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Large Eddy Simulations of gaseous flames in gas turbine combustion chambers

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A B S T R A C T

Recent developments in numerical schemes, turbulent combustion models and the regular increase of computing power allow Large Eddy Simulation (LES) to be applied to real industrial burners. In this paper, two types of LES in complex geometry combustors and of specific interest for aeronautical gas turbine burners are reviewed: (1) laboratory-scale combustors, without compressor or turbine, in which advanced measurements are possible and (2) combustion chambers of existing engines operated in realistic operating conditions. Laboratory-scale burners are designed to assess modeling and fundamental flow aspects in controlled configurations. They are necessary to gauge LES strategies and identify potential limitations. In specific circumstances, they even offer near model-free or DNS-like LES computations. LES in real engines illustrate the potential of the approach in the context of industrial burners but are more difficult to validate due to the limited set of available measurements. Usual approaches for turbulence and combustion sub-grid models including chemistry modeling are first recalled. Limiting cases and range of validity of the models are specifically recalled before a discussion on the numerical breakthrough which have allowed LES to be applied to these complex cases. Specific issues linked to real gas turbine chambers are discussed: multi-perforation, complex acoustic impedances at inlet and outlet, annular chambers... Examples are provided for mean flow predictions (velocity, temperature and species) as well as unsteady mechanisms (quenching, ignition, combustion instabilities). Finally, potential perspectives are proposed to further improve the use of LES for real gas turbine combustor designs.

Keywords:

Large Eddy Simulations
Complex geometry
Swirled flows
Gaseous combustion
Turbulent combustion
Gas turbine

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1. Introduction

Aeronautical turbulent reacting flows involve a wide range of scales and complexities caused by the specific shapes of engines and the combustion regimes encountered in these devices. Because of the space and weight constraints, designers need to develop burners which ensure maximum efficiency and compactness. Over the years, manufacturers have gained significant experience and existing designs largely rely on flow recirculations to increase mixing and flow-through times despite a reduced size combustion chamber. In parallel, pollutant emissions and regulations have induced changes of technology with the emergence of partially premixed and premixed burners. Multi-point fuel injection systems and staging are also being implemented as potential solutions to the new regulations. All these concepts increase the complexity of the flow and lead to specific flow dynamics and combustion responses. Although these designs are being routinely evaluated

by Computational Fluid Dynamics (CFD), most present modeling strategies rely on Reynolds Average Navier-Stokes (RANS) approaches developed for mean stationary flows [1–10]. Such models benefit from extensive research and developments from the scientific community and have been successfully calibrated on simple fundamental configurations. However, the complexity of flows in modern gas turbines adds multiple constraints on RANS and limits their precision, Fig. 1. Alternative numerical solutions are thus needed to further increase the share of CFD and decrease the number of real engine tests and design iterations.

CFD alternatives to RANS for aeronautical gas turbine applications must justify the increase in development, maintenance and computer costs. These new tools need also to be compatible with existing industrial knowledge and conception rules. The use of new CFD approaches and their future in the design chain is still unclear. It will probably depend on the computing power available to engineers as well as their ability to master and analyze ever more

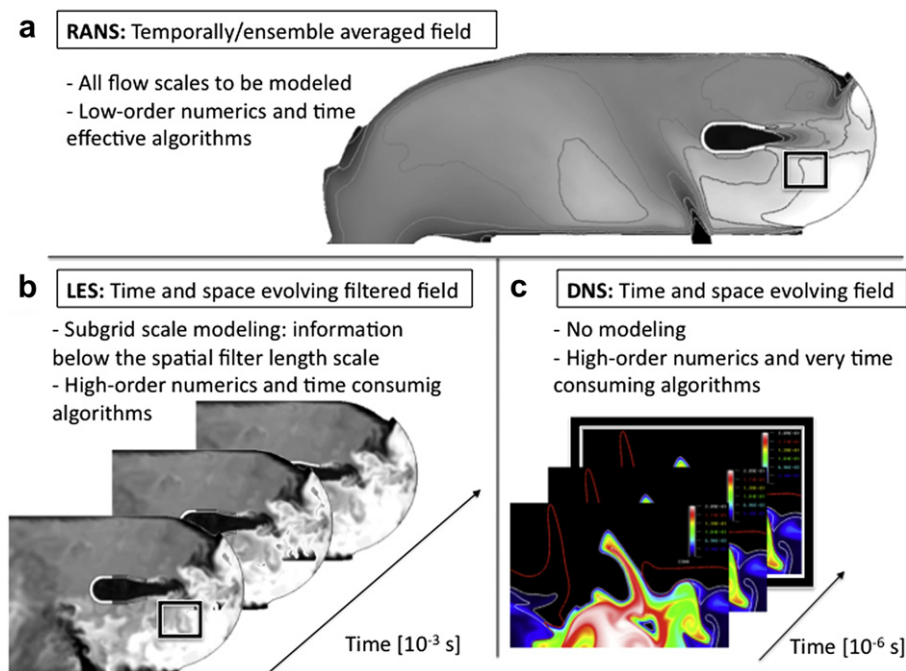


Fig. 1. Schematic representation of the three numerical methods used to simulate turbulent reacting flows: (a) RANS provide access to a temporally/ensemble averaged field representing the flow field in complex systems (extracted from [319]); (b) LES give access to a temporally and spatially evolving set of fields representative of the spatially filtered governing system of equations (extracted from [320,360]) and (c) DNS provide the exact spatially and temporally evolving field obtained by directly solving the governing equations (extracted from [361]).

complex predictions. From a modeling point of view, combustion CFD scientists improved numerical predictions by focusing their efforts on time and space dependent descriptions of the problems. The main objective of such unsteady simulation is to relax the modeling constraints by taking into account unsteadiness and inhomogeneities which are very difficult to model with RANS [11–13]. Two fully unsteady computing and modeling strategies are currently available for turbulent reacting flows: (1) Direct Numerical Simulations (DNS) and (2) Large Eddy Simulations (LES). While DNS, Fig. 1(c), suppress any notion of modeling [14–19] aside from the chemical model which needs to be supplied, LES, Fig. 1(b), introduce a scale separation between the large and small scale flow motions [20–30]. Small scale effects on the large scales are thus to be mimicked by a model.

With DNS all scales must be resolved and computing costs grow with the flow turbulent Reynolds and Damköhler numbers [31], respectively noted $Re_t = u' l_t / \nu$ and $Da = \tau_t / \tau_c$. These two numbers involve the turbulent velocity fluctuation, u' , its characteristic length scale, l_t , and time scale, τ_t as well as the dynamic fluid viscosity, ν and a chemical time scale, τ_c . For DNS of non-reacting turbulent flows, every flow scale is to be resolved so scaling laws read: $Re_t < N^{(4/3)}$, where N is the number of cells in each direction and $Re_t^3 < M$, where M is the number of temporal integration steps [30,32]. Both criteria are needed to ensure a proper statistical representation of the larger flow scale as well as of the dissipative scales (Kolmogorov length scale, η_K [33,34]). For turbulent premixed reacting flows, the spatial scaling is $Re_t Da < (N/Q)^2$ for a one-step irreversible reaction, Q being the number of grid points in the thin reaction zone (of the order of 20 for simple chemical schemes) [32]. DNS is hence limited by two ratios: l_t / δ_l^0 , the turbulent to flame thickness ratio and u' / s_l^0 , the turbulent to chemical speeds. For DNS of turbulent non-premixed reacting flows, mixing and chemical times need to be both accurately represented since τ_c is controlled not only by the mixing of the adjacent streams of fuel and oxidizer but also by the consumption rate (that locates around the stoichiometric line). However both phenomena are flow dependent and clear numerical constraints are difficult to obtain unless more constraints are provided [35–37]. Current computing strategies [37–41] with adaptive meshing and high-order numerical schemes [42–46] allow to cover simple configurations which are used for model validations and understanding of fundamentals of turbulent reacting flows [43,46,47]. Aeronautical reactive flows remain out of reach because of the high Reynolds number, $O(10^8)$, and the highly energetic fuels yielding l_t / δ_l^0 to be of the order of 10 to 1000 and u' / s_l^0 to range from 0.5 to 500 depending on the target application.

LES put less stringent limits for the computational size by filtering out all flow small scales. Ideally for non-reacting flows, Sub-Grid Scale (SGS) velocity models impose that the scale separation or filter cut-off frequency lies in the inertial range of the turbulent spectrum yielding $Re_t < (qN)^{(4/3)}$ where q is the proportionality factor between η_K , the Kolmogorov length scale, and the cut-off length of the filter, Δ_F . Hence, and aside from the chemistry problem, LES allow simulating turbulent flows with turbulent Reynolds numbers approximately 500 times larger than DNS with the same number of points. Evaluation of the proper scaling for turbulent reacting LES remains unclear and often depends on the turbulent combustion model used to close the corresponding SGS terms.

Over the two last decades, DNS and LES have grown very rapidly thanks to the large increase in computing power and the rise of massively parallel architectures. LES codes and models have appeared as a clear scientific alternative to RANS and they are routinely being developed and bench-marked by the turbulent combustion scientific community. Recent developments of this approach now focus on transient flow phenomena with added

complexities: *i.e.* multi-phase flows, ignition and extinction sequences.... Note however that numerical and modeling prerequisite conditions are usually not available in the literature especially for real aeronautical burner simulations. The actual contribution of such methods and their potential use in the context of the industrial design chain are thus still to be investigated and tested.

This review intends to highlight available strategies and models that have allowed LES of aeronautical applications as well as their validations thereby underlying the current status and potential limitations or need for developments [48]. To do so the document focuses only on recent years gaseous reacting LES in light of the industrial need. Details on LES (modeling, numeric, massively parallel computations...) are first provided. Specificities related to industrial flow burners are then given along with the step-by-step recent LES validations for industry-like experimental burners. The advent of highly resolved LES (almost DNS-like computations) are specifically discussed for these simplified yet complex burners and issues pertaining to the modeling hypotheses of LES sub-models are underlined. Real burner LES are then reviewed to yield the industrial state-of-the-art and highlight potential directions for future developments.

2. Fundamentals of LES for complex burner simulations

This section describes the basics of LES: *i.e.* fundamentals, models for very high Reynolds number flows and the limiting case of low Reynolds number flows, discretization of the governing model equations, boundary conditions and their impact on the predictions. A specific subsection is dedicated to the current state-of-the-art computing facilities and their impact on LES strategies.

2.1. The filtering approach: implicit, explicit and no-model approaches

LES usually rely on spatial filtering where a filter $\mathcal{G}(\mathbf{x}, \mathbf{x}'; \mathbf{t})$, not yet defined, is convoluted with the instantaneous evolution equations or flow variables. Explicit and implicit approaches yield different classes of LES: the former introduces the filter explicitly, applies it to the governing equations and then discretizes the problem to obtain the solution numerically; the latter associates the discretization of the governing equations by an under-resolved grid to a filtering procedure which is undetermined hence implicit. This section describes the differences between the two approaches and highlights their links with the numerical schemes. Details about the closure problem and the current LES models available are given in the following sub-sections.

Filtering can be performed in the space or frequency domains. Although LES in the frequency domain have interesting properties, their use in industry-like configurations is unrealistic. The discussion is limited here to spatially dependent filters and temporally invariant functions [49]: *i.e.* $\mathcal{G}(\mathbf{x}, \mathbf{x}'; \mathbf{t}) = G(\mathbf{x} - \mathbf{x}'; t)$. To ease manipulation of the governing equations, the filter is usually selected to satisfy the conservation of constants (linearity), local invariance and whenever possible to commute with derivation [30,50]. These constraints are strong and often do not apply to bounded non-uniform non-homogeneous flow problems. Commutation errors are usually imbedded in real flow LES formalisms [51] unless specific filters are introduced [52–55]. To ensure a proper flow representation, generic properties of the initial set of Navier–Stokes equations for reacting mixtures are also to be conserved. Typically, Galilean, time, rotational, reflection invariances and material indifference [56–58] of the filtered quantities are desirable. The unfiltered Navier–Stokes equations satisfy these constraints but the LES equations may not because of the models proposed as closures. For reacting flows, the most

important property is probably the need to conserve bounds to ensure that filtered species mass fractions do not go above one and below zero. That simple mathematical property translates in the need for positive filters.

Although the notion of filters is mastered in the mathematical context, its use in LES codes is not well defined (see chapter 7 of [30] for details). The previous discussions on the desired filter properties are justified in the explicit filtering context: *i.e.* the filter is well defined and the governing equations are the result of the convolution operation. However, solving for the filtered transport equations requires numerical schemes that solve a discrete representation of the continuous problem. This discretization introduces new spatial scales into the problem especially if the grid is non-uniform (typical of industrial LES solvers). Changes of cell topologies or mesh stretching are known problems from the purely numerical point of view. In a fully explicit LES approach, the discretization should not introduce un-controlled uncertainties so that high-order non-dissipative and non-dispersive schemes are mandatory [59]. Their use whenever possible however increases the computer cost of such simulations. To reduce spurious numerical oscillations implicit filtering or pre-filtering can be introduced in the simulation (at every time step for example) [58,59] but again an additional overhead is inferred. Theoretically, the notion of effective filter issued by a simulation can be introduced [23] irrespectively of the numerics or filter used to derive the governing equations. In this context, LES can be viewed as a combination of one subgrid model and one numerical scheme, leading to an unknown filter also called 'effective filter' [30,60]. The fact that subgrid models and numerical schemes are intrinsically linked has led to alternative solutions where all the dissipation is provided by the numerics only and no sub-grid model is used [61,62]. In that case, "no model" LES or MILES (Monotone Integrated Large-Eddy Simulation) can be performed if the numerical scheme is constructed adequately [63].

Despite various potential frameworks, the risk with all these LES approaches is clear. It essentially stems from the ratio between the simulation inertia forces to the simulation dissipative forces (resolved field convective force over the model plus numerical dissipation forces): if this effective simulation Reynolds number locally depends explicitly on the grid or model and differs from the real flow Reynolds number, large flow differences are expected. Such criteria are particularly critical for transitioning flows or specific regions of a turbulent flows (near wall region). Despite this limit, recent developments prove LES to be a promising tool for complex applications. Its strong ties with numerics and modeling essentially yields a difficult environment for scientists to adequately evaluate and assess potential paths of improvements toward fully controlled and mastered LES at an engineering level.

2.2. LES transport equations and sub-models

Due to the non-linear nature of the governing equations of turbulent reacting flows, spatial filtering yields a closure problem where sets of new unknown terms need to be modeled for the problem to be solved numerically [1–10]. Denoting the Favre filtered field by,

$$\bar{\rho} \tilde{f}(\mathbf{x}; t) = \int \rho(\mathbf{x} - \mathbf{x}'; t) f(\mathbf{x} - \mathbf{x}'; t) G(\mathbf{x} - \mathbf{x}') d\mathbf{x}', \quad (1)$$

ρ standing for the fluid density, the following recursive properties apply to any quantity τ combining primitive variables [64],

$$\tau(f, g) = \tilde{f}g - \tilde{f}\tilde{g}, \quad (2)$$

$$\tau(f, g, h) = \tilde{f}g\tilde{h} - \tilde{f}\tilde{g}\tilde{h} - \tilde{f}\tau(g, h) - \tilde{g}\tau(f, h) - \tilde{h}\tau(f, g) \quad (3)$$

These terms and the LES models used for them must satisfy the desired filter properties described in Section 2.1. For turbulent reacting flows, the filtered compressible, multi-species governing LES equations read:

Species α mass fraction, Y_α , conservation:

$$\frac{\partial \bar{\rho} \tilde{Y}_\alpha}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \tilde{Y}_\alpha \tilde{u}_j) = -\frac{\partial}{\partial x_j} [\bar{J}_{j,\alpha} + \bar{J}_{j,\alpha}^t] + \bar{\omega}_\alpha, \quad (4)$$

Momentum conservation:

$$\frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \tilde{u}_i \tilde{u}_j) = -\frac{\partial}{\partial x_j} [\bar{P} \delta_{ij} - \bar{\tau}_{ij} - \bar{\tau}_{ij}^t], \quad (5)$$

Total energy, E , conservation:

$$\frac{\partial \bar{\rho} \tilde{E}}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \tilde{E} \tilde{u}_j) = -\frac{\partial}{\partial x_j} [\bar{u}_i (P \delta_{ij} - \bar{\tau}_{ij}) + \bar{q}_j + \bar{q}_j^t] + \bar{\omega}_T, \quad (6)$$

where, P is the pressure, u_i the i th component of the flow velocity vector, $J_{j,\alpha}, \tau_{ij}, q_j$ are respectively used to denote the species diffusion fluxes, the viscous stress tensor and heat flux [32]. Finally, $\bar{\omega}_\alpha$ and $\bar{\omega}_T$ are the species source terms and heat release rate issued by the chemical process taking place in the flame. The ideal gas law and mixing laws are also needed to close the problem. In Eqs. (4)–(6), three classes of unknown terms can be distinguished. They involve correlations between velocity components, u_i , species mass fractions, Y_α , temperature, T , (denoted by the superscript t in Eqs. (4)–(6)) as well as chemical source terms, $\bar{\omega}_\alpha, \bar{\omega}_T$:

- Second-order correlations, $\bar{\tau}_{ij}^t = -\bar{\rho} \tau(u_i, u_j)$, $\bar{J}_{j,\alpha}^t = \bar{\rho} \tau(u_i, Y_\alpha)$... appear when rewriting the convective terms of the filtered governing equations. These quantities are usually associated with the loss of information due to filtering fields containing a large range of length and time scales as encountered in a turbulent flow (*i.e.* without chemical reaction) [65,66]. The associated terms are the so-called Sub-Grid Scale (SGS) Reynolds stresses appearing in the filtered momentum conservation equations, Eq. (5), and the SGS scalar fluxes appearing in the filtered species conservation equations [10,32,67], Eq. (4).
- When solving for the mixture species equations, higher order correlation terms arise from the filtering of the highly non linear chemical reactions that control the consumption and production of species and heat release: *i.e.* $\bar{\omega}_\alpha, \bar{\omega}_T$. These chemical source terms need to be addressed accurately if combustion is to be properly predicted. Such terms involve complex products of species mass fraction to given powers, exponential functions of temperature and they cannot be simply approximated by the substitution in the respective expressions of the filtered fields [68–70]. The art of turbulent combustion modeling is a key ingredient of models for $\bar{\omega}_\alpha$ and $\bar{\omega}_T$: many models actually express source terms as functions of new quantities such as the scalar dissipation rate or the flame surface density, depending on the combustion regime [71–78] and require additional conservation equations.
- Other terms such as $\tau(u_i, u_j, u_k)$ or $\tau(u_i, p)$ are often disregarded and will not be comprehensively discussed here unless specifically needed. For example, transport of any energy which includes the flow kinetic energy, yields, after filtering, a third order term which needs closure [79]. Compressible flow governing equations also involve pressure-velocity terms that may be of importance [80–82]. Similarly, it is of usual practice to neglect molecular properties fluctuations (such as viscosity, diffusivity...).

2.2.1. Turbulence models for velocity and scalars

2.2.1.1. *SGS Reynolds stress models.* Conventional SGS Reynolds stress models are based on the Boussinesq hypothesis and the notion of turbulent viscosity. These are probably the most popular models and almost exclusively used in industrial flow LES. A non-comprehensive view of such closures is provided below with emphasis on their derivation and the target properties, Table 1. Since real reacting flow LES mainly treat the problem in physical space, only models applicable in this context are addressed, although spectral models do provide important information. For more information or a current status on developments in LES turbulent modeling, readers are referred to more fundamental reviews [22,25,29,30].

The Smagorinsky model [83] is probably the most popular turbulent closure model when dealing with complex configurations because of its simplicity and robustness. Derived in the context of isotropic decaying turbulence [84] with an assumption of equilibrium between kinetic energy fluxes at all turbulent scales, its advantages and weaknesses are well known. Its first weakness is that it is not suitable for transitioning flows or to treat wall turbulence. Improvements of the original model cover its use for anisotropic grids [85,86], automatic estimations of the model constant [87–89] through the use of dynamic procedures based on Germano’s identity [64] or the use of a transport equation for the SGS kinetic energy to account for non-equilibrium of the turbulent field [90,91]. Although dynamic procedures clearly improve LES predictions and extend the general use of the SGS closures, their implementation in general purpose LES codes usually requires some local averaging [88,89,92,93] and an implementation of a test filter.

In the context of wall bounded or transitioning flows with a strong impact of the mean shear, the conventional eddy viscosity estimates over-predict the turbulent diffusivity resulting in excessive turbulent diffusion and artificial re-laminarization of the LES filtered field. Based on turbulent properties and invariants [56,94–96], new expressions are possible to improve the model behavior as produced by [30,97–100] similarly to RANS approaches [101]. An alternative for wall bounded flows which still remains a weak point of LES SGS models is the use of correction functions or specific wall modeling [102–105].

A second class of SGS velocity model relies on the similarity hypothesis between the resolved field and a test scale [106].

Linearization of the similarity hypothesis yields the so-called tensorial eddy viscosity model or non-linear model [107]. Deconvolution methods have also appeared [108] and although these new models have shown great potential in *a priori* validations, a *posteriori* use proved them to be insufficiently dissipative. Mixed closures relying on the first set of Smagorinsky like models and similarity expressions have been proposed [109,110].

Other approaches [63,86,90,111–117] with various degrees of maturity illustrate the still on-going effort to propose reliable and more general turbulent LES closures.

2.2.1.2. *SGS species and scalar flux models.* Similarly to the velocity SGS term, the species or scalar SGS flux is usually modeled based on the Boussinesq hypothesis and the use of turbulent Schmidt or Prandtl numbers [118]. Such numbers are necessary to deal with the differences in physics that govern the evolution equations: *i.e.* turbulent fluctuations cover all scales from integral to Kolmogorov scales [33,119] while scalar fluctuations go all the way to the Batchelor or Corrsin ($Sc < 1$) scales [3,70,120]. Dynamic procedures have also been proposed [80,121,122] following the formalism discussed above. Extensions using different tensorial relationships have also been developed [123,124].

For species and temperature turbulent mixing, alternatives to the gradient diffusion hypothesis are scarce and mainly reduce to the Linear Eddy Mixing (LEM) model [125–133] or transported FDF approaches [134,135]. It is important to note that most of the mixing LES models available today strongly rely on the accuracy of the turbulent viscosity closure. Furthermore these are usually directly applied to turbulent reacting flows despite the clear limitations and the strong link that exists between mixing and chemical reactions [68,70,136].

2.2.2. Combustion models

The filtered species and energy equations cannot be closed without a significant modeling effort. They are not only governed by turbulence but also depend on mixing: different combustion regimes are possible depending on the flow configuration, type of fuel and injection system present in the burner [76]. Combustion regimes are usually introduced to characterize the physical processes that dominate a flame and to choose closure models: *i.e.* flamelet,

Table 1
Tentative classification of the turbulent SGS LES closures available.

	Fundamental hypothesis	Model properties				
		Near wall behavior	Solid rotation	Pure shear	Contraction/expansion	
					Axisymmetric	Isotropic
Eddy viscosity closures^a						
Smagorinsky [83]	Production = dissipation	$O(y^0)$	0	1	$\approx 3:46$	$\approx 2:45$
Wale [97]	Production = dissipation	$O(y^3)$	$\approx 0:9$	0	$\approx 0:15$	0
Vreman [98]	Production = dissipation	$O(y^1)$	$\approx 0:71$	0	$\approx 1:22$	1
Sigma [100]	Production = dissipation	$O(y^3)$	0	0	0	0
Dynamic closures^b						
Germano [64,88]	Production = dissipation	$O(y^0)$	To be determined – strong dependency on the homogenization procedure			
Similarity based closures						
Bardina [106]	Scale similarity hypothesis	To be determined – strong dependency on the homogenization procedure				
Mixed closures						
Samgorinsky-Bardina [109,110]	Scale similarity hypothesis & Production = dissipation	To be determined – strong dependency on the homogenization procedure				
Non-linear closure						
Tensorial viscosity [107], de-convolution model [108]	Production = dissipation	To be determined				
One or more equation closures						
Additional transport eqns [89–91]	Production \neq dissipation	To be determined				

^a Conclusions and observations extracted from [100].

^b Conclusions and observations extracted from [369].

distributed reaction, thickened flame... [76,137]. Laminar flames are the natural starting point to distinguish combustion modes present in real applications: diffusion flames, premixed flames and partially premixed flames. The addition of turbulence is usually introduced through the notion of combustion diagrams to justify the use of a specific turbulent combustion model knowing the combustion mode. Contrarily to RANS, LES models must also degenerate naturally to filtered laminar flames in zones where turbulence is low: they must preserve a large part of the flame structure.

A brief overview of major LES combustion/chemistry sub-models is provided below relying on the presentations of [76] and [137]. More comprehensive reviews on this specific problem are available in [77,78,138–144].

2.2.2.1. Chemical description. Combustion is a multi-scale phenomenon involving a broad range of time scales which find their root at the atom level. Starting with the atomic bounds around 10^{-15} s, Arrhenius laws allow to reduce time scales of leading species from 10^{-15} s to 10^{-10} s and of the order of seconds for slower reactions. Integrating such sparse systems remains a challenging task especially when they need to be coupled to transport phenomena. To start addressing such issues prior to their use in a CFD code, different methods are available and one needs to choose from a wide range of strategies [137]:

- Constitutive relations: such as Arrhenius laws for rate constants aim at representing atomistic process by a continuum.
- Chemical mechanism reduction: intent to identify most important species and reaction steps in order to decrease significantly the number of species and reactions needed to represent the initial skeletal mechanism. Different techniques exist for reductions: *i.e.* Quasi-steady state (QSS), Partial Equilibrium (PE), Computational Singular Perturbation (CSP) [145], Intrinsic Low-dimensional Manifold (ILDM) [146,147].
- Stiff chemistry integrators: aim at removing stiffness in the set of ordinary or partial differential equations [148,149] needed to describe the reduced chemical system in the presence of transport.
- Storage chemistry approaches: aim at accelerating the chemistry integration while the CFD integration proceeds. Tabulation of pre-computed laminar or turbulent flames falls in this class of approaches. Flamelet Generated Manifolds (FGM) [150,151] or Flamelet Prolongation of ILDM (FPI) [152–156] tables are typical examples. In Situ Adaptive Tabulation (ISAT) [157], Piece-wise Reusable Implementation of Solution Mapping (PRISM) [158] or Artificial Neural Networks (ANN) [159] are other examples where the tables adapt automatically during the CFD computation.

The main limitation behind all reduction methods is the reduced range of applicability. The final kinetic model also ends up having strong links with the turbulent combustion model which itself contains underlying assumptions (often related to specific combustion modes and regimes). All these observations seriously reduce the extent and generality of some of the tables or schemes obtained.

2.2.2.2. Turbulent combustion models. The turbulent combustion closures available to the CFD LES community are numerous and are often direct extensions of RANS turbulent combustion models. Such direct extensions are legitimate since all scales associated to the flame are usually below the LES filter length scale. Care is however needed as discussed in Section 2.2.3.

Like RANS modeling, LES turbulent combustion models rely heavily on the combustion fundamental theories and combustion modes: *i.e.* fully premixed, non-premixed [70] for which the basic properties are recalled below.

- Premixed flames are combustion modes where fuel and oxidizer are fully mixed before reacting: the flame front separates the unburnt premixed gases from the fully burnt reactants. Species and temperature transport is important only in the vicinity of the flame. Under the assumption of a simplified chemical description and transport, the governing equations can be recast into a single global equation for the progress variable characterizing the state of reaction. This progress variable is usually noted c and non-dimensionalized to equal zero in fresh gases and one in burnt gases. The *unfiltered* or *exact* evolution equation of c reads:

$$\frac{\partial \rho c}{\partial t} + \frac{\partial}{\partial x_j} [\rho u_j c] = \frac{\partial}{\partial x_l} \left[\rho D \frac{\partial c}{\partial x_l} \right] + \dot{\omega}_c. \quad (7)$$

If the flame is thin and can be described as a propagating front, Eq. (7) can be written in a propagative form called the G -equation by tracking the position of an iso- c surface [75]:

$$\frac{\partial \rho c}{\partial t} + \frac{\partial}{\partial x_j} [\rho u_j c] = \rho s_L^0 \frac{\partial c}{\partial x_l} \quad (8)$$

where the laminar flame speed noted s_L^0 appears explicitly and characterizes the propagation of the local premixed flame elements [160–162].

- For non-premixed or diffusion flames, fuel and oxidizer are separated and combustion occurs in the diffusive region where molecular and turbulent transport allow mixing of the two components prior to reaction. Here again, laminar and SGS mixing terms are essential since they control the rate of consumption [70,75,76]. For simplified transport and kinetics, Schwab-Zeldovich [163] variables or conserved scalars (noted Z) transport equations can be derived to represent the state of mixing within the flame independently of reaction. Its *unfiltered* or *exact* transport equation reads:

$$\frac{\partial \rho Z}{\partial t} + \frac{\partial}{\partial x_j} [\rho u_j Z] = \frac{\partial}{\partial x_l} \left[\rho D \frac{\partial Z}{\partial x_l} \right]. \quad (9)$$

Similarly to premixed modes, a coordinate attached to the stoichiometric iso- Z surface allows to recast the species transport equations with specific diffusive terms: scalar dissipation rate $\chi = D(\partial Z/\partial x_l)(\partial Z/\partial x_l)$, species transport across iso- Z lines in the normal and tangential iso- Z directions and reaction. Neglecting curvature effects, local composition within the flame appears to be controlled by the scalar dissipation rate.

- All regimes which are nor premixed neither non-premixed are called partially premixed. They are encountered quite often in gas turbines and are much less understood. Such regimes control auto-ignition problems, flame stabilization in the near field of burners, local quenching or re-ignition mechanisms. A simplified partially premixed flame prototype is the triple flame configuration [164–171]. Classical models developed for premixed or purely diffusion flames should not be used unless specifically adapted.

Based on these fundamental developments for specific combustion modes, many turbulent combustion models are available for LES. Following the classification of [31,76,172], one distinguishes between the purely geometrical and pseudo-statistical/statistical type of closures.

In purely geometrical approaches, the governing equations are in a propagation form and unclosed terms are provided in light of the combustion regimes characterized by the turbulent Reynolds

number, Re_t , turbulent Damköhler number, Da_t and/or the turbulent Karlovitz number, Ka_t . These models are usually linked to the flamelet assumption: *i.e.* the reaction zone is thin compared to turbulent length scales and only curvature, stretch and wrinkling effects impact the filtered rate of reaction in the iso- c or iso- Z formulations. For this class of models the notion of flame surface density per unit volume, Σ , or flame wrinkling factors, Ξ , can also be introduced to evaluate the filtered value of the reaction rate [173–178] using: $\widetilde{\omega}_k = \widetilde{Q}_k \widetilde{\Sigma}$, where \widetilde{Q}_k stands for the local density filtered burning rate of species k per unit flame area, or using a model for $\Xi = \widetilde{Q}_k / \sqrt{(\partial \widetilde{c} / \partial x_i)(\partial \widetilde{c} / \partial x_i)}$, the wrinkling factor. Both approaches require information on the flame inner structure and chemistry.

In statistical models, SGS terms can be constructed using the Filtered Density Function (FDF) or Probability Density Function (PDF) associated with the filtering operation of LES [179] (cf. [180] for further discussions on the differences between FDF and PDF formalisms in LES). The main result of such formalisms is that unknowns can be obtained by direct integration of the FDF or PDF. In these methods, two distinct sub-classes are to be distinguished:

- The presumed approach where the FDF/PDF form is fixed a priori and usually parameterized by the local value of the filtered quantity and its second filtered central moments. Conventional presumed PDF shapes are the delta, beta, Gaussian and Log-Normal functions. Within the same working frame, the Conditional Filtered Moment Closure (CMC) [137,181] approach consists in solving the transport equations of the conditionally filtered terms (some of which appear in the evolution equation of the FDF/PDF) which can then be multiplied by a presumed FDF/PDF and integrated to yield estimates of the SGS unknowns.
- The transported FDF/PDF approach solves directly for the governing equation of the function and performs the numerical integration of the estimated FDF/PDF to yield values for

the filtered unclosed terms present in the LES transport equations.

From a modeling point of view each approach implies the closure of specific terms appearing in the evolution equations. In that respect, SGS or equivalent terms involving mixing through velocity/species are to be addressed unless the velocity/species FDF/PDF is considered [113,134,135,182–185]. Scalar dissipation rate or equivalent terms must be closed too [67,74,186].

One last type of model for LES of turbulent reacting flows can also be somehow classified as pseudo-statistical approach: the Linear Eddy Model (LEM) [131,132]. In this approach, the initial filtered profiles (or coarse-grained structures) are mapped onto a so-called triple-map structure which is parameterized by a PDF and aims at representing the effect of eddies on mixing as well as stirring all the way to the viscous range. Reaction is taken into account in this 1-D space. Each physical process is taken into account using a splitting operator technique preserving specific time scales of each process.

Table 2 summarizes possible LES turbulent combustion models with specificities and context of development. For further information, readers are redirected to [76,137] and turbulent combustion books [31,75,119,187].

2.2.3. LES and the DNS limit modeling constraint

Originally LES models were constructed for non reacting flows and derived in the high Reynolds number limit to ensure the existence of the inertial range within the turbulent spectrum. Another constraint is that LES model contributions should vanish as the filter size tends to zero to recover the DNS limit of the simulations: *i.e.* a fully resolved simulation where all of the dissipative scales are captured. Likewise and to properly recover the DNS limit or transition regions, models should distinguish regions of potential low Reynolds number from fully turbulent ones, go to zero near walls and flows in solid rotation or purely sheared, in axisymmetric or isotropic expansion (or contraction). With such constraints dynamic filtering [64,85,87,88,188] and

Table 2
Tentative classification of the turbulent reacting LES closures available.

Formalism	Modeling	Solved quantity	Closures	Reaction source term	Range of application
Geometric	G-field	G	S_T	$\rho_u S_T \sqrt{\frac{\partial \widetilde{G}}{\partial x_i} \frac{\partial \widetilde{G}}{\partial x_i}}$	Premixed
	Flame surface density	$\widetilde{\Sigma}$	\widetilde{Q}_k	$\widetilde{\Sigma} \widetilde{Q}_k$	All modes
	Flame wrinkling	\widetilde{c}	$\widetilde{\Xi}$	$\rho_u S_T^0 \widetilde{\Xi} \sqrt{\frac{\partial \widetilde{c}}{\partial x_i} \frac{\partial \widetilde{c}}{\partial x_i}}$	All modes
	Thickened flame	\widetilde{Y}_k	E (Efficiency function)	$\frac{E \widetilde{\omega}_k(\widetilde{Y}_k, \widetilde{T})}{\mathcal{F}}$ (\mathcal{F} , thickening factor)	All modes
Statistical	Presumed FDF/PDF	$\widetilde{Z}; (\widetilde{Z} - \widetilde{Z}\widetilde{Z})$	$(\widetilde{u}_i \widetilde{Z} - \widetilde{u}_i \widetilde{Z}); D \frac{\partial \widetilde{Z}}{\partial x_k} \frac{\partial \widetilde{Z}}{\partial x_k} - D \frac{\partial \widetilde{Z}}{\partial x_k} \frac{\partial \widetilde{Z}}{\partial x_k}$	$\int \widetilde{\omega}_k(Z) f(Z) dZ$	Non-premixed
		$\widetilde{c}; (\widetilde{c}c - \widetilde{c}\widetilde{c})$	$(\widetilde{u}_i \widetilde{c} - \widetilde{u}_i \widetilde{c}); D \frac{\partial \widetilde{c}}{\partial x_k} \frac{\partial \widetilde{c}}{\partial x_k} - D \frac{\partial \widetilde{c}}{\partial x_k} \frac{\partial \widetilde{c}}{\partial x_k}$	$\int \widetilde{\omega}_k(c) f(c) dc$	Premixed flames
	CMC	All of the above	All of the above	$\int \int \widetilde{\omega}_k(c, Z) f(c, Z) dcdZ$	All modes
		$\widetilde{Y}_k \widetilde{c}$	$\widetilde{u}_i \widetilde{c}; \widetilde{u}_i \widetilde{Y}_k \widetilde{c}; D \frac{\partial \widetilde{Y}_m}{\partial x_l} \frac{\partial \widetilde{Y}_n}{\partial x_l} \widetilde{c}$	$\int \widetilde{\omega}_k \widetilde{c} f(c) dc$	Premixed
		$\widetilde{Y}_k \widetilde{Z}$	$\widetilde{u}_i \widetilde{Z}; \widetilde{u}_i \widetilde{Y}_k \widetilde{Z}; D \frac{\partial \widetilde{Y}_m}{\partial x_l} \frac{\partial \widetilde{Y}_n}{\partial x_l} \widetilde{Z}$	$\int \widetilde{\omega}_k \widetilde{Z} f(Z) dZ$	Non-premixed
		$\widetilde{Y}_k \widetilde{Z}, c$	$\widetilde{u}_i \widetilde{Z}, c; \widetilde{u}_i \widetilde{Y}_k \widetilde{Z}, c; D \frac{\partial \widetilde{Y}_m}{\partial x_l} \frac{\partial \widetilde{Y}_n}{\partial x_l} \widetilde{Z}, c$	$\int \int \widetilde{\omega}_k \widetilde{c}, \widetilde{Z} f(c, Z) dcdZ$	All modes
	LEM	\widetilde{Y}_k	$f(l)$	$\overline{\widetilde{\omega}_k(Y_k)}$	All modes
	Transported FDF/PDF	$f(\xi)$	$\widetilde{u}_i \xi; D \frac{\partial \widetilde{c}}{\partial x_l} \frac{\partial \widetilde{c}}{\partial x_l} \xi$	$\int \widetilde{\omega}_k(\xi) f(\xi) d\xi$	Premixed
		$f(\eta)$	$\widetilde{u}_i \eta; D \frac{\partial \widetilde{Z}}{\partial x_l} \frac{\partial \widetilde{Z}}{\partial x_l} \eta$	$\int \widetilde{\omega}_k(\eta) f(\eta) d\eta$	Non-premixed
		$f(\psi_k)$	$\widetilde{u}_i \psi_k; D \frac{\partial \widetilde{Y}_m}{\partial x_l} \frac{\partial \widetilde{Y}_n}{\partial x_l} \psi_k$	$\int \widetilde{\omega}_k(\psi_j) f(\psi_j) d\psi_j$	All modes

higher order tensor relationships constitute clear benefits since introducing increased locality through test filtering or improved asymptotic behaviors of the SGS modeling issued by a better qualification of the local resolved flow state, Table 1.

For reacting flows, the situation is more complicated because combustion almost always takes place at scales which are not resolved today. For example, LES of experiments with low Reynolds but large Damköhler numbers corresponds to flows where all turbulent scales can be resolved but the flame front reaction zone is still under-resolved. Typically, in real applications the fuel consumption rate of highly energetic species such as kerosene is large and leads to Damköhler numbers larger than one (*i.e.* very thin reaction fronts). Within the same flow, the Damköhler number associated with the chemical steps of *NO* or *CO* are usually close to one. Modeling is thus confronted with the major difficulty of ensuring the proper description of the interaction of the kerosene consumption in a thin front strongly affected by turbulence while providing the missing information for slowly evolving chemical reactions in an inhomogeneous hot medium that is weakly turbulent. Modeling is thus essential: even-though models are usually derived for flames in highly turbulent flows, they must also be able to propagate quasi-laminar fronts in low turbulence zones as well as in highly resolved intense turbulent fields. In some configurations, neglected terms in the exact filtered transport equations may play non-negligible roles [189–192]. Such issues are becoming of greater importance with the advent of massively parallel computers: in a few years, these computing resources will allow to resolve flow and flame structures in such regimes and SGS models must degenerate to true DNS [189–192].

A final observation which further underlines the potential difficulty of mastering the DNS limit of LES comes from recent works on LES modeling for stationary turbulent flows [193–196]. From these

studies, it is clear that defining a clear procedure to unanimously qualify different LES approaches is not an easy task. Indeed, it is now well accepted that a LES prediction is the result of combined modeling and numerical errors and the behavior of such a cocktail is often counter-intuitive [193–196]. For example, if one accepts to change the value of the Smagorinsky constant noted C_S , an optimal grid resolution C_S pair exists which provides good quality predictions [193,194,196] at minimum cost and away from the DNS limit. Because of such observations, multiple LES quality indices have been proposed [193,197–199] for non-reacting and reacting [200] flows. These issues are still to be answered to systematically qualify LES SGS modeling and numerical strategies for an improved understanding of LES predictions in complex geometries.

2.3. Numerical methods

LES flow solvers are either incompressible, fully compressible or based on the low Mach number approximation. Each approach imposes different algorithms, computer costs and numerical schemes which may not be compatible with LES basic constraints. However and based on current state-of-the-art LES solvers, key features seem to emerge and this section summarizes them with a specific emphasis on their advantages and disadvantages. Readers can find details associated to High Performance Computing for CFD on massively parallel systems in recent review papers [201,202]. A non-comprehensive list of codes dedicated to LES of reacting flows is provided in Table 3 along with the numerical characteristics retained in each case and the research groups involved in their developments.

2.3.1. High-order schemes, mesh type and complex geometries

Despite recent controversies, there is little doubt that high-order schemes are desirable for LES to minimize numerical dispersion and

Table 3
Tentative survey of the massively parallel codes available for complex reacting LES applications.

Code	Formalism	Numerics	Target burners
PUFFIN (Loughborough University)	Low Mach, conserved scalar/progress variable Navier-Stokes	Second order in time and space, multi-block structured code	Lab-scale burners
FLOWSI (Imperial College)	Low Mach, conserved scalar/progress variable Navier-Stokes	Second order in time and space, multi-block structured code	Lab-scale burners
BOFFIN (Imperial College)	Low Mach, conserved scalar/progress variable Navier-Stokes	Second order in time and space, multi-block body fitted structured code	Lab- and real-scale burners
FASTEST 3D (Technical University of Darmstadt)	Low Mach, conserved scalar/progress variable Navier-Stokes	Second order in time and space, multi-block body fitted structured code	Lab-scale burners
CDP (Stanford University)	Low Mach, conserved scalar/progress variable Navier-Stokes	Second order in time and space, unstructured meshes	Lab- and real-scale burners
NGA (Stanford University; University of Colorado; Cornell University)	Low Mach, multi-species (transported PDF) Navier-Stokes	Second order in time and space, multi-block structured code	Lab-scale burners
RAPTOR (Sandia National Lab. code)	Compressible, multi-species Navier-Stokes	Second order in time and space, multi-block structured code	Lab- and real-scale burners
LESLIE (Georgia Inst. of Tech.)	Compressible multi-species, conserved scalar/progress variable Navier-Stokes	Second order in time, fourth order in space, multi-block hybrid (structured, unstructured) code	Lab- and real-scale burners
Penn. State University code	Compressible, multi-species Navier-Stokes	Second order in time and space, multi-block structured code	Lab-scale burners
YALES2 (CORIA)	Low Mach, conserved scalar/progress variable Navier-Stokes	Explicit, third order in time and space, unstructured/hybrid meshes	Lab- and real-scale burners
AVBP (CERFACS)	Compressible, multi-species Navier-Stokes	Explicit, third order in time and space, unstructured/hybrid meshes	Lab- and real-scale burners
PRECISE (Rolls-Royce)	Low Mach/compressible, conserved scalar/progress variable & multi-species Navier-Stokes	Third order in time and space, unstructured meshes	Lab- and real-scale burners
THETA (Deutsches Zentrum fuer Luft- und Raumfahrt e. V -DLR)	Multi-species Navier-Stokes	Second order in time and space, unstructured meshes	Lab- and real-scale burners
OpenFoam	Low Mach/compressible, conserved scalar/progress variable & multi-species Navier-Stokes	Second order in time and space, unstructured meshes	Lab- and real-scale burners
FLUENT (ANSYS)	Low Mach/compressible, conserved scalar/progress variable & multi-species Navier-Stokes	Second order in time and space, unstructured meshes	Lab- and real-scale burners

dissipation to preserve good quality unsteady flow predictions. That constraint imposes the use of high-order centered spatial schemes with the addition of artificial viscosity on highly disturbed grids. In a world where only simple geometries would be computed (for example laboratory-scale systems), this constraint would lead to structured grid methods on which high-order schemes (from 4 to 8th order) are easily built. Unfortunately, most real combustion chambers have geometries which are so complicated that meshing them with a multi-block structured grid takes too much time. In addition, good strong scaling requires balanced decomposition of the computational domain which is difficult with a pre-decomposed block-structured approach in geometries other than cubes and assimilated. As a result, most recent LES codes for real gas turbines are developed using unstructured or hybrid grids. Interestingly, on such grids, developing high-order numerical schemes is a challenge. Most existing solvers on unstructured grids are limited to second-order spatial accuracy except the (expensive) TTGC scheme developed by Oxford and CERFACS [203]. Combining the accuracy of high-order schemes developed for structured grids with the flexibility of unstructured grid solvers is a key issue in the construction of combustion LES solvers.

2.3.2. Implicit versus explicit time integration, incompressible, low Mach and fully compressible approaches

Combustion codes are easily written in compressible form: a simple explicit technique allows to develop rapidly a solver for LES of reacting flows on unstructured grids. The main disadvantage of such solvers is that their time step is limited by the CFL condition which is controlled by the sound speed. For very slow flames, computing a flow-through time can require too many time iterations and lead to a very slow computation (or large turn around time). This discussion is actually complicated by different issues:

- Alternative solutions to explicit compressible codes are: (1) time implicit compressible methods and (2) low-Mach formulations. In time implicit compressible methods, the full compressible equations are solved implicitly to remove the CFL constraint. In low-Mach formulations, acoustics are removed from the conservation equations and a Poisson solver is needed to obtain pressure at each instant. For approaches (1) and (2), large implicit systems must be solved at each iteration. Both approaches are found in combustion codes [204,205] and constitute interesting and efficient alternative methods.

- The fully compressible solvers still have a few advantages [201,206]: being explicit, they are efficient on very massively parallel systems. They also capture acoustics naturally, a property which is mandatory to study certain instabilities. In practice, implicit compressible solvers are stable over a wide range of CFL numbers but they are not precise and must be run for constrained CFL values (smaller than 10), making them potentially as slow as explicit solvers. The main reason for such a behavior is linked to the matrix inversion which is time consuming. Low-Mach number codes offer more convincing performances especially for very slow flames where acoustics is not important and the linear system to be solved of reasonable size.
- In practice, in most gas turbine combustion chambers, Mach numbers are not small. Within swirlers, Mach numbers of the order of 0.3 are common. At chamber outlets, a throat created by the high pressure stator usually creates a choked region. For these cases, compressible solvers remain mandatory.

2.3.3. Boundary condition treatment and stability

The numerical treatment of the flow boundary conditions is also of importance and can result in artificially stable or unstable computations. These treatments differ depending on the set of equations solved (*i.e.* compressible or incompressible Navier–Stokes equations). An important conclusion reached in the last ten years is that the flow within the combustion chamber can be computed with much more precision using LES than RANS and that, at this level of precision, outlet and inlet conditions become critical. Typically, a proper LES of a combustion chamber should include a description of the mean and turbulent flow at the inlet (resolved in time). Moreover, the acoustic impedances at inlet (compressor side) and outlet (turbine side) should also be known: this is not the case today and work is required on these questions because these boundary conditions probably control the flow more than the details of the SGS models used within the chamber.

2.4. The massively parallel context, mesh generation and data management

Real burner LES imply the use of high end massively parallel super-computers which process data subsets of the same problem in parallel. Efficient and minimum exchanges of information are mandatory to ensure scalability up to thousands of cores. Message

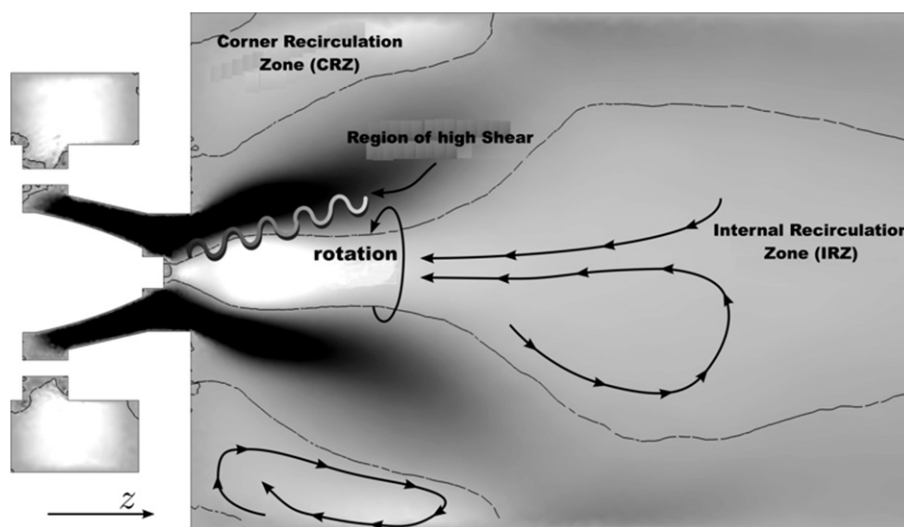


Fig. 2. Typical view of the main flow structures present outside a swirler from a real aeronautical burner provided by Turbomeca and mounted on an experimental test bench [362–364].

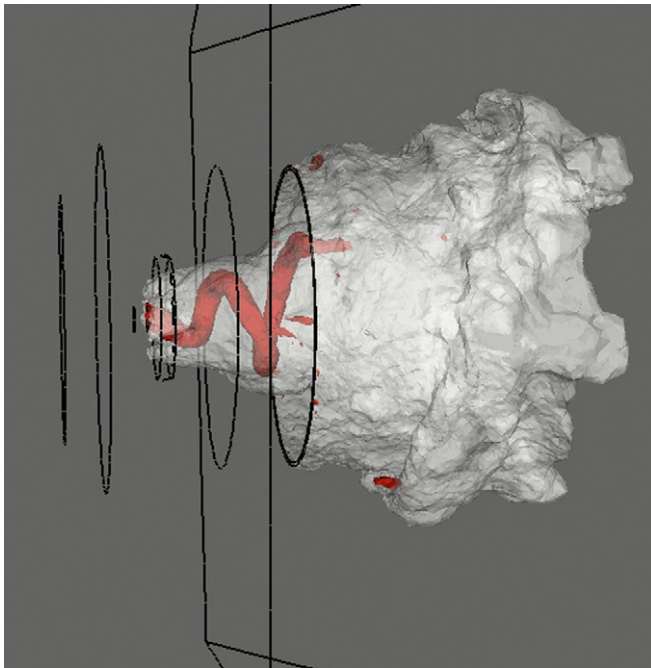


Fig. 3. Typical view of a PVC (red iso-surface) as provided by the LES of a real gas turbine combustion chamber. The light grey iso-surface depicts the flame position. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

passing coding is at the root of parallel computing. As mentioned above, efficient message passing for CFD in gas turbine chambers requires using fully unstructured grids. However, the generation of large unstructured meshes and their partitioning become bottlenecks today [201,202,207]. I/O must also be totally reorganized to avoid slowing down the code. These issues raise specific questions about the exploitation and maintenance of the codes: maintaining a code which can run efficiently both on distributed and shared memory machines with thousand of cores is extremely difficult.

All of the above mentioned modeling and algorithmic issues are clearly of major importance for real applications. Building codes aiming at simulating real industrial problems often imposes a list of sacrifices: spatial accuracy and modeling are usually sacrificed to ensure robustness and performance, thereby giving access to LES predictions of such complex flows with a bounded and reasonable computer cost. These choices are clearly needed and based on specific and scientifically identified pros and cons; simple models may be favored provided that their limits are well understood and implications on the LES predictions well anticipated. The difficulty inherent to such choices and model limitations are thus of foremost importance to ensure valuable exploitation and understanding of the obtained results to contribute to the decision making. The next section reviews aspects of the validation steps followed by researchers and industry to better qualify the different LES numerical and modeling strategies available today.

3. Laboratory-scale burner simulations

Laboratory-scale burners are usually designed to assess modeling and fundamental flow aspects in controlled

Table 4
Tentative survey of laboratory scale burner LES's.

Ref.	Code	Turbulence	Combustion	Target applications
[231]	FLOWSI	Dyn. Smag.	Non-reacting	Bluff body flame
[229,230,232,233]	Unknown	Dyn. Smag.	Non-reacting	Swirl flames series
[241]	Unknown	Smag.	Non-reacting	TECFAM burner
[242–244,250]	Penn. State	Smag.	Non-reacting	Swirl Mixer (US Patent)
[370]	CDP	Dyn. Smag.	Z, c, flamelet	Non-prem. coaxial jet comb.
[181]	Unknown		CMC	Bluff body
[371,184]	Unknown	Smag.	FDf	Bluff body
[372–375]	Unknown	Dyn. Smag.	c, flamelet	Swirl flames series
[376]	LESLIE	LDKM	LEM	Swirl flames series
[377]	PRECISE	Smag.	FDf	Swirl flames series
[378]	SiTCom	Smag.	PCM-FPI	Lifted flame, vitiated coflow
[379]	LESLIE	LDKM, Mixed	Flamelet G, EDC finite rate	Swirl stratified prem. flame
[302]	AVBP	Wale	Thick. Flame, reduced chem.	Prem. swirl comb.
[275,276]	AVBP	Wale	PCM-FPI	Prem. swirl comb.
[192,205]	YALES2	Dyn. Smag.	C, tabulated chem.	Prem. swirl comb.
[190]	AVBP	Wale	FTACLES - FPI	Prem. swirl comb.
[247]	AVBP	Smag.	Thick. Flame	Prem. swirl comb.
[380–382]	Unknown	LDKM	Reduced chem., wrinkling [383]	GE LM 6000 comb.
[384]	Unknown	Smag.	c, flame wrinkling, flamelet	Prem. swirl comb.
[385]	Unknown	Smag.	G and Z, flamelet	Partially prem. comb.
[381]	Unknown	Unknown	G	Swirl comb.
[246,386]	AVBP	Smag.	Thick. Flame, reduced chem.	Siemens burner
[387,293]	AVBP	Smag.	Thick. Flame, reduced chem.	Prem. staged burner
[388–391]	Penn. State	Smag.	G, flamelet	Prem. swirl comb.
[392]	LESLIE	LDKM	Reduced chem.	GE DACRS comb.
[393,394]	Unknown	Smag.	EBU, reduced chem.	DOE-NETL prem. comb.
[395,396]	LESLIE	LDKM	G and Z, flamelet	Prem. comb.
[397]	Unknown	Smag.	Level-set, flamelet	Prem. comb.
[384]	Unknown	Smag.	Flamelet	Swirl comb.
[337]	Unknown	LDKM, Mixed	Z, c, flamelet [383]	TARS burner [264]
[398,399]	Unknown	Smag.	G, flamelet	GE-LM 6000, DOE-HAT
[400,401]	LESLIE	LDKM	LEM-LES, reduced chem.	GE-1
[402]	AVBP	Smag.	Thick. Flame, reduced chem.	Alstom swirl
[296,303,304]	AVBP	Smag.	Thick. Flame, reduced chem.	Siemens stratified

configurations. They are necessary to assess LES strategies as well as their limitations, reliability, capability and orient future developments.

3.1. Key geometrical features

To be representative of real gas turbines, laboratory burners need to retain specific features and cover a wide range of turbulent flows and physics that are present in gas turbine engines. For example, it is desirable to use real swirlers and to mount them on experimental test facilities heavily equipped with flow and combustion diagnostics. Since 2000, multiple laboratories have followed this path. In the following mainly swirled flames are discussed as they are typical of the next generation of gas turbine combustion chambers. Swirl is used to generate large recirculation zones that are usually located right outside the injection system, improve mixing, ease flame stabilization and locally increase the flow residence time thereby reducing the size of the combustion chamber [208,209]. Simple laboratory burners [208,210–221] with such features have been used to qualify LES.

3.2. Flow validations

Swirling flows without combustion have been heavily investigated (see reviews [208,222–228]). They nonetheless remain

difficult to compute [209] even though recent LES [229–233] allow confidence in this modeling strategy for more complex geometries. The main specificities of swirl confined flows are illustrated on Fig. 2. The two main non-dimensional numbers controlling simple swirled flows are the Swirl number, S and the Reynolds number, Re :

$$S = \frac{G_\phi}{RG_x} = \frac{\int_0^R UW r^2 dr}{R \int_0^R U^2 r dr}, \quad (10)$$

$$Re = \frac{U_0 R}{\nu}. \quad (11)$$

S measures the ratio of the axial flux of the swirl momentum, G_ϕ [$\text{kg m}^2 \text{s}^{-2}$], to the axial flux of the axial momentum, G_x [$\text{kg m}^2 \text{s}^{-2}$] multiplied by a characteristic length of the swirl annulus, R [m]. In Eq. (10), U [m/s] and W [m/s] are the mean axial and tangential velocities measured in a plane usually located at the exit of the swirler generator. The Reynolds number is defined using the bulk axial velocity, U_0 , the swirler annulus radius, R , and the fluid kinematic viscosity, ν . Other definitions are possible for the swirl number and originate from purely geometrical considerations [215]. For example, the geometrical swirl,

$$S_g = \frac{W_0}{U_0}, \quad (12)$$

is often encountered.

Depending on Re and S , the following mechanisms and structures appear:

- Inner Recirculating Zone (IRZ): This region is created by the intense swirl. Usually located right along the axis of the swirler,

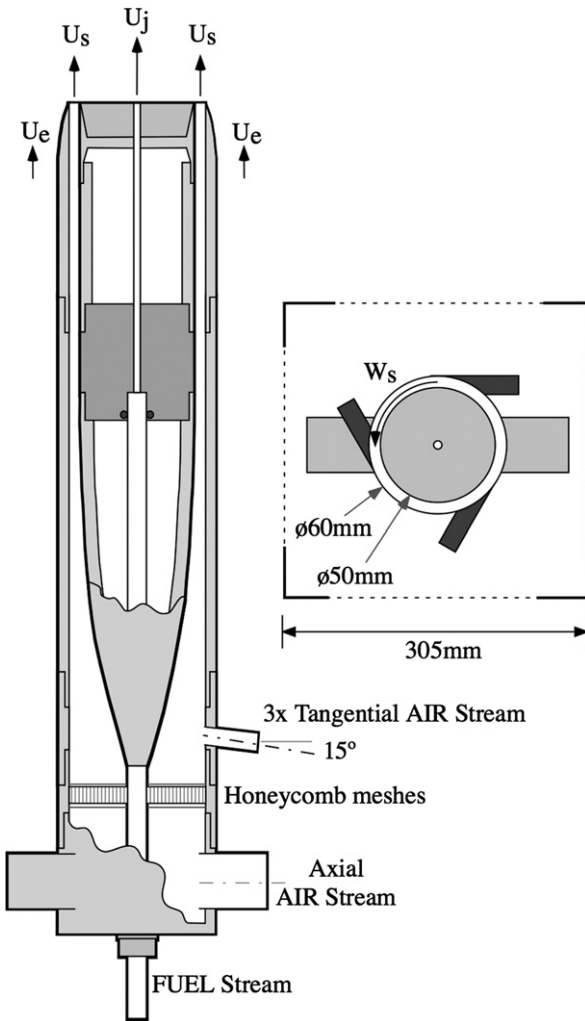


Fig. 4. Geometrical setup of the Sandia swirl combustor used to validate LES [215,245].

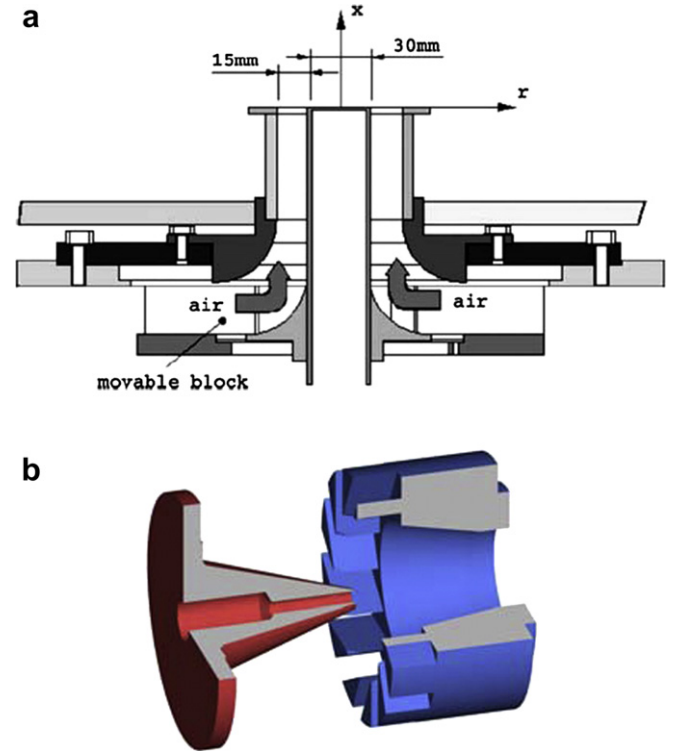


Fig. 5. Geometrical setup of (a) the TECFLAM swirler combustor and (b) TIMECOP-AE swirler used to validate LES [239–241].

this recirculation bubble appears for large values of S (typically above 0.6). It results from the radial pressure gradient generated by the guided rotating flow (large tangential velocity component of the flow) and the flow expansion through a nozzle at the chamber inlet: the radial pressure gradient and axial velocity components suddenly decay producing a negative axial pressure gradient and a reverse flow or IRZ [208,222–228].

- Corner Recirculating Zones (CRZ): In confined configurations, the sudden expansion of the flow at the chamber inlet is partly controlled by flow recirculating bubbles present at the outer edges of the dump [208,224,234].
- Processing Vortex Core (PVC) and Vortex Breakdown (VB): Under specific conditions (still not clearly mastered) the central vortex core present in the inner parts of the swirler or the IRZ becomes unstable giving rise to the PVC [224]. This destabilization can induce oscillations of the IRZ in the axial

and azimuthal directions. The PVC believed to be at the origin of such oscillations coincides with a vorticity tube of helical shape located at the outer rim of the IRZ. This thin vortex tube has a helicoidal shape can be co- or counter-rotative to swirl, Fig. 3. It then can turn around the swirler axis in the swirl or opposite direction. In some cases several vortex tubes may co-exist at the same time [235–237]. Finally, this specific structure is highly dependent of the CRZ and IRZ interactions [238], which are controlled by the details of the swirler and dump configurations.

LES flow predictions and validations on unconfined swirled configurations provide an evaluation of LES codes on flows typical of real burners (Table 4). A typical experiment proposed for this specific purpose is the Sandia burner described on Fig. 4 and investigated numerically by [229,232,233] in its non-reacting operation. Other swirl injector systems [239–241], Fig. 5, have

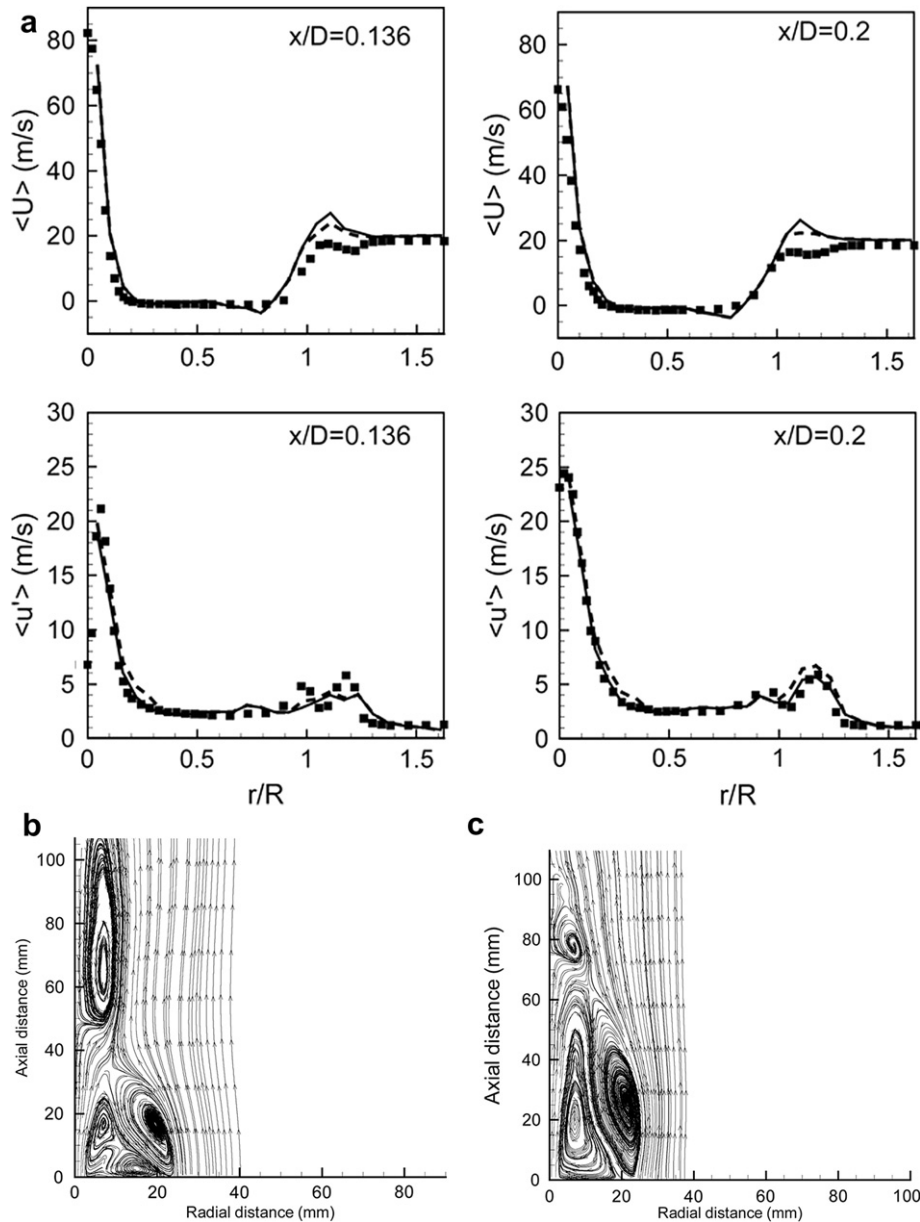


Fig. 6. Mean and RMS profiles as well as flow streamlines obtained by LES for a bluff-body swirl flow, Fig. 4: (a) — 1 million grid point LES, - - 1.44 million grid point LES, ■ experimental measurements; (b) high swirl and (c) moderate swirl numbers [232].

been also investigated numerically [230,242–244] with the same observations as the one produced below.

3.2.1. Predictions of the mean statistical flow features

For large swirl number and moderate swirl Reynolds number flows [215,245], LES of the configuration illustrated on Fig. 4 is found to be quite insensitive to grid resolution and SGS modeling, Fig. 6(a). At moderate swirl numbers, near Vortex Breakdown (VB): *i.e.* $S \approx 0.5$, predictions are more sensitive but remain interesting and encouraging, Fig. 6(b). SGS dynamic procedure and grid resolution may be of importance in such critical flow conditions. Inflow conditions are also suspected to be of primary importance and should at least reproduce the unsteady effect of a turbulent flow entering the computational domain. LES still remain the only current modeling strategy that correctly predicts mean statistical (*i.e.* mean and Root Mean Square (RMS)) flow features in strongly swirled flows.

3.2.2. Predictions of the unsteadiness of swirl flow features

The unsteady structures characteristic of swirl flows control the fuel and air mixing process and the interactions between the flame and the flow. These unsteady characteristics are much less investigated or validated because of the difficulty in properly characterizing such motions experimentally or numerically. The PVC is known to be weakly dependent of Re and its frequency f can be expressed in terms of a Strouhal number, fDe/U_0 where De stands for the swirler exit diameter [228]. Evaluations of the LES issued PVC's [239,240] and their Strouhal numbers for the cold flow configuration of Fig. 5(a) are overall in agreement with

experimental findings [241,246–250], Fig. 7; typical results report approximately 10% errors when comparing LES PVC frequencies with experimental measurements.

For these well documented geometries [215,239,240,245, 251–253], LES predictions are encouraging, Figs. 6 and 7. In configurations where S is above the critical swirl number of 0.5–0.6 where VB is expected [238,254,255], results are very satisfactory (*cf.* Sandia configuration of Fig. 6). In most reacting systems, the fuel injection location within the swirler is of importance not only because it determines mixing, but also because of the interaction and potential flow topology change they may have on the swirling flow. Jet/IRZ interaction is often present and impacts the evolution of the PVC and even the IRZ itself as discussed by [228] (*cf.* Fig. 6 for a typical example).

3.3. Reacting flow validations

LES of reacting flows is a relatively new research topic which only emerged in the early nineties and became a focus point of CFD research in the late nineties. This contrasts with research on ‘pure’ LES (*i.e.* without reaction) that appeared in the sixties in the weather-forecast community [83,90]. Many reasons explain these differences. The first one probably originates from the turbulent combustion community itself which dedicated a lot of effort in implementing new combustion models for RANS and naturally lags the turbulent community. Second, computer power and the emergence of highly efficient machines and algorithms only recently allowed to address even simple laboratory-scale configurations which was a necessary step for the turbulent reacting LES

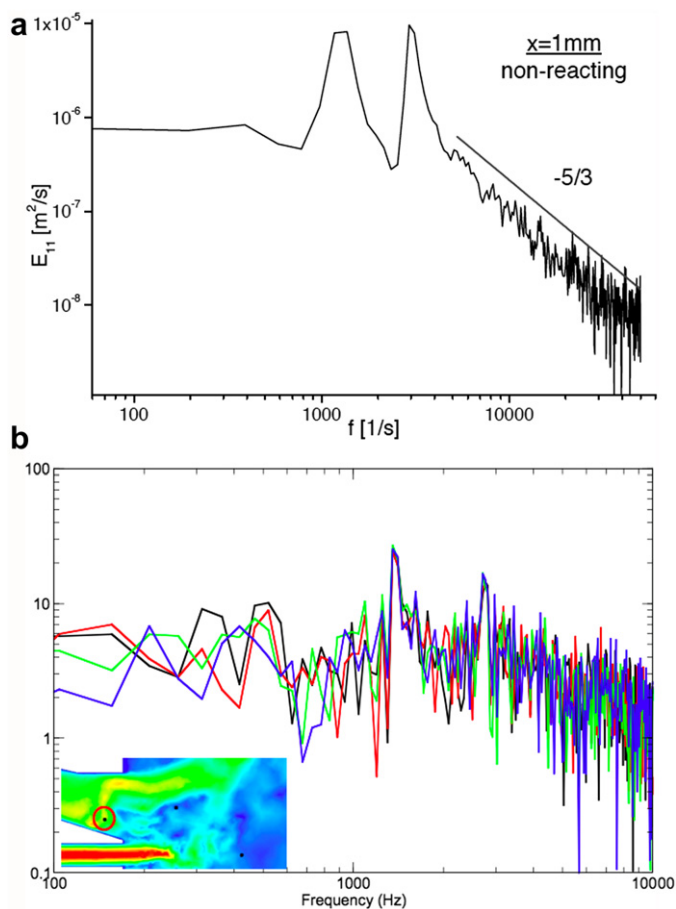


Fig. 7. Spectral analysis of the swirl flow motions in the TECFLAM swirler combustor [239–241]: (a) experimental measurements and (b) corresponding LES spectra.

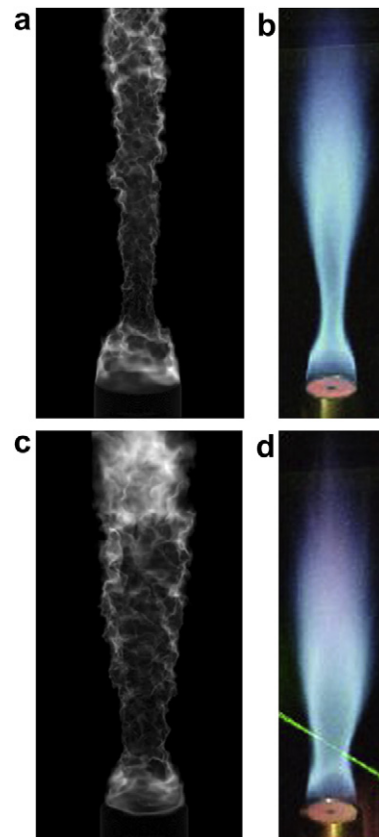


Fig. 8. Geometrical setup of a typical swirled flame for which detailed measurements are available. First row, SMH1 flame: instantaneous views of (a) Imperial College LES predictions [258] and (b) the associated experiment [215]. Second row, SMH2 flame: instantaneous views of (c) Imperial College LES predictions [258] and (d) the associated experiment [216].

concept to be validated. Finally, very few theories or mathematical models are available for turbulent reacting flows for conceptual validations. This is clearly not the case for turbulent non-reacting flows. In parallel to these efforts, new flame modeling concepts appeared and the turbulence as well as the turbulent combustion communities started to recognize the role of the most energetic flow structures in CFD. All these developments are transcribed in the TNF workshop series [256] or Combustion Symposium series [257] which mainly addressed RANS modeling validations until the 1990s and now focus almost exclusively on LES.

The following discussion focuses on some of the recent contributions and efforts in the field of reacting LES in swirled configurations. First, statistically stationary unconfined and confined

simple configurations are reviewed followed by a discussion on recent leading-edge LES applications to illustrate the possibilities of massively parallel architectures. Finally, two specific research subjects benefiting from these recent LES developments are discussed: (1) thermo-acoustic instabilities often encountered in real gas turbine engines and (2) transient phenomena (ignition, extinction sequences...).

3.3.1. Statistically stationary flow conditions

Many LES contributions mainly aim at validating SGS turbulent combustion models (*cf.* the series of TNF workshop proceedings and comments on the matter). Swirled flames have been computed only recently, Fig. 8 (predictions for the Sandia burner of Fig. 4), and

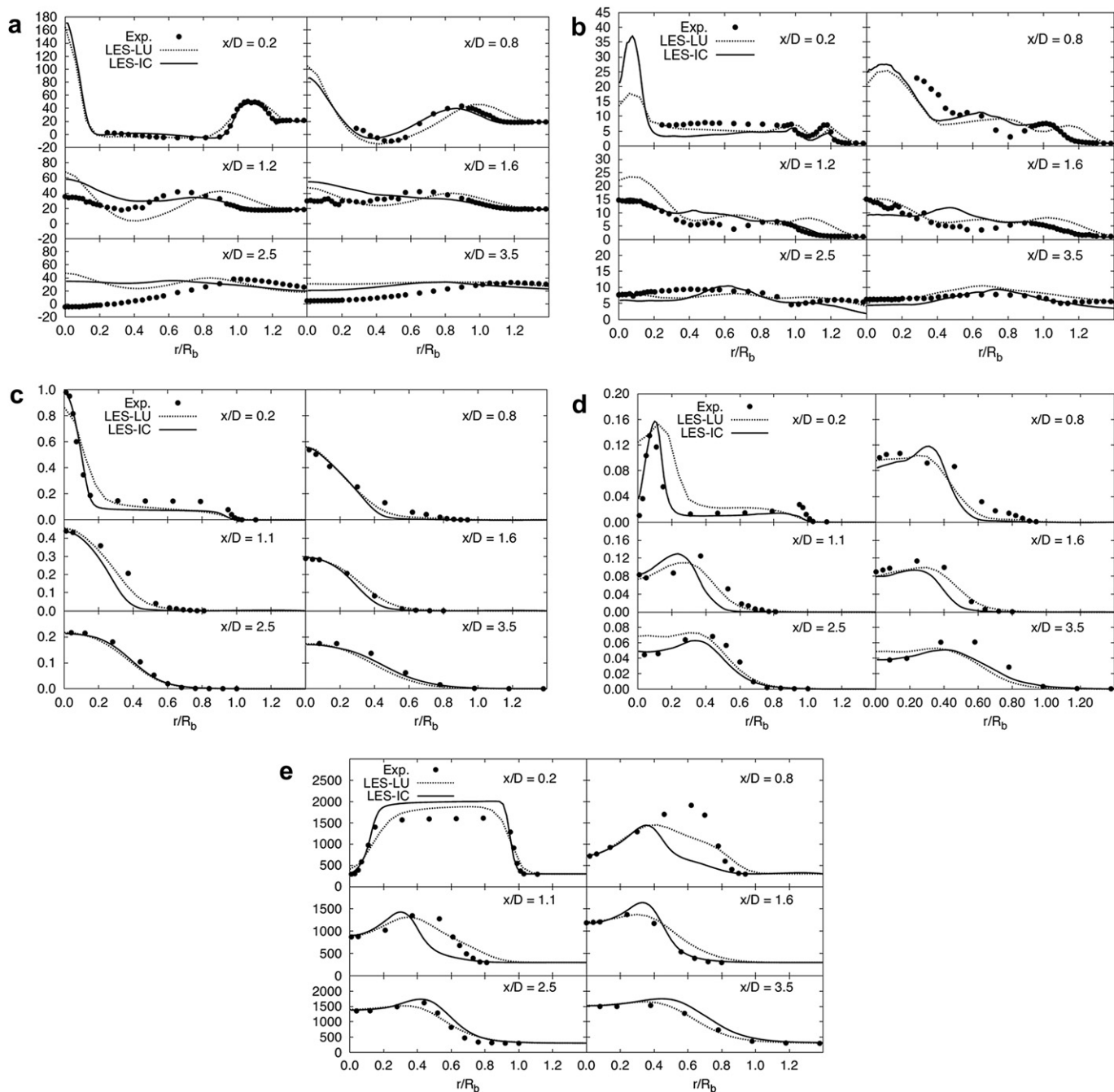


Fig. 9. Comparisons of the numerical predictions [258] with experimental data [215] for SMH1: (a) mean axial velocity profiles, (b) mean mixture fraction and their respective RMS (b) and (d) along with (e) the mean temperature profiles.

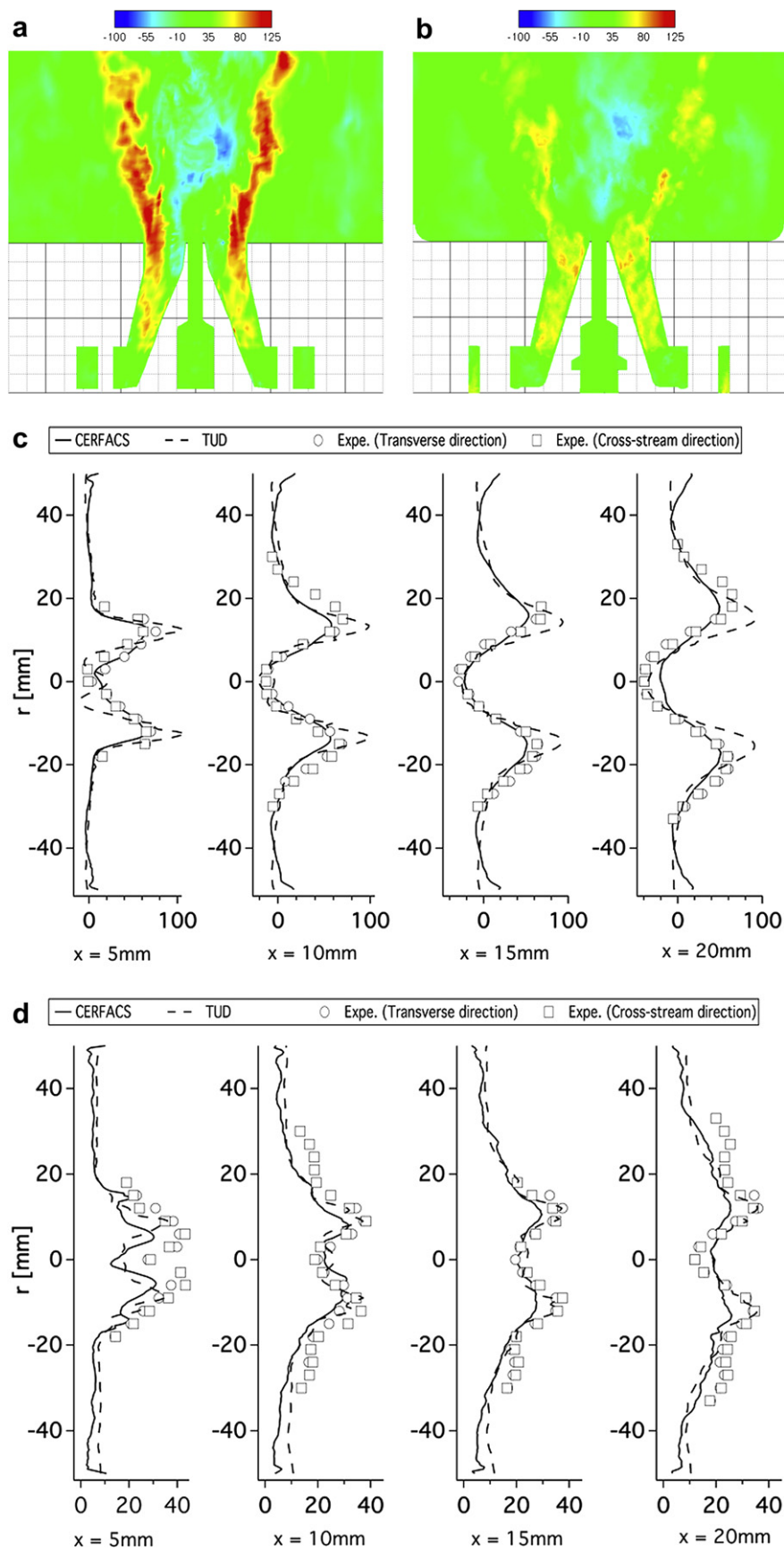


Fig. 10. Instantaneous views of the axial velocity component: (a) TUD, (b) CERFACS. Temporal averages of the two LES's are compared to experimental measures for (c) the mean axial velocity component and (d) its RMS at four axial locations within the chamber (courtesy of P. Pantangi, A. Sadiki, M. Haeger and A. Dreizler from TUD University, Germany).

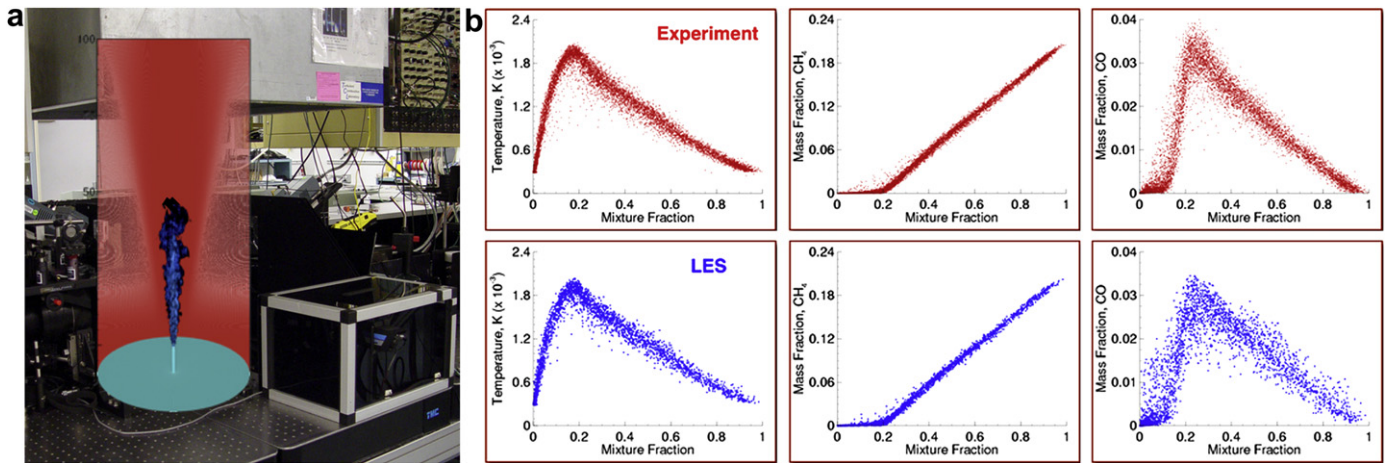


Fig. 11. (a) DLR-A flame configuration and highly resolved LES mesh along with (b) a comparisons of scatter plots obtained by measurements (red) and LES (blue) [273]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

most of the effort has been concentrated on jet flames. A list of the identified productions for laboratory swirled flames is given in Table 4. These studies generally demonstrate the superiority of LES for turbulent reacting flows. The main reason originates in the natural unsteady nature of the governing LES equations which dynamically reproduce the interactions that control mixing and turbulence/flame interactions over a wide range of applications. These laboratory tests have however a major limitation to qualify SGS sub-models for mixing and turbulent flame interactions: their Reynolds number is low inducing a significant overlap of the inertial and dissipative scales thereby violating the scale separation hypothesis needed for most SGS closures. Conclusions are thus difficult to extrapolate to real gas turbine flows where the flow Reynolds number is much higher. Fuels are also much more energetic (thinner flame fronts) and grid resolutions much lighter. All of these issues have been identified and highlighted in Section 2.2.2. The difficulty reduces in discriminating turbulent combustion models based on reliable quality criteria in the context of mixing and reaction. This last subject still remains an open issue despite recent contributions [200].

Recent LES publications on gaseous laboratory scale swirl flames for the Sandia burner illustrated on Fig. 4 and complementing the predictions of Fig. 8, provide first insights on the importance of modeling strategies. This example was produced jointly by researchers in England (Imperial College and Loughborough University) [258] with two low Mach number structured (in cartesian and/or cylindrical formulations) LES codes. Results are provided on Fig. 9 for the swirled experiment of [215,216] operated with methane, air and hydrogen for two swirl numbers ($S_g = 0.32$ for SMH1 and $S_g = 0.54$ for SMH2, respectively). Although the codes differ, they theoretically use the same LES formalism. The simulations are produced on different grids and slightly different boundary conditions. SGS velocity closures rely on dynamic closures [24] with numerical regularization for unphysical model coefficients (clipping). Turbulent combustion modeling relies on single or multiple flamelet approaches, Eq. (9). Chemical terms are obtained from two chemistry models with variable or constant strain rates and a subgrid scalar variance, $\tilde{Z}Z - \tilde{Z}\tilde{Z}$, is coupled to a presumed Beta PDF. Depending on the LES method available in each code, the scalar variance is closed either by use of [259] or [260].

Mean velocity, mixture fraction and temperature predictions for SMH1 are compared to experimental data on Fig. 9. When available RMS profiles are added. Similar results for SMH2 are also available

[258]. Overall, both LES codes and strategies provide similar behaviors. Grid sensitivity and inflow boundary specifications are specifically highlighted underlying the difficulty of clearly differentiating one modeling strategy compared to another one (see above discussion on boundary condition effects). However, both approaches provide very satisfactory predictions of the mean and RMS flow fields irrespectively of the flow swirl number. Mixing is well predicted for both experimental conditions. Mean temperature profiles are consistent with experiments.

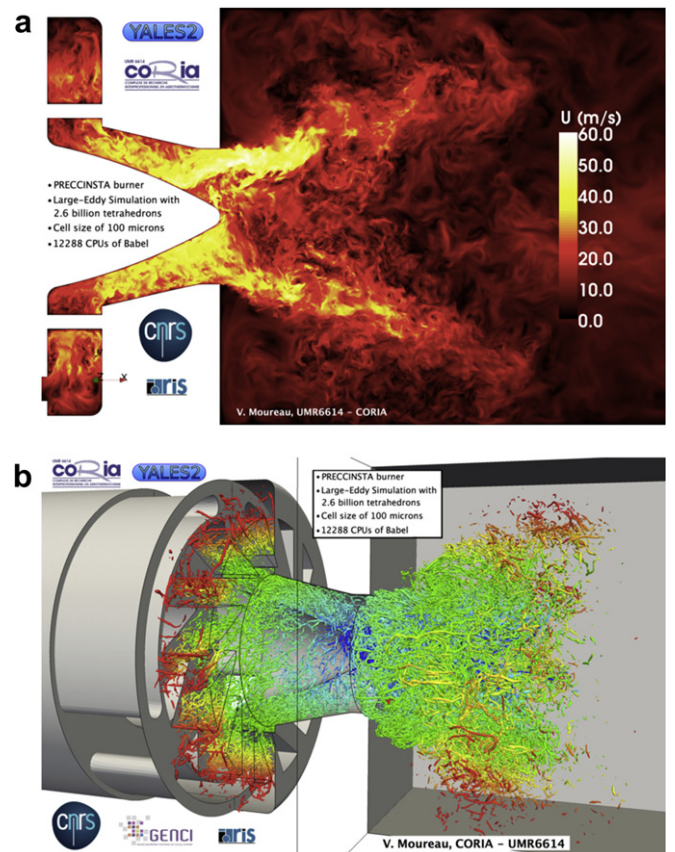


Fig. 12. Massively parallel LES predictions of the Precinsta burner: (a) instantaneous velocity field and (b) turbulent structures [205].

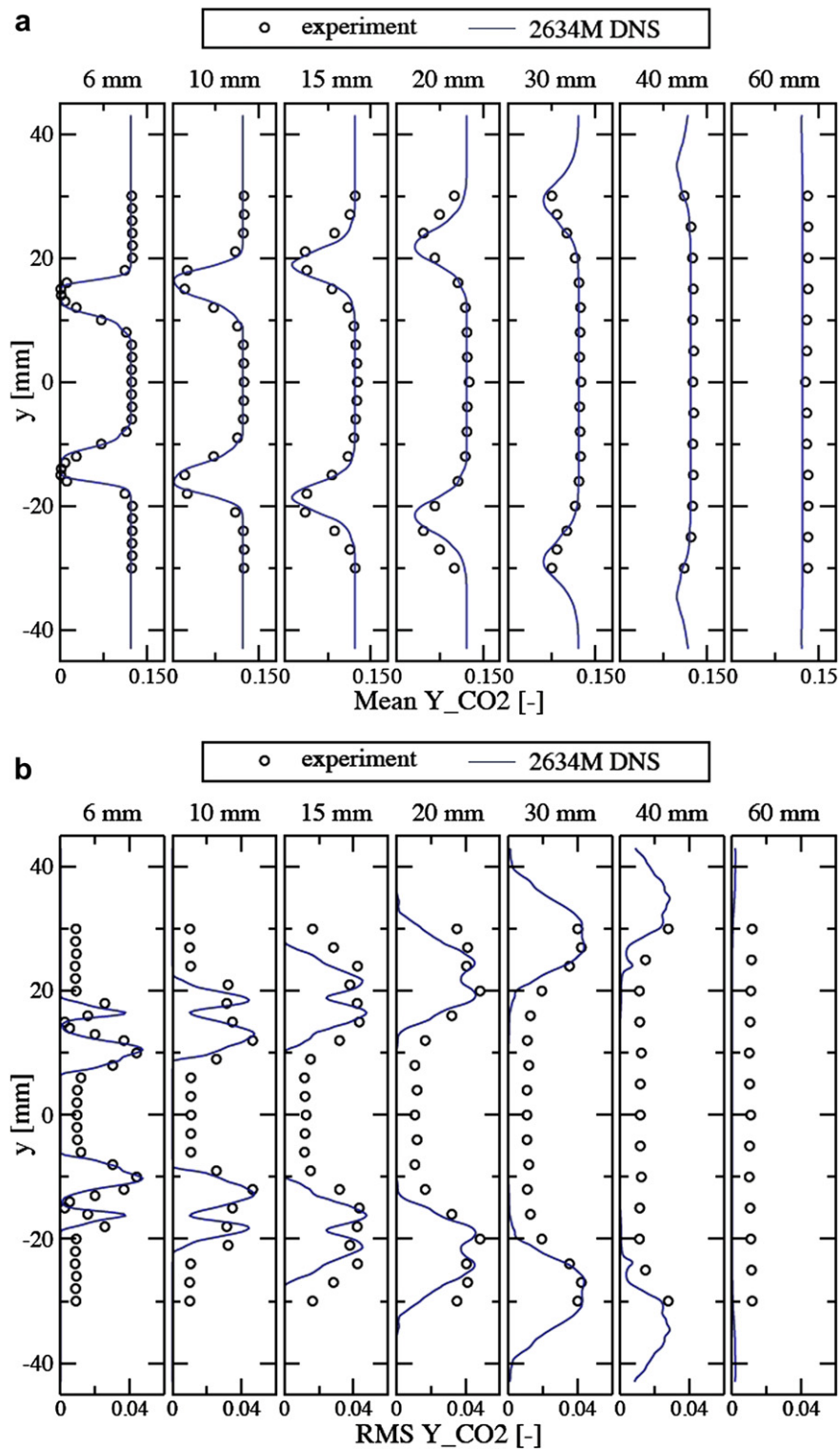


Fig. 13. Mean profiles of (a) the CO_2 mass fraction and (b) its RMS obtained by LES and in the experiment at different stations within the combustor.

Similar validations against laboratory scale burners with more complex geometries start to appear. Such a comparison has recently been produced within the context of the TIMECOP AE European project¹ involving major European aeronautical engine

manufacturers. This specific study follows the MOLECULES European project² where the same injector was studied but operated with gaseous methane [240,241]. For the case discussed here, pure gaseous kerosene is injected separately from the swirled air stream

¹ TIMECOP AE stands for Toward Innovative Methods for Combustion Prediction in Aero-Engines, FP6-2005-Aero-1.

² MOLECULES stands for Modeling of Low Emissions Combustors using Large Eddy Simulations, GRD1-2000-2522.

and the rig is operated at different mean pressures. Details on the swirler geometry are visible on Fig. 5(b). Both computations include the fuel injection system: i.e. the swirler veins and the fuel axial pipe, Fig. 10(a) & (b), to avoid specifying inflow conditions which may impact the predictions. Here again mesh resolution, numerics and formalisms differ. The flow solver from Technische Universität Darmstadt (TUD) is a low Mach number, second order accurate in time and space, multi-block solver. CERFACS's code is third order in time and space, explicit and fully compressible. Turbulent combustion modeling also differ. The latter relies on tabulated chemistry and a conserved scalar [144] approach while the former uses a reduced two-step mechanism [261] coupled to the Dynamic Thickened Flame model [262]. The main outcome is a weak but observable difference in exit swirler axial velocity and RMS profiles, Fig. 10(c) & (d), obtained with the two codes. The main reason for such findings is the relative difference in axial momentum flux at the fuel jet exit predicted by each simulations (different jet profiles). These small flow differences impose changes in IRZ topologies which in turns impact the flame stabilization mechanism and localization. In fact one LES prediction produces a lifted flame when the other predicts a flame anchored slightly inside the swirler. Of course turbulent combustion modeling and most likely chemistry is involved in such stabilization processes. However the importance is not clear. Despite these observations, mean and RMS flow predictions agree with experimental data and uncertainties do not exceed 10% if compared to experimental findings which also contain measurement errors. Note that direct views of the operating burner could not clearly discriminate between a lifted or anchored flame.

An open question which seems relevant in light of the previous comparison is the actual IRZ and PVC dynamics in confined and complex reacting flows. No clear experimental assessment of PVC behavior in combustions conditions is currently available although experimental investigations specifically point to such issues [240,253,263]. Certain LES results confirm the presence of a PVC in cold flow conditions and observe its presence or disappearance in reacting flows. Such a structure is of critical importance especially for complex systems where fuel injection is usually located in the near region of the PVC [264]. The PVC will play a role in the flame stabilization process or flow transition from one operating condition to another. This is an additional difficulty to qualify LES in an industrial context since such behaviors may be amplified or damped by modeling and discretization errors. At least the question emerges due to the potential benefit of simulating fully unsteady features by LES which is not possible with RANS.

3.3.2. Leadership-class LES modeling and predictions

In recent years, the advent of massively parallel machines offering PetaFlops (one million billions of floating point operations, 10^{15} , per second) [206,265] or projections for ExaFlops (10^{18}) capabilities in 2020. Such new horizon and machines infer a new impetus to code developers and new coding strategies or data management schemes to better benefit from the added computing power. The net result is the emergence of new LES codes able to manage efficiently hundred thousand and even billion points LES. With such capabilities new modeling constraints appear and help understanding or assessing each model contribution in specific and well mastered circumstances by seriously reducing the impact of numerics for example. This recent environment yields new types of LES results that are presented here. Issues pertaining to the nearly fully resolved fields are also illustrated.

3.3.2.1. Highly resolved LES predictions. This brute-force method has produced quite successful results for the DLR-A flame [266–268] and the PRECCINSTA burner [269–271]. Fig. 11 presents

(a) the DLR-A set-up along with (b) scatter plots of temperature, methane and CO mass fractions as functions of the mixture fraction space, Z , at given axial stations in the jet obtained by measurements and LES [272,273]. Predictions and measurements are in excellent

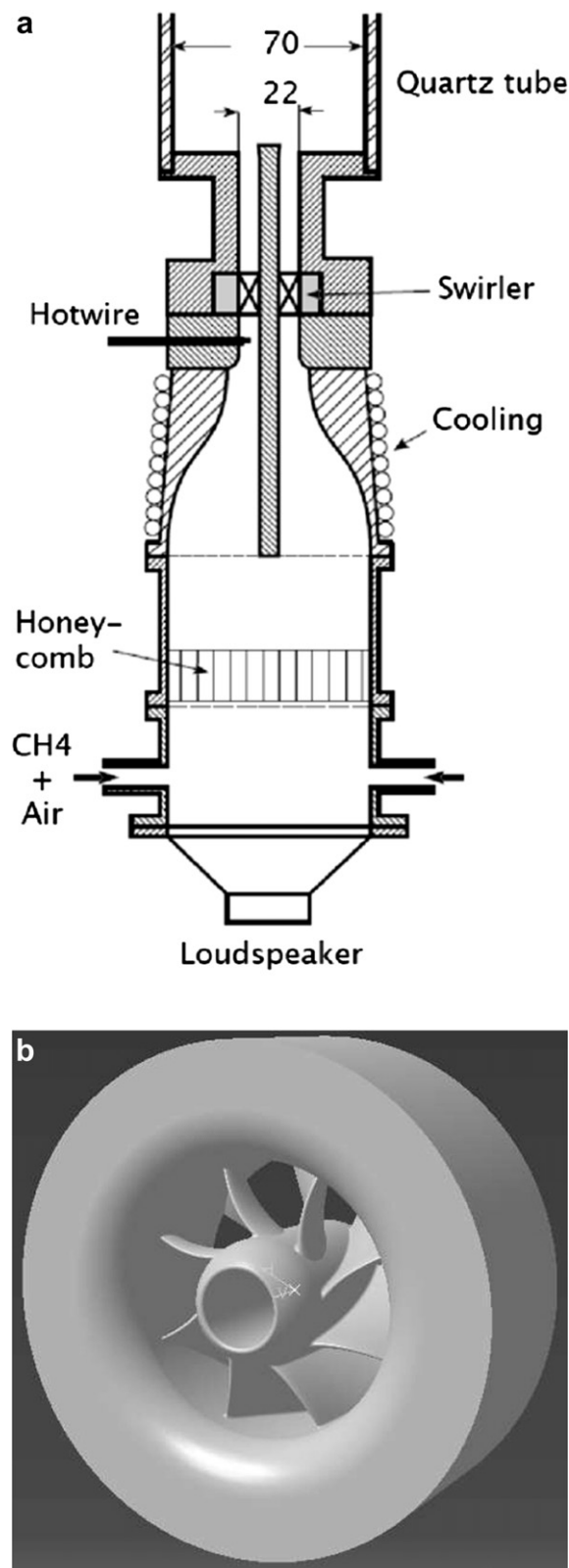


Fig. 14. Views of (a) the experimental burner and (b) swirler used to determine FTF/FDF's experimentally and numerically [302].

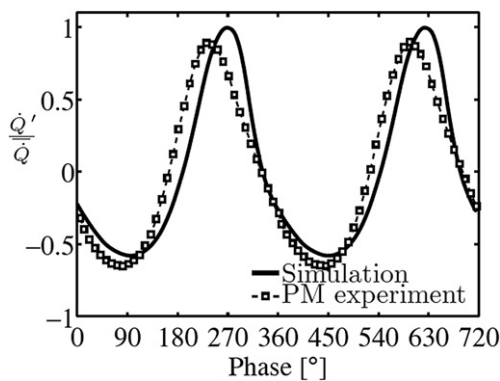


Fig. 15. Temporal evolution of the heat release rate fluctuation obtained by LES and experimentally [302] for the above configuration, Fig. 14.

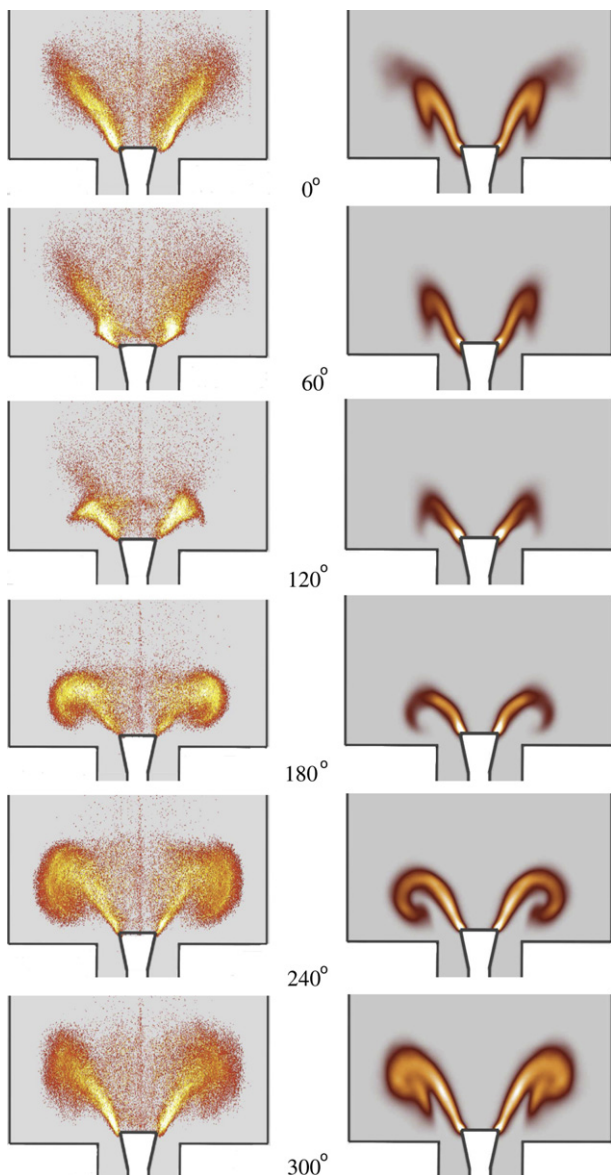


Fig. 16. Direct comparisons of experimental observations (left column) and LES predictions (right column) for a swirl stabilized premixed flame subject to an external acoustic forcing. All snapshots are taken at equal phase angles and allow to retrieve leading mechanisms governing the FDF [302].

agreement. This is also confirmed for mean and RMS velocity profiles at multiple axial locations [273]. Higher order quantities that are usually required and of importance for higher Reynolds number flames (here the reported value is $Re \approx 15,000$) can be probed accurately in the simulation and in the experiment [273] to validate the modeling strategy. Conclusions derived from these analyses are useful but only constitute a first step toward higher order and model validations for real industrial configurations that use much more complex fuels and operate at much higher Reynolds numbers.

More complex configurations such as the PRECCINSTA burner [269–271] for which $Re = 40,000$ have also been treated with such codes [190,192,205,247,274–277]. In [192,205] and although modeling hypotheses are still present, numerical predictions and mesh independence have been obtained for mean flow quantities and RMS in cold flow conditions, Fig. 12. For the stations which target the IRZ, various LES grid resolutions are obtained with a highly resolved LES (claimed to be a DNS provided that modeling can be disregarded) using 2.6 billion tetrahedral cells by use of a low Mach number code relying on a fourth-order finite volume scheme. For this cold flow condition, LES converges reasonably well with first and second order mean flow statistics becoming independent of the grid resolution for 329 million or more tetrahedra. In reacting conditions, convergence can also be reached but at a higher cost: *i.e.* for this tool and modeling at least ≈ 450 million cells are needed [192]. Such findings are very encouraging since they confirm that even outside the theoretical framework for which models are derived and specifically in near realistic experimental setups, convergence is accessible. The next step is to clearly assess the importance of the modeling hypotheses by a *posteriori* validations and identifications of the various terms

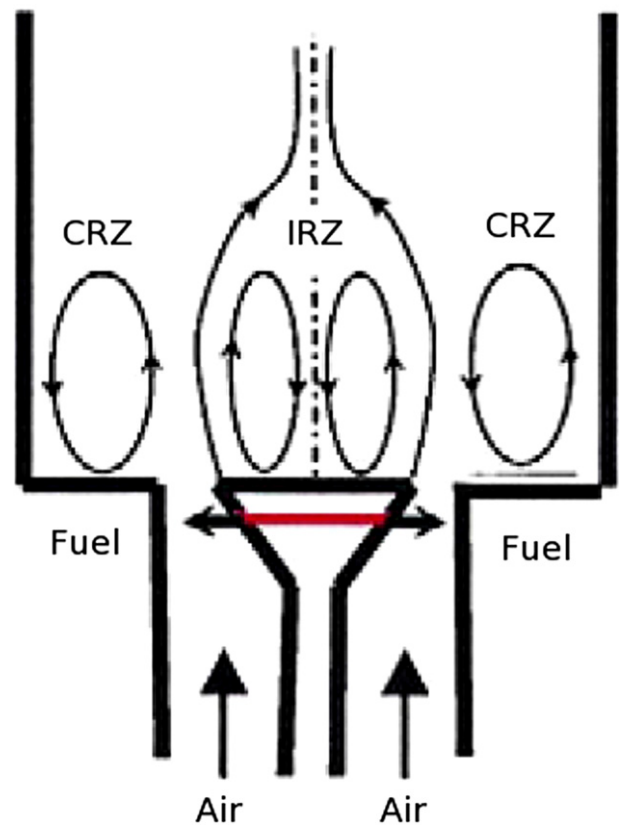


Fig. 17. Configuration to study ignition in a bluff body case [314].

at play in such predictions (*i.e.* detailed estimates of balance equations).

Comparisons of mean combustion quantities with experimental findings, Fig. 13(a), provide excellent agreement for all major species profiles. Uncertainties remain present for RMS values of species mass fractions, Fig. 13(b). Issues pertaining to the actual accuracy of the measurements for these quantities need to be precisely known. For example, the experimental measurement volume is larger than the cell size currently used in the computation. Likewise the time scales integrated and represented by both diagnostics differ: LES can only reasonably compute few flow-through times (generally milliseconds) when measurements run over several minutes. Finally, modeling is still required in such LES. Typically, the hypothesis of a perfectly premixed burner is assumed in this work when recent experimental and numerical findings show that incomplete mixing is present in the experiment [271,277]. Tabulation is also required and specific closures valid under the purely premixed combustion are used. Another interesting question (especially from a pure industrial point of view) is: what modeling terms among the numerics, LES models, boundary conditions... provide the leading contribution to these predictions and to what level?

3.3.2.2. Highly resolved LES modeling. New code capabilities conjugated with massively parallel machines offer an alternative view to the conventional turbulent combustion LES modeling strategy. Indeed, in the long term, LES (and even DNS) grid independent solutions will be applicable to some real industrial flow problems. In other words, we will simply compute most of phenomena that are today modeled. Even though this perspective is exciting and will certainly become true in the next years for simplified lab-scale burners [205,278], it remains probably a very long term option in gas turbines. First, as pointed out in [190,279], conventional approximations provided for the filtered viscous stress tensor may not be sufficient to fully recover expected flame behaviors in the context of fully resolved premixed flames [191]. LES models do not converge to DNS when the number of grid points increases because SGS models usually neglect certain effects. For example, Schmidt numbers are often assumed to be equal in LES, an assumption which may be acceptable for LES but not for DNS. Similarly many SGS models derived for premixed turbulent flows cannot capture a laminar or well resolved laminar front. Finally, technological devices present in real gas turbine combustors (effusion cooling, two-phase flows) will require modeling even on a petascale machine.

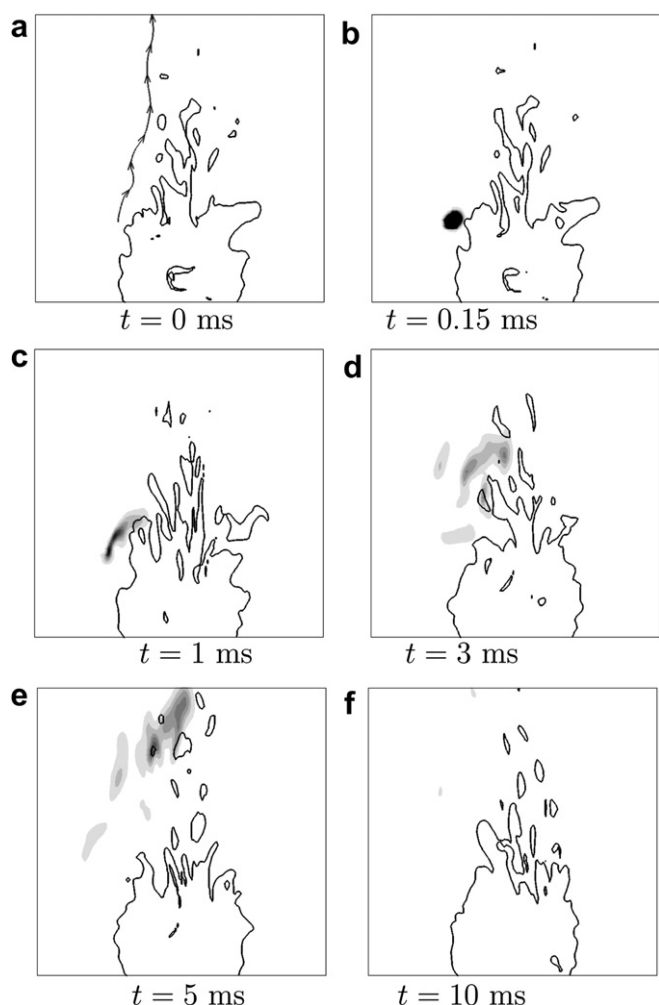


Fig. 18. Failed ignition sequence as observed in time by LES [310] of the experimentally diagnosed configuration of [314] shown on Fig. 17. Snapshots correspond to the instantaneous field of temperature (dark iso-contours) after sparking at $t = 0$ ms. Each instant also shows the isostoichiometric line.

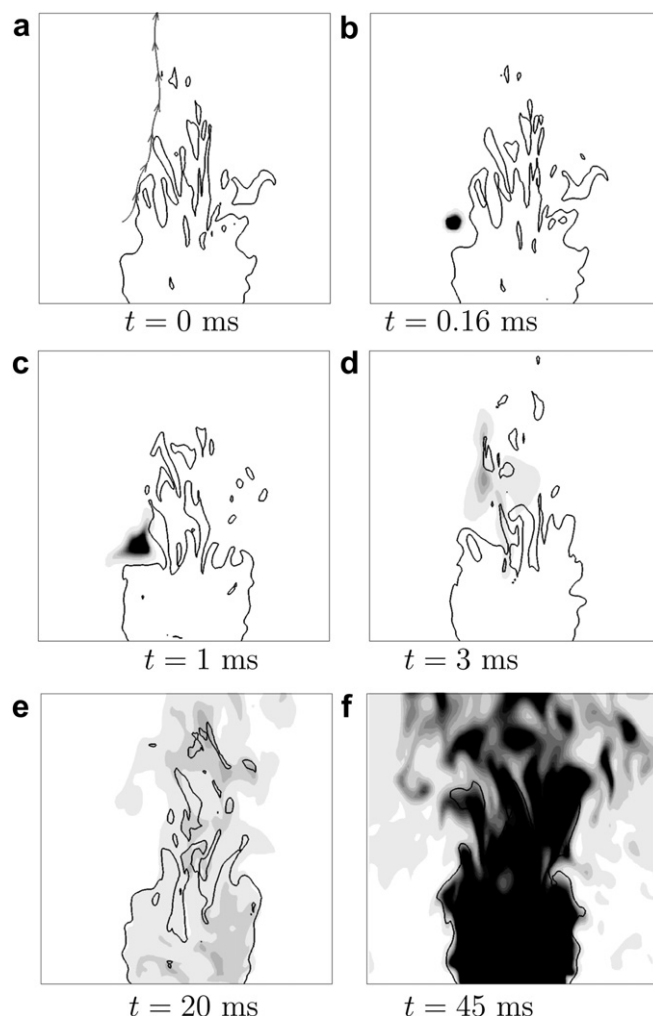


Fig. 19. Successful ignition sequence as observed in time by LES [310] of the experimentally diagnosed configuration of [314] and shown on Fig. 17. Snapshots correspond to the instantaneous field of temperature (dark iso-contours) after sparking at $t = 0$ ms. Each instant also shows the isostoichiometric line.

3.3.3. Thermo-acoustic instabilities

Thermo-acoustic stability of gas turbine combustors has been the subject of intense research due to the potential constraints imposed by new regulations on pollutant emissions [280,281]. To meet these new objectives, conventional designs have to operate in lean premixed modes: *i.e.* fuel and oxidizer enter the swirler as a partially premixed gas. However such configurations are known to be prone to thermo-acoustic instabilities [31,282]. These oscillatory operating conditions must be avoided since they reduce considerably the life-time of the engine. The difficulty in predicting such physics is that the driving force involves the coupling (in phase and space) of heat release fluctuations and acoustic perturbations as evidenced by the Rayleigh criterion [31,282–288]. The combustion response to flow perturbations (acoustic or hydrodynamic) is thus the triggering mechanism.

Two numerical strategies essentially relying on LES can be adopted to investigate the thermo-acoustic response of a design. The first approach directly simulates the experimental or real geometry by use of compressible LES. The aim is to rely on the LES behavior: *i.e.* growth or damping of acoustic fluctuations by the computations and the eventual limit-cycle typical of a saturated thermo-acoustic operating mode. Although this approach presents some interest in real applications where no data is available, such predictions will clearly be influenced by all the parameters

identified previously. It will also depend on the modeler ability to properly treat the acoustic boundary conditions of its simulation [289–293]. Extending this procedure to the entire range of conditions of a real burner is still too costly. The alternative approach is to use thermo-acoustic Helmholtz solvers and model or obtain the so called Flame Transfer Function (FTF) [286] or Flame Describing Function (FDF) [294] by use of acoustically forced LES [293,295,296]. This approach allows to separate the acoustic problem (which is handled by the acoustic solver) and the flame response problem (which can be computed by LES on smaller domains).

For swirled flames, the best tools to evaluate the flame response are LES or experiments [219–221,296–302]. Recent publications [219,221,302] highlight the influence of swirl. In particular, the conversion of acoustic energy into vortical energy across the swirler is evidenced [302]. The main impact on the flow is a fluctuating component of the swirl number due to the convected vortical structures generated within the swirler. For such flows, the flame response not only contains the acoustic response but also a non-linear component imposed by the flow modification. A direct consequence is that the flame response depends not only on the acoustic perturbation frequency but also its amplitude. Combining LES and laser diagnostics on a laboratory swirled flame, Fig. 14, Palies et al. [302] have studied the constructive or destructive interactions of the phenomena determining the flame response.

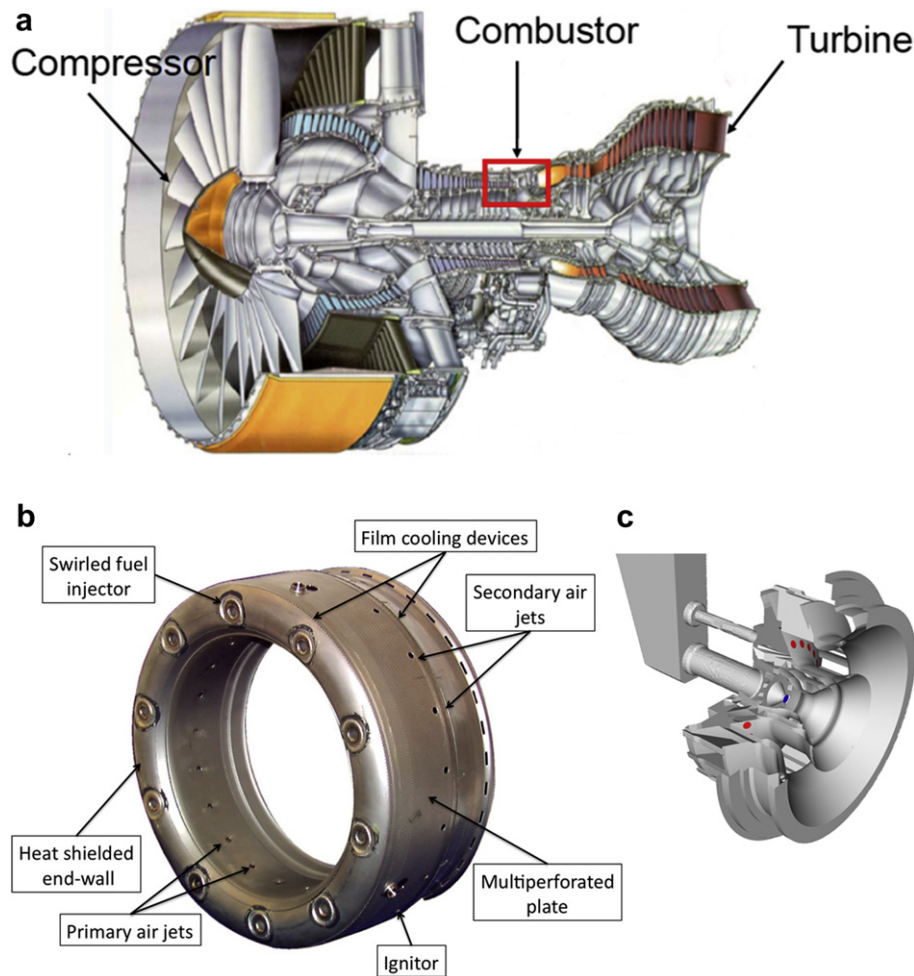


Fig. 20. Typical view of an aeronautical gas turbine engine: (a) full engine, (b) combustor flame tube and (c) a detailed view of a recent swirler design. Fuel injection points are here visualized through red dots. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 5

Tentative survey of real scale burner LES's.

Ref.	Code	Turbulence	Combustion	Target applications
[316]	BOFFIN	Smag.-Lilly	Z, presumed PDF, flamelet	Rolls-Royce Tay engine [403,404]
[349,405]	AVBP	Smag.	Thicken Flame, reduced chemistry	Siemens burner
[406,318,317]	CDP	Dyn. Smag.	\bar{Z} , \bar{c} , presumed PDF's, complex kinetics [370]	Pratt & Whitney combustor
[48]	PRECISE	Dyn. Smag.	EDB, two-step chemistry	Rolls-Royce development burner
[400]	LESLIE	LDKM	LEM-LES, three-step chemistry [400,407]	TAPS (GE-2) combustor [400,408]
[319,360,320]	AVBP	Smag.	Thicken Flame model, reduced chemistry	Helicopter combustion chamber from Turbomeca
[312]	BOFFIN	Dyn. Smag.	Transported PDF	Ignition sequence of a Rolls-Royce burner
[343]	PRECISE	Dyn. Smag.	CMC or EBU, one-step chemistry [409]	Ignition sequence of a Rolls-Royce burner
[365,366]	Unknown	Unknown	Unknown	Honeywell burners
[306,307]	AVBP	Smag.	Thicken Flame, reduced chemistry	Ignition sequence of Turbomeca burner
[355-357,342]	AVBP	Smag.	Thicken Flame, reduced chemistry	Helicopter combustion chamber from Turbomeca
[358]	Openfoam	LDKM, Mixed model	PaSR-LES [70,410], reduced chemistry	CESAR combustion chamber

LES are here quite successful in reproducing the experimental observations, Fig. 15. Detailed comparisons between phase averaged views of the forced burner and LES, Fig. 16, confirm the suitability of the approach (at least for the frequency of interest) to reproduce the flame response.

In parallel to these theoretical developments which now rely on the experimental diagnostics and LES, more realistic burners are treated numerically. The PRECCINSTA burner, Fig. 12, has been specifically designed for thermo-acoustic studies and preliminary LES results [247] reproduce certain unstable operating conditions. Some unstable cases could however not be recovered with fully premixed LES. In fact recent simulations [277] confirm experimental evidence that partial pre-mixing is of importance in triggering the oscillation under specific conditions ($\Phi = 0.7$) [271,303,304]. Such results emphasize the need for comprehensive studies of the LES ability and modeling capabilities or sensitivity to properly understand thermo-acoustic instabilities.

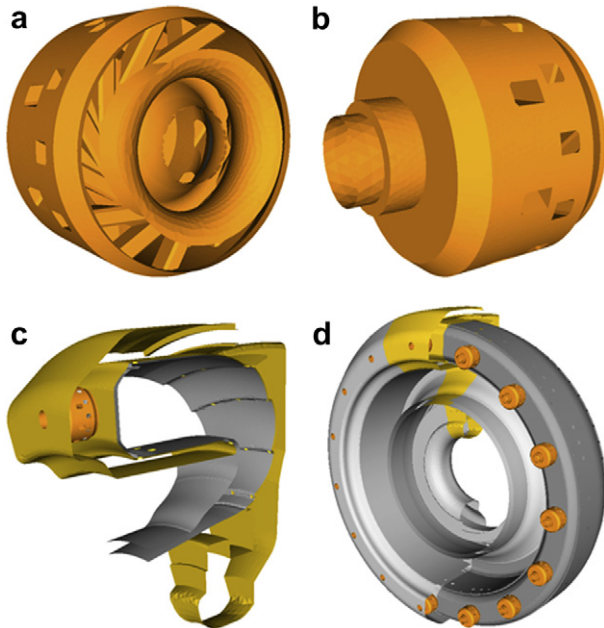


Fig. 21. Geometrical elements of a gas turbine for which LES is reported: (a) & (b), swirler; (c) single sector domain taking into account the swirler (orange), the flame tube (dark grey) and the chamber casing (yellow); (d) full annular configuration with all sectors, swirlers and the entire flame tube. The particular example corresponds to a reverse combustor design. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3.3.4. Transient operating conditions

LES being intrinsically unsteady, one single fully transient flow (*i.e.* non-statistically stationary) can be obtained. Although such predictions usually require averaging multiple LES [305], individual predictions can guide our understanding of complex phenomena otherwise inaccessible by conventional or industrial numerical tools.

Two transient reacting flows of great interest to the gas turbine manufacturers are the forced ignition or re-ignition and extinction phases of gas turbine engines. Forced numerical ignition studies for aeronautical applications have recently started [306-312] to complement theoretical and experimental works [305,311,313].

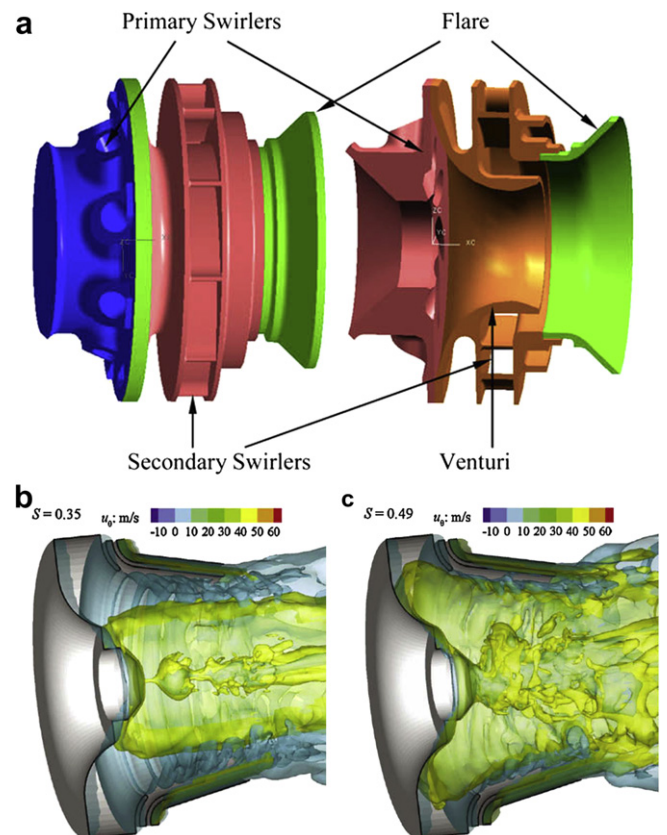


Fig. 22. Real swirler design as reported by [244,250,335]: (a) description of the different elements present on a design for the CFM56 engine and LES predictions of an industrial swirler and whose swirl number is (b) $S = 0.35$ and (c) $S = 0.49$ (instantaneous views of the azimuthal velocity component vector).

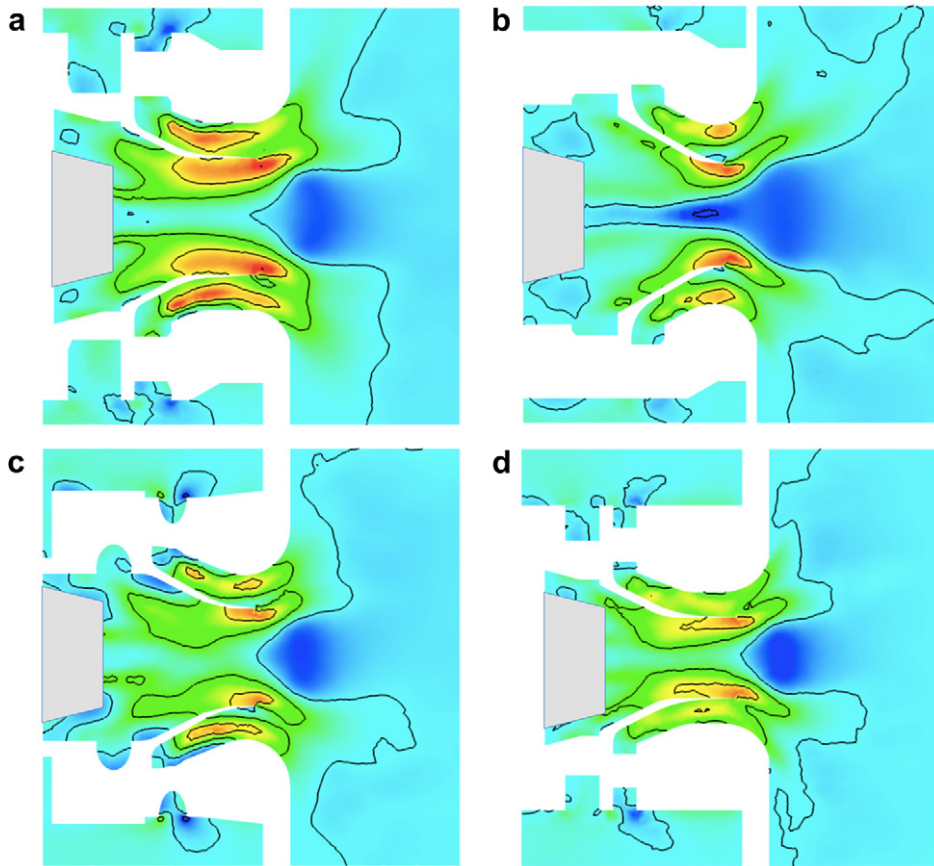


Fig. 23. Real swirler designs evaluated through LES of non-reacting flow: views of the axial mean velocity component, the flow going from left to right (dark blue corresponds to large values of back flow; courtesy of A. Roux from Turbomeca, Safran group). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Laboratory test cases providing a direct comparison between measurements and LES are very promising. An example of forced ignition studied experimentally by [314], Fig. 17, and computed with LES by [310] are presented in Figs. 18 and 19. Two sequences

are shown on Figs. 18 and 19 which illustrate the capacity of the approach to address the notion of success or failure the flame stabilization process. For a given energy deposit at a given position but at two different instants of a statistically stationary flow,

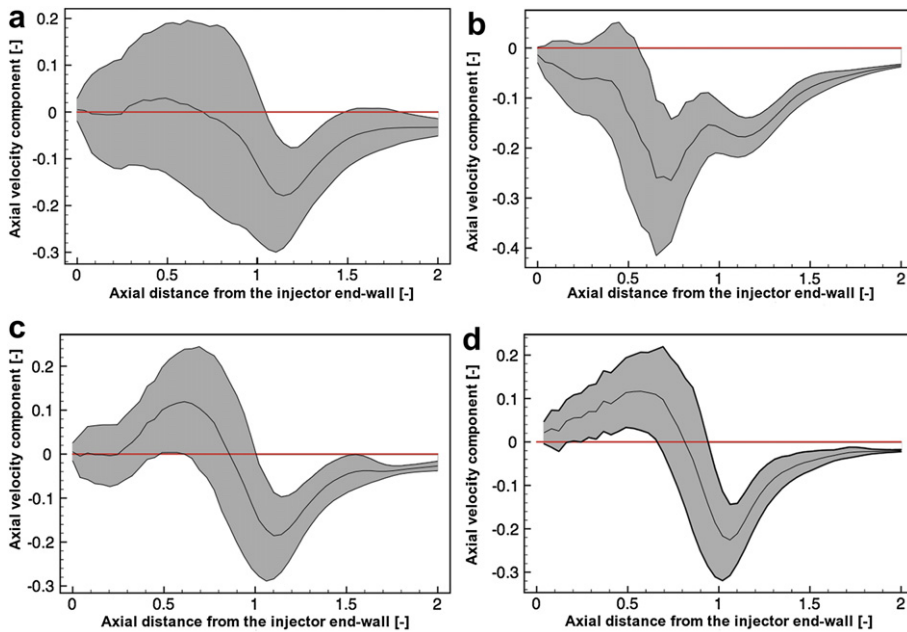


Fig. 24. Mean axial velocity profiles along the symmetry axis of the swirlers of Fig. 23. The superposed grey envelop represents the extent of the potential local and instantaneous value of the axial velocity component obtained by LES (courtesy of A. Roux from Turbomeca, Safran group).

one observes two distinct transient predictions: a failed one, Fig. 18, and a successful ignition, Fig. 19. Multiple parameters play determining roles in the initially formed flame kernel (if created by a spark or a laser ignitor). Local values of the flammability limits are of course of importance, but so is the turbulence at the flame surface. Naturally, legitimate questions arise from such simulations [308,309,315]. LES remain nonetheless the only tool capable of producing such predictions for high Reynolds number flows.

4. Real engine combustor simulations

LES in real engines first appeared in 2004–2008 [48,316–320]. These simulations are more difficult to validate due to the limited set of available measurements and the extreme simplifications introduced in comparison to real operating burners. This section reviews recent LES performed for gaseous reacting flows in real engine combustors. The primary objective of the discussion is to provide a status and highlight the added value of LES for industrial flows.

4.1. Specific features and missing links

Performing LES in real chambers imposes geometrical or physical simplifications:

- Boundary conditions (inflow, outflow conditions and impedances): real burners are fed by a compressor and blow into a turbine, Fig. 20(a). These devices control the inflow and outflow conditions of the combustion chamber. Acoustically they induce impedances which need to be evaluated if LES are to be representative of a real operating condition of the engine. Contrarily to most experimental set-ups, fuel and air enter the combustor through multiple inlets and yield a partially premixed environment. The major consequence of these technological choices is that the local flame combustion modes and regimes or the local value of the equivalence ratio are not known.
- Cooling devices: Technological effects (multi-perforated plates, dilution jets, films...), Fig. 20(b), are present in all parts of the combustion chamber to make sure that the combustor walls can durably sustain the hot product temperatures issued by combustion.
- Fuels (liquid and heavily carbonated fuels or even multi-component fuels): Most real aeronautical engines operate on liquid fuels, Fig. 20(c). Modeling sprays raises a new and significant difficulty (compared to gas combustion). An additional complexity is due to the kinetic models required for such fuels: describing the chemistry of hydrogen in an academic experiment can be done but this exercise remains impossible

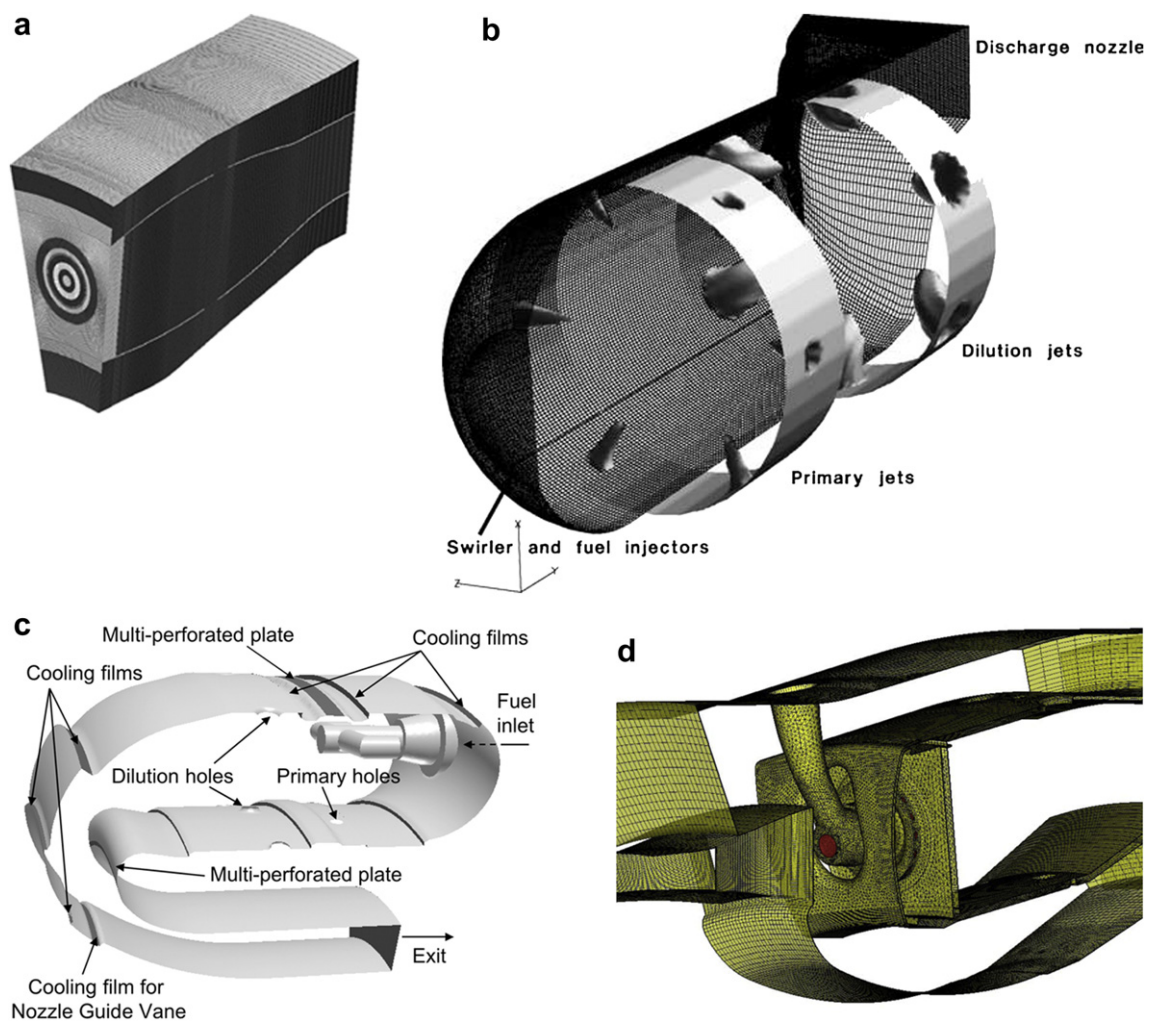


Fig. 25. Single sector LES: views of the different computational domains treated by LES [48,316–320].

today for the multi-component fuels with long carbon chains found in gas turbines.

On-going researches focus on these difficulties through various mathematical tools and developments that may be integrated in a LES solver targeting real gas turbines. Interested readers are pointed to the homogenization techniques for multi-perforated walls [321–323], Euler/Euler [324–328] and Euler/Lagrange [329,330] formalisms for two-phase flows as well as on-going developments in the field of primary and secondary atomization [331]. In parallel and to improve the prediction of the thermal environment of such devices, recent contributions address multi-physics type of solutions where wall heat transfer [332,333] and/or radiation [334] are coupled to LES.

4.2. Current state-of-the-art LES for real engine combustors

Provided that modelers accept necessary simplifications, real engine LES are currently possible as detailed in Table 5. Since the first published results [48,316–320], multiple laboratories and industrial groups have produced such simulations with different tools and modeling strategies. The results can be divided into three categories related to the geometrical extent of the computation domain and the technological device characterized numerically:

- Non reacting swirler simulations: One of the first steps for combustion chamber engineers is to guaranty that the swirlers, Fig. 21(a) & (b), will provide the desired flow field and fuel distribution for a proper flame stabilization. Because of the complexity of this device, Figs. 20(c) and 21(a) & (b), direct flow

visualization by cheap diagnostics is not accessible to industry. So swirler design and characterization rely on RANS predictions. These results can be improved or complemented by LES predictions which are more expensive but provide a unique access to the cold flow dynamics before final assembly and testing.

- Single sector simulations: Simulations of the flame tube alone, with the swirler and including or not the chamber casing, are used to predict flame positions, local flame dynamics and exit temperature profiles, Fig. 21(c).
- Full annular burner simulations: The previous simulations can be extended by taking into consideration the entire combustion chamber: *i.e.* its full complexity in the azimuthal direction, Fig. 21(d). These computations are usually relevant to industry for azimuthal burner thermo-acoustic stabilities, potential long distance flame interactions, fully transient phenomena or geometrical singularities preventing any hypothesis on the azimuthal periodicity of the flow.

4.2.1. Swirler simulations

Real swirlers include complex flow passages or veins with multiple obstacles and wing profiles that impose a rotating motion to the air streams. Current technologies involve an increasing number of veins that can be co or counter rotating, axially or radially oriented, Fig. 22(a). In order to guide the flow and control the mass flow rate (*i.e.* flow split), the shape of the veins is usually highly convoluted and optimized. These Venturi, flares and separators, Fig. 22(a), control the swirl number of each passage. Most importantly, the swirler is the component of the combustor through which fuel feeds the burner. Fuel injection points may be multiple, located at various locations within the swirler and based

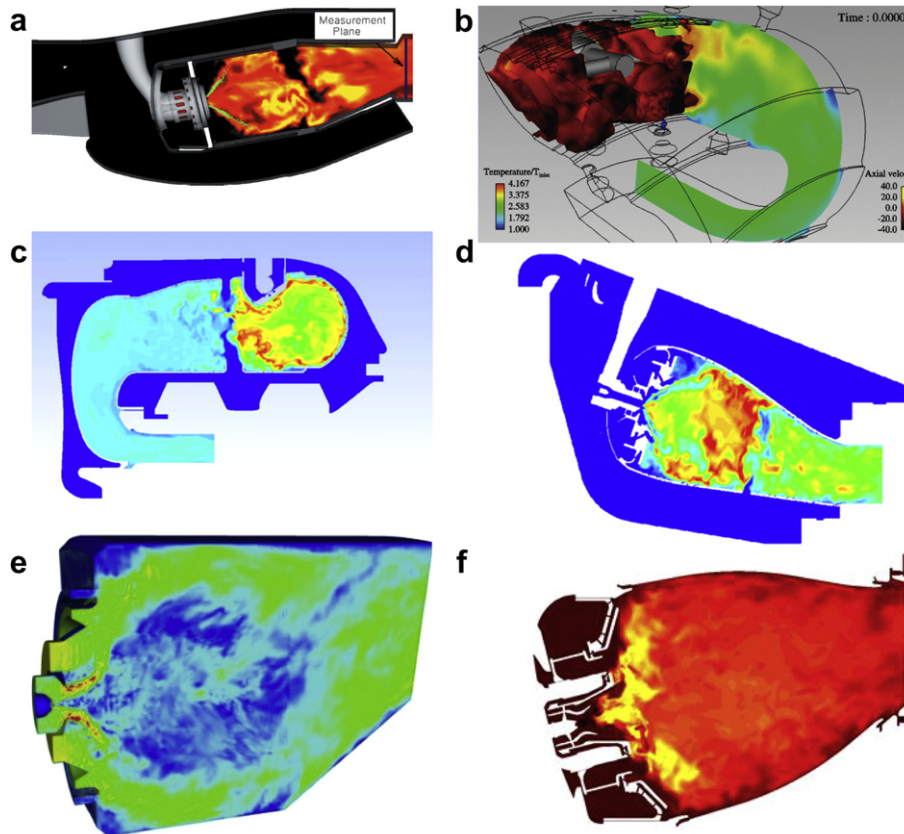


Fig. 26. Typical views of aeronautical gas turbine engine LES: instantaneous field prediction for (a) Pratt & Withney engine [318], (b) a helicopter engine from Turbomeca (Safran group) [319], (c) and (d) two combustor concepts from Honeywell [365,366], (e) a Rolls-Royce lean burn engine [367] and (f) a GE aviation engine [368].

on various types of technologies. In real applications, these injection points can also be operated simultaneously or independently depending on the operating condition of the engine (idle, ignition sequence, cruise or full power). In more advanced designs, fuel staging is also used in control strategies to avoid thermo acoustic instabilities.

With such complex systems, the main difficulty resides in the engineer's ability to guaranty a clear understanding of the mean flow patterns. The main difficulty is to properly predict the IRZ relative position with respect to the chamber end-wall or inner end-wall of the swirler. Two swirled flows (Fig. 22(b) & (c)) were investigated by LES [242,243] to assess flow dynamics and more specifically the position and breakdown of the IRZ as discussed in details in [244]. A second example is shown on Fig. 23 which presents the mean axial velocity component predicted by LES for four different concepts of swirler. The position of the IRZ in front of the swirler, can change even for small geometrical modifications. For example, Fig. 23(b) shows that in some cases, the IRZ penetrates all the way inside the component producing different flow opening angles at the exit of the device. The added value of LES is also to give reliable access to the unsteady features of the flow as illustrated on Fig. 24: the mean axial velocity component profile along the axis of the four designs with the envelope of resolved fluctuating component (*i.e.*: $+/-u'$). Of course, qualifying one design compared to another is a matter of choice and depends on the objective of the engineer.

A typical example related to thermo-acoustic instabilities, is the analysis and assessment of the swirler flow reactivity to external forcing. For such analyses [335], the flow response to various fluctuations is possible by LES. In [335], the response of

a radial swirl injector to mass flow rate perturbations is obtained for a wide range of frequencies. Similarly to [302] but in non-reacting conditions, the flow response to acoustic forcing provides understanding of the leading mechanisms governing the acoustic admittance of the devices. These computations help to qualify the impact such oscillations can have on the mixing of air and fuel [250,277,302] prior to combustion in the main chamber.

4.2.2. Single sector simulations

Pioneering studies on real industrial combustion chamber [48,316,318,319] mainly focused on a single sector description of the full annular gas turbine burner (Figs. 25 and 26) thereby imposing an axi-periodic hypothesis on the flow realization. This simplification is mainly justified by the need to reduce the computational overhead of LES. The primary objective of such computations, Fig. 26, is to qualify LES strategies and codes on industrial problems and assess the gain offered by the method when compared to RANS tools.

Since experimental measurements of the reacting flow in real configurations are scarce, industrial design criteria are used to assess any new numerical approach. Very limited information is known on real engines and design parameters are only indirect diagnostics: typically, while academic experiments provide velocity, temperature and species fields in the whole combustor, real combustor data correspond only to a few temperature measurements at the chamber outlet and one value of the total flow rate. One important quality parameter scrutinized by engineers is the mean exit temperature field of the combustion chamber because it controls the lifetime of the turbine blades. Improved

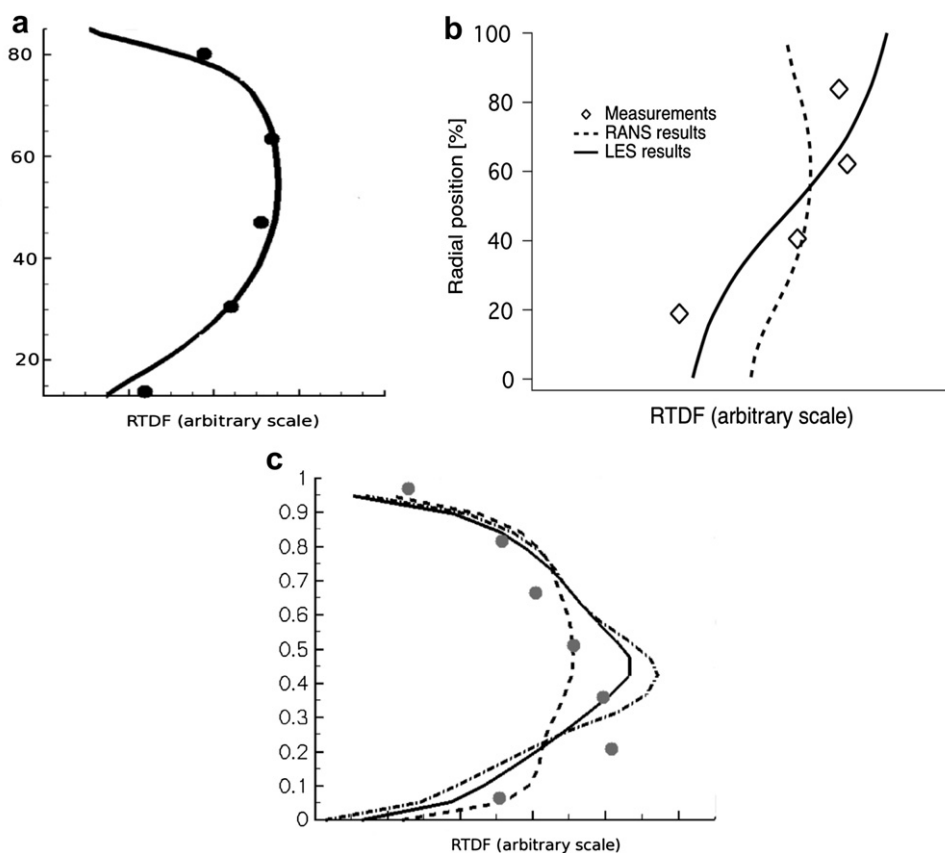


Fig. 27. Outlet normalized temperature profiles (RTDF) obtained numerically by LES, RANS and experiments for different real combustors: (a) Pratt & Whitney engine (LES, line and experiment, symbols) in the plane identified in Fig. 26 [318] (a); (b) Turbomeca engine (LES, RANS and experiment) [319] and (c) Rolls-Royce engine (LES with and without the casing and RANS with the casing) [48]: Symbol, rig measurements; —, LES of liner plus annuli; - -, LES of liner only; and -.-, RANS simulation of liner only.

estimates of this field induce better known limits of the engine operation and effectively translate in turbine blades that are more effectively designed and cooled. Ideally the exit combustor temperature profile should be well homogenized thanks to an efficient mixing in time and space of the hot products of combustion with cooler air coming from dilution jets and wall films [336]. Of course this optimal point is difficult to reach in combustors

which are more and more compact and require larger amount of air to go through the swirler (to improve premixing needed for pollutant reduction). Long and difficult design iterative loops based on RANS are thus needed to locate the primary and secondary jet positions that meet specific objectives.

Preliminary LES of single sector computations with similar geometrical constraints as the one encountered with RANS were

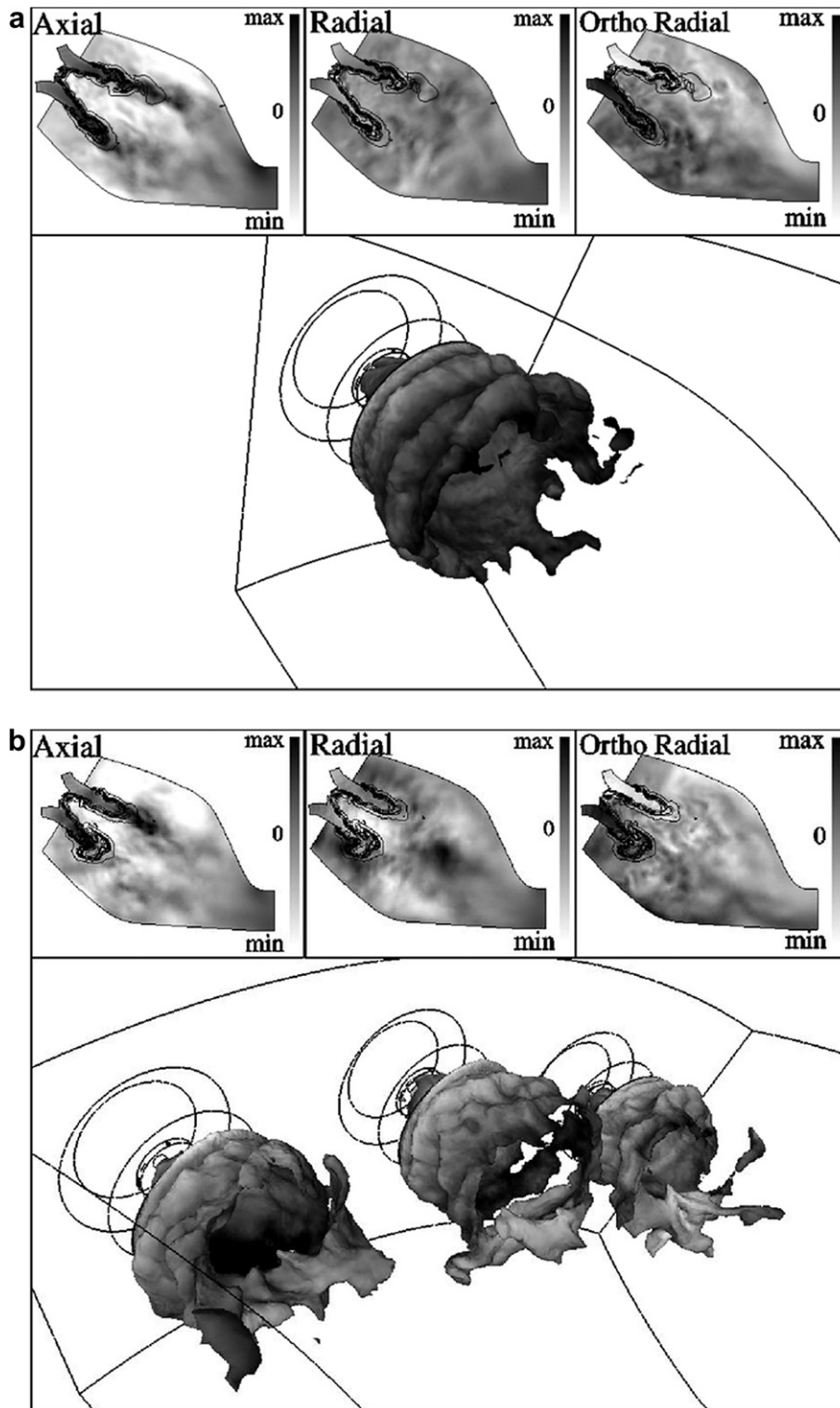


Fig. 28. FTF predictions of an industrial burner by treating (a) one single and (b) three burner forced LES [349].

rapidly produced [48,316,318,319]. Comparisons are obtained for the Radial Temperature Distribution Function (RTDF), which corresponds to the radial representation of the mean azimuthal variations of the temperature elevation relative to a reference value [319]. Fig. 27 shows that LES provide better representations of this crucial parameter for different types of engines. These preliminary studies, also point out that to capture the mixing process between hot and fresh gases, the flame stabilization mechanism, the swirler [316] as well as part of the casing [48] may need to be included in the computational domain [337]. To reduce the boundary condition impact, an ad-hoc strategy is to extend the computational domain to zones where the boundary conditions are known [338–341].

Local mesh resolution effects have also been investigated [320,342]. Preliminary conclusions are in agreement with theoretical analyses and observations obtained on laboratory-scale burners. However, in complex configurations, mean fields seem relatively insensitive to mesh resolution when looking at velocity statistics. Conclusions are not as clear for reacting LES especially for combustion quantities. Turbulent combustion modeling effects have been obtained [343] for a fixed grid resolution and show reasonable agreement between all mean fields irrespectively of the model used (CMC-LES [344] or EBU-LES [343]). All these findings confirm the robustness of LES for an industrial use.

Similarly to the predictions obtained in the research context of thermo-acoustic instabilities, single sector real engine LES provide estimates of FTF's that are otherwise not accessible or very costly to produce experimentally [345,346]. These FTF's can then be used along with thermo-acoustic solvers (Helmholtz or network models) to determine the thermo-acoustic stability map of a burner [298,299,347,348]. For such LES [349], the compressible solver needs to consider an acoustically open numerical setup (no acoustic reflection at the inflow and outflow conditions of the LES computational domain). Acoustic forcing of the inflow (or outflow) [290–292,350] is then applied to determine the frequency dependent function that is the FTF [351–353], Fig. 28. It is important to underline at this point that current FTF estimates obtained with such single sector LES impose specific constraints. Typically, such FTF's are representative of a flame response issued by acoustic modes inducing flow perturbations going through the studied burner. In some situations, azimuthal acoustic modes, often triggered in annular combustors, may arise from flame interactions issued by purely azimuthal modes that can be potentially transparent from the swirler passage point of view. These issues and potential limits are clearly unanswered today and are being investigated by different institutions in the world.

Finally, fully transient phases as encountered in the ignition of a burner are also available in real burners [312,344,354]. These predictions are more difficult to assess.

4.2.3. Full annular burner simulations

The increase in computing power combined with the potential simplification of the boundary conditions has led to computations of complete chambers. Typically, full annular combustor demonstrations have been produced recently [342,355–358]. Such extended computational domains are justified only if information proceeds in the azimuthal direction and can not be properly captured with a single sector hypothesis: for example to simulate flame propagation from a burner to the next after ignition of a flame kernel [307], neighboring flames that interact with each other or the existence of an azimuthal thermo-acoustic instabilities [356].

Fig. 29 presents snapshots of two full burner LES obtained by considering the entire geometry of the burner located between the

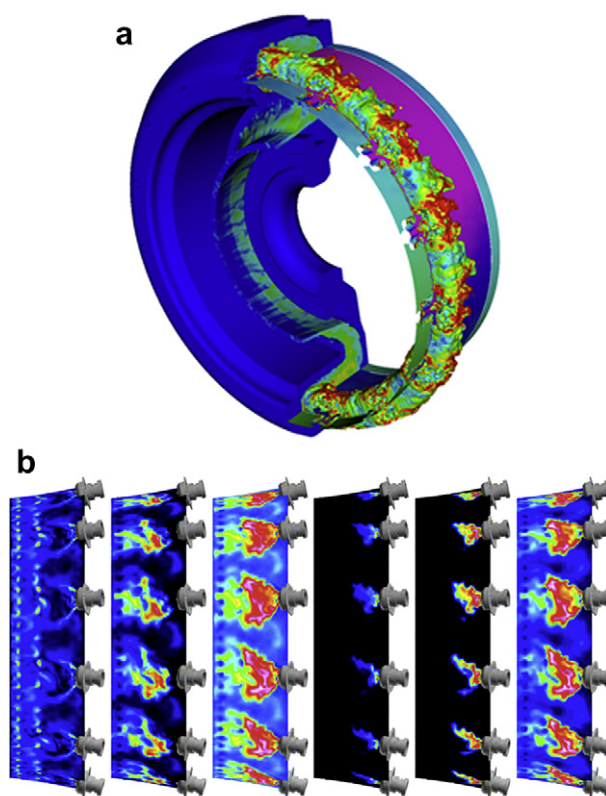


Fig. 29. Instantaneous snapshot of LES predictions in full annular real combustors: (a) from [358] and (b) from [342,355–357].

compressor and the turbine for (a) a Turbomeca burner [342] and (b) the CESAR combustor [358]. In both predictions pressure oscillations are reported and linked to azimuthal thermo-acoustic modes. Compared to single sector LES [355,357,358], the aerodynamics within the combustion chamber differ and the secondary jet mixing with the hot products changes from burner to burner. Reports on the LES behaviors and indirect observations on the real engines confirm the good quality of the predictions thereby providing some confidence in the ability of LES to at least reproduce macroscopic unsteady flow in real engines. These results not only provide a demonstration of the current status of advanced LES solvers when used on massively parallel computers but also give access to new sets of data. Indeed such unsteady fields need now to be studied to feed the design chain and complement design assessments based on RANS. From a theoretical point of view, these simulations greatly contribute to our understanding of azimuthal thermo-acoustic instabilities in complex burners and open new perspectives in terms of investigations. Indeed and despite the fact that great care still needs to be taken, resulting limit-cycles and their triggering mechanisms are clearly of great importance.

Finally, it is interesting to discuss the first simulation of ignition in a full annular chamber [307]. For this demonstration, the problem of burner to burner flame propagation is specifically addressed: two opposite torch ignitors located in an annular combustion chamber provide the initial energy to the non reacting flow mixture of air and fuel, Fig. 30(a). As time proceeds the resulting flames propagate in opposite directions to ignite the different neighboring sectors of the entire combustor, Fig. 30(b)–(c). Such a sequence is inherently unsteady and goes from a statistically stationary cold flow engine to a fully statistically stationary reacting flow corresponding to an engine ready for operation. LES [307] offer a first view of that phase that could be envisioned by engineers only indirectly. Indeed, test-

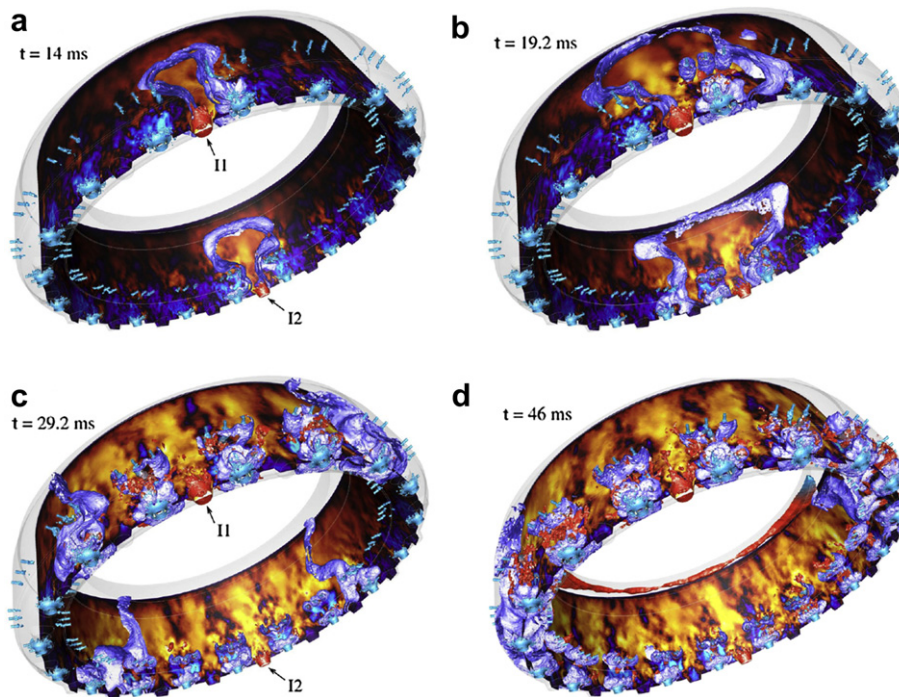


Fig. 30. LES of a light-around sequence obtained for an annular combustion chamber [307].

bench facilities can in this case at best provide an integrated view of the process or indirect diagnostics such as the engine pressure and power output while the light-around proceeds. Results reveal that the whole ignition process takes around 40 ms, that the flame front is propagating azimuthally at 20 m/s and is mainly driven by gas expansion.

5. Conclusions and perspectives

Over the last decade LES of turbulent reacting flows has received great interest from the scientific community. This interest now spreads to companies because LES are the only alternative to the two extreme numerical tools available to the combustion community: Direct Numerical Simulations (DNS) and Reynolds Average Navier-Stokes Simulations (RANS) [359].

- In the former, the turbulent reacting flow is addressed in a brute force way and all scales are resolved in space and time by the numerical scheme which needs to be highly accurate and used in very small well-controlled computational domains to ensure reliable and stable simulations.
- In the latter, the governing equations are first mathematically recast into a new set of equations (usually time-independent) governing the spatial evolution of the mean flow quantities. All scales of interactions require modeling which is a quite difficult task.

Despite its intrinsic difficulties, RANS has rapidly entered the design chain of current industrial gas turbine manufacturers. It has benefited from intense researches as well as modeling contributions and is today very cost effective. DNS is almost model free but is not applicable to industrial applications because it is too computer intensive and unable to deal with Reynolds numbers or the geometrical complexities of real applications. The computing power for such exercises is simply not accessible. It is nonetheless a very valuable scientific tool that benefits from the current and

future evolutions of super-computing and has become a key element of the recent modeling strategies.

LES represent partially the unsteady dynamics of the flows present in DNS and required for improved predictability while alleviating the modeling effort needed with RANS. Being a fully unsteady formalism, LES computer cost is typically 100 times larger than RANS. However, since only the large-scale flow quantities are solved for, the Reynolds number and Damkhöler number flows accessible to LES are greatly extended compared to DNS. As a result, if one takes into account the increasing computing power, LES can be performed today for many burners, in one night (like RANS a few years ago) with a precision which is not very far from DNS.

Despite such progresses, comprehensive and detailed studies are still needed to further extend LES to real combustors. Aside from the recurring difficulty of properly addressing turbulent reacting flows, which is independent of the numerical approach, grid resolution plays a determining role in LES model assessments. This specific issue becomes more complicated in industrial applications where combustion modes and regimes are a priori unknown and grid resolution is far from being uniform. The influence of boundary conditions on the predictions also appears to be of importance for LES of laboratory or real engine configurations. Recent developments in algorithmic open new perspectives on the effective contributions of modeling compared to numerics. The new generations of solvers allow to include geometrical complexities appearing in real applications thereby providing more realistic and predictive simulations. They also lead to more systematic analyses of the modeling hypotheses needed to produce LES closures with reduced numerical impact. At the same time and because of the reduced size of the local mesh resolution accessible to these codes, new LES closures can arise focusing on new formalisms oriented toward mixing and flame inner structures subject to quasi-fully resolved turbulent fields.

The last decade has seen many new applications of LES to real combustors. Transient reacting flows such as forced ignition sequences of burners are now numerically addressed in parallel to

advanced experiments with temporally resolved diagnostics. Thermo-acoustic instability is also a field which now relies as much on numerics as on experiments. LES of both problems have appeared to assess real engine combustion chambers and preliminary results are very encouraging. More simulations are being produced within industry to complement RANS predictions and improve the design cycle of the next generation of aeronautical engines. However, this ultimate objective still requires further modeling to be able to take into account the multi-phase flow nature of the fuel alimentation of real engines. Better estimates of the thermal environment will probably require considering conduction, radiation and technological solutions that are still difficult to address directly numerically with the available grid resolution or even theoretically in the context of LES.

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