Degrees of Belief as Basis for Scientific Reasoning?

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1. The Bayesian Approach to Scientific Reasoning

Bayesianism is the claim that scientific reasoning is probabilistic, and that probabilities are adequately interpreted as an agent's actual subjective degrees of belief measured by her betting behaviour.

Confirmation is one important aspect of scientific reasoning. The thesis of this paper is the following: Given that scientific reasoning (and thus confirmation) is at all probabilistic, the subjective interpretation of probability has to be given up in order to get right confirmation, and thus scientific reasoning in general.

This will be argued for as follows: First, an example will be considered which is an instance of a more general version of the problem of old evidence, POE. This suggests to look whether the existing solutions to POE provide a solution to the more general problem called C.

The first result is that the existing solutions to POE are no genuine ones, because they do not provide a solution to C.

More importantly, the attempts to solve C all have in common that they essentially depend on the agent's absolutely first guess, her first degree of belief function p_0.

Therefore, C leads to the problem of prior probabilities, POPP. However, the standard solution to POPP – the "washing out of priors" relying on convergence to certainty and merger of opinion – is not applicable here, because the solutions to C never get rid of the agent's first degree of belief function p_0.

By the subjective interpretation of probability, p_0 is any arbitrary assignment of values in [0,1] to the atomic propositions of the underlying language. Thus, by choosing an appropriate p_0 one can obtain more or less any degree of confirmation. In case evidence E is known and logically implied by hypothesis H and background knowledge B, the degree of confirmation is even uniquely determined by the agent's first guesses in H and E.

The only way out is some kind of objective or logical probability function the agent could adopt as her first degree of belief function p_0. However, the difficulty of determining such a logical probability function just was the reason for turning to the subjective interpretation of probability.

2. Bayesian Confirmation Theory

According to Bayesian confirmation theory, the agent's degree of confirmation of hypothesis H by evidence E relative to background knowledge B is measured by some function c_p such that

$$c_p(H, E, B) \begin{cases} > 0 & \Leftrightarrow \quad p(H \mid E \wedge B) > p(H \mid B) \\ = 0 & \Leftrightarrow \quad p(H \mid E \wedge B) = p(H \mid B) \\ < 0 & \Leftrightarrow \quad p(H \mid E \wedge B) < p(H \mid B), \end{cases}$$

where p is the agent's degree of belief function. Any such function c_p is called a *relevance measure* (based on p).

An example is the *distance measure* d_p,

$$d_p(H, E, B) = p(H \mid E \wedge B) - p(H \mid B).$$

3. The Example

An agent with degree of belief function p considers the hypothesis

H = All Scots wear kilts.

At time t_1 she has the impression to see her friend Stephen wearing a kilt. As the agent is not wearing her glasses, her degree of belief in

E = Stephen wears a kilt

is not very high, say

$$p_1(E \mid B_1) = .6,$$

where p_1 is her degree of belief function at t_1. B_1 is her background knowledge at that time containing the information that Stephen is Scot.

Because of knowing that H and B_1 logically imply E, the agent gets interested in whether Stephen is indeed wearing a kilt. So she takes on her glasses and has a careful second look at Stephen, who still seems to wear a kilt – this happening at time t_2.

In passing from t_1 to t_2 the only change in the agent's degrees of belief is in E. Moreover, for some reason she cannot express her observation in terms of a proposition. So her degree of belief in E increases *exogenously*, say to

$$p_2(E \mid B_2) = .9,$$

where p_2 is the agent's degree of belief function at t_2. Her background knowledge B_2 at t_2 is the same as at t_1, because the only change is in E, and that change is exogenous, i.e. not due to any proposition on which the agent could condition. So B_1 is logically equivalent to B_2, $B_1 \equiv B_2$.

4. The Less Reliable the Source of Information, the Higher the Degree of Bayesian Confirmation

Let us compare the agent's degrees of confirmation at time t_1 and at time t_2.

As the agent knows that H and B_1 logically imply E (and does not forget this and that Stephen is Scot),

$$p_j(E \mid H \wedge B_j) = 1, \text{ for all points of time } t_j, j \geq 0,$$

even if it is not assumed that she is logically omniscient in the first sense that all logical truths are transparent to her (cf. Earman 1992, 122).

Given Jeffrey conditionalisation (JC), i.e. assuming

$$p_1(H \mid \pm E \wedge B_1) = p_2(H \mid \pm E \wedge B_2),$$

it follows that

H is more confirmed by E relative to B_1 at t_1 than (relative to B_2) at t_2 if and only if the agent's degree of belief in E at t_1 is smaller than at t_2, i.e.

$$d_{p1}(H, E, B_1) > d_{p2}(H, E, B_2) \Leftrightarrow p_2(E \mid B_2) > p_1(E \mid B_1).$$

More generally,

C $\quad d_{p1}(H, E, B_1) > d_{p2}(H, E, B_2) \Leftrightarrow$
$p_1(E \mid H \wedge B_1) > p_1(E \mid B_1)$ and
$p_2(E \mid B_2) > p_1(E \mid B_1)$

or

$p_1(E \mid H \wedge B_1) < p_1(E \mid B_1)$ and $p_2(E \mid B_2) < p_1(E \mid B_1)$,

where the only change in the agent's degrees of belief in passing from t_1 to t_2 is exogenous and in E, whence $B_1 \equiv B_2$, and JC is used. Here and in the following the probabilities of all contingent propositions involved are assumed to be positive.

C holds for the distance measure d_p, the *log-likelihood ratio* l_p, and the *ratio measure* r_p,

$l_p(H, E, B) = \log[p(E \mid H \wedge B)/p(E \mid \neg H \wedge B)]$,

$r_p(H, E, B) = \log[p(H \mid E \wedge B)/p(H \mid B)]$.

The measure s_p,

$s_p(H, E, B) = p(H \mid E \wedge B) - p(H \mid \neg E \wedge B)$,

is invariant w.r.t. exogenous belief changes in E (which yield $B_1 \equiv B_2$), i.e.

$s_{p1}(H, E, B_1) = s_{p2}(H, E, B_2)$.

In case of c_p,

$c_p(H, E, B) = p(H \wedge E \wedge B) \cdot p(B) - p(H \wedge B) \cdot p(E \wedge B)$,

something different (but not much better) holds:

C' $\quad c_{p1}(H, E, B_1) > c_{p2}(H, E, B_2) \Leftrightarrow$

$p_1(E \mid H \wedge B_1) > p_1(E \mid B_1)$ and $p_1(E \wedge B_1)/p_2(E \wedge B_2) > p_2(\neg E \wedge B_2)/p_1(\neg E \wedge B_1)$

or

$p_1(E \mid H \wedge B_1) < p_1(E \mid B_1)$ and $p_1(E \wedge B_1)/p_2(E \wedge B_2) < p_2(\neg E \wedge B_2)/p_1(\neg E \wedge B_1)$.

For the different measures and the problem of measure sensitivity cf. Fitelson 2001.

5. A More General Version of the Problem of Old Evidence

C is a more general version of the problem of old evidence, POE. POE is that evidence E which is old in the sense of being assigned a degree of belief of 1 cannot provide any confirmation, since for any p, H, E and B:

$p(H \mid E \wedge B) = p(H \mid B)$, if $p(E \mid B) = 1$.

POE is a problem, because there are historical cases where old evidence did provide confirmation (for an excellent discussion cf. chapter 5 of Earman 1992).

And: If POE is a problem, then so is C.

This is important, because a Bayesian could simply refuse to consider C as counterintuitive. Is it not rational, she might say, that I take positively relevant E to provide the less confirmation for H, the more I already believe in E and have built this belief into my belief in H?[1]

[1] This point was made by Luc Bovens in personal correspondence.

This reply is perfectly reasonable, but applies equally well to POE. However, a brief look at the literature shows that POE is taken to be a problem.

Let us therefore look whether the existing solutions to POE give rise to a solution to C. Generally, there are two ways of approaching POE:

1) Conditioning on the entailment relation: Garber 1983

2) Counterfactual strategy: Howson and Urbach 1993

6. Conditioning on the Entailment Relation

The idea here is to distinguish between a historical and an ahistorical POE, and to solve the former by noting that

> what *increases* [the agent]'s confidence in [H] is not E itself, but the *discovery* of some generally logical or mathematical relationship between [H] and E. (Garber 1983, 104)

Then one shows that even if $p(E \mid B) = 1$,

> the discovery that [H entails E] *can* raise [the agent]'s confidence in [H]. (Garber 1983, 123)

Conditioning on the entailment relation does not provide a solution to C, for in the example the agent is interested in E just because of knowing that H and B_1 logically imply E (and does not forget this and that Stephen is Scot), whence

$p_j(H \text{ entails } E \mid B_j) = 1$, for every point of time t_j, $j \geq 0$.

Moreover, by substituting 'H entails E' for E one gets another instance of C.

7. The Counterfactual Strategy

Concerning POE, Howson and Urbach write:

> the support of [H] by E is gauged according to the effect which one believes a knowledge of E *would now have* on one's degree of belief in [H], on the (counter-factual) supposition that one does not yet know E. (Howson and Urbach 1993, 404-405)

Suppose $B - E$ is the logically weakest proposition such that

$(B - E) \wedge E \equiv B$,

so that $p(X \mid B - E)$ is the agent's degree of belief in X "on the (counter-factual) supposition that [she] does not yet know E".

Then, if $p(E \mid B) = 1$, the agent's degree of confirmation is given by

$d'_p(H, E, B) = p(H \mid B) - p(H \mid B - E)$,

"actual" – "counterfactual".

However, in case E is not known, it cannot be dropped from B. Therefore one has to generalize from the case of POE where $p(E \mid B) = 1$ to the case of C where $p(E \mid B)$ need not be 1.

The question is, of course, how the counterfactual strategy is adequately generalized. Apart from the above, there are the following (and uncountably many more) formulations of $d'_p(H, E, B)$:

$d'_p(H, E, B) = p(H \mid (B - E) \wedge E) \cdot p(E \mid B) + p(H \mid (B - E) \wedge \neg E) \cdot p(\neg E \mid B) - p(H \mid B - E)$

$= p(H \mid (B - E)\wedge E)\cdot p(E \mid B) - p(H \mid B - E)$

$= p(H \mid (B - E)\wedge E) - p(H \mid B - E)$

$= p(H \mid B\wedge E) - p(H \mid B - E)$

8. Generalizing the Counterfactual Strategy

Instead of considering

> the (counter-factual) supposition that one does not yet know E (Howson and Urbach 1993, 405)

the quote suggests to consider

> the (counter-factual) supposition that one does not yet *believe in E to degree* $p(E \mid B)$.

However, the background knowledge at t_1 and at t_2 is the same, because the change in the agent's degree of belief in E is exogenous. Therefore one cannot just drop something (say, all information bearing on E) from B_2 to get a counterfactual supposition $B_2 \setminus E$ which could play a role analogous to that of $B_2 - E$ in the special case where $p_2(E \mid B_2) = 1$.

Instead, one really has to adopt a new probability function p^E! Suppose $p^E(X \mid B)$ is the agent's degree of belief in X on the counterfactual supposition that she does not yet believe in E to degree $p(E \mid B)$.

Then there are the following (and uncountably many more) ways of generalizing d':

$g_{1p}(H, E, B) = p^E(H \mid B\wedge E)\cdot p(E \mid B) + p^E(H \mid B\wedge\neg E)\cdot p(\neg E \mid B) - p^E(H \mid B)$

$g_{2p}(H, E, B) = p^E(H \mid B\wedge E)\cdot p(E \mid B) - p^E(H \mid B)$

$g_{3p}(H, E, B) = p^E(H \mid B\wedge E) - p^E(H \mid B)$

$g_{4p}(H, E, B) = p(H \mid B\wedge E) - p^E(H \mid B)$

$g_{5p}(H, E, B) = p(H \mid B) - p^E(H \mid B)$

9. The Result to Follow – and a Necessary and Sufficient Condition for it

According to Bayesian intuitions, the result to follow is that

> H is more confirmed by E relative to B_2 at t_2 than (relative to B_1) at t_1 if and only if the agent's degree of belief in E at t_2 is greater than at t_1, i.e.
>
> $c_{p2}(H, E, B_2) > c_{p1}(H, E, B_1) \Leftrightarrow p_2(E \mid B_2) > p_1(E \mid B_1)$,
>
> provided E is positively relevant for H given B_1 ($\equiv B_2$).

More generally, this means either D_C or D_A, depending on how one construes "positively relevant":

D_C $\quad c_{p2}(H, E, B_2) > c_{p1}(H, E, B_1) \Leftrightarrow$
$p_1^E(E \mid H\wedge B_1) > p_1^E(E \mid B_1)$ and $p_2(E \mid B_2) > p_1(E \mid B_1)$

or

$p_1^E(E \mid H\wedge B_1) < p_1^E(E \mid B_1)$ and $p_2(E \mid B_2) < p_1(E \mid B_1)$

D_A $\quad c_{p2}(H, E, B_2) > c_{p1}(H, E, B_1) \Leftrightarrow$
$p_1(E \mid H\wedge B_1) > p_1(E \mid B_1)$ and $p_2(E \mid B_2) > p_1(E \mid B_1)$

or

$p_1(E \mid H\wedge B_1) < p_1(E \mid B_1)$ and $p_2(E \mid B_2) < p_1(E \mid B_1)$.

Before continuing, note that it is plausible to assume that counterfactual degrees of belief are stable over time, i.e.

E $\quad p_1^E(H \mid B_1) = p_2^E(H \mid B_2)$.

The reason is that in going from t_1 to t_2 the only change is exogenous and in E, and $p_i^E(H \mid B_i)$ is the agent's degree of belief in H on the counterfactual supposition that she does not yet believe in E to degree $p_i(E \mid B_i)$.

Interestingly, E sheds positive light on g_1 and g_5 (here and in the following the index of the background knowledge is dropped, because $B_1 \equiv B_2$):

1) E is necessary and sufficient for g_1 to satisfy D_C, assuming "counterfactual Jeffrey conditionalisation", i.e. $p_1^E(H \mid \pm E\wedge B) = p_2^E(H \mid \pm E\wedge B)$, and

2) E is necessary and sufficient for g_5 to satisfy D_A, assuming JC.

Moreover, E sheds negative light on $g_{2\text{-}4}$: Given counterfactual JC,

3) E is necessary and sufficient for g_2 to satisfy F, and

4) E is necessary and sufficient for g_3 to satisfy G_C.

Given JC,

5) E is necessary and sufficient for g_4 to satisfy G_A.

Here

F $\quad c_{p2}(H, E, B) > c_{p1}(H, E, B) \Leftrightarrow p_2(E \mid B) > p_1(E \mid B)$,

G_C $\quad c_{p2}(H, E, B) = c_{p1}(H, E, B) = p_i^E(H \mid B\wedge E) - p_i^E(H \mid B)$,

G_A $\quad c_{p2}(H, E, B) = c_{p1}(H, E, B) = p_i(H \mid B\wedge E) - p_i^E(H \mid B)$.

F is odd, because it does not matter whether E is positively relevant for H given B. G_C and G_A are odd for a Bayesian, because confirmation is invariant w.r.t. exogenous belief changes in E.

All things considered it seems fair to say that the proper generalisation of d' is g_1 or g_5. In order to get confirmation right they both require counterfactual degrees of belief to be stable over time.

So g_1 and g_5 reduce to

$g_{1pi}(H, E, B) = p_0^E(H \mid B\wedge E)\cdot p_i(E \mid B) + p_0^E(H \mid B\wedge\neg E)\cdot p_i(\neg E \mid B) - p_0^E(H \mid B)$,

$g_{5pi}(H, E, B) = p_i(H \mid B) - p_0^E(H \mid B)$.

10. Actual Degrees of Belief

Whether or not the preceding generalisations are appropriate, they are not satisfying, because it remains questionable how $p^E(X \mid B)$ is determined and related to the agent's actual degree of belief function $p(X \mid B)$. This question being unanswered, the counterfactual strategy is concluded to provide no genuine solution to C either.

Let us therefore consider an account solely in terms of actual degrees of belief (and providing a possible answer to the mentioned question).

Generally, the example in section 3 is one where E is positively relevant for H given B, and the agent's degree of belief in E changes exogenously as time goes by. If there is an increase (decrease) in the agent's degree of belief in

E, her degree of belief in H increases (decreases), too; and conversely, if E is negatively relevant for H given B.

All Bayesian accounts of confirmation measure in some way the difference between

$p(H \mid E \wedge B)$ and $p(H \mid B)$.

Given Bayes or strict conditionalisation, this is just the difference between the agent's prior and posterior degree of belief in H when she learns E and nothing else.

The counterfactual strategy measures the difference between the agent's actual or posterior degree of belief in H and her counterfactual one – the latter replacing her prior. The reason is that the prior and posterior degrees of belief coincide if E was already known.

Solving C requires something more general, because there one does not learn or know E; there is only a change in the agent's degree of belief in E.

This suggests to consider the agent's prior and posterior degree of belief in H when the only change is exogenous and in E.

However, one cannot simply take the difference between

$p_i(H \mid B)$ and $p_{i-1}(H \mid B)$.

(B is the same, because all changes are exogenous.)

For suppose the agent's degree of belief in E increases enormously in going from t_{i-2} to t_{i-1}, say from

$p_{i-2}(E \mid B) = .01$ to $p_{i-1}(E \mid B) = .9$;

and then it increases again in going to t_i, but only slightly, say to

$p_i(E \mid B) = .91$.

Then the difference between

$p_{i-2}(H \mid B)$ and $p_{i-1}(H \mid B)$

is much greater than the difference between

$p_{i-1}(H \mid B)$ and $p_i(H \mid B)$.

Consequently, the difference between the prior and posterior degree of belief in H at t_{i-1} is much greater than that at t_i, although the agent's degree of belief in E at t_{i-1} is smaller than at t_2, i.e.

$p_i(H \mid B) - p_{i-1}(H \mid B) < p_{i-1}(H \mid B) - p_{i-2}(H \mid B)$

and

$p_i(E \mid B) > p_{i-1}(E \mid B)$,

where E is positively relevant for H given B, and all belief changes are exogenous.

What one has to consider instead is the difference between the agent's current degree of belief in H, $p_i(H \mid B)$, and her *first* degree of belief in H, $p_0(H \mid B)$, where the only change in going from t_0 to t_i is exogenous and in E.

The proposal therefore is

$g_{6pi}(H, E, B) = p_i(H \mid B) - p_0(H \mid B)$

$= p_0(H \mid E \wedge B) \cdot p_i(E \mid B) + p_0(H \mid \neg E \wedge B) \cdot p_i(\neg E \mid B) -$
$- p_0(H \mid B)$ $\quad$ i times JC,

which satisfies D_A.

g_1, g_5, and g_6 coincide, if

$p_0^E(H \mid \pm E \wedge B) = p_0(H \mid \pm E \wedge B)$ and $p_0^E(H \mid B) = p_0(H \mid B)$.

11. The Common Knock-Down Feature *or* Anything Goes

All three measures g_1, g_5, and g_6 have in common that their values essentially depend on the agent's first degree of belief function p_0.

In case E is known and logically implied by H and B, the agent's degree of confirmation of H by E relative to B at time t_i (measured by g_6) is even uniquely determined by her first guesses in E and H, $p_0(E \mid B)$ and $p_0(H \mid B)$!

Why the exclamation mark?

First, because this shows that the idea behind any Bayesian theory of confirmation – namely to determine the degree of confirmation by the agent's actual subjective degrees of belief – is shown to fail.

Second, because – by the subjective interpretation – p_0 is any arbitrary assignment of values in [0,1] to the atomic propositions of the underlying language, whence by choosing an appropriate p_0 one can obtain more or less any degree of confirmation.

12. The Problem of Prior Probabilities

Thus we are back at the problem of prior probabilities, the standard solution to which I take to be the "washing out of priors" relying on convergence to certainty and merger of opinion (cf. Earman 1992, esp. 57-59).

However, the latter is not applicable here, because g_6 and company never get rid of the agent's first degree of belief function p_0.

The only way out is some kind of objective or logical probability function the agent could adopt as her p_0.

Yet the difficulty of determining such a logical probability function just was the reason for turning to the subjective interpretation. [2] [3]

References

Earman, J. (ed.) 1983 *Testing Scientific Theories*, Minnesota Studies in the Philosophy of Science, vol. 10, Minnesota: University of Minnesota Press.

Earman, J. 1992 *Bayes or Bust? A Critical Examination of Bayesian Confirmation Theory*, Cambridge, Mass.: The MIT Press.

Fitelson, B. 2001 *Studies in Bayesian Confirmation Theory*, University of Wisconsin-Madison: Dissertation.

Garber, D. 1983 "Old Evidence and Logical Omniscience in Bayesian Confirmation Theory", in J. Earman (ed.), *Testing Scientific Theories*, Minnesota Studies in the Philosophy of Science, vol. 10, Minnesota: University of Minnesota Press, 99-131.

Howson, C. and Urbach, P. 1993 *Scientific Reasoning: The Bayesian Approach*, 2nd ed., Chicago/La Salle: Open Court.

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