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A rationale for the payback criterion An application of almost stochastic dominance to capital budgeting

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A rationale for the payback criterion

–An application of almost stochastic dominance to capital budgeting–

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Abstract: Textbooks on financial management have emphasized the shortcomings of the payback criterion for decades. However, empirical evidence suggests that in actual capital budgeting procedures the payback method is used quite regularly. Mostly, it is implemented supplementary to net present value or internal rate of return, but small companies tend to rely on payback times as single criterion. A convincing theoretical foundation for the observed use of the payback criterion is lacking.

Consequently, our goal is to provide such an explanation for the payback criterion's popularity. We demonstrate from a decision theoretical perspective how relying on payback times simplifies investment decisions in modern organizations. Gathering information from different management levels and ensuring the utilization of individual skills requires a multi-stage capital budgeting process. Accordingly, we consider fundamental organizational features of this process with respect to their impact on the payback method's use.

For this purpose, we built upon almost stochastic dominance (ASD) as modeling device. Firstly, we show that applying this concept allows to include the risk preferences of all relevant decision makers into the analysis. Secondly, we illustrate that the criteria derived from this model help conveying these preferences to those who do the preparatory work preceding the final decision. To some extent, these new criteria are generalizations of payback times. This finding provides a potential explanation for the payback's persisting prominence.

Key Words: capital budgeting, decision procedures, investment appraisal, net present value, payback method, almost stochastic dominance, risk aversion, uncertainty

JEL classification: G31, D81, (D 70)

1 Introduction

According to the *Bureau of Economic Analysis* in 2005 the "[w]orldwide capital expenditures by U.S. [multinational companies] totaled $ 478.1 billion; capital expenditures in the United States by U.S. parents accounted for $ 340.8 billion ... ."[1] Nevertheless, investment activities' relevancy does not merely result from the volume of capital spent but from their impact on profitability as well. A recent magazine article explains the unprecedented record earnings of *Fortune* 500 companies in 2006 by "[t]he productivity improvement ... partially due to the capital expenditure boom of the '90s."[2] Hence, capital budgeting decisions are among the most important ones made by financial managers.[3] Most companies, especially divisionalized ones, have installed centralized capital budgeting procedures to structure these resource allocation decisions and to enforce company policies.[4] Here, standardized key figures are implemented to communicate comparable investment proposals over all divisions supporting top-level management decisions.

Despite huge academic effort dedicated to developing theoretical concepts and practical procedures for enhancing efficiency and effectiveness of capital budgeting decisions[5], a considerable gap between theory and practice remains.[6] Recent studies confirm that –in contrast to recommendations from financial theory– the payback criterion still enjoys great popularity among practitioners. For instance, *Graham/Harvey* (2002) state that in their sample "... the payback period was the most frequently used capital budgeting technique (...). This result is surprising in the sense that financial textbooks have stressed the shortcomings of the payback criterion for decades ..."[7] In fact many companies use payback time supplementary to net present value or internal rate of return.[8] This kind of use seems to become even more important over time.[9] From a theoretical point of view the wide-spread appli-

[1] Bureau of Economic Analysis (2007, p. 1).

[2] Tully (2007).

[3] Cf. Ryan/Ryan (2002, p. 355), Klammer et al. (1991, p. 113).

[4] Cf. Ryan/Ryan (2002, p. 358), Ann et al. (1987, p. 112).

[5] In this sense Pinches (1982, p. 6) states: "Tremendous strides have been made by both the business and academic communities in terms of our understanding and acceptance of capital budgeting techniques."

[6] Cf., e.g., Arnold/Hatzopoulos (2000, p. 622), O'Brien (1997, p. 173), Kim et al. (1986, p. 50).

[7] Graham/Harvey (2002, pp. 11).

[8] Cf. Lefley (1996, p. 208).

[9] Cf. Yard (2000, p. 155) who states: "[The payback method's] use as a single criterion seems to have decreased over time, ... [but] it is still commonly used as a secondary measure. In fact this type of use seems to have increased."

cation of the payback method is remarkable. As a matter of fact, no consensus exists which forces are driving this phenomenon. Some previous papers try to explain the payback method's popularity by interpreting it as a means of risk controlling or liquidity planning. Further, some authors argue that the payback method's simplicity in combination with management's lack of familiarity with more sophisticated methods of investment appraisal results in the persistence of this rule of thumb.[10] However, a convincing theoretical foundation for the observed use of the payback criterion is still lacking.

Accordingly, our goal is to provide an explanation for the payback criterion's popularity. A major contribution of our approach is to recognize the importance of fundamental organizational aspects, like delegation or hierarchical subordination, for investment decision processes.[11] This contrasts with previous theoretical contributions attempting to justify the application of the payback criterion, because they concentrate on a single decision maker.[12] Hence, these papers do not capture a key issue of modern capital budgeting procedures – the interaction of managers from different hierarchy levels and professional backgrounds. Due to the complexity of modern investment projects many real world capital budgeting decisions require expertise from various different areas like, e.g., technology, engineering, sales, finance, and sociology. Moreover, preparatory tasks, like data gathering, preparing proposals etc., are delegated within organizations. Thus, it is reasonable to assume that investment decisions are subject to several personal views and depend on the organizational context at hand. To secure consistency, in many companies these multi-person multi-level capital budgeting procedures are governed by a formal screening and review body, like an investment committee. Moreover, they are structured by formal guidelines, schedules, and manuals intended to secure a broad support for a successful investment project.[13] In such an environment the use of appraisal techniques acceptable for all relevant decision makers simplifies reaching mutually agreed upon decisions. However, note that we do not search for an automatism identifying the best investment decision for a given data set, i.e., we are not aiming at replacing the complete decision making process within the organization by a black box. Rather, we are looking for a decision theoretical justification of

[10]Cf. Graham/Harvey (2002, p. 12), Ryan/Ryan (2002, p. 355).

[11]E.g., Lefley (1996, p. 214) notes the role of the payback criterion in communication.

[12]Cf., e.g., Narayanan (1985, pp. 311), Mepham (1975, pp. 869), Gordon (1955, pp. 253).

[13]Cf. Arnold (2005, p. 171), Pike (1988, p. 343), Klammer et al. (1991, p. 128). See also the case study in Ross (1986).

the payback method as an instrument to ease decision making on risky investment projects, accepting its application as key investment figure in practice.

We will demonstrate that the payback criterion may be a mutually acceptable tool in the sense discussed above. By its implementation a multitude of personal risk preferences can be represented in the investment process simplifying the coordination of decision making.[14] For modeling this representation problem we use almost stochastic dominance (ASD) introduced by Leshno and Levy (2002). Firstly, we show that this concept allows to include the risk preferences of the relevant decision makers into the analysis. Secondly, it can be shown that conveying these preferences to those who do the preparatory work preceding the final decision can be achieved by means of ASD as well. This finding provides a potential explanation for the payback's persisting prominence.

The remainder of the paper is organized as follows: In section 2 an overview of the empirical and theoretical literature focusing on the payback method is given. In section 3, we consider fundamental organizational features of the capital budgeting process in a stylized manner, i.e., we identify the impact of organizational aspects on the importance of the payback criterion. In section 4 the modeling device of almost stochastic dominance is introduced, the new investment appraisal criterion (*Leshno-Levy* criterion) is derived, and its relation to earlier research on payback times by, e.g., *Gordon* (1955) and *Levy* (1968) is clarified. The paper concludes with a brief summary in section 5.

2 Literature review

From an academic perspective net present value (NPV) has been argued to be the favorable capital appraisal technique over decades. Nevertheless, early studies dating from the '60s find the discounted cash-flow techniques –including NPV– to be the least popular ones.[15] Moreover, some of these studies identify the payback method (PB) as the most popular appraisal technique, irrespective of its well-documented shortcomings, like ignoring time value of money, ignoring cash flows beyond the cutoff date, and setting the cut-off period arbitrarily. Even today the payback

[14]Similarly, Arnold (2005, p. 156): "There is an indication in the literature that [...] simpler methods are used for purposes such as communicating project viability and gaining commitment throughout an organisation."

[15]For an overview of these early studies cf. Ryan/Ryan (2002, p. 357).

method is still used quite regularly as many empirical studies show.[16] These studies have isolated a number of factors with explanatory power for the prominence of the payback method. Besides capital budget size[17] and the degree of shareholder-value orientation[18] the most prominent factors are:

- firm size: Empirical observations indicate that small companies tend to rely more often on the payback method.[19] However, as *Lefley* (1996, p. 208) points out, the results are controversial.[20]

- CEO/CFO age and tenure respectively CEO/CFO level of education: The younger a CEO/CFO and the higher his degree of education the more often sophisticated appraisal techniques, like NPV or internal rate of return (IRR), are used, meaning the relative importance of PB decreases.[21]

- firms' capital constraints: *Pike* (1983, pp. 666 and 669) claims that the payback-criterion is used more often, and profitability measures less often, the more restricted financial resources are. However, neither *Brounen et al.* (2004, p. 8) nor *Graham/Harvey* (2002, p. 12) can confirm this hypothesis.

- nationality: European companies are more hesitant to apply discounted cash flow techniques than American ones.[22] For Asian companies the payback method even dominates in terms of application frequency and importance.[23]

Although nationality has an influence on payback method's application frequency its use is nevertheless a world-wide phenomenon, see the international overview provided in *Horngren et al.* (2006), which is extended and up-dated in Table 1.[24] Here the application frequencies of the most popular techniques are listed by country:

[16] For an overview of such empirical studies from 1970 to 1995, cf. Lefley (1996, pp. 217).
[17] Cf. Ryan/Ryan (2002, pp. 359).
[18] Cf. Brounen et al. (2004, p. 8).
[19] Cf. Brounen et al. (2004, p. 8), Graham/Harvey (2002, p. 12), Ryan/Ryan (2002, p. 361).
[20] Cf. Arnold/Hatzopoulos (2000, p. 605), Block (1997, p. 289), Sangster (1993, p. 318) as well.
[21] Cf. Brounen et al. (2004, p. 8), Graham/Harvey (2002, p. 12).
[22] Cf. Brounen et al. (2004, p. 5).
[23] Cf. Ann et al. (1987, pp. 116). Hermes et al. (2005, p. 7) argue that a host country's stage of economic development should influence the companies' capital budgeting. However, regarding payback method they cannot confirm this claim given their sample of Dutch and Chinese firms.
[24] Cf. Horngren et al. (2006, p. 735). The column-wise accumulated percentages add up to more than 100%, meaning that companies regularly apply two or more methods simultaneously.

	USA[25]	AUS	CDN	GER[26]	IRE[27]	JPN	RSA[28]	SCO	UK[29]
NPV[30]	75%	45%	41%	48%	84%	6%	17%	48%	80(97)%
IRR	76%	37%	62%	42%		4%	32%	58%	81(84)%
PB[31]	57%	61%	50%	50%	84%	52%	17%	78%	70(66)%

Table 1: International comparison of capital budgeting methods

The prominence of the payback method is confirmed in other studies as well, like, e.g., in a recent survey on European companies.[32] The same holds true for Asian companies. In Malaysia the payback criterion dominates, in Hong Kong and Singapore the accounting rate of return is similarly popular.[33]

Despite differences in sample sizes, firm sizes and considered industries, Table 1 provides a general empirical insight. Obviously, profitability measures, like NPV and IRR, and the payback criterion are used side by side. However, according to *Pike* (1996, p. 83) only NPV and IRR are used as substitutes indicating that the payback method is not regarded as a simple rule of thumb replacing more sophisticated profitability measures.[34] Thus, the payback method's persistence cannot be explained satisfactorily by its simplicity in combination with lack of familiarity with more sophisticated methods. Further, if this argumentation were true, with the improvement of CEOs' and CFOs' education the gap between theory and practice should narrow, i.e., the use of the payback method should vanish. But, longitudinal UK data do not support this assumption. Table 2 shows that the prominence of the payback method has not been decreasing over time.[35]

[25]Cf. Graham/Harvey (1999, p. 27).

[26]Cf. DeBrounen et al. (2004, p. 7).

[27]Cf. also O'Brien (1997, pp. 180) for the widespread use of the payback criterion in Ireland.

[28]Cf. Hall (2000, p. 361). Note that the data refer to the most important method to be used.

[29]Cf. Arnold/Hatzopoulos (2000, p. 605). In brackets are given the data for the 100 largest companies included in the Times 1000.

[30]Remember the abbreviations for Net-Present-Value (NPV), Internal Rate of Return (IRR), and payback time (PB).

[31]Percentages refer to the use of both variants of the pay-back-method –static and discounted version.

[32]The study by Brounen et al. (2004) reproducing the set-up used by Graham/Harvey (2001) is presented in Table 8 in appendix A.

[33]Cf. Ann et al. (1987, pp. 116).

[34]In contrast, Arnold/Hatzopoulos (2000, p. 608) find that NPV and IRR gain at the expense of the payback method to a minor extent.

[35]Similar overviews for the USA –given in appendix A as Table 9– confirm the observation extracted from the UK data.

	Pike (1996), data for				Sangster	Lefley	Arnold/Hatzo-
	1975	1980	1986	1992	(1993)	(1994)	poulos (2000)
NPV	32%	39%	68%	74%	48%	52%	80%
IRR	44%	57%	75%	81%	58%	55%	81%
PB	73%	81%	92%	94%	78%	94%	70%

Table 2: Capital budgeting in the UK

Obviously, the payback method's relative importance has decreased, because of the discounted cash flow techniques' growing prominence.[36] However, total application of the payback method has increased from 73% in 1975 to 94% in 1992. The more recent studies starting in 1993 confirm the observation that the payback method is applied for virtually all investment appraisals. But note that these studies only account for the application frequency and not for the methods' importance.[37] With the increasing prevalence of discounted cash-flow techniques, the payback method has become less important. Nowadays it is typically used as a secondary criterion or as a constraint.[38] An example for the use as a constraint is given when a required payback period –subjectively determined based on past experiences and project risk estimations– is used in the sense of a hurdle-rate. An example for the use as a secondary criterion is given, when projects of the same profitability are ranked by their pay-off period. While PB as a secondary criterion allows for an additional fine-tuning in the projects' ranking, the application as a constraint sorts out projects independent of their profitability.

Early analytical papers try to justify the payback criterion as a proxy for profitability. *Gordon* (1955, pp. 253) shows that the reciprocal of the payback period can be interpreted as an estimation of the IRR. His analysis has been extended by *Levy* (1968 and 1971) and *Sarnat/Levy* (1969). *Mepham* (1975, p. 869) points out that selecting investment projects from a whole bundle of alternatives according to the NPV rule and according to the PB method results in comparable selection decisions. However, this finding suffers from the restrictive assumption of constant cash-inflows. Further, simulations have been run, in order to test how the PB

[36] Cf. Pike (1988, p. 347).

[37] The difference between use and importance is elaborated in Lefley (1996, p. 208). He finds that use of PB is positively and importance of PB is inversely related to firms' capital budget size.

[38] Pike (1983) finds that 90% of firms implementing multiple methods use PB. Similarly, Shields et al. (1991) suggest that U.S. companies maximize net present value or rates of return subject to payback constraints.

method performs in identifying promising projects compared to more sophisticated techniques.[39]

With rise of the agency paradigm, the application of the PB method has been explained by incentive effects due to institutional properties. For instance, *Narayanan* (1985) concludes that a manager benefits from applying the payback method to investment selection problems. By choosing projects with early cash-inflows he improves the expectation about his skills and hence the present value of his remuneration.[40] A similar principal-agent conflict is analyzed by *Thakor* (1990). Here the payback method is used as a means of harmonizing the goals of actual shareholders and future investors.

Moreover, payback times are considered as a means of accounting for liquidity constraints. *Weingartner* (1969) states that the application of the PB method selects projects with high cash flows in the early periods of the useful life. In this sense the PB method minimizes the risk of foregoing other investment projects due to lack of capital, i.e., PB increases the probability of being able to invest in unforeseen future investment alternatives. This means, payback times account for the management's restricted forecasting ability.[41] *Wambach* (2000) analysis a situation, where the investment outlay instead of future cash-inflows is uncertain. Based on a real-option approach the payback period is used as an indicator for the optimal time instance for investment activities given required capital outlays decrease over time.[42]

Our approach combines three dimensions of the literature reviewed above. Firstly, we concentrate on riskiness of investment projects, like *Weingartner* (1969) or *Wambach* (2000). Secondly, we incorporate into our model important organizational features of typical capital budgeting processes, extending the view of agency theory. Thirdly, we shall find that the central appraisal criterion deduced following our approach may be seen as a generalization of the contributions by *Gordon* (1955) and *Levy* (1968). Interestingly, the evidence provided in Table 1 is consistent with our risk-controlling justification of the payback method: Interpret NPV and IRR as profitability measures and PB as a risk figure. Then, a simple explanation for the simultaneous use of NPV or IRR on the one hand and PB on the other hand is

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[39] Cf., e.g., Hertz (1968).

[40] Similar arguments can be found in Pike (1985, p. 50) and Chen/Clark (1994, p. 123). For an overview of papers dealing with incentive effects caused by reputational concerns, see Hirshleifer (1993, pp. 148).

[41] Cf. Lefley (1996, p. 209).

[42] Cf. Wambach (2000, pp. 253)

that investments are evaluated in a fashion similar to the $\mu - \sigma$–principle frequently applied in financial theory.

3 Organizational embedding of capital budgeting procedures

Since technological knowledge, administrative skills as well as product- and customer-related information are distributed among many individuals, capital budgeting in modern corporations is a multi-person process. The spectrum of decision making bodies in organizations ranges from a single decision maker over a group of decision makers originating from different management levels interacting to find a consensus, like, e.g., in investment committees, to a group of peers in charge of the final investment decision, like executive boards. For categorizing different organizational forms of capital budgeting processes and their impact on the payback method's role distinctive characteristics have to be identified. We focus on the degree of delegation of decision competences, the size of the decision making body and communication needs resulting from preparatory work preceding the final decision making. Here, on the one hand we consider the degree of delegation as a continuous attribute, because, e.g., the preparation of an investment proposal typically includes eliminating unfavorable alternatives, meaning decision competences are (partially) exerted at this stage. On the other hand the decision making body may consist of a single individual or a multitude of decision makers. In the latter case the final decision may be a consensual process, where preferences have to be communicated among the decision makers in order to moderate a compromise. Alternatively, the final multi-person decision may be subject to a voting procedure, where majorities have to be organized by co-ordinating preferences. This is reflected in Table 3, depicting four stylized investment process scenarios, by the degree of co-ordination:

	degree of coordination	
degree of delegation	low	high
low	entrepreneurial decision	board decision
high process	representative's decision	investment committee decision

Table 3: Different investment decision scenarios

The characteristic of communication needs is not exhibited explicitly in Table 3 as they may occur in each setting depending on the delegation of preparatory tasks. Note however that since we focus on investment projects with uncertain future cash flows, communication means primarily conveying risk preferences.

For the four scenarios different justifications for the payback method may hold true:

- In the entrepreneurial decision scenario only one decision maker exists. Thus, no problem of coordinating different decision makers arises. Further, as no investment competences are delegated no communication of requirements occurs. However, key figures describing investment projects will be needed for communication, if data are collected by other individuals on a preparatory stage. This scenario describes small family-owned companies or start-ups where, e.g., engineers develop and market a product from an invention without a strong background in business knowledge. Here, the use of the payback method might be explained by the knowledge hypothesis, i.e., the payback method is used because of its simplicity.

- The scenario 'representative's decision' is very similar to the entrepreneurial one. However, companies are slightly bigger, so that the generation of ideas for investment projects and the subsequent appraisal and selection process are (partly) delegated. Minimum requirements, e.g., profitability hurdle rates, are communicated via key figures serving as appraisal criteria. Due to the existence of a hierarchically superior decision maker the representative needs to justify his decision according to these key figures. This implies that a critical payback time may result from the entrepreneur's individual preferences or experiences. Hence, the payback method is a rule of thumb rather than a theoretically based criterion.

- The scenario 'board decision' indicates that top-level decision makers decide upon the investment projects to be funded. Because data gathering is delegated and pieces of information are aggregated in a proposal, an unbiased communication via key indicators is required. Further, the decision process needs co-ordination, because, e.g., power can be distributed asymmetrically among board members. Therefore, inter-subjectively reliable measures are needed to make a compromise or to find a majority on the board. In this case, PB might be used as a secondary criterion.

- An ‘investment committee’ bundles the investment competences for the whole firm relieving top level management. It is formed by members of various skills from different departments, like technicians, accountants, and engineers. They have to communicate subjective assessments and predictions in such a manner that they can be reproduced and understood by third parties. Further, due to uncertainty of cash flows an inter-subjective definition of acceptable risk has to be found. Here, the payback method typically serves as a constraint, i.e., as a tool for communicating the decision maker’s risk preferences to the applicant or the management accountant.

Identifying the need for a certain investment, the search for corresponding investment alternatives, and the preparation of the investment proposal typically belong to the competences of the operating divisions (decentralized phase). However, depending on the required capital expenditure, the authority for the final project selection is delegated to a centralized committee (centralized phase).[43] We will consider the interface between the decentralized and the centralized phase of the budgeting process, because this is a crucial point in both the ‘board decision’ and the ‘investment committee proposal’ scenario. Note however, the budgeting process should not be thought of as a linear one. Especially, the preparation of the proposals will typically include repeated discussion of preliminary versions, modifications of the project, and re-estimation of cash flows. This may explain empirical evidence indicating that companies consider project definition and cash flow estimation the most difficult steps of the capital budgeting process.[44] Moreover, management accountants discussing the project with applicants have an important pre-decision function. If they qualify proposals as unsatisfactory, these proposals will barely be presented to the investment committee or the board. This is consistent to the very high approval rates observable for those projects finally presented to the decision instances.[45]

[43]For a formal description cf. Taggart (1987, pp. 179).
[44]Cf. Scott/Petty (1984, p. 113).
[45]Cf. Arnold (2005, p. 171).

4 A criterion for investment appraisal based on almost stochastic dominance

4.1 The general idea of applying almost stochastic dominance to organizationally embedded investment choices

In this section we will briefly introduce the concept of almost stochastic dominance. We will demonstrate that it becomes possible by this concept to include the preferences of all relevant decision makers into the analysis. Note that this is a necessary step in modelling capital budgeting processes for organizational settings represented by the ‘board decision’ or the ‘investment committee proposal’. Moreover, we emphasize that ASD allows conveying the decision makers’ risk preferences to subordinates responsible for the early and intermediate stages of the capital budgeting process.

We focus on the choice between a risk-free investment in bonds and a real investment alternative with uncertain future cash flows, henceforth referred to as the risky investment project. Let the risky investment project be represented by the sequence of uncertain net cash flows, C_t, $t = 0, 1, 2, ..., T$, where T is the last period of the project’s useful life. Assume the initial capital outlay, $C_0 \in \mathbb{R}$, to be given and certain, whereas the subsequent periods’ net cash flows, C_t, are random variables. The risk-free investment is characterized by the certain initial capital outlay c_0^F and the risk-free interest rate, $i \geq 0$. This defines its cash flows in the subsequent periods to be $c_t^F = ic_0^F, t = 1, 2, ..., T^F - 1$ and $c_{T^F}^F = (1 + i)c_0^F$, with T^F being the bonds’ maturity period. Assume that both investment opportunities require the same initial capital outlay, $C_0 = c_0^F$, and are finished in the same period, i.e., $T = T^F$. Due to the cash flows’ uncertainty the NPV of the risky alternative is a random variable, V. In the following, we will assume a positive expected value of the random NPV, $E(V) > 0$. Figure 1 shows the cumulative distribution functions (CDF) of the two alternatives’ NPVs.

The CDF of the risky investment project’s NPV, $G(\cdot)$, is assumed to be continuous on its finite support $[\underline{v}, \overline{v}], \underline{v} < 0 < \overline{v}$. As we will consider its risk explicitly, the NPV is calculated by discounting with the interest rate of the risk-free alternative. The same procedure applied to the risk-free alternative yields a stepwise CDF, $H(\cdot)$, which assigns probability one to an NPV of zero, see Figure 1.

Given an organizational setting discussed in section 3, the choice between the

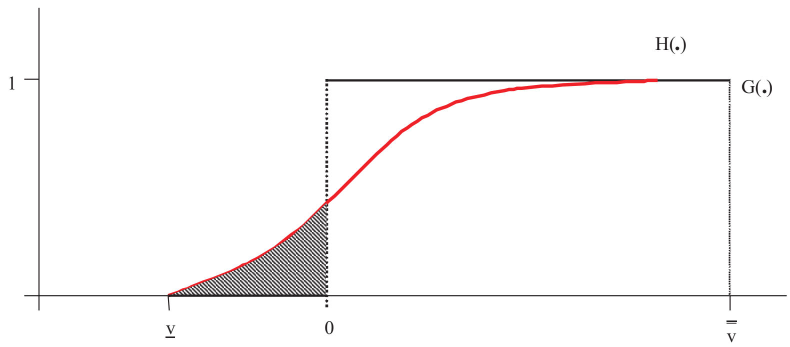

Figure 1: The non-applicability of first order stochastic dominance

risky and the risk-free investment project requires taking into account the risk preferences of a multitude of involved agents. From a decision theoretical perspective, stochastic dominance is an appropriate device for this purpose. Its main idea is summarized as follows: If a risky alternative A, described by its wealth consequences, dominates an alternative B in terms of first order stochastic dominance, A is preferred to B by every rational and non-satiable decision maker. These decision makers are represented by non-decreasing differentiable real-valued utility function.[46] Let $\mathcal{U}$ denote the set of these utility functions. Typically, $\mathcal{U}$ would include the subset of decision makers relevant for the investment decision process in the sense discussed in section 3 or more precisely the corresponding utility functions. Hence, if the risky investment stochastically dominates the risk-free one, all relevant decision makers, as a group representing the firm, prefer the risky alternative. Moreover, an agent preparing the corresponding investment proposal would know they do so, if he discerns the dominance relation. Thus, generating projects which are likely to be accepted in the capital budgeting process becomes possible.

Note that stochastic dominance remains a reasonable decision criterion even if some of the axioms of expected utility theory do not apply. In this sense stochastic dominance is robust under variations of the rationality concept, making it even more attractive for practical application.[47] But a look at Figure 1 reveals that first order stochastic dominance is not applicable to the comparison of the two investment projects under consideration. The reason is that $G(\cdot)$, representing the risky investment project, assigns positive probability to wealth reductions as well as to wealth increases. Therefore, the requirement for first order stochastic dominance to hold, i.e., no intersection of the two CDFs, is violated. In Figure 1, the hatched

[46]Cf., e.g., Levy (1992, p. 557).
[47]Cf. Levy (1992, p. 560) for a discussion of this aspect.

area indicates the difference between the two CDFs in the region of violation which is restricted to the negative part of the CDF's support. However, recently *Leshno/Levy* (2002) proposed almost stochastic dominance (ASD), a modified version of stochastic dominance that alleviates this problem.

ASD differs from first order stochastic dominance in the way of handling the violation indicated above. For first order stochastic dominance the existence of such a violation means it is not applicable. In contrast, ASD measures the risky investment project's degree of violation by a parameter ε_G. This parameter represents the cumulated difference between the two CDFs in the region of violation (e.g., the size of the hatched area in Figure 1) relative to their cumulated difference over the entire support (e.g., the size of the hatched plus the dotted area in Figure 1). In a second step, under ASD the set of utility functions is reduced, starting from the set $\mathcal{U}$ of all rational and non-satiable utility functions with $u'(x) > 0$ for all x. The process of reducing $\mathcal{U}$ is controlled by a second parameter, ε.[48] The set of utility functions remaining after the reduction, $\mathcal{U}(\varepsilon)$, represents all those decision makers who, despite the violation of first order stochastic dominance, prefer alternative $G(\cdot)$ to $H(\cdot)$, given the degree of violation of the non-intersection requirement is bounded by ε. Note that $\mathcal{U}(\varepsilon) \subset \mathcal{U}$ because some decision makers in $\mathcal{U}$ will not tolerate the violation and will thus be eliminated from $\mathcal{U}$. In an application to the capital budgeting procedure, the remaining set $\mathcal{U}(\varepsilon)$ obviously should contain the set of the relevant decision makers' utility functions as a subset.

Finally, the risky investment, or its CDF $G(\cdot)$, is said to dominate the risk-free investment, or CDF $H(\cdot)$, by ε-almost stochastic dominance (ε-ASD), if and only if $\varepsilon_G \leq \varepsilon$. More formally denoted, $\varepsilon_G \leq \varepsilon \Leftrightarrow G(\cdot) \succeq_\varepsilon H(\cdot)$.

To explain the idea in greater detail, we describe mathematically the degree of violation of the non-intersection requirement and have a closer look at the process of reducing the set of decision makers.

The degree of violation of the non-intersection requirement, ε_G

Firstly, in general the size of the hatched area in Figure 1 is given by

$$\int_{\underline{v}}^{0} G(v) - H(v)dv = E(V^-) \tag{1}$$

[48]Note that the distinction between ε and ε_P does not occur in Leshno/Levy (2002), but is introduced here for ease of presentation.

while the size of the dotted region may be written as

$$\int_0^{\overline{v}} H(v) - G(v)dv = E(V^+) \tag{2}$$

where $V^- = -\min\{V, 0\}$ is the negative part and $V^+ = \max\{V, 0\}$ is the positive part of the net present value, V. Both of these random variables are almost surely non-negative. Hence, their respective expected values $E(V^-)$ and $E(V^+)$ are non-negative and given by the areas indicated in Figure 1.

Following the ideas of *Leshno/Levy* (2002), parameter ε_G measuring the investment project's degree of violation of the non-intersection requirement is therefore given by [49]

$$\varepsilon_G = \frac{E(V^-)}{E(V^+) + E(V^-)} = \frac{1}{1 + \frac{E(V^+)}{E(V^-)}} \tag{3}$$

where $\frac{E(V^+)}{E(V^-)} > 1$.[50] $E(V^-)$ is the expected value of random variable $|V|$ conditional on $V \leq 0$. Hence, it measures the risk of a potential loss in wealth caused by the risky investment project. Similarly, $E(V^+)$ measures the chance of earning a potential profit. Therefore, (3) can be interpreted as a chance-risk-relationship.

ε-almost stochastic dominance

Secondly, we return to the idea of eliminating from $\mathcal{U}$ utility functions $u(\cdot)$, where the magnitude of the reduction is controlled by a predetermined (maximal) degree of violation of first order stochastic dominance, ε. ASD was proposed by *Leshno/Levy* (2002) for cumulative distribution functions with finite support, $[\underline{x}, \overline{x}]$. Restrict $\mathcal{U}$ to twice differentiable functions and the subset $\mathcal{U}(\varepsilon) \subset \mathcal{U}$ remaining after the elimination process is given by

$$\mathcal{U}(\varepsilon) = \left\{ u \in \mathcal{U} : u'(x) \leq \left[\frac{1}{\varepsilon} - 1\right] \min\left\{u'(z) : z \in [\underline{x}, \overline{x}]\right\}, x \in [\underline{x}, \overline{x}] \right\} \tag{4}$$

For utility functions on $\mathbb{R}$ exhibiting risk aversion, i.e., $u'(x) > 0, u''(x) < 0$ for all $x \in [\underline{x}, \overline{x}]$, the defining inequality in (4) can be reformulated to

$$u'(\underline{x}) \leq \left[\frac{1}{\varepsilon} - 1\right] u'(\overline{x}) \tag{5}$$

Hence, utility functions excluded from $\mathcal{U}(\varepsilon)$ "... assign a relatively high marginal

[49] Cf. Leshno/Levy (2002, p. 1080), formula (5).

[50] Note that $E(V^+) > E(V^-) > 0$ follows from reasons given on page 19.

utility to very low values or a relatively low marginal utility to large values of x".[51] Note that $\mathcal{U}(\varepsilon)$ contains utility functions of different types, like risk-averse, risk-neutral or S-shaped.

Example: To give a simple example, consider the family of exponential utility functions $u_e(x) = 1-\exp\{-rx\}$, where $r > 0$ is the Arrow-Pratt measure of absolute risk aversion. If we restrict the analysis to this family, condition (5) reads

$$\begin{aligned} u_e'(\underline{x}) &\leq \left[\frac{1}{\varepsilon} - 1\right] u_e'(\overline{x}) \\ \Rightarrow r\exp\{-r\underline{x}\} &\leq \left[\frac{1}{\varepsilon} - 1\right] r\exp\{-r\overline{x}\} \\ \Leftrightarrow r &\leq \frac{\ln(\frac{1}{\varepsilon} - 1)}{(\overline{x} - \underline{x})} =: r(\varepsilon) \end{aligned} \tag{6}$$

For predetermined $\varepsilon > 0$ all decision makers with an exponential utility function remain in $\mathcal{U}(\varepsilon)$ as long as their degree of absolute risk aversion does not exceed the upper bound, $r(\varepsilon)$, determined by (6), i.e., as long as $r \leq r(\varepsilon)$ holds. Hence, for exponential utility functions, the (maximally admissible) degree of absolute risk aversion, $r(\cdot)$, is a function of parameter ε.[52] The derivative of $r(\cdot)$ is

$$\frac{dr(\varepsilon)}{d\varepsilon} = -\frac{1}{(\overline{x} - \underline{x})(\varepsilon - \varepsilon^2)} < 0 \tag{7}$$

This negative relationship shows that for the family of exponential utility functions an increasing ε implies a decreasing critical risk aversion. Consequently, for a growing (maximal) degree of violation of the non-intersection requirement, ε, decision makers with an ever smaller risk aversion have to be eliminated from $\mathcal{U}$, i.e., the greater ε, the smaller $\mathcal{U}(\varepsilon)$.[53] Actually, *Leshno/Levy* (2002, p. 1079) show that if $\varepsilon \to 1/2$, $\mathcal{U}(\varepsilon)$ eventually contains linear utility functions only.

For a predetermined ε *Leshno/Levy* (2002) proof in their Theorem 1 that if a risky project's degree of violation of first order stochastic dominance does not exceed ε, i.e., $\varepsilon_G \leq \varepsilon$, all decision makers represented by $\mathcal{U}(\varepsilon)$ prefer the risky to the risk-

[51] Cf. Leshno/Levy (2002, p. 1079).

[52] Note that the effect of ε can be observed so nicely, because exponential utility functions have a constant absolute risk aversion (CARA).

[53] Since in a decision process the set of decision makers is given, the admissible risk might be adjusted to achieve a majority, see table 4.

free investment project. More precisely, $G(\cdot)$ dominates $H(\cdot)$ by ε-ASD if and only if all decision makers in $\mathcal{U}(\varepsilon)$ prefer $G(\cdot)$ to $H(\cdot)$.

Accounting for the organization of capital budgeting when choosing the set $\mathcal{U}(\varepsilon^*)$

Up to this point, the (maximal) degree of violation, ε, for the definition of $\mathcal{U}(\varepsilon)$ was chosen arbitrarily. Next we look at the fixed and given set $D \subset \mathcal{U}$ of utility functions representing non-risk seeking decision makers being relevant for the capital budgeting process according to the organizational environment described in section 3. To avoid the case of pure risk neutrality assume that at least one decision maker in D is not risk neutral. Let ε^*, $0 < \varepsilon^* < 0.5$, be the largest degree of violation the decision makers in D accept in terms of almost stochastic dominance.[54] Hence, the utility functions of all relevant decision makers are included in the set $\mathcal{U}(\varepsilon^*)$, $D \subset \mathcal{U}(\varepsilon^*) \subset \mathcal{U}$. The set $\mathcal{U}(\varepsilon^*)$ of utility functions, thus, serves as a simple model of the firm's decision making structure. For example, if the decision has to be made by a board of directors or an investment committee in a consensual manner, every member of this group should be represented by $\mathcal{U}(\varepsilon^*)$. Under these circumstances the risky project is preferred to the risk-free investment by all relevant decision makers if $\varepsilon_G \leq \varepsilon^*$. This means, it is almost stochastic dominant at level ε^* (ε^*-ASD), if $\varepsilon_G \leq \varepsilon^* \Leftrightarrow G(\cdot) \succeq_{\varepsilon^*} H(\cdot)$.

Example: Assume the investment committee consists of seven members, $j = 1, ..., 7$, with exponential utility functions $u_j(x) = 1 - \exp\{-jx\}$.[55] The committee might in general decide based on the consensus principle or on majority voting. Suppose the individual voting behaviour exclusively depends on the member's risk assessment of a project. Thus, the appropriate ε^* depends on the voting scheme –e.g. unanimity, qualified majority (for instance 75 % or $\frac{2}{3}$), or simple majority– and on the risk preferences of the single decision makers. The pivotal committee member, i.e., the one whose approval ensures the required majority, is indicated by j^*. Assume $\overline{x} - \underline{x} = 0.1$. In this simple example the ε^* values in Table 4 apply:
Having stated the advantages of almost stochastic dominance, some theoretical considerations shall identify problems to be coped with when implementing ASD in real budgeting procedures:

[54] See Appendix B for a formal definition of this value.

[55] In this example, all decision makers are risk averse so that an application of Almost Stochastic Second Order Dominance (ASSD) becomes possible, see Leshno/Levy (2002, p. 1080). Since we intend to include non-risk-averse decision making behaviour, too, we for the sake of generality apply Almost Stochastic First Order Dominance (ASD).

majority rule	$j^* = r_{j^*}$	ε^*	D
unanimity	7	0.332	$u_1, u_2, ..., u_7$
three-quarter majority	6	0.354	$u_1, u_2, ..., u_6$
two-thirds majority	5	0.378	$u_1, u_2, ..., u_5$
simple majority	4	0.401	$u_1, u_2, ..., u_4$

Table 4: ε^* depending on majority rule and committee members' preferences

Technically, almost stochastic dominance of a risky project over the risk free alternative means that all relevant decision makers in D prefer $G(\cdot)$ over $H(\cdot)$. Since D is a proper subset of $\mathcal{U}(\varepsilon^*)$ this is not a necessary condition for an unanimous preference.[56] Cases might exist, where a project's ε_G exceeds ε^*, although on an individual level for all decision makers in D the expected utility for the risky project calculated according to their individual utility function exceeds the expected utility for the risk free alternative. This means, the risky project is mistakenly rejected. Nevertheless, in our view this drawback is balanced by the ease of communicating risk preferences via the parameter ε^*. Note that the opposite mistake, accepting mistakenly a too risky project, is impossible under almost stochastic dominance. Thus, in a multi-person context almost stochastic dominance is a cautious heuristic aggregating individual risk preferences.

Moreover, the relationship between the support $[\underline{x}, \overline{x}]$, used for the definition of $\mathcal{U}(\varepsilon^*)$ in (4), and the support of the random NPV's probability distribution $[\underline{v}, \overline{v}]$ warrants a discussion, because ε^* might vary depending on delegation. To ease the discussion, ε^* is replaced by ε^*_x, given $[\underline{x}, \overline{x}]$ is the support used in the definition of $\mathcal{U}(\varepsilon^*)$ and by ε^*_v, given $[\underline{v}, \overline{v}]$ is the support of the NPV's probability distribution in the following.

The literature indicates that investment competences are delegated to different hierarchy levels depending on project size. For example, Table 5 shows the distribution of critical expenditures requiring formal capital budgeting analysis for the companies included in *Fortune* 1000.[57]

Consider a large project to be authorized by a board level committee, i.e., a situation corresponding to the 'board decision' scenario in section 3. In this case we may assume $[\underline{x}, \overline{x}] = [\underline{v}, \overline{v}]$, meaning that the project is accepted or rejected based

[56] Cf. Leshno/Levy (2002, p. 1080, Theorem 1).

[57] Cf. Ryan/Ryan (2002, p. 358). See also Ferreira/Brooks (1988, p. 23) and Arnold (2005, p. 171). However, note budgeting systems again may vary according to cultural backgrounds, cf., e.g., Bailes/Assada (1991, pp. 133).

< 10.000$	10.000 – 99.999$	100.000 – 500.000$	> 500.000$	n. def.
21.2%	27.3%	31.8%	19.2%	0.5%

Table 5: Critical capital expenditures for *Fortune* 1000 companies

exclusively on its own characteristics. Accordingly, the parameter ε_v^* depends on the project specific bounds $\underline{v}$ and $\overline{v}$. Thus, ε_v^* can be used to generate a cut-off payback period or a cut-off IRR as described in the following. Its value properly represents the committee's attitudes toward the risky project.

For small investment projects completely authorized by a lower management level this statement may not hold true. Here, the interval $[\underline{x}, \overline{x}]$ may serve to define 'small projects' by the condition $[\underline{v}, \overline{v}] \subset [\underline{x}, \overline{x}]$. In this case parameter ε_x^* is again determined for those decision makers responsible at the board level, according to the interval $[\underline{x}, \overline{x}]$. Cut-off values deduced from ε_x^* may be imposed on all small projects to restrict the discretion of lower-level management. Accordingly, a controlled delegation of decision competences, like in the 'investment committee' scenario presented in section 3, becomes possible. However, a small project although rejected by the lower-level decision-makers, might have been authorized by the board, had the board considered it based on its individual interval $[\underline{v}, \overline{v}]$. This is due to the fact that $\varepsilon_x^* \leq \varepsilon_v^*$ for $\underline{x} < \underline{v} < \overline{v} < \overline{x}$.[58] As a consequence it may be reasonable to advice the investment committee respectively the representative to submit projects to the top hierarchy level for a final decision, given any doubts remain.

4.2 The *Leshno-Levy* criterion as a tool for the appraisal of risky investment projects

For a thorough analysis of risky investment projects the time structure of cash flows has to be considered. Therefore, let the risky investment project be defined by the sequence of net cash flows, C_t, $t = 0, 1, 2, ..., T$, as defined at the beginning of this section. Assume the initial capital outlay, $C_0 \in \mathbb{R}$, to be given and certain, whereas the subsequent periods' net cash flows C_t are non-degenerated random variables with probability distribution on the finite interval $[\underline{c}_t, \overline{c}_t]$. Moreover, the risk-free investment is characterized by the certain initial capital outlay c_0^F and the risk-free interest rate, $i \geq 0$. With this notation, the random NPV, V, of the risky investment

[58]See Appendix B for a comprehensive mathematical analysis.

is calculated by applying the discount factor, $q = 1 + i$, to the cash flows

$$V = \sum_{\tau=0}^{T} q^{-\tau} C_\tau \tag{8}$$

The CDF of random variable V, $G(\cdot)$, lives on the finite support $[LB, UB]$ (see Figure 2). The lower bound, LB, of the NPV is given by $LB = C_0 + q^{-1}\underline{c}_1 + ... + q^{-T}\underline{c}_T$, while the upper bound is $UB = C_0 + q^{-1}\overline{c}_1 + ... + q^{-T}\overline{c}_T$. In the following, we will assume a positive expected value of the random NPV, $E(V) > 0$. The reasons for this assumption are that, firstly, the NPV criterion applied to deterministic capital budgeting situations excludes projects with non-positive NPV as disadvantageous. Secondly, *Leshno/Levy* (2002, Proposition 2) show that a necessary condition for one alternative dominating the other one in terms of ASD is that the mean of the CDF representing the dominating alternative is strictly larger than the mean of the dominated one. By definition, the mean of the risk-free alternative's NPV is zero. Therefore, $E(V) > 0$, i.e., the classical textbook profitability criterion, is a necessary condition for ASD to hold for a risky investment project.

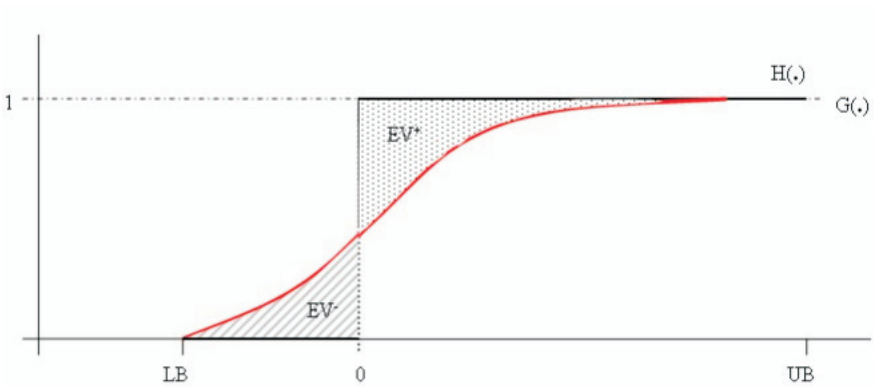

Figure 2: ASD of risky investment project's NPV

Similar to Figure 1, $E(V^-)$ and $E(V^+)$ are depicted by the hatched and the dotted areas in Figure 2, respectively. For $E(V^-) = 0$ the risky investment project dominates the risk-free investment in the sense of (simple) first order stochastic dominance. However, $E(V^-)$ symbolizes the expected loss in wealth due to choosing the risky instead of the risk-free project. If this expected loss was zero, the probability of a negative NPV, $P(V < 0)$, would vanish, too. To exclude such trivial situations with no economic risk, we assume $E(V^-) > 0$, and consequently $LB < 0$. As we have already seen, for this situation first order stochastic dominance does not apply.

The expected value of any random variable may be written as[59]

$$E(V) = E(V^+) - E(V^-) \tag{9}$$

Consequently, a positive expected NPV, $E(V) > 0$, means that the degree of violation of the non-intersection requirement by the risky investment project, i.e., parameter ε_G as defined by (3)

$$\varepsilon_G = \frac{E(V^-)}{E(V^+) + E(V^-)},$$

is restricted to $0 < \varepsilon_G < \frac{1}{2}$.[60] Moreover, by use of (9) we may replace (3) by

$$\varepsilon_G = \frac{E(V^-)}{E(V) + 2E(V^-)} \tag{10}$$

Because of $E(V^-) > 0$, (10) can be rearranged to the final version of the degree of violation of first order stochastic dominance of the risky investment project

$$\varepsilon_G = \frac{1}{2 + \frac{E(V)}{E(V^-)}} = \frac{1}{2 + \frac{1}{\frac{E(V^+)}{E(V)} - 1}} \tag{11}$$

where $\frac{E(V^+)}{E(V)} > 1$. In addition, we choose control parameter ε^* so that the utility functions of all relevant decision makers are included in $\mathcal{U}(\varepsilon^*)$, i.e., $D \subset \mathcal{U}(\varepsilon^*)$. Then the risky project dominates the risk-free one in terms of ASD at level ε^*, if and only if $\varepsilon_G \leq \varepsilon^*$, or

$$\frac{1}{\varepsilon^*} - 2 \leq \frac{1}{\frac{E(V^+)}{E(V)} - 1} \tag{12}$$

In other words, ε^*-ASD of the risky project, i.e., $G(\cdot) \succeq_{\varepsilon^*} H(\cdot)$, is equivalent to the following criterion.

Proposition 1 *Let the expected NPV, $E(V)$, and the expected loss in wealth, $E(V^-)$, be strictly positive. Then*

$$E(V^-) \leq \left[\frac{1}{\frac{1}{\varepsilon^*} - 2}\right] E(V) \Leftrightarrow G(\cdot) \succeq_{\varepsilon^*} H(\cdot). \tag{13}$$

[59] Cf. Wolff (1989, p. 19)

[60] As is required by Leshno/Levy (2002).

Compared to the risk-free alternative, a risky investment project thus is attractive to the decision makers, represented by the set of utility functions $D \subset \mathcal{U}(\varepsilon^*)$, if its expected loss in wealth is bounded by a multiple of the expected net present value. This multiple $\gamma(\varepsilon^*) = \frac{1}{\frac{1}{\varepsilon^*}-2}$ exclusively depends on the attitude toward risk commonly agreed upon by all relevant decision makers.[61] Obviously, for risk-neutral decision makers, i.e., $\varepsilon^* \to \frac{1}{2}$, the restriction on the right hand side is rather weak, whereas it becomes tight for strongly risk-averse decision makers. For $\varepsilon^* \to 0$, the right hand side of (13) approaches zero. Henceforth, we refer to the inequality in (13) as the *Leshno-Levy* criterion. It is well known that $E(V^-) = P(V \leq 0)E(-V|V \leq 0)$, see Ogryczak/Ruszczynski (1999). By using this expression the *Leshno-Levy* criterion becomes

$$P(V \leq 0)E(-V|V \leq 0) \leq \gamma(\varepsilon^*)E(V) \tag{14}$$

Note that we did not assume stochastically independent net cash flows. Typically, one would expect sales volume, prices of materials, or labor etc. and hence corresponding cash flows of subsequent periods to be correlated. Under such circumstances the most advisable approach for evaluating the distribution of the cash flows and, hence, the NPV in practical terms is Monte-Carlo simulation.

Example: Suppose that for a risky investment project the initial capital outlay is 50.000 \$ and each of its random cash flows, $C_t, t = 1, 2, ..., 5$, follows a triangular distribution with parameters minimal value, α_{min}, modus, α_{mod}, and maximal value, α_{max}. Let the interest rate be $i = 0.1$. In table 6 numerical values are given for these parameters, where $\alpha_{min} = \alpha_{mod}/3$ and $\alpha_{max} = 1.5\alpha_{mod}$. The calculated lower bounds and expected values of the NPV as defined in (8) for successive conceivable lengths of the investment project's useful life are given in columns six and seven of table 6. Column eight displays the corresponding values for the upper bound on the expected loss in wealth. We assume that the utility functions of all relevant decision makers are included in $\mathcal{U}(0.001)$. In this case, $\gamma(0.001) = 0,001002004$.

[61] Recently, the finance literature discussed the Expected Shortfall as a risk measure to replace the criticized Value-at-Risk. Denote again by $G(0) = P(V < 0)$ the probability of a negative NPV V. If both sides of the inequality in (13) are divided by this probability, the left-hand side reads $E(V^-)/G(0)$. This is the Expected Shortfall at confidence level $G(0)$, see Acerbi/Tasche (2002). Thus, the Proposition says that the risky investment project is ε^*-ASD, if and only if its Expected Shortfall at confidence level $G(0)$ is bounded by $\gamma(\varepsilon^*)\frac{E(V)}{G(0)}$. The Expected Shortfall may be rewritten in terms of a mean-risk model with the weighted mean deviation from quantile as the risk measure, see, e.g., Choi/Ruszczynski (2008). Hence the above interpretation of (13) holds for the mean-risk model as well.

The last column of table 6 shows the values for the expected loss resulting from a Monte-Carlo simulation with 10.000 iterations. Here we assume that the correlation of any pair of successive random cash flows, $(C_t, C_{t+1}), t = 1, 2, 3, 4$, equals 0.3.

	Cash Flow Distribution				Expected NPV and r. h. s. of (13)			
t	α_{min}	α_{mod}	α_{max}	$E(C_t)$	LB_t	$E(V_t)$	$\gamma(\varepsilon^*)E(V_t)$	$E(V_t^-)$
1	6000	18000	27000	17 000.00	-44 545.45	-34 545.45	-34.62	34 568.25
2	7666.67	23000	34500	21 722.22	-38 209.37	-16 593.20	-16.63	16 637.35
3	9666.67	29000	42500	27 388.89	-30 946.66	3 984.47	3.99	2 155.23
4	8333.33	25000	37500	23 611.11	-25 254.88	20 111.18	20.15	131.41
5	6333.33	19000	28500	17 944.44	-21 322.38	31 253.27	31.32	6.99

Table 6: Parameter values of triangular cash flow distributions, lower bound and mean of NPV, right hand side of (13), and simulation results for expected loss

Let the actual useful life of the investment project be five years. The corresponding expected loss then turns out to be 6.99 which is below the critical upper bound of 31.32. Consequently, the risky project dominates the risk-free one in terms of ASD at the given level of $\varepsilon^* = 0.001$. Note that when carrying out the simulation experiment with the same initial seed for the random number generator we observed an expected loss of 1.15 for pairwise uncorrelated cash flow variables, while it increased to 29.59 for a correlation equal to 0.6.

4.3 An interpretation of (13) as a generalization of the payback criterion

While the *Leshno-Levy* criterion (13) is easily applied in a simulation context, it does not provide much insight into the underlying economic mechanism. Hence, we specify the risky investment in greater detail to gain additional insight. The first assumption enables us to derive a simple and intuitive upper bound for the expected loss, $E(V^-)$, of the risky investment project.[62] The following interpretations of (13) will build upon this upper bound.

1. Assumption: Convexity of CDF $G(\cdot)$ on $[LB, 0]$

[62] Ogryczak/Ruszczynski (1999) yield an alternative upper bound. They discuss absolute semideviation $\delta = 0.5 \int_{\infty}^{-\infty} |v - E(V)|\, G(dv)$ as a measure of risk. From their Corollary 2 follows $E(V^-) \leq \delta$. Therefore it is easy to derive a mean-risk criterion from the Leshno-Levy criterion (13) which ensures ε^*-ASD of the risky project: $0 \leq E(V) - \gamma(\varepsilon^*)^{-1}\delta \Rightarrow G(\cdot) \succeq_{\varepsilon^*} H(\cdot)$.

Assume that the net present value, V, is a continuous random variable with CDF $G(\cdot)$. If moreover CDF $G(\cdot)$ is convex on $[LB, 0]$ we get the following upper bound

$$E(V^-) = P(V \leq 0)E(-V|V \leq 0) \leq \frac{1}{2}|LB|G(0) \tag{15}$$

This upper bound is determined by the probability that the NPV is non-positive, $G(0) = P(V \leq 0)$, i.e., a kind of loss probability, and the maximal potential loss in wealth, LB.[63] Thus, $\frac{1}{2}|LB|$ serves as an upper bound for $E(-V|V \leq 0)$.

Whether or not (15) holds is easily checked by evaluating the corresponding simulation results. For instance, the Monte-Carlo simulation considered above yields a $G(0)$ of $0,00224851$. With LB taken from table 6 we calculate the upper bound to be $\frac{1}{2}|LB|G(0) = 23.97$. Since for the example the expected loss amounts to 6.99, see table 6 again, (15) holds. Note that the quality of the upper bound for the expected loss crucially depends on the characteristics of $G(\cdot)$ in the negative part. Especially, if $G(\cdot)$ is rather flat, but shows a significant probability of very small losses, the upper bound of (15) poorly approximates the true $E(V^-)$.

2. Assumption: Non-negative cash flows

Condition (15) becomes even simpler, if all future cash flows are non-negative, i.e., $\underline{c}_t = 0$ for all periods $t \geq 1$. This assumption seems to be restrictive only at first glance.[64] Remember that C_t is the net cash flow originating from the risky project in period t. Hence, a straightforward interpretation of this assumption is that management anticipating a negative net cash flow in the future will discontinue project operations.

With the simplifying assumption $\underline{c}_t = 0$ for all periods $t \geq 1$ and the amount of capital originally tied up in the investment project denoted by $I = |C_0|$, obviously $|LB| = I$. First, we thus conclude from the *Leshno-Levy* criterion (13) and the upper bound (15) the following.

Corollary 1 *Under assumptions 1. and 2.,*

$$\frac{1}{2}G(0)I \leq \left[\frac{1}{\frac{1}{\varepsilon^*} - 2}\right] E(V) = \gamma(\varepsilon^*)E(V) \tag{16}$$

[63] Remember $P(V < 0) = P(V \leq 0)$ for continuous random variables V.

[64] A sensible application of the payback criterion under uncertainty relies on the implicit assumption of non-negative cash flows beyond the cut-off date. Hence, the empirically observable use of the payback method indicates that such investment projects satisfying the assumption of non-negative cash flows in the long run are rather common.

is a sufficient condition for the risky investment project to dominate the risk-free alternative by ε^*-ASD, $G(\cdot) \succeq_{\varepsilon^*} H(\cdot)$.

Formula (16) is a generalization of the simple dynamic version of the payback criterion, a fact we will focus on in this section. Further, we will emphasize in section 4.4 that it can be interpreted as a risk-adjusted rate of return as well.

Second, non-negative cash flows allow an interpretation of (13) in terms of the dynamic payback criterion's deterministic version. This dynamic payback criterion inspects periods $1 \leq t \leq T$ to see whether or not the discounted cumulated cash flows realized hitherto exceed the initial capital outlay I. The payback period is the first period when no capital remains tied up in the project. In order to extend this procedure to the current situation, we define the time-dependent random net present values $V_t, t = 1, ..., T$, analogously to the definition of NPV, V, in (8)

$$V_t = \sum_{\tau=0}^{t} q^{-\tau} C_\tau \tag{17}$$

The CDF $G_t(\cdot)$ of random variable V_t lives the on finite support $[LB_t, UB_t]$, with the lower bound $LB_t = C_0 + q^{-1}\underline{c}_1 + ... + q^{-t}\underline{c}_t$ and the upper bound $UB_t = C_0 + q^{-1}\overline{c}_1 + ... + q^{-t}\overline{c}_t$. Figure 3 shows the CDFs of the time-dependent random NPVs $V_1, V_2, ..., V_5$ based on the Monte-Carlo simulation described in the example of section 4.2 for the case of stochastic independent cash flows.

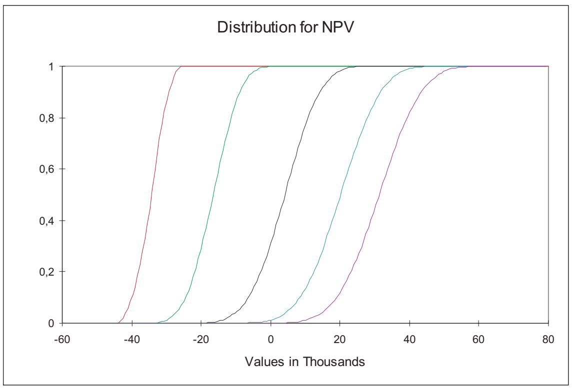

Figure 3: CDFs of NPVs $V_1, V_2, ..., V_5$, from left to right.

To proceed with the payback interpretation of (13) for a given fixed value for ε^*, see figure 4. The straight line in figure 4 starting at the origin is the set of all those combinations (y, z) of expected loss in wealth, $y \geq 0$, and expected NPV, $z \geq 0$, for

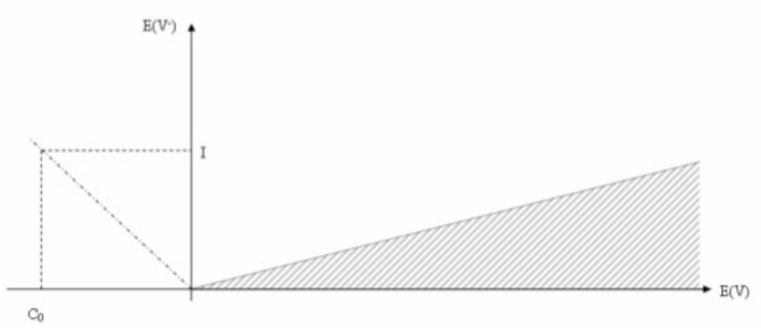

Figure 4: Graphical representation of admissible solutions

which (13) holds with equality, $y = \gamma(\varepsilon^*)z$. If the risky investment project dominates the risk-free alternative in terms of ε^*-ASD, the combination $\big(E(V_T^-), E(V_T)\big)$ is an element of the set of feasible solutions of (13), indicated in figure 4 by the hatched area.

Suppose now that, following the payback procedure, $\big(E(V_t^-), E(V_t)\big)$, $t = 0, 1, ..., T$, are calculated successively. Then, the initial combination $\big(E(V_0^-), E(V_0)\big)$ is not an element of that set of admissible solutions because $E(V_0) = C_0 < 0$. Hence, a period t^*, $1 \leq t^* \leq T$ exists, when (13) holds for the first time. Combinations calculated for later periods, $\big(E(V_{t^*+n}^-), E(V_{t^*+n})\big)$, $n = 1, 2, ..., T - t^*$, may in general leave the set of feasible solutions again. However, for non-negative cash flows $C_t, t > t^*$, this is ruled out, because in this case $E(V_{t+1}^-) \leq E(V_t^-)$ and $E(V_t) \leq E(V_{t+1})$. Thus, t^* reminds of the payback period: All decision makers represented by $\mathcal{U}\ (\varepsilon^*)$ prefer the risky investment project to the risk-free investment, if the former's useful life exceeds t^*, i.e., $t^* \leq T$.

For example, the simulation results displayed in table 6 show that $\big(E(V_t^-), E(V_t)\big)$ are not elements of the set of admissible solutions for $t = 0, 1, ..., 4$, while obviously $\big(E(V_5^-), E(V_5)\big) = (6.99, 31253.27)$ is. Therefore, $t^* = 5$ can be interpreted as a payback period, if the project's useful life is sufficiently long.

Hitherto, we only assumed non-negativity of cash flows and the convexity of CDFs $G(\cdot)$ on $[LB, 0]$. If we impose on the risky investment project the additional assumption that expected cash flows are identical, the relationship between the *Leshno-Levy* criterion (13) and the payback criterion becomes even more obvious.[65]

3. Assumption: Random cash flows with identical expected value

[65] Note that this is a generalization of the original assumption used by Gordon (1955) in his attempt to justify the payback criterion.

Assume in the following a zero interest rate for the risk-free investment, $i = 0$, as the ordinary static payback criterion does. Further, let expected cash flows be identical, $E(C_t) = \hat{c} > 0$, for all periods $t \geq 1$. Under these assumptions the expected NPV of the risky project is given by $E(V) = T\hat{c} - I$, and (16) becomes

$$\frac{G(0)}{2\gamma(\varepsilon^*)} \leq \frac{(T\hat{c} - I)}{I} \tag{18}$$

Inequality (18) means that the risky investment project dominates the risk-free project by ε^*-ASD, i.e., $G(\cdot) \succeq_{\varepsilon^*} H(\cdot)$. Denoting the static project-specific payback period in the common way by $\beta_G = \frac{I}{\hat{c}}$ and rearranging (18) results in

Corollary 2 *Under assumptions 1., 2., and 3.*

$$\beta_G \leq \frac{T}{\frac{1}{2}\left(\frac{1}{\varepsilon^*} - 2\right) G(0) + 1} = \beta^* \leq T \tag{19}$$

is a sufficient condition for ε^*-ASD dominance of the risky investment project, $G(\cdot) \succeq_{\varepsilon^*} H(\cdot)$.[66]

In (19) β^* is the cut-off value which the project-specific payback period β_G should not exceed. If the common static payback period β_G is not greater than β^*, the risky investment project is the ε^*-dominant alternative. Thus, (19) reflects the way the common payback criterion is used as a restriction in capital budgeting processes, see section 3: If the payback period β_G is small enough relative to the cut-off value, then the scrutinized risky investment project is acceptable. ε^*-almost stochastic dominance as a tool for modeling certain aspects of organizational structures of the company in the way described in this paper, therefore, may conceptually explain the use of the payback criterion in many real-world capital budgeting processes.

Expectedly, the cut-off period β^* depends on the risk parameter ε^* that represents the decision makers attitude toward risk. Further, it is a function of the project specific useful life T and the probability of eventually suffering a loss, $G(0) = P(V < 0)$, too.[67] The smaller ε^*, i.e., the more risk averse the relevant decision makers are, and the greater the loss probability as a parameter measuring

[66] Note that this Corollary is easily modified to cover the alternative scenario used by Levy (1968) in his analysis of the payback criterion. Moreover, we can adjust for the case of non-identical but positive expected cash flows.

[67] That the cut-off period should depend on the risk of the project under scrutiny is well known in the literature, cf. Horngreen et al. (2006, p. 731).

the project's inherent risk, the more restrictive is the cut-off value for a given useful life. The longer, c. p., the useful life, the weaker is the requirement given by (19).[68]

4.4 An interpretation of (13) as a generalization of a rate of return criterion

For this interpretation, assume again that the CDF, $G(\cdot)$, of the final NPV, V, is continuous on its finite support, convex on $[LB, 0]$, and that cash flows are non-negative. Hence, condition (16) is applicable to show the risky investment project's ε^*-ASD. Rearranging this condition yields the following

Corollary 3 *Under assumptions 1. and 2.*

$$\hat{\rho} = \frac{G(0)}{2\gamma(\varepsilon^*)} \leq \frac{E(V)}{I} = \rho_G \tag{20}$$

is a sufficient condition for ε^*-ASD dominance of the risky investment project, $G(\cdot) \succeq_{\varepsilon^*} H(\cdot)$.

$E(V) > 0$ is the expected net present value representing the increase in wealth resulting from the risky investment project. Since the net present value of the risk free investment project is by definition zero, ρ_G may be interpreted as a multi-period risk premium, defined for T periods. Hence, the risky investment project dominates the risk-free one in terms of ε^*-ASD, if the project's multi-period risk premium exceeds the cut-off value $\hat{\rho}$.[69] Phrased differently, the decision makers demand a compensation of at least $\hat{\rho}I$ for the risk they take. While the expected NPV is commonly calculated as

$$E(V) = \sum_{\tau=0}^{T} q^{-\tau} E(C_\tau) = -I + \sum_{\tau=1}^{T} q^{-\tau} E(C_\tau) \tag{21}$$

the risky project is dominant by ε^*-ASD if according to (20)

$$0 \leq -\hat{\rho}I + E(V) = -(1+\hat{\rho})I + \sum_{\tau=1}^{T} q^{-\tau} E(C_\tau) \tag{22}$$

[68] For example, if the useful life of an investment project is $T = 10$ years and the probability of a loss in wealth is $G(0) = 0.2$, the board determining the cut-off payback period to be 3 years implies that $\varepsilon^* = 0.04$, i.e., the set of all relevant decision makers, D, is a subset of $\mathcal{U}(0.04)$.

[69] This cut-off value is transformed into a required markup to the interest rate soon.

This means, the decision makers expect the project to generate (expected and discounted) cash flows which cover not only the initial capital outlay, I, but also the multi-period risk compensation, $\hat{\rho}I$. Note that the critical value, $\hat{\rho}$, and thus the compensation, depend on the project's risk in terms of the probability of a loss in wealth, $G(0)$, and parameter $\gamma(\varepsilon^*)$, representing the risk attitude of the decision makers.

Inequality (20) provides a key to delegating at least part of the investment decision process to lower management levels. Once parameter $\gamma(\varepsilon^*)$ is determined for the investment committee, other managers may collect and analyze project-data and subsequently calculate the loss probability, the expected net present value $E(V)$, and the initial capital outlay I. Finally, they can compute the project-specific multi-period risk premium ρ_G and compare it to the specific cut-off risk premium $\hat{\rho}$ derived from (20). If the cut-off premium is exceeded, the risky investment project is preferable to the risk-free alternative from the investment committee's point of view.

Example: Table 7 depicts the values of parameter ε^* computed from (20) for different values of the specific cut-off rate $\hat{\rho}$ and the probability of a loss in wealth $G(0)$. As an example suppose that the relevant decision makers expect a multi-period risk premium of 0.12 and the probability of suffering a loss in wealth is 0.10. Thus, all decision makers in $\mathcal{U}(0.227)$ prefer the risky investment project to the risk-free one. If at least one relevant decision maker was more risk averse, i.e., the corresponding ε^* was smaller than 0.227, the risk premium would have to be increased accordingly.

	cut-off risk premium $\hat{\rho}$							
$G(0)$	0.1	0.11	0.12	0.13	0.14	0.15	0.2	0.25
0.001	0.00495	0.00450	0.00413	0.00382	0.00355	0.00331	0.00249	0.00199
0.01	0.04545	0.04167	0.03846	0.03571	0.03333	0.03125	0.02381	0.01923
0.05	0.16667	0.15625	0.14706	0.13889	0.13158	0.12500	0.10000	0.08333
0.1	0.25000	0.23810	0.22727	0.21739	0.20833	0.20000	0.16667	0.14286
0.2	0.33333	0.32258	0.31250	0.30303	0.29412	0.28571	0.25000	0.22222
0.3	0.37500	0.36585	0.35714	0.34884	0.34091	0.33333	0.30000	0.27273

Table 7: ε^* for given critical value and loss probability $G(0)$

Table 7 can also be read in another way. From Table 4 we know that in the example of exponential utility functions all seven decision makers are included in $\mathcal{U}(\varepsilon^*)$ for

$\varepsilon^* = 0.332$. Thus, if the loss probability is estimated at $G(0) = 0.3$, a multi-period risk premium above 0.15 is required by the investment committee, given almost stochastic dominance is used as the basis for the final decision.
As stated above the critical risk premium, $\hat{\rho}$, is a multi-period rate, i.e., it is not an interest rate for a single period but for the project's whole useful life. In order to emphasize the appropriate interpretation of this multi-period risk premium, it will be transformed into an annual rate. Equating the right hand side of (22) to zero yields the project's risk-adjusted internal rate of return, $\hat{\imath}$, that accounts for the compensation of project risk. Consequently, the risky project is preferred to the risk-free one by ε^*-ASD, if this risk-adjusted IRR exceeds the risk-free investment's interest rate i, $\hat{\imath} \geq i$,.

Calculating the risk-adjusted IRR of a project with identical expected cash flows, $E(C_t) = \hat{c}$ for all $t \geq 1$, is particularly simple, as in this case (22) reduces to[70]

$$0 = -(1+\hat{\rho})I + \alpha(\hat{\imath}, T)\hat{c}, \tag{23}$$

Note that we get

$$\alpha(\hat{\imath}, T) = (1+\hat{\rho})\, \alpha(i^*, T) \tag{24}$$

from (23), if i^* is the common IRR of the investment project calculated from equating (21) to zero, i.e., ignoring the demanded risk compensation, for identical expected cash flows $\hat{c}$

$$0 = -I + \alpha(i^*, T)\hat{c} \tag{25}$$

Example: For a given cut-off risk-premium of $\hat{\rho} = 0.12$, a useful life of $T = 6$ and a common IRR of $i^* = 0.15$, we derive from (24)

$$\alpha(\hat{\imath}, 6) = 1.12\alpha(0.15, 6) = 4,238620617 \tag{26}$$

or a risk-adjusted IRR $\hat{\imath} = 0.10934$. Hence, this per annum IRR is about 0.04 smaller than the common IRR of $i^* = 0.15$. Put differently, this means that instead of levying a risk compensation of $\hat{\rho}I$ the interest rate of the risk-free investment, i, could be increased by 0.04 to yield the appropriate cut-off value for the common IRR.[71]

[70] $\alpha(i, T) = \frac{(1+i)^T - 1}{(1+i)^T i}$ denotes the annuity factor for T periods and interest rate i.

[71] Note that Pike (1984, p. 344) empirically estimates the risk premium added to the risk-free interest rate between 7% and 16% for typical-risk investment projects.

5 Summary and conclusions

Initiated by the payback criterion's prominence in business practice the following research questions have been raised:

1. In the analytical literature on capital budgeting the organizational embedding of investment decision processes has been neglected hitherto. If taken into account, can it contribute to explaining the widespread use of key investment figures, like, e.g., payback periods or hurdle rates defined on profitability?
2. How may investment decisions within a multi-person context be represented from a decision theoretical perspective? Does almost stochastic dominance provide a suitable modeling device?
3. The information required for communicating risk attitudes via ASD is less extensive compared to that under an EUT regime. Can this fact contribute to the explanation of the payback criterion's popularity in business practice?

Following the review of a large body of empirical papers in section 2 which emphasize the high relevance of the problem at hand, we develop in section 3 a framework of representative organizational environments of investment decision processes. In contrast to most contributions analyzing the use of key figures in capital budgeting like, e.g., the payback period, we allow for these organizational aspects to play an important role in decision making. On the one hand, we account for the fact that investment responsibility is typically delegated to a group of individuals. Such groups need to agree on the merits of a risky investment project as compared to a risk free investment, demanding descriptions of risk attitudes which can be easily discussed and compared. On the other hand, we take into consideration that preparation of proposals, pre-selection of projects, and decision making are (partly) delegated to lower-level management. Hence, it is a requirement that descriptions of risk attitudes can be easily communicated.

Having derived the importance of the investment decisions' organizational embedding, we introduce in section 4.1 the concept of almost stochastic dominance for describing the risk attitudes of the relevant decision makers. From a theoretical perspective, a risky investment project's profitability is best described by its stochastic net present value. But even if the corresponding distribution function was given and known by all decision makers, the corresponding risk might individually be perceived in different ways due to varying utility functions. This complicates the modeling of capital budgeting decisions in a multi-person context. However, as demonstrated in section 4.2 of this paper, ε-ASD may be extended to represent the risk preferences of a specific group of decision makers characterized by the parameter value ε^*. Thus, the participation of a multitude of decision makers is accounted for in a natural way: ε^*-ASD of a risky investment project over the risk-free alternative means that all relevant decision makers prefer the first project. We show under very general assumptions that this ε^*-ASD is equivalent to a criterion that balances the risky project's expected loss with its expected NPV, weighted by a factor that depends on the decision makers' attitudes toward risk. This criterion is very easily applied to real-world investment projects using simple spread sheet simulation as demonstrated in section 4.2. Note moreover, that determining ε^* as the parameter describing the decision makers' risk attitudes might be easier than estimating von-Neumann-Morgenstern utility functions from a decision analytical point of view.

The simplification of information processing activities inherent to this approach becomes evident, when, an application of the *Leshno-Levy* criterion or the even simpler payback criterion as stated in Corollary 2 is compared to decision making based on Expected Utility Theory (EUT) in a decentralized firm. Consider a group of decision makers in charge of the capital budgeting decision which wants to delegate preparatory tasks, like, e.g., the pre-selection of viable projects, or the final decision on 'small projects', to the lower ranks of the management hierarchy. Following EUT, the set of individual utility functions representing the relevant decision makers would have to be communicated to those managers responsible for the preparatory stages of the decision making process or selecting 'small' investment projects. In contrast, the use of the ASD payback criterion is fairly simple and secures acceptability from all committee members' point of view. By means of this tool risk preferences are simply conveyed and delegating project selection to lower-level management is easily controlled. Hence, the number of pieces of information to be exchanged is reduced significantly. Instead of communicating a set of utility functions respectively the probability distribution of the NPV only the cut-off value, β^*, or the project specific payback period, β_G, is transmitted.

The more detailed analysis in section 4.3 shows our criterion to be a generalization of the common payback criterion. Similar considerations apply to the criterion's interpretation as a risk-adjusted internal rate of return in section 4.4. Developing criteria from almost stochastic dominance which coincide with simple key indicators contributes to explaining the payback criterion's prominence. The reason is that these criteria allow to identify the investment alternative preferable by a group of relevant decision makers and simplify communication of risk preferences as well as project characteristics.Since they are derived under fairly general organizational circumstances these criteria are applicable to real-life capital budgeting processes. Moreover, formulae for calculating cut-off values of these key indicators like in Corollary 2 may support reasonable decision making. Under the corresponding assumptions decisions based on these criteria turn out be rational in terms of decision theory. Further, these criteria permit identifying investment projects which are commonly acceptable in terms of risk, even at the initial stages of the appraisal process. Thus, they provide reasonable instruments to reduce the burden resting on final decision making bodies with respect to routinely approved projects.

Note that our analysis focuses solely on simple investment appraisals, i.e., the choice between a risky investment project and a risk-free alternative. This is no limitation for the majority of 'small investment projects' involving decision-delegation to lower and middle management. Here, restrictions on liquidity are of minor interest, meaning the projects will be assessed based exclusively on their individual merits. Considerations will be different for 'large investment projects' challenging a company's financial resources. Here, with two or more projects competing for liquidity, the choice will be between different risky investment projects. In this case, the role of the payback criterion may be that of a necessary condition, meaning the criterion has to be met by the project for being considered as a reasonable alternative. Moreover, for projects of this importance a formal investment proposal should be required. Here, the payback criterion may serve as one important indicator, but the final decision will depend on a number of additional aspects, like strategic, managerial, technological or others.

A Additional evidence on PB's prominence

Recent data from a European survey study are presented in Table 8:[72]

	GER	FRA	NL	UK
NPV	47.6%	35.1%	70.0%	47.0%
IRR	42.2%	44.1%	56.0%	53.1%
PB	50.0%	50.9%	64.7%	69.2%

Table 8: Capital budgeting in Europe

Further, additional American surveys are given in Table 9:

	Gitman et al. (1977)	Schall et al. (1978)	Kim et al. (1990)	Cooper et al. (1992)	Graham/ Harvey (2001)
NPV	36%	56%	45%	46%	75%
IRR	68%	65%	64%	85%	76%
PB	53%	74%	54%	58%	57%

Table 9: Capital budgeting in the US

B Facts and definitions

For reading convenience we put together the following definition and facts

(D1) For $\underline{y}, \overline{y} \in \mathbb{R}, \underline{y} < \overline{y}$, the set $\mathcal{U}(\varepsilon;\, y)$ of twice differentiable, non-satiable, and real valued utility functions is defined by

$$\mathcal{U}(\varepsilon; y) = \left\{ u \in \mathcal{U} : u'(y) \leq \left[\frac{1}{\varepsilon} - 1\right] \min\left\{u'(z) : z \in [\underline{y}, \overline{y}]\right\}, y \in [\underline{y}, \overline{y}] \right\}$$

(F1) $\mathcal{U}(\varepsilon;\, y) \subset \mathcal{U}$ for all $0 < \varepsilon \leq 1/2$

(F2) $\varepsilon_1 < \varepsilon_2 \Rightarrow \mathcal{U}(\varepsilon_2; y) \subset \mathcal{U}(\varepsilon_1; y)$; see *Leshno/Levy* (2002, p. 1079)

(F3) Let $M \subset \mathbb{R}$ be an interval and $[\underline{y}, \overline{y}] \subset M$. For twice differentiable utility functions on M the defining inequality in (D1) may be reformulated:

[72] DeBrounen et al. (2004, p. 7).

$$\max\left\{u'(z) : z \in [\underline{y}, \overline{y}]\right\} \leq \left[\frac{1}{\varepsilon} - 1\right] \min\left\{u'(z) : z \in [\underline{y}, \overline{y}]\right\}$$

$$\Leftrightarrow \varepsilon \leq \left[1 + \frac{\max\{u'(z)\}}{\min\{u'(z)\}}\right]^{-1} =: e_u \in \left(0, \frac{1}{2}\right]$$

where $\varepsilon = 0.5$ iff the utility function is linear. Let $D \subset \mathcal{U}$ be the finite set of utility functions representing the relevant decision makers. To avoid a situation where the evaluation of the risky project based on expected values is sufficient, exclude the case of only risk neutral decision makers. Note that for risk neutral utility functions the bound defined in (F3) becomes $e_u = 1/2$. Further, let $\mathcal{E}(D; y)$ denote the set of parameter values ε with $D \subseteq \mathcal{U}(\varepsilon; y)$:

$$\mathcal{E}(D; y) = \{\varepsilon : 0 < \varepsilon < 1/2, D \subseteq \mathcal{U}(\varepsilon; y)\} \tag{27}$$

Lemma 1 *$\mathcal{E}(D; y)$ is a non-empty set and bounded from above by $\hat{e} < \frac{1}{2}$.*

Proof: Let $[\varepsilon_n, n \in \mathbb{N}]$, $0 < \varepsilon_n < 1/2$, be a sequence of numbers with $\lim_{n\to\infty} \varepsilon_n = 0$. Due to (F2) $[\mathcal{U}(\varepsilon_n; y), n \in \mathbb{N}]$ is a non-decreasing sequence of sets. Now let $\hat{e} = \min\{e_u : u \in D\}$ be the smallest of the bounds defined in (F3) for any utility function in the set D. $0 < \hat{e} < 1/2$ holds true because there is at least one non-linear utility function in D. Hence, there is an $n_{\hat{e}} \in \mathbb{N}$ so that for all $n \geq n_{\hat{e}}$ we have $\varepsilon_n < \hat{e}$. For all these ε_n, due to (F3), $D \subseteq \mathcal{U}(\varepsilon_n; y)$. Thus from definition (27) follows that $\mathcal{E}(D; y)$ is non-empty. Moreover, $\mathcal{E}(D; y)$ is bounded by 0 and $\hat{e} < 1/2$.

Hence, $\sup \mathcal{E}(D; y) < \frac{1}{2}$ exists. Consequently, define the largest ε, so that the set $\mathcal{U}(\varepsilon;\, y)$ contains the utility function of all relevant decision makers, i.e. ε^*_y, by

$$\varepsilon^*_y := \sup \mathcal{E}(D; y) \tag{28}$$

With this definition we get the following result for intervals $[\underline{v} < \overline{v}] \subset [\underline{x} < \overline{x}]$ discussed in section 4.1

Proposition 2 *For $[\underline{v} < \overline{v}] \subset [\underline{x} < \overline{x}]$ define ε^*_x and ε^*_v by (28). Then $\varepsilon^*_x \leq \varepsilon^*_v$.*

Proof: Definitions (28) and (D1) yield $D \subseteq \mathcal{U}(\varepsilon^*_x; x) \subset \mathcal{U}(\varepsilon^*_x; v)$. Hence, $\varepsilon^*_x \in \mathcal{E}(D; v)$ and, therefore, $\varepsilon^*_x \leq \sup \mathcal{E}(D; v) = \varepsilon^*_v$.

References

Acerbi, C., D. Tasche [2002]: "On the coherence of expected shortfall", in: *Journal of Banking and Finance*, 26, pp. 1487-1503

Ann, W.K., E.J. Farragher, R.K.C. Leung [2001]: "Capital investment practices: A survey of large corporations in Malaysia, Singapore and Hong Kong", in: *Asia Pacific Journal of Management*, 4, pp. 112-123

Arnold, G.C. [2005]: "Corporate Financial Management", 3rd ed., Prentice Hall, Harlow et al.

Arnold, G.C., P.D. Hatzopoulos [2000]: "The theory-practice gap in capital budgeting: Evidence from the United Kingdom", in: *Journal of Business Finance & Accounting*, 27, pp. 603-626

Bailes, J.C., T. Assada [1991]: "Empirical differences between Japanese and American budget and performance evaluation systems", in: *The International Journal of Accounting*, 26, pp. 131-142

Block, S. [1997]: "Capital budgeting techniques used by small business firms in the 1990s", in: *The Engineering Economist*, 42, pp. 289-302

Brounen, D., A. de Jong, K. Koedijk [2004]: "Corporate finance in Europe –Confronting theory with practice", Workingpaper, *Erasmus Research Institute of Management*, ERS-2004-002-F&A

Bureau of Economic Analysis – U.S. Department of Commerce [2007]: "Summary estimates for multinational companies: Employment, sales and capital exdpenditures for 2005", in: *http://www.bea.gov/newsreleases/international/mnc/mncnewsrelease.htm*

Chen, S., R. Clark [1994]: "Management compensation and payback method in capital budgeting: A path analysis", in: *Accounting and Business Research*, 24, pp. 121-132

Choi, S., Ruszczynski [2008]: "A risk-averse newsvendor with law invariant coherent measure of risk", in: *Operations Research Letters*, 36, pp. 77-82

Cooper, W.D., M.F. Cornick, A. Redmon [1992]: "Capital budgeting: A 1990 study of Fortune 500 company practices", in: *Journal of Applied Business Research*, 8, pp. 20-23

Ferreira, E.J., L. Brooks [1988]: "Capital budgeting: A key management challenge", in: *Business*, 38, pp. 22-29

Gitman, J., J.R. Forrester, Jr. [1977]: "A survey of capital budgeting techniques used by major U.S. firms", in: *Financial Management*, 6, pp. 66-71

Gordon, M.J. [1955]: "The payoff period and the rate of profit", in: *Journal of Business*, 28, pp. 253-260

Graham, R., C.R. Harvey [2002]: "How do CFOs make capital budgeting and capital structure decisions?", in: *Journal of Applied Corporate Finance*, 15, pp. 8-23

Graham, R., C.R. Harvey [2001]: "The theory and practice of corporate finance: Evidence from the field", in: *Journal of Financial Economics*, 60, pp. 1-27

Hall, J.H. [2000]: "Investigating aspects of the capital budgeting process used in the evaluation of investment projects", in: *South African Journal of Economic and Management Sciences*, 3, pp. 353-368

Harris, M., A. Raviv [1996]: "The capital budgeting process: Incentives and information", in: *Journal of Finance*, 51, pp. 1139-1174

HERMES, N., P. SMID, L. YAO [2005]: "Capital budgeting practices: A comparative study of the Netherlands and China", Working Paper, *Faculty of Economics, University of Groningen*

HERTZ, D.B. [1968]: "Investment policies that pay off", in: *Harvard Business Review*, 46, pp. 96-108

HIRSHLEIFER, D. [1993]: "Managerial reputation and corporate investment decisions", in: *Financial Management*, 22, pp. 145-160

HORNGREN, C.T., S.M. DATAR, G. FOSTER [2006]: "Cost accounting – a managerial emphasis", 12th ed., Prentice Hall, Upper Saddle River

KIM, S.H., T. CRICK, S.H. KIM [1986]: "Do executives practice what academics preach?", in: *Management Accounting*, 68, pp. 49-52

KIM, I., J. SONG [1990]: "U.S., Korea, & Japan – accounting practices in three countries", in: *Management Accounting*, 72, pp. 26-30

KLAMMER, T., B. KOCH, N. WILNER [1991]: "Capital budgeting practices – a survey of corporate use", in: *Journal of Management Accounting Research*, 3, pp. 113-130

LEFLEY, F. [1994]: "Capital investment appraisal of advanced manufacturing technology", in: *International Journal of Production Research*, 32, pp. 2751-2776

LEFLEY, F. [1996]: "The payback method of investment appraisal: A review and synthesis", in: *International Journal of Production Economics*, 44, pp. 207-224

LESHNO, M., H. LEVY [2002]: "Preferred by "all" and preferred by "most" decision makers: Almost stochastic dominance", in: *Management Science*, 48, pp. 1074-1085

LEVY, H. [1971]: "A comment on payback: A reply", in: *Journal of Financial and Quantitative Analysis*, 6, p. 1161

LEVY, H. [1968]: "A note on the payback method", in: *Journal of Financial and Quantitative Analysis*, 3, pp. 433-443

LEVY, H. [2006]: "Stochastic Dominance", 2nd ed., Springer

MEPHAM, M.J. [1975]: "A payback interpretation of the annuity tables", in: *Accounting Review*, 50, pp. 869-870

NARAYANAN, M.P. [1985]: "Observability and the payback criterion", in: *Journal of Business*, 58, pp. 309-323

O'BRIEN, M. [1997]: "Payback: A gulf between managerial accounting and financial theory in practice – A view from accountants and finance officers in Ireland", in: *International Journal of Accounting*, 32, pp. 173-186

OGRYCZAK, W., A. RUSZCZYNSKI [1999]: "From stochastic dominance to mean-risk models: Semideviations as risk measures", in: *European Journal of Operational Research*, 116, pp. 35-50

PIKE, R.H. [1983]: "The capital-budgeting behaviour and corporate characteristics of capital-constrained firms", in: *Journal of Business, Finance & Accounting*, 10, pp. 663-671

PIKE, R.H. [1985]: "Owner-manager and the role of the payback method", in: *Accounting and Business Research*, 16, pp. 47-51

PIKE, R.H. [1988]: "An empirical study of the adoption of sophisticated capital budgeting practices and decision-making effectiveness", in: *Accounting and Business Research*, 18, pp. 341-351

PIKE, H. [1996]: "A longitudinal survey on capital budgeting practices", in: *Journal of Business Finance & Accounting*, 23, pp. 79-92

PINCHES, G. [1982]: "Myopia, Capital Budgeting and Decision Making", in: *Financial Management*, 21, pp. 6-19

ROSS, M. [1986]: "Capital Budgeting Practice of Twelve Large Manufacturers", in: *Financial Management*, 15, pp. 15-22

RYAN, P.A., G.P. RYAN [2002]: "Capital budegting practice of the Fortune 1000: How have things changed", in: *Journal of Business and Management*, 8, pp. 355-364

SANGSTER, A. [1993]: "Capital investment appraisal techniques: A survey of current usage", in: *Journal of Business Finance & Accounting*, 20, pp. 307-332

SARNAT, M., H. LEVY [1969]: "The relationship of rules of thumb to the internal rate of return: A restatment and generalization", in: *Journal of Finance*, 24, pp. 479-490

SCHALL, L.D., G.L. SUNDEM, W.R. GEIJSBEK JR. [1978]: "Survey and analysis of capital budgeting methods", in: *Journal of Finance*, 33, pp. 281-287

SCOTT, D.F., J.W. PETTY [1984]: "Capital budgeting practices in large american firms: A Retrospective analysis and synthesis", in: *The Financial Review*, 19, pp. 111-123

SHIELDS, M.D., C.W. CHOW, Y. KATO, Y. NAKAGAWA [1991]: "Management accounting practices in the U.S. and Japan: Comparative survey findings and research implications", in: *Journal of International Financial Management and Accounting*, 3, pp. 61-77

TAGGART JR., R.A. [1987]: "Allocating capital among a firm's disvisions: Hurdle rates vs. budgets", in: *Journal of Financial Research*, 10, pp. 177-189

THAKOR, A.V. [1990]: "Investment 'myopia' and the internal organization of capital allocation decisions", in: *Journal of Law, Economics, and Organization*, 6, pp. 129-154

TULLY, S. [2007]: "A profit gusher of epic proportions", in: *http://money.cnn.com/magazines/fortune/fortune_archive/2007/04/30/8405391/index.htm*

WAMBACH, A. [2000]: "Payback criterion, hurdle rates and the gain of waiting", in: *International Review of Financial Analysis*, 9, pp. 247-258

WEINGARTNER, H.M. [1969]: "Some new views on the payback period and capital budgeting decisions", in: *Management Science*, 12, pp. 594-607

WOLFF, R. W. [1989]: Stochastic modelling and the theory of queues, Prentice Hall, Englewood Cliffs, New Jersey

YARD, S. [2000]: "Developments of the payback method", in: *International Journal of Production Economics*, 67, pp. 155-167

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