
Published conference paper

Sarhosis, V, Sheng, Y and Garrity, SW (2010) *Computational modelling of clay brickwork walls containing openings.* International Masonry Society Proceedings 11, Volume 3 . 1743 - 1752. ISSN 0950-9615

8th International Masonry Conference 2010 in Dresden

Computational modelling of clay brickwork walls containing openings

SARHOSIS, V.[1]; SHENG, Y.[2]; GARRITY, S.W.[3]

ABSTRACT

The use of the Distinct Element Method to simulate the response of single leaf clay brickwork walls with openings to vertical, in-plane, static loading is described. The walls were modelled as an assemblage of stiff yet deformable bricks with mortar joints as zero thickness interfaces. Conventionally, the results of tests on small specimens are used to determine the material or interface parameters. These values usually need to be adjusted to allow for inherent variations in the materials, workmanship effects and differences in the boundary conditions of the small-scale tests compared with those in the larger structure. In this research the material and interface parameters were determined by applying a manual optimisation to the results of a series of laboratory tests carried out on full-scale wall panels. The computational model was then used to predict successfully the behaviour of a longer span wall panel constructed from a similar brick and mortar combination.

Keywords: *Distinct element modelling, masonry walls, optimisation*

1 INTRODUCTION

This paper describes the development of a computational model for masonry that will be used to study different strengthening systems for single leaf brick wall panels containing openings. As many of the brickwork walls in need of strengthening were constructed of low strength materials or they have deteriorated with time, cracking tends to be along the brick/mortar interfaces and failure usually results from de-bonding of the bricks. As a result the authors decided to use a computational model based on the Distinct Element Method which was developed by Cundall [1] in 1971. The method was developed for commercial use by Cundall and Itasca Limited [2] for 2-dimensional structures in the form of the software UDEC (Universal Distinct Elements Code). Initially DEM was applied to rock engineering projects where continuity between the separate blocks of rock did not exist. More recently it has been used to model masonry structures [3, 4 and 5] in which the failure mechanism is governed primarily by the masonry unit/mortar interface characteristics.

1.1. An overview of UDEC and masonry modelling

UDEC [2] is a numerical program based on the distinct element method for discontinuous modelling and can simulate the response of discontinuous media subjected to either static or dynamic loading. When used to model brickwork structures, the bricks are represented as an assemblage of rigid or deformable distinct blocks which may take any arbitrary geometry. Rigid blocks do not change their

---

1) PhD student, School of Civil Engineering, University of Leeds, UK, v_sarhosis@yahoo.com
2) Lecturer in Civil Engineering, School of Civil Engineering, University of Leeds, UK, y.sheng@leeds.ac.uk
3) Hoffman Wood Professor of Architectural Engineering, School of Civil Engineering, University of Leeds, UK, s.w.garrity@leeds.ac.uk

geometry as a result of any applied loading and are mainly used when the behaviour of the system is dominated by the mortar joints. Deformable blocks are internally discretised into finite difference triangular zones and each element responds according to a prescribed linear or non-linear stress-strain law. These zones are continuum elements as they occur in the finite element method (FEM). Mortar joints are represented as zero thickness interfaces between the blocks. These interfaces can be viewed as interactions between the blocks and are governed by appropriate stress-displacement constitutive laws. Interaction between the blocks is represented either by sets of point contacts or by sets of edge to edge contacts, with no attempt to obtain a continuous stress distribution through the contact surface. The mechanical interaction between the blocks is simulated at the contacts by spring like joints with normal ($\mathbf{J}_{\mathbf{kn}}$) and shear stiffness ($\mathbf{J}_{\mathbf{ks}}$) as well as frictional ($\mathbf{J}_{\mathbf{fric}}$), cohesive ($\mathbf{J}_{\mathbf{coh}}$) and tensile strengths ($\mathbf{J}_{\mathbf{ten}}$), as shown in Figure 1.

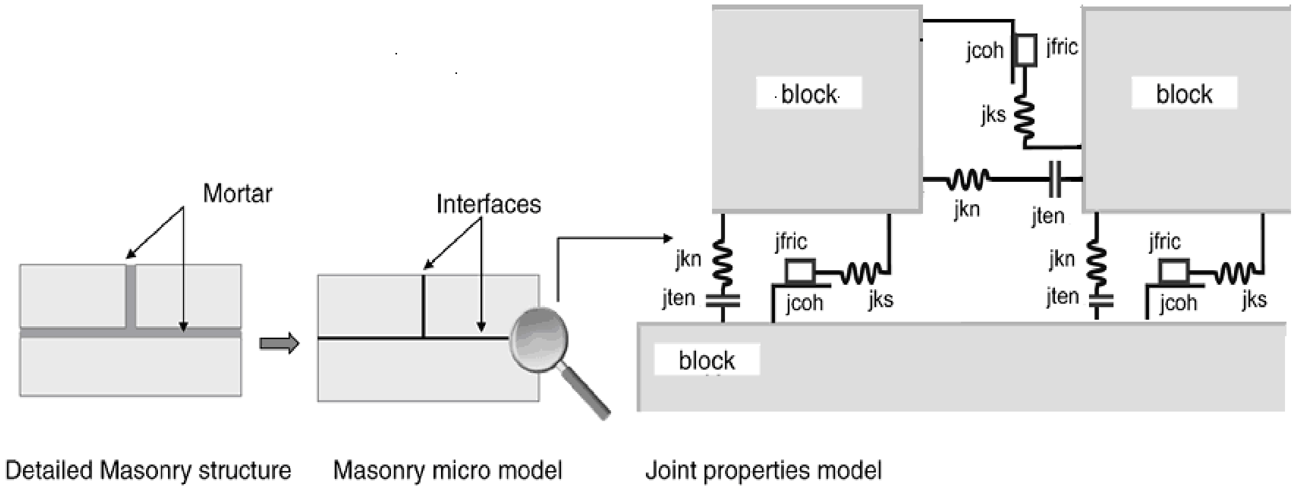

Figure 1. Interface model in UDEC [2] (adopted and altered by the authors)

As with the Finite Element Method, the unknowns are the nodal displacements and rotations of the blocks. However, unlike FEM, DEM is a dynamic process and the unknowns are solved explicitly by the differential equations of Newton’s Second law of motion at all bricks or nodes and the force-displacement law at all contacts. The force-displacement law is used to find the contact forces from known displacements while Newton’s second law gives the motion of the blocks resulting from the known forces acting on them. In this way, large displacements along the mortar joints and the rotations of the bricks are allowed with the sequential contact detection and update of tasks automatically. Furthermore, UDEC can model both static and dynamic load effects. The static solution is achieved by artificial damping similar to the dynamic relaxation method where the equations of motion are damped to reach the equilibrium state. Also, UDEC can employ time-stepping algorithms either in real time scale or as a numerical device to solve quasi- static problems [2].

2 MATERIAL PARAMETER IDENTIFICATION FOR MASONRY CONSTITUTIVE MODELS

Conventionally the material parameters for masonry constitutive models are obtained directly from the results of compressive, tensile, and shear tests on small masonry prisms. Some of these parameters are very variable and sensitive to the method of testing. This is likely to be due to the combined effects of eccentric loading, stress concentrations and variations in the resistance to applied stress that are likely to exist in the test specimens. Some researchers also carry out separate tests on masonry units and/or mortar specimens. In such cases the effects of boundary conditions such as platen restraint and the shape and size of the test specimen can have a significant influence on the magnitude of the measured parameter. Although the testing of small specimens is simple, relatively inexpensive and involves little specialist equipment there are a number of limitations of such tests. In particular the simple conditions under which the small specimens are tested in the laboratory do not usually reflect the more complex boundary conditions, the combinations of stress-state types and load spreading effects that exist in a large scale masonry structure. In addition, to cater for the aforementioned eccentric load effects, stress concentrations and inherent variations in resistance, it is usually necessary to test large numbers of small specimens. The situation is made more complex when workmanship is considered. Usually a much higher standard and consistency of workmanship will be achieved when constructing small scale test specimens when compared with the construction of larger scale masonry structures. Such variations in workmanship will not be captured if the material parameters are based on the results from the testing of small scale specimens. As a result of these difficulties it is often necessary to adjust the material parameter values obtained from small scale experiments before they can be used in the numerical model. The authors have found a further complication when using UDEC to model masonry. As the material parameters define the characteristics of the zero thickness interfaces between the mortar joints and the blocks, they can be difficult to measure directly from physical tests.

Many of the difficulties in determining representative material parameters for masonry constitutive models have previously been identified by Toropov and Garrity [6]. In an attempt to address these difficulties Toropov and Garrity proposed that the material parameters could be obtained from the responses of relatively complex or “non-trivial” large scale masonry structures to externally applied loads. It was envisaged that such tests would be carried out in the laboratory and the large scale structures selected for this purpose would contain a variety of different stress states. The responses measured in the laboratory would normally be deflections or distortions rather than surface strains. A computational model would also be used to predict the response of the same masonry structure to the applied loads. The initial material parameters used in the constitutive model would be based on values obtained either from the literature, small scale specimen tests carried out by other researchers or from codes of practice. The material parameters in the computational model would then be “tuned” using an optimisation process so that the predicted response of the structure would agree sufficiently closely with that obtained from physical testing. Essentially, the material parameter identification problem can be defined in terms of an optimisation problem in which the function to be minimised is an error function that expresses the difference between the response measured from the large scale experiments and that obtained from the numerical analysis. This approach was used in the research described in this paper. Accordingly the remainder of this paper is structured as follows:

Section 3: A brief description is provided of the laboratory testing of 4 single skin clay brickwork wall panels containing 2.025 m span openings and a wall panel containing a 2.925m opening.

Section 4: A description is given of the computational modelling of similar wall panels (each with a 2.025m opening) using UDEC.

Section 5: The determination of the material parameters using a manual optimisation process is given. This includes a series of numerical experiments to guide the optimisation. The load to cause first visible cracking, the failure load and the load vs midspan displacement relationship obtained from the laboratory testing of the 4 wall panels with a 2.025m opening, described in section 3, are compared with the numerical predictions obtained from UDEC. The material parameters are then optimised to achieve similar responses to those obtained in the laboratory.

Section 6: This section describes how the UDEC model with the newly determined material parameters is used to predict the behaviour of a wall panel with a 2.925m opening. The numerical results are compared with the development of the crack pattern under incremental loading; the load at first visible cracking; the failure load; the failure mechanism and the load vs. deflection relationship obtained from the testing of a wall panel with a 2.925m opening in the laboratory.

3 EXPERIMENTAL TESTS ON WALL PANELS WITH OPENINGS

Five single leaf unreinforced masonry wall panels were tested in the laboratory at the University of Bradford by one of the authors (Garrity). These results are part of a larger test programme and have not yet been published. The wall panels were developed to represent the clay brickwork outer leaf of an external cavity wall containing openings for windows. Four of the wall panels (S1, S2, S3 & S4) had an opening of 2.025m. The fifth panel (L1) had an opening of 2.925m. Typical details of the panels with a 2.025m opening are shown in Figure 2. All five panels were built with a soldier course immediately above the opening with the remainder of the brickwork being constructed in stretcher bond. The bricks were UK standard size (215mm x 102.5mm x 65mm) Ibstock Artbury Red Multi Stock bricks with a water absorption of 14% and a sand faced finish. The joints were all 10mm thick, 1:12 (opc:sand) weigh-batched mortar. The bricks and mortar were selected to produce brickwork with a low bond strength, the aim being to represent low quality, high volume wall construction which, in the authors' experience, is fairly typical of low rise domestic construction in the UK. Each wall panel was subjected to a single vertical point load applied at the top of the wall at midspan. The point load was distributed through a steel spreader plate. The load was applied to each wall incrementally. The midspan deflection was recorded at each load increment and each wall was inspected visually for signs of cracking throughout the test. The test results are summarised in Table 1.

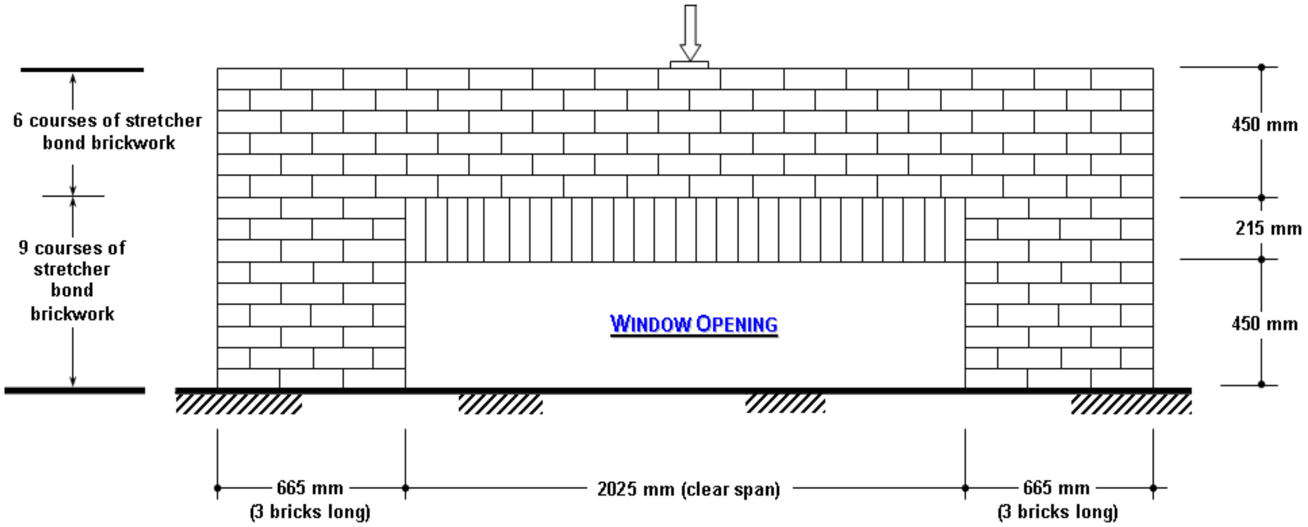

Figure 2. Typical masonry wall panel with 2.025m span opening tested in the laboratory

Table 1. Masonry wall panel test results

Panel	Clear opening (mm)	Mortar compressive strength (MPa)	Load at first visible crack (KN)	Failure load (KN)
S1	2025	0.72	0.72	3.69
S2	2025	0.79	1.6	4.6
S3	2025	0.86	1.6	5.1
S4	2025	1.18	1.71	5.67
L1	3025	0.64	0.1	1.6

4 COMPUTATIONAL MODELLING OF THE MASONRY WALL PANELS WITH UDEC

4.1. UDEC wall panel geometry

A UDEC model of the wall panel with a 2.025m opening, shown in Figure 2, was created in which each brick was represented by a deformable block separated by a zero thickness interface at each mortar bed and perpend joint. To allow for the 10mm thick mortar joints in the real wall panels, each deformable block was based on the nominal brick size used in the laboratory built panels increased by 5mm in each face dimension to give a UDEC block size of 225mm x 102.5mm x 75mm. The UDEC model, which consisted of 182 distinct blocks to represent each brick, is shown in Figure 3.

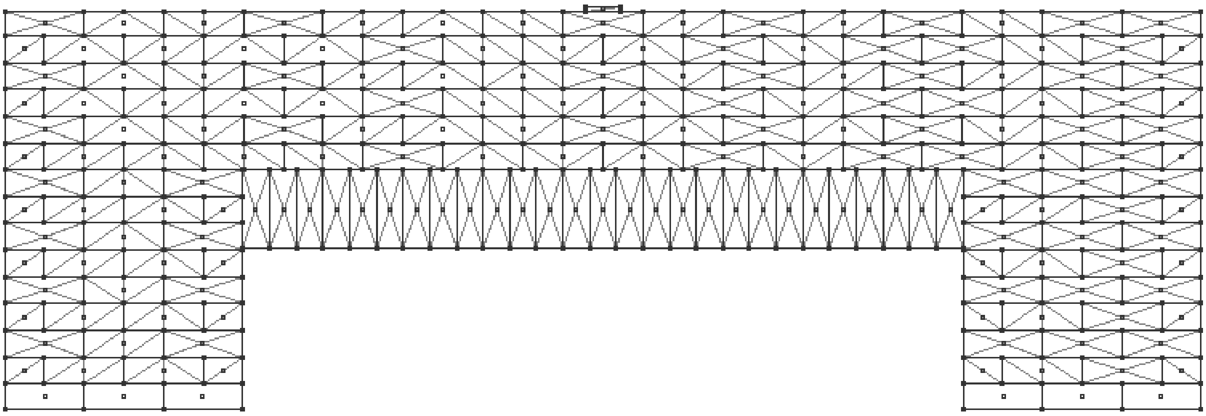

Figure 3. UDEC geometric model of a masonry wall panel with a 2.025m opening

4.2. Block and interface details

The bricks were modelled as deformable blocks, thus allowing any deformation to occur both in the blocks and in the joints. It was assumed that the bricks would exhibit linear stress-strain behaviour and that slip along the mortar joints would be the predominant failure mechanism. The mortar joints were represented by interfaces modelled using UDEC's *elastic-perfectly plastic coulomb slip-joint area contact* option [2]. This provides a linear representation of the mortar joint stiffness and yield limit and is based upon elastic normal ($\mathbf{J_{kn}}$) and shear ($\mathbf{J_{ks}}$) stiffness, frictional ($\mathbf{J_{fric}}$), cohesive ($\mathbf{J_{coh}}$) and tensile ($\mathbf{J_{ten}}$) strengths, as well as the dilation ($\mathbf{J_{dil}}$) characteristics of the mortar joints. If, in the numerical calculation, the bond tensile strength or shear strength is exceeded, then the tensile strength and cohesion are reduced to zero in accordance with the Mohr-Coulomb relationship ($\tau = J_{coh} + \sigma_n \tan J_{fric}$). Initial values of the material parameters for the constitutive model were selected from values reported in the literature [7, 8].

4.3. Boundary conditions and load simulation

The bottom edges of the UDEC wall panel model were modelled as rigid supports in the vertical and horizontal direction whilst the vertical edges of the wall panel were left free. A high damping ratio was assigned to the model to simulate static loading. Initially the model was brought into a state of equilibrium under its own self weight and then the externally applied central point load was applied incrementally.

In order to determine the collapse load, Itasca [2] advises that displacement-controlled boundary conditions should be used rather than a force-controlled approach. As a result, a constant vertical velocity was applied at the load spreader plate on the top of the wall panel. The velocity was converted to a vertical displacement and the force acting on the spreader plate for each load increment. Hence a load versus midspan displacement relationship was determined for the panel. This was later compared in the optimisation process to the experimental results obtained in the laboratory. It is important to note that convergence tests were carried out on the magnitude of the velocity to be applied to the spreader plate to make sure that a quasi-static loading condition was achieved.

5 DETERMINATION OF THE MATERIAL PARAMETERS

A manual optimisation procedure was used to determine the material interface parameters to be used in the UDEC model. Numerical simulations of the load versus midspan deflection relationships were compared with the corresponding experimental results for panels S1, S2, S3, S4. In addition, comparisons were made with the load at which first visible cracking occurred (assuming a value of 0.2mm) and the failure load. The development of the surface cracking pattern predicted using UDEC was also compared with that observed in the laboratory experiments.

In order to carry out the optimisation process a series of numerical experiments had to be carried out to determine the influence of the various material parameters on the predicted behaviour of the wall panels. These are described in more detail below.

5.1. The influence of the brick properties

A parametric study was carried out to assess the effect of the density, the elastic modulus and Poisson’s ratio of the brick on the predicted mechanical behaviour of the wall panels. Using a range of values from the literature [7, 8], each property was separately varied by 10% in the numerical model. As expected it was found that these parameters had no significant effect on the mechanical behaviour of the wall panels.

5.2. The influence of the joint interface properties

Two different numerical experiments were carried out to investigate the effect of different joint interface parameters on the mechanical behaviour of the wall panels. The first experiment considered the influence of the inelastic joint interface material properties (i.e. the joint friction angle, cohesion, and tensile strength), while the second experiment investigated the influence of the elastic joint interface properties (i.e. the normal and shear stiffness). In both cases, the dilation angle was assumed to be zero as, according to Lourenço [7], this assumption provides a conservative prediction of the load vs. displacement relationship. This is considered further in 5.2.3.

5.2.1 Inelastic joint interface parameters

The literature [2, 3, 4, 9] indicates that for the bricks and mortar used in the wall panel tests, the value of $\mathbf{J_{fric}}$ could range from 20-40 degrees while the values of $\mathbf{J_{coh}}$ and $\mathbf{J_{ten}}$ could range from 0.05 to 0.55MPa. Table 2 demonstrates the effect of the inelastic parameters derived from the numerical analysis with UDEC. In all simulations, the $\mathbf{J_{kn}}$ and $\mathbf{J_{ks}}$ values were kept constant and equal to 83 and 36 GPa/m, respectively, as suggested by Lourenço [7]. The results from this investigation indicated the following:

a). The joint tensile strength has a significant influence on the occurrence of the first visible crack.
b). The joint friction and joint cohesion have a significant influence on the load at which failure is predicted to occur.
c). The initial displacement of the wall panel due to self weight effects remains constant and is independent of the inelastic parameters.
d). The behaviour of the wall panel up to first cracking is linear, thereafter the load vs. displacement relationship is non-linear.

5.2.2 Elastic interface parameters

The joint stiffness parameters ($\mathbf{J_{kn}}$ and $\mathbf{J_{ks}}$) describe the stress deformation characteristics of the joints. The joint normal stiffness ($\mathbf{J_{kn}}$) characterises the response of the model to normal loading and is a measure of the normal stress per unit closure of the joint. It has been proposed [10] that $\mathbf{J_{kn}}$ may be influenced by the initial actual contact area between the two joint surfaces, the joint wall roughness, the strength and deformability of any surface roughness features, the thickness of the joint and the type and physical properties of the joint infill material. The joint shear stiffness ($\mathbf{J_{ks}}$)

characterises the response to shear loading and is measured as the ratio of the shear stress to the shear deformation. The joint shear stiffness depends on the roughness of joint surfaces which can be determined by the distribution of the amplitude and the inclination of the surface roughness features, the properties of the joint filling material and the length of joints [10].

Table 2. Results of numerical simulations to investigate the influence of the inelastic interface parameters

No.	J_{fric} (deg)	J_{coh} (MPa)	J_{ten} (MPa)	J_{dil} (deg)	Initial displacement (mm)	First crack		Failure	
						Load (kN)	Disp. (mm)	Load (kN)	Displacement (mm)
1	25	0.05	0.05	0	0.026	0.28	0.032	2.81	0.200
2	25	0.1	0.05	0	0.026	0.28	0.032	4.49	0.240
3	25	0.15	0.05	0	0.026	0.28	0.032	6.28	0.362
4	25	0.05	0.1	0	0.026	1.27	0.057	3.45	0.257
5	25	0.1	0.1	0	0.026	1.25	0.056	6.35	0.566
6	25	0.15	0.1	0	0.026	1.25	0.056	8.20	0.537
7	25	0.05	0.15	0	0.026	2.32	0.089	3.96	0.337
8	25	0.1	0.15	0	0.026	2.22	0.080	7.04	0.525
9	25	0.15	0.15	0	0.026	2.21	0.080	10.03	0.752
10	35	0.05	0.05	0	0.026	0.28	0.032	4.78	0.565
11	35	0.1	0.05	0	0.026	0.28	0.032	6.51	1.021
12	35	0.15	0.05	0	0.026	0.28	0.032	9.41	0.987
13	35	0.05	0.1	0	0.026	1.36	0.060	3.26	0.242
14	35	0.1	0.1	0	0.026	1.25	0.056	7.72	1.064
15	35	0.15	0.1	0	0.026	1.25	0.056	9.20	0.587
16	35	0.05	0.15	0	0.026	2.46	0.095	4.11	0.311
17	35	0.1	0.15	0	0.026	2.30	0.083	9.88	1.270
18	35	0.15	0.15	0	0.026	2.21	0.080	10.53	0.685

The literature [3, 4, 5, 9] indicates that the values of **J_{kn}** and **J_{ks}** can range from 8.3 to 8300 GPa/m and 3.6 to 3600 GPa/m, respectively. A series of numerical experiments was carried out to identify the effect of varying the elastic interface parameters properties. The results are summarised in Table 3. From this study, assuming that the inelastic interface parameters remain constant, it was found that **J_{kn}** and **J_{ks}** have a bigger influence on the displacement of the wall panel at both initial cracking and at failure, than on the magnitude of the load at first cracking or failure.

Table 3. Results of numerical simulations to investigate the influence of the elastic interface parameters

Scale	J_{kn} (GPa)	J_{ks} (GPa)	Initial displacement (mm)	Load at first crack (kN)	Displacement at first crack (mm)	Failure load (kN)	Displacement at failure (mm)
1	8.3	3.6	0.189	0.24	0.24	2.39	1.26
10	83	36	0.026	0.27	0.03	2.62	0.28
100	830	360	0.009	0.36	0.01	2.76	0.07
1000	8300	3600	0.002	0.82	0.01	3.70	0.05

As part of this study the influence of the **J_{kn}/J_{ks}** ratio was also explored as the two parameters are inter-dependent [7]. It was found that the higher the ratio used in the model, the lower the predicted load and displacement at both first cracking and at failure. The value of the ratio also had a small influence on the initial displacement due to self weight effects.

5.2.3 Effect of the angle of dilation

Dilation is only likely to be a factor in the case of the wall panels when cracks can propagate through the bed joints under low levels of normal stress but comparatively high levels of slip (i.e. non-elastic sliding). In the panels tested in the laboratory, this situation occurred close to the failure condition when horizontal cracking of the support brickwork and uplift occurred. In a sensitivity analysis carried out by the authors, the angle of dilation was found to increase the failure load as the brickwork could sustain greater uplift.

5.3. The results of the optimisation exercise

Using a manual optimisation process, guided by the results of the aforementioned parametric studies, the material parameters shown in Table 4 were obtained. The parameters were then used to model the behaviour of panels S1 to S4, inclusive; the results are shown in Figure 4. As expected a good correlation was obtained between the UDEC prediction and the experimental results, bearing in mind the variation in the mortar compressive strengths measured in the laboratory (see Table 1).

Table 4. UDEC input parameters derived from the manual optimization process

	Brick Properties	**Symbol**	**Value**	**Units**
Elastic parameters	Density	d	2000	Kg/m^3
	Elastic modulus	E	1670	MPa
	Poisson’s ratio	v	0.14	---
	Mortar Joint Properties			
	Joint normal stiffness	J_{kn}	55	GPa/m
	Joint shear stiffness	J_{ks}	38	GPa/m
Inelastic parameters	Joint friction angle	Φ	33	Degrees
	Joint cohesion	J_{coh}	0.052	MPa
	Joint tensile strength	J_{ten}	0.10	MPa
	Joint dilation angle	Ψ	12	Degrees

6 COMPARISON OF UDEC PREDICTIONS WITH EXPERIMENTAL RESULTS

In order to check the validity of the material parameters, UDEC was used to predict the behaviour of an additional wall panel with an opening of 2.925m (Panel L1; see Table 1). UDEC predicted the crack pattern close to failure as shown in Figure 5. The crack pattern and the development of the cracks at different stages of applied loading were very similar to the behaviour observed in the laboratory. Further details of the qualitative output from UDEC has been reported elsewhere [11]. The load vs. displacement relationship predicted using UDEC for panel L1 is compared with that measured in the laboratory in Figure 6. It should be noted that the material parameters were based on an optimisation of the results obtained from 4 wall panel tests in the laboratory in which the mortar strengths varied from 0.72MPa to 1.18MPa. In spite of such variation, the behaviour of panel L1 predicted using UDEC is very similar to that observed experimentally. In summary, UDEC was able to predict the crack development; the crack pattern at failure; the mode of failure; the load at first visible cracking; the collapse load and the load vs displacement response with an acceptable degree of accuracy.

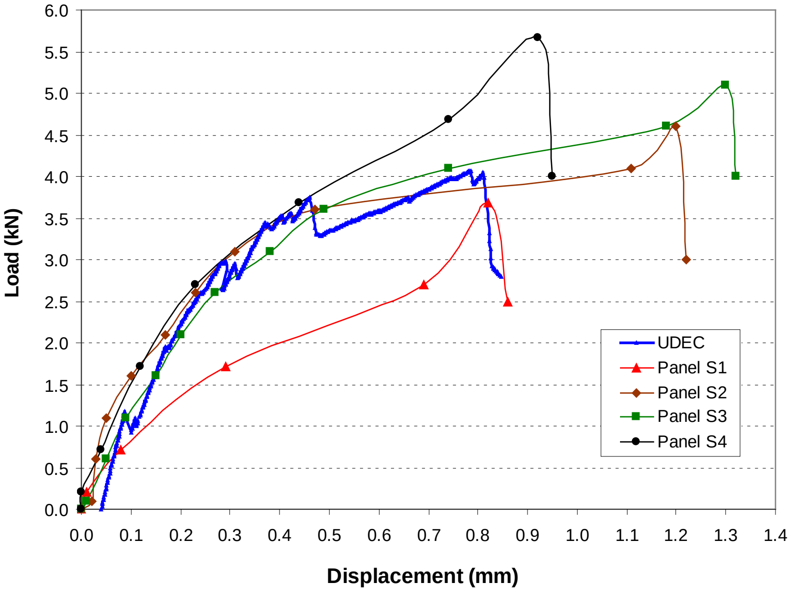

Figure 4. Comparison of experimental and numerical results for wall panels S1 to S4

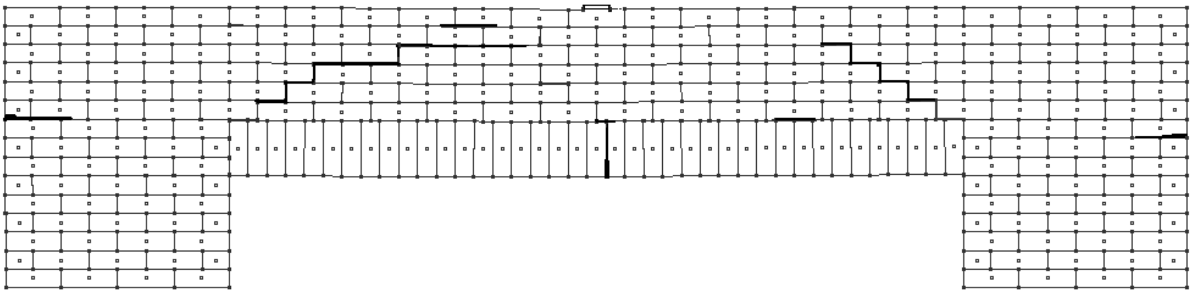

Figure 5. The crack pattern in wall panel L1 at failure, predicted using UDEC.

7 SUMMARY OF THE PRINCIPAL FINDINGS

The distinct element method in the form of UDEC has been used to model the behaviour of single leaf brickwork wall panels with openings under vertical in-plane loading. A method of determining the material parameters for the constitutive model using the results of large-scale experiments and a manual optimisation approach, informed by a series of parametric studies, has been presented. The proposed model was then used to predict the behaviour of a wall panel with a larger opening. Good correlation was achieved between the predicted behaviour of the larger wall panel and that observed in the laboratory. The next phase of the research will make use of optimisation software to obtain an improved tuning of the material parameters. Reinforcement will also be added to the computational model which will be used to assess the effectiveness of different forms of strengthening.

ACKNOWLEDGEMENTS

The work presented in this paper is supported by an EPSRC Comparative Award in Science and Engineering (CASE/CAN/07/22) and Bersche-Rolt Ltd. Particular thanks are due to George Edscer and Chris Smith of Bersche-Rolt Limited for their continued support and encouragement.

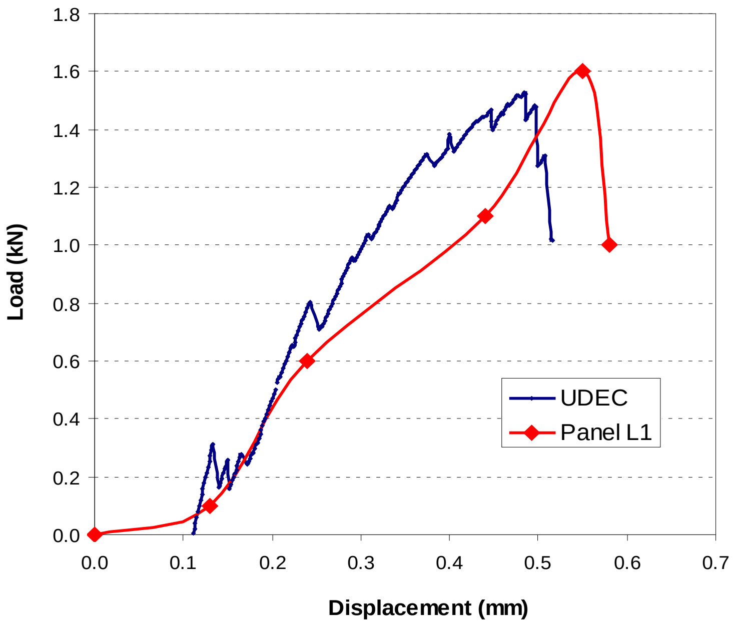

Figure 6. The experimental and UDEC predicted load vs displacement relationships for panel L1

REFERENCES

[1] Cundall P.A.: A computer model for simulating progressive, large scale movements in blocky rock systems, *Proceedings of the Symposium of the International Society of Rock Mechanics*, Nancy, France, 1971, Volume 1, paper II-8.

[2] ITASCA: *UDEC – Universal Distinct Elements Code Manual*. Theory and Background, Itasca consulting group, Minneapolis, USA, 2004.

[3] Schlegel, R. & Rautenstrauch, K.: *Failure analysis of masonry shear walls. Numerical modelling of discrete materials in engineering*, Bauhaus University Weimar, Taylor and Francis, Germany, 2004, 33-42.

[4] Rots, J. G.: Structural masonry: An experimental numerical basis for practical design rules, A.A. Balkema: Netherlands 1997, 96-106.

[5] Zhuge, Y.: Micro-modelling of masonry shear wall panels with distinct element approach. In: *Proceedings of the 17th Australian Conference on the Mechanics of Structures and Materials*, Gold Coast, Australia, 2002, 131-136.

[6] Toropov, V.V. & Garrity, S.W.: Material parameter identification for masonry constitutive models, *Proceedings of the Eighth Canadian Masonry Symposium*, Jasper, Alberta, Canada, 1998, 551-562.

[7] Lourenço, P. B. B.: *Computational strategies for masonry structures*. PhD-Thesis, Delft University of Technology, 1996. Delft University Press: Delft 1996.

[8] Hendry, A. W.: *Structural masonry*, 2nd edition, Palgrave Macmillan, London, UK, 1998.

[9] Lourenço, P. B. & Rots J. G.: A multi-surface model for the analysis of masonry structures. *Journal of Engineering Mechanics*, 123(7), 1997, 660-668.

[10] Idris J., Al-Heib M., Verdel T.: Numerical modelling of masonry joints degradation in built tunnels, *Tunnelling and Underground Space Technology*, 24(2009), 617-626.

[11] Sarhosis, V. Garrity, S. W. & Sheng, Y.: Distinct element modelling of masonry wall panels with openings. Paper no. 17 in: *Proc. of the 9th International Conference on Computational Structures Technology*, Athens 2008.