow is studied in horizontal plane as if it were a vertical fracture of the same length as the horizontal fracture of the well. The pressure drop in this 2D-xy flow has been determined by Giger [5] and Joshi [6] from potential-fluid-flow theory as shown in Eq. (17)

$$P_e - P_F = \frac{Q'\mu B_0}{2\pi K_h h}\cosh^{-1}(X) \tag{17}$$

where, X is a parameter, which depends on shape and dimensions of area drained by well.

For ellipsoidal drainage area

$$X = 2a/L$$

whereas for horizontal drainage area

$$X = \cosh \ln\left[\frac{a + \sqrt{a^2 - (L/2)}}{L/2}\right]$$

$$X = \cosh \ln\left[\frac{2a}{L} + \sqrt{\left(\frac{2a}{L}\right)^2 - 1}\right]$$

Therefore, the pressure drop in horizontal plane is given in Eq. (18)

$$P_e - P_F = \frac{Q'\mu B_0}{2\pi K_h h}\ln\left[\frac{2a}{L} + \sqrt{\left(\frac{2a}{L}\right)^2 - 1}\right] \tag{18}$$

The additional pressure drop term (2D-yz), $P_F - P_H$, in the vicinity of the well is derived by Giger [5] and given as

$$P_F - P'_H = \frac{Q'\mu B_0}{2\pi K_h L}\ln\left[\frac{h}{2\pi r_w}\right] \tag{19}$$

The approximate solution for the pressure drop of both inflows by combining Eqs. (17) and (18) becomes

$$\Delta P_{\text{fluid}} = P_e - P_F + P_F - P'_H = \frac{Q'\mu B_0}{2\pi K_h h}\left[\cosh^{-1}(X) + \frac{h}{L}\ln\left(\frac{h}{2\pi r_w}\right)\right] \tag{20}$$

The reduction of one-phase flow problem in an anisotropic porous medium to flow in "an equivalent isotropic medium" uses the transformation dictated by dimensional analysis. In this transformation, the well becomes elliptical and its radius; r_w has to be changed to $r_w(1+\beta)/2\beta$ to have the same section [14]. Several solutions are available in the literature [1,4–6,13]. After reflecting anisotropy of formation, Eq. 20 becomes

$$\Delta P_{\text{fluid}} = P_e - P'_H = \frac{Q'\mu B_0}{2\pi K_h h}\left[\cosh^{-1}(X) + \frac{\beta h}{L}\ln\left(\frac{h}{2\pi r'_w}\right)\right] \tag{21}$$

where

$$r'_w = \left[\frac{1+\beta}{2\beta}\right] r_w \tag{22}$$

Introducing the skin factor into Eq. 21, Giger [5] expressed the pressure drop due to fluid flow through horizontal well as

$$\Delta P_{\text{fluid}} = \frac{Q'\mu B_0}{2\pi K_h h}\left[\cosh^{-1}(X) + \beta\frac{h}{L}\left(\frac{h}{2\pi r'_{we}}\right)\right] \tag{23}$$

where

$$r'_{we} = \left[\frac{1+\beta}{2\beta}\right] r_w \exp(-S_V) \tag{24}$$

Specific Productivity Index With Flow Restriction

Cho and Shah [14] reported that inflow performance of the well in terms of the productivity index per unit length of producing horizontal section and drawdown at each position along the section provides the following equation [13,14]:

$$q_s(x) = J_s(x)[P_e - P_w(x)] \tag{25}$$

where, P_e is the constant pressure at the outer boundary condition and P_w (x) is the pressure varying along the wellbore due to all possible pressure losses. J_s (x) is the specific productivity index per unit length of the wellbore. It depends on geometry of well, formation characteristics (permeability), and flow patterns (spherical or radial flow). It is assumed that the specified productivity index per unit length of the wellbore is constant.

Mass balance linking the change in well rate, q_w (x) at x along the well gives the following equation:

$$\frac{d}{dx}q_w(x) = -q_s(x) \tag{26}$$

Combining Eqs. 25 and 26 gives

$$\frac{d}{dx}q_w(x) = -J_s(x)[P_e - P_w(x)] \tag{27}$$

Differentiating Eq. 27 with constant $J_s(x)$ and P_e results in

$$\frac{d^2 q_w(x)}{dx^2} = J_s(x)\frac{dP_w(x)}{dx} \tag{28}$$

The following boundary conditions are applied on the differential equation:

$$\left[\frac{dq_w}{dx}\right]_{x=0} = J_s(x)[P_e - P_w(0)] = J_s(x)\Delta P \tag{29}$$

$$[q_w(x)]_{x=L} = 0 \tag{30}$$

Dikken [4] represented that the semi-experimental relationship between pressure gradient inside the well and the actual well rate at each point as

$$\frac{dP_w(x)}{dx} = R_s q_w(x)^{2-\alpha} \tag{31}$$

where, R_s is the flow resistance incorporating friction and α is the experimental constant for effective roughness in the wellbore.

Solving Eq. 31 numerically with the boundary conditions, Dikken [4] suggested the following expressions for the flow rate:

$$q_w(x) = \frac{2J_s(x)\Delta P(L-x)}{\exp(L\sqrt{J_s(x)}R_s)} \tag{32}$$

$$J_s(x) = \frac{Q'}{(P_e - P'_H)} \tag{33}$$

Specific Productivity Index With Pressure Losses in Horizontal Wellbore

The conventional productivity is calculated by the flow (Q'), which is not considered the pressure differences between wellbore and heel point of the well, to the reservoir drawdown pressure. In this calculation, a main assumption is that there is no pressure difference in wellbore end and heel point. For relatively short horizontal well length (less than 2000–3000 ft), the assumption is applicable. But for the longer horizontal wells (over 3000 ft), the pressure between wellbore end and heel point should be taken into account in calculation of the flow rate.

The flow rate (Q_{tp}) is estimated using the consent proposed by Cho and Shah [14].

The friction factor is a function of Reynolds number and effective roughness (ε_e). Reynolds number is defined as [14]

$$N_{RE} = \frac{\rho V_x D}{\mu} = 0.1231\frac{\rho Q}{\mu D} \tag{34}$$

For laminar flow, fanning friction factor is defined as [14]

$$f = \frac{16}{N_{RE}} \tag{35}$$

For turbulent flow, the following correlations are reported by various researchers [13–16]:

Dikkens

$$f = 0.079 N_{RE}^{-\alpha} \tag{36}$$

Table 1 Fluid and reservoir parameters used in this study at reservoir condition

Boundary pressure	$P_e = 3000$ psia
Oil viscosity,	$\mu_o = 1$ cp
Effective roughness	$\varepsilon/D = 0.1$
Formation volume factor	$B_o = 1.2$ rbl/stb
Horizontal permeability	$K_h = 20$ md
Vertical drainage aArea	32 (acre)
Drainage type	elliptical
Drawdown pressure	150 psi
Well length	L = 4000 ft
Fluid density	53.1 lbm/ft^3
Vertical permeability	$Kv = 2$ md
Formation thickness	H = 50 ft
Time period	10 days
Skin factor	5
Empirical coefficient	$\alpha = 0.25$

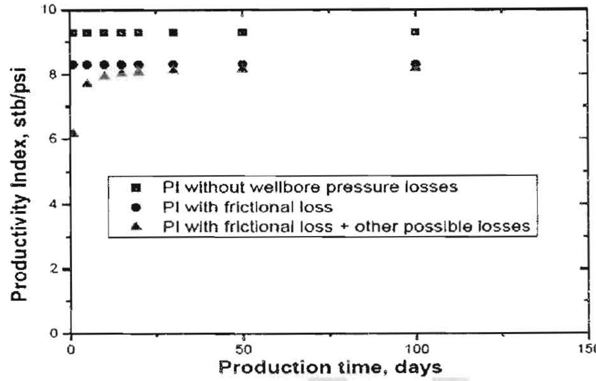

Fig. 1 Productivity index versus time

Siens

$$f = 0.25\{1.8 * \log[6.9/N_{RE} + (\varepsilon/3.7D)^{10/9}]\}^{-2} \tag{37}$$

Jain

$$f = 0.25\{1.14 - 2\log(\varepsilon/D + 21.25N_{RE}^{-0.9})\}^{-2} \tag{38}$$

The pressure drop due to friction in a well can be expressed in terms of traditional Fanning friction factor, f [16]

$$\Delta P_{\text{fric}} = \frac{2f\rho V_x^2}{g_c D} dL \tag{39}$$

The pressure drop due to accumulation can be written as [17]

$$\Delta P_{\text{acc}} = \frac{2\rho V_x}{g_c}\frac{dL}{dt} \tag{40}$$

The pressure drop due to convective acceleration or kinetic energy change can be written as [17]

$$\Delta P_{K.E} = \frac{2\rho V_x^2}{g_c} \tag{41}$$

The detailed of the fundamental equations governing flow in horizontal pipes is expressed in the Appendix A.

Once all the pressure drop terms are obtained, the new specific productivity index which takes into consideration the friction

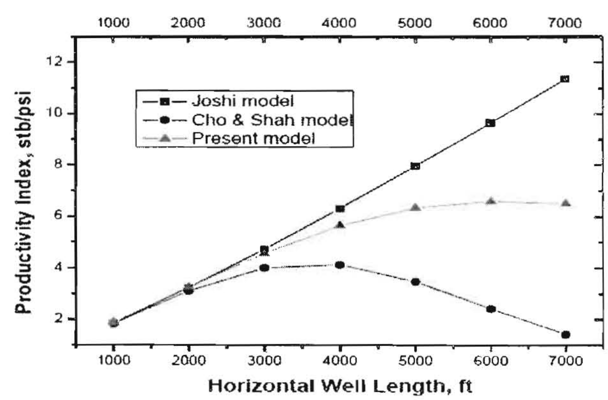

Fig. 2 Productivity index versus horizontal well length at diameter = 0.25 ft

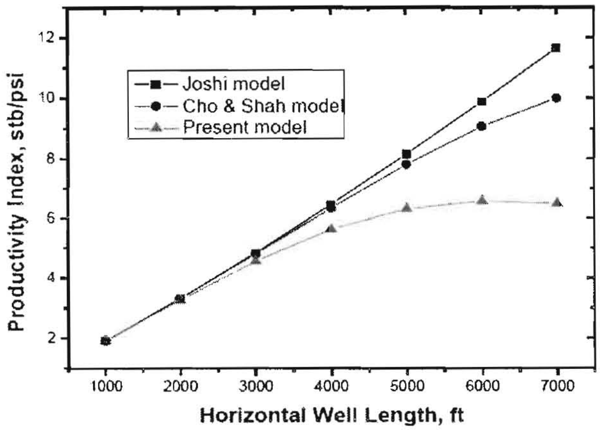

Fig. 3 Productivity index versus horizontal well length at diameter = 0.50 ft

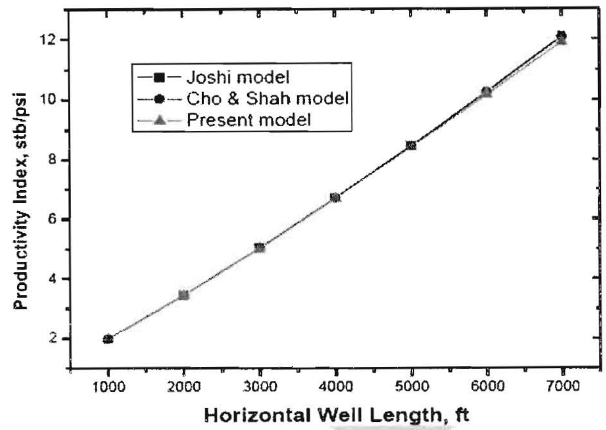

Fig. 5 Productivity index versus Horizontal Well Length at diameter = 1.35 ft

pressure effect, pressure loss due to fluid accumulation and pressure loss due to kinetic energy is defined as

$$J_h = \frac{Q}{\Delta P_T} = \frac{Q_{tp}}{\Delta P_{\text{fluid}} + \Delta P_{\text{dam}} + \Delta P_{\text{fric}} + \Delta P_{\text{acc}} + \Delta P_{K.E}} \tag{42}$$

The detailed application of this model has been demonstrated in Appendix B.

Model Analysis

Using the same data (Table 1) provided by Cho and Shah [14] in their paper, MS EXCEL software was used to calculate productivity index for long horizontal well considering all possible forms of losses such as kinetic energy change and fluid accumulation Also, the optimum ratio of well diameters to well length that could compensate for pressure losses in horizontal wellbore was estimated.

Discussion of Results

Figure 1 shows the variation of productivity index with time for long horizontal well bore using various models. The figure depicts that the flow rate increases from 0 to 50 days and then stabilizes above 50 days of production time. The difference in productivity with frictional loss only and productivity with all possible losses is the amount of flow restricted by both kinetic energy change and fluid accumulation. This difference is less significant at the later time of production. Thus, it is evident that there exists an initial transience at the onset of flow which later stabilizes with time.

Figure 2 shows the effects of increasing horizontal well length on productivity index profile as predicted by the modified and existing models (Joshi and Cho). It is observed that as the well length increases, an increasing deviation of the modified models from the existing one was obtained with a larger deviation from Joshi model. The large deviation from Joshi model implies that Joshi model over-predicts the productivity index more due to its failure in considering pressure losses due to friction, kinetic change and fluid accumulation while the smaller deviation observed in Cho model was due to inconsideration of pressure losses due to kinetic change and fluid accumulation.

However, to illustrate the effects of diameter on this deviation, the effects of increase in diameter from was investigated as shown in Figs. 3–5. It is observed from these figures that as the length increases, the diameter must increase to compensate for the pressure losses that caused deviation among the models until an optimum diameter and length combinations are achieved. Therefore, the effects of variation of diameter and horizontal well length on productivity index was investigated to obtain the optimum combinations of diameter to well length and it was found to be as shown in Fig. 6. The PI is maximum at the corresponding well length and diameter when the pressure losses due to friction, kinetic change, and fluid accumulation have been compensated for and all the models agree.

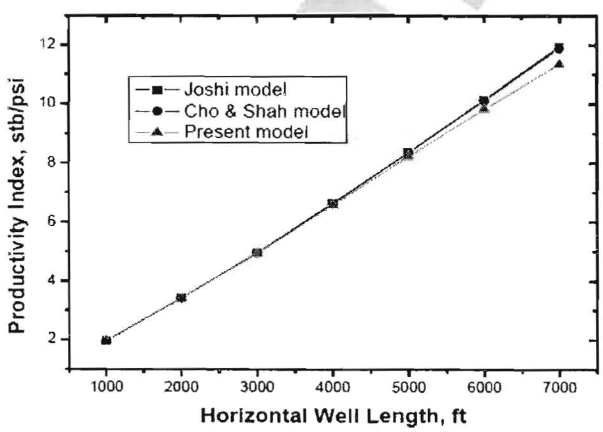

Fig. 4 Productivity index versus horizontal well length at diameter = 1.0 ft

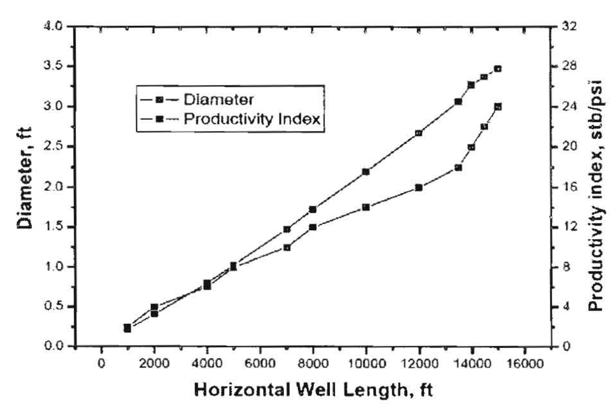

Fig. 6 Optimum diameter corresponding to horizontal well length

Conclusion

An analytical model that takes into account pressure losses due to friction, kinetic change, and fluid accumulation was developed to estimate productivity index of long horizontal wellbore. The model was compared with existing models that did not take into account the additional pressure losses and found that the existing models over-estimate well productivity. However, as wellbore diameter increased, an effective diameter at which the productivity index response predicted by the modified model approaches that predicted by the existing models.

It can be concluded that the effects of wellbore pressure losses due to increase in horizontal well length can be compensated for by an optimum wellbore diameter to length ratio.

Nomenclature

a = half major axis of drainage ellipse, ft
B_o = Formation volume factor
D = Inner diameter of wellbore, ft
f = Fanning friction factor
g_c = Conversion factor, 32.17 lb m ft/lb f s^2
h = formation thickness, ft
J_s = Areal productivity index (PI), stb/day/psi
$J_s(x)$ = Productivity index per unit length, stb/day/psi/ft
K = Isotropic formation permeability, md
K_e = Effective reservoir permeability, md
K_h = Horizontal permeability, md
K_v = Vertical permeability, md
L = Horizontal well length, ft
N_{Re} = Reynolds number, dimensionless
P_e = External boundary pressure, psi
P_F = Intermediate arbitrary pressure in wellbore, psi
ΔP_f = Pressure drop due to frictional losses in the horizontal portion of the well, psi
$\Delta P_{K.E}$ = Pressure drop due to kinetic energy change, psi
ΔP_{acc} = Pressure drop due to fluid accumulation, psi
P_H' = Pressure at the heel without friction loss, psi
P_H = Pressure at the heel with friction loss, psi
P_w = Pressure in the wellbore
Q = Oil production rate with friction loss, stb/day
Q' = Oil production rate without friction loss, stb/day
q_s= Inflow into the well unit length, rbl/day/ft
q_w = Flow rate in the wellbore, rb/day
RF = Recovery factor
R_s = Flow resistance of the well, Dimensionless
r_e = Radius of drainage area, ft
r_s = Radius of a invaded zone around wellbore, ft
r_w = Wellbore radius, ft
r_{we} = Effective wellbore radius, ft
r'_{we} = Effective wellbore radius in anisotropic, ft
S_H = Horizontal skin factor, dimensionless
S_v = Vertical skin factor, dimensionless
t = Production lasting time, year
V_x = Superficial oil velocity, ft/sec
x = Distance along the well coordinator, ft
X = Drainage configuration parameter specified in Eq. 11, dimensionless
α = Empirical coefficient for flow resistance
β = Anisotropy (Kh/Kv), dimensionless
ΔP_o = Drawdown at the heel of the well, psi
ε = Absolute roughness, ft
ε_e = Effective roughness, dimensionless
ρ = Oil density, lbm/cuft
μ = Fluid viscosity, cp

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Appendix A

Considering fluid flow in a pipe with uniform cross-sectional area using the mass conservation principle, conservation for a control system that includes mechanical energy and fluid dynamic forces can be expressed as following:

(Momentum of entering flow on at control surface) − (momentum of exiting flow at control surface) + (fluid normal force control surface) + (fluid tangent force on control surface) + (gravitational on control volume) + (mechanical forces force on control volume) = (rate of change of momentum in the control volume).

The momentum equation governing the flow in wellbore or pipe is obtained

$$PA - PA - \frac{d}{dL}(PA)dL - \tau_o \pi D dL - \rho g A dL\left(\frac{dy}{dL}\right) - \rho A dL u\frac{du}{dL} = \rho A dL\frac{du}{dt} \tag{43}$$

The parameter τ_o is the shear stress between the fluid and the pipe wall. This wall shear stress can be evaluated from a force balance between pressure forces and viscous forces defined by:

$$\tau_o = \frac{D}{4}\left(\frac{dP}{dL}\right)_f \tag{44}$$

where

$(dP/dL)_f$ is the pressure gradient due to viscous shear or frictional losses

and is defined as

$$\left(\frac{dP}{dL}\right)_f = \frac{2f'\rho u|u|}{D} \tag{45}$$

Equation 45 is the Fanning equation and f' is the Fanning friction factor. In terms of the Moody friction factor, $f = 4f'$

$$\frac{2f'\rho u|u|}{D} = \frac{f\rho u|u|}{2D} \tag{46}$$

Recognizing that $dy/dL = \sin\alpha$, Eq. 43 reduces to

$$-A\frac{dP}{dL} - \tau_o\pi D - \rho g A\sin\alpha - \rho A u\frac{du}{dL} = \rho A\frac{du}{dt} \tag{47}$$

and

$$w = \frac{f\rho u^2}{8}\pi D \tag{48}$$

$$\rho\frac{du}{dt} + \rho u\frac{du}{dL} + \frac{dP}{dL} = -\frac{w}{A} - \rho g\sin\alpha \tag{49}$$

Introducing field units Eq. 49 becomes

$$\frac{\rho}{g_c}\frac{du}{dt} + \frac{\rho u}{g_c}\frac{du}{dL} + \frac{dP}{dL} = -\frac{fu^2\rho}{2g_c D} - \frac{\rho g}{g_c}\sin\alpha \tag{50}$$

The 1D form of the energy equation for gas flow can be written as

$$\frac{dP}{dL} = -\frac{g}{g_c}\rho\sin\alpha - \frac{\rho u}{g_c}\frac{du}{dL} - \frac{f\rho u^2}{2g_c D} - \frac{\rho}{g_c}\frac{du}{dt} \tag{51}$$

The total pressure gradient is made up of four distinct components:

$$\frac{dP}{dL}=\left(\frac{dP}{dL}\right)_{\rho L}-\left(\frac{dP}{dL}\right)_{ke}-\left(\frac{dP}{dL}\right)_{f}-\left(\frac{dP}{dL}\right)_{\rm acc} \tag{52}$$

where

$\left(\frac{dP}{dL}\right)_{\rho L}=\frac{g}{g_c}\rho\sin\alpha$ is the component due to elevation or potential energy change.

$\left(\frac{dP}{dL}\right)_{ke}=\frac{\rho u}{g_c}\frac{du}{dL}$ is the component due to convective acceleration or kinetic energy change.

$\left(\frac{dP}{dL}\right)_{f}=\frac{f\rho u^2}{2g_cD}$ is the component due to frictional losses.

$\left(\frac{dP}{dL}\right)_{\rm acc}=\frac{\rho u}{g_c}\frac{du}{dt}$ is the component due to accumulation.

For horizontal pipelines

$$\frac{1}{\rho}\frac{dP}{dL}=-\frac{u}{g_c}\frac{du}{dL}-\frac{fu^2}{2g_cD}-\frac{u}{g_c}\frac{du}{dt} \tag{53}$$

Appendix B: Sample Calculation

Objective: To calculate the actual productivity index that includes all possible losses in horizontal wellbore using equations derived with the given conditions below as shown in Table 1.

Solution Procedure

Step 1: Horizontal drainage area

$$R_{ev}=\sqrt{A_v*43560/\pi}=666.06\,ft$$

$$A_h=\pi(L/2+R_{ev})(R_{ev})/43560=128.1\ \text{(acre)}$$

$$a=\frac{L}{2}\left[0.5+\sqrt{0.25+(2R_{eh}/L)^4}\right]^{0.5}=21621\,ft$$

Step 2: Basic calculation

$$\beta=\sqrt{K_h/K_v}=3.16$$

$$\cosh^{-1}(X)=\ln\left[\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]=0.40$$

Step 3: Calculation of flow rate (without friction effect)

$$Q'=\frac{0.007078*K_hh\Delta P/\mu}{\cosh^{-1}(X)+\frac{\rho h}{L}\ln\left(\frac{h}{2\pi r'_{we}}\right)}=1393(\text{stb/d})$$

Step 4: calculation of flow resistance

$$R_s=2.921*10^{-15}L^{1.86}\left(\frac{\mu D}{\rho}\right)^{\alpha}\frac{\rho}{\pi^{1.75}D^5}=4.1*10^{-6}$$

Step 5: calculation of flow rate with flow resistance

$$J_s(x)=\frac{QB_o}{\Delta PL}=2.78*10^{-3}(\text{rbl/psi/day/ft})$$

$$Q_{x=0}=\frac{J_s(x)\Delta P(L-X)}{\cosh(L\sqrt{J_s(x)R_s})}=\frac{1668}{1.075}=1551(\text{rbl/d})$$

$$Q=\frac{1551}{1.2}=1293(\text{stb/d})$$

Step 6: calculation of Reynolds's number

$$V_x=\frac{4Q}{\pi D^2}=0.76(ft/s)$$

$$N_{RE}=\frac{\rho V_xD}{\mu}=0.1231\frac{Q\rho}{\mu D}=24278$$

$$Q=\text{bbl/d},\rho=\text{lbm/ft}^3,\mu=\text{cp},D=\text{ft}$$

Step 7: calculation of friction pressure

$$f=0.25\left[1.14-2\log(\varepsilon/D+21.25N_{RE}^{-0.9})\right]^{-2}=0.026$$

$$\frac{dP_w}{dx}=\frac{2f\rho V_x^2}{Dg_c}=0.132(lbf/ft^2/ft)$$

$$\Delta P_f(x)_{x=L}=0.132*4000/144=3.67(\text{psi})$$

Step 8: calculation of pressure drop due to fluid accumulation

$$\Delta P_{\rm acc}=\frac{2\rho V_x^2}{g_c}\frac{dL}{dt}=5.157$$

Step 9: calculation of pressure drop due to kinetic energy change

$$\Delta P_{K.E}=\frac{2\rho V_x^2}{g_c}=1.856$$

Step 10: calculation of the PI

$$P_E-P_H=P_e-P_F+P_F-P'_H+\Delta P_f(L)+\Delta P_{K.E}+\Delta P_{\rm acc}$$

$$J'_s=\frac{Q'}{P_e-P_F+P_F-P'_H}=9.28(\text{stb/psi/d})$$

(Conventional PI with friction loss effect, pressure drop due fluid accumulation and pressure drop due to kinetic energy)

$$J_s=\frac{Q}{P_e-P_F+P_F-P'_H+\Delta P_f(L)+\Delta P_{K.E}+\Delta P_{acc}}=7.94(\text{stb/psi/d})$$

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