On Mental Transformations

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Abstract

The paper presents an alternative interpretation of the experimental data published by Kahneman and Tversky in their 1992 study "Advances in Prospect Theory", which describes the Cumulative version of their Prospect Theory from 1979. It was assumed that, apart from the operations made during the initial stage of problem resolution, which Prospect Theory defines as Editing (here generalized as Mental Adaptation), other mental transformations such as Prospect Scaling (resulting from Focused Attention) and Logarithmic Perception of Financial Stimuli should be considered when analyzing the experimental data. This led to the design of an explicit, simple and symmetric solution without the use of the probability weighting function. The double S-type function obtained (the decision utility) resembles the utility curve specified by the Markowitz hypothesis (1952) and substitutes the fourfold pattern of risk attitudes introduced by Cumulative Prospect Theory. The results may signal a return to a description of people's behavior that only relies on the utility-like function.

Keywords: *Prospect/Cumulative Prospect Theory, Markowitz Utility Hypothesis, Mental Processes, Adaptation & Attention Focus, Aspiration Level*

JEL classification: *D03, D81, C91*

1 Introduction

The first approach based on a utility curve was proposed by Nicolas Bernoulli as early as 1734. However, it was von Neumann and Morgenstern (1944) who showed that the expected

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utility hypothesis could be derived from several axioms which assumed that human decisions are rational. Since then, expected utility theory has become the dominant hypothesis in the economic thought of that time. As early as 1948, Friedman and Savage argued that the curvature of the utility function varies in order to explain buying lottery tickets and insurances. Further developments were proposed by Markowitz (1952), who considered the shape of the utility function around the "customary" level of wealth. Later on, in Subjective Utility Theory (Savage, 1954) the classical definition of probability was replaced with the subjective one.

However, the growing amount of experimental data indicated that no utility function could correctly explain human behavior. The most famous was the Allais paradox (1953). This led to the creation of several theories collectively referred to as Non-Expected Utility Theories. As Prospect Theory (1979) was met with objections from a mathematical point of view, an enhanced version was created - Cumulative Prospect Theory, CPT (Tversky & Kahneman, 1992). Prospect Theory, and its extended version, gave rise to the concepts of the value function and the probability weighting function. The value function is supposed to evidence risk aversion for gain prospects and risk seeking for loss prospects, as well as a general aversion to losses. The probability weighting function is supposed to show the non-linear transformation of probabilities when making decisions, which would explain people's willingness to participate in lotteries as well as their tendency towards less risky investments in the case of average probabilities. Prospect Theory gave rise to new research trends. Much attention was focused on the probability weighting function (Camerer and Ho, 1994; Wu and Gonzalez, 1996, 1999; Prelec, 1998; Tversky and Wakker, 1995).

Prospect Theory has also met with criticism. Nwogugu (2006) has compiled a large collection of objections and draws on a bibliography of 131 titles to support his claims. The author asserts that Prospect Theory was derived using improper methods and calculations and that it is not consonant with natural mental processes. Shu (1995) shows that it is wrong to assume the existence of probability weights. Neilson and Stowe (2002) demonstrate that Cumulative Prospect Theory cannot simultaneously explain participation in lotteries and the Allais paradox. Blavatsky (2005) claims that the theory does not explain the St. Petersburg paradox, a classic problem of decision making under conditions of risk. Levy and Levy (2002) state that their experimental results negate Prospect Theory and confirm the Markowitz hypothesis.

The present paper, too, is critical of Prospect Theory. However, it is not criticizing individual components or individual methodological assumptions, but is rather focused on analyzing the entire process of how the end results of the 1992 study were obtained from the experimental data. It has been stated that apart from the operations made at the initial stage of problem resolution, which Prospect Theory defines as Editing (in this study generalized as Mental Adaptation), any analysis of the experimental data should include other mental transformations such as Prospect Scaling (resulting from Focused Attention) and Logarithmic Perception of Financial Stimuli. This assumption finds its explanation in psychology, in particular cognitive psychology, and in research at the sensory and neuronal levels.

On the basis of the assumptions stated above and using exactly the same experimental data that were used to derive Cumulative Prospect Theory, an explicit, simple and symmetric solution was obtained without the use of the probability weighting function. A function was obtained which describes a direct relationship between the probability and relative certainty equivalent. The resulting curve (named the decision utility function) has a symmetric double S-type shape consistent with the Markowitz hypothesis (1952). More importantly, the decision utility function explains how people's attitude towards risk depends on their state of mind and their aspiration levels. The explanation of risk attitudes given by the convex-concave-convex-concave shape of the decision utility function substitutes the fourfold pattern introduced by CPT. Summarizing the results presented in this study provide a basis of negating Prospect Theory as a theory that correctly describes how decisions are made under conditions of risk.

The paper is organized as follows. Section 2 describes the mental transformations which form the basis of the derivation presented in the following part of the study. These transformations include Probability Weighting, Mental Adaptation, Prospect Scaling and Logarithmic Perception of Financial Stimuli. Section 3 provides a solution using the Mental Adaptation and Prospect Scaling transformations. Direct S-shaped relationships between the probability and relative certainty equivalents are obtained separately for gain and loss prospects. In Section 4, the results for gain and loss prospects are combined to produce a single solution named the "decision utility" function. The obtained curve strongly resembles the utility function specified by the Markowitz hypothesis (Section 5). Section 6 of the paper presents the derivation of the model for multi-outcome lotteries. Section 7 presents the solution with the additional consideration of the Stimuli Logarithmic Transformation. Section 8 summarizes the study.

2 Review of Mental Transformations

2.1. Transformation of Probabilities.

That perception of probabilities is distorted is simultaneously one of the key assumptions and key results of Prospect Theory. The concept of decision weights was introduced into the first version of Prospect Theory in 1979. Even at that early stage, Kahneman and Tversky were stating that decision weights were not probabilities and did not comply with the axioms of probability. This led to serious mathematical objections (failure to comply with the First Order Stochastic Dominance). As a result, Rank-Dependent Expected Utility Theory (Quiggin) was developed as early as 1982 to remedy the shortcomings of its predecessor. The key concepts of that theory were later adopted by Cumulative Prospect Theory (Tversky, Kahneman, 1992). The axiomatization is based on pretty complex topological models and Choquet integrals (Schmeidler, 1989, Wakker 1989, 1990; Kahneman and Tversky, 1992 and appendix to their publication).

It is important to note that Kahneman and Tversky distinguish *overestimation* (often encountered when assessing the probability of rare events) and *overweighting* (as a feature of decision weights) (Prospect Theory, 1979). The latter phenomenon lacks psychological justification to the extent that the former has it (for instance by dint of insufficient knowledge). It is difficult to explain in psychological terms how a decision regarding an event whose probability is known seems to assume a different probability value. This is what the probability weighting function addresses. Furthermore no mechanism was posited to explain why this effect of probability transformation only manifests itself at the moment a decision is made. A failure to distinguish between *overestimation* (which can be referred to as a kind of *subjective* view of events whose probabilities are not known) and *overweighting* (an artificial concept to explain the results of experiments regarding events whose probabilities are known) leads to the commonly accepted view that the probability weighting function has a profound psychological justification. The next part of this study shows that the probability weighting function (i.e. the entire probability transformation concept) is not necessary to explain the results of the experiments conducted by Kahneman and Tversky.

2.2. Mental Adaptation[3]

Evolutionary adaptation was first described by British natural theologians John Ray

[3] For a broader review please refer to K. Kontek (2010b).

(1627–1705) and William Paley (1743–1805). The theory was later refined by Charles Darwin (1809–82). Peter Medawar, winner of the Nobel Prize for Medicine and Physiology in 1960, describes the term as "*a process allowing organisms to change to become better suited for survival and reproduction in their given habitat"*. The Oxford Dictionary of Science defines adaptation as "*any change in the structure or functioning of an organism that makes it better suited to its environment"*. More definitions can be found in Rappaport (1971) and Williams (1966). Summarizing *"adaptation can refer to a trait that confers some fitness on an animal, but it also represents the process by which that trait has come about"* (Greenberg, 2010).

"Neural or sensory adaptation is a change over time in the responsiveness of the sensory system to a constant stimulus. More generally, the term refers to a temporary change of the neural response to a stimulus as the result of preceding stimulation". This Wikipedia definition is close to those met in academic texts: "*Adaptation in the context of sensation refers to the fact that a prolonged and uniform sensory stimulus eventually ceases to give rise to a sensory message*" (Medawar, 1983, more in Laughlin, 1989 and Hildebrandt, 2010). The best example of neural adaptation is eye adaptation. Similar mechanisms are well attested for smell, temperature, taste, pain and touch (Gregory, Colman, 1995, Medawar, 1983).

The definitions presented so far all assume that it is the living organism which adapts to changing environmental conditions. However, from the standpoint of a human being, adaptation may be seen as a process of changing the external world to suit its requirements. This was best expressed by Leakey (1981) as follows: *"Animals adapt themselves to environment, hominids adapt environment to themselves using tools, language and complex cooperative social structures*". Mutual human - environment interaction was described by the famous Swiss psychologist Jean Piaget, who *"considers in fact intelligence rising from mental adaptation, where the adaptation is the equilibration of the action of an organism on the environment (assimilation) and of the action of the environment on the organism (accommodation)"* (Maniezzo, Roffilli, 2005).

In the author's opinion, the term "mental adaptation" is best expressed as *"the state of not thinking about certain phenomena"*. This definition follows the Sulavik (1997) paper on mental adaptation to death in the case of professional rescuers, although it can easily be extended to cover many other situations like stress, major illness, bereavement, financial loss, immigration (Jasinskaja-Lahti, 2006), disasters (Leon, 2004), and even space travel (NASA). It should be borne in mind that mental adaptation occurs in positive situations as well – financial windfalls, professional achievements, falling in love etc. "Hedonic treadmill" is another term for mental adaptation coined by Brickman and Campbell (1971) *"to describe the now widely accepted notion that though people continue to accrue experiences and objects that make them happy – or unhappy – their overall level of well-being tends to remain fairly static."* (Mochon et al., 2008, also Kahneman, 1999). There are several other meanings of adaptation encountered in the literature (e.g. social adaptation). A wide coverage of hedonic adaptation examples is given by Frederick and Loewenstein (1999). Nevertheless, most of them have a common feature, viz. they signify a shift of either the organism's structure or its perception system to a new level. As a result, people (and animals) become better suited to external conditions, do not sense any more external stimuli, and cease to think about certain phenomena.

2.3. Prospect Scaling

Prospect Scaling, as the mental transformation resulting from focusing attention, is of key significance for deriving the solution presented in the following part of this study. The springboard for discussion is the Weber law[4], one of the fundamental laws of psychophysics. The law states that the Just Noticeable Difference is a constant proportion of the initial stimulus magnitude7. It follows from the Weber law that the same change in stimulus (for instance 0.2 kg) can be strongly felt, slightly noticed or not perceived at all depending on the magnitude of the initial stimulus. It further follows that an unambiguous and absolute perception level of a specific stimulus change cannot be determined, as this depends on the situational context. This applies equally to financial stimuli. The human sensory system adapts itself to financial quantities, just as it does to physical ones. This means that when looking at financial prospects (projects, investments, lotteries etc.), the reference value (size of the investment, major lottery prize) becomes a value of reference in the entire mental process, causing an absolute amount of money (say 10 USD) to be relevant (for instance when shopping) or irrelevant (when buying a house), i.e. depending on the context. This conclusion constitutes a fundamental difference to Prospect Theory, which regards profits and losses in absolute terms, and tries to draw a value function as a function of absolute amounts of money.

The mechanism responsible for this mental transformation is attention – one of the most thoroughly examined concepts in cognitive psychology. According to one definition, attention is the process of selectively concentrating on a single perceived object, source of stimulation, or

[4] Not to be confused with the Weber-Fechner Law discussed in 2.4.

topic from among the many available options (Nęcka, 2007). The existence of attention is indispensable on account of a living organism's need to adapt to the demands of the environment (Broadbent, 1958) and on account of the finite ability of the brain to process information (Duncan, Humphreys, 1989). Several models of attention division are discussed, especially in relation to Focused Attention. The entire mechanism can be explained by such aspects of attention as Selection and Gain (Amplification) Control, the existence of which is evidenced by attention research at the neuronal level. Others, including Hillyard et al. (1998), state that attention has a gain (amplification) control character which aims to increase the signal to noise ratio of the stimuli on which attention is focused. The signal of most interest to the brain is maintained at a stable and optimal level as a result. Further, it is assumed that the amplification control mechanism operates at a higher mental level as well. This leads to problems differing in scale being perceived as equally significant when attention is focused. It is not difficult to conclude that the mathematical equivalent of amplification is homothety or scaling.

The arguments cited indicate that the attention focused on a specific payment in the conducted experiments seems to be a natural effect that has to be factored into any analysis of the results. This is especially the case under experimental conditions as those surveyed are remunerated for their participation; it means they are paid to focus their whole attention on the analyzed problems. The assumption that the value of a prospect payment becomes a reference value in the conducted experiments leads to a completely different solution than that which Prospect Theory proposes.

2.4. Logarithmic Perception of Financial Stimuli

Logarithmic perception of financial stimuli is the last mental transformation significant to deriving the results presented in the following part of the study. Here, the reference point for discussion is also a fundamental psychophysical law, viz. the Weber-Fechner law, which concerns the logarithmic perception of stimuli.[5] Logarithmic perception of financial figures is not considered in Prospect Theory despite there being known examples of logarithmic and exponential functions being used in financial applications.[6] Instead, the authors of Prospect Theory used the relationship introduced by Stevens (1957), who stated that stimuli perception was determined by a power function. That type of function was included in Prospect Theory to describe the value

[5] Hearing described using the decibel scale is an example of this sort of perception.
[6] Suffice it to mention compound interest, logarithmic index charts and logarithmic rates of return.

function. Surprisingly the difference in approach turns out to be insignificant since both functions (logarithmic and power) have an almost identical shape for low x-coordinate values.[7] This leads to the conclusion that the perception of monetary amounts used in Kahneman and Tversky's experiments could be equally well described using a logarithmic curve. Some arguments in favor of a logarithmic perception of financial stimuli are provided by other results presented in CPT, which states that mixed prospects are accepted when gains are at least twice as high as losses. This effect may be easily explained by noticing that in logarithmic terms, a 100% profit corresponds to a 50% loss. The experimental results presented in the CPT article also show that, for a probability of 0.5, the certainty equivalents appear to be around 0.41[8] of the payment value irrespective of the type of prospect (gain or loss), the presence or absence of riskless components, or the payment amount after deducting the riskless components. This effect can be interpreted as being a result of a logarithmic perception of payment value. A very strong argument in support of a logarithmic, rather than a "power", perception of monetary amounts is given in Section 6 of this paper.

3 Solution Using Mental Adaptation & Prospect Scaling Transformations

This section contains the alternative analysis of the experimental data presented by Kahneman and Tversky in their 1992 paper. This analysis is based on the assumption that apart from Mental Adaptation Transformation, Prospect Scaling should be also considered when analyzing the experimental data. It is here assumed that the reference values for the certainty equivalents under examination were the prospect payments (outcomes) themselves.

During the experiment conducted by Kahneman and Tversky, certainty equivalents CE were collected for the prospects of payment $\$P_{min}$ with probability $1 - p$ or payment $\$P_{max}$ with probability p, where:

$$|P_{min}| < |P_{max}| \tag{3.1}$$

The payment $\$P_{min}$ should be interpreted as the riskless component. The experimental results are presented in Table 3.3 of the original CPT publication (1992). It is assumed that there is

[7] Within the range [0, 0.6], $x^{0.88}/1.34$ is the best approximation of the $ln(1+x)$ function using a power function. The coefficient 0.88 is exactly the same as the power coefficient of the value function in Prospect Theory.

[8] This is exactly $\sqrt{2}-1$.

a function F such that:

$$CE = F(P_{min}, P_{max}, p) \tag{3.2}$$

The variables CE' and P_{max}' are now introduced to account for the mental adaptation process. These are a P_{min} translation of CE and P:

$$CE' = CE - P_{min} \tag{3.3}$$

$$P' = P_{max} - P_{min} \tag{3.4}$$

If $P_{min} = 0$, we refer to the prospect as having no riskless component and then $CE' = CE$ and $P' = P_{max}$. Introducing these new variables presupposes the existence of a function G such that:

$$CE' = G(P', p) \tag{3.5}$$

At this point (3.5) is transformed in such a way that probability p, and not CE', becomes the value to be determined. Due to the fact that CE' is monotonic with respect to p, it may be assumed that there is an inverse function H such that:

$$p = H(CE', P') \tag{3.6}$$

In order to take Prospect Scaling into account, it is assumed that the value of payment P' becomes the reference value for the certainty equivalent CE' and that the equivalent values are scaled by a coefficient $1/P'$. As a result, a variable $r = CE' / P'$ is introduced as the relative certainty equivalent with a value in the range [0,1]. This also supports the existence of the following D function defined over the range [0,1]:

$$p = D(CE' / P') = D(r) \tag{3.7}$$

For example, for the specific values listed in Table 3.3 of Kahneman and Tversky's paper, the relationships $D(9/50) = 0.10$, $D(21/50) = 0.50$, and $D(37/50) = 0.90$ are obtained for the prospect (0, 50), and the relationships $D(14/100) = 0.05$, $D(25/100) = 0.25$ are obtained for the prospect (0, 100). For the prospect with the riskless component (50, 150), the relationships $D((64-50)/100) = D(14/100) = 0.05$, $D((72.5-50)/100) = D(22.5/100) = 0.25$, and $D((86-50)/100) = D(36/100) = 0.5$, are obtained after the Mental Adaptation Transformation.

The obtained values are plotted on the graph $p = D(r)$ and approximated using the least squares method with the assistance of the Cumulative Beta Distribution $I_r(\alpha, \beta)$ (i.e. regularized incomplete beta function). This particular function was selected because it is defined in the domain [0,1] and because of the extraordinary flexibility the two parameters α and β give its shape.

Approximations were made separately for the loss ($P < 0$)[9] and gain ($P > 0$) prospects. The results are presented in Figure 3.1.

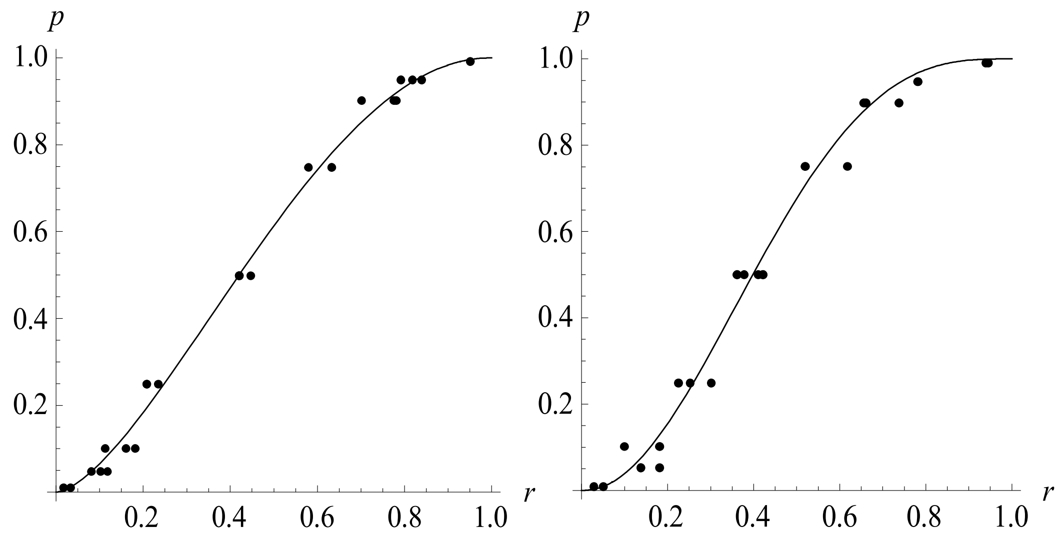


Figure 3.1 Transformed experimental points and approximation $p = D(r)$ using cumulative beta distribution function for loss prospects (left) and for gain prospects (right).

The approximations obtained for the function $p = D(r)$ for loss and gain prospects allow the following conclusions to be drawn:

1. The function $p = D(r)$ is S-shaped for both loss and gain prospects.
2. The respective values of the parameters α and β are 2.24 and 3.22 for gain prospects and 1.59 and 2.09 for loss prospects. The disparity between the parameters α and β in both cases confirms the asymmetry of the function $D(r)$ with respect to the center point $(p, r) = (1/2, 1/2)$.
3. The intersection of the approximation functions $p = D(r)$ with the straight line $p = r$ occurs when r has a value of approximately 0.25. This value is called the aspiration level, as (in case of gains) the risk seeking attitude is present for lower values of the relative outcome r, and risk aversion is present for greater values of r. This implies a change of attitude to risk at the aspiration level, which is in accordance with the generally accepted notion of this term. The pattern for losses is reversed.

Assuming Focused Attention and the resulting Prospect Scaling Transformation led to a different solution than that presented by Prospect Theory. First of all, the entire description has been reduced to the relationship $p = D(r)$, and the value function and the probability weighting

[9] It should be noted that for the loss prospects, the value of relative certainty equivalent r is also positive, as *CE'* and *P'* are both negative in this case.

function have disappeared altogether as they are not needed to describe the experimental results. Secondly, the relative certainty equivalent r is directly transformed into probability p (and vice versa). This means that in order to determine the certainty equivalent CE for probability p, the value of r need only be read directly from the graph (Figure 3.2) and multiplied by the value of payment P'. For example, $r = 0.75$ for $p = 0.95$. Hence, $CE' = 75$ for $P' = 100$ (the value obtained experimentally was 78). In case of prospects with riskless components, e.g. (50, 150), the value of the certainty equivalent $CE = CE' + P_{min} = 75 + 50 = 125$ (the value 128 was obtained in the experiment).

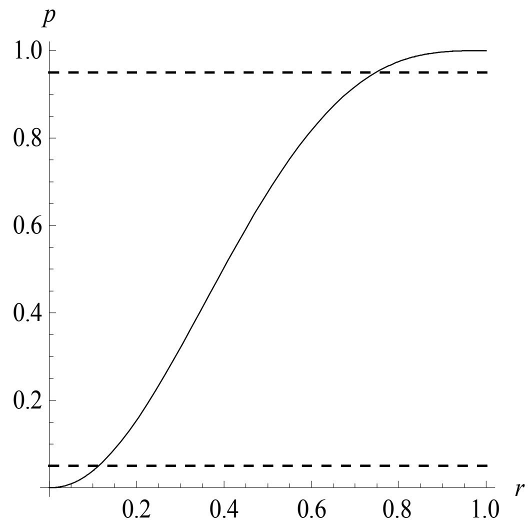


Figure 3.2 Relationship $p = D(r)$ for gain prospects with plotted lines $p = 0.05$ and $p = 0.95$.

Finally, nonlinear changes of the certainty equivalents (especially within high and low probability ranges) can be presented simply (see Figure 3.2). Increasing probability from 0 to 0.05 causes the relative certainty equivalent r to increase from 0 to 0.11. Increasing probability from 0.95 to 1 causes the relative certainty equivalent r to increase from 0.75 to 1. A similar explanation could be presented for loss prospects.

4 Combining Gains and Losses

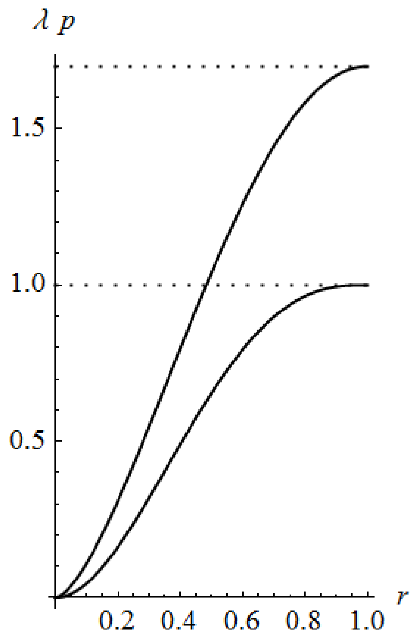
The solutions obtained so far comprise two $p = D(r)$ functions with one describing losses, the other gains. The loss and gain prospects need to be scaled before the two functions can be presented together. The simplest assumption has been adopted, similar to the Prospect Theory approach when defining the value function, namely:

$$D_p = \lambda D_n \tag{4.1}$$

where D_p is the curve for gains, and D_n is the curve for losses. In order to determine the scale factor λ, Kahneman and Tversky conducted additional experiments, the results of which are presented in Table 3.6 of the original publication. The obtained results indicate that the mixed prospects are accepted if the profit is at least twice as great as the loss. An exact ratio value of 2.07 as the mean value of Θ resulting from problems 1-6 in the Table 3.6 (Tversky and Kahneman, 1992), is assumed for further calculations. Taking into account that $D_p(1) = 1$, and $D_n(0.483) \approx 0.589$ we obtain:

$$\lambda = 1/0.589 \approx 1.70 \tag{4.2}$$

Now, let us present this result graphically. Figure 4.1a shows functions D_p and D_n (the latter multiplied by λ). It is evident that D_n is now equal to 1 for $r = 0.483$, and that the loss and gain curves are scaled. Figure 4.1b presents both functions in different form. The function D_n for the loss prospects is presented within a range of [-1,0], and the scale factor λ has a value of -1.70.



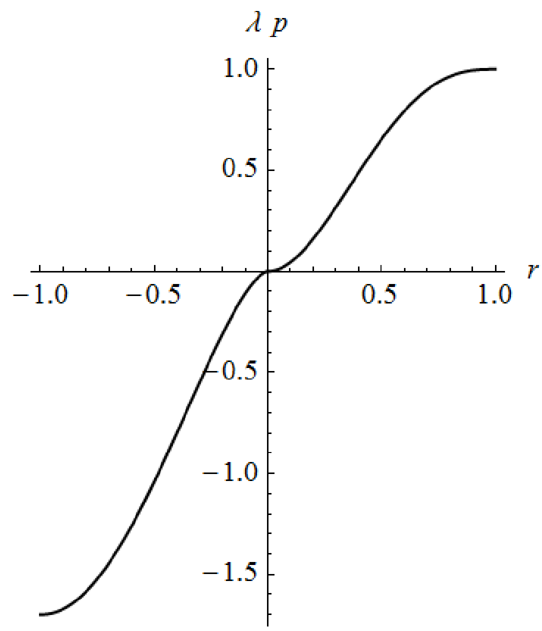


Figure 4.1 Functions $D_p(r)$ and $D_n(r)$ presented together on a single graph. (Left) within a range of r [0,1]; function $D_n(r)$ multiplied by the constant λ = 1.70, (right) function $D_n(r)$ within a range of r [-1,0] and multiplied by the constant λ = -1.70.

The question of how to interpret the curve presented in Figure 4.1b may now be posed. We call this decision utility and it needs to be stated that this curve presents the sum total of all the knowledge that has come out of Prospect Theory and its cumulative version.

1. The fourfold pattern of risk attitudes, which was presented by CPT and confirmed in other studies, is evident:

a). in case of gain prospects, the curve is convex for probabilities below 30% (corresponding to risk taking), and becomes concave for probabilities above 30% (corresponding to risk aversion);

b). in case of loss prospects, the curve is concave for probabilities below 20% (corresponding to risk aversion), and becomes convex for probabilities above 20% (corresponding to risk seeking).

2. The convex-concave-convex-concave shape of the decision utility substitutes therefore the fourfold pattern of risk attitudes described by CPT.

3. The function's more linear shape for loss prospects confirms the results of other studies that people's attitude to risk for losses is rather neutral in nature[10].

4. Both parts of the curve (for loss and gain prospects) describe the results of experiments without having to resort to the probability weighting function.

5. Both parts of the curve are scaled, which means that mixed prospects can also be analyzed.

5 The Markowitz Utility Function Hypothesis

In 1952, Markowitz published an article "The Utility of Wealth" presenting his hypothesis on the shape of the utility function. While this article was known to Kahneman and Tversky, they believed that neither this nor any other utility function could explain certain psychological experiments. This led to the development of Prospect Theory as an alternative to classical economic theories based on utility functions. That the decision utility curve so closely resembles the curve presented in the Markowitz article (Figure 5.1) is highly surprising given the result was obtained using the same experimental data used to derive Cumulative Prospect Theory.

Markowitz specified the utility function as follows: *The utility function has three inflection points. The middle inflection point is defined to be at the "customary" level of wealth. The first inflection point is below customary wealth and the third inflection point is above it. The distance between the inflection points is a non-decreasing function of wealth. The curve is monotonically increasing but bounded from above and from below; it is first concave, then convex, then concave, and finally convex. We may also assume that* $|U(-X)| > U(X)$, $X > o$ *(where* $X = o$

[10] See Wakker (2003) for reference, who confirms that the pattern for losses is less clear than in the case of gains.

is customary wealth).[11]

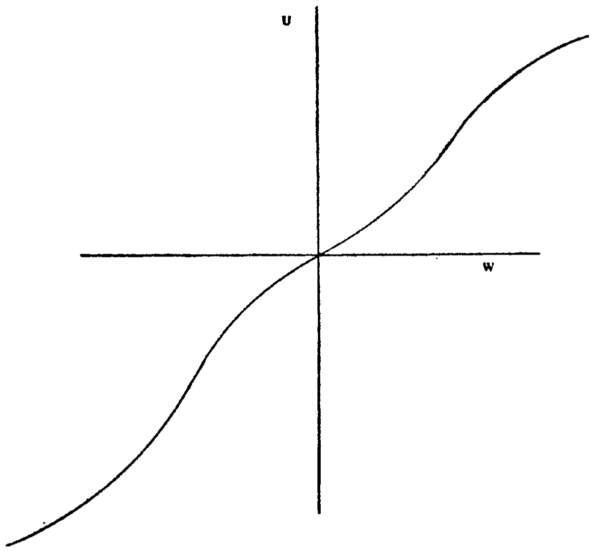


Figure 5.1 The shape of the utility function according to the Markowitz hypothesis of 1952.

It is clear that all but one of the requirements of the utility curve expressed by Markowitz in his hypothesis are met by the curve presented in Figure 4.1b. The decision utility curve has three inflection points right where Markowitz predicted they would be. The function is monotonically increasing and is limited from the top and from the bottom. Concavities and convexities occur in the order assumed by Markowitz. The condition related to the function value for X values having opposite signs is also met (which Figure 4.1a verifies). The only condition not met is that the distances between the inflection points depend on people's wealth. Markowitz noted: *If the chooser were rather rich, my guess is that he would act as if his first and third inflection points were farther from the origin. Conversely, if the chooser were rather poor, I should expect him to act as if his first and third inflection points were closer to the origin.* In the Markowitz hypothesis the position of inflection points changes because the *w* (wealth)-axis is expressed in absolute terms. This differs from the decision utility curve, where the *r*-axis is expressed relative to the size of the prospect.

There is no reason to regard this approach as being in any way anomalous. People commonly say "I have gained 15% on my stock investments" rather than "I have gained 5% *of my wealth* on my stock investments". It is clear enough that the former sentence assumes the value of

[11] One more requirement was important for Markowitz: in the case of recent windfall gains or losses the second inflection point may, temporarily, deviate from present wealth. This requirement does not influence the shape of the curve but is important when considering the dynamics of people's behavior.

the stock investment as the reference for gain/losses considerations. Moreover, according to Thaler (1985), people keep mentally separate accounts, so that investments and expenditures are considered as separate parts rather than as a whole. As a result, instead of saying "I have lost 2% of my wealth on my stock and real estate investments" people typically consider "I have lost 5% on my house but I have gained 15% on stocks" despite the fact that the absolute values of stock and house investments may differ substantially. It follows that the decision utility applies for each separate account, albeit with different reference values established by the attention focus. This may mean that people could be risk seeking and risk averse at the same time depending on the status and prospects of each account.

Markowitz's assumption that the shape of the utility curve corresponds with the value of wealth precluded his curve (however tempting its shape) from being able to explain experiments on financial payments which were not directly related to the wealth of the people being studied. This is what led Kahneman and Tversky to reject the Markowitz hypothesis and develop Prospect Theory. The result presented here, however, may signal a return to an approach based on the utility-like function and lead to a negation of Prospect Theory. Accepting that gains and losses need not be considered in relation to wealth, but to any other value depending on where a person's attention is focused, is all that it would take to come back to this earlier concept. The payoff is a simpler and more accurate description of people's behavior.

6 The Model for Multi-Outcome Lotteries

Derivation of the model for multiple outcomes is straightforward. It is assumed that for each lottery with more than two outcomes in the range [0,1], there exists an equivalent lottery with two outcomes of 0 and 1 only. The probability of winning this equivalent lottery can be determined as follows. Each outcome r_i of the multi-outcome lottery can be considered the certainty equivalent of a lottery, whose outcomes are 0 and 1, and whose probability of winning is $p_i^{'} = D(r_i)$. The joint probability of winning the equivalent lottery is therefore $\sum_{i=1}^{n} p_i \, p_i^{'}$, which leads to:

$$p_{eq} = \sum_{i=1}^{n} p_i D(r_i) \tag{6.1}$$

This p_{eq} probability is called the equivalent probability of the multi-outcome lottery. The relative certainty equivalent of the multi-outcome lottery can be found by inverting (6.1):

$$r = D^{-1}\left(\sum_{i=1}^{n} p_i D(r_i)\right) \tag{6.2}$$

Please note that (6.1) and (6.2) do not require the concept of probability weighting in either its basic or cumulative form. Please also note the strong resemblance of the equivalent probability formula (6.1) to the Expected Utility valuation. In fact, the decision utility model follows Expected Utility Theory with a transformed outcome domain. Decision utility is expressed in terms of probability and does not require any hypothetical "utils" to describe people's behavior. To put it more straightforwardly, the equivalent probability is the decision utility. Once accustomed to this seemingly strange notion, everything can be considered at the basic probability theory level.

A more detailed analysis of the model for multi-outcome lotteries is presented in Kontek (2010a). It is enough to state here that the decision utility model presents similar results to Cumulative Prospect Theory in the case of two-outcome lotteries. However, the results differ for multi-outcome lotteries. It is shown that CPT can make some "strange" predictions for a simple multi-outcome lottery. This result is largely inexplicable and calls the correctness and applicability of this theory into question. The result obtained using the decision utility function is not similarly disadvantaged. This is because it uses the classical notion of probability.

7 Including Logarithmic Perception of Financial Stimuli

An important objection raised by one of the referees was that the proposed model fails to interpret the following case. Let us suppose that $P_{min} = 0$ and $p = 0.5$. The model implies that CE/P is constant. The referee does not think that this is realistic, as he expects this ratio to decrease as P becomes very large. For instance, somebody may well be indifferent between a certain \$40 and a 50% chance of winning \$100, but will definitely prefer a certain \$40 million to a 50% chance of winning \$100 million.

We note first that (quite surprisingly) Cumulative Prospect Theory likewise fails to explain this type of case. According to this theory, the weight w associated with a probability of p is constant. Cumulative Prospect Theory then assumes the value function to be described by a power function that conforms to Stevens Law, i.e.:

$$v(x) = x^{\alpha} \tag{7.1}$$

It follows that the indifference in the first case is described as:

$$w(0.5)100^{\alpha} = 40^{\alpha} \tag{7.2}$$

Multiplying both sides of (7.2) by a constant $1{,}000{,}000^{\alpha}$ we obtain

$$w(0.5)100{,}000{,}000^{\alpha} = 40{,}000{,}000^{\alpha} \tag{7.3}$$

which also describes indifference in the second case. This means that increasing the prospect size does not change the preference, according to Cumulative Prospect Theory (in contrast to common belief). This preference change can only be explained by assuming a value function of decreasing elasticity, whereas the power function is of constant elasticity (Scholten & Read, 2010). They propose using the logarithmic function $v(x)$ = 1/a log(1 + a x) in order to explain similar observations. This is exactly the function which appeared in the first version of this paper, although in a slightly different context. It follows that the following formula should be used to calculate the *perceived* relative outcome when considering bigger amounts of money:

$$r = \frac{u(CE) - u(P_{\min})}{u(P_{\max}) - u(P_{\min})} \tag{7.4}$$

where u denotes the perception utility. As has been shown, this utility cannot be described using a power function, as postulated by Cumulative Prospect Theory. A logarithmic function is therefore preferred. Please note that:

$$\frac{\ln(1 + a\,40)}{\ln(1 + a\,100)} < \frac{\ln(1 + a\,40{,}000{,}000)}{\ln(1 + a\,100{,}000{,}000)} \tag{7.5}$$

meaning that the perceived relative outcome in the second case is greater than in the first one, and corresponding to the risk aversion part of the decision utility function. This explains the preference for a sure payment in the case of a million dollar lottery.

Parameterization of the perception utility is, however, highly unlikely using the original data set of Tversky and Kahneman as \$400 is the maximum payment they consider. This task is therefore left for further research using other experimental data. In any case, a linear perception utility can safely be assumed for small monetary amounts as in the derivation presented in Section 3.

8 Summary

The article presents an alternative interpretation of the experimental data published by

Kahneman and Tversky in their 1992 paper "Advances in Prospect Theory". Mental transformations, crucial to deriving the results, were discussed in the introduction section. Later, the solution was derived without using the probability weighting function. The obtained function has a double S-type shape that strongly resembles the utility curve specified by the Markowitz hypothesis (1952). The presented decision utility function shows that risk seeking appears for relative outcomes below the aspiration level. On the other hand risk aversion is present for relative outcomes greater than aspiration level. This pattern is reversed for losses. The explanation of risk attitudes given by the convex-concave-convex-concave shape of the decision utility function substitutes the fourfold pattern introduced by CPT. The paper shows that the perception utility should be described using the logarithmic function, rather than the power function as assumed by Cumulative Prospect Theory. This enables the change of preferences with increasing prospect sizes to be explained. The results presented provide a basis for negating Prospect Theory as the theory which best describes decision-making under conditions of risk and may foreshadow a return to describing people's behavior solely by using utility functions. The main problem with Prospect Theory is the probability weighting concept, which makes this theory so difficult to apply to more complex applications (for instance multi-outcome lotteries). Coming back to the classical notion of probability should make it possible to use it to model real world conditions.

References:

Allais, M., (1953). *Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école Américaine.* Econometrica 21, 503-546.

Blavatsky, P., (2005). *Back to the St. Petersburg Paradox?,* Management Science 51, pp. 677-678.

Brickman, P., Campell, D., (1971). *Hedonic relativism and planning the good society*. In M. H. Appley (Editor.), *Adaptation-level theory: A symposium* (pp. 287-302). New York Academic Press.

Broadbent, D.E., (1958). *Perception and Communication.* London: Pergamon.

Camerer, C. F., Teck Ho., (1994). *Violations of the Betweenness Axiom and Nonlinearity in Probabilities.* Journal of Risk and Uncertainty, 8, 167-96.

Darwin, C. (1859). "Origin of Species by Means of Natural Selection: Or, the Preservation of the Favoured Races in the Struggle for Life".

Duncan, J., Humphreys, G.W., (1989). *Visual search and stimulus similarity.* Psychological Review, *96*, 433-458.

Frederick, S., Loewenstein, G., (1999). *Hedonic Adaptation*. In Kahneman, D., Diener, E. Schwarz, N. (Editors) *Well-Being. The Foundations of Hedonic Psychology*. Russell Sage Foundation. pp. 302-329.

Friedman, M., Savage, L.P., (1948). *The Utility Analysis of Choices involving Risk.* Journal of Political Economy, Vol. 56, p.279-304.

Greenberg, N., *Adaptation defined.* https://notes.utk.edu/bio/greenberg.nsf .

Gregory, R. L., Colman, A. M., (1995). *Sensation and Perception*. Longman Group Limited.

Hildebrandt, K. J., (2010). *Neural adaptation in the auditory pathway of crickets and grasshoppers*. Dissertation, Mathematisch-Naturwissenschaftlich Fakultaet, Humboldt-Universitaet, Berlin.

Hillyard, S.D., Hoff, K., Propper, C.R., (1998). *The water absorption response: a behavioral assay for physiological processes in terrestrial amphibians*. Physiol. Zool., 71, 127–138.

Jasinskaja-Lahti, I., (2008). *Long-term immigrant adaptation: Eight-year follow-up study among immigrants from Russia and Estonia living in Finland.* International Journal of Psychology, 43 (1), pp. 6-18.

Kahneman, D. (1999). *Objective Happiness*. In Kahneman, D., Diener, E. Schwarz, N. (Editors) *Well-Being. The Foundations of Hedonic Psychology*. Russell Sage Foundation. pp. 302-329.

Kahneman, D., Tversky, A., (1979). *Prospect theory: An analysis of decisions under risk.* Econometrica, 47, 313-327.

Kontek, K., (2010a). *Decision Utility Theory: Back to von Neumann, Morgenstern, and Markowitz*. MPRA Working Paper http://mpra.ub.uni-muenchen.de/27141/, available at: http://ssrn.com/abstract=1718424

Kontek, K., (2010b). *Two Kinds of Adaptation*. MPRA Working Paper http://mpra.ub.uni-muenchen.de/25169/, available at: http://ssrn.com/abstract=1679447

Laughlin, S. B., (1989). *The role of sensory adaptation in the retina*. Journal of Experimental Biology, 146, pp. 39-62.

Leakey, R. E., (1981). *Making of Mankind*. E P Dutton.

Leon, G., R., (2004). *Overview of the psychosocial impact of disasters.* Prehosp Disast Med, 19(1), pp. 4-9.

Levy, M., Levy, H., (2002). *Prospect Theory: Much Ado About Nothing?*. Management Science, v.48 n.10, p.1334-1349, October

Maniezzo, V., Roffilli, M., (2007). *A Psychogenetic Algorithm for Behavioral Sequence Learning.* International Journal on Artificial Intelligence Tools, 16, pp. 195-217.

Markowitz H., (1952). *The Utility of Wealth.* Journal of Political Economy, Vol. 60, p.151-8.

Medawar, P. B., Medawar, J.S., (1983). *Aristotle to Zoos. A Philosophical Dictionary of Biology.* Harvard University Press, Cambridge, Massachusetts.

Mochon, D., Norton, M. I., Ariely, D., (2008). *Getting off the hedonic treadmill, one step at a time: The impact of regular religious practice and exercise on well being*. Journal of Economic Psychology, 29, pp. 632-642.

Neilson, W. S, Stowe, J., (2002). *A Further Examination of Cumulative Prospect Theory Parameterizations.* Journal of Risk and Uncertainty, Springer, vol. 24(1), pages 31-46, January

von Neumann J., Morgenstern O., (1944). *Theory of Games and Economic Behavior*, Princeton University Press.

Nęcka, E., et. al., (2007). *Psychologia poznawcza,* Wydawnictwo Naukowe PWN i ACADEMICA Wydawnictwo SWPS 2008.

Nwogugu, M., (2006). *A further critique of cumulative prospect theory and related approaches.* Applied Mathematics and Computation 179(2): 451-465

Quiggin, J., (1982). *A theory of anticipated utility*. Journal of Economic Behavior and Organization 3(4), 323–43.

Prelec, D., (1998). *The Probability Weighting Function.* Econometrica, 66:3 (May), 497-527.

Rappaport, R.A., (1971). *Ritual, sanctity, and cybernetics.* American Anthropologist, 73(1), pp. 59-76.

Savage, L. J., (1954). *The Foundations of Statistics.* John Wiley and Sons, New York.

Schmeidler, D., (1989). *Subjective Probability and Expected Utility without Additivity.* Econometrica, 57: 571-587.

Scholten, M., Read, D., (2010). Anomalies to Markowitz's Hypothesis and a Prospect-Theoretical Interpretation. SSRN Working Paper http://ssrn.com/abstract=1504630

Shu, Li., (1995). *Is there a decision weight [pi]?.* Journal of Economic Behavior & Organization, 1995, vol. 27, issue 3, pages 453-463

Stevens, S. S., (1957). *On the psychophysical law.* Psychological Review 64(3):153–181. PMID 13441853.

Sulavik, J., (1997). *An elementary form of mental adaptation to death.* Human Affairs, 7, 1997, 2, 113-117.

Thaler, R. H., (1985). *Mental accounting and consumer choice*. Marketing Science, 4, 199-214.

Tversky, A., Wakker, P., (1995). *Risk Attitudes and Decision Weights.* Econometrica 63, 1255-1280

Tversky, A., Kahneman, D., (1992). *Advances in Prospect Theory: Cumulative Representation of Uncertainty.* Journal of Risk and Uncertainty, Springer, vol. 5(4), pages 297-323, October.

Wakker, P., (1990). *Characterizing optimism and pessimism directly through comonotonicity.* Journal of Economic Theory, Elsevier, vol. 52(2), pages 453-463, December.

Wakker, P., (1989). *Continuous subjective expected utility with non-additive probabilities.* Journal of Mathematical Economics, Elsevier, vol. 18(1), pages 1-27, February.

Williams, G. C., (1966). *Adaptation and natural selection; a critique of some current evolutionary thought.* Princeton.

Wu, G., Gonzalez, R., (1996). *Curvature of the Probability weighting function.* Management Science 42, 1676-1690.

Wu, G., Gonzalez, R., (1999). *On the Shape of the Probability weighting function.* Cognitive Psychology, 1999, 38(1), pp. 129-66. http://dx.doi.org/10.1006/cogp.1998.0710