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Determination of point displacements in the geodetic network

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Abstract

The article describes the procedure for testing the statistical significance of point displacements in the geodetic network as the intermediate stage between the adjustment of respective epochs measurements and an in-depth deformation analysis. The cumulative distribution function of the test statistic, presenting the relation between the displacement and the displacement accuracy, is determined by simulations. On the basis of this cumulative distribution function a critical value of the test statistic for a selected significance level is determined. In the null hypothesis it is assumed that the point is stable. A comparison of the critical value to the test statistic value is made and the actual risk level for rejecting the null hypothesis is estimated. Further on, a practical example of implementing the test in a simulated network is given. The test statistic proved to be simple and applicable: the points with significant displacements were identified successfully.

KEYWORDS: deformation analysis, simulations, hypothesis testing, significant displacements.

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1 Introduction

Basically, deformation analysis is the procedure for determining displacements of assumed stable points and determining significant displacements in geodetic networks. Inaccurate presumptions about assumed stable points in a geodetic network can bring about grave consequences in interpretation of established displacements or when predicting the downfall of buildings. In the process of identification of displacements, the test statistic is very important. A detailed knowledge of deformation analysis methods as well as practical experience are essential for an appropriate interpretation of the estimated point displacements.

In everyday use, the test for determining the statistical significance of a displacement is a function of the point displacement and the respective accuracy. The calculated value is then usually increased by a factor of safety of 3 or 5 or more, which makes the estimation of significant displacements too gross. For the proposed method, simulations of an actual probability distribution function are determined, providing the basis for calculating the right critical value at a chosen significance level. In this way, statistically significant point displacements may be determined far more accurately.

When assessing point displacements, the information on the actual risk of making the error when rejecting the true null hypothesis is very useful and a calculation of this value is advisable. Based on the assumption that the distribution function is established in detail, the suggested test statistic is simple and fit for day-to-day use and refers to the first estimation of the geodetic network. Therefore, it can be carried out right after a two-epoch adjustment and accordingly, the need for carrying out the deformation analysis is identified.

2 Single epoch analysis

For identification of point displacement by way of geodetic observations, the reference points need to be chosen. Characteristic points on the object are tested for displacements. According to the required accuracy of point displacement determination, the execution of observations must be carried out carefully with proper tools while following standard observational approaches. The observations in the geodetic network are adjusted and the network quality estimated.

Importantly, in networks for displacement identification a network quality estimation is carried out prior to the measurements examining the accuracy, reliability, sensitivity and the cost effectiveness of setting up a network (Caspary 2000). In identification of displacements, network reliability and sensitivity are of primary importance, thus great effort must be made in detecting the presence of undisclosed gross errors. In the planning and optimization phase, the sensitivity of observations needs to be enabled, thereby increasing the probability of detecting outliers.

A well projected network for displacement detection should enable a high degree of detection and elimination of gross errors in observations as well as minimize the effect of potentially undetected outliers influencing the unknowns. Testing the relation between the a posteriori variance $\hat{\sigma}_0^2$ and the a priori reference variance σ_0^2 is called the *global model hypothesis testing*. At the same time, the presence of gross error observations in the network is tested, which is in turn possible only by having a reliable knowledge of the a priori reference variance. In case of incongruence between the observations and the model in the course of the global testing, the Baarda's Data Snooping method for examination, detection and elimination of outliers in observations is introduced. The Pope's Data Screening approach or the Danish approach is used when the a priori reference variance is not reliably known.

After a careful analysis and quality estimation of single epochs, the displacements are estimated and the accuracy of estimating the two-epoch displacements is calculated. In everyday engineering work the difference estimation of point positions between two epochs provides a sufficient amount of information on displacements. This is applicable with a sufficient number of stable points and with displacement that are several times the size of the displacements standard deviations. However, in specific and precise geodynamic research the implementation of a detailed deformation analysis according to one of the several known approaches is essential (i. e. the Delft, Fredericton, Hannover, Karlsruhe, München method etc.).

3 Testing the significance of displacements

The basis for displacement determination of a man-made object or any given object on the surface of the earth is to identify the displacements of characteristic points of an object. The points comprise networks, which are monitored in time intervals called *epochs* that are set out in advance. The point displacements between two epochs can be inferred only from *identical points*, measured in two epochs. However the points are often damaged or they have to be included into the network due to changes of circumstances. Non-identical points are eliminated in the adjustment procedure or with S-transformation, respectively (Mierlo 1978). After the two-epoch adjustment the point displacements and its standard deviations are estimated.

3.1 Displacement estimation and displacement accuracy estimation

In geodetic networks set up for determining displacements, the requirement that standard deviations for displacements of geodetic points be provided is very essential. If the estimated displacements are several times the size of the displacement standard deviations, the most probable displacements can be inferred from the differences in point positions. In addition to determining the magnitude and direction of the displacements, the hypothesis testing for the displacemrnt is also necessary. Consequently, these corresponding calculations must be performed.

Point displacements are determined on the basis of comparing point coordinates in two epochs. Let us assume the point coordinates $T(y,x)$ in a plane and time t and $t+\Delta t$. In order to calculate the estimation accuracy of point displacements the covariance matrix of point coordinates for respective epochs must be known. $T_t(y_t,x_t)$ represents the position of point T in time t, and $\mathbf{\Sigma}_t$ is the corresponding covariance matrix, and $T_{t+\Delta t}(y_{t+\Delta t},x_{t+\Delta t})$ represents the coordinates of point T in time $t+\Delta t$ with the corresponding covariance matrix $\mathbf{\Sigma}_{t+\Delta t}$. This can be expressed as

$$\mathbf{\Sigma}_{T_t} = \begin{bmatrix} \sigma_{y_t}^2 & \sigma_{y_t x_t} \\ \sigma_{y_t x_t} & \sigma_{x_t}^2 \end{bmatrix} \quad \text{and} \quad \mathbf{\Sigma}_{T_{t+\Delta t}} = \begin{bmatrix} \sigma_{y_{t+\Delta t}}^2 & \sigma_{y_{t+\Delta t} x_{t+\Delta t}} \\ \sigma_{y_{t+\Delta t} x_{t+\Delta t}} & \sigma_{x_{t+\Delta t}}^2 \end{bmatrix}.$$

We assume that the coordinates in time t are not correlated with the coordinates in time $t + \Delta t$. Thus, the covariance matrix of coordinates of identical points y_t, x_t, $y_{t+\Delta t}, x_{t+\Delta t}$ can be written as

$$\mathbf{\Sigma}_{T_t T_{t+\Delta t}} = \begin{bmatrix} \sigma_{y_t}^2 & \sigma_{y_t x_t} & 0 & 0 \\ \sigma_{y_t x_t} & \sigma_{x_t}^2 & 0 & 0 \\ 0 & 0 & \sigma_{y_{t+\Delta t}}^2 & \sigma_{y_{t+\Delta t} x_{t+\Delta t}} \\ 0 & 0 & \sigma_{y_{t+\Delta t} x_{t+\Delta t}} & \sigma_{x_{t+\Delta t}}^2 \end{bmatrix}. \tag{1}$$

The displacement of point T may be evaluated as

$$d = \sqrt{\Delta y^2 + \Delta x^2} = \sqrt{\left(y_{t+\Delta t} - y_t\right)^2 + \left(x_{t+\Delta t} - x_t\right)^2} \ . \tag{2}$$

Further on the displacement variance is determined by

$$\sigma_d^2 = \mathbf{J}_d \mathbf{\Sigma}_{T_t T_{t+\Delta t}} \mathbf{J}_d^T , \tag{3}$$

where the Jacobi matrix $\mathbf{J}_d$ equals:

$$\mathbf{J}_d = \begin{bmatrix} \dfrac{\partial d}{\partial y_t} & \dfrac{\partial d}{\partial x_t} & \dfrac{\partial d}{\partial y_{t+\Delta t}} & \dfrac{\partial d}{\partial x_{t+\Delta t}} \end{bmatrix} = \begin{bmatrix} -\dfrac{\Delta y}{d} & -\dfrac{\Delta x}{d} & \dfrac{\Delta y}{d} & \dfrac{\Delta x}{d} \end{bmatrix}. \tag{4}$$

By inserting the equations (1) and (4) into equation (3) we get the representation for displacement variance of point T

$$\sigma_d^2 = \left(\frac{\Delta y}{d}\right)^2 \left(\sigma_{y_t}^2 + \sigma_{y_{t+\Delta t}}^2\right) + 2\frac{\Delta y}{d}\frac{\Delta x}{d}\left(\sigma_{y_t x_t} + \sigma_{y_{t+\Delta t} x_{t+\Delta t}}\right) + \left(\frac{\Delta x}{d}\right)^2 \left(\sigma_{x_t}^2 + \sigma_{x_{t+\Delta t}}^2\right), \quad (5)$$

that is used for testing displacements by a test statistic given in equation (6) described in the next section.

3.2 Determining the distribution function of test statistic with simulations

In deformation analysis single epochs are usually adjusted as free networks. In this way the best linear unbiased estimation of the unknowns and independence of test statistic regarding the chosen network datum is enabled. After adjusting at least two epochs it is possible to determine the displacement of point d according to equation (2) and standard deviation of displacement σ_d according to equation (5). Since these two parameters can be calculated prior to a detailed deformation analysis, they are rightly used in the statistical testing.

When estimating displacements the test statistic is often calculated as:

$$T = \frac{d}{\sigma_d} \quad (6)$$

and compared to the critical value according to the chosen significance level α. Point displacements are established with an appropriate probability only when the displacements are significantly larger than the estimation accuracy of displacements.

Assuming that the errors of observations are distributed normally $\varepsilon \sim N(0,\sigma^2)$, then the parameters being the linear functions of the observations $\hat{\mathbf{x}} \sim N(\mu_{\hat{\mathbf{x}}},\sigma_{\hat{\mathbf{x}}}^2)$ are distributed normally as well. The point displacement is calculated with equation (2). Since Δy and Δx are calculated as the difference of two normally distributed random unknowns, the Δy and Δx are distributed normally, too. This, however, is not the case with point displacement d, which is a nonlinear function of Δy and Δx. Consequently, it is difficult to analytically determine the form and the type of the distribution of the test statistic (6). The distribution function for the discussed test statistic is therefore determined by simulations (Rubinstein 1981; Savšek-Safić 2002).

The coordinates differences Δy and Δx are normally distributed random variables with variance-covariance matrix as follows:

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_{\Delta y}^2 & \sigma_{\Delta y \Delta x} \\ \sigma_{\Delta y \Delta x} & \sigma_{\Delta x}^2 \end{bmatrix}. \tag{7}$$

The standard deviations of coordinates differences in two epochs are calculated as

$$\begin{aligned} \sigma_{\Delta y} &= \sqrt{\sigma_{y_t}^2 + \sigma_{y_{t+\Delta t}}^2} \\ \sigma_{\Delta x} &= \sqrt{\sigma_{x_t}^2 + \sigma_{x_{t+\Delta t}}^2} \end{aligned}, \tag{8}$$

where $\sigma^2_{y_t}, \sigma^2_{y_{t+\Delta t}}, \sigma^2_{x_t}, \sigma^2_{x_{t+\Delta t}}$ are coordinate variances of $y_t, y_{t+\Delta t}, x_t, x_{t+\Delta t}$. The covariance is calculated as:

$$\sigma_{\Delta y \Delta x} = \sigma_{y_t x_t} + \sigma_{y_{t+\Delta t} x_{t+\Delta t}}, \tag{9}$$

where $\sigma_{y_t x_t}$ and $\sigma_{y_{t+\Delta t} x_{t+\Delta t}}$ are covariances of the coordinates in both epochs.

The basic idea for generating a sample of dependent normally distributed random variables is to generate a sample of *independent* normally distributed random variables and then use a linear transformation to obtain a sample of *dependent* random variables.

For generating the sample of the normally distributed random variables the Box and Müller approach was applied (Box et al., 1958; Press et al., 1992). Let us assume that u_{1i} and u_{2i}, $i = 1,...,n$ are samples of two independent and uniformly distributed random variables U_1 and U_2, and n is the number of simulations. The sample of two independent normally distributed random variables Z_1 and Z_2 is calculated as follows:

$$\mathbf{z}_i = \begin{bmatrix} z_{1i} \\ z_{2i} \end{bmatrix} = \begin{bmatrix} \sqrt{-2\ln u_{1i}}\,\sin(2\pi\, u_{2i}) \\ \sqrt{-2\ln u_{1i}}\,\cos(2\pi\, u_{2i}) \end{bmatrix}, \quad i = 1,...,n. \tag{10}$$

For generating a sample of dependent normally distributed random variables a linear transformation is needed. The variance-covariance matrix $\mathbf{\Sigma}$ is decomposed by Cholesky decomposition

$$\mathbf{\Sigma} = \mathbf{U}^T \mathbf{U} . \tag{11}$$

In our case $\mathbf{U}$ takes the following form

$$\mathbf{U} = \begin{bmatrix} \sigma_{\Delta y} & \dfrac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y}} \\ 0 & \sigma_{\Delta x} \sqrt{1 - \left(\dfrac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y} \sigma_{\Delta x}} \right)^2} \end{bmatrix} . \tag{12}$$

For transformation of a sample of independent normally distributed random variables to a sample of dependent random variables the linear transformation

$$\mathbf{y}_i = \mathbf{U}^T \mathbf{z}_i , \quad i = 1,...,n \tag{13}$$

is used.

In our case the coordinate differences are generated by the following equations

$$\begin{aligned} \Delta y_i &= z_{1i} \sigma_{\Delta y} \\ \Delta x_i &= z_{1i} \frac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y}} + z_{2i} \sigma_{\Delta x} \sqrt{1 - \left(\frac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y} \sigma_{\Delta x}} \right)^2} \end{aligned} \tag{14}$$

where it is assumed that the means of Δy and Δx are zero ($\mu_{\Delta y} = \mu_{\Delta x} = 0$) and $i = 1,...,n$.

The standard deviations of point coordinates in respective epochs vary from point to point. Therefore the distribution function of the test statistic (6) takes on a different form for each point in each two epochs. By using the simulated normally distributed random variables (14), d is calculated using equation (2) and σ_d using equation (5). Consequently in n simulations, this procedure allows us to determine the empirical cumulative probability distribution function of the test statistic (6) for individual points.

Critical value T_{crit} and actual risk α_T are determined from obtained empirical cumulative distribution function by the following procedure (see Figure 1):

1. generate coordinate differences Δy_i, Δx_i; $i = 1,...,n$ (Equation 14)
2. calculate displacement d_i (Equation 2), its standard deviation σ_{d_i} (Equation 5) and test statistic T_i; $i = 1,...,n$ (Equation 6)
3. form empirical cumulative probability distribution function F_T^* by sorting T_i;

   $F_T^*(T_i) = \frac{i}{n}$; $T_i \leq T_{i+1}$
4. determinate critical value T_{crit} from F_T^*: $T_{crit} = T_{i-\lceil(1-\alpha)n\rceil}$

   or

   determinate actual risk α_T from F_T^*: $\alpha_T = 1 - \frac{i}{n}$ for such i that $\min T_i > T$.

Figure 1: Empirical cumulative distribution function of the test statistic $T = d / \sigma_d$

The test statistic is then tested according to the given null hypothesis and its alternative hypothesis:

H_0 : $d = 0$; the point is stable between two epochs, and

H_a : $d \neq 0$; the point has changed its position.

The test statistic (6) is compared to critical value acquired from empirical cumulative distribution function. If the test statistic value is smaller than the critical value at a chosen significance level α, then the risk of rejecting the true null hypothesis is too high. Accordingly, it is established that the displacement is not statistically significant. If the test statistic value exceeds the critical value, the risk of rejecting the true null hypothesis is lower than the chosen significance level α. Therefore, the null hypothesis is rightly rejected and the statistical significance of the displacement is thereby confirmed.

This decision is supported by calculating the actual risk α_T of rejecting the true null hypothesis (the probability of committing *Type I Error*). Two possibilities are examined:

- $T > T_{crit}$ i.e. $\alpha_T < \alpha$: the null hypothesis is rejected; the point displacement is statistically significant and
- $T < T_{crit}$ i.e. $\alpha_T > \alpha$: the null hypothesis is not rejected; the point displacement is statistically non-significant.

Regarding the actual risk and the consequences of making the wrong decision, it is up to the user to decide upon the risk level of acceptability. As a consequence, a point is thereupon considered as stationary or displaced.

4 Case example of significant displacement testing in a test network

In this case a simple test network is established. The displacements of points 1, 2, 3 and 7 are assumed as known. The observations are generated as independent normally distributed with standard deviation of $\sigma_{\alpha} = 1''$ for angle observations and $\sigma_{s} = 5\,mm$ for distance observations (see Table 1). The geodetic datum of the network is determined as a free network datum. Two epochs are examined with identical types and number of observations (see Figure 2). In the procedure of testing the null hypothesis $H_0 : d = 0$ and significance level $\alpha = 5\%$ are chosen. The empirical cumulative distribution functions are generated by the Premik software (Ambrožič et al 2002) for each point, where the number of simulation is set to 100000. The simulation is carried out on the basis of 100000 iterations. The existing Premik software was enhanced by adding hypothesis testing which enables the user to determine the statistical significance of the displacement of a particular point. The calculated displacements are compared to the known values. In the following section all the necessary input data for adjustments as well as the adjusted values of point coordinates in single epochs are given.

Table 1: Simulated observations of two epochs

Table 2: Known displacements of points 1, 2, 3 and 7 between two epochs

Figure 2: Test network and displacements

Table 3: Approximate coordinates equal in both epochs

Table 4: Point coordinates adjustment in a free network adjustment of single epochs

The empirical cumulative distribution function is determined by simulations for the test statistic (6) for each point. Figure 3 shows the cumulative distribution function calculated for Point 2 in a test network. The cumulative distribution function is different for each network point. Regarding Point 2, Figure 3 illustrates the critical value $T_{crit} = 2.384$ at the chosen significance level $\alpha = 5\%$. If the values of T_{crit} are taken to be 3 or 5 as are generally used as a "rule of thumb", the actual risk is $\alpha_{T(3)} = 1.18\%$ and $\alpha_{T(5)} = 0.00\%$, respectively. Thus the actual risk α_T at rejecting the true null hypothesis is set to minimum.

Figure 3: Distribution function of test statistic for Point 2: $T = d / \sigma_d = 4.724$

Importantly, in the testing procedure one must compare the calculated value of the test statistic to the critical value, T_{crit} of the test statistic (6) at a chosen significance level. In the test network presented in this paper (Figure 2) the critical values calculated at the significance level of $\alpha = 5\%$ ranged from 2.376 to 2.894 (Table 5).

Table 5: Significance displacement testing in a test network

As inferred from Table 5, the displacements of statistical significance are undoubtedly present at Points 1 and 7, since $T > 10$. The test statistic value is considerably higher than its critical value, therefore the actual risk α_T of rejecting the true null hypothesis is minimal. The suggested test statistic reveals a displacement at Point 2, since $T > 4$ with minimal actual risk of rejecting the true null hypothesis. The actual displacement at Point 3 is not big enough to be statistically significant, since $T < 2$. The actual risk of rejecting the true null hypothesis at Point 3 is $\alpha_T = 24.66\%$, thus the displacement is not revealed, owing to statistical non-significance. The actual risks for rejecting the true null hypothesis at assumingly stable points 4, 5 and 6 exceed 30% which is substantially more than the chosen significance level $\alpha = 5\%$. Thus, it is not possible to claim that the points had moved.

As illustrated, the critical values for individual points are not equal. Therefore, it is of great importance to determine the distribution function of the test statistic accurately for each network point and to avoid the indiscriminate use of those critical values that are used most frequently.

5 Conclusion

A contractor of geodetic works is expected to present not only data on point displacements, but also to provide insurance in terms of the quality of displacement estimation. In addition to the assumed null hypothesis $H_0 : d = 0$ and the chosen significance level $\alpha = 5\%$, the actual risk of rejecting the true null hypothesis is crucial. The participation of the commissioning party in the process evaluating the estimated displacements is highly recommended. The decision upon risk acceptability is then in the hands of the commissioner.

As has been shown, test statistic (6) along with the empirical cumulative distribution function are appropriate tools for testing the significance of point displacements in a geodetic network. Since the displacement and its respective accuracy are acquired by a simple method, the suggested procedure is appropriate and provides good results that furnish a good first estimate of the situation in the discussed network. The test example illustrates that the estimation of displacement significance is directly dependent upon the critical value at a chosen significance level α. An accurate displacement estimation is achieved only if the critical value is determined according to the actual distribution function of the test statistic. This is a considerable advance with respect to the "rule of thumb" values for $T_{crit} = 3$ to 5 which were generally used in practical analyses. Having in mind the difficulty level of the assignment and its consequences, the decision must be made whether there is the need for a detailed deformation analysis carried out by one of the known approaches.

Notation

The following symbols are used in this paper:

d	=	displacement of point T between two epochs
H_a	=	alternative hypothesis
H_0	=	null hypothesis
T	=	actual value of test statistic
T_{crit}	=	critical value of test statistic
F_T^*	=	empirical cumulative distribution function
n	=	number of simulation repetitions
i	=	simulation index
$T_t(y_t, x_t)$	=	coordinates of point T in time t
$T_{t+\Delta t}(y_{t+\Delta t}, x_{t+\Delta t})$	=	coordinates of point T in time $t+\Delta t$
t	=	time index of 1st epoch
$t+\Delta t$	=	time index of 2nd epoch
U_1, U_2	=	sample of two independent uniformly distributed random variables
Z_1, Z_2	=	sample of two normally distributed random variables
α	=	significance level
α_T	=	actual risk of rejecting the true null hypothesis
Δy , Δx	=	coordinates differences between two epochs
Δy_i , Δx_i	=	simulated coordinates differences between two epochs
$\mathbf{\Sigma}_t$	=	variance-covariance matrix of point coordinates in time t
$\mathbf{\Sigma}_{t+\Delta t}$	=	variance-covariance matrix of point coordinates in time $t+\Delta t$

$\mathbf{\Sigma}_{T_t T_{t+\Delta t}}$ = variance-covariance matrix of coordinates of identical points in time t and $t+\Delta t$

$\mathbf{\Sigma}$ = variance-covariance matrix of coordinates differences Δy, Δx

σ_0^2 = a priori reference variance

$\hat{\sigma}_0^2$ = a posteriori reference variance

σ_d^2 = displacement variance of point T between two epochs

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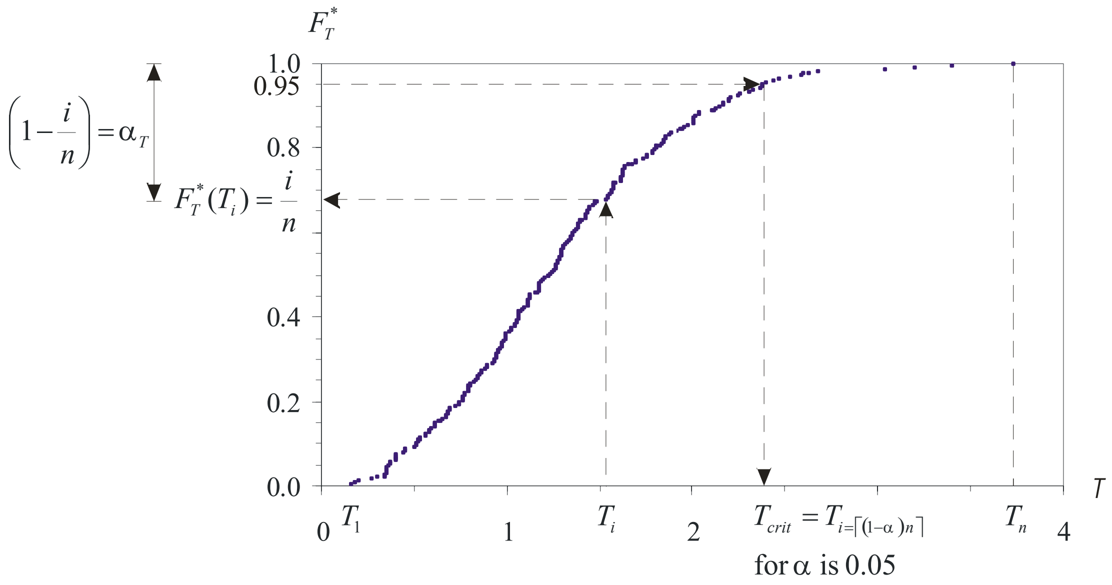
Figure captions

Figure 1: Empirical cumulative distribution function of the test statistic $T = d / \sigma_d$

Figure 2: Test network and displacements

Figure 3: Distribution function of test statistic for point 2: $T = d / \sigma_d = 4.724$



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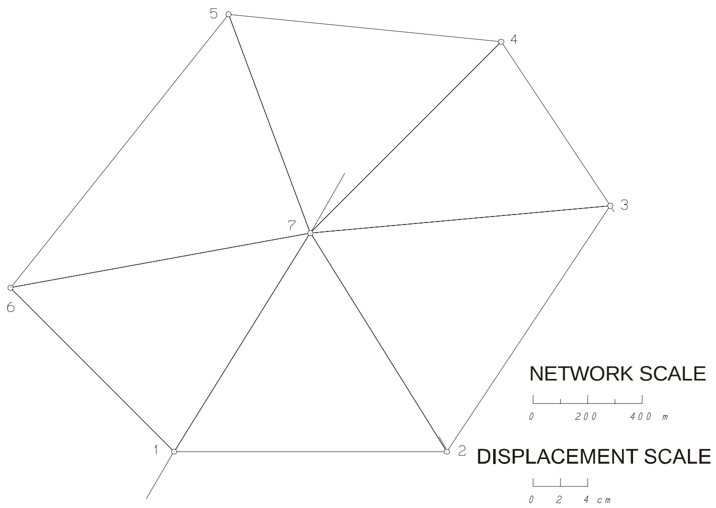

Figure 2: Test network and displacements

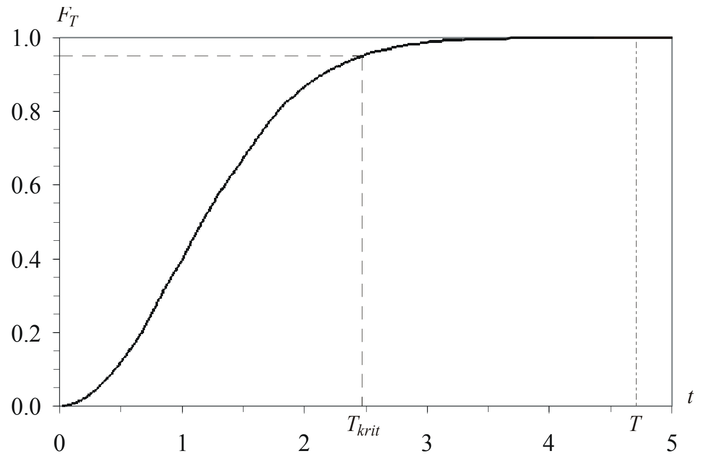

Figure 3: Distribution function of test statistic for Point 2: $T = d / \sigma_d = 4.724$

Table 1: Simulated observations of two epochs

Point		Null epoch				Epoch 2			
From	To	Direction			Length	Direction			Length
		0	'	"	[m]	0	'	"	[m]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	6	314	59	58.6	848.5203	315	00	08.3	848.5437
1	7	32	00	18.4	943.4058	32	00	18.0	943.4930
1	2	90	00	00.6	1000.0017	89	59	48.8	1000.010
2	1	269	59	58.1	1000.0077	269	59	50.2	1000.003
2	7	327	59	41.6	943.3963	327	59	50.8	943.4170
2	3	33	41	24.9	1081.6692	33	41	27.8	1081.660
3	2	213	41	23.2	1081.6572	213	41	27.7	1081.666
3	7	264	48	19.6	1104.5400	264	48	28.5	1104.507
3	4	326	18	35.0	721.1132	326	18	35.0	721.1192
4	3	146	18	33.4	721.1152	146	18	34.9	721.1152
4	7	224	59	59.9	989.9525	225	00	00.3	989.9073
4	5	275	42	39.1	1004.9917	275	42	37.1	1004.999
5	4	95	42	37.9	1004.9861	95	42	36.1	1004.986
5	7	159	26	39.7	854.4009	159	26	29.0	854.3696
5	6	218	39	36.1	1280.6231	218	39	35.9	1280.621
6	5	38	39	35.0	1280.6242	38	39	34.6	1280.626
6	7	79	41	43.7	1118.0403	79	41	36.3	1118.074
6	1	134	59	59.5	848.5338	135	00	10.4	848.5325

7	6	259	41	42.2	1118.0366	259	41	36.6	1118.068
7	5	339	26	38.3	854.4000	339	26	28.6	854.3591
7	4	45	00	00.9	989.9507	45	00	03.6	989.8993
7	3	84	48	21.1	1104.5387	84	48	29.6	1104.505
7	2	147	59	40.6	943.3984	147	59	50.6	943.4008

Table 2: Known displacements of points 1, 2, 3 and 7 between two epochs

Point (1)	Displacement - d [mm] (2)	Azimuth - ν [^{0}] (3)
1	40	210
2	12	330
3	5	150
7	50	30

Table 3: Approximate coordinates equal in both epochs

Point	Approximate coordinates	
	y_0	x_0
(1)	(2)	(3)
1	1000.0000	1000.0000
2	2000.0000	1000.0000
3	2600.0000	1900.0000
4	2200.0000	2500.0000
5	1200.0000	2600.0000
6	400.0000	1600.0000
7	1500.0000	1800.0000

Table 4: Point coordinates adjustment in a free network adjustment of single epochs

Point	Null epoch		Epoch 2		Coordinate difference	
	$\hat{y}_1$ [m]	$\hat{x}_1$ [m]	$\hat{y}_2$ [m]	$\hat{x}_2$ [m]	$d_{\hat{y}}$ [m]	$d_{\hat{x}}$ [m]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	999.9988	999.9995	999.9821	999.9599	-0.0167	-0.0396
2	2000.0013	1000.0012	1999.9899	1000.0085	-0.0114	+0.0073
3	2600.0037	1899.9984	2600.0039	1899.9942	+0.0002	-0.0042
4	2200.0004	2500.0000	2200.0015	2500.0007	+0.0011	+0.0007
5	1199.9988	2600.0007	1199.9983	2599.9966	-0.0005	-0.0041
6	399.9973	1599.9989	399.9991	1599.9972	+0.0018	-0.0017
7	1499.9997	1800.0013	1500.0252	1800.0429	+0.0255	+0.0416

Table 5: Significance displacement testing in a test network

Point	Simulated displacement		Actual displacement d [mm]	σ_d [mm]	T	T_{crit}	α_T (%)
	d_{sim} [mm]	Displacement					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	40.0	yes	**43.0**	2.7	15.931	2.382	**0.00**
2	12.0	yes	**13.5**	2.9	4.724	2.384	**0.00**
3	5.0	yes	4.2	2.6	1.646	2.391	24.66
4	0.0	no	1.3	2.6	0.499	2.894	88.22
5	0.0	no	4.1	2.8	1.466	2.376	32.66
6	0.0	no	2.5	2.7	0.903	2.384	65.55
7	50.0	yes	**48.8**	1.9	25.838	2.387	**0.00**