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Reply to the discussion by Khabbazian, M., Meehan, C.L., and Kaliakin, V.N., on "Geosynthetic-encased stone columns: Analytical calculation model" [Geotextiles and Geomembranes 29(1), 2011, pp. 29-32]

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The discussion presents some finite element results and comparisons investigating the capability of the proposed analytical method to yield predictions of the behaviour of encased and non-encased stone column reinforced ground. The main emphasis is on settlement reduction, stress concentration ratio and hoop forces in the geosynthetic encasement. The authors reported that there exists a disparity between the analytical model and the finite element results, which should be particularly evident for encased columns installed in very soft soil. As this is not consistent with previous studies and finite element analyses, which were performed during our research, some additional analyses were made in order to compare the results from analytical and finite element methods. The analyses were made by using the same data set as presented in the discussion. Therefore, the tensile stiffness of the geosynthetic (J), the depth of the unit cell (H), the diameter of the encased column (d_c) were assumed to be 3000 kN/m, 8.0 m, 0.8 m, respectively. Other model parameters, consistent with those used in the discussion, are presented in Table 1.

Table 1. Model parameters

Alternatively, the soil was considered as an elastic material in order to match the assumption of the analytical method and to analyze the effect of soil yield on the overall performance of the model.

The finite element analyses (FEM) were performed with commercial Plaxis finite element code for plane strain and axi-symmetric modelling of soil and rock (Brinkgrave, 2002). The unit cell was modelled in axi-symmetric condition by using high-order 15-node triangular elements for solids and 5-node linear elements for the geosynthetic encasement elements. Boundary conditions were as described in the original paper and small-strain theory was adopted for the analyses.

The initial stress state has crucial influence on the final results, as it determines the behaviour of the elasto-plastic model. Therefore, it was determined exactly as proposed in the analytical method by using "k_0" procedure with adopted stress ratios K_{ini} as shown in Table 1. In order not to introduce imbalance of the radial stresses along the soil/column interface, initial vertical stresses in the soil $\sigma_{zs,ini}(z) = \gamma_s z$, were multiplied by factor $K_{ini} = 0.8$ and vertical stresses in the column $\sigma_{zc,ini}(z) = \gamma_c z$ by $K_{ini} = 0.8 \left(\frac{\gamma_s}{\gamma_c}\right) = 0.533.$ Before any load was applied to the model, the "null" calculation step was performed to verify equilibrium and compliance of the calculated initial stresses with the analytical method. It is important to note that finite element programs differ and applying self-weight load (e.g. "gravity" loading) in the first calculation stage does not necessarily lead to initial stress state as proposed in the analytical method. In such cases direct stress initialization should be used instead.

Two different approaches were used to simulate the loading of the "unit-cell". For the first approach (rough conditions) 0.2 m thick weightless elastic concrete raft (E = 30 GPa) was placed on the top of the "unit-cell". To satisfy the assumption about equal settlements on the top of the "unit-cell", rotations of the raft at the inner and outer boundary of the calculation model were prevented and the raft was loaded with a load of 100 kPa. In order to simulate the perfectly rough conditions no interface elements were used between the raft, stone column and soil. The second approach (smooth conditions) was similar as the one proposed by the authors of the discussion. The vertical displacements were prescribed to the nodes on the top boundary of the model, while radial displacements were kept free in order to simulate perfectly smooth conditions. As vertical displacements were applied to the top of the model, the calculated reactive vertical force of the whole model was monitored until it reached the value of the equivalent load 100 kPa. This approach is similar to the one used in the discussion, where, accordingly, resultant stresses were recorded as opposed to the total reactive vertical force.

The comparison of vertical settlements for column spacing ratios ($\frac{d_s}{d_c}$) from 2 to 5 for three different sets of calculations are shown in Table 2.

Table 2. Comparison of vertical settlements for various column spacings

As expected, the calculated settlements for the perfectly smooth top boundary are almost the same as or just slightly higher than the values obtained for perfectly rough top boundary. It can also be seen that for the adopted data (possible) yield in soil has no influence on the calculated settlements. The absolute settlement differences were found to be between 0.42 and 0.5 cm with values of vertical settlements being 2-7% lower in the analytical solution (Table 2.) Obviously, these results are not in line with the values reported in the discussion where analytically obtained vertical settlements were found to be 10-24% lower as compared to finite element calculations using Abaqus code and 4-noded quadrilateral elements.

As higher disparity from 7 to 53% between calculated settlements obtained by analytical and finite element method was reported in the discussion for higher ratios of elastic modulus $\frac{E_C}{E_S}$, additional finite element analyses with the elastic modulus of the soil E_S varied to achieve ratios $\frac{E_C}{E_S}$ = 15, 30, 60, 120 and 240 were performed with constant spacing ratio $\frac{d_s}{d_c}$ = 3. The comparison of maximum vertical settlements is shown in Table 3.

Table 3. Comparison of vertical settlements for various ratios $\frac{E_C}{E_S}$

The absolute differences in the calculated displacements were found to be between 0.16 and 2.41 cm with values of vertical settlements being only 2 to 7% lower in the analytical solution. The differences in the calculated settlements are considerably lower than those presented in the discussion (7 to 53%).

Similarly, no such large disparities were found for stress concentration factors and hoop forces in the geosynthetic encasement. In the discussion the stress concentration factors on the top of the unit cell were calculated as the ratio of average stresses on top of the column to the average stress on top of the unit cell as obtained by finite element analyses. Such an approach seems reasonable, but can lead to unreliable results since the stresses at any selected cross-section are not exact, but interpolated within every single finite element. The accuracy of the stresses in a cross-section is therefore highly dependent on the type and size of the finite elements and the stress state within them (elastic or plastic). Additionally, to calculate the average stresses corresponding circular areas should also be taken into account.

A comparison of vertical stresses $\Delta\sigma_z$ on the cross-section at the top of the unit cell (*z* = 0 m) and at the depth *z* = 4 m according to the analytical method and as calculated by finite element method (perfectly rough conditions) for moduli ratios $\frac{E_c}{E_s}$ = 30 and 240 is shown in Figs. 1 and 2, respectively. In order to investigate the effect of finite element mesh density on the final results, two different meshes (coarse and fine) were adopted for the finite element analysis.

Fig. 1. Comparison of vertical stresses $\Delta\sigma_z$ ($\frac{E_c}{E_s}$ =30, $\frac{d_e}{d_c}$ = 3)

Fig. 2. Comparison of vertical stresses $\Delta\sigma_z$ ($\frac{E_c}{E_s}$ = 240, $\frac{d_e}{d_c}$ = 3)

Figs. 1 and 2 show that calculated vertical stresses in the soil, which remain predominantly in elastic state and represent 88.9% of the total surface area, are in close agreement with the analytical method. Vertical stresses in the stone column are in the range of the analytical solution, but heavily influenced by the density of the mesh and the interpolation of the stresses within yielded finite elements, with the calculated settlements almost the same regardless of the mesh density. From Figs. 1 and 2 it is also obvious that averaging/integration of calculated stresses in cross-sections can lead to unreliable results, especially if the analysis is displacement controlled and corresponding circular areas are not taken into account.

Radial deformations and hoop forces have already been discussed in the original paper (see Fig. 5 of the original paper), where the comparison of radial deformations, which are directly related to encasement hoop forces ($F_R = J\varepsilon_r$), is shown. As a consequence of column yield and strain localization (formation of shear bands in the stone column) in the finite element model the analytical method is not able to match the maximum (or minimum) radial deformations or hoop forces. However, the analytically determined radial deformations are found to be in good agreement when they are compared to the average deformations as obtained by finite element analyses. To demonstrate this, the hoop forces along the column depth as calculated by analytical and finite element method are shown in Fig. 3. The analyses were made for the basic set of data, constant spacing ratio $\frac{d_e}{d_c}$ = 3 and moduli ratios $\frac{E_c}{E_s}$ = 30 and 240. Perfectly rough conditions were applied and coarse mesh was used for the finite element analyses. Analytically calculated maximum hoop forces are found to be up to 20% smaller as compared to numerically obtained maximums. Again, the disparity in calculated hoop forces is considerably smaller than that in the discussion.

Fig. 3. Comparison of hoop forces ($\frac{d_e}{d_c}$ = 3)

As we were not able to reproduce the results from the discussion, some attention was paid to the 4-node quadrilateral element, which was used in the analyses presented in the discussion. The 4-node quadrilateral element is developed on the basis of bilinear interpolation of displacements in radial and vertical direction. Therefore, the radial deformation $\varepsilon_r = \frac{\partial u_r}{\partial r}$ varies linearly along the depth coordinate, but remains constant along the radius at all locations within the element and thus, it cannot exactly model the problems where $\varepsilon_r \propto r$. This can lead to the lack of accuracy for the computed stresses in cases where the mesh of finite elements is not fine enough (Buchanan, 1995). In our case, where the derivate $\frac{\partial u_r}{\partial r}$ is significant result as it affects the calculation of stresses, the finite element analysis can be improved significantly by using higher order elements. However, the disparity in the results is too high to be attributed only to the selected finite element and mesh density.

To sum up, we were not able to reproduce the results presented in the discussion. Our computational analyses have shown good agreement with the analytical method regardless of the type of loading (stress or displacement controlled analysis) and mesh density for various column spacings $\frac{d_e}{d_c}$ and modulus ratio $\frac{E_c}{E_s}$, as long as the computations were in accordance with the assumptions of the analytical method. Moreover, the analytical method was compared to the method of Raithel and Kempfert (2000) for non-encased stone columns in the original paper and both sets of results were in good agreement.

Since any errors in the Abaqus or Plaxis finite element codes are unlikely, there must be some difference in the calculation process and/or in the evaluation of results. Some possible causes for the disparity, such as generation of the initial stresses, evaluation of the final load for displacement controlled analysis and evaluation of stress concentration factors, are mentioned above, but only on the basis of the data presented in the discussion no clear conclusion on the causes of disparity in results is possible.

We would like to encourage all those who are interested in analytical and numerical modeling of the behaviour of stone columns to make their own numerical analyses and verify the analytical results as well as the results of numerical results from the paper and discussion.

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Table 1. Model parameters

Part	Model	γ (kN/m^3)	ϕ (deg.)	c (kPa)	ψ (deg.)	E (MPa)	ν	K_{ini}
Stone column	Mohr-Coulomb	22.5	40	0	0	60	0.3	0.533
Soil	Mohr-Coulomb	15	25	0	0	2	0.3	0.8

Table 2. Comparison of vertical settlements for various column spacings

Column spacing ratio ($\frac{d_e}{d_c}$)	Analytical method U_z (cm)	FEM (rough) U_z (cm)	FEM (smooth) U_z (cm)	FEM (rough) Elastic soil U_z (cm)
2	6.90	7.20	7.40	7.19
3	13.91	14.32	14.40	14.30
4	18.69	19.04	19.11	19.04
5	21.78	22.07	22.21	22.07

Table 3. Comparison of vertical settlements for various ratios $\frac{E_c}{E_s}$

Moduli ratio ($\frac{E_c}{E_s}$)	Analytical method U_z (cm)	FEM (rough) U_z (cm)	FEM (smooth) U_z (cm)	FEM (rough) Elastic soil U_z (cm)
15	8.44	8.56	8.60	8.55
30	13.91	14.32	14.40	14.30
60	20.83	21.67	21.70	21.67
120	27.87	29.52	29.30	29.33
240	33.57	35.98	35.60	35.94

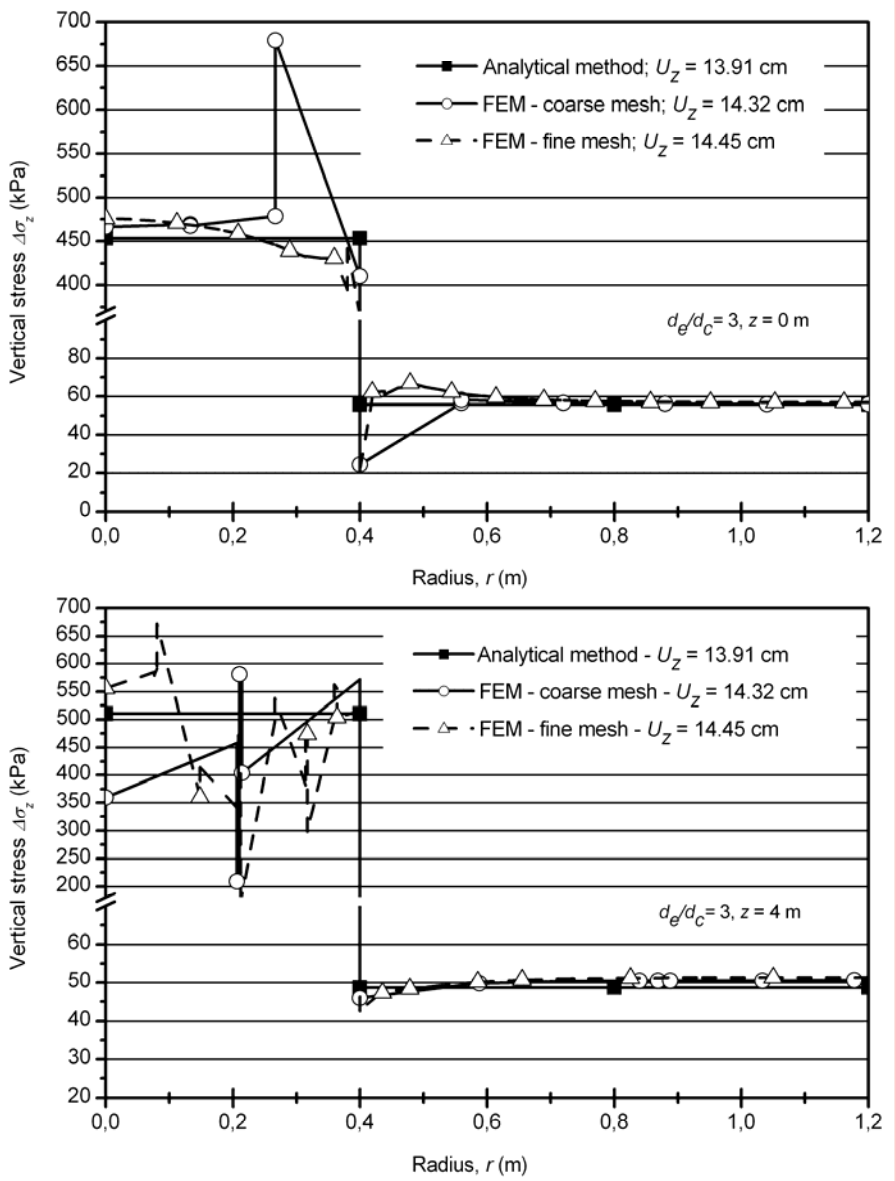

Fig. 1. Comparison of vertical stresses $\Delta\sigma_z$ ($\frac{E_c}{E_s}$ =30, $\frac{d_e}{d_c}$ = 3)

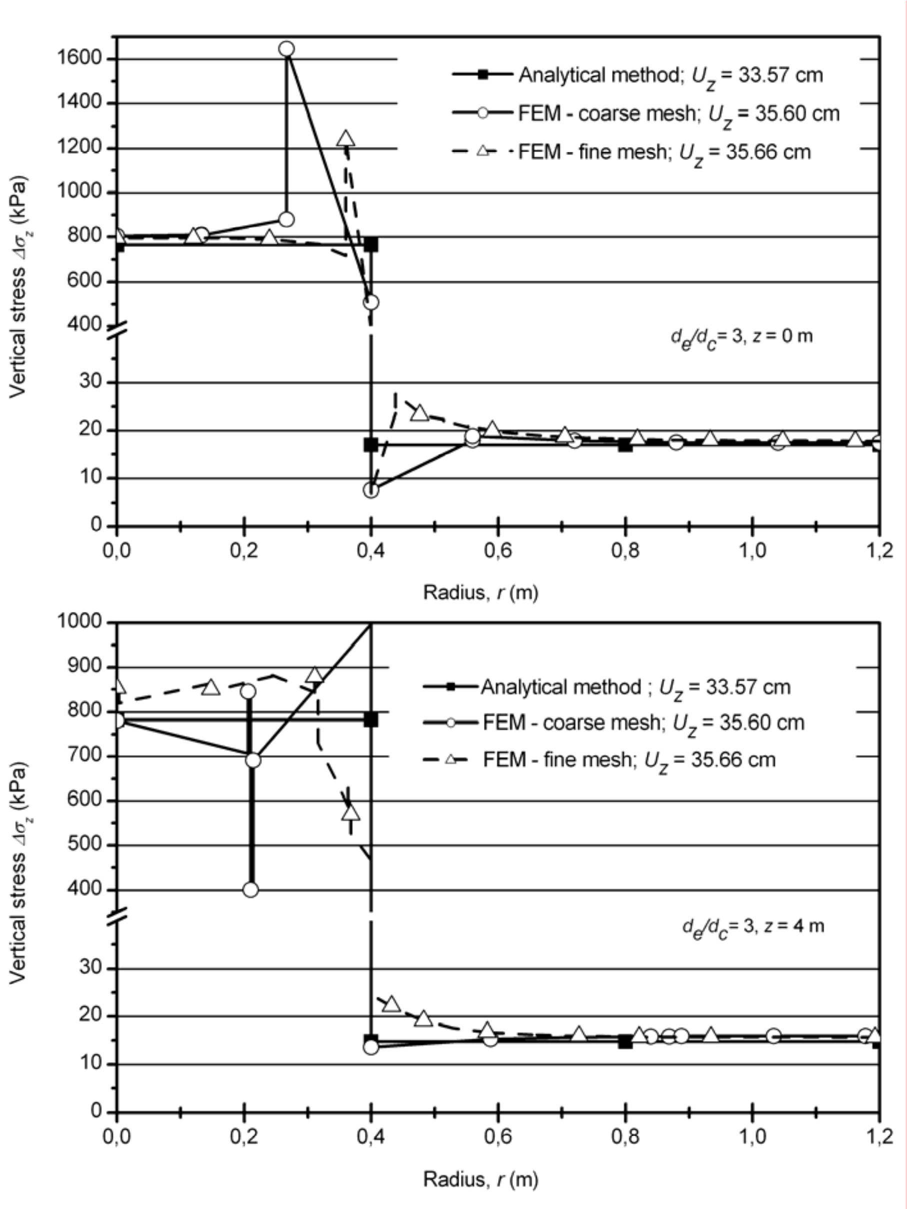

Fig. 2. Comparison of vertical stresses $\Delta\sigma_z$ ($\frac{E_c}{E_s}$ = 240, $\frac{d_e}{d_c}$ = 3)

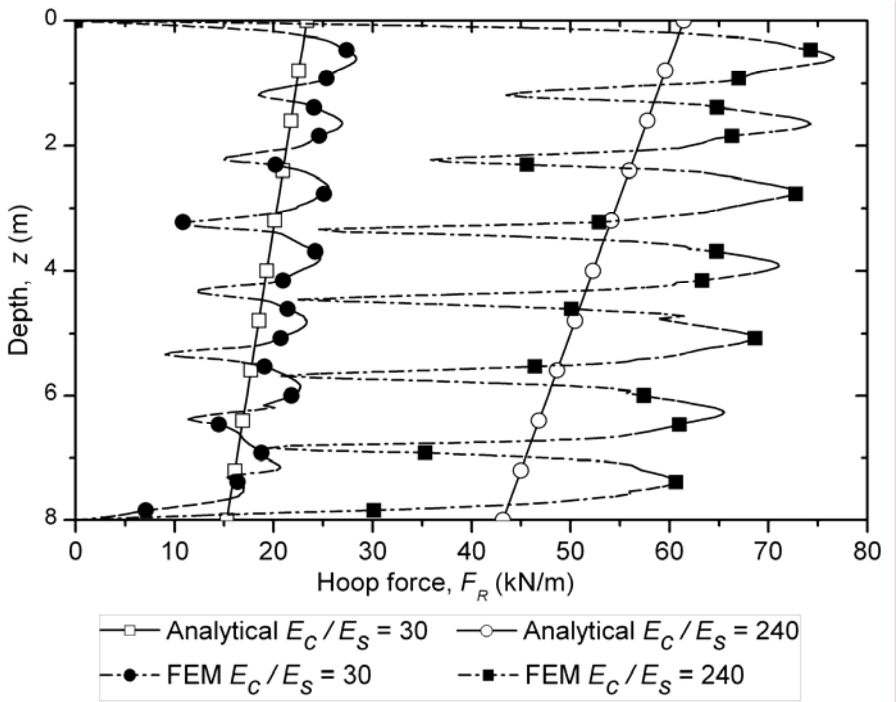

Fig. 3. Comparison of hoop forces ($\frac{d_e}{d_c}$ = 3)