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Eprints ID: 5303

**To link to this article:** DOI:10.1016/j.compchemeng.2010.01.011  
URL: <http://dx.doi.org/10.1016/j.compchemeng.2010.01.011>

**To cite this version:** Haït, Alain and Artigues, Christian *On electrical load tracking scheduling for a steel plant.* (2010) Computers & Chemical Engineering, 34 (11). pp. 1899-1903. ISSN 0098-1354

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# On electrical load tracking scheduling for a steel plant

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## Abstract

Nolde and Morari (2010) study a steel manufacturing scheduling problem where the tasks must be scheduled such that electricity consumption matches to a pre-specified periodic energy chart. They propose a continuous time integer linear programming formulation to solve the problem. In this note, we present an alternative continuous time formulation, focused on the relative positions of tasks and time periods, that improves significantly the computation time.

### Keywords:

Steel plant, Mixed integer linear programming, Load tracking

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## 1. Introduction

This paper deals with production scheduling of a steel plant with energy cost. Due to demand increase and to environmental considerations, the reflexion about a better energy consumption in production is a subject of growing interest. New constraints and objectives challenge the classical production models, from long term to short term management and from industrial networks to single plants (Gibbs and Deutz, 2007; Cheung and Hui, 2004; Agha et al., 2010; Haït and Artigues, 2009; Castro et al., 2009).

Energy consumption is an important subject in steel plants. Electrical devices may induce a high power demand if the schedule is not focused on energy. Various types of energy bills exist, calculated from energy consumption, power limits and fix or time variable energy cost. Few papers have studied this aspect for steel scheduling. Boukas et al. (1990) present a hierarchical approach to minimize the makespan of a set of batches while respecting a global power limit. Nolde and Morari (2010) propose a MILP scheduling model to track a pre-specified periodic load curve. In this paper we propose an alternative model, based on Haït and Artigues (2009), to solve the latter problem.

## 2. Problem statement

Nolde and Morari (2010) study a particular steel plant

called mini-mill. The layout is organized as an hybrid flowshop inspired from Harjunkoski and Grossmann (2001) where a four-step batch process transforms scrap metal into cast steel: first melting in Electrical Arc Furnaces (EAF), then Argon Oxygen Decarburization (AOD), alloying and refinement in Ladle Furnace (LF) and finally casting in a Continuous Casting Machine (CCM). The plant is composed of two identical EAF, then one equipment for every other step.

A crane is used to move the batches from one equipment to another. In this model, the crane is viewed as an additional equipment. The empty displacements of the crane are not taken into account. Hence we will consider a seven-task process for each batch, four "processing" tasks and three "transportation" tasks inserted between them. For each task a processing time interval is given, and no delay is allowed between the tasks. Finally, no idle time is allowed between two batches on the continuous casting machine.

The main originality of the problem comes from the objective function. The cost of a schedule depends on the energy consumption of the equipments, calculated on a periodic basis. A target consumption is negotiated between the electricity provider and the plant owner for every 15-minute interval. Any variation from this contracted consumption is highly penalized and the objective is to minimize the sum of these penalties.

Given the particular objective function, modeling the scheduling problem is not straightforward. A 15-minute

discretization is not enough because of the task durations and the accuracy required to obtain good solutions. Nolde (2008) proposed a discrete-time and a continuous-time model for this application. In the discrete-time model, the intervals are subdivided into smaller periods for scheduling needs. The comparison has shown that the continuous-time model is more efficient. Nolde claims that it is due to the high number of time points where a task may potentially start or end. It has also been shown that the computation time is linked to the wideness of task duration intervals, with refer to energy consumption intervals.

### 3. Continuous-time scheduling formulation

The scheduling model is given in Nolde and Morari (2010). We remind it briefly in the following.

- All tasks are indexed with  $l \in \mathcal{L}$ . Tasks starting and ending times are denoted  $t_l^s$  and  $t_l^e$ .
- All jobs are indexed by  $k \in \mathcal{K}$ . The tasks of job  $k$  are given by set  $\mathcal{L}_k$ .
- Machines are indexed by  $i \in \mathcal{I}$ , where  $\mathcal{I}$  is the set of all machines, including the crane. The power consumption of machine  $i$  is given by  $p_i$ . The tasks running on machine  $i$  are given by set  $\mathcal{L}_i$ .
- The task sequence is stored in the set  $\mathcal{S}$  that contains 2-tuples with all pairs of subsequent tasks  $(l', l'') \in \mathcal{S}$ , where task  $l'$  precedes task  $l''$ .

The processing time of a task belongs to a predefined interval:

$$T_l^{\min} \leq t_l^e - t_l^s \leq T_l^{\max} \quad (1)$$

In a sequence of tasks, the steel making process shall not be interrupted:

$$t_{l'}^e \leq t_{l''}^s \quad \forall (l', l'') \in \mathcal{S} \quad (2)$$

Given that the Electrical Arc Furnaces (first task of a job) are identical, and that the jobs are also identical, we can affirm without loss of generality that the sequence of tasks on the EAF is known. So, the cumulative constraint on the set of EAF may be converted into disjunctive constraints on each EAF. Consequently, the resource constraints of the problem are all disjunctive:

$$t_{l_1}^e \leq t_{l_2}^s \text{ OR } t_{l_2}^e \leq t_{l_1}^s \quad \forall (l_1, l_2) \in \mathcal{L}_i^2, i \in \mathcal{I}, l_1 < l_2 \quad (3)$$

A binary variable is associated to each couple  $(l_1, l_2)$  defined as above, and the classical Big-M technique is used to model the disjunction.

Additional binary variables  $\delta_{l_1, l_2}^{CCM}$  are used to model the working without interruption of the continuous casting machine (CCM).

$$t_{l_1}^e \neq t_{l_2}^s \Rightarrow \delta_{l_1, l_2}^{CCM} = 0 \quad \forall (l_1, l_2) \in \mathcal{L}_{CCM}^2 \quad (4)$$

$$\sum_{l_2 \in \mathcal{L}_{CCM}} \delta_{l_1, l_2}^{CCM} = 1 \quad \forall l_1 \in \mathcal{L}_{CCM} \quad (5)$$

### 4. A new task/interval overlap formulation

The objective function is based on a discretization of the time horizon at regular 15-minute intervals. The energy consumption of task  $l$  during interval  $n$  results in the multiplication of power consumption  $p_i$  of the equipment used ( $l \in \mathcal{L}_i$ ) by the time overlap  $o_{l,n}$  between the task and the interval. In the following, interval  $n \in \mathcal{N}$  starts at time  $t_{n-1}$ , finishes at time  $t_n$  and lasts  $D = t_n - t_{n-1}$  constant. The beginning of the first interval is  $t_0 = 0$  and the end of the last interval  $t_{|\mathcal{N}|}$  corresponds to the time horizon  $\mathcal{T}$ .

The model we propose as an alternative to the one of Node and Morari (2010) relies on binary variables that represent the relative position of an event and an interval. These variables are then used to model the overlaps between tasks and intervals.

#### 4.1. Time/interval binary variables

The following formulation, initially introduced by Pritsker and Watters (1968), uses binary variables to indicate whether or not an event takes place before or during an interval. Variable  $\gamma_{l,n}^s$  is equal to 1 if task  $l \in \mathcal{L}$  begins before or during interval  $n$ , 0 otherwise (Fig. 1). The corresponding constraints are:

$$t_l^s \geq t_n(1 - \gamma_{l,n}^s) \quad n = 1, \dots, |\mathcal{N}| - 1 \quad (6)$$

$$t_l^s \leq t_n + \mathcal{T}(1 - \gamma_{l,n}^s) \quad n = 1, \dots, |\mathcal{N}| - 1 \quad (7)$$

$$\gamma_{l,n+1}^s \geq \gamma_{l,n}^s \quad n = 1, \dots, |\mathcal{N}| - 1 \quad (8)$$

$$\gamma_{l,|\mathcal{N}|}^s = 1 \quad (9)$$

In case of regular intervals of duration  $D$ , equations (6) and (7) can be written:

$$t_l^s \geq D.n(1 - \gamma_{l,n}^s) \quad n = 1, \dots, |\mathcal{N}| - 1 \quad (10)$$

$$t_l^s \leq D.n + \mathcal{T}(1 - \gamma_{l,n}^s) \quad n = 1, \dots, |\mathcal{N}| - 1 \quad (11)$$

Note that if  $t_l^s$  exactly coincides with interval bound  $t_n$ ,  $\gamma_{l,n}^s$  can either take value 0 or 1. This is not a problem because the constraints described in the following section ensure the global consistency of time overlap.

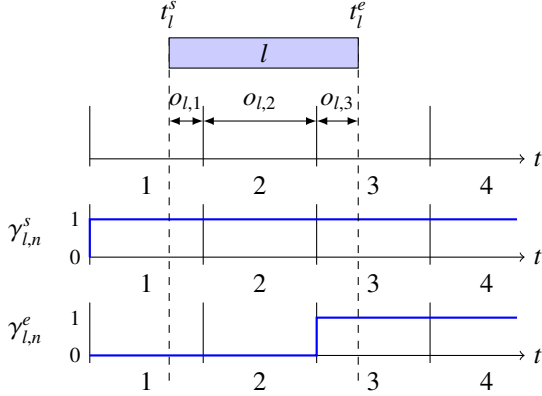


Figure 1: Task/interval overlap and time/interval variables.

#### 4.2. Task/interval overlap

For each couple (task  $l$ , interval  $n$ ), there are six possible configurations (Fig. 2) that correspond to the relative position of the task and the interval. In order to get the overlap  $o_{l,n}$ , the type of configuration should be determined. Nolde and Morari (2010) use one binary variable for each configuration and ensure that only one configuration is chosen. Then they determine the appropriate overlap value. The number of binary variables is thus  $6 \times |\mathcal{N}| \times |\mathcal{L}| = 6 \times |\mathcal{N}| \times \sum_{k \in \mathcal{K}} |\mathcal{L}_k|$ . Moreover, the selection of the right configuration induces many big-M constraints. Using the time/interval variables, less binary variables are needed and also less big-M constraints.

For a given task  $l$ , scheduled between  $t_l^s$  and  $t_l^e$ , the task/interval overlaps  $o_{l,n}$  will only be greater than zero for the intervals where  $\gamma_{l,n}^s - \gamma_{l,n-1}^e = 1$  (Fig. 1). In the same way, all the configurations can be described with time/interval variables. The following constraints give the overlaps between tasks and intervals according to these variables (for all task  $l \in \mathcal{L}$  and interval  $n \in \mathcal{N}$ , in case of regular interval of duration  $D$ ):

$$0 \leq o_{l,n} \leq D(\gamma_{l,n}^s - \gamma_{l,n-1}^e) \quad (12)$$

$$o_{l,n} \geq D(\gamma_{l,n-1}^s - \gamma_{l,n}^e) \quad (13)$$

$$o_{l,n} \geq t_l^e - D.n + D.\gamma_{l,n-1}^s - \mathcal{T}(1 - \gamma_{l,n}^e) \quad (14)$$

$$o_{l,n} \geq D.n(1 - \gamma_{l,n-1}^s) - t_l^s - D\gamma_{l,n}^e \quad (15)$$

$$\sum_n o_{l,n} = t_l^e - t_l^s \quad (16)$$

Equations (13)-(15) respectively match configurations (d), (e) and (f) while the other configurations and the global consistency are given by (12) and (16). Given that there is no interruption between the successive tasks of a same batch, i.e.  $t_l^e = t_{l+1}^s$ , the number of binary

variables can be reduced:  $\gamma_{l,n}^e = \gamma_{l+1,n}^s$ . We just need to use variables  $\gamma_{l,n}^s$  for the starting dates and add one binary variable for the end of the last task of each job. Consequently, the number of binary variables is:  $(|\mathcal{N}| - 1) \times \sum_{k \in \mathcal{K}} (|\mathcal{L}_k| + 1)$ .

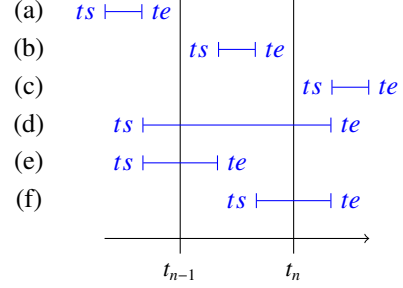


Figure 2: Overlap configurations of a task and an interval.

#### 4.3. Energy consumption by interval

The mean energy consumption of a task over an interval is given by the following constraint:

$$w_{l,n} = p_i.o_{l,n} \quad \forall i \in \mathcal{I}, l \in \mathcal{L}_i \quad (17)$$

The objective function is the sum of all tracking errors:

$$\min \sum_{n \in \mathcal{N}} \left| q_n - \sum_{l \in \mathcal{L}} w_{l,n} \right| \quad (18)$$

where  $q_n$  is the target energy consumption for interval  $n$ . The energy cost is neglected in the objective function as the difference is low between two admissible solutions whereas the penalties on tracking errors are very high.

## 5. Results

The example from Nolde and Morari (2010) consists in scheduling 15 identical batches in order to track a given energy load curve on an horizon  $\mathcal{T} = 1440$  min divided into 96 intervals. Table 1 gives the power consumption of the equipments. Table 2 gives the sequence of tasks and their processing time min/max intervals.

The contracted load curve, resulting from a negotiation with the energy provider, is shown in Figure 3 (top). The model is written in OPL and CPLEX 12.2 is used to solve the MILP. Tests have been performed on an Intel 2.66 GHz CPU. The new proposed formulation is solved to optimality in less than 4 minutes. Figure 3 (bottom) presents the deviation from the target load curve. Table 3 compares the performance of the new formulation with the one of Nolde and Morari (2010) obtained on a 3 GHz CPU.

Table 1: Power consumption of the equipments

Unit $i$	Power consumption $p_i$ [energy unit / min]
Electric arc furnace (EAF)	1000
Crane (C)	10
Argon oxygen decarburization (AOD)	80
Ladle furnace (LF)	150
Continuous casting machine (CCM)	50

Table 2: Tasks for the production of one steel batch

Task no.	Unit	Duration [min]	
		Min	Max
1	EAF	110	110
2	C	5	10
3	AOD	76	150
4	C	5	10
5	LF	18	24
6	C	5	10
7	CCM	48	86

The main advantage of our formulation, focused on the representation of the relative position of tasks and intervals, is to significantly reduce the number of binary variables and the number of constraints (especially big-M constraints). This leads to a more efficient resolution of the MILP model. One can also note that the binary variables are linked by constraints like (8) and other cuts that improve the computation time.

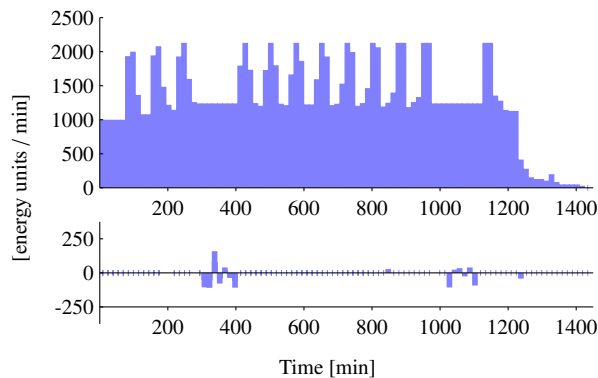


Figure 3: Load curves: contracted load (top) and optimal deviation (bottom).

## 6. Conclusion and future work

In this paper we propose a new continuous-time MILP formulation for the steel scheduling problem with energy constraints. This formulation turns out to be very efficient on the case study from Nolde and Morari

Table 3: Performance comparison

	Nolde & Morari	Haït & Artigues
Constraints	233701	100465
Continuous variables	10387	10387
Binary variables	60480	13890
CPU	35 days	4 min.
Objective	1160	1037 (opt.)

(2010). However, the scheduling part is quite simple in this case study. It would be interesting to test the same model on more difficult problems like the steel scheduling problem from Harjunkoski and Grossmann (2001). Future work will focus on decomposition approaches to solve such problems. A first application is given in Haït and Artigues (2011) that present a two-level CP/MILP approach to solve parallel machine scheduling with energy costs. Additionally, this formulation may be used as a part of more complex models, for example RTN-based models like the one proposed by Castro et al. (2009) for process scheduling under variable electricity cost.

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