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MARC ARTZROUNI – JOHN KOMLOS

POPULATION GROWTH THROUGH HISTORY AND THE ESCAPE FROM THE MALTHUSIAN TRAP: A HOMEOSTATIC SIMULATION MODEL

1. INTRODUCTION

The study of patterns of population growth through history has provided fertile research grounds for demographers, historians, and anthropologists. Going sufficiently far back in time, such investigations are within the realm of historical demography and paleodemography, which resorts to the study of skeletal remains (1), artifacts (2), food remains (3), and habitat space (4) in order to produce estimates of population counts as far back as several million years B.C. (5). For the period 8000 B.C. to 1975 A.D., Durand (6) and Biraben (7) have produced "indifference ranges" (8) that are generally in agreement. Little is known before the Neolithic agricultural revolution (9) (circa 8000 B.C.) except that population grew extremely slowly prior to that era. With a total of about 5-10 million in 8000 B.C., the world's population grew at a rate of approximately 0.5% per decade and reached 700 million at the beginning of the eighteenth century. Thereafter, the growth rate

(1) S.F. Cook, *Prehistoric Demography*, Module #16 (Reading, Mass.: Addison-Wesley, 1972).

(2) J.G.D. Clark, *Starr Carr: A Case Study in Bioarcheology*, Module #10 (Reading, Mass.: Addison-Wesley, 1972).

(3) F.A. Hassan, *Demographic Archeology* (New York: Academic, 1981).

(4) G.A. Johnson, *Local Exchange and Early State Development in Southwestern Iran*, Anthropol. Pap. Mus. Anthropol., Univ. Mich., (1973).

(5) E.S. Deevey, *The Human Population*, Sci.Am., 203 (1960), pp. 195-205. For a review of anthropological methods in paleodemography and an extensive bibliography, see R.M. Schacht, *Estimating Past Population Trends*, Am. Rev. Anthropol., 10 (1981), pp. 119-140.

(6) A.D. Durand, *Historical Estimates of World Population: An Evaluation*, Popul. and Dev. Rev., 3 (1977), pp. 253-296.

(7) J.N. Biraben, *Essai sur l'évolution du nombre des hommes*, Population, 1 (1979), pp. 13-25.

(8) A.D. Durand, loc. cit. in footnote 6, p. 260.

(9) J.N. Biraben, loc. cit. in footnote 7, p. 23.

increased and reached an average level of about 5% per decade for the period 1750 A.D. to 1900 A.D., resulting in a total population of 1.6 billion in 1900. The decennial growth rate has increased since and has reached 20% per decade in the 1970s.

Following Coale (10) we may then divide the demographic history of the world into two periods: a period of slow growth until the eighteenth century followed by a "population explosion" which was subsequently accentuated by the demographic transition (11).

The 10,000 years extending from the Neolithic agricultural revolution to the Industrial Revolution can be thought of as a unity because the economy was overwhelmingly agricultural throughout that period (12). Indeed, North does not see the Industrial Revolution of the late eighteenth century as a radical break with the past: "Instead, ... it was the evolutionary culmination of a series of prior events. The real revolution occurred much later, in the last half of the nineteenth century" (13). If North is correct, and the nine or ten millennia that elapsed between the two economic revolutions do, indeed, constitute a unity in some sense, then one ought to be able to capture the dynamics of economic and population growth from circa 8000 B.C. to 1900 A.D. with a model whose structural equations are invariant with respect to time (14).

Although our purpose is to simulate the dynamics of economic and population growth for the world, we will posit an economic model inspired by the experience of the European economies. This is not a contradiction, first because these economies were, after all, the ones which produced an industrial revolution, and second, because the patterns of European and world population growth were qualitatively similar prior to the eighteenth century.

2. THE MALTHUSIAN ASSUMPTIONS

In this study we propose a Malthusian simulation model in which a homeostatic regulating mechanism captures the struggle between population growth and

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(10) A.J. Coale, *The History of the Human Population*, Sci. Am., 231 (1974), p. 43.

(11) A.J. Coale, loc. cit. in footnote 10, p. 48.

(12) E.L. Jones, *The European Miracle, Environments, Economies, and Geopolitics in the History of Europe and Asia* (Cambridge: Cambridge University Press).

(13) D.C. North, *Structure and Change in Economic History* (New York and London: W.W. Norton and Company, 1981), p. 162. For R. Cameron the Industrial Revolution is a "long, drawn out process, in no sense inevitable, which scarcely deserves the epithet revolutionary" (R. Cameron, *The Industrial Revolution: A Misnomer*, The History Teacher, 15 (1982), p. 383).

(14) R. Cameron, *The Logistics of European Economic Growth: A Note on Historical Periodization*, The Journal of European Economic History, 2 (1973), pp. 145-148. R. Cameron, *Economic History Pure and Applied*, The Journal of Economic History, 36 (1976), pp. 3-27.

society's resource base (15).

Few scholars have generated more controversy and more widely diverging interpretations than Malthus (16). However his exegetes broadly agree on the core of his demographic theory, that is the notion that "population is food controlled" (17). Although by no means original (18), this premise is based on Malthus' assumption that "food is necessary to the existence of man" (19) and that the "power of population is indefinitely greater than the power in the earth to produce subsistence for man" (20). Malthus posited a regulating mechanism, which adjusted the number of people to the food supply. He explains this mechanism in the following way (21):

> We will suppose the means of subsistence in any country just equal to the easy support of its inhabitants. The constant effort towards population, which is found to act even in the most vicious societies, increases the number of people before the means of subsistence are increased. The food, therefore, which before supported eleven million, must now be divided among eleven million and a half. The poor consequently must live much worse, and many of them be reduced to severe distress. [...] During this season of distress, the discouragements to marriage and the difficulty of rearing a family are so great that the progress of population is retarded. In the meantime, the chepness of labour, the plenty of laborours [sic], and the necessity of an increased industry among them, encourage cultivators... till ultimately the means of subsistence may become to the same proportion to the population at the period to which we set out. The situation of the labourer being then again tolerably comfortable, the restraints to population are in some degree loosened; and after a short period, the same retrogade and progressive movements, with respect to happiness, are repeated.
>
> This sort of oscillation will not probably be obvious to common view.

This quote brings us to the heart of the matter: as a result of the "incessant contest" between population growth and the production of the means of sub-

(15) For a discussion of homeostatic equilibria, see S. Cotts Watkins and E. Van de Walle, *Nutrition, Mortality, and Population Size: Malthus' Court of Last Resort,* Journal of Interdisciplinary History, (1983), pp. 205-226.

(16) Essays on Malthus are often accompanied by an entire chapter on mistakes and misundertandings concerning his work or his biography. See for instance Chapter 4 on *Minor Quibbles and Gross Misunderstandings* in the classic *Malthus,* by William Peterson (Cambridge, Mass.: Harvard University Press, 1979). There is even disagreement concerning basic facts of Malthus' biography. For a revealing discussion, see Chapter VIII *(Mistakes about Malthus)* in G. F. McCleary's, *The Malthusian Population Theory* (London: Faber and Faber Limited, 1953), p. 94; K. Smith's, *The Malthusian Controversy* (New York: Octagon Books, 1978) discusses the various interpretations of Malthus' work.

(17) N. Keyfitz, *The Evolution of Malthus's Thought: Malthus as a Demographer* in *Malthus Past and Present,* edited by J. Dupaquier and E. Grebenik (London and New York: Academic Press, 1983), p. 3.

(18) See K. Smith, op. cit. in footnote 16, p. 7.

(19) T.R. Malthus, *On Population,* edited and introduced by Gertrude Himmelfarbe, (New York: The Modern Library, 1960), p. 8.

(20) T.R. Malthus, op. cit. in footnote 19, p. 9.

(21) T.R. Malthus, op. cit. in footnote 19, p. 161.

sistence, the population tends to oscillate in a homeostatic mechanism resulting from the conflict between the population's natural tendency to increase and the limitations imposed by the availability of food.

It is well documented that the fluctuations experienced by the world's population throughout history did not have a regular, cyclical pattern, but were, to a large extent, brought about by randomly determined demographic crises (wars, famines, epidemics, etc.) (22). As McKeown and others have pointed out, the main cause of these fluctuations of the past were mortality crises (23). There are four kinds of crises: subsistence crises, epidemic crises, combined crises (subsistence/epidemic), and finally crises from other causes, which are mainly exogenous (wars, natural or other catastrophes) (24). We believe, with Flinn (25) that combined crises were probably the most frequent manifestations of the Malthusian mechanism described above.

The economic-demographic simulation model we propose postulates a Malthusian regulating mechanism by which the positive checks operate (during subsistence crises only) in the form of randomly determined demographic crises. Demographic and economic simulation models are not new (26), however "The

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(22) M.W. Flinn, *The European Demographic System, 1500-1820* (Baltimore: The Johns Hopkins University Press, 1981), p. 57. More generally, for a thorough discussion on mortality crises see H. Charbonneau and A. Larose (editors), *The Great Mortalities: Methodological Studies of Demographic Crises in the Past* (IUSSP Proceedings; Liège: Ordina Editions, 1979).

(23) T. McKeown, *The Modern Rise of Population* (New York: Academic Press, 1976), p. 5. See also R.D. Lee, *Econometric Studies of Topics in Demographic History* (New York: Arno Press, 1978), p. 16.

(24) S. Sogner, *Nature and Dynamics of Crises (Including Recent Crises in Developing Countries)* in A. Charbonneau and A. Larose, op. cit. in footnote 22, p. 313.

(25) M. Flinn, op. cit. in footnote 22, p. 95.

(26) I. Hodder, *Simulation in Population Studies.* In *Simulation Studies in Archeology*, pp. 59-62, Ed. I. Hodder (Cambridge: Cambridge University Press, 1978). An example of a complex simulation model applied to modern times is provided by D.H. Meadow et al. in *Limits to Growth* (New York: Universe Books, 1972). Alternatives are proposed in H.S.D. Cole et al., *Models of Doom* (New York: Universe Books, 1973). More economic models can be found in H. Leibenstein, *A Theory of Economic-Demographic Development* (Princeton: Princeton University Press, 1954), and R.R. Nelson, *A Theory of the Low Level Equilibrium Trap in the Underdeveloped Economies,* American Economic Review, 46 (1956), pp. 894-908. More recently, a simulation model in which population growth induces technological change was proposed by G. Steinmann, *A Model of the History of Demographic-Economic Growth,* in *Economic Consequences of Population Change in Industrialized Countries,* edited by G. Steinmann, *Studies in Contemporary Economics,* vol. 8 (Berlin and Heidelberg: Springer-Verlag, 1984). A distinction is made (p. 30) between normal death rates and "supplementary 'epidemic' death rates". Steinmann suggests that normal death rates depend on per capita income and technological knowledge. His model depicts the ability of the population to avert the Malthusian menace. Steinmann's model tests the effect of a single epidemic on the long-run evolution of population; his model shows that under plausible assumptions the population can, indeed, recover from a crisis. See also J. Simon, *The Present Value of Population Growth in the Western World,* Population Studies, 37 (1983), pp. 5-21.

systematic study of population crises has scarcely been attempted previously" (27), and a Monte Carlo Malthusian simulation model which captures the major stylized facts of economic and population growth over a period of then millennia is original.

We believe that the "incessant contest" between population growth and the production of food supplies characterized the period extending from the Neolithic to the Industrial Revolution. Indeed, crises followed by periods of population decline during which the nutritional status of the population improved gave rise to fluctuations which testify to the continued existence of the "Malthusian trap" (28): population could not grow beyond its carrying capacity for long, and when it did, the resulting overshoot was followed by a "crash" (29) (i.e. the positive checks such as diseases, famines, wars, etc.).

We divide the economy into two sectors: a subsistence sector and a sector producing all other goods, including capital (hereafter called the capital-producing sector). In our model the "escape" from the Malthusian trap depends on a sufficient accumulation of capital and a buildup of the population in the capital-producing sector. The production of food is then sufficient to sustain a population that grows unhindered. The resulting demographic explosion that paralleled the Industrial Revolution is therefore not arbitrarily built into the model but is the natural outcome of a long-run demographic and economic process.

3. THE MODEL SPECIFICATION

The time variable is t and $t = 0$ corresponds to the beginning of the period under consideration (8000 B.C.). The unit of time is the decade, and the endpoint is $t = 990$, which is 1900 A.D. The outputs $Q_A(t)$ and $Q_I(t)$ of the subsistence sector and the capital-producing sector at period t are described by equations of the Cobb-Douglass type:

$$Q_I(t) = C_1\, K(t)^{\alpha_1}\, P_I(t)^{\alpha_2} \qquad [1]$$

$$Q_A(t) = C_2\, K(t)^{\beta_1}\, P_A(t)^{\beta_2} \qquad [2]$$

where

1. $K(t)$, the aggregate capital stock at period t, is accumulated through the following process

(27) T.H. Hollingsworth, *An Introduction to Population Crises*, in H. Charbonneau and A. Larose, op. cit. in footnote 22, p. 17.

(28) E. Van de Walle, *Malthus Today*, in Dupaquier and E. Grebenik, op. cit. in footnote 17, p. 239.

(29) R.Schacht, loc. cit. in footnote 5, p. 134.

$$K(t{+}1) = K(t) + \lambda(t)\, Q_I(t) \qquad [3]$$

where $\lambda(t)$ is the savings rate prevailing at period t.

2. $P_I(t)$ and $P_A(t)$ are the populations of the two sectors at time t and may be thought of as the urban and rural populations; $P(t) = P_I(t) + P_A(t)$ is the total population.
3. $C_1, C_2, \alpha_1, \alpha_2, \beta_1, \beta_2$ are positive constants with $\alpha_1 + \alpha_2 = \beta_1 + \beta_2 = 1$.

Several aspects of these equations need further explication. Land is not explicitly included in eq. [2], because the land surface of the earth has remained almost constant during the period under consideration, and improvements of land as well as discoveries of new land are conceptualized as part of the capital stock.

Capital in this model includes, in a cumulative fashion, not only human and psysical capital but also knowledge broadly conceived. This includes the creation of institutions conducive to efficient production because they improve the efficiency of markets (30). Human capital should, in addition to education, include such gains made by the human species as the increased resistance to disease slowly acquired through time (31).

An idiosyncratic aspect of the model is that the capital stock enters both production functions. Thus we avoid having to assume two different rates of capital accumulation, which would not add much conceptually to the model. In any event, the aggregate capital stock, as broadly conceived as it is in this model, can enter the production functions of both sectors without loss of generality.

We now turn our attention to the demographic process, which we will model in accordance with the Malthusian hypothesis discussed earlier (i.e., the "incessant contest" between population growth and available resources). We will formulate the dynamics of the world's population in the simplest possible manner by considering the aggregate relationship

$$P(t+1) = (1 + r(t+1)) \cdot P(t) \qquad [4]$$

where $r(t+1)$ is the decennial growth rate at period $t+1$. We indicated earlier that population growth in the past was to a large extent determined by the fluctuations of mortality; on the other hand, fertility remained, in historical terms, relatively stable. Therefore the variations of $r(t)$ largely reflect the evolution of mortality rates.

We next define the per capita output of the subsistence sector as

$$S(t) = Q_A(t)/P(t) = C_2\, K(t)^{\beta_1}\, P_A(t)^{\beta_2}/P(t) \qquad [5]$$

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(30) D.C. North and T. Roberts, ***The Rise of the Western World*** (Cambridge: Cambridge University Press, 1973).

(31) W.H. McNeill, ***Plagues and Peoples*** (New York: Anchor Press, 1977); T.W. Schultz, ***Capital Formation by Education***, Journal of Political Economy, 68 (1960), pp. 571-583.

We now express the Malthusian assumptions by postulating that $r(t+1)$ is equal to a constant r^* as long as $S(t)$ remains above some critical value S^*. Hence the population grows unhindered when subsistence is larger than some biologically determined minimum (for instance $S(t)$ can be thought of as the number of calories available per capita; S^* is then the minimum level deemed sufficient to sustain a growing population). The quantity r^* is also the "escape rate" because it is the growth rate experienced by the population when it emancipates itself from the Malthusian trap.

If $S(t)$ drops below S^*, the population is in a Malthusian crisis. The population is then susceptible, in a random fashion, to lowered growth rates, which can become negative and take on disastrous proportions. More precisely, we have

$$r(t+1) = r(t) - e(t) \qquad [6]$$

where $e(t)$ is a nonnegative random variable generated by a Monte Carlo type simulation that will be described later in the text.

In sum, we have

$$S(t) \geqslant S^* \Rightarrow r(t+1) = r^* \qquad [7a]$$

$$S(t) < S^* \Rightarrow r(t+1) = r(t) - e(t) \qquad [7b]$$

In effect, our model is a mixed model, which is deterministic (eq. [7a]) when the population is not in crisis and becomes stochastic (eq. [7b]) when the population is in a crisis (however, the overall process is stochastic). It is now clear that we are modeling the reaction of the population to a subsistence crisis, although the population was clearly subject to epidemic crises even when the per capita agricultural output was above the critical level. We believe that this stylization of the Malthusian mechanism is not unreasonable given the intimate interdependence of nutritional status and susceptibility to disease (32).

Finally we need to specify the allocation of the total population between the two sectors of the economy. We will consider two different situations, depending on whether the population is in a crisis ($S(t) < S^*$) or not ($S(t) \geqslant S^*$). In the latter case we postulate that both sectors of the population grow at the same rate r^*. When the population is in a Malthusian crisis, we recall eq. [6] and observe that the growth rate $r(t)$ decreases with t. As long as $r(t)$ is still larger than 0 (i.e., $e(t)$ is not too large and we are still at the early stages of the crisis) we postulate that $P_A(t)$ remains stationary, and that the capital-producing sector absorbs the excess population. This can be thought of as migratory movement of the rural population driven away by the subsistence crisis. Indeed, until the nineteenth century, cities

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(32) For a discussion of the relationship between nutritional status and disease, see McKeown, op. cit. in footnote 23, chapter 7.

were known to be "population sumps" (33) which often drained the countryside of young adults (34). As the crisis persists, the growth rate $r(t)$ eventually becomes negative and we postulate that the capital-producing sector absorbs all the decrease in the total population. This is in keeping with the notion that population density in urban areas, once it reaches a critical level after several years in a Malthusian crisis, leaves the cities highly susceptible to catastrophic epidemics (the Black Death in fourteenth-century Europe, and more generally endemic outbusts of bubonic plague, cholera, smallpox, etc. are well-documented examples of such epidemics in early modern Europe) (35). Another example is ancient Egyptian cities, which were known to have been totally deserted following an epidemic (36). Hence, the declining population of the urban areas at the advanced stage of a crisis is conceptualized primarily as an excess level of mortality brought about by a long period of increasing population, which leaves the capital-producing sector vulnerable to diseases (this vulnerability being of course exacerbated by the drawn-out subsistence crisis).

In sum, when the population is in a Malthusian crisis, we have

$$P_A(t) = P_A(t-1) \qquad [8]$$

$$P_I(t) = P_I(t-1) + (P(t) - P(t-1)) \qquad [9]$$

Equations [8] and [9] express our assumption that the capital-producing sector absorbs any change in the population. If the population is declining and $P_I(t)$ becomes negative in eq. [9], then $P_I(t)$ is reset to 0 and $P_A(t)$ is equal to $P(t)$. This allocation of the population between the two sectors may not be fully realistic, however we feel that too little is known on the subject, and that the stylization proposed here is a plausible approximation of the past.

We will now specify the random mechanism that will generate the perturbations $e(t)$ of eq. [6] when the population is in a Malthusian crisis (i.e., $S(t) < S^*$). A first step will determine whether there is a perturbation $(e(t) > 0)$ or not $(e(t) = 0)$. Indeed a demographic reaction to a subsistence crisis is in no way ineluctable and we feel that the probability of $e(t) = 0$ should be strictly positive.

We first define the quantity

$$U(t) = \frac{1}{1 + 4e^{\cdot 400(S^* \cdot S(t))}} \qquad [10]$$

(33) W.H. McNeill, *Human Migration: Historical Overview*, in *Human Migration*, W.H. McNeill and R.S. Adams (editors) (Bloomington & London: Indiana University Press, 1978), p. 6.

(34) W.H. McNeill, *Historical patterns of migration*, Current Anthropology, (1979), p. 96.

(35) M. Flinn, op. cit. in footnote 22, p. 55.

(36) See W.H. McNeill, loc. cit. in footnote 34, p. 101.

and we observe that $U(t)$ is smaller than 1 and approaches 1 as $S^*-S(t)$ becomes larger. We now draw a random number $u(t)$ uniformly distributed between 0 and 1. If $u(t) > U(t)$, then $e(t) = 0$, and if $u(t) \leqslant U(t)$, then $e(t)$ will be positive with a value determined by a second random procedure. Hence the probability of a crisis increases with the severity of the crisis (i.e., $S^*-S(t)$).

If the above procedure resulted in a strictly positive perturbation $e(t)$, we draw a random number $v(t)$ from a truncated (the positive side) normal distribution with mean 0 and variance 1. We then postulate that $e(t)$ is proportional to $v(t)\ U(t)$. In addition $e(t)$ increases with $y(t)$, the number of periods (decades) the population has been in a crisis; $e(t)$ is given by

$$e(t) = 0.1 * v(t) U(t)\ (1 + e^{0.15(y(t)-5)}) \qquad [11]$$

As a consequence of this process, population during a crisis is susceptible to small fluctuations as well as to the catastrophic collapses which have characterized population of the past.

Finally the exponents α_1, α_2, β_1, β_2 are set equal to 0.5 and the constants C_1 and C_2 are equal to 1.300 and 0.213. The value S^* was fixed at the arbitrary value of 0.08 in conjunction with an initial value $K(0)$ of 0.4. These values need not be realistic because we do not claim to plausibly simulate the values of those economic variables about which very little is known. We do however, purport to approximate a realistic mechanism and a numerically plausible pattern of population growth for the period 8000 B.C. to 1900 A.D. Hence we chose an intial value $P(0)$ of six million for the population of the world in 8000 B.C. and no population in the capital-producing sector (i.e., $P_I(0) = 0$). The growth rate r^* was set equal to 0.05 per decade; we have chosen this value because it is the escape rate that prevailed at the beginning of the Industrial Revolution.

For the savings rate $\lambda(t)$ we have chosen a function of the form

$$\lambda(t) = 0.01 + 1.778 * 10^{-26}\ e^{0.05756t} \qquad [12]$$

which corresponds to values of $\lambda(t)$ growing slowly from one percent per decade in 8000 B.C. to four percent in 1700 A.D. and eleven percent in 1900 A.D.

The model is now completely specified and the simulation was run a number of times. Our expectation was that the buildup of capital stock and population would lead to the escape from the Malthusian trap. In Figure 1 we have plotted a typical trajectory for the population of the world and the 95% confidence interval resulting from one hundred runs of the simulation (we verified that for any given decade the resulting estimates of the population were approximately normally distributed). In order to better visualize the escape we have plotted the population counts only for the period 1500 B.C. to 1900 A.D.; from 8000 B.C. to 1500 B.C. the growth is very slow. This pattern conforms well to our knowledge of population growth during that period; indeed for every run of the simulation the escape occurs

FIG. 1 *Simulation of population growth*
(A trajectory and 95% confidence interval)

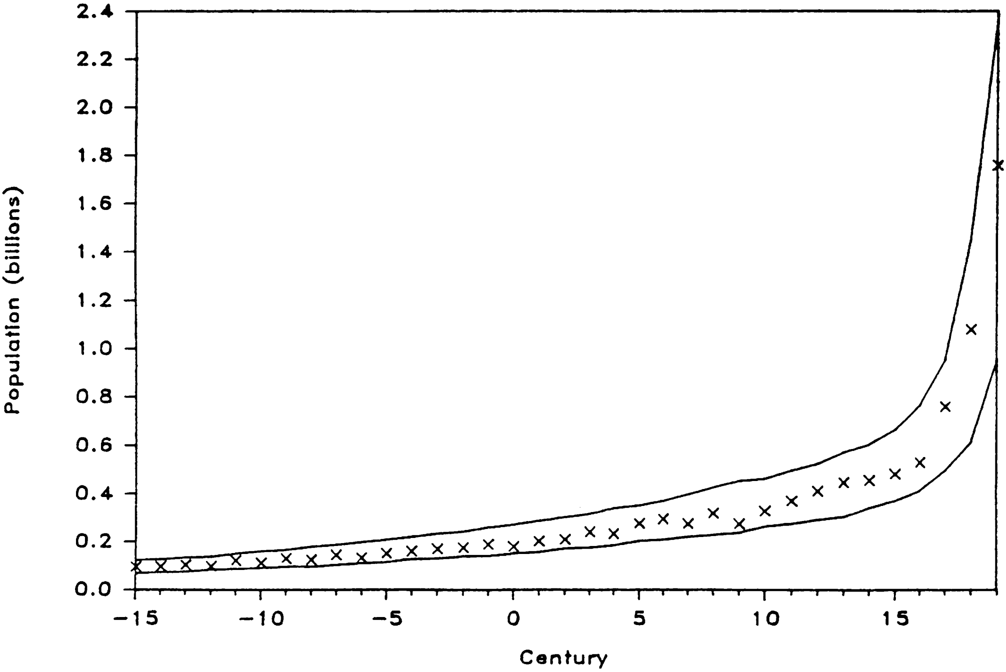
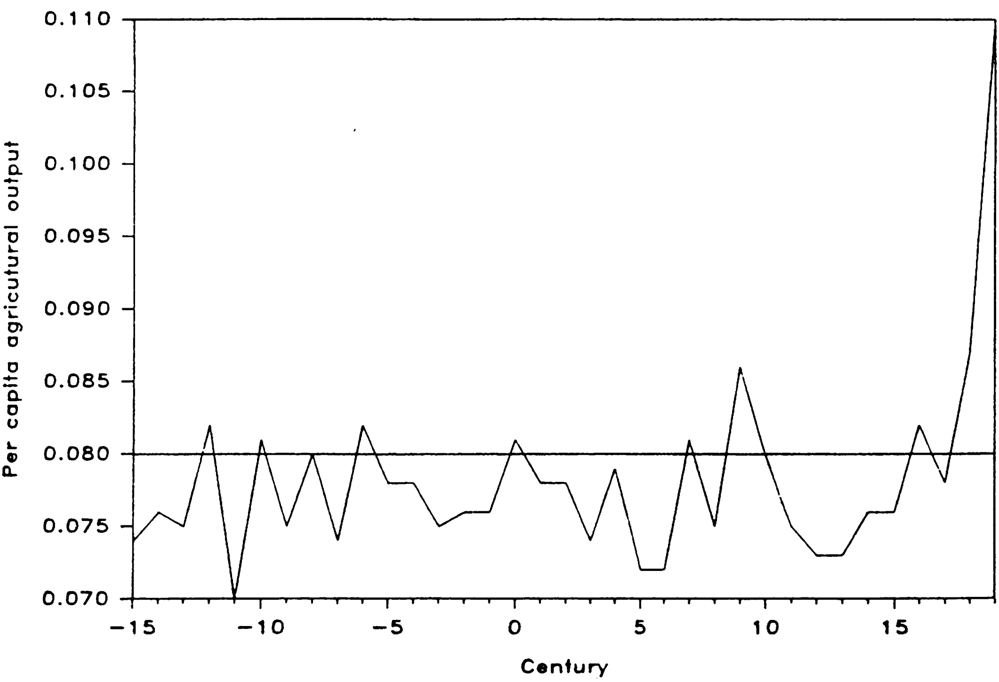


FIG. 2 *Per capita agricultural output*



during the seventeenth or eighteenth century. The 95% confidence intervals guarantee that the escape will occur during that period, regardless of the particular sequence of random events. The aggregate capital stock $K(t)$ follows a similar pattern of monotone and slow growth until the seventeenth century, followed by the explosion that we conceptualize as the Industrial and Demographic Revolutions.

The escape is confirmed in Figure 2, where we have plotted (for the particular outcome considered in Figure 1) the per capita agricultural output $S(t)$: $S(t)$ oscillates about $S^* = 0.08$ until the time of escape during the eighteenth century.

In Figures 3 and 4 we have plotted the population of the two sectors for the period 1500 B.C. to 1900 A.D. The population of the subsistence sector increases relatively smoothly; on the other hand the population of the capital-producing sector is regularly decimated, which reflects the fact that capital grew intermittently. However, during the Middle Ages a permanent capital-producing sector was established, and this coincides with a growth of the corresponding population (Figure 3).

We have now seen how the escape mechanism functions and how the Industrial Revolution could be considered as the outcome of an accumulation process by which the aggregate capital stock and the population reach critical values resulting in a per capita agricultural output that remains above the minimum level. We will now examine the more formal aspects of this question, specifically the conditions on $P_I(t)$ and $P_A(t)$ that will ensure the escape from the Malthusian trap.

4. THE ESCAPE CONDITIONS

In this section we explore conditions under which the population remains above its minimum subsistence level, i.e., conditions under which $S(t) \geqslant S^*$ for all t larger than some value t_0 (t_0 will be fixed, for convenience, at $t_0 = 0$).

We recall the definition of $S(t)$ in eq. [5] and observe that a sustained value of $S(t)$ depends not only on a sufficiently large population $P_A(t)$ in the subsistence sector but also on a sustained growth of the aggregate capital stock $K(t)$. Indeed, if $K(t)$ does not increase, consideration of eq. [5] shows that $S(t)$ cannot remain above S^* for long; this is due to the diminishing returns to labor (i.e., the exponent β_2 of $P_A(t)$ in eq. [5]). Because of the contribution of $P_A(t)$ to eq. [5] and the contribution of $P_I(t)$ to the accumulation of capital, it is now apparent that some compromise is necessary between the two sectors to ensure an escape. We now examine this question in greater detail.

Given that the population remains above subsistence, the process is deterministic; indeed the random mechanism (which operates only in times of crises) has no bearing on the conditions for an escape, i.e., the initial conditions on $P_I(0)$, $P_A(0)$, $K(0)$ and $\lambda(0)$ ($Q_A(0)$ and $Q_I(0)$ are "derived variables" inasmuch as they are functions of the previously defined quantities).

For simplicity, we will assume that the savings rate remains constant and equal to λ once the escape has occurred. It is clear that if the escape obtains with a

FIG. 3 *Simulation of population growth: capital-producing sector*

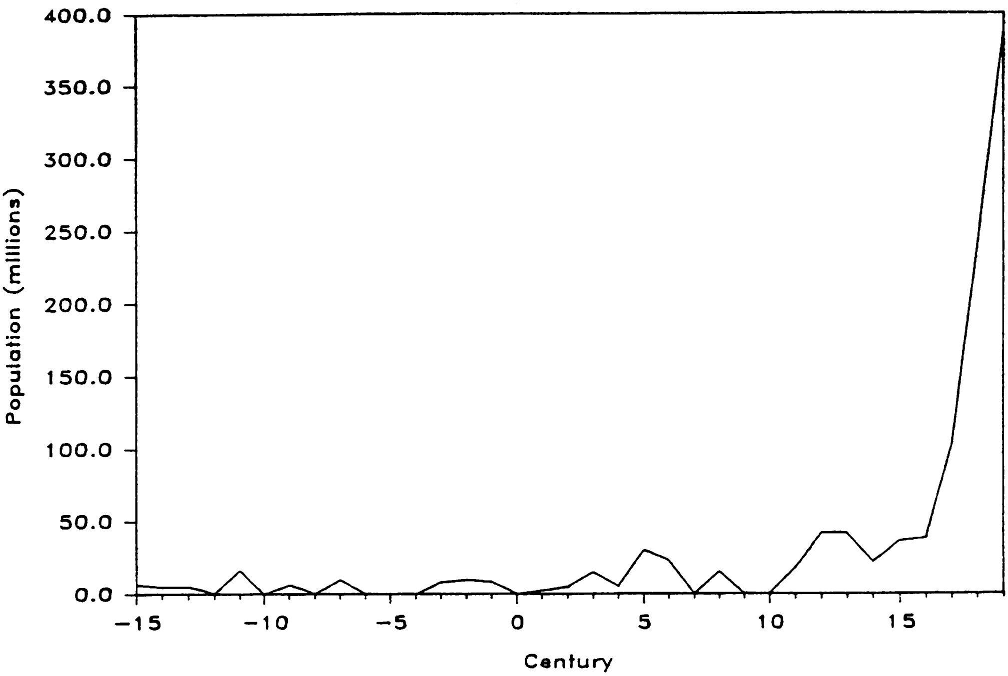
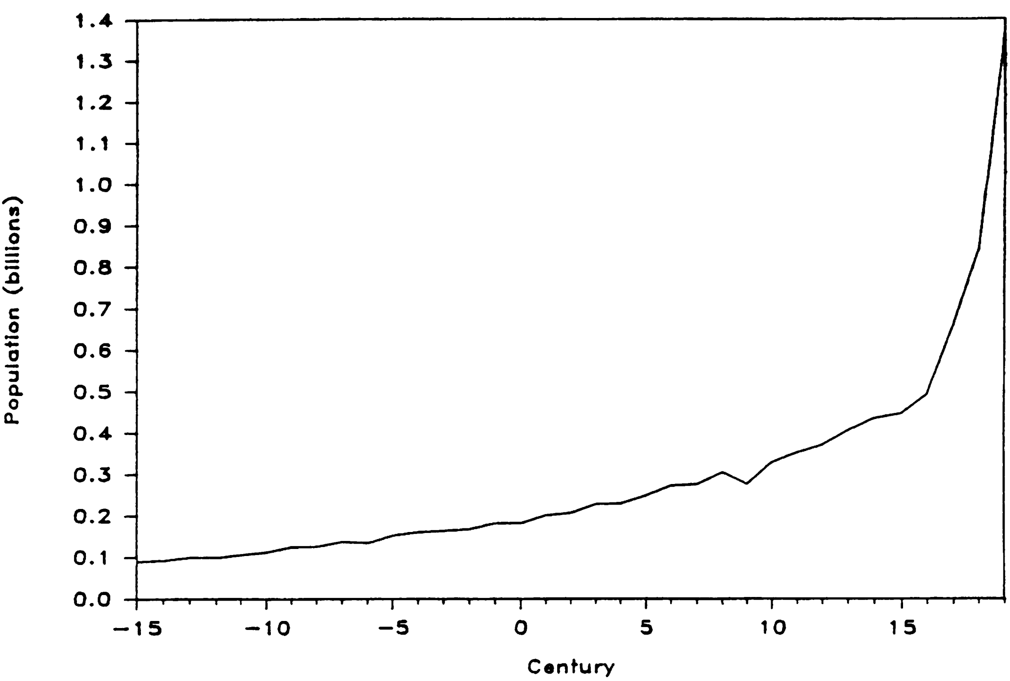


FIG. 4 *Simulation of population growth: subsistence sector*



constant savings rate, then it will occur a fortiori if the savings rate continues to increase after the time of escape. In any event, the savings rate cannot increase indefinitely and therefore it is not unreasonable to assume that it becomes constant at some point in time.

Because all variables are considered at period *0* (the time of escape) we replace the functional notations $P(0)$, $P_I(0)$, $P_A(0)$, $K(0)$ by P, P_I, P_A and K. Bearing in mind that the savings rate $\lambda(t) = \lambda$ is assumed fixed, we define

$$\theta_1 = \lambda C_1 / K^{\alpha_2} \qquad [13]$$

$$\theta_2 = C_2 K^{\beta_1} / S^* \qquad [14]$$

In the Appendix we show that the population remains above its subsistence level (i.e., $S(t) \geqslant S^*$ for $t > 0$) if and only if the pair (P_I, P_A) belongs to the area of the plane defined by

$$E_1 = \left\{ \begin{array}{ll} P_I \geqslant (r^*/\theta_1)^{1/\alpha_2} & [15] \\ P_I \leqslant \theta_2 P_A^{\beta_2} - P_A & [16] \end{array} \right.$$

and

$$E_2 = \left\{ \begin{array}{ll} P_I \leqslant (r^*/\theta_1)^{1/\alpha_2} & [17] \\ C_2 (\lambda C_1)^{\beta_1/\alpha_2} P_I^{\beta_1} P_A^{\beta_2} / S^*(P_A + P_I) \geqslant (r^*)^{\beta_1/\alpha_2} & [18] \end{array} \right.$$

In order to explore this locus $E_1 + E_2$ of escape conditions on (P_I, P_A), we fix the parameters α_1, α_2, β_1, β_2, C_1, C_2, S^*, r^* at their previous values; the capital stock K and the savings rate λ prevailing at the time of escape will be fixed at 100 and 0.04 respectively, which are approximately the values observed at the time of escape in the simulations of the previous section. Under these circumstances [18] defines a cone that is equivalently described by

$$P_A \left[\frac{\varphi - \sqrt{\varphi^2 - 4}}{2} \right] \leqslant P_I \leqslant P_A \left[\frac{\varphi + \sqrt{\varphi^2 - 4}}{2} \right] \qquad [19]$$

where

$$\varphi = \left[\frac{C_1 C_2 \lambda}{r^* S^*} \right]^2 - 2 \qquad [20]$$

In Figure 5 we plotted the locus of escape conditions $E_1 + E_2$ corresponding to these values (the escape region is shaded); $P_I^* = (r^*/\theta_1)^2$ is the value of P_I at which a horizontal line separates the two regions E_1 and E_2. Hence the region E_1 is bounded by the curve C defined by eq. [16] and by the horizontal line $P_I = P_I^* = 92.4$; E_2 is bounded by this line and by the lines L_1 and L_2 that define the cone of [19]. In view of [19] it is apparent that φ must be larger than 2 for the locus to be nonempty. Hence the savings rate λ at the time of escape must satisfy

$$\lambda \geqslant 2r^*S^*/C_1C_2 = 0.029 \qquad [21]$$

If $\lambda = 0.029$ then the cone is completely closed and the locus reduces to a straight line between the origin and $[(\theta_2/2)^2, (\theta_2/2)^2]$; at this point the horizontal line $P_I = P_I^*$ is tangent to the curve C defined by [16].

In Figure 6 we plotted the same locus of escape conditions as in Figure 5 except that the capital K was set equal to 150 (instead of 100). The cone defined by lines L_1 and L_2 remains unchanged (because φ of eq. [20] is independent of K). The horizontal line $P_I = P_I^*$ has moved upward and the curve C has changed in such a way that the escape region $E_1 + E_2$ is strictly larger than in Figure 5. This shows the crucial role played by the capital stock $K(t)$ since, everything else held constant, the escape region grows with $K(t)$.

The value

$$P_A^* = P_I^*[\varphi + \sqrt{\varphi^2 - 4}]/2 \qquad [22]$$

is the abscissa of the point at which the curve C, the horizontal line $P_I = P_I^*$, and the line L_1 intersect. Therefore, P_A^* is the largest possible value of the population in the subsistence sector compatible with an escape.

Given a total population P_0, the range of feasible values for P_A and P_I (such that $P_A + P_I = P_0$) can be found by drawing a 45° line from the point $P_I = P_0$ on the y-axis to the point $P_A = P_0$ on the x-axis. The segment of that line that is in the escape region $E_1 + E_2$ defines the feasible range for P_A and P_I. In Figure 6 this line is drawn for a total population P_0 of 300. The feasible values of P_I and P_A are on the segment (a,b). The range of faesible values for the two sectors of the population can now be determined graphically by reading the coordinates of a and b. The abscissas 88 and 255 of a and b give the minimum and maximum values of P_A; the ordinates 45 and 212 give the minimum and maximum of P_I.

In summary, we have examined analytically the conditions under which the population escapes the Malthusian trap. Our investigation has confirmed the expectation concerning a necessary compromise between the two sectors of the population: the relative share of each group is suggested by the cones of Figures 5 and 6. Total numbers are kept within the limits imposed by the curve C. We recall from eq. [8b] of the Appendix that this bound corresponds to an initial values of $S(t)$ larger than S^*. Bearing in mind the expression of $S(t)$ given in eq. [5] it is apparent that P_I cannot be too large if $S(t)$ its to be larger than S^*. Similarly P_A cannot be too large either, because of diminishing returns to labor in the agricultural sector.

FIG. 5 *Locus of escape conditions on* (P_I, P_A)
$(K = 100;\ \lambda = 0.04)$

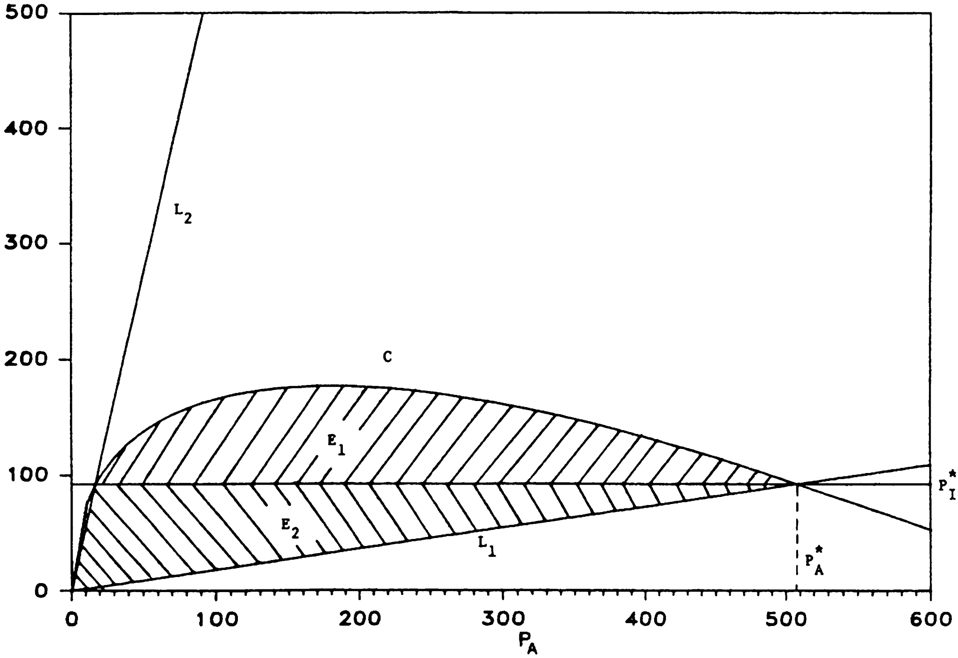
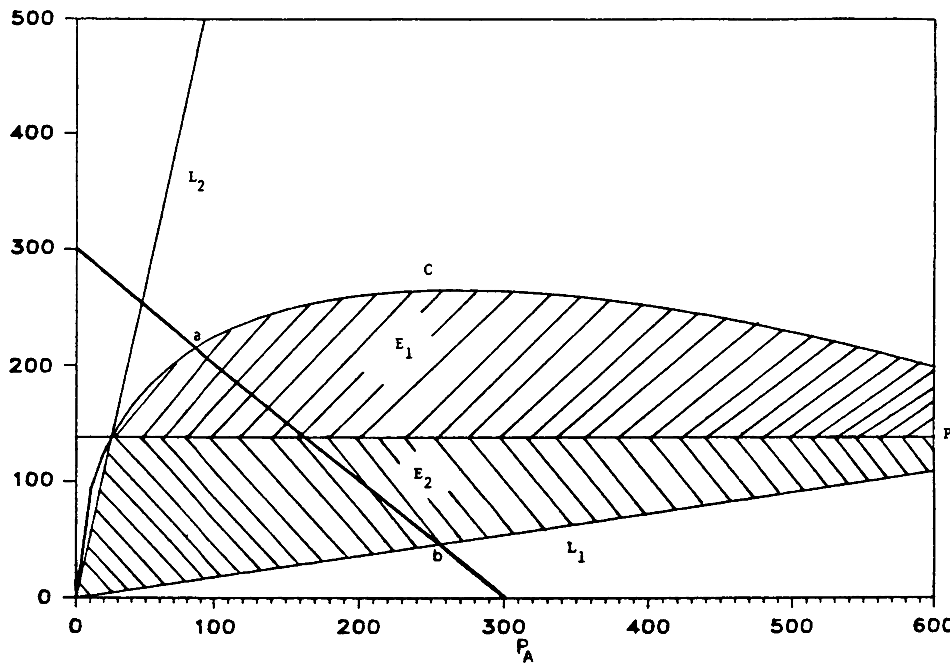


FIG. 6 *Locus of escape conditions on* (P_I, P_A)
$(K = 150;\ \lambda = 0.04)$



5. DISCUSSION AND CONCLUSION

We have described a Malthusian simulation model that captures the salient features of demographic and economic growth for the period extending from the agricultural to the Industrial Revolution. Our purpose was to conceptualize these 10,000 years as a unity and to show how a slow accumulation of capital stock brought about the Industrial Revolution, thus enabling the population to emancipate itself from the Malthusian menace.

Although we feel that we have successfully modeled the dynamics of the Malthusian equilibrium, we realize that at times we may have sacrificed realism for the sake of simplicity. For example, it may not have been fully realistic to assume that in the absence of a Malthusian crisis the population grew automatically and deterministically at a constant rate r^*. Also, the intersectorial migration of the population could have been modeled in more complicated terms. Some economic aspects of the model could have been specified in more elaborate ways. For example, we could have disaggregated capital and considered different rates of accumulation in the two sectors. We also could have introduced economies of scale. Yet, before we know more about the long-run evolution of the European economies, such efforts seem pointless.

Also, it may not have been realistic to embed the population of the world within a single economic unit, thus ignoring the fact that for most of man's history, the economic development of the different continents were to a large extent independent. However, the global subsistence crises described in this paper can be thought of as the contributions of local crises to the demographic history of the world. To be sure our model could be tested on a disaggregated scale and would presumably hinge on the premise that "the smaller the entity, the greater the magnitude of the demographic catastrophes" (during the Middle Ages, Europe is known to have lost up to one-third of its population; the population of some cities may have been wiped out completely by epidemics). Our major goal was to specify a demographic regime which could evolve within a hostile economic environment, and we feel that none of these complexities would aid in our understanding of long-term economic and demographic growth patterns.

In brief, our model captures the dynamics of a homeostatic equilibrium in which the population reacts to subsistence crises in the form of randomly determined demographic perturbations that can take on catastrophic proportions. The escape from the Malthusian menace results from a slow accumulation of capital and hinges on a subtle compromise between the populations of the two sectors of the economy. It is hoped that this model will contribute to a better understanding of one of the most momentous events in man's history: the Industrial Revolution of the eighteenth century and the concurrent demographic explosion that followed the escape from the Malthusian trap that had shackled humanity for millions of years.

APPENDIX

We define

$$D(t) = S(t)/S^* \qquad [1b]$$

$$s = (1 + r^*)^{\alpha_2} \qquad [2b]$$

and the growth rate $G(t)$ of the capital stock

$$G(t) = K(t+1)/K(t) - 1 \qquad [3b]$$

We recall that the savings rate λ is assumed constant once the escape has occurred and that both sectors of the economy grow at the same rate r^*. Our purpose is then to investigate conditions under which $D(t)$ remains larger than 1 when $D(0) > 1$.

In views of eqs. [1] - [3] and [5] the variables $G(t)$ and $D(t)$ satisfy, for every t

$$G(t+1) = \frac{sG(t)}{[G(t)+1]^{\alpha_2}} \qquad [4b]$$

$$D(t+1) = D(t)\left[\frac{1+G(t)}{1+r^*}\right]^{\beta_1} \qquad [5b]$$

$$D(t) = \left[\frac{G(0)}{G(t)}\right]^{\beta_1/\alpha_2} D(0) \qquad [6b]$$

It can be seen that r^* is a fixed point of the iterative process defined by eq. [4b], i.e. if $G(0) = r^*$, then $G(t) = r^*$ for every $t > 0$. Furthermore if $G(0) \geqslant r^*$ (resp. $G(0) < r^*$) then the sequence $G(t)$ $(t = 1, 2, ...)$ decreases (resp. increases) monotonically and converges to the fixed point r^*. Hence if $G(0) \geqslant r^*$ and $D(0) \geqslant 1$, eq. [5b] shows that $D(t)$ will remain larger than 1. Therefore a first region of escape is

$$E_1 = \left.\begin{array}{ll} G(0) \geqslant r^* & [7b] \\ D(0) \geqslant 1 & [8b] \end{array}\right\}$$

Similarly it can also be seen that the population will escape the Malthusian trap if the pair $[G(0), D(0)]$ belongs to

$$E_2 = \left.\begin{array}{ll} G(0) \leqslant r^* & [9b] \\ D(0) \geqslant \left[\dfrac{r^*}{G(0)}\right]^{\beta_1/\alpha_2} & [10b] \end{array}\right\}$$

These two regions are those of eqs. [15]-[16] and [17]-[18] in the text where the variables $G(0)$ and $D(0)$ are expressed as functions of P_I and P_A.

SUMMARY

A Malthusian simulation model is proposed to describe the growth of human population from the Neolithic through the Industrial Revolution. The economy is composed of a subsistence sector and a capital-producing sector. Our model captures the "incessant contest" between population growth and the means of subsistence. When the per capita agricultural output falls below a biological minimum, the growth rate of the population is subject, in a random fashion, to perturbations that can take on disastrous proportions. The slow accumulation of capital (and the buildup of the population of the capital-producing sector) eventually enables the population to overcome the contraints of the hostile economic environment. Our simulations (complete with confidence intervals) yield numerically realistic estimates of the population that eventually escapes from the Malthusian menace and grows unhindered during the Industrial Revolution.

RIASSUNTO

Viene proposto un modello di simulazione malthusiano per descrivere la crescita della popolazione umana dal neolitico alla rivoluzione industriale. L'economia si compone di un settore di sussistenza e di un settore producente capitale. Il modello si impernia su l'"incessante disparità" tra crescita della popolazione e produzione dei mezzi di sussistenza. Quando la produzione agricola pro-capite scende al di sotto di un minimo biologico, il tasso di crescita della popolazione è soggetto, in un modo casuale, a perturbazioni che possono raggiungere proporzioni disastrose. La lenta accumulazione di capitale (e lo sviluppo della popolazione del settore che produce capitale) infine possono mettere in grado la popolazione di superare i vincoli dell'ambiente economico ostile. Le simulazioni (complete di intervalli di fiducia) producono stime numeriche realistiche della popolazione che alla fine sfugge alla minaccia malthusiana e cresce senza ostacoli durante la rivoluzione industriale.

RESUME

Un modèle simulant l'homéostasie malthusienne permet d'expliquer l'histoire démographique depuis l'ére néolithique jusqu'à la révolution industrielle. Nous considérons une économie à deux secteurs: un secteur produisant les moyens de subsistance et un secteur produisant tout le reste y compris le capital. Le modèle rend compte du conflit entre la tendance à la croissance démographique et la disponibilité des moyens de subsistance. Lorsque la production agricole par tête tombe en dessous d'un minimum vital, des crises de mortalité aléatoires tendent à faire baisser la population. Dès que la production agricole redevient suffisante, la population croît exponentiellement. Nos simulations recréent de facon numériquement réaliste 10000 ans d'histoire démographique et montrent comment l'accroissement de la population dans le secteur produisant le capital (et la lente accumulation du capital) ont permis à la population d'échapper définitivement à la menace malthusienne.