An Inventory Model for Deteriorating Commodity under Stock Dependent Selling Rate

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Abstract. *Economic order quantity (EOQ) is one of the most important inventory policy that have to be decided in managing an inventory system. The problem addressed in this paper concerns with the decision of the optimal replenishment time for ordering an EOQ to a supplier. This Model is captured the affect of stock dependent selling rate and varying price. We developed an inventory model under varying of demand-deterioration-price of commodity when the relationship of supplier-grocery-consumer at stochastic environment. The replenishment assumed instantaneous with zero lead time. The commodity will decay of quality according to the original condition with randomize characteristics. First, the model is addressed to solve a problem phenomenon how long is the optimum length of cycle time. Then, an EOQ of commodity to be ordered by will be determined by model. To solve this problem, the first step is developed a mathematical model based on reference's model, and then solve the model analytically. Finally, an inventory model for deteriorating commodity under stock dependent selling rate and considering selling price was derived by this research.*

Keywords: *deterioration commodity, expected profit, optimal replenishment time stock dependent selling rate*.

1. INTRODUCTION

The deteriorating commodity is defined as a commodity with decay or loss of quality marginal value that results in the decreasing usefulness from the original condition (Nahmias, 1982; Dave, 1991, and Raafat, 1991). Firms selling commodity whose quality level deteriorates over time often face difficult decisions when unsold inventory remains. Since the leftover commodity is often perceived to be of lower quality than the new commodity, carrying it over offers the firm a second selling opportunity and also reduced selling price. In this case, firms should choose optimal strategy as trade of order quantity from supplier and selling price down policy as impact of deterioration rate.

Unfortunately, some commodities, like fresh foods, vegetables, fruits and cooking spices, will be perishable or spoiled when the lifetime is over. In contrast, Grosser faces a market potential with a stochastic environment for instant cost per unit, life time and demand of commodity. They should obtain inventory policy to maximize their profit.

A survey of literature on inventory models for deteriorating items was given by Raafat (1991) and Nahmials (1992). This research is started with concerning a variety of inventory models from previous researches that consider the effect of deterioration and perishable with time dependent. The following scheme is used to categorize the various models for instant: (i) decision criteria: total cost or total profit; (2) consider varying demand; (3) consider deterioration varying, and consider cost/price varying **(Table 1).**

Bose et al (1995), Giri et al (1996), Jamal et al (1997), dan Ghosh & Chaudhuri (2004) developed inventory models with time dependent deteriorate rate in deterministic of deterioration varying. Meanwhile, Aggoun et al (2001) was conduct research by assuming accelerate of deterioration varying is a stochastic process from some fast alternatives with randomize characteristics. Some of the recent researches has been done by Chang & Dye (1999), Aggoun et al (2001), Mehta & Shah (2003), Teng et al (2003), Ghosh & Chaudhuri (2004) and Boukhel et al (2005). They are consider the decline of demand or sales quantity due to the decreasing usefulness from the original condition.

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Table 1: Summary of the related research

Author (s) (Published)	Decision Criteria	Varying Demand	Varying Deterioration	Varying Cost/Price	Back-Ordering
Dave (1989)	**TC**	YES, Time Linier-Increasing	NO, Constant	NO	NO
Bose et al (1995)	**TC**	YES, time-varying	YES, *Exponential*	YES	YES
Shah and Jha (1991)	**TP**	YES, Selling price-dependent	-	YES	NO
Goswami & Chaudhuri (1991)	**TC**	YES, time-varying	NO, Constant	NO	YES
Chung & Ting (1993)	**TC**	YES, time-varying	NO, Constant	NO	-
Padmanabhan & Vrat (1995)	**TP**	YES, Selling price-dependent	YES, Exponential	YES	NO
Giri et al (1996)	**TC**	YES, time-varying	YES, *Linier*	YES	YES
Benkherouf & Mahmoud (1996)	**TC**	YES, time-varying	NO, Constant	NO	-
Jamal et al (1997)	**TC**	NO, Constant	YES, Exponential	NO	YES
Chang & Dye (1999)	**TC**	YES, time-varying	NO, Constant	NO	YES
Aggoun et al (2001)	**TC**	YES, Stochastic	YES, stochastic	NO	NO
Mehta & Shah (2003)	**TC**	YES, Exponential	NO, Constant	YES	NO
Teng et al (2003)	**TC**	YES, time-varying	NO, Constant	NO, Constant	NO
Ghosh & Chaudhuri (2004)	**TC**	YES, *Time Quadratic*	YES, Weibull	NO, Constant	NO
Boukhel et al (2005)	**TC**	YES, Inventory Level	NO, Constant	NO	YES
Roy (2008)	**TP**	YES, Selling price-dependent	NO, Constant	YES	NO
Present Paper	*TP*	*YES, Selling Price-Dependent*	*YES, Exponential*	*YES*	*YES*

In most previous models, integration among supply depend on selling price with price policy in stochastic market environment was not conducted yet. In most models do not considered selling price. Then, the minimize total cost were used as decision criteria. Shah & Jha (1991) have started to developing inventory model with maximize profit as decision criterion, however the model do not be related to the impact of deterioration rate. Aggoun et al (2001) have conducted to integrate among deterioration with inventory level in one particular market environment which is stochastic. However, they do not considering cost or price varying yet as the impact of deterioration rate.

In the present paper, we have developed an inventory model for deteriorating commodity under stock dependent selling rate and considering selling price. The performance criterion of this model is to maximize profit by simultaneously determining the selling price.

2. THE PROBLEM FORMULATION

The novelty we will be taking into consideration in this research is developed an inventory model under varying of demand-deterioration-price of commodity when the relationship of supplier-grocery-consumer at stochastic environment. First, the model is addressed to solve a problem phenomenon how long is the optimum length of cycle time. Then, an EOQ of commodity to be ordered by will be determined by model during the length of cycle time to get maximum profit. The framework of a stochastic inventory model for deteriorating commodity was shown in Figure 1.

Figure 1 is described the characteristic on natural of inventory system. It was depicted by the following:

(i) this system is constructed by single supplier, single grosser and many consumers;
(ii) the item is a single commodity. Each commodity will decay of quality according to the original condition with randomize characteristics; and
(iii) all stock outs (shortages) are lost and not recovered, and
(iv) the excess stocks is expired and no value.

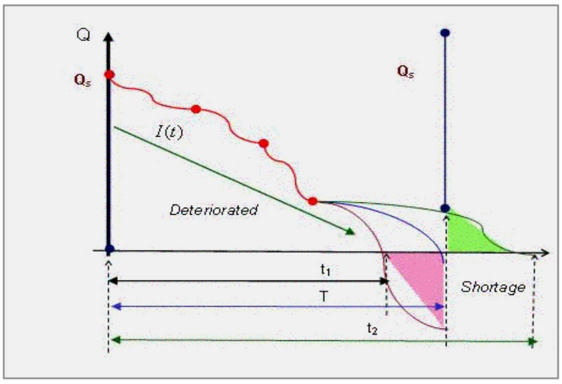

Figure 1: Framework of an inventory model

Mathematical Abbreviations and Symbols

The mathematical model is derived with the following assumptions and notations:

(1). In a cycle of planning horizon, the costs of the model is as follows:
- i). a fixed ordering cost, A;
- ii). a holding cost per unit in stock per unit of time that proportional with price and time, h;
- iii). a shortage cost per unit of item per unit of time, c_k, and N is the total quantity shortage items.

iv). a purchase cost per unit of item, p

(2). The replenishment assumed instantaneous (uniform) with zero lead time, $L = 0$.

(3). The single commodity deteriorates with rates, θ_j, a specific density function.

(4). $I_{(t)}$, is the inventory level at time t.

(5). T_D, is total demand where the demand rate (the selling rate), $D_{(t)}$, at time t, that may occur as long as T, depends on the change in inventory level, $I_{(t)}$ and parameter of stock dependent selling, b is assumed $D_{(t)} = a + b.I_{(t)}$.

(6). The excess of stocks, Q_u, is expired and no value.

(7). Selling price, P,

Decision Variables :

T_R	:	is total revenue from selling a single commodity.
T_C	:	is total of Inventory cost
T_P	:	is total profit that calculated by T_R-T_C.
T	:	is the length of the cycle time.
EOQ	:	The quantity of commodity to be ordered by grosser is an economic order quantity, $Q_s = Q^*$ during the length of the cycle time.

3. THE MATHEMATICAL MODEL

We first analyze the model of Aggoun el al (2001) as reference and then develop a new stochastic inventory model based on *periodic review system* (P Model), Aggoun et al (2001) dan Shah & Jha (1991). The mathematical model formulation is developed with the following steps:

(1). total of inventory cost

The total of inventory costs is consists the total of purchase cost (O_b), ordering cost (O_p); holding cost (O_s), shortage cost (O_k) and excess cost (O_u). In consequence the total cost in this inventory system can be written as:

$$T_c = O_b + O_p + O_s + O_k + O_u \qquad (1)$$

(2). total of purchase cost and ordering cost

The total purchase cost is the purchase cost per unit of item times the quantity of commodity to be ordered by grosser. The ordering cost is obtained as a single cost to place in order divide by the length of the cycle time. Total purchasing cost will depend on the optimum length of cycle time in each order.

$$O_b = pxQ_s \qquad (2)$$

$$O_p = A/T \qquad (3)$$

(3). total holding cost

In this inventory system, the total of average inventory is sum of the two components (the total of quantity to be ordered, T_D and the total of quantity didn't serviced, N) minus average total demand per the length of the cycle time.

$$O_s = h(Q_s - \tfrac{1}{2}T_D + N) \qquad (4)$$

(4). total shortage cost and total excess cost

The total shortage cost is expected the total quantity of customer's order didn't service times a shortage cost per unit of item during the length of the cycle time. The total quantity shortage items are calculated by total demand minus total sales. Any stock above the economic time supply should be sold will expire**.** The total excess cost is the purchase cost per unit of item times the quantity of excess.

$$O_k = c_k N.T \qquad (5)$$

$$O_u = p(Q_s - T_D), \text{ if } Q_S \geq T_D \qquad (6)$$

(5). the inventory level

The inventory level is decreases with time dependent due to selling rate and deterioration rate. When the stocks are positive selling rate is stock dependent. The opposite, when the stocks is negative the demand rate is constant. Therefore, the inventory level decreases due to stock dependent selling as well as deterioration during the period: $0 \leq t \leq t_1$.

Then, during the period $t_1 \leq t \leq T$, demand is backlogged. Furthermore, during the period $T \leq t \leq t_2$, when the stocks is positive, the excess of stocks is expired and no value. From Padmanabhan & Vrat (1995), the basic model with varying rate of deterioration that describes this model is given by:

$$\frac{\partial I_{(t)}}{\partial t} = -(a + bI_{(t)}) - \theta_t I_{(t)},\ 0 \leq t \leq t_1 \qquad (7)$$

$$\frac{\partial I_{(t)}}{\partial t} = -a,\ t_1 \leq t \leq T \qquad (8)$$

(6). Calculate the level of inventory

We consider the level of inventory at time, ***t***, $I_{(t)}$, the length of the cycle time. The solution of equation (7) and (8), using Linier First Order Equation Theorem (see Appendix A1), for the boundary condition $I_{(t)} = 0$, is

$$I(t) = \frac{a}{b+\theta}\{e^{(b+\theta)(t_1 - t)} - 1\},\ 0 \leq t \leq t_1 \qquad (9)$$

$$I(t) = a(t_1 - t),\ t_1 \leq t \leq T \qquad (10)$$

(7). Calculate the total holding cost and total shortage cost

To calculate the total holding cost and total shortage cost, we require to be determined how many is the total quantity shortage items. The total quantity shortage during length of the cycle time is given by $N = T_D - Q_s$, if $T_D > Q_s$ and $N = 0$, if $T_D \le Q_s$.

The probability of shortage and the mean number short can readily be calculated for the multiple reorder point policy. Since Z is a random variable of demand distribution function under the assumption that $f(Z)$ is normal distribution, we have the total quantity shortage is

$$N = \int_{Q_s}^{\infty} (z - Q_s) f(z) dz \tag{11}$$

(8). Calculate the total demand

The total demand, where the demand rate depends on the change in inventory level and parameter of stock dependent selling, we obtained by solve equation (12).

$$T_D = \int_0^T \{a + b.\mathrm{I(t)}\}\mathrm{dt} \tag{12}$$

Then, Substitute $I_{(t)}$ by equation (9) and (10), we can calculate ***T*$_D$**, it can be simplified and expressed by equation (13).

$$\begin{aligned} T_D &= \int_0^{t1} \{a + b[(\frac{a}{b+\theta})\{e^{(b+\theta)(t_1 - t)} - 1\}]\}\mathrm{dt} \\ &\quad + \int_{t_1}^{T} \{a + b[a(t_1 - t)]\} dt \\ &= at_1 + \frac{ab}{(b+\theta)^2}\{e^{(b+\theta)t_1} - 1 - t_1(b+\theta)\} \\ &\quad + a(T - t_1) \end{aligned} \tag{13}$$

(9). Calculate the Total profit

Total profit is equal to total revenue less than total cost. Total revenue is obtained by multiplication demand per length of the cycle time to selling price then subtracting to equation (2)-(6), the total profit is

$$\begin{aligned} T_P = \frac{1}{T} P.T_D - \langle\ & p.Q_s + \frac{A}{T} \\ & + h\{Q_s - \frac{1}{2}T_D + N\} + c_k\, NT \\ & + p\{Q_s - T_D\}\ \rangle \end{aligned} \tag{14}$$

Hence, We have the profit function $Tp(T, t_1, Q_s)$. Our objective is to maximize the profit function $Tp(T, t_1, Q_s)$ by define T, t_1, Q_s. From the function (14), we can easily extend to the more function using three objective variables, T, t_1, Q_s and expressed by equation (15):

$$\begin{aligned} & T_P(T, t_1, Q_s) \\ & = (P+p)\frac{1}{T}[at_1 + \frac{ab}{(b+\theta)^2}\{e^{(b+\theta)t_1} - 1 - t_1(b+\theta)\} \\ & + a(T - t_1)] \\ & + (\frac{1}{2}h)\frac{1}{T}[at_1 + \frac{ab}{(b+\theta)^2}\{e^{(b+\theta)t_1} - 1 - t_1(b+\theta)\} \\ & + a(T - t_1)] - 2pQ_s - \frac{1}{2}hQ_s - \frac{A}{T} \\ & -(h + c_k T)[\int_{Q_s}^{\infty}(z - Q_s) f(z) dz \end{aligned} \tag{15}$$

4. SOLUTION PROCEDURE

Using convexity rule, we can obtained three objective variable: T^*, t_1^*, Q_s^*. The necessary conditions and the sufficient condition for equation maximizing $Tp(T, t_1, Q_s)$ are

$$\begin{aligned} & \frac{\partial T_P(T, t_1, Q_s)}{\partial T} = 0; \frac{\partial T_P(T, t_1, Q_s)}{\partial t_1} = 0; \\ & \frac{\partial T_P(T, t_1, Q_s)}{\partial Q_s} = 0 \end{aligned} \tag{16}$$

$$\begin{aligned} & \frac{\partial^2 T_P(T, t_1, Q_s)}{\partial T^2} < 0; \frac{\partial^2 T_P(T, t_1, Q_s)}{\partial {t_1}^2} < 0; \\ & \frac{\partial^2 T_P(T, t_1, Q_s)}{\partial Q^2{}_s} < 0 \end{aligned} \tag{17}$$

From equation (15), first we can derivative the equation to $\partial Tp / \partial Qs = 0$, with implies

$$\begin{aligned} & \frac{\partial T_P(T, t_1, Q_s)}{\partial Q_s} = 0; \\ & -(h + 2p) + (h + \frac{c_k}{T}) \int_{Q_s}^{\infty} f(z) dz = 0 \end{aligned} \tag{18}$$

Hence, from equation (18), we can defined the probability of stock out, α , as

$$\int_{Q_s}^{\infty} f(z) dz = \frac{(h + 2p)}{(h + \frac{c_k}{T})} = \alpha \tag{19}$$

Second, we can derivative the equation (15) to $\partial Tp / \partial t_1 = 0$ then we get to simplify following relation (see Appendix A2):

$$\frac{\partial TP(T,t_1,Q_s)}{\partial t_1}=0$$

$$=\frac{1}{T}(P+\frac{1}{2}h+p)\left[\frac{ab}{(b+\theta)}(e^{(b+\theta)t_1}-1)\right] \quad (20)$$

Then $e^{(b+\theta)t_1}\approx\frac{(b+\theta)}{ab}$

Then, we can define t_1 using logarithmic rule, as

$$t_1=\frac{\ln[(b+\theta)/ab]}{(b+\theta)} \quad (21)$$

Third, we can derivative the equation (15) to $\partial Tp/\partial T=0$. The optimum value of T can be obtained by solving the equation (15). It is require two substitutions considerably more computational, so we can easily to solve the derivative problem.

Let $\lambda_1=(P+p);$ and (22)

$$\lambda_2=\frac{ab}{(b+\theta)^2}\{e^{(b+\theta)t_1}-1-t_1(b+\theta)\}; \quad (23)$$

The derivative of the latter, according the product-difference rule, after some simplification (see Appendix A3), is

$$\frac{\partial T_P(T,t_1,Q_s)}{\partial T}=0$$

$$-\frac{1}{T^2}\frac{1}{2h}\lambda_1\lambda_2-\frac{A}{T^2} \quad (24)$$

$$+c_k\int_{Q_s}^{\infty}(z-Q_s)f(z)dz=0;$$

$$T^*=\sqrt{\frac{(\lambda_1\lambda_2+2hA)}{2hc_k\int_{Q_s}^{\infty}(z-Q_s)f(z)dz}}. \quad (25)$$

The optimum value of T can be obtained from expression (20). The value of $\partial^2T_p/\partial T^2$ is always negative to satisfy the sufficient condition for maximizing T_P. The sufficient condition for maximum value of profit is

$$\frac{\partial^2T_P(T,t_1,Q_s)}{\partial T^2}=-\frac{1}{T^3}\frac{1}{2h}\lambda_1\lambda_2-\frac{A}{T^3} \quad (26)$$

For T > 0, expression (26) always negative.

The optimum value of T can be obtained by solving equation (20). It requires considerably more computation than a non recursive procedure. As a consequence, we suggest slightly modifying the **Hadley-Within** algorithm as follows:

- **Step 1.** Start with assumption that t_1=T; then find expected T_D by solving equation (13). Compute $T_0=\sqrt{2A/Dh}$ based on Wilson's model.
- **Step 2.** Compute the probability of stock out, α, by solving equation (19).
- **Step 3.** Compute Q_s by assume that demand is normally distributed, $Q_s=T_D+z_\alpha\sqrt{T}$.
- **Step 4.** Compute the total profit, T_{P0}, by solving equation (14).
- **Step 5.** Go to **step 1.** To make iteration by increasing T, substitute $T_i=T_0+\Delta T_0$ and compute T_{Pi} by performing Step 2 and Step 3. If $T_{Pi}>T_{P0}$, let $T_{i+1}=T_i+\Delta T_i$ then compute T_{Pi+1}. If $T_{Pi}<T_{P0}$ STOP iteration with then go to **Step 6.**
- **Step 6.** To make iteration by decreasing T, substitute $T_i=T_0-\Delta T_0$ and compute T_{Pi} by performing Step 2 and Step 3. If $T_{Pi}>T_{P0}$, let $T_{i+1}=T_i-\Delta T_i$ then compute T_{Pi+1}. If $T_{Pi}<T_{P0}$ STOP iteration. Hence, the uniqueness of the optimum replenishment policy, T, can be provided by choosing the best T_{Pi}.

5. NUMERICAL EXAMPLE AND ANALYSIS

To illustrate the present model, the following examples are considered. The problem to be solved here is that a Grosser sell a single commodity, i.e. garlic, such as in agricultural production, where the output of harvesting is a stochastic environment (i.e. life time and quality).

Example:

Find the optimum replenishment policy, here we have parameters as follows: a=10; b=0.05; A=25,000; h=500; c_u=10,000; θ =0.1; P=10,000; and e=2.71828.

Compute the optimum replenishment policy by the proposed algorithm above:

(1). Compute T_0 and T_D:

After we calculate based on the proposed algorithm above, after some simplification, then we get:

b+θ	=	0.15
Ab	=	0.5
(b+θ)/ab	=	0.3
Ln(b+θ)/ab)	=	3.401197
t_1={Ln(b+θ)/ab}/(b+θ)	=	22.67465
Find T_0	≈	23
λ_1	=	15,250
λ_2	=	568.86
T_D	=	798.86

(2). Find α, Zα, T_R, Q_s and T_P:

Table 2 is viewed the result of iteration algorithm to find the optimum replenishment policy.

Table 2. The optimum replenishment policy

		T_0	T_1	T_2	T_3	T_4	T_5	T_6
Find T_i	=	0.354	0.36	0.37	0.35	0.34	0.33	0.32
Find α	=	0.3650	0.3713	0.3814	0.3612	0.3510	0.3409	0.3307
Z_α	=	0.35	0.30	0.25	0.35	0.38	0.40	0.43
f(z)	=	0.3752	0.3814	0.3867	0.3752	0.37175	0.3683	0.3355
Ψ(z)	=	0.2481	0.2668	0.2863	0.2481	0.23925	0.2304	2137.2304
T_C	=	5.95	6.00	6.08	5.92	5.83	5.74	5.66
Qs	=	282.85	287.77	295.73	279.81	271.83	263.85	255.88
T_P	=	31,252,255	30,611,717	29,619,116	31,656,271	32,757,512	33,920,423	33,320,761

Computed results and the optimal replenishment policy are shown in Table 2. The optimal value of T and T_P are T* = 0.33 and T_p= 33,920,423. This numerical example capture that the grosser should obtained the optimal replenishment policy compare to optimal quantity of commodity to be ordered. Then, based on the optimal replenishment policy, we proposed how many quantity of commodity to be ordered by grosser to get maximum profit.

In our comparison with reference model, it is clear that this model obtains the length of the cycle time then we calculate the quantity of commodity to be ordered by grosser. The reference model derives optimal operating characteristics of the expected cost per unit time. Then, the reference model make the approximate policy by calculate the average deviation to each parameters.

6. CONCLUSION

A stochastic inventory model for deteriorating commodity under stock dependent selling rate and considering selling price was derived by this research. In particular, the optimal replenishment time was derived. Moreover, the numerical examples were shown to evidence the usefulness of the proposed model. By integrating demand depend on selling price between price policy in stochastic market environment, Firms or Grosser can maximize the profit through determine the optimal replenishment time.

In reality, many of the inventory systems dealing with foods items, vegetables, and meats can be tacked by the present model; in which the optimal replenishment time can measured per day, per weeks, etc. due to the commodity's characteristics. Besides that, the unit in stock can measured per kg, per ton, per stock keeping units (SKU) and etc.

For further research, this model could be extended to other characteristics of deteriorations problems in grosser, in examples with capacitated stored, considering treatment cost to pursuing decay of quality, consider transaction scheme to supplier.

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APPENDIX

Appendix A1: The derivation of equation (7)

$$\frac{\partial I_{(t)}}{\partial t} = -(a + bI_{(t)}) - \theta_t I_{(t)},\ 0 \le t \le t_1$$

Linier First Order Equation Theorem

$$\tfrac{dy}{dx} + vy = z \ \approx\ y(t) = e^{-\int vdt}(A + \int ze^{\int vdt}dt)$$

$$\text{Let } v = (\theta_t + b);\ y(t) = I_{(t)} \text{ and } z = -a$$

The specific value of the integration can be expressed as,

$$I_{(t)} = e^{-\int(\theta+b)dt}(A + \int(-a)e^{\int(\theta+b)}dt)$$

Substituting this limit, $0 \le t \le t_1$ for the equation above and the get the specific value of A, define $I_{(0)=}0$, then find the specific value of $I_{(t)}$, proof of expression (9):

$$I(t) = \frac{a}{b+\theta}\{e^{(b+\theta)(t_1-t)} - 1\}$$

Appendix A2: The proof of equation (18)

Let equation (18),

$$\frac{\partial T_P(T, t_1, Q_s)}{\partial t_1} =$$

$$(P + \frac{1}{2}h + p)\frac{1}{T}[at_1 + \frac{ab}{(b+\theta)^2}\{e^{(b+\theta)t_1} - 1 - t_1(b+\theta)\}$$

$$+ a(T - t_1)] - (h + 2p)Q_s - \frac{A}{T}$$

$$- (h + c_k T)\int_{Q_s}^{\infty}(z - Q_s) f(z)dz$$

We can easily extend to the more function using the role of the derivative of the power-function rule generalized:

$$\text{If } y = c.f(t_1) \text{ then } \frac{\partial y}{\partial t_1} = c.f'(t_1)$$

Let us consider a function:

$$f(t_1) = \{at_1 + \frac{ab}{(b+\theta)^2}\{e^{(b+\theta)t_1} - 1 - t_1(b+\theta)\} + a(T - t_1)\}$$

$$f'(t_1) = \{a + \frac{ab}{(b+\theta)^2}\{e^{(b+\theta)t_1}(b+\theta) - (b+\theta)\} - a\}$$

$$= \frac{ab}{(b+\theta)}(e^{(b+\theta)t_1} + 1)$$

Thus, it is also valid to write:

$$\frac{\partial T_P(T, t_1, Q_s)}{\partial t_1} =$$

$$\frac{1}{T}(P + \frac{1}{2}h + p)\left[\frac{ab}{(b+\theta)}(e^{(b+\theta)t_1} - 1)\right] = 0$$

$$\frac{1}{T}(P + \frac{1}{2}h + p)\{\frac{ab}{(b+\theta)}(e^{(b+\theta)t_1}\} = \frac{1}{T}(P + \frac{1}{2}h + p);$$

The equation can be expressed alternatively as

$$e^{(b+\theta)t_1} = \frac{(b+\theta)}{ab};$$

So that a different value of t1 will result in different value of the derivate, such as

$$t_1 = \frac{\ln[(b+\theta)/ab]}{(b+\theta)}$$

Appendix A3: the proof of equation (24)

From equation (15), we can easily extend to the more function using the role of the derivative of the product of two functions (product role):

$$\text{If } y(t) = u(t).v(t) \text{ then } \frac{\partial y}{\partial t} = u.v' + vu'$$

$$\text{Let } u = \frac{1}{T}(P + \frac{1}{2}h + p);$$

and

$$v = \{at_1 + \frac{ab}{(b+\theta)^2}\{e^{(b+\theta)t_1} - 1 - t_1(b+\theta)\} + a(T - t_1)\}$$

Find the derivatif of u and v

$$u' = \{-\frac{1}{T^2}(P + \frac{1}{2}h + p)\}; \; v' = a$$

The derivative of the latter, according the product-difference rule, is

$$\frac{\partial T_P(T, t_1, Q_s)}{\partial T} = 0,$$

Let $\lambda_1 = (P + p);$ and

$$\lambda_2 = \frac{ab}{(b+\theta)^2}\{e^{(b+\theta)t_1} - 1 - t_1(b+\theta)\};$$

$$\frac{\partial y}{\partial t} = u.v' + vu'$$

$$= \frac{1}{T}(P + \frac{1}{2}h + p)a +$$

$$[\frac{ab}{(b+\theta)^2}\{e^{(b+\theta)t_1} - 1 - t_1(b+\theta)\} + aT][-\frac{1}{T^2}(P + \frac{1}{2}h + p)]$$

Thus, it is also valid to write:

$$T^* = \sqrt{\frac{(\lambda_1\lambda_2 + 2hA)}{2h\,c_k \int_{Q_s}^{\infty} (z - Q_s)\, f(z)dz}}\,.$$

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