

# Ontological Relativity reconsidered: Quine on Löwenheim-Skolem, Davidson on Quine

Herbert Hrachovec

Quine's thinking shares important traits with postmodernism. Robert J. Fogelin has recently contrasted an austere side of his philosophy to an "open or soft side", most famously exhibited by claims about the indeterminacy of translation and the inscrutability of reference. Toughness is pitted against openness in an "apparent reconciliation of seemingly competing viewpoints" which, according to Fogelin, "gives Quine's position much of its attractiveness and resilience." [1, 544] It is because of his characteristically rigorous anti-positivism, one might speculate, that many of Quine's fellow analytic philosophers allow him to get away with the doctrine of "ontological relativity" which is anathema when proclaimed by French theorists.<sup>1</sup>

In view of the dramatic contrast between speculative thought dressed in fashionable jargon and Quine's sober accounts of the scientific *status quo* it might seem frivolous even to suggest that his work exhibits a postmodern touch. The present paper will, nevertheless, focus on Quine's usage of the Löwenheim-Skolem theorems as a prominent example of ontological relativity and will attempt to show that Quine's treatment is unattractive to philosophers of mathematics and – more generally – untenable within the very methodology arising from Quine's basic approach. After examining the doctrine of relativity as applied to the Löwenheim-Skolem result two recent reflections on its theoretical impact will be discussed. Those contributions make a strong case against the kind of attitude exemplified by Quine, but they do not address the general issue of how to deal with relativism. Donald Davidson has provided a remarkably attractive strategy to counter relativistic moves. Stripping away postmodernist

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<sup>1</sup> There is indeed a disturbing fuzziness in quotes like the following one picked at random from the writings of Felix Guattari: "The relativity of points of view of space, time and energy do not, for all that, absorb the real into the dream. The category of Time dissolves into cosmological reflections on the Big Bang even as the category of irreversibility is affirmed." [2, 52]

Compare this to one of Quine's remarks on space-time: "Maintaining the even tenor of our ways, we can leave the time dimension independent of the spatial ones as we always have. But we do well still to keep it firmly alongside, treating the world and its denizens tenselessly as fourdimensional, simply for logical clarity and quite apart from relativity." [3, 199]

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pretensions this strategy challenges the peculiar Quinean mix of methodological austerity and anti-foundationalism.

## Quine on Skolem

Quine's philosophical approach to the Löwenheim-Skolem theorems is based upon his famous epistemological scenario of radical interpretation. One quick way to put the general idea is that there are no independent "meanings" that could guide anyone in trying to make sense of initially incomprehensible utterances. Sensory stimulations, processed by our conceptual apparatus, is all we have at our disposal. We are, consequently, not entitled to assume ontologically autonomous features of the world. Conceptual processing of stimulation enshrouds people within cognitive restrictions that force them to suspend judgement on ultimate facts. A naturalistic approach – to jump right into Quine's argument in "Ontological Relativity" – does not provide the means to decide whether the sentence "Grass is green" presumes the existence of a certain colour or of an instantiated property.

The problem is that OS tension cannot distinguish between abstract singular and concrete general terms. Pointing at grass and uttering the word "green" may just as well be taken to refer to a quality of grass or to the property of green exemplified by grass. The latter is, as Quine explains, a case of deferred OS tension. "It occurs when we point at the gauge, and not the gasoline, to show that there is gasoline." [4, 40] Quine's next remark takes this kind of argument straight into the philosophy of logic. "Another such example is afforded by the Gödel numbering of expressions." [4, loc.cit.] The gauge is to gasoline as a number to some letter under a Gödel mapping. The indeterminacy of reference affects talk about worldly objects as well as talk about symbols. Quine switches easily from his empiristic scenario to reflections on set-theoretical constructs.

Always, if the structure is there, applications will fall into place. As paradigm it is perhaps sufficient to recall again this reflection on expressions and Gödel numbers: that even the pointing out of an inscription is no final evidence that our talk is of expressions and not of Gödel numbers. We can always plead deferred ostension." [4, 44]

The Löwenheim-Skolem theorems fit nicely into this account.

Given an uninterpreted formal system that can be provided with a model in an uncountable domain this very same system has a countable model. We cannot, by examining the logical pattern manifest in a set of sentences, decide whether those sentences refer to the full array of real numbers or some appropriately manipulated countable subset. The so-called Skolem paradox arises if one holds a traditional semantic theory according to which a particular *signum* rigidly refers to its *designatum*. Quine's "principle of relativity" resolves the quandary.

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What makes sense is to say not what the objects of a theory are, absolutely speaking, but how one theory of objects is interpretable and reinterpretable in another. [4, 50]

This is because the theory forms at our disposal do not – auto-magically as it were – pick out their intended models. Investigating their semantics *we* have to provide such models. “Paraphrase in some antecedently familiar vocabulary, then, is our only resource; and such is ontological relativity.” [4, 54]. Relativism proves its worth in the Löwenheim-Skolem case. Talk about *the* reference of terms of a theory is devoid of meaning, unless relativised to a framing theory. [4, 60] The message seems clear (and clearly favourable to postmodernism): one cannot count on any basic tool-set to describe the world. Assertions always depend on the circumstances of language use.

## Uneasy Dialogues

In his paper quoted at the outset Robert J. Fogelin points to an inherent tension in Quinean methodology. A thoroughly pragmatic naturalist will hardly be impressed by the contrived cases of indeterminacy constructed by Quine. She will, in all likelihood, assume that translation manuals tend to converge and refuse to worry about esoteric ontological principles.

A surprising feature of many of Quine’s arguments is that they employ something very like Cartesian skeptical scenarios – though applied to issues concerning meaning and reference rather than to knowledge. [1, 563]

The force of Quine’s attack on well-established meanings depends on highly artificial moves to shatter our common confidence in the working of language. It is, for example, difficult to see why a naturalist should be worried by cosmic counterparts and proxy functions.<sup>2</sup> Such reservations are borne out by two recent discussions of the philosophical impact of the Löwenheim-Skolem result, both of which employ the rhetorical device of fictional (meta-mathematical) dialogues.

It is Quine himself who prepared the ground for using dialogues as a philosophical ploy. Radical interpretation is just a fancy name for trying to understand complete strangers. Clearly, this is a limiting case in everyday linguistic practice and we might relax those radical constraints to enquire into the conditions of partial dialogical incomprehension. A clash of meta-mathematical doctrines is a case in point. Since there is a lot of common ground among the participants of such discussions the original gavagai scenario is ill suited to model the situation triggered by the Löwenheim-Skolem proof. Still, it makes good Quinean

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<sup>2</sup> The idea of cosmic counterparts is that a speaker could refer to *everything but* a particular state of affairs using a term conventionally referring to this state. Proxy functions are systematic, truth-value preserving permutations of a given interpretation.

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sense to construct an idealised encounter of two language communities that are at odds on how to understand each other's use of certain terms, i.e. "number" or "set". Stuart Shapiro imagines the following controversy between two meta-mathematical "tribes".

A guy names "Second" introduces standard second order semantics, the values of the property variables of his syntax ranging over all sets of numbers. He proves that, given this interpretation of the underlying language, the real numbers are uncountable. His contrahent, called "First", balks at the phrase "all *sets* of numbers". She does not treat its meaning as an obvious extension of the agreed upon notion of "all numbers" and asks for clarification. Second complies, producing an axiomatized version of his initial, informal semantical account. But the formal system he proposes is itself in need of interpretation and so the dialogical mismatch repeats itself. Whereas Second regards his formal construction as an explication of his semantic meta-theory for second order logic, First takes it to be just another first order theory, open to a number of competing interpretations. The meaning of "uncountable" in particular, remains controversial. One party to the conflict takes it to be fixed by ordinary mathematical practice, possibly supplemented by explanatory formalisation. But once such formal constructions are put forward, the opposing party is free to pick alternatives, i.e. an interpretation satisfying the predicate "uncountable" in a countable domain. This is precisely Quine's point: What an expression refers to depends on the framing theory used to discuss its occurrences in the object language.

If both sides persist they will systematically talk past each other, dramatically exemplifying the threat of all-pervasive mutual incomprehension between language users. But this is obviously not the way such difficulties are ordinarily worked out. Shapiro, at this juncture, has an excellent point referring to Wittgensteinian rule following. Mere words can never force an unique interpretation upon us. Quine's rejection of the "museum mythos" echoes this part of Wittgenstein's analysis. But this analysis does not rest content with assigning different meanings to given expressions, depending on framing theories and thus inviting relativism. Wittgenstein is, at the same time, insisting on the legitimacy of given language uses. Discursive practice, it is true, can be interrupted by misunderstanding at almost every point. Speakers, nevertheless, usually manage to resolve the ensuing puzzlement. If unsuccessful, they keep using their respective language, without solving the conflict, but not without right. Shapiro joins Putnam in stressing the fact that

There is an important aspect of mathematical practice – the *use* of mathematical discourse – that is not captured by first-order languages . . . [5, 256]

Any attempts to brush away an ordinary mathematician's untroubled understanding of "number" or "set" can, in Putnam's lucky phrase, "only have crazy solutions" [6, 24]. Stuart Shapiro:

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My thesis here is that, for better or worse (well, for better), the attitude underlying the practice of mathematicians is that skepticism is false. [5, 260]

Informal language can give rise to uncertainty and conflict, requiring clarification in a more formal mode. But there is no appeal to anything but informal understanding when repeated attempts at clarification fail.

Shapiro's attitude is to reject relativism in favour of existing use. Some questions are, inevitably, begged before so-called "unintended interpretations" of certain phrases can be proposed. This is, however, an awkward way to put the issue. It seems to re-establish a skeptical outlook on top of the attempt to put it at rest. Isn't the relativist saying precisely that existing uses, i.e. framing theories, are always begging questions? A refinement of the argument is called for. It will be given in two steps, the first one offering a closer look at question-begging within the philosophy of mathematics. This is, as has already become evident, but a special case of the general problem of relativism which will be addressed in the concluding section.

Paul Benacerraf's essay "What Mathematical Truth Could Not Be - I" is an extended reflection on, as he puts it, extracting "philosophical juice from metamathematical results". [7, 36] His opening parable sets the stage. Lapidus, the matchmaker, has considerable difficulties in convincing the Cohens that he has got the right girl for their son. Beauty and wealth cannot overcome their opposition to a gentile woman. It is only when he reveals her identity that he secures their approval: they cannot resist the appeal of their son marrying Princess Margaret of England. Having fixed the deal Lapidus mutters to himself: "Whew. Well, that's *half* the job. [7, 34] Cheerfully exploiting this moral tale Benacerraf traces the impact of the "Princess Margaret premiss" in philosophical accounts of the limitative theorems in meta-mathematics. Impressing the audience with a formal proof is the easy part, like overwhelming the Cohens with a Royal name. Such proofs are not, however, in themselves philosophical moves. In order to acquire their intended importance they have to be embedded in philosophical argument. How this is to be done is not fixed by the formal proof, just as the powerful appeal of a name is quite different from the behaviour of the name's bearer. Applied to the Löwenheim-Skolem debate Benacerraf's challenge is this: We should not be unduly impressed by the mere possibility of reading "uncountable" as countable.

Imagine, again, a dialogue involving Georgia, a student, and her teaching assistant, a Skolemite. Her use of Cantorian techniques is challenged by her tutor who claims that "she cannot write a set of first order axioms that are satisfiable in an *uncountable* domain but not in any *countable* domain." [7, 61] Georgina tries in vain to give a first-order characterisation of her Cantorian understanding of the term under discussion. All her attempts to capture her pre-formal intuitions about undenumerability are squashed by the Skolemite who claims to be able to come up with a countable model for any axiom system Georgina might propose to formalise the semantic assumptions underlying her employment of set theory. The Skolemite's rules "will always permit interpretations

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that place only demonstrably finite and countable sets in its (i.e. the predicate's 'uncountable') extension." [7, 65]. Relativism looms again.

Like Stuart Shapiro Benacerraf rejects the Skolemite by insisting on mathematical practice, but his argument is more sophisticated. Cantorian persuasions are not justified because, as Shapiro puts it, "in the present situation, it looks like questions must be begged". [5, 247] Drawing attention to the "Princess Margaret Premiss" is an attack on the relativist's standing as partner in a dialogue. The point is that it does not suffice to utter the words "Princess Margaret" or "uncountable" to be entitled to be taken seriously. While Georgina cannot prohibit strange uses of the term "uncountable" by the Skolemite, *she* is the unconditional authority on how *her* language is to work and hence on how to handle problem cases. She might be forced to express her beliefs in an idiom that can be cleverly re-interpreted in order to subvert her cognitive arrangement. But this does not indicate that her words lack definite meaning. An axiomatized system is an empty shell and cannot serve to overthrow a mathematical practice that happens to be richer than its expressive capacities. Benacerraf puts his point in a nutshell:

The best comparison I can suggest is with trying to persuade someone who understands ordinary counting that she really has no concept of 17 and 18 as distinct and intelligible cardinalities because both get translated as 'more' into a language that has only '1', '2', '3', ... '13', 'more' (and no more) as the available choices." [7, 72]

The use of "more" in this scenario is no match to the use of "18". It's not a case of unavoidable question begging but rather of the opponents failure to achieve common ground.. Relativism falls flat without something the opposing views are opposing each other *about*. Benacerraf's defence of the operations of a meta-language against attempts to fiddle with some of its predicates can be generalised.

## Davidson on Quine

The preceding summary of Paul Benacerraf's contribution contains a fair amount of Davidson. His criticism of Quine on ontological relativity is reminiscent of Hegel's objections to Kant's *Ding an sich*. Some factor X is supposed to play an essential role in providing outside support for theoretical projections, even though we can, within Kant's epistemological design, never grasp any of its qualities. Hegel regards this as a strange case of theoretical self-mutilation and overrides Kant's restrictions within his own peculiar holism. Donald Davidson has, likewise, found a strange impediment to holism in Quine, namely his dualism of conceptual form and empirical content. It seems that Quine, in order to call himself an empiricist, has to assume the *extra-theoretical* existence of something that comes to be mediated conceptually. But language, Davidson replies, cannot sensibly refer to an extra-linguistic given other than by the use

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of language. This move seems, at a first glance, to dismantle the accountability of theories towards “the real world”. But the impression is mistaken. Davidson’s move is, in fact, the cornerstone of a strong anti-relativistic position.

In rough outline the argument is this. Relativism plays off multiple, mutually exclusive, world views against each other. This strategy, however, can only work, if those views share a common point of reference, namely “the world”, otherwise they cannot be said to be mutually exclusive. Just listing different language games is an activity outside the scope of the present problem. But now, assuming unsurmountable conflict, “the world” can be seen as a dubious *Ding an sich*. Its function is to serve as a common denominator for theoretical approaches that are, at the same time, thought to be incompatible to each other. Davidson proposes to cut through this conceptual tangle and take one of two positions. Either there *is* substantial common ground between competing world views or so-called “competing world views” are mis-named since they do not meet the minimum condition for there *to be* a competition, rather than, for example, a struggle for survival of the fittest. In both cases the threat of epistemological relativism disappears. This is but a sketch of the implications of doing away with the form-content-distinction and cannot be further pursued here. One of Davidson’s explanatory constructions will have to suffice to link these considerations to the Löwenheim-Skolem case.

A speaker, call him A, uses an expression, i.e. “Wilt”. B tries to understand A and in doing so arrives at two conjectures.[8, 234] According to the first one, “Wilt” refers to a person named Wilt; the alternative hypothesis correlates the term with Wilt’s shadow. Both options seem feasible in view of the empirical data at a given time. Should we infer that A’s linguistic utterance lacks reference or that B is confused about persons and shadows? Of course not. One level up a speaker C can easily disambiguate B’s claim that “By uttering ‘Wilt’ A means either Wilt or Wilt’s shadow.” The term “Wilt” works differently according to the competing hypotheses which B, tentatively, ascribes to A. From C’s point of view those hypotheses are different attempts, by B, to systematise A’s linguistic activity. B’s use of “Wilt” and “Wilt’s shadow“ are entirely unaffected by B’s hesitation regarding her semantic task. B must, in fact, be able to refer to persons and shadows making up the environment she shares with A, if she wants to get *any* hold on A’s pronouncements. Her inability to uniquely determine the reference of a term in the object language does not imply that her own use of the term, and in particular of terms translating the term under investigation into her own language, is indeterminate. B can perfectly well distinguish between Wilt and Wilt’s shadow, even though she is unclear about whether A means one or the other by his use of a given expression.

The situation is suggestively similar to Shapiro’s and Benacerraf’s dialogues. Bracket the physical connotations and substitute “uncountable” for “Wilt”. A certain term can then be observed to systematically occur in an array of patterns of language-like inscriptions. What’s his meaning? A Davidsonian has an attractive answer which comes in three steps, corresponding to the hermeneutical setup just rehearsed. At level A “uncountable” has no meaning, since meanings are theoretical concepts developed in a meta-language. In the present

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case speaker B ascribes meanings to expressions of an uninterpreted calculus, Benacerraf's "empty shell". There is just one way she can do this, namely by employing her language capacities, finding an appropriate gloss for "uncountable" within her idiom. As it happens, that might not be a straightforward affair. Refer to Shapiro and Benacerraf to fill in the details. But notice that – given this setting – the appeal of relativistic doubts simply dissolves. It is up to B to decide upon her theoretical investment, for instance to regard "uncountable" as a predicate satisfied by what she cannot but call the reals. This can be perspicuously expressed at level C. Several hypotheses are at B's disposal to understand the role played by the term in the object language. Unless B wants to get completely confused she is well advised to distinguish between different interpretations she might want to impose upon a given linguistic phenomenon and their possible indeterminacy as far as some hermeneutical setting is concerned. Observe the contribution "uncountable" makes to the object language, check alternative interpretations, take your choice. Relativism is replaced by different commitments to radical translation.

Davidson's dictum is entirely convincing: "Truth is relative to an object language, but not to a metalanguage." [8, 233] In order to get a hold on truth and meaning we need the asymmetrical setup of levels A and B.<sup>3</sup> The only way for B to turn into a relativist is to become overly impressed by her own abilities to develop allegedly incommensurable interpretational schemes, disregarding the fact that those schemes are of her own making. Recall Davidson's general maxim: For world views to diverge they must be built on shared assumptions. If they have nothing in common they do not merit to be called different views *of the world*. As Wittgenstein famously put it: If lions could speak, we could not understand them [9, 536]

All of this is in direct contradiction to Quine's position and would, at least, demand another paper for discussion. This one concludes with a quick hit against postmodernism. Arguments in analytical philosophy, even those that propose ontological relativity, bear systematic scrutiny. A close look at Quine's paper reveals him to be somewhat careless in his use of "theory". The "empty shell" we have been talking about is, in his parlance, a "theory form" (Quine 1969, 53) and he observes, correctly, that we cannot, given such a form, but guess at the intended model. But he goes on to argue:

It is thus meaningless within the theory to say which of the various possible models of our theory form is our real or intended model. [4, 54]

This passage is the analytic equivalent of the vernacular claim that we are caught within the confines of our language, unable to attain a stance transcending this contingency. The sentiment is certainly widespread. It is, however,

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<sup>3</sup> Such constructions can, of course, be iterated, turning the utterances of speaker B into an object language. Such a ploy *does not* relativise B's treatment of A. It just re-iterates the only strategy available to deal with initially meaningless signs, i.e. to consider alternative interpretations and to eventually apply one or the other.



unfounded as can be seen by resolving a striking indeterminacy in Quine's use of the term "theory" in the preceding passage. As long as semantical issues remain undecided "within the theory" means: tentatively translating a theory form. Such an enterprise does not suggest relativity. In order to arrive at ontological relativity "within the theory" must *also* mean "as claimed by a certain theory". The core of relativism is this strategic fusion of non-commitment and commitment into an apparently coherent position. Quine's doctrine collapses Davidson's levels. He fails to distinguish between activities that try to make sense of strange data and the second level awareness that there is, in general, no unique solution to this kind of enterprise.

One can admit of doubts about the reference of component expressions of theory forms. And one might well be able to overthrow many of the theories built upon interpretations of theory forms. Here is one thing one cannot do: Make sense of puzzling phenomena while abstaining from established language use. There are no "free" doubts, just as there is no free dinner. Even though the proposition sounds old-fashioned, its truth seems unassailable: substantive conflict, as distinct from opposing slogans, demand considerable common effort.

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