The Rationality of Faith

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1. Introduction

According to Franklin (1998, 109) Pascal's (1952) wager and Leibniz's theory that this is the best of all possible worlds are latecomers in the Faith-and-Reason tradition. Yet they have remained interlopers; for they have never been taken as seriously as the older arguments for the existence of God and other themes related to faith and reason. Yet Pascal's wager is of interest for historians of probability and decision theorists for its first instance of explicitly decision theoretic reasoning in print and its invocation of infinite utility. Moreover, it is of interest for psychologists for its discussion of voluntarism and for philosophers of religion and theologians as a putative proof that belief in God is an obligation of rationality (Hajek 2000, 1). Furthermore, as decision theory and the mathematics of infinities are flourishing and have advanced rapidly over the last couple of years (McClennen 1994, 115; Sobel 1996, 23; Vallentyne 2000) and as belief in God has grown over the last terrorist attacks and religions have multiplied over the last decades - just look at the new world religion of the Bahai's -, it might be worthwhile to look at Pascal's wager to get a new evaluation of the situation.

So let's have a look at one version of Pascal's wager: suppose you are confronted with the following decision problem. There are two actions to choose from: either to bet on God, that is, a_1, or to bet against God, that is, a_2. Two states of the world can be the case: either God exists, that is, s_1, or God doesn't exist, that is, s_2. If one combines the respective actions with the respective states of the world, the following outcomes ensue: infinite life, that is, o_{11}, wretchedness, that is, o_{21}, the life of a Christian, that is, o_{12}, and the life of a non-Christian, that is, o_{22}. So how should one decide rationally in this kind of a problem?

	s_1: God exists.	s_2: God doesn't exist.
a_1: to bet on God.	o_{11}: infinite life (∞)	o_{12}: the life of a Christian (-10)
a_2: to bet against God.	o_{21}: wretchedness (-1000)	o_{22}: the life of a non-Christian (+10)

Figure 1. Decision matrix for Pascal's wager.

According to Rescher (1985, 7) - and I agree with him - Pascal's wager doesn't answer the question whether God exists, but whether we should accept that he exists. Moreover, this version of the wager also doesn't deal with the question which kind of God exists; whether it is the God of the Protestants, of the Catholics, of Islam, of Judaism, of the Bahai's, of a God who punishes those who bet on him and who supports those who don't bet on him, whether the nontheistic view of Hinduism is true or the pantheism of Spinoza, etc. Yet this important issue which deals with the adequacy of the states of the world leads to the many-Gods objection (Jordan 1994). Furthermore, one might also want to question whether the actions are the right ones. For one might want to question the moral adequacy of these actions (Quinn 1994) and claim that adequate actions might be to belief in God and not to belief in God. Yet as Armour (1993, 2) has already observed: „We must constantly remember that one makes one's bet in this case not by putting one's money down at the two-dollar window but by *acting as if* God exists." Moreover, Morris (1994, 57) has pointed out that „Belief is not under our direct voluntary control." So the decision maker cannot simply choose to believe or not to believe, but has to bet on God's existence or non-existence and act as if God exists. Yet whether this is true has to be discussed. Hence for decision problems like Pascal's wager decision theory has two problems to solve: (1) which states of the world and which actions should figure in decision situations. (2) after having specified the states of the world and the actions which action should the decision maker decide for.

Yet because of space restrictions I start with the above formulation of Pascal's wager and take it as granted. Now the question arises how is one supposed to argue in this decision problem? One solution is to follow the principle of maximizing expected utility, where the expected utility for the respective actions can be calculated as follows (Savage 1954/1972): $U(a_i) = \sum_{j=1}^{m} p(s_j)u(o_{ij})$, that is, the utility U of an action a_i is the sum of the weighted utilities of the outcomes $u(o_{ij})$, where the weights are the probabilities of the states of the world $p(s_j)$. The expected utility for a_1 is then the following, if the probability for s_1 is 0.1 and the probability for s_2 is 0.9: $U(a_1)=0.1\text{x}(\infty)+0.9\text{x}(-10)$=infinite. The expected utility for a_2 is: $U(a_2)=0.1\text{x}(-1000)+0.9\text{x}(+10)$=finite. Because the expected utility for a_1 is much bigger than the expected utility for a_2 (even if the probability for God's existence is very small!), and because it is rational to maximize one's expected utility, one should decide for a_1. Therefore it is rational to bet on God.

2. Objections against Pascal's Wager

The following objections arise against this solution: (1) decision theory only allows for finite utility, so it cannot solve Pascal's wager (Jeffrey 1983, 150; McClennen 1994); (2) Bernoulli's law of the diminishing marginal utility of money precludes infinite utilities (Sorensen 1994, 143); (3) certain assignments of vague probabilities to God's existence scotch the wager (Hajek 2000); (4) satisficing one's utility is better than maximizing one's utility (Slote 1989); (5) the utility of an action doesn't only depend on the utilities of the outcomes, but also on other factors like the decision maker's attitude towards risk and the amount of possible change from the decision maker's reference point (Gärdenfors and Sahlin 1988); (6) the many-Gods objection: there are more and/or other possibilities than God exists and God doesn't exist as states of the world (Jordan 1994); (7) it is morally objectionable to bet on God (Franklin 1998; Quinn 1994).

Because of space restrictions I will only look at objection (3): Hajek (2000, 2-3) has pointed out that Pascal's wager depends on the decision maker assigning a probability greater than zero to God's existence. Hence decision makers who can only assign a zero probability to God's existence maximize their expected utility by betting against God, and decision makers who cannot assign any probability to God's existence cannot calculate their expected utility at all. One might object the following to these two counterexamples: Pascal neither wants to convince strict theists nor strict atheists by his wager, but those which remain suspended between a state of faith and one of unbelief. Moreover, one might want to ask are there really any strict atheists in this world? For if one is really desperate, doesn't one usually pray to God? If one is at death's door, doesn't one think it is better to believe in God? Even John von Neumann, one of the pioneers of classical game theory, had converted to Catholicism by the time he was confined to bed by an advanced and incurable cancer, and he was reported to have said that Pascal had a point (Macrae 1992, 379). Furthermore, even if the probability the decision maker assigns to God's existence is very small, to bet on God still obtains the best result. However, according to Jordan (1994, 108) the product of an infinitesimal and an infinite number is infinitesimal. If this is true (which it isn't) and if one assigns an infinitesimal to God's existence, then the principle of maximizing expected utility leads to bet against God. Yet Hajek (2000, 3) has said that Pascal's argument is addressed to human beings. And in fact human beings don't assign infinitesimal probabilities to propositions. One may object to that although human beings in fact don't assign infinitesimal probabilities to propositions, they can assign such probabilities. For suppose a dart is thrown at a unit square. Can't I assign an infinitesimal probability to its hitting a particular point in the square? Hence even though one in fact does not, one can also assign an infinitesimal probability to God's existence.

With regard to the case where decision makers cannot assign any probability to God's existence the following can be said: Pascal's wager simply doesn't arise for them. Unfortunately Hajek doesn't tell us which kind of decision makers he has in view. Very young children or mentally retarded people might not be able to assign any probability to God's existence. For they might not have any concept of probability and/or of God yet. With regard to grown ups with a sound mind they might not be willing to assign any probability to God's existence, but if one presses them to do so, I doubt that they cannot come up with some probability assignment. Hence this argument isn't valid.

Hajek (2000, 3) has pointed out that the belief states of humans are vague, that is, humans cannot assign probability, precise to indefinitely many decimal places, to all propositions. Moreover, Hajek (2000, 4) claims that vague beliefs will typically be represented by probability intervals. Yet with the exception of Kyburg (1980) all other decision theorists conceptualize their decision theories not in terms of interval probabilities, but in terms of point probabilities, so that they must have a justification for doing so.

Two reasons for point probabilities and against interval probabilities come to my mind:

(1) one should prefer a simpler decision theory to a more complicated one, if the former theory is as adequate as the latter for solving the problems in its field. Because a decision theory with point probabilities is simpler than a decision theory with interval probabilities, and because there is no reason why a decision theory with point probabilities shouldn't be able to deal with all the problems in its field, whereas a decision theory with interval probabilities is already limited in its applicability, a decision theory with point probabilities should be preferred to a decision theory with interval probabilities.

(2) If one uses interval probabilities one has to provide a justification for both borders, whereas if one uses point probabilities one just has to justify one point. The former can be seen by the following: if the decision maker claims that his knowledge situation that it is very likely that there is good weather tomorrow is best represented by the probability interval (0.7, 0.9), one can ask the decision maker why his knowledge situation isn't best represented by the probability interval (0.69, 0.91)? In the case of point probabilities one can only ask the decision maker why he uses this point, for example, P = 0.9 in the weather example, and not any other one, like P = 0.91. Because in the case of interval probabilities and in the case of point probabilities it is difficult to provide justifications, and because it is better to need as few justifications as possible, point probabilities should be preferred to interval probabilities.

Moreover, Kyburg (1974, 264-267) admits in opposition to point probabilities interval probabilities only fulfil in certain special cases the axioms of a generalized mathematical probability calculus for intervals. This is a serious defect of interval probabilities. For it is a minimal requirement in decision theory that the decision maker's probabilities are coherent. This can be achieved by fulfilling the axioms of the mathematical probability calculus. If the decision maker's probabilities are incoherent, he is in a position to face a betting situation which has become known as "Dutch book". Therefore Kyburg's interval probabilities are limited in their applicability to certain special cases. Yet Hajek (2000, 5-6) objects that a sure way to avoid being Dutch booked is to remain totally vague, which is true, but has as a consequence that the decision maker isn't willing to bet on anything anymore, which leads to complete inaction. I don't think it is ideally rational never to decide anything in your whole life. However, one might be able to rewrite the axioms of the mathematical probability calculus in such a way that they can accommodate interval probabilities. Moreover, Dutch books might be avoided, if there is a suitable decision rule for use of interval-valued probabilities. Decision rules can look at bets already made and stay away from new bets that would make a Dutch book.

Yet if one uses interval probabilities instead of point probabilities, why doesn't one also use interval utilities instead of point utilities? After all in the game show *The Price Is Right*, humans have problems assigning prices to everyday goods, too, that is, they have vague conceptions which kind of price might be the right one with respect to particular goods. In my opinion using interval utilities instead of using point utilities leads to unnecessary complications of decision theory. That's probably also the reason why one doesn't find any decision theory with interval utilities. [1]

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