

Measuring the Uncertain – Remarks on Entropy and Information

Jan Werszowiec Plazowski, Marek Suwara, Kraków

1. Entropy as the measure of our uncertainty in knowing the World

According to original Shannon's concept entropy is the measure assigned to the spectrum of possible states of a given system. The formula

$$S = \sum_{i=1}^N p_i \log p_i$$

where p_i is the probability of state i reaches the null value for the case when the cognitive subject knows exactly in which state the system is. The practical use of this formula strongly depends upon the actual ability to determine ALL possible states of the system and ability to estimate ACTUAL PROBABILITIES of those states, which is possible only for highly idealized and reduced systems. Thus direct application of such measure to our general knowledge of the world seems rather impossible unless we suggest some general procedures to reduce the number of 'possible worlds'. The way philosophy maintains such a task is metaphysics of the world.

A cognitive subject can reduce his/her uncertainty through gaining some information. Incoming information can change the subject's knowledge in two ways: it may either eliminate some possible states or it can change the way the subject 'partitions' the world. The former situation corresponds to gaining some knowledge in the way of empirical examining the world – the result of an experiment reduces our uncertainty of the world by selecting of the whole spectrum of expectable answers to the "experimental question" those which may be the most probable. We claim that the expectable answers are not declarations stated by "the World" but result from our ability predict "the World's" behavior. Such predictions are based upon a certain metaphysical model of "the World" as well as upon the theoretical background.

The latter would rather resemble the change in 'paradigm' suggested by T.Kuhn and thus is characteristic to the discovery processes. It is evident that the both are not independent.

2. Relative entropy concept

The way a cognitive subject computes the entropy depends on the way he/she "partitions" the world i.e. what number and kinds of possible states of the world are possible according to the metaphysical representation of the world.

Provided one and only one metaphysical representation of the world is possible and this one is 'uniquely filled out' with the 'scientific theories' one can try to evaluate the entropy. In the more probable case, where there are competitive theories or, which is even worse, competitive metaphysical representations, the problem can be handled with the use of the concept of relative entropy between two probability distributions $P(x)$ and $Q(x)$

$$D(P||Q) = \sum_x P(x) \ln \frac{P(x)}{Q(x)}$$

In general the relative entropy is not symmetric under interchange of the distributions $P(x)$ and $Q(x)$

$$D(P||Q) \neq D(Q||P),$$

Both distributions $P(x)$ and $Q(x)$ are defined over the same alphabet A_x

The measure of information resulting from the relative entropy formula is lower than the one resulting from original Shannon's formula. As the formula can be reversed one can get the measure of world partitioning and consequently a measure of information content embedded in a certain metaphysical hypothesis. This, however requires the existence of clean, separate metaphysical pictures of the world, which does not seem to be the case in any period of history of philosophy. What is more, one can hardly expect such situation in the future.

Therefore one should note that the relative entropy formula does not, and cannot, contain the complete probability. In order to use the latter we would have to be in possession of the complete knowledge of the world. The ability to possess such a knowledge is a myth present in philosophy since Aristotle. Mythical character of this assumption results from at least two premises:

the world seems open

our knowledge does not partition the world uniquely

Here we would like to stress that standard concept of cognition based upon the assumption that a cognitive subject is spiritual and "isolated" from the world and information can be considered as spiritual substance only. As long as such assumption remains the base of epistemology no progress is possible.

3. Information and its usefulness

The relative entropy concept does not cover all aspects of information usefulness. Pragmatic approach to information requires also including the actual use of the information. As this is often considered subjective we would like to concentrate on some objective characteristics of information use. When we think of information as a means of subject's fitness to the real world. The thesis, that cognitive subjects are subject to natural selection bridges the gap between abstractive cognition and "physical" reality. The cognitive subject no longer is a "purely spiritual" entity but is treated as a specific subsystem of the world. A specific system because of his/her homeostatic functions, but a subsystem since he/she is connected to the world with multiple feedback relations.

Understanding information should be based upon its realistic function as evolutionary fitness. As evolution theory suggests, we must thus introduce a certain "fitness factor".

The problem seems to be of nonlinear character where methods of complexity analysis would be more appropriate. Nevertheless, continuing the examples

coming from other sciences we suggest some linearization:

$$I = -S \cdot F(\xi) = \left(-\sum_{i=1}^N p_i \log p_i\right) \cdot F(\xi)$$

The factor $F(\xi)$ should show the following features:

tend to zero when fitness of information tends to zero

tend to Shannon formula when fitness of information tends to infinity

be negative when fitness becomes negative

The above requirements can be fulfilled for instance by a formula:

$$F(\xi) = (1 - e^{-\xi})$$

The suggested formula has some advantages:

it reveals the real value of information not subjective one

it covers all interactions of the cognitive subject with his/her environment; no matter what partitioning are

it lets for accounting of negative fitness

it shows that control processes can be treated as the information edge.

it gives qualitative factor ready for simulation of cognitive processes

Apparently the ability to include both concepts: relative entropy and evolutionary "fitness factor" should give a means to estimate "objective" assessment of metaphysics. It would also allow to treat such assessment dynamically. This would be especially beneficial for describing the culture flow.

4. Conclusion – why "objective"?

For centuries philosophy have taught us how to search the most objective and the most certain knowledge. On the other side information theory gives us a means to measure not what we know about the world but what we can expect i.e. what we do not know in such strong meaning of this word as we are used to assign to logic.

Our considerations in this topic lead to the following conclusions:

it is impossible to construct a homomorphism between the knowledge and the world: first - information comes to a subject dynamically, second – the environment (the world) changes, last but not least – any attempt to estimate the complete probability must fail because the classes established by different metaphysics are not separate – thus correspondence concept of truth must be abandoned

the latter does not mean that any subject can construct the world's image freely (which is often called relativization of knowledge)

our knowledge – or as we would rather say: information about the real world – is not certain but rather probable but still useful because of its influence on our fitness to the world

the objective does not necessarily mean anti-relative – as a subsystem of the Universe trying to learn its structure we are both cognitive subjects and objects within this Universe – this has several consequences: a need for "new language of science", which would reflect the complexity of systems examined, based on *Principia Cybernetica* rather than *Principia Mathematica*, a need for revision of classical philosophical divisions such as *the spiritual* and *the material*, *the mental* and *the mechanical*,

there is a need for changing the objectives of science: the aim of science is not finding the truth about the universe but rather deciding which of the theoretical pictures of the world is better fitted to reality.

References

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