Macroeconomic Impact of Population Aging in Japan: A Perspective from an Overlapping Generations Model

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Abstract

Due to a sharp decline in the fertility rate and a rapid increase in longevity, Japan's population aging is the furthest advanced in the world. In this study we explore the macroeconomic impact of population aging using a full-fledged overlapping generations model. Our model replicates well the time paths of Japan's macroeconomic variables from the 1980s to the 2000s and yields future paths for these variables over a long horizon. We find that Japan's population aging as a whole adversely affects GNP growth by dampening factor inputs. It also negatively impacts on GNP per capita, especially in the future, mainly due to the decline in the fraction of the population of working-age. For these findings, fertility rate decline plays a dominant role as it reduces both labor force and saver populations. The effects of increased longevity are expansionary, but relatively minor. Our simulations predict that the adverse effects will expand during the next few decades. In addition to closed economy simulations, we examine the consequences of population aging in a small open economy setting. In this case a decline in the domestic capital return encourages investment in foreign capital, mitigating the adverse effects of population aging on GNP.

JEL Classification: E20, J11

Keywords: Population Aging; Overlapping Generations Model; Capital Flow.

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1 Introduction

In recent years, much attention has been paid to the macroeconomic consequences of population aging across the world. Table 1 displays the dependency ratios for G7 countries as well as developing countries, based on projections reported by the United Nations.[1] The dependency ratio exhibits a clear upward trend in all countries. In particular, Japan is ahead of other countries in terms of the pace of its population aging. In 1990 Japan's dependency ratio was 17%, which was the lowest among G7 countries. However, it went up to 35% in 2010, the highest in the group, and is expected to increase to 53% in 2030. The rise in the dependency ratio stems from two factors: (i) a decline in the fertility rate and (ii) an increase in longevity. Table 2 shows that Japan's population aging has been most rapid due to its lower fertility rate and greater longevity, respectively the lowest and highest among listed countries in recent years. Japan can be seen as an illustrative example for population aging.

There is vast literature exploring the link between demographic composition and economic activity. One pioneering work within a general equilibrium framework is Auerbach and Kotlikoff (1987), who build an Overlapping Generations (hereafter OG) model calibrated to the U.S. economy and evaluate the impact of demographic transitions on economic activity. They demonstrate that the OG model is considered the workhorse model for analyzing the economic consequences of demographic transitions and the associated fiscal policy. Miles (1999) also utilizes an OG model to explore demographic impact, focusing on the U.K. and European countries.[2] In recent years Japan's population aging has been investigated based on an OG model in two strands of literature.[3] The one strand focuses on movements in the saving rate in Japan. The Japanese saving rate has displayed

[1] Here the dependency ratio is defined as the 'old-age' dependency ratio, the number of individuals aged above 64 divided by the number of individuals aged 15 to 64.

[2] Within a growth accounting framework, for instance, Maddaloni et al. (2006) analyze the effects of population aging on economic growth, financial markets and public finance in the Euro area, considering the fertility rate, longevity, and immigration.

[3] As more recent work, Ikeda and Saito (2012) analyze, based on a dynamic stochastic general equilibrium model, the implications of demographic changes for real interest rate dynamics in post-war Japan. From a different perspective, Fujiki et al. (2012) conduct an empirical investigation of how population aging affects households' asset portfolio allocations, particularly between stocks and other assets, in Japan.

a downward trend since the 1990s, while it was much higher than the U.S. saving rate in the past.[4] Chen et al. (2007) and Braun et al. (2009) demonstrate that this can be partially accounted for by population aging. Other things being equal, since a lower saving rate results in less capital accumulation, the saving rate decline associated with population aging may cause output decline. The other strand, for example, Ihori et al. (2005, 2011), focuses on the effects of population aging on the fiscal balance including public pension and health insurance systems, as well as on debt sustainability.[5] If deficits in these systems are to some extent financed by the general government through distortional taxes, then population aging may severely dampen private economic activity.

In this study, we investigate quantitatively the overall impact of population aging on current and future economic growth. Our study provides a clear picture of economic outlooks in aging countries using a full-fledged OG model calibrated to the Japanese economy where population aging is proceeding at the fastest pace in the world. Our analysis incorporates two new ingredients into otherwise standard OG models. First, we explicitly introduce public medical benefits provided by a health insurance system. Since the current health insurance system in Japan is partly organized in a pay-as-you-go style similar to the public pension system, an increase in public medical benefits due to population aging raises the payroll income tax rate, distorting households' labor supply decisions. Most existing studies do not explicitly incorporate this aspect, and thus potentially underestimate the impact of population aging on economic growth and the fiscal balance.[6] Second, we adopt the 'bond-in-utility' specification developed in Hansen and İmrohoroğlu (2012), when incorporating government bonds into the model. In Japan, the return on government bonds has been lower than the return on private capital, and the spread has widened since the

[4] Horioka (1997), Horioka and Watanabe (1997), Dekle (2000), and Koga (2005) are examples of sophisticated empirical work providing evidence for the Japanese saving rate decline.

[5] Dekle (2003) and Kozu et al. (2003) offer a broad discussion. İmrohoroğlu and Sudo (2011a, b) and Braun and Joines (2012) utilize neoclassical growth and OG frameworks to address these issues from various angles.

[6] Most studies on the Japanese economy, including Cheng et al. (2007) and Braun et al. (2009), abstract from the public health insurance system. A few exceptions, for example, Ihori et al. (2005, 2011) and Oguro and Shimasawa (2011, Chapter4) introduce Japanese public health insurance in an OG model. Their models, however, assume an exogenous labor supply and are therefore immune from the distortionary effects of population aging on economic growth through the social security tax.

middle of the 1990s in spite of the massive increase in public debt. This fact reflects a strong preference for holding government bonds in the domestic private sector. Most existing studies assume that households are indifferent between government bonds and private capital. However, this assumption fails to capture the actual movements in the return on government bonds and hence in the corresponding interest repayments. In long-term projections especially, this assumption generates a tendency to overestimate government debt levels and the corresponding national burden.[7]

Assuming perfect foresight in the model, we compute the equilibrium transition paths of allocations and factor prices from 1982 onward. Our model successfully replicates the actual time paths of GNP and other main macroeconomic variables, such as the national saving rate and the real interest rate up until 2010. In addition, we make what we call an out-of-sample baseline projection from 2011 onward based on a scenario where future paths for exogenous variables are plausibly chosen. To evaluate quantitatively the impact of population aging on the economy, we simulate the model under hypothetical scenarios in which population aging is arrested in terms of first the fertility rate, and then longevity.[8] We compare the equilibrium paths generated under the alternative scenarios with those obtained under the baseline scenario. In order to clarify the mechanism through which population aging affects economic growth, we divide the effects of population aging into fertility-driven effects and longevity-driven effects. We evaluate not only the effects on GNP growth, but also the effects on GNP per capita in order to isolate the mechanical impact of lower population growth. In addition to simulations based on a closed economy model, we explore the macroeconomic impact of population aging in an open economy setting with complete capital mobility. To this end, we extend our baseline model to a small open economy model to compute the responses of key macroeconomic variables to a decline in the fertility rate and an increase in longevity.

[7]In carrying out our long-term projections where the government debt to GNP ratio is endogenously determined, we find that this assumption impedes the convergence of the computed equilibrium transition path. This suggests that introducing the spread between private capital and government bonds is crucial for evaluating the sustainability of government debt.

[8]Since the youngest households in our model are 21 years old, we define the 'fertility rate' in the model as the growth rate of the age-21 population. This definition is different from common definitions of the 'total fertility rate', expressing the average number of children per woman during her lifetime.

The remainder of this paper is organized as follows. Section 2 describes the setup of our model. Section 3 explains the data sources and our calibration methodology. Section 4 provides our quantitative findings on the macroeconomic impact of Japan's population aging. Section 5 extends the model into an open economy setting. Section 6 concludes.

2 Model

In this section, we present our OG model. The economy consists of four types of agents: households, firm, social security system, and general government. We incorporate two new features into an otherwise standard OG model: (i) a public medical benefit provided by the health insurance system and (ii) a 'bond-in-utility' specification as developed in Hansen and İmrohoroğlu (2012).

2.1 Demographics

The time period of the model is discrete and annual. The economy consists of 80 generations of different ages denoted by $j = 21, ..., 100$. In each period t, a new generation aged 21 is born into the economy, while the other existing generations each shift forward by one. The oldest generation, $j = 100$, which we assume as the maximum age, dies out deterministically in the subsequent period. The growth rate of the new generation (age-21 households) in period t is denoted by ρ_t, which we will hereafter refer to as the fertility rate in the model. Then the age-21 population in period t, expressed as $P_{21,t}$, is given by

$$P_{21,t} = (1 + \rho_t)\, P_{21,t-1}. \tag{1}$$

All households face a mortality risk that is common within the same cohort but may differ across cohorts. They survive to the subsequent period with conditional survival probability $\psi_{j,t}$, which is the probability that households aged $j-1$ in period $t-1$ survive to become age j households in period t. Note that $\psi_{101,t} = 0$ by assumption. Then the population,

$P_{j,t}$, and cohort share, $\mu_{j,t}$, of age j households in period t are expressed as follows:

$$P_{j,t} = \psi_{j,t} P_{j-1,t-1}, \tag{2}$$

$$\mu_{j,t} = \frac{P_{j,t}}{\sum_{i=21}^{100} P_{i,t}} \quad \text{for } j = 21, ..., 100. \tag{3}$$

2.2 Households

Until mandatory retirement at age $j = 65$, households supply labor to firms and earn wage income according to their age-specific labor efficiency from age 21 to 65.[9] Working households pay not only the usual labor income tax, but also payroll income taxes covering social security benefits, namely their public pension and medical benefits, in every period. Households aged $j > 65$ withdraw from the labor force, and receive public pension benefits from the social security system. Households own capital and rent it to firms throughout their lives. They also purchase one-period government bonds issued by the general government, and receive interest payments from the general government. They consume the rest of their income.

In period T, newly born households, whose age is j in a subsequent period $t \equiv T-21+j$, choose sequences of consumption, labor, capital and bond holdings, to maximize their expected lifetime utility (discounted by the subjective discount factor β):

$$\max \sum_{j=21}^{100} \beta^{j-21} \left[\prod_{i=21}^{j} \psi_{i,T-21+i} \right] u\left(c_{j,t}, 1-h_{j,t}, b_{j+1,t+1}\right), \tag{4}$$

subject to the budget constraints over their lifetime:

$$\begin{aligned} &(1+\tau_{c,t})\, c_{j,t} + k_{j+1,t+1} + b_{j+1,t+1} \\ &= \left[1 + (1-\tau_{k,t})\, r_t^K\right] k_{j,t} + (1+r_t^B) b_{j,t} \\ &= \left[1 + (1-\tau_{k,t})\, r_t^K\right] k_{j,t} + (1+r_t^B) b_{j,t} + pb_{j,t} + \tau_t + \xi_t \quad \text{for} \quad j > 65, \end{aligned} \tag{5}$$

[9] Currently, mandatory retirement in Japanese firms is not necessarily at age 65, but is usually set between 60 and 65. In our projections, however, there is little labor supplied by those aged 63 to 65. The results of the analysis may therefore be expected to change little even if we lower the assumed age of mandatory retirement by a few years.

$$
\begin{aligned}
&(1+\tau_{c,t})\,c_{j,t}+k_{j+1,t+1}+b_{j+1,t+1}\\
&=\left[1+(1-\tau_{k,t})\,r_t^K\right]k_{j,t}+(1+r_t^B)b_{j,t}+pb_{j,t}+\tau_t+\xi_t \quad \text{for} \quad j>65,
\end{aligned}
\tag{6}
$$

where $c_{j,t}$ is consumption and $h_{j,t}$ is labor input. $k_{j,t}$ and $b_{j,t}$ denote capital and bond holdings of age j households at the beginning of period t. w_t is the wage rate, r_t^K is the before-tax real rate of return on private capital, and r_t^B is the after-tax real rate of return on government bonds. $\tau_{c,t}$ is the consumption tax rate, $\tau_{h,t}$ and $\tau_{k,t}$ are the tax rates on income from labor and capital, and τ_t is a lump-sum transfer in period t.[10] ξ_t is a lump-sum transfer associated with accidental bequests, which are left by households who die in the preceding period $t-1$. $\tau_{s,t}$ and $\tau_{m,t}$ are the payroll income tax rates (contribution rates) for public pension and health insurance, respectively. $pb_{j,t}$ is public pension benefits that retirees aged j in period t receive, described later. ε_j is age-specific labor efficiency from age 21 to 65, which we assume time-invariant. We assume that a new household born in period t has no initial assets[11]: $k_{21,t}=b_{21,t}=0$. In addition, no household surviving to the maximum age 100 carries over any assets to the next period: $k_{101,t}=b_{101,t}=0$, so there is no bequest motive. We also assume that the government collects all accidental bequests including capital income in period $t-1$, and redistributes them equally among all households alive in period t. The total amount of accidental bequests in period t is given by

$$
\xi_t=\sum_{j=22}^{101}(1-\psi_{j,t})\left[\left\{1+(1-\tau_{k,t-1})\,r_{t-1}^K\right\}k_{j-1,t-1}+(1+r_{t-1}^B)b_{j-1,t-1}\right]P_{j-1,t-1}. \tag{7}
$$

Following Hansen and İmrohoroğlu (2012), we introduce government bond holdings into the utility function. The basic idea behind this 'bond-in-utility' specification is to incorporate households' preference for the liquidity and safety characteristic of government bonds.[12] The functional form of households' utility is assumed to be separable in terms of

[10] This corresponds conceptually to what is categorized as the sum of the net current transfers and the capital transfers in the Japanese National Income Account.

[11] The borrowing constraint is not imposed on households' assets in our model. In other words, households are allowed to borrow against their future income.

[12] As an alternative specification, some studies, such as Arai and Ueda (2012), introduce an exogenous

its arguments, which are given as

$$u\left(c_{j,t}, 1-h_{j,t}, b_{j+1,t+1}\right) = \log c_{j,t} + \gamma_t \log\left(1-h_{j,t}\right) + \eta_t \log b_{j+1,t+1} \quad \text{for } j \leq 65, \tag{8}$$

$$u\left(c_{j,t}, 1-h_{j,t}, b_{j+1,t+1}\right) = \log c_{j,t} + \eta_t \log b_{j+1,t+1} \quad \text{for } 65 < j < 100, \tag{9}$$

$$u\left(c_{j,t}, 1-h_{j,t}, b_{j+1,t+1}\right) = \log c_{j,t} \quad \text{for } j = 100, \tag{10}$$

where γ_t and η_t are time-variant variables representing households' preferences for leisure and government bond holding in period t. Higher γ_t or η_t implies that households put a higher value on leisure or government bond holding. When households receive utility from government bond holding ($\eta_t > 0$), the first order conditions of age j households in period t with respect to government bond holdings yield the following equation:

$$(1-\tau_{k,t})r_t^K - r_t^B = \frac{\eta_{t-1}}{\beta}(1+\tau_{c,t}) \sum_{j=21}^{100-1} \mu_{j,t} \frac{c_{j,t}}{b_{j+1,t+1}}. \tag{11}$$

This equation demonstrates that, other things being equal, the spread between private capital and government bonds increases with the preference parameter η_t and decreases with the average ratio of government bonds to consumption. A higher preference for government bonds (higher η_t) leads to a wider spread and smaller interest repayments by the general government. A higher government debt level relative to consumption level lowers the marginal utility from government bond holding and thereby narrows the spread.

2.3 Firm

There is a representative firm producing final goods with the Cobb-Douglas production technology. In perfectly competitive spot-markets the firm rents capital and hires labor from households so as to maximize its profit:

$$\max \Pi_t = A_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - w_t L_t, \tag{12}$$

spread between private capital and government bonds. An important feature of our 'bond-in-utility' model is that the model's spread is endogenous and its size is negatively related to the outstanding amount of government debt.

where α is the capital share of output, K_t is aggregate capital stock, L_t is aggregate labor input, and R_t is the rental rate of private capital. A_t denotes the total factor productivity (hereafter TFP) in period t, and we assume that the TFP grows at the rate of g_t in every period:

$$g_t \equiv (A_t/A_{t-1})^{1/(1-\alpha)}. \tag{13}$$

In equilibrium, the factor prices are given by

$$R_t = \alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} \equiv r_t^K + \delta_t; \tag{14}$$

$$w_t = (1-\alpha) A_t \left(\frac{K_t}{L_t}\right)^{\alpha}, \tag{15}$$

where δ_t is the depreciation rate of private capital. Aggregate demand for capital and labor inputs are equalized to aggregate supply of these primary inputs, so as to clear the respective markets in every period:

$$K_t = \sum_{j=21}^{100} P_{j,t} k_{j,t}, \tag{16}$$

$$L_t = \sum_{j=21}^{65} P_{j,t} \varepsilon_j h_{j,t}. \tag{17}$$

Here, the evolution of the aggregate capital stock is given by

$$K_{t+1} = I_t + (1-\delta_t) K_t, \tag{18}$$

where I_t is aggregate investment in period t.

2.4 Social Security System

The social security system is divided into two sections: public pension and health insurance. We assume that the public pension benefit $pb_{j,t}$ provided to age j households in period t depends on their historical wage income. It is proportional to the average wage income that households receiving the benefit earned during their working years. The public pension benefit provided by the social security system to a new retiree in period t, $pb_{65+1,t}$, is defined as

$$pb_{65+1,t} \equiv \theta \frac{1}{65+1-21} \sum_{i=21}^{65} w_{t+i-65-1} \varepsilon_i h_{i,t+i-65-1}, \tag{19}$$

where θ is the replacement ratio that determines the size of the public pension benefit relative to the past wages. The public pension benefit that age j households in period t receive is formulated as

$$\begin{aligned} pb_{j,t} &= 0 && \text{for } j = 21, 22, ..., 65, \\ pb_{j,t} &= pb_{65+1,t+65+1-j} && \text{for } j = 65+1, ..., 100. \end{aligned} \tag{20}$$

The per-capita medical costs, denoted by mb_j, are assumed to be different across age j and time-invariant.[13] The age-specific profile of the medical costs is exogenously given and grows along a balanced growth path. We define the medical benefit $mb_{j,t}$ for age j households in period t as follows:

$$mb_{j,t} = (1+g_t) mb_{j,t-1} \quad \text{for } t = 2, 3, ..., \qquad mb_{j,t} = mb_j \quad \text{for } t = 1. \tag{21}$$

For both public pension benefits and medical benefits, part of the costs is covered by the general government, and the rest is covered by the relevant section of the social security system. Taking as given the coverage ratios of transfers/expenditures financed by the general government, which we denote as $\kappa_{s,t}$ and $\kappa_{m,t}$, the social security system adjusts the contribution rates for public pension and health insurance, $\tau_{s,t}$ and $\tau_{m,t}$, so that budget balance is separately maintained for both the public pension and health insurance in every period:

$$\tau_{s,t} = \frac{(1-\kappa_{s,t}) \sum_{j=65+1}^{100} P_{j,t} pb_{j,t}}{w_t \sum_{j=21}^{65} P_{j,t} \varepsilon_j h_{j,t}}, \tag{22}$$

$$\tau_{m,t} = \frac{(1-\kappa_{m,t}) \sum_{j=21}^{100} P_{j,t} mb_{j,t}}{w_t \sum_{j=21}^{65} P_{j,t} \varepsilon_j h_{j,t}}. \tag{23}$$

[13] The main reason is due to data availability. The profile could reasonably be considered time-variant because of fast progress in medical technology.

2.5 General Government

The general government raises revenues by newly issuing one-period government bonds and levying taxes on households' consumption, labor income, and capital income, to finance its spending that is the sum of government purchases, transfers/expenditures to the social security system, interest repayments on government bonds, and other lump-sum transfers. Taking the sequences of government revenues and spending as given, government bond issuance is adjusted so that the following consolidated budget constraint holds in every period:

$$
\begin{aligned}
&(1+r_t^B)B_t + G_t + \kappa_{m,t}\sum_{j=21}^{100} P_{j,t}mb_{j,t} + \kappa_{s,t}\sum_{j=65+1}^{100} P_{j,t}pb_{j,t} + \sum_{j=21}^{100} P_{j,t}\tau_t + \sum_{j=21}^{100} P_{j,t}\xi_t \\
&= B_{t+1} + \tau_{c,t}C_t + \tau_{h,t}(1-\tau_{s,t}-\tau_{m,t})w_t L_t + \tau_{k,t}r_t^K K_t, \qquad (24)
\end{aligned}
$$

where G_t and B_t are government purchases and government bonds at the beginning of period t, respectively. Note that the supply of government bonds is equalized to the sum of households' bond holdings in each period:

$$
B_t = \sum_{j=21}^{100} P_{j,t}b_{j,t}. \qquad (25)
$$

2.6 Competitive Equilibrium

Given the initial distribution of private capital stock and government bonds $\{k_{j+1,0}, b_{j+1,0}\}_{j=21}^{100}$, the paths of the fiscal policy variables $\{G_t, \tau_t, \tau_{c,t}, \tau_{k,t}, \tau_{h,t}, \tau_{s,t}, \tau_{m,t}, \kappa_{s,t}, \kappa_{m,t}\}_{t=0}^{\infty}$, demographics $\{\rho_t, \{\psi_{j,t}\}_{j=21}^{100}\}_{t=0}^{\infty}$, TFP growth rate $\{g_t\}_{t=0}^{\infty}$, depreciation rate $\{\delta_t\}_{t=0}^{\infty}$, and preference disturbances $\{\gamma_t, \eta_t\}_{t=0}^{\infty}$, a competitive equilibrium consists of sequences of prices $\{w_t, r_t^K, r_t^B\}_{t=0}^{\infty}$, allocations $\{C_t, L_t, K_t, B_t\}_{t=0}^{\infty}$, and households' decision rules $\{\{c_{j,t}, h_{j,t}, k_{j+1,t+1}, b_{j+1,t+1}\}_{j=21}^{100}\}_{t=0}^{\infty}$ such that, in each period, (i) households maximize their lifetime utility (4), subject to (5) and (6), given prices; (ii) the firm maximizes its profits, (12), given prices; (iii) the budget constraints of the social security system and the general government, (22), (23), and (24), hold; (iv) the market clearing conditions hold for the capital input, labor input, and government bonds, (16), (17), and (25); (v) the resource constraint holds:

$$Y_t \equiv A_t K_t^{\alpha} L_t^{1-\alpha} = C_t + I_t + G_t + \sum_{j=21}^{100} P_{j,t} m b_{j,t}, \tag{26}$$

where Y_t is aggregate output and C_t is aggregate consumption:

$$C_t = \sum_{j=21}^{100} P_{j,t} c_{j,t}. \tag{27}$$

3 Data, Calibration, and Assumptions

We take 1982 as the starting point for our simulations, because this is the first year for which national account series with a consistent set of definitions are available. The last period for which we have data on all variables is 2010. The model thus employs observed values of its exogenous inputs for the 1982-2010 period, and assumed values of these inputs for 2011 and beyond. We assume that the economy will reach its steady state far in the future, and carry out an iterative computation to calculate an equilibrium transition path that connects the economy of 1982 and the long-run steady state. In this section, we first discuss, in detail, the calibration of the model's structural parameters, as well as the definition and construction of the exogenous inputs used in both in-sample simulation and projection. We then explain the assumptions about the fiscal regime in the long-run that guarantees the household and general government transversality conditions, as well as the assumptions governing the long-run steady state values.

3.1 Constant Parameters

The calibrated constant parameters are given in Table 3. The three parameters α, β, and θ are constant throughout our analysis. Following closely the data construction methodology proposed in Hayashi and Prescott (2002) and İmrohoroğlu and Sudo (2011a, b), we use the sample average for the income share of capital in GNP α from 1982 to 2010. We choose the value of the subjective discount factor β so that our benchmark model replicates well the time path of the capital-output ratio for the period from 1982 to 2010. The replacement ratio θ for the public pension is set to 0.3 so that the model can reproduce the historical average of the actual pension benefits to GNP ratio from 1982 to 2010.

In addition, we assume that the age-specific labor efficiency ε_j and age-specific medical costs mb_j are constant over time. The labor efficiency values are taken from Braun et al. (2009), while the values for individual medical costs are taken from the 2009 Report of National Medical Expenditure (NME) by the Ministry of Health, Labour and Welfare.

3.2 Inputs of Exogenous Variables for 1982-2010 and Beyond

In order to conduct perfect foresight projections from 1982 and beyond, we need to specify parameter values characterizing the demographic structure; these include the growth rate of the age-21 population (i.e., 'fertility rate' in our model) $\{\rho_t\}_{t=1982}^{\infty}$, conditional survival probability $\{\{\psi_{j,t}\}_{j=21}^{100}\}_{t=1982}^{\infty}$, and sequence of exogenous macroeconomic variables $\{G_t/Y_t$, τ_t/Y_t, $\tau_{c,t}$, $\tau_{h,t}$, $\tau_{k,t}$, $\kappa_{s,t}$, $\kappa_{m,t}$, δ_t, g_t, γ_t, $\eta_t\}_{t=1982}^{\infty}$. The detailed definition and construction methodologies for these variables are given in Appendix A.

Figures 1 and 2 display, respectively, the age-specific life-cycle profiles that characterize households, and the time series of exogenous variables fed into the baseline model. As in related studies on other countries, the labor efficiency exhibits a hump-shape peaking around age 55, the medical costs monotonically increase with age, and the conditional survival probability decreases with age and grows steadily over time. The growth rate of the age-21 population was above zero in the 1980s but has been continuously negative in the current years, around -2%.

3.3 Steady State Assumptions

The calibrated steady state values of the key variables are presented in Table 4. In order to ensure the transversality condition of the general government over a long horizon, and to maintain the convergence of the dynamic paths in the economy, we consider, following Hansen and İmrohoroğlu (2012), a class of fiscal regime switch that will take place in 2050. We assume that the general government adjusts the government bond issuance endogenously from 1982 to 2049 so as to fulfill the government budget constraint (24), taking the sequence of the other fiscal instrument variables $\{G_t/Y_t$, τ_t/Y_t, $\tau_{c,t}$, $\tau_{h,t}$, $\tau_{k,t}$,

$\kappa_{s,t},\ \kappa_{m,t}\}_{t=1982}^{\infty}$ as given. From 2050 and beyond, the new fiscal regime is implemented and the general government adjusts the size of a lump-sum tax levied on all households so as to maintain the government bond to GNP ratio at its target level. We assume that the target level of the ratio decreases linearly from its value in 2050 to 0.441 by 2100, its historical average over 1982 to 2010. Except for the growth rates of TFP and the age-21 population, the steady state values of the other variables are basically the sample averages from 1982 to 2010. The steady state value of TFP growth is 1%, which is the historical average of the growth rate of the Solow residual from 1995 to 2007.[14]

4 Quantitative Findings

In this section, we document the quantitative results of simulations. Under the baseline assumptions, we conduct perfect foresight projections where newly-born households are informed of the sequence of exogenous variables in their birth period and beyond. We first demonstrate the in-sample performance of our model by comparing the model-generated main macroeconomic variables with the data counterparts. We also provide out-of-sample projections for 2011 onward. Next, we formulate counterfactual simulations to evaluate the impact of population aging on current and future GNP per capita as well as GNP growth. Since a decline in the fertility rate and an increase in longevity are both important driving forces of Japan's population aging, we investigate these two effects separately.

4.1 Baseline Long-Term Projection

We first demonstrate the basic performance of the model by comparing the actual and model-generated series of key macroeconomic variables over the sample period 1982-2010. Figure 3 displays the time paths of the GNP growth rate, GNP per capita (defined as real GNP per total population), national saving rate, real return on capital, capital-labor ratio, capital share of total assets, pension benefits to GNP ratio, medical benefits to GNP

[14] Here we drop the subsample periods that include the 'bubble boom' during the 1980s and the global financial crisis from 2008 to 2009 in constructing our benchmark future path for TFP. The average value over the whole sample period is 1.7%, which is slightly higher than our benchmark assumption. In Appendix C, we conduct a sensitivity analysis to see how our result is affected by the assumption on TFP growth.

ratio, primary balance to GNP ratio, and net government debt to GNP ratio. The thin lines depict the actual series, and the thick lines depict the model-generated series. Our model replicates the dynamics of the macroeconomic variables well. In particular, the long-lasting output growth slowdown that started with the burst of the asset bubble in 1991 is reproduced in the model series. As discussed in existing studies such as Chen et al. (2007) and Braun et al. (2009), the national saving rate and real return to capital also decrease in the most recent two decades, and the model-generated data closely tracks the corresponding actual time series. The general patterns of revenues and expenditures for both the social security system and the general government are also captured by the present model. In both actual data series and model-generated series, persistent increases in the pension benefit and medical benefit, deterioration in the primary balance, and accelerated government debt accumulation are observed.

Figure 4 shows out-of-sample projections for the macroeconomic variables from 2011 onwards under the baseline scenario. The Japanese economy experiences zero GNP growth in the next decade, as 1% TFP growth is almost exactly offset by declines in labor and capital inputs. From the late 2020s and beyond, however, GNP is seen to fall gradually, since the factor inputs decline at a faster rate than before. As indicated by an increase in average working hours in these periods, the reduction in the labor input is driven by a decrease in the labor force population. A decline in the capital input is induced by changes in households' saving behavior as well as an increased proportion of dis-savers. Although workers raise their saving rate, the rise is dominated by the decline in the saving rate of retirees. Redistribution by the general government will rise over a long horizon, as shown in the expansion of public pension and medical benefits. Although the level of GNP declines in the long-run, GNP per capita maintains a gradual increase, reflecting the increase in working hours and 1% TFP growth. Behind these movements, the after-tax rate of return on private capital stays stable mainly because 1% TFP growth compensates for the increase in the capital-labor ratio induced by population aging. On the other hand, the real return on government bonds rises gradually over time because of deterioration in the fiscal balance caused by population aging. Consequently, the spread between private capital and government bonds shrinks through the forecast horizon. Of course, this movement in the spread depends strongly on the assumed future paths of households' preference for government bond holding.[15]

4.2 Counterfactual Simulation

Next we examine how population aging affects economic activity in Japan. To this end, we conduct counterfactual simulations in which population aging is arrested. We investigate separately the two effects behind population aging in Japan, namely a decline in the fertility rate and an increase in longevity.

Decline in Fertility Rate

In Figure 5, we display the time paths of macroeconomic variables in the economy where the population aged 21 is held constant from 1995 and beyond.[16] This hypothetical fertility rate sequence is substantially higher than the actual counterpart, which has been almost continuously negative since 1995. The discrepancy between the baseline scenario, denoted by the solid lines, and this counterfactual scenario, denoted by the dotted lines, displays the quantitative contribution made by the decline in the fertility rate after 1995. On the one hand, reduced fertility dampens GNP growth, the growth rates of the two factor inputs, the national saving rate, and the capital return. On the other hand, it raises the pension/medical benefits to GNP ratio. Its qualitative effects on workers' saving behavior and leisure choice change over the simulation period.

A fertility rate decline contributes to the slowdown of GNP growth through three distinct channels. First, it mechanically reduces the labor force population. Second, by raising the proportion of retirees in the economy, it aggravates the social security burden on labor income. As a result, the contribution rates, $\tau_{s,t}$ and $\tau_{m,t}$, are raised so as to satisfy the budget balance requirements of the social security system (22) and (23), thus distorting workers' labor supply and saving decisions. Third, marginal propensities to

[15] In Appendix C, we conduct a sensitivity analysis to see how our result is affected by the assumption on households' preference for government bond holding.

[16] This means that $\{\rho_t\}_{t=1995}^{\infty} = 0$.

consume are higher among retirees than workers. On average, workers are savers and retirees are dis-savers under both scenarios. The decline in the fertility rate, therefore, increases the proportion of dis-savers among households, reduces the national saving rate, and hampers capital accumulation. The first and third channels drive GNP growth down primarily through reductions in the population of the labor force and savers. In contrast, the second channel influences GNP growth by affecting average working hours and the workers' saving rate.

In the baseline scenario, the real return on capital is lowered as the labor input drops more quickly than the capital input because of the fertility rate decline. Meanwhile, the after-tax return on labor (the effective wage) is reduced more as a result of higher contribution rates. Average working hours and workers' capital saving rate are higher in the baseline scenario than in the counterfactual scenario. In the baseline scenario working-age households work more and save more because their lifetime income decreases substantially due to the lower factor prices (effective wage and return on capital) caused by the fertility rate decline. As Figure 6 shows, GNP per worker is thus higher in the baseline scenario. GNP per capita, however, is lower in the baseline scenario because the increase in GNP per worker is more than cancelled out by the decline in the proportion of the working-age population. This relative decrease in the working-age population is the dominant factor depressing GNP per capita in the next few decades. It is noteworthy that these adverse effects on GNP per capita will expand sharply through the forecast horizon. Population aging also affects the fiscal balance. Figure 7 displays the impact of a fertility rate decline on the primary balance and net government debt level. The fertility rate decline deteriorates the primary balance because it lowers tax revenues of the general government. As a result, government debt accumulates more quickly over time.

Increase in Longevity

In Figure 8, we display the time paths of macroeconomic variables in the economy where the household survival probability, instead of increasing, is held constant from 1982 onward.[17] As shown in the upper panel of Figure 1, the conditional survival probability shifts upward over time, with the increased survival probability of retirees being particularly remarkable. The higher longevity impact positively on variables such as the growth rates of the two factor inputs, GNP growth, and GNP per capita, but not on the real rate of return on capital or government bonds, nor on the retirees' saving rate.

Two reasons can be adduced to explain why increased longevity results in economic expansion. The one reason is the absence of the predominant contractionary channel above (the first of the three channels mentioned), which does not operate here since the labor force population is almost unaffected by a change in longevity. The other is the operation of a further (the fourth) channel, through which increased longevity influences households' precautionary motive. As pointed out in previous studies, such as Chen et al. (2007) and Braun et al. (2009), households expecting to survive longer have an added incentive to work and save so as to insure themselves against a longer life after retirement. Consequently, average working hours and the capital saving rate of workers are both higher in the baseline scenario. Admittedly, the second and third channels discussed above also operate when longevity increases. As with the fertility rate decline, these channels dampen the hours worked by younger generations and reduce the proportion of savers in the economy. Because the increases in average working hours and saving rate stemming from the fourth channel dominate the adverse effects stemming from the second and third channels, GNP per capita is higher and GNP grows at a slightly faster rate under the baseline scenario. However, Figure 9 shows that the expansionary effects on GNP per capita will recede gradually through the forecast horizon as the proportion of workers to total population declines. Figure 10 indicates that increased longevity deteriorates the primary balance because it increases social security benefits. In the fourth panel of Figure 7, however, increased longevity leads to a sizable fall in the real interest rate because households save more in response to a longer life span. Consequently, interest repayments by the general government become much smaller compared with the case of a fertility rate decline, and the impact on government debt becomes moderate.

[17] This means that $\{\psi_{j,t}\}_{j=21}^{100} = \{\psi_{j,1982}\}_{j=21}^{100}$ for the entire simulation period.

According to the results in this section, we find that the expansionary effects of increased longevity on GNP growth are relatively small in the next few decades, while the negative effects of a decline in the fertility rate are large. This means that, although Japan has witnessed a significant increase in longevity, the impact of population aging as a whole adversely affects GNP per capita as well as GNP growth in the future. However, this might not be the case in other countries. As indicated in Table 2, some countries, such as Canada, France, the U.K., and the U.S., have not seen declines in their fertility rates but have faced increased longevity. In these countries, it is possible that expansionary effects may outweigh adverse effects. The simulations presented in this section suggest that the relative size of the fertility rate decline and the longevity increase is a crucial determinant of the macroeconomic impact of population aging.

5 Extension to an Open Economy Setting

In the sections above, we discussed the macroeconomic impact of population aging in a closed economy. We assumed implicitly that households had no access to foreign asset markets. In this section, we relax this assumption, extending the current model to a small open economy setting. We modify the model structure by introducing foreign capital with an exogenously-determined rate of return. The amounts of domestic and foreign capital held by Japanese households are determined at the equilibrium where the rates of return from the two types of capital are equalized. Notice that, since domestic saving and domestic investment are not equalized in an open economy model, there occur capital inflows (outflows) from (to) abroad, namely variation in the current account.

In Figure 11, the dotted lines show the macroeconomic consequences when the fertility rate drops permanently by one percentage point from its steady state level.[18,19] For comparison, we depict by the solid line the consequences of the same demographic change in our

[18] Here, we assume that the economy is initially at the steady state, which corresponds to the terminal steady state described in the previous sections.

[19] Most demographic changes are anticipated by households through various future population projections made by government or private institutions and they rarely materialize as shocks. To capture this effect, in this section, we assume that the demographic change is anticipated by households ten years before it materializes in period $t = 1$.

baseline closed economy. As we discuss above, in a closed economy, the fertility rate decline induces a rise in the capital-labor ratio for a while, thus decreasing the rate of return on capital. In a small open economy, this makes foreign capital more attractive than domestic capital, and households accumulate foreign asset holdings, reducing domestic investment. As a result, the trade balance improves in the short run. However, it deteriorates in the long run as capital outflows to overseas lead to decreasing domestic production compared with GNP. Meanwhile, the income balance monotonically increases as households receive positive income inflows from overseas, as is currently the case for Japanese households. The positive effects on the income account dominate the negative effects on the trade balance in the long-run as well, and therefore the current account maintains improved. Such positive income inflows from overseas mitigate the adverse effects of a fertility rate decline on domestic production, and GNP per capita in the open economy setting is larger than in the closed economy.[20]

Lastly, we demonstrate the consequences of increased longevity in an open economy framework in Figure 12. We consider a scenario where the conditional survival rates of households aged 65 and older rise permanently by 0.5 percentage points. As in the discussion above, higher longevity encourages household saving as insurance against longer lives. In contrast to the closed economy case, however, household saving is also invested in foreign capital, resulting in larger foreign asset holdings. Through the same mechanism illustrated in Figure 11, the domestic capital stock falls faster and GNP per capita is larger in the open economy than in the closed economy.

The above exercise indicates that population aging has important implications for current account dynamics. Specifically, a decline in the fertility rate, an increase in longevity, or a combination of both can act to foster investment in foreign capital by lowering the return on domestic capital.[21] In Figure 13, we chart the evolution of the current account

[20] Admittedly, the assumed rate of return on foreign capital is critical in determining the extent both of capital outflows and income inflows. For example, in an alternative case where the rate of return on foreign capital declines gradually, as it would if population aging were also taking place abroad, foreign capital also becomes less attractive than before. As long as the foreign return is higher than the domestic one, income flows remain positive; but they will be smaller than those shown in Figure 9 and the decline in GNP due to population aging will be correspondingly less mitigated.

[21] Our result is consistent with the findings of Ferrero (2010), who evaluates the demographic effects on

together with movements in population growth and average life span for the U.S., Japan, and the average of advanced countries (AAC).[22] The discrepancy between the current accounts of the two countries was minimal around 1980, clearly widening subsequently. That is, during the last two decades, Japan has maintained a current account surplus while the U.S. has persistently experienced current account deficits. As far as the two countries are concerned, therefore, Japanese households invest in foreign capital and U.S. households receive capital investment from overseas.

Turning our attention to demographic transitions, from the 1960s to the 1980s population growth in the U.S., AAC, and Japan is seen to have evolved along similar trajectories. This suggests that demographic change in this period would have had minimal effects on the current accounts of either the U.S. or Japan. Since the 1980s, however, the population in the U.S. has grown quicker than in AAC, while in Japan it has grown more slowly. Meanwhile, average life span in Japan has been longer than in the U.S. and AAC, where it has evolved roughly in parallel. According to the simulations conducted above, these demographic changes would have been expected to reduce the return on capital investment in Japan relative to AAC, leading to capital flow from Japan to AAC and a current account surplus in Japan.[23] An equivalent but opposite mechanism would have been expected to operate for the U.S. What we see, therefore, is that the observed current account dynamics and demographic changes in Japan and the U.S. during the last few decades are broadly consistent with our model's implications.

6 Conclusion

In this study, we quantitatively explore the macroeconomic impact of Japan's population aging using a full-fledged overlapping generations model. We calibrate the model to the Japanese economy over the sample period 1982-2010. Under a set of plausible assumptions

U.S. current account developments, based on the life-cycle model of Gertler (1999).

[22] See footnote to Table 1 for the definition of "more developed countries".

[23] Admittedly, our discussion here abstracts from the TFP movements considered key determinants of current account dynamics in existing studies such as Ferrero (2010) and Chen et al. (2009). In these studies, a difference in TFP growth rate between the two countries plays a dominant role in their current account dynamics as it substantially affects their respective returns to capital.

about future paths of TFP as well as demographics, we make out-of-sample projections for the next three decades. To gauge the impact of population aging on GNP growth and GNP per capita, we simulate our model under the counterfactual assumption in which population aging is arrested. Since population aging in Japan is caused by both a decline in the fertility rate and an increase in longevity, we evaluate these two effects separately.

We find that Japan's population aging as a whole adversely affects GNP growth by dampening factor inputs. It also negatively impacts on GNP per capita, especially in the future, mainly due to the decline in the fraction of the population of working-age. For these findings, a decline in the fertility rate plays a quantitatively larger role in lowering GNP growth because it reduces the labor force population and the proportion of savers in the economy. Although working households mitigate these adverse effects by increasing their working hours and saving rates, this is not enough to compensate for the effects of the shrinking working-age population. The effects of increased longevity are expansionary, but relatively small. Our simulations also predict that the negative effects of fertility rate decline will expand during the next few decades. Furthermore, in a small open economy setting, we show that when domestic households have access to the foreign asset market, they may mitigate the adverse effects of population aging on GNP by investing their savings in foreign capital.

Our results imply that the ongoing and predicted demographic transition may be expected to have long-lasting adverse effects on the Japanese economy. Our present analysis, however, does not incorporate economic and institutional changes that may occur when a society responds to population aging and that can potentially increase factor inputs; these could include a higher female labor participation rate or social security reforms such as postponing the retirement age. Extending the current model in these directions is left for our future research.

A Construction of Exogenous Variables

This appendix documents the detailed definitions and construction methodologies for the exogenous variables used in our baseline scenario simulation. The in-sample time series of the variables $\{G_t/Y_t,\ \tau_t/Y_t,\ \tau_{c,t},\ \tau_{h,t},\ \tau_{k,t},\ \kappa_{s,t},\ \kappa_{m,t}\ \rho_t,\ \{\psi_{j,t}\}_{j=21}^{100},\ \delta_t,\ g_t\}_{t=1982}^{2010}$ are computed from the actual data, and the sequences of the two parameters $\{\gamma_t,\ \eta_t\}_{t=1982}^{2010}$ are chosen so that the model-generated government bond yield and labor input capture the actual counterparts successfully as described below. We make the following assumptions about the out-of-sample time series of these variables.

- $\{G_t/Y_t, \tau_t/Y_t\}_{t=1982}^{\infty}$: We construct the "government purchases to GNP ratio," G_t/Y_t, from "gross fixed capital formation + government consumption expenditure - social transfers in kind, payable," divided by GNP, and the "transfers to GNP ratio," τ_t/Y_t, from "other current transfers (receivable) - other current transfers (payable) + capital transfers (receivable) - capital transfers (payable)," divided by GNP. For the purposes of in-sample simulation, we construct the corresponding series using the actual data. For the purposes of projection, we assume that the government purchases to GNP ratio reverts linearly over the three decades from 2011 towards its historical average and that the lump-sum transfers to GNP ratio stays constant at the steady state level from 2011 onwards.

- $\{\tau_{c,t}\}_{t=1982}^{\infty}$: The consumption tax in the model is assumed to rise from zero to 3% in 1989, and to 5% in 1997. For the forecast period, we assume that the consumption tax rate is raised from 5% to 8% in 2014 and to 10% in 2015, unchanged thereafter.

- $\{\tau_{h,t}, \tau_{k,t}\}_{t=1982}^{\infty}$: Closely following Hayashi and Prescott (2002), tax revenue from capital income is calculated as "direct tax on financials + direct tax on non-financials + .5 $\times$ indirect tax - .5 $\times$ value added taxes (VAT)." Tax revenue from labor income is calculated as "direct tax on households + .5 $\times$ indirect tax - .5 $\times$ VAT." Similarly, capital income is constructed as ".5 $\times$ indirect tax - .5 $\times$ VAT + operating surplus in corporate sector + operating surplus in housing non-corporate sector + .2 $\times$ operating surplus in non-housing non-corporate sector + net factor payments + adjustment for statistical discrepancy." Labor income is constructed as ".5 × indirect tax - .5 × VAT + compensation of employees + .8 × operating surplus in non-housing non-corporate sector + adjustment for statistical discrepancy." The tax rates are computed from the tax revenues divided by the corresponding income. For out-of-sample projections, we assume that over the three decades each tax rate linearly converges to its historical average during 1982 to 2010.

- $\{\kappa_{s,t},\ \kappa_{m,t}\}_{t=1982}^{\infty}$: We assume that the coverage ratios of general government expenditures/transfers to total public pension benefits and to total public medical benefits are maintained at their 2010 levels (41.3% and 37.8%).[24]
- $\{\rho_t, \{\psi_{j,t}\}_{j=21}^{100}\}_{t=1982}^{\infty}$: The age-21 population growth rate is computed based on the Annual Report on Current Population Estimate by the Statistics Bureau of the Ministry of International Affairs and Communication, for the sample period up until 2010. For the future horizon, these are computed based on the Projection of Future Population by the National Institute of Population and Social Security Research (IPSS). The age-specific conditional survival probabilities are computed from the same data.
- $\{g_t, \delta_t\}_{t=1982}^{\infty}$: The TFP growth rate up to 2010 is computed from the Solow residual series, $A_t = Y_t/(K_t^a L_t^{1-a})$, that is constructed in line with Hayashi and Prescott (2002) and İmrohoroğlu and Sudo (2011a, b). For the periods beyond 2010, we assume that the TFP growth rate is constantly 1%, which is the historical average of the growth rate of the Solow residual from 1995 to 2007. The depreciation rate is set to its actual value up to 2010, and linearly converges to 0.071, which is the historical

[24] We define the public pension benefits as the sum of Welfare Pension, National Pension, Pensions of Seamen's Insurance, Long-term entitlements from the Federation of National Public Personnel Mutual Aid Associations, Long-term entitlements from the Pension Fund Association for Local Government Officials, and Long-term entitlements from Others. We define the public medical benefits as the sum of Health Insurance, Medical benefits of Seamen's Insurance, National Health Insurance, the New Medical Care System for the Elderly, Health insurance run by Private Mutual Associations, the Japan Health Insurance Association, Short-term entitlements from the Federation of National Public Personnel Mutual Aid Associations, Short-term entitlements from the Pension Fund Association for Local Government Officials, and Short-term entitlements from Others.

average from 1982 to 2010, over the next three decades.

- $\{\gamma_t, \eta_t\}_{t=1982}^{\infty}$: The utility weight on households' leisure is assumed to rise from unity in 1988 to 1.37 in 1993 in a quadratic fashion, so as to incorporate the effect of institutional changes affecting labor input into the model. As discussed by Hayashi and Prescott (2002), a mandatory reduction of working hours was established in the late 1980s, and the length of the working week drops from 44 hours in 1988 to 40 hours in 1993.[25] The in-sample sequence of utility weights on government bond holding is set so that the model-generated government bond yield matches the corresponding data series.[26] The weight is set at the 2010 value in the forecast horizon.

B List of Equations

This appendix summarizes key equations used for computing transition equilibrium paths.

B.1 Demographic Structure

- Evolution of population:

$$P_{21,t} = (1 + \rho_t)\, P_{21,t-1}. \tag{B-1}$$

$$P_{j,t} = \psi_{j,t} P_{j-1,t-1} \quad \text{for } j = 21, ..., 100. \tag{B-2}$$

- Cohort share in total population:

$$\mu_{j,t} = \frac{P_{j,t}}{\sum_{i=21}^{100} P_{i,t}} \quad \text{for } j = 21, ..., 100. \tag{B-3}$$

[25] Based on a simple growth model, Hayashi and Prescott (2002) point out that the fall in workweek length as well as the slowdown in TFP growth is important for the Japanese economic stagnation during the 1990s.

[26] We construct the government bond yield series by dividing the nominal 'net property income' of the general government by the nominal 'net financial liabilities' of the general government and the growth rate of the GNP deflator.

B.2 Households' Optimality Conditions

- Budget constraints:

$$
\begin{aligned}
& (1+\tau_{c,t})\, c_{j,t} + k_{j+1,t+1} + b_{j+1,t+1} \\
= \; & \left[1 + (1-\tau_{k,t})\, r_t^K\right] k_{j,t} + (1+r_t^B) b_{j,t} \\
& + (1-\tau_{h,t})\,(1-\tau_{s,t}-\tau_{m,t})\, w_t \varepsilon_j h_{j,t} + \tau_t + \xi_t \qquad \text{for } j \leq 65; \quad \text{(B-4)} \\
& (1+\tau_{c,t})\, c_{j,t} + k_{j+1,t+1} + b_{j+1,t+1} \\
= \; & \left[1 + (1-\tau_{k,t})\, r_t^K\right] k_{j,t} + (1+r_t^B) b_{j,t} + p b_{j,t} + \tau_t + \xi_t \quad \text{for } j > 65. \quad \text{(B-5)}
\end{aligned}
$$

- First order conditions with respect to consumption, labor input, capital and bond holdings:

$$\frac{(1+\tau_{c,t+1})\, c_{j,t+1}}{(1+\tau_{c,t})\, c_{j,t}} = \beta \psi_{j+1,t+1} \left[1 + (1-\tau_{k,t+1})\, r_{t+1}^K\right], \tag{B-6}$$

$$\frac{\gamma_t}{1-h_{j,t}} = \frac{(1-\tau_{j,t})\,(1-\tau_{s,t}-\tau_{m,t})\, w_t \varepsilon_j}{c_{j,t}}, \tag{B-7}$$

$$\eta_t \frac{1}{b_{j,t+1}} + \frac{\beta \psi_{j+1,t+1}(1+r_{t+1}^B)}{(1+\tau_{c,t+1})\, c_{j,t+1}} = \frac{1}{(1+\tau_{c,t})\, c_{j,t}}, \tag{B-8}$$

$$(1-\tau_{k,t}) r_t^K - r_t^B = \frac{\eta_{t-1}}{\beta}(1+\tau_{c,t}) \sum_{j=21}^{100-1} \mu_{j,t} \frac{c_{j,t}}{b_{j+1,t+1}}. \tag{B-9}$$

- Other conditions:

$$k_{21,t} = b_{21,t} = 0. \tag{B-10}$$

$$k_{101,t} = b_{101,t} = 0. \tag{B-11}$$

B.3 Firm's Optimality Conditions

- Firm's production of final goods with the Cobb-Douglas technology:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \tag{B-12}$$

- First order conditions with respect to capital demand and labor demand (real wage and rental rates):

$$R_t = \alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} \equiv r_t^K + \delta_t; \tag{B-13}$$

$$w_t = (1-\alpha) A_t \left(\frac{K_t}{L_t}\right)^{\alpha}. \tag{B-14}$$

B.4 Social Security System

- Budget balance for public pension:

$$(1-\kappa_{s,t}) \sum_{j=65+1}^{100} P_{j,t} pb_{j,t} = \tau_{s,t} w_t \sum_{j=21}^{65} P_{j,t} \varepsilon_j h_{j,t}. \tag{B-15}$$

- Budget balance for public health insurance:

$$(1-\kappa_{m,t}) \sum_{j=21}^{100} P_{j,t} mb_{j,t} = \tau_{m,t} w_t \sum_{j=21}^{65} P_{j,t} \varepsilon_j h_{j,t}. \tag{B-16}$$

B.5 General Government

- Budget constraint:

$$\begin{aligned}(1+r_t^B)B_t + G_t + \kappa_{m,t} \sum_{j=21}^{100} P_{j,t} mb_{j,t} + \kappa_{s,t} \sum_{j=65+1}^{100} P_{j,t} pb_{j,t} + \sum_{j=21}^{100} P_{j,t} \tau_t + \sum_{j=21}^{100} P_{j,t} \xi_t \\ = B_{t+1} + \tau_{c,t} C_t + \tau_{h,t}(1-\tau_{s,t}-\tau_{m,t}) w_t L_t + \tau_{k,t} r_t^K K_t.\end{aligned} \tag{B-17}$$

B.6 Market Clearing Conditions and Resource Constraint

- For capital, labor, government bonds, and final goods,

$$K_t = \sum_{j=21}^{100} P_{j,t} k_{j,t}, \tag{B-18}$$

$$L_t = \sum_{j=21}^{65} P_{j,t} \varepsilon_j h_{j,t}, \tag{B-19}$$

$$B_t = \sum_{j=21}^{100} P_{j,t} b_{j,t}, \tag{B-20}$$

$$Y_t = C_t + K_{t+1} - (1 - \delta_t)K_t + G_t + \sum_{j=21}^{100} P_{j,t} mb_{j,t}. \tag{B-21}$$

C Sensitivity Analysis

In the main text, we have shown that population aging, particularly a decline in the fertility rate, adversely affects capital and labor inputs, lowering GNP growth. We have also shown that the adverse effects are expected to expand in the next few decades. These results are based on a set of assumptions that we have made about the future paths of exogenous variables and the household utility function. In this section, we consider how our results change as we alter these assumptions. In the first case we examine how much the baseline forecast varies with the assumption about the future TFP growth rate. In the second and third cases we examine how much the impact of a fertility rate decline is affected by the specification of the household utility function, specifically the labor supply elasticity and preference for government bond holding.

Sensitivity to TFP Growth

The dotted lines in the top four panels in Figure 14 depict the time paths of macroeconomic variables when TFP grows at a rate of 2% instead of 1% from 2011 onward. For comparison, the solid lines depict the time paths of the same variables under the baseline assumption. Since the assumptions except TFP growth are identical in these two cases, the discrepancies between these two lines are solely attributable to the different assumptions on TFP growth. In the baseline projection with 1% TFP growth, the adverse effects of population aging gradually influence factor inputs, leading to negative GNP growth from the mid-2020s. In the alternative projection with 2% TFP growth, labor input growth falls temporarily in 2011 before rising in subsequent periods, as higher TFP growth permanently enhances households' lifetime income generating wealth effects on working hours. From 2020 onward, the labor input growth rate converges to that in the baseline scenario. In the alternative scenario, the saving rate and capital input growth rate are slightly higher from the mid-2010s than in the baseline scenario, because higher TFP growth generates a higher return on capital, giving households an incentive to save more. However, these effects on GNP growth are quite limited. GNP growth in the alternative scenario is higher than in the baseline scenario by approximately 1%. A positive growth rate is maintained until 2040 in the alternative scenario, although it is dampened by population aging over time.

Sensitivity to Labor Supply Elasticity

Second, we ask if the labor supply elasticity matters for the impact of population aging. Existing studies, for example, Trabandt and Uhlig (2011) and İmrohoroğlu and Kitao (2009), investigate the importance of the size of the labor supply elasticity parameter for the effects of social security reforms or tax reforms within a general equilibrium framework. Here we replace the functional form of the present utility function (8) with the following utility function:

$$u\left(c_{j,t}, 1-h_{j,t}, b_{j+1,t+1}\right) = \log c_{j,t} - \gamma_t \frac{h_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \eta_t \log b_{j+1,t+1} \quad \text{for } j \leq 65,$$

where ν is the Frisch elasticity of the labor supply. In general, ν is supposed to be a positive number less than unity.

In Case II of Figure 14 we compute the equilibrium paths of GNP per capita and labor input growth under this alternative utility function with $\nu = 0.5$. The fertility rate declines in the population aging scenario, but not in the alternative non-aging scenario. Comparison with Figure 5 demonstrates how the specification of the labor supply elasticity affects the impact from the fertility rate decline. With this specification the variations in labor input are less volatile in both the aging and non-aging scenarios than those in the baseline scenario of Figure 5. However, the discrepancies between the two scenarios are much the same as those observed in Figure 5. We can conclude that the macroeconomic impact of declining fertility is not very sensitive to the elasticity of the labor supply.

Sensitivity to Utility from Government Bond Holding

Lastly, we ask if households' preference for government bond holding matters for the impact of population aging. In our baseline simulation, we calibrate the utility weight that households place on government bond holding η_t so that the model-generated return on government bonds r_t^B traces the data perfectly. Other things being equal, higher η_t implies a lower government bond yield, since households are willing to hold government bonds even if the spread is wider. A lower government bond yield reduces government interest repayments, moderating government bond accumulation through equation (24).

In Case III of Figure 14 we compute the equilibrium paths for the case where households put a larger weight on government bond holding. Specifically from 2011 onward we set the value of η_t at double its value in the original specification. Comparison with Figure 5 demonstrates how the specification of households' utility preference for government bonds affects the impact of population aging. The discrepancies between the aging and non-aging scenarios are not significantly different from Figure 5. However, since households hold government bonds at a low (or even negative) rate of return, the government bond yield remains at lower levels under both the aging and non-aging scenarios.

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Table 1: Dependency Ratios

	Dependency Ratio (over 65 age/15-64 age pop., %)		
	1990	2010	2030
Canada	17	20	38
China	9	11	24
France	21	26	39
Germany	22	31	48
Italy	22	31	44
Japan	17	35	53
Russia	15	18	29
U.K.	24	25	34
U.S.	19	20	33
More Developed	19	24	36
Less Developed	8	9	15

(Source) United Nations, "World Population Prospects, the 2010 Revision."
(Note) More developed regions comprise Europe, Northern America, Australia, New Zealand and Japan, and Less developed regions comprise all regions of Africa, Asia (excluding Japan), Latin America and the Caribbean plus Melanesia, Micronesia and Polynesia.

Table 2: Total Fertility Rates and Elderly Survival Rates

	Total Fertility Rate		Survival Rate of 85-89 age pop.	
	1985-90	2005-10	1990	2009
Canada	1.62	1.65	0.62	0.69
China	2.63	1.64	0.45	0.50
France	1.80	1.97	0.63	0.71
Germany	1.43	1.36	0.53	0.65
Italy	1.34	1.38	0.57	0.68
Japan	1.66	1.32	0.61	0.73
Russia	2.12	1.44	0.49	0.51
U.K.	1.84	1.83	0.57	0.65
U.S.	1.89	2.07	0.62	0.67

(Source) United Nations, "World Population Prospects, the 2010 Revision,"
World Health Organization, "Global Health Observatory Data Repository (Life Table)."
(Note) Total fertility rate is defined as the number of children per woman.

Table 3: Constant Parameters

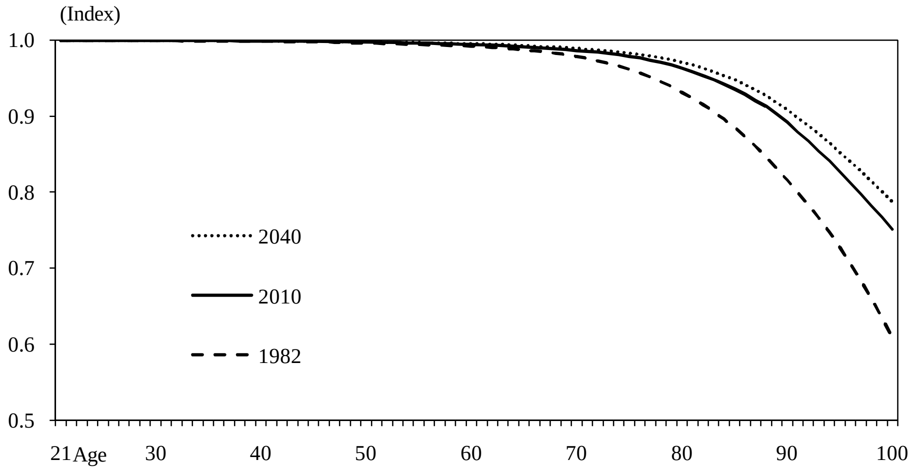
α	β	θ	$\{\varepsilon_j\}_{j=21}^{65}$	$\{mb_j\}_{j=21}^{100}$
0.406	0.972	0.30	Braun et al. (2009)	NME (2009)

Table 4: Steady State Values of Key Variables

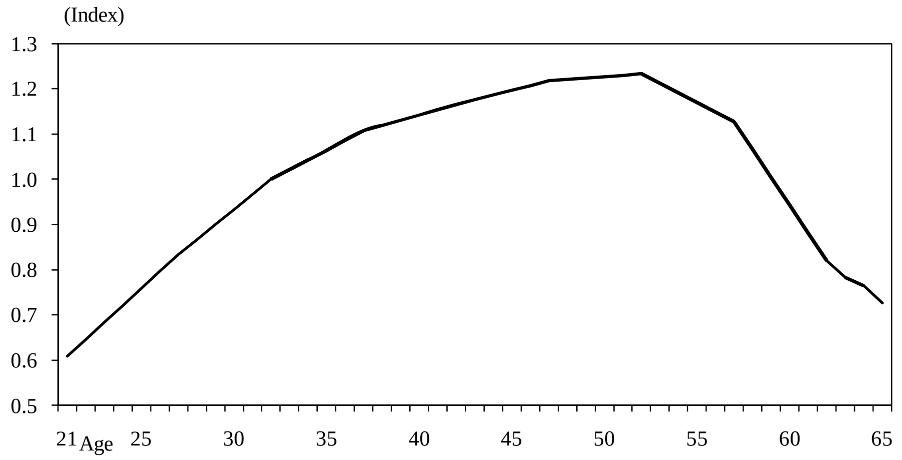
G/Y	B/Y	τ/Y	τ_c	τ_h	τ_k	κ_s	κ_m	g	ρ
0.142	0.441	0.022	0.10	0.15	0.316	0.413	0.378	0.01	0.00

Figure 1: Age-Specific Profiles

(1) Conditional Survival Probability



(2) Labor Efficiency



(3) Individual Medical Cost

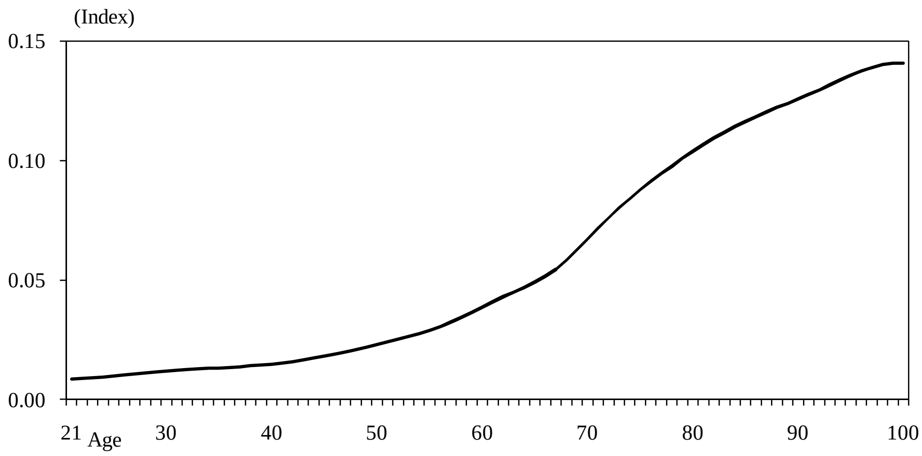
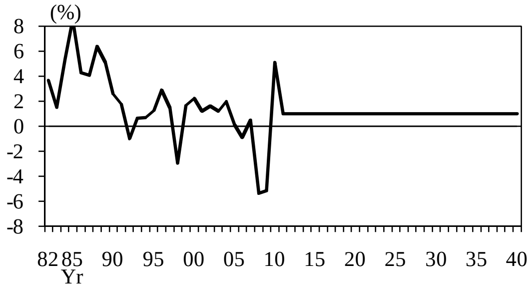
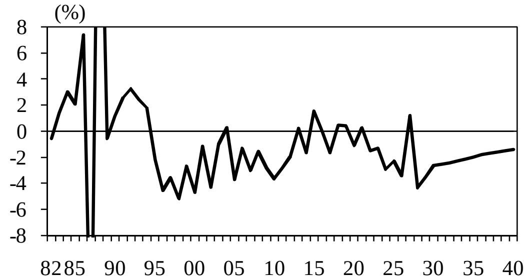


Figure 2: Exogenous Time Series

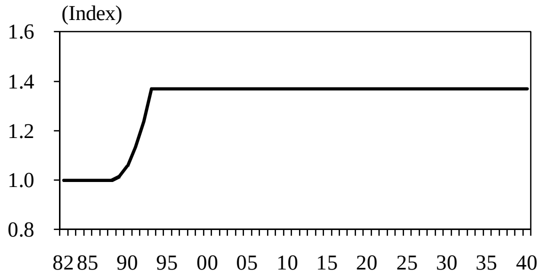
(1) Growth Rate of TFP Factor



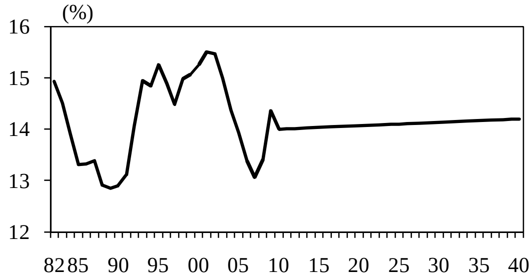
(2) Fertility Rate (Age-21 Population Growth)



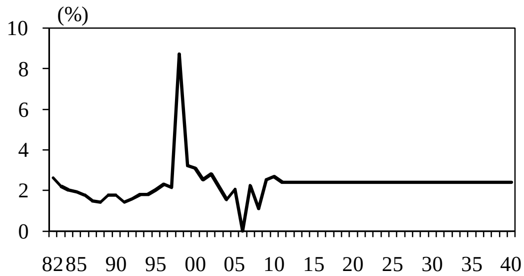
(3) Preference for Leisure



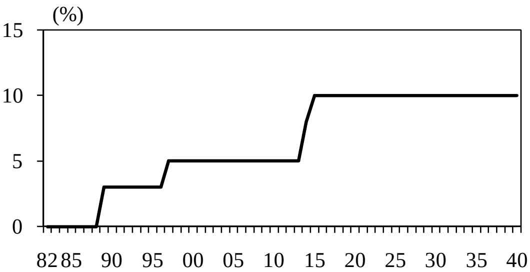
(4) Government Purchases-GNP Ratio



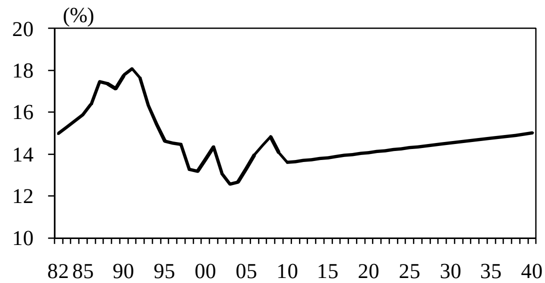
(5) Current/Capital Transfers-GNP Ratio



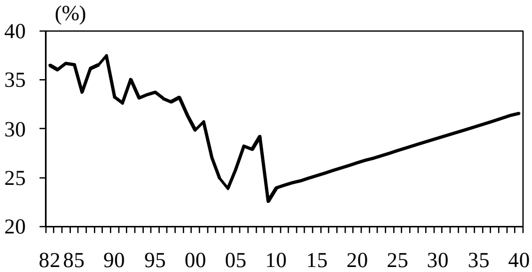
(6) Consumption Tax Rate



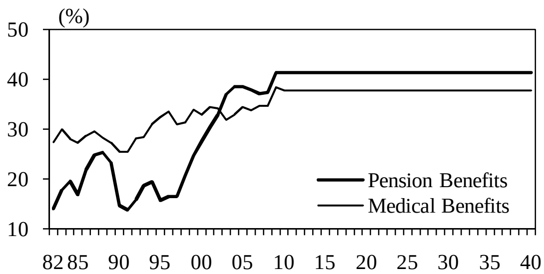
(7) Labor Income Tax Rate



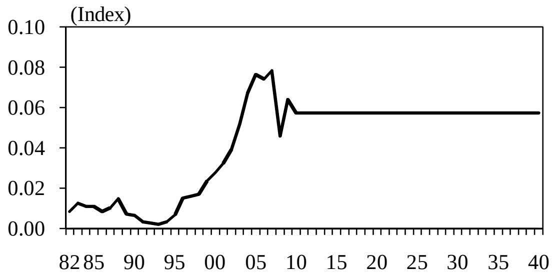
(8) Capital Income Tax Rate



(9) Coverage Ratios of General Government



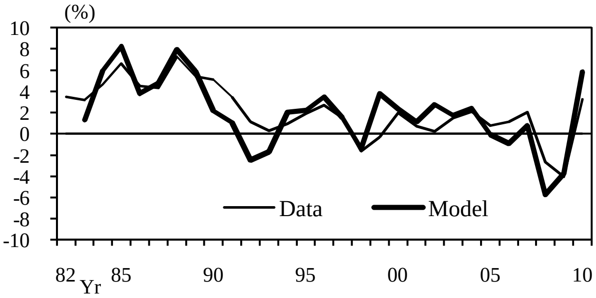
(10) Preference for Bond Holdings



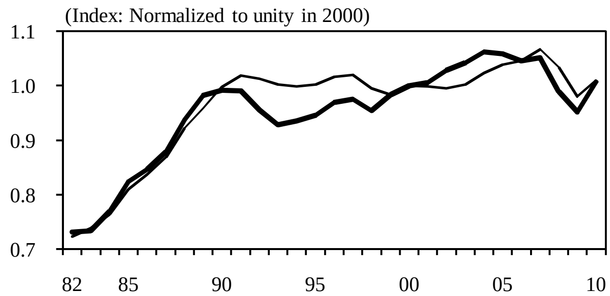
(Note) Government purchases of (4) do not include public medical benefits.

Figure 3: Model Performance

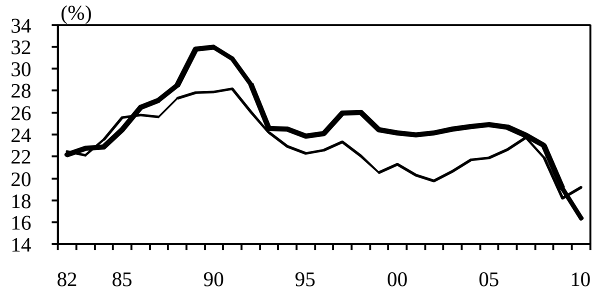
(1) Growth Rate of Real GNP



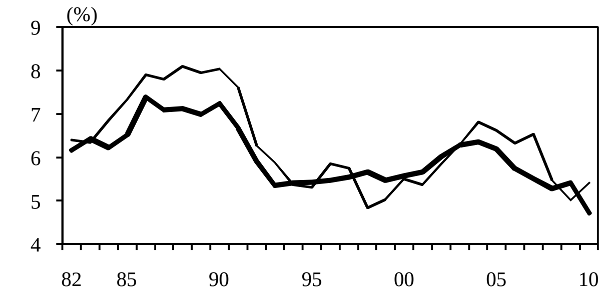
(2) Real GNP per Capita



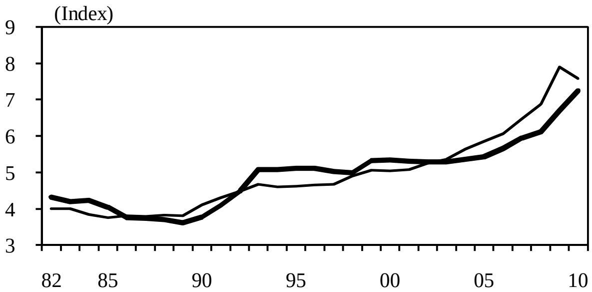
(3) Gross National Saving Rate



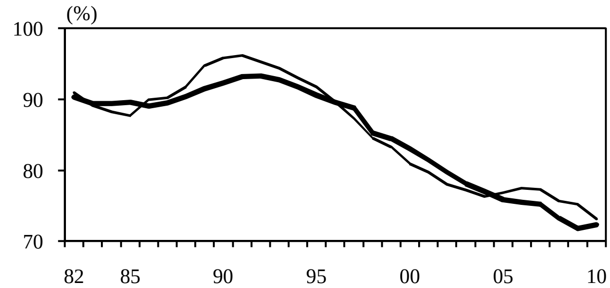
(4) After-Tax Rate of Return on Capital



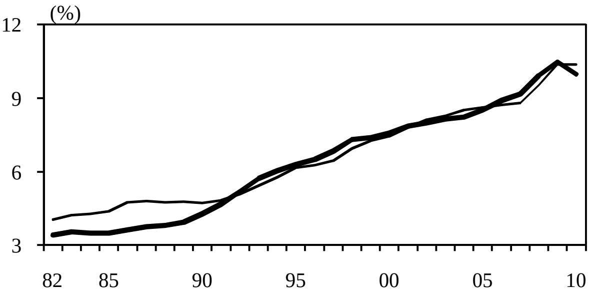
(5) Capital-Labor Ratio



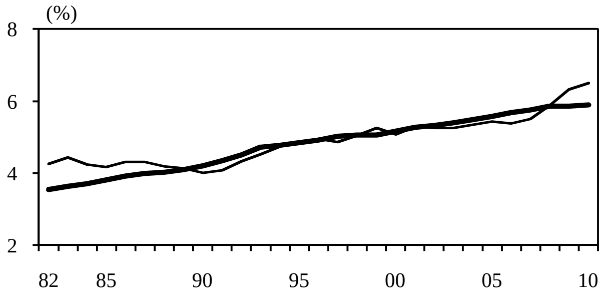
(6) Capital Share of Total Assets



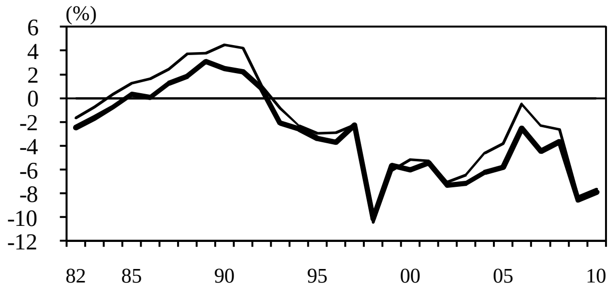
(7) Pension Benefits-GNP Ratio



(8) Medical Benefits-GNP Ratio



(9) Primary Balance-GNP Ratio



(10) Net Government Debt-GNP Ratio

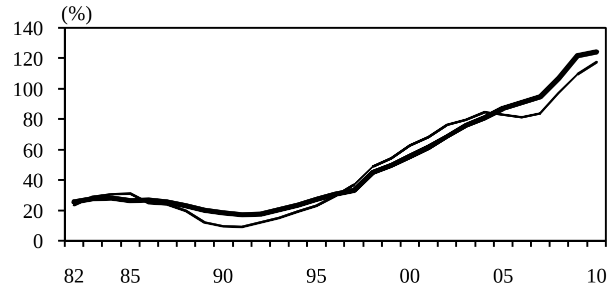
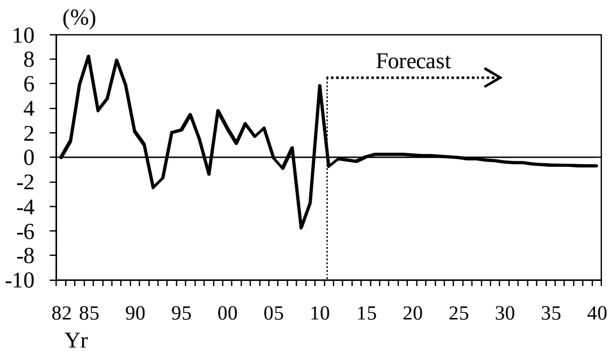
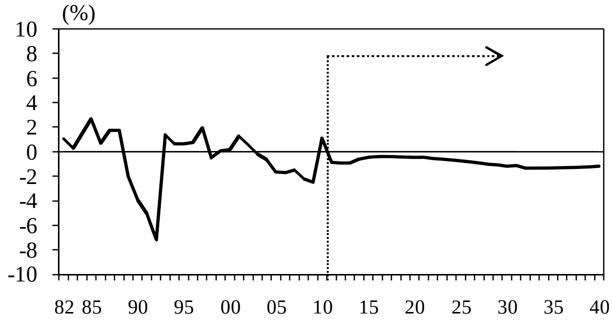


Figure 4: Baseline Long-Term Projection

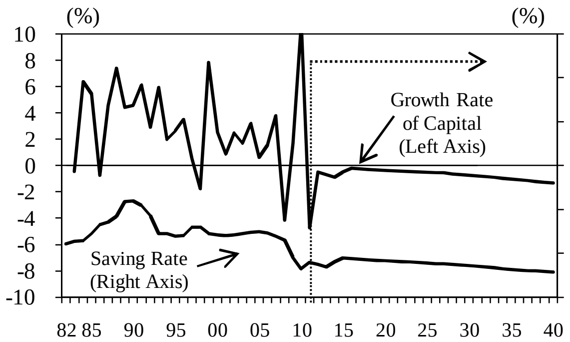
(1) Growth Rate of Real GNP



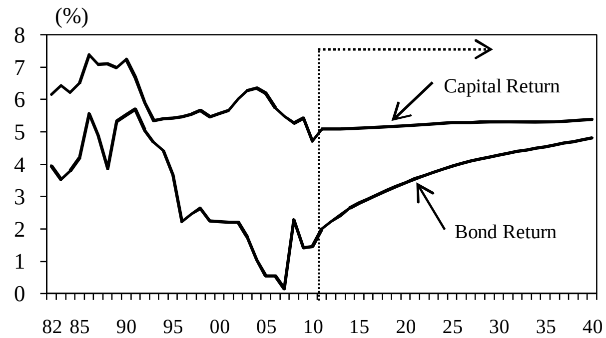
(2) Growth Rate of Labor



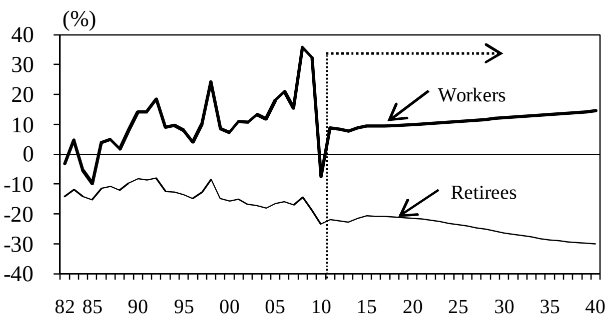
(3) Growth Rate of Capital and Gross Saving Rate



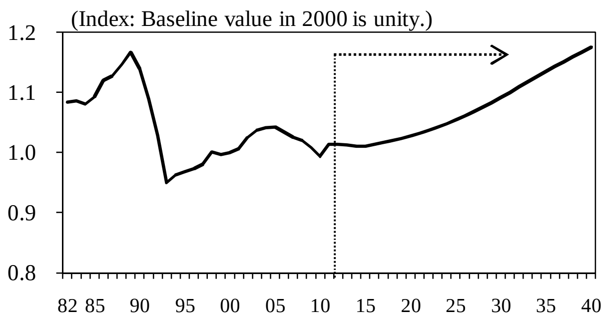
(4) Real Returns on Capital and on Bonds



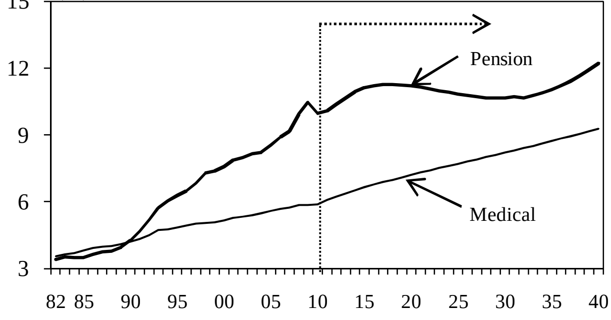
(5) Saving Rates of Workers and Retirees



(6) Average Hours Worked



(7) Pension/Medical Benefits-GNP Ratio



(8) Real GNP per Capita

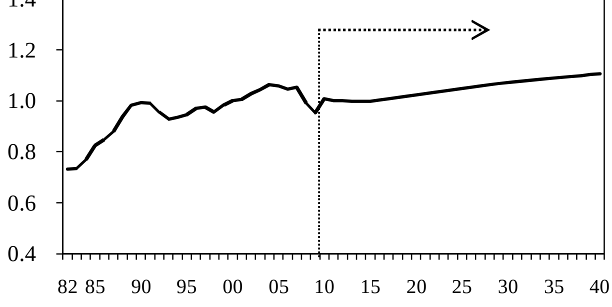
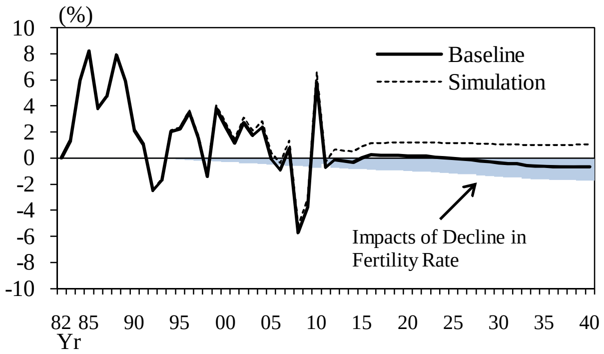


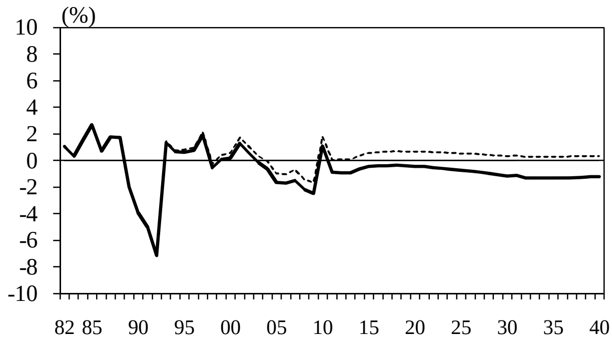
Figure 5: Impact of Decline in Fertility Rate (Counterfactual Simulation)

[Assumption] No fertility rate decline: the size of the age-21 population is held constant from 1995. Other exogenous series are identical to Figure 4.

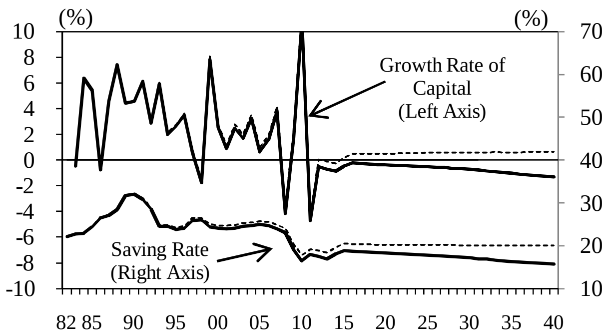
(1) Growth Rate of Real GNP



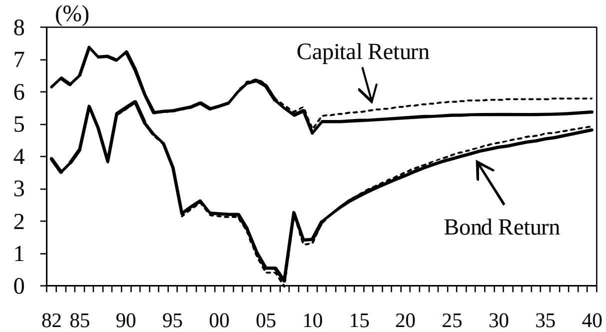
(2) Growth Rate of Labor



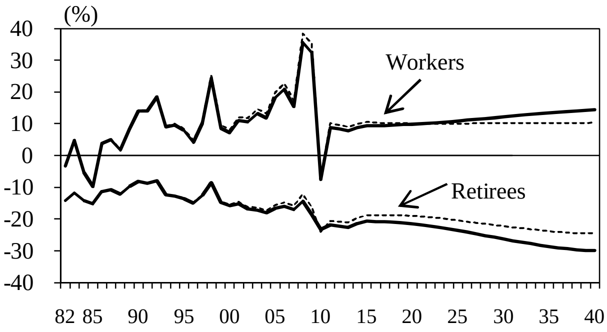
(3) Growth Rate of Capital and Gross Saving Rate



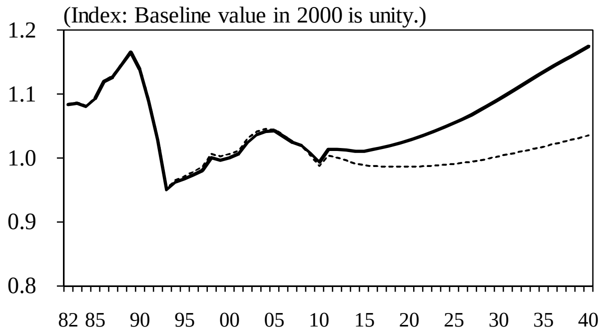
(4) Real Returns on Capital and on Bonds



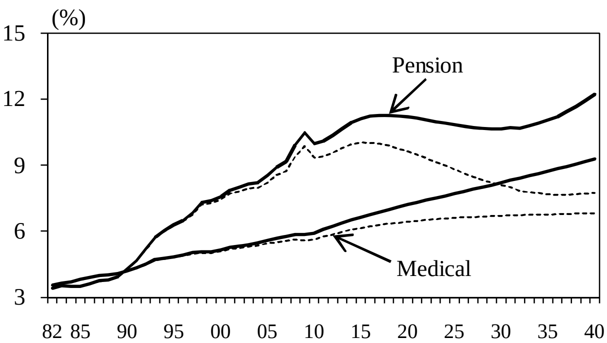
(5) Saving Rates of Workers and Retirees



(6) Average Hours Worked



(7) Pension/Medical Benefits-GNP Ratio



(8) Real GNP per Capita

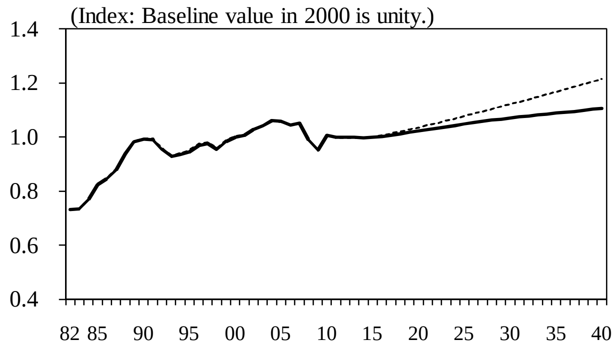


Figure 6: Impact of Decline in Fertility Rate on Real GNP per Capita

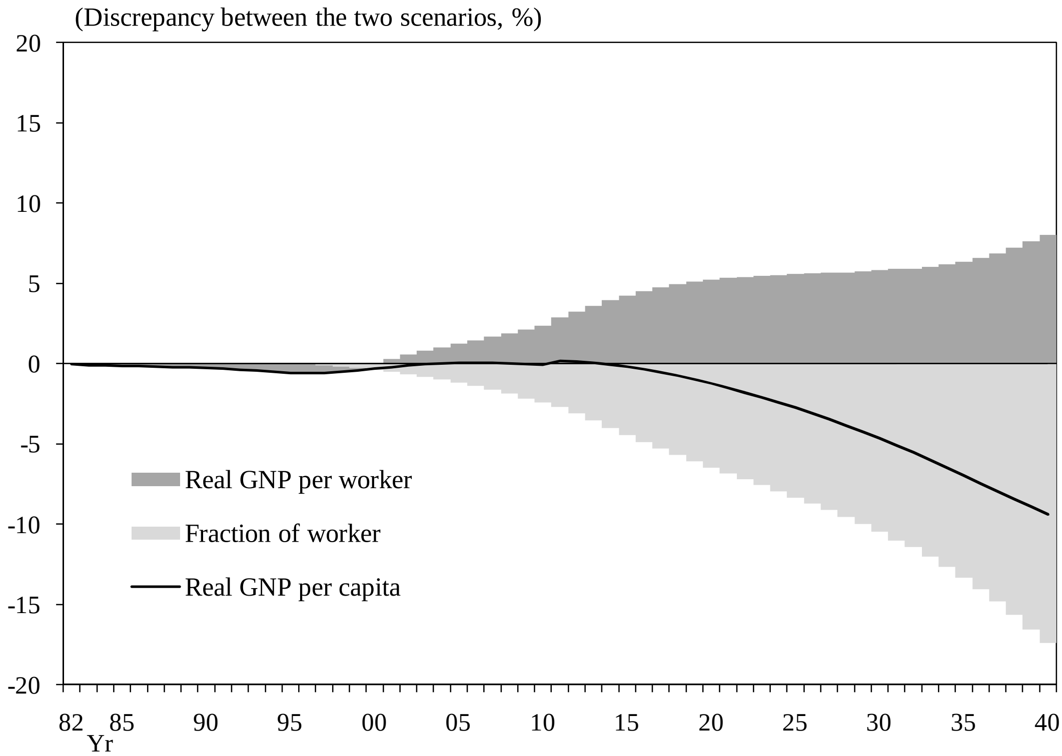
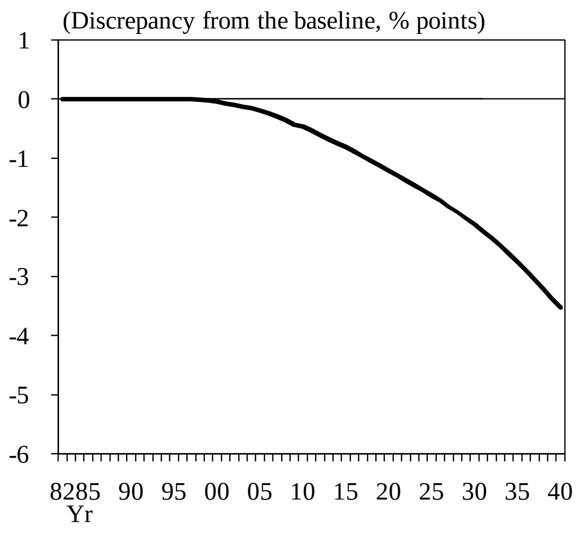


Figure 7: Impact of Decline in Fertility Rate on Fiscal Balance

(1) Primary Balance-GNP Ratio



(2) Net Government Debt

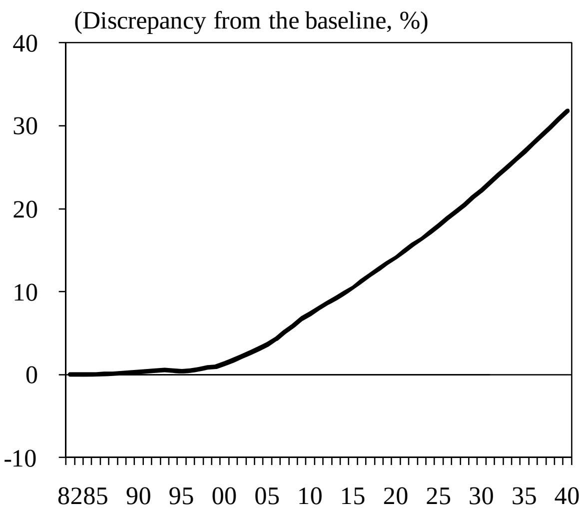
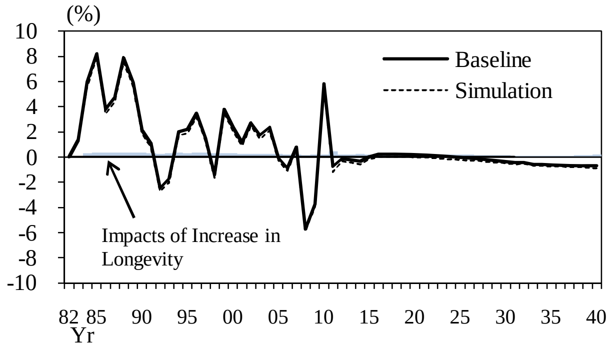


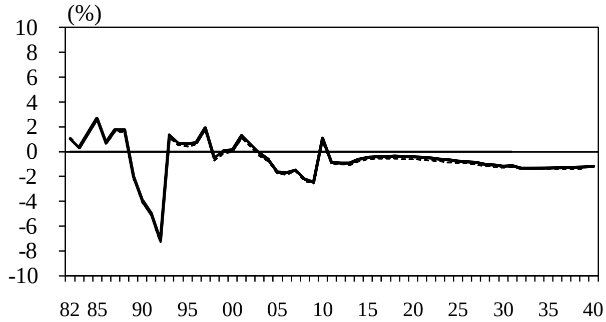
Figure 8: Impact of Increase in Longevity (Counterfactual Simulation)

[Assumption] No increase in longevity: households' survival probability is held constant from 1982 onward. Other exogenous series are identical to Figure 4.

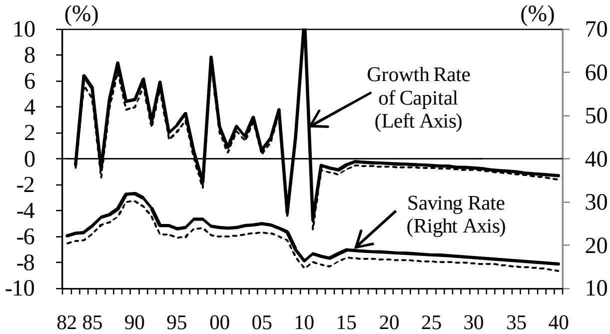
(1) Growth Rate of Real GNP



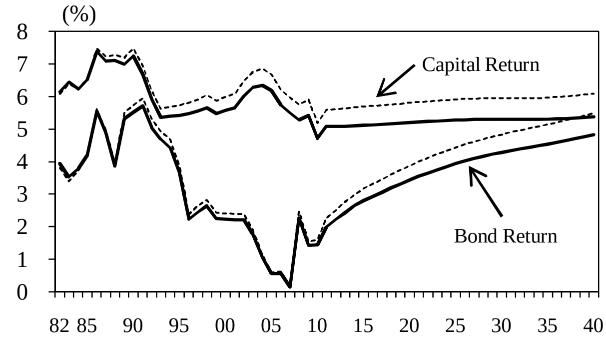
(2) Growth Rate of Labor



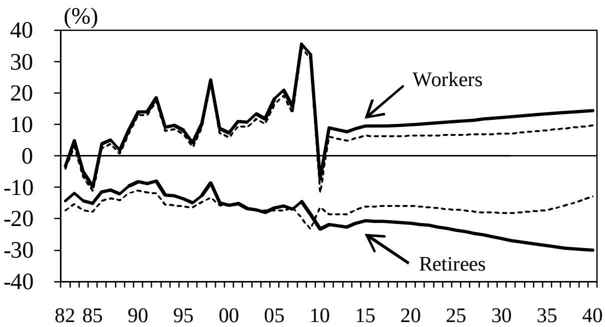
(3) Growth Rate of Capital and Gross Saving Rate



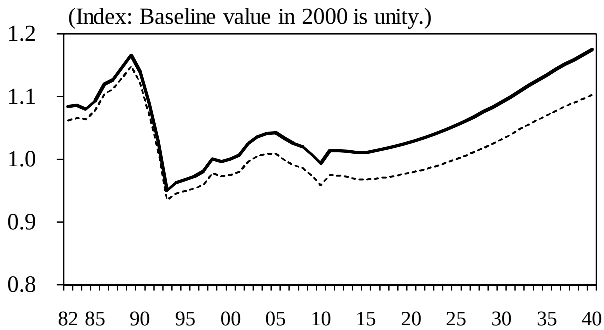
(4) Real Returns on Capital and on Bonds



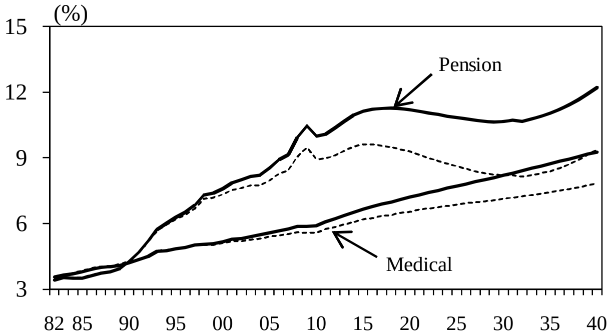
(5) Saving Rates of Workers and Retirees



(6) Average Hours Worked



(7) Pension/Medical Benefits-GNP Ratio



(8) Real GNP per Capita

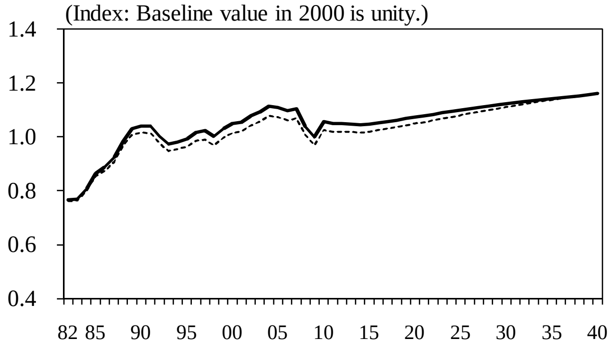


Figure 9: Impact of Increase in Longevity on Real GNP per Capita

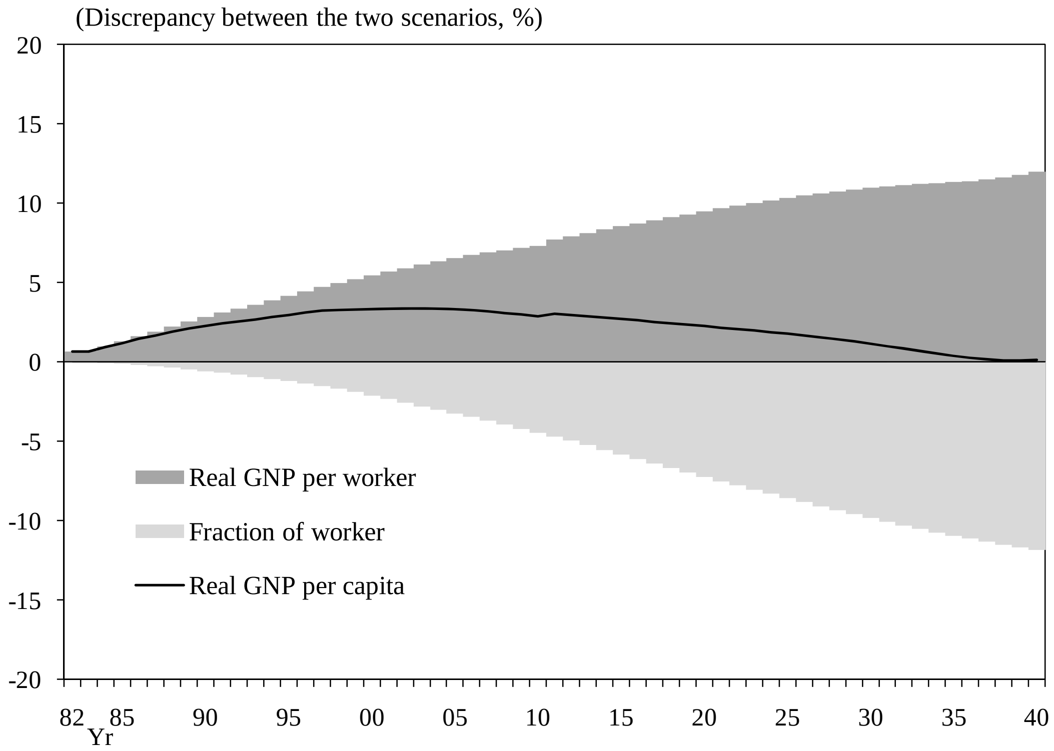
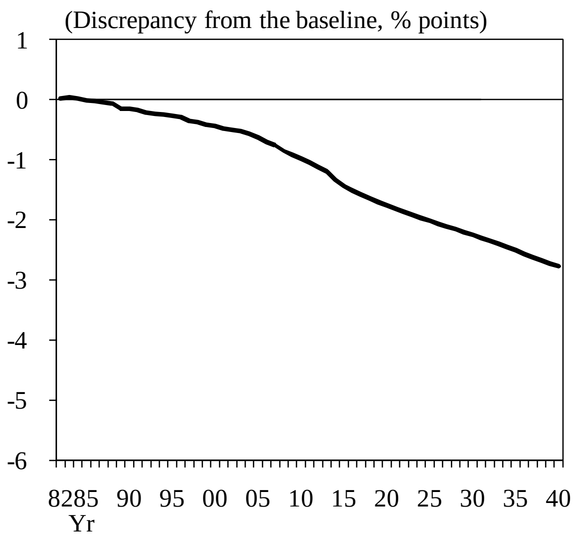


Figure 10: Impact of Increase in Longevity on Fiscal Balance

(1) Primary Balance-GNP Ratio



(2) Net Government Debt

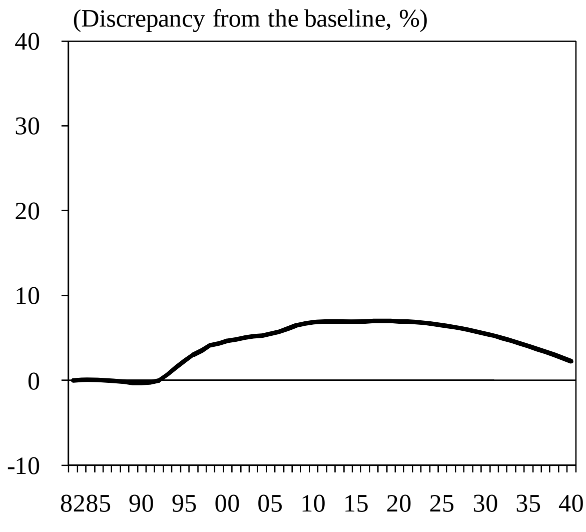
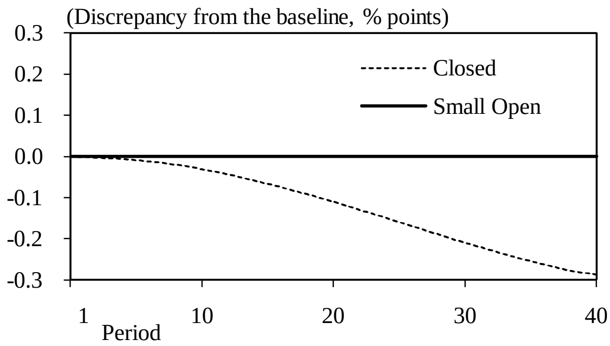


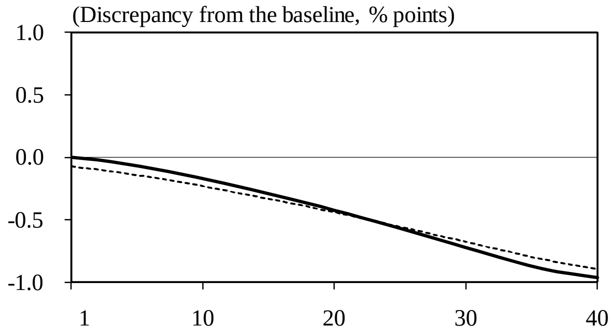
Figure 11: Impact of Decline in Fertility Rate on Current Account

[Assumption] The growth rate of age-21 population declines permanently by 1.0% point.
Domestic return on capital is equalized to foreign return on capital.

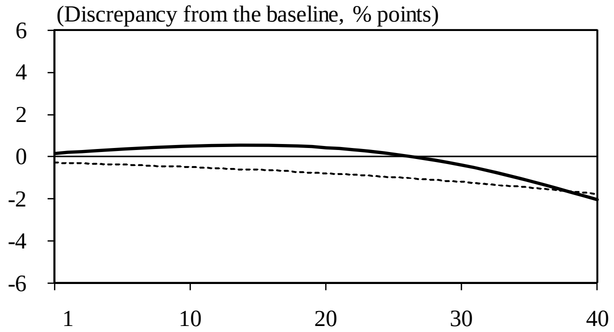
(1) Return on Capital



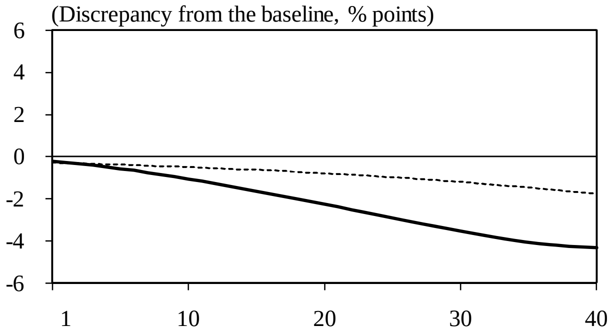
(2) Growth Rate of Real GNP



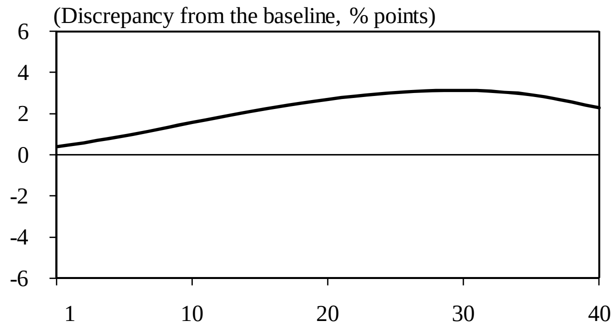
(3) Saving Rate



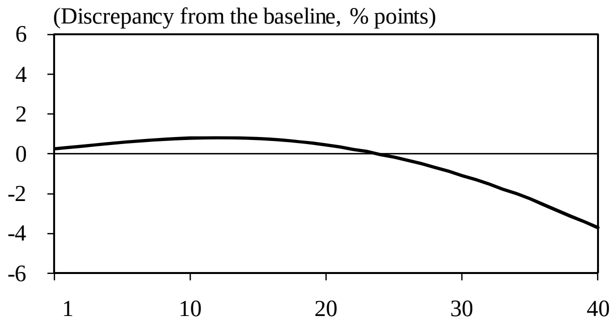
(4) Investment Rate



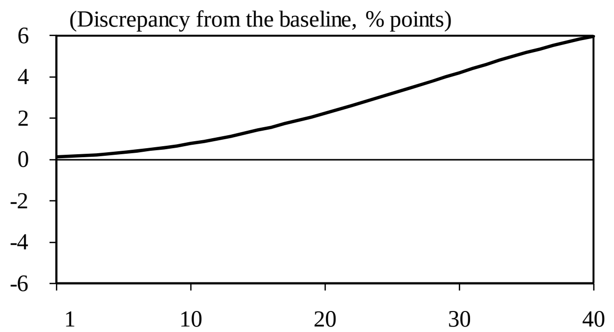
(5) Current Account-GNP Ratio



(6) Trade Balance-GNP Ratio



(7) Income Account-GNP Ratio



(8) Real GNP per Capita

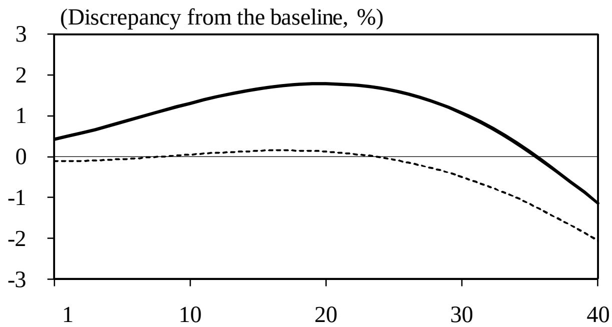
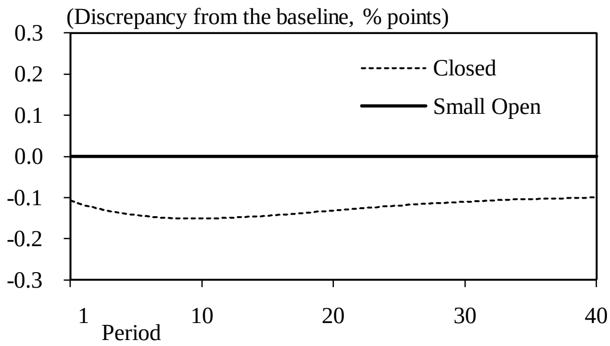


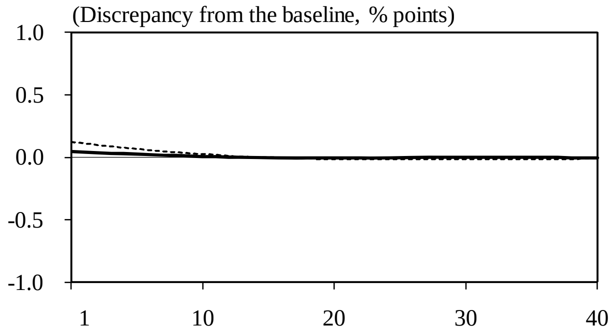
Figure 12: Impact of Increase in Longevity on Current Account

[Assumption] The conditional survival probabilities of age 65 over households rise permanently by 1.0% point. Domestic return on capital is equalized to foreign return on capital.

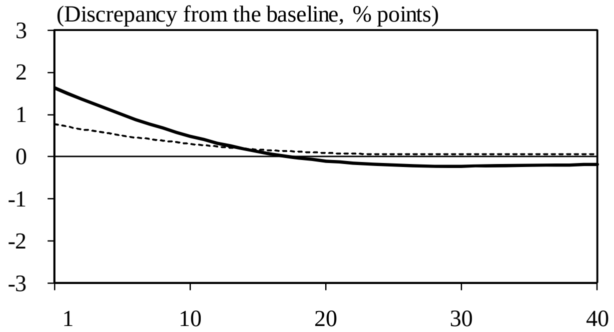
(1) Return on Capital



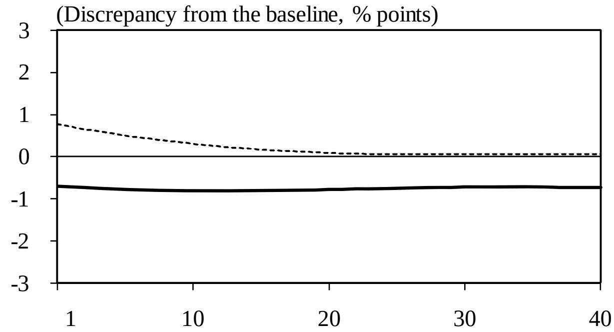
(2) Growth Rate of Real GNP



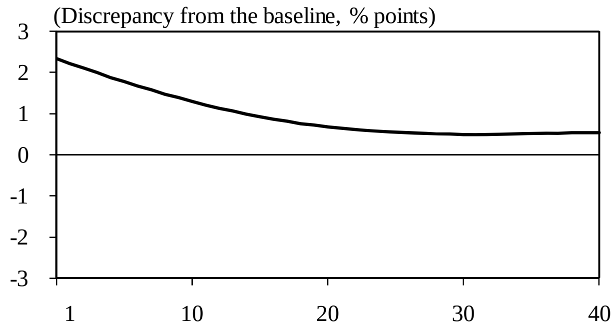
(3) Saving Rate



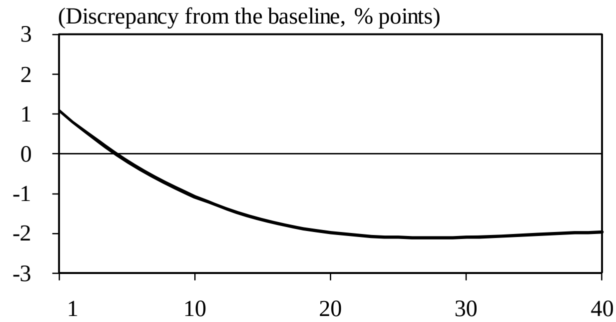
(4) Investment Rate



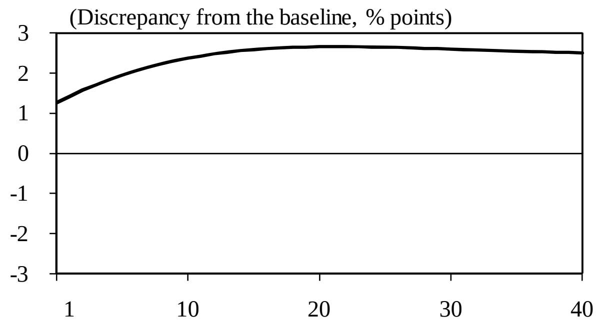
(5) Current Account-GNP Ratio



(6) Trade Balance-GNP Ratio



(7) Income Account-GNP Ratio



(8) Real GNP per Capita

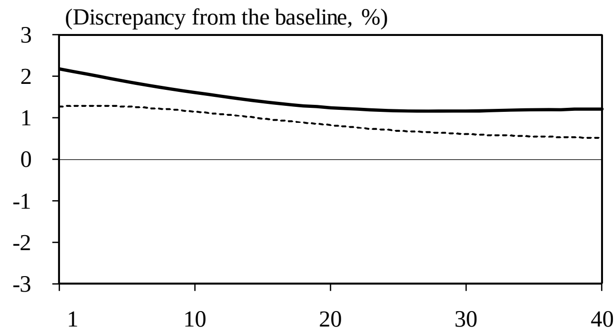
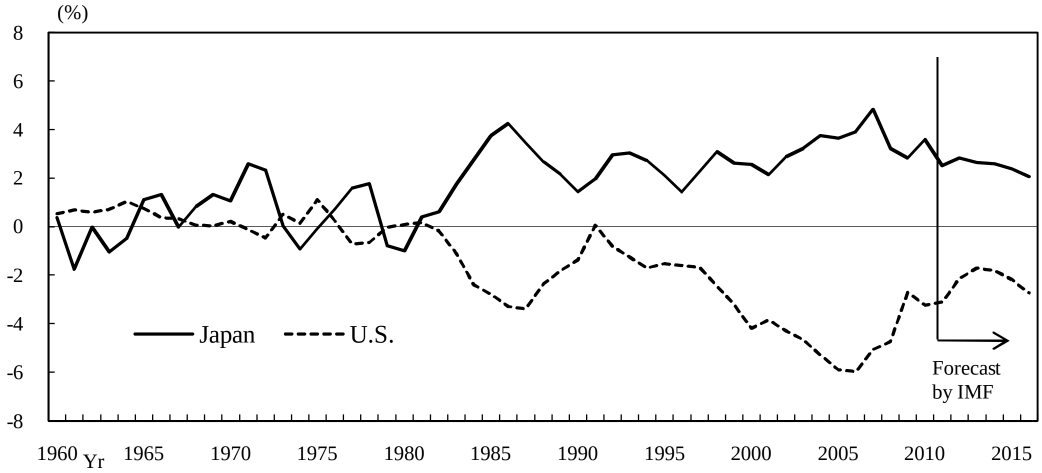
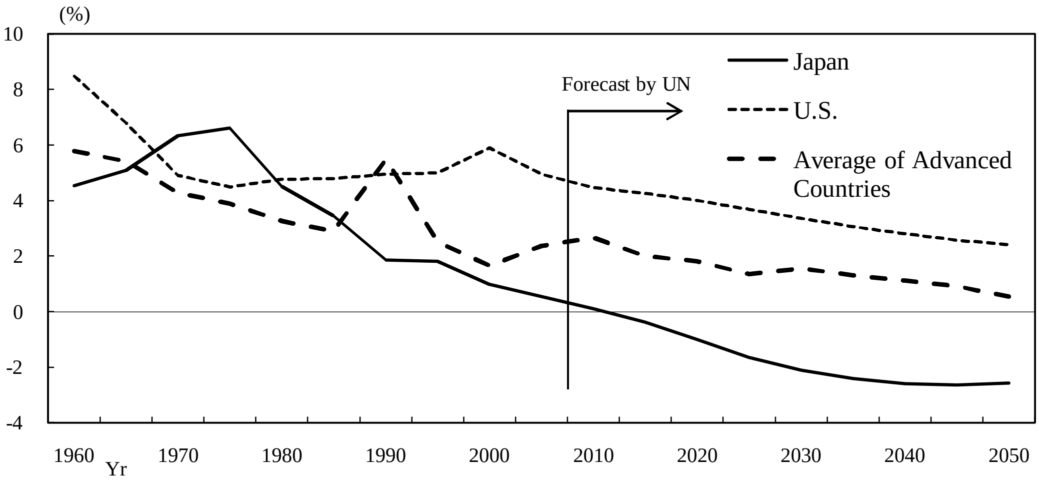


Figure 13: Current Account and Demographics in Japan and the U.S.

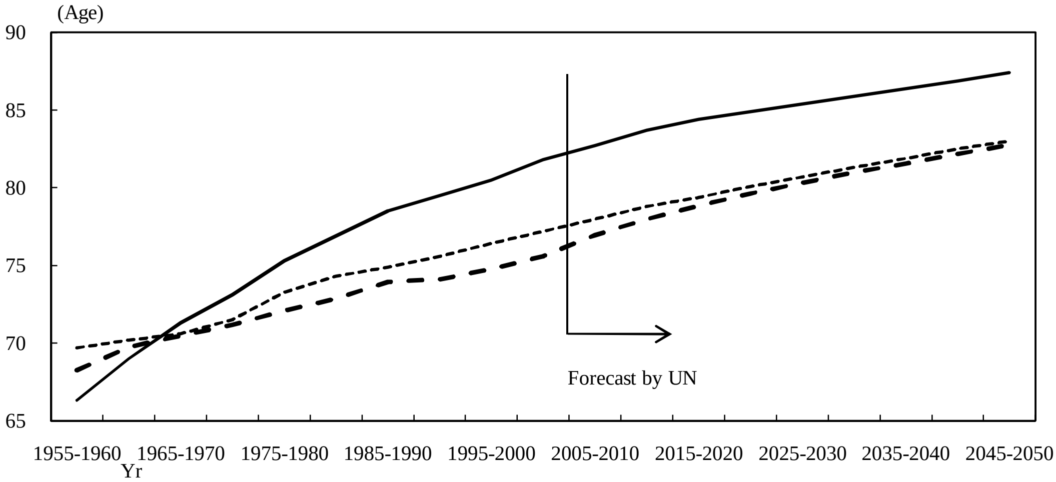
(1) Current Account (to GDP Ratio)



(2) Population Growth Rate



(3) Average Life Span



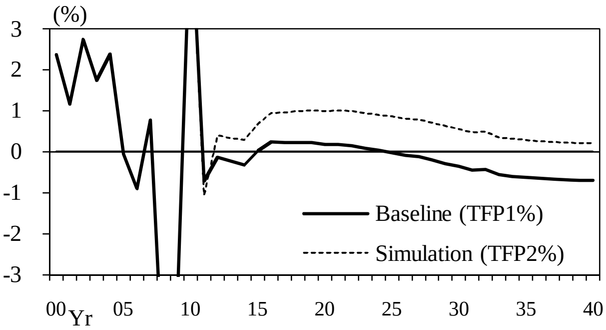
(Source) IMF "World Economic Outlook," United Nations "World Population Prospects (Medium Forecast)," Japanese Cabinet Office "National Accounts," U.S. Department of Commerce "National Income and Product Accounts."

(Note) 'Average of Advanced Countries' corresponds to the category 'More Developed Regions' by United Nations "World Population Prospects (Medium Forecast)."

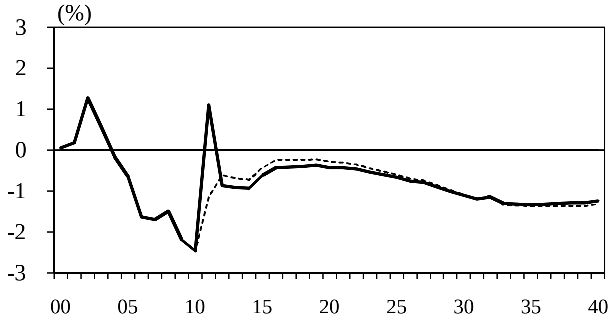
Figure 14: Sensitivity Analysis

< Case I > Higher TFP Growth Rate (2%)

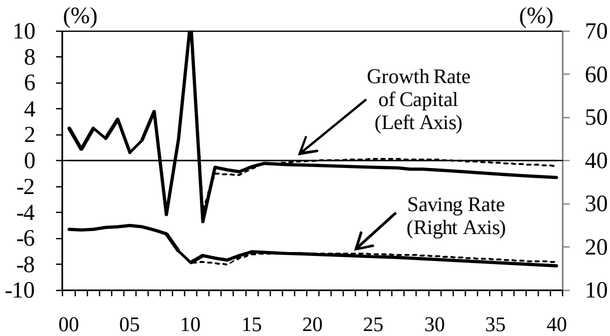
(I-1) Growth Rate of GNP



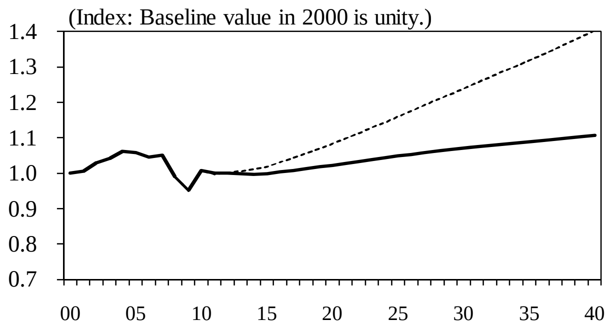
(I-2) Growth Rate of Labor



(I-3) Growth Rate of Capital and Gross Saving Rate

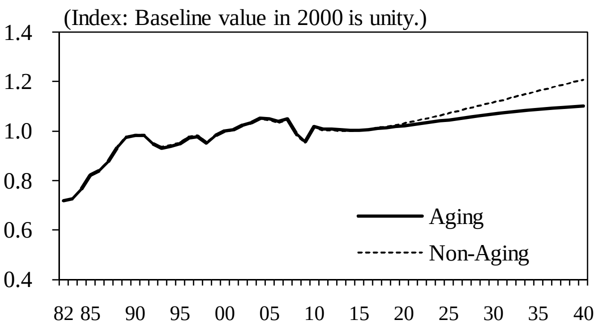


(I-4) Real GNP per Capita

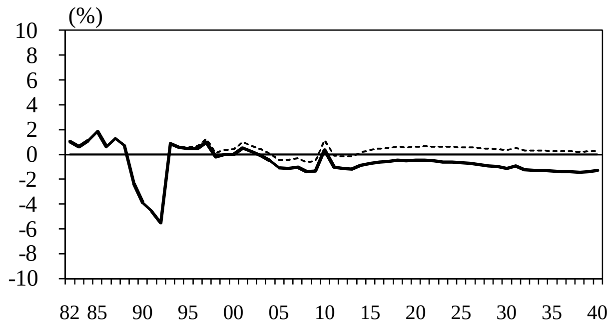


< Case II > Less Elastic Labor Supply (Frisch Elasticity = 0.5)

(II-1) Real GNP per Capita

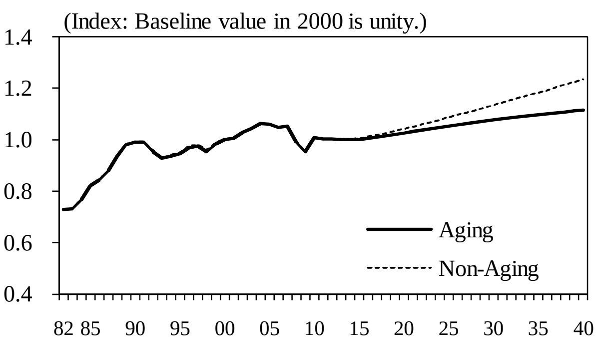


(II-2) Growth Rate of Labor



< Case III > Higher Preference for Bond (Twofold η_t ever after 2011)

(III-1) Real GNP Per Capita



(III-2) Real Returns on Capital and on Bonds