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# Firm entry, credit availability and monetary policy

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## Abstract

This paper presents a dynamic general equilibrium model that incorporates firm entry under credit rationing. Goods-producing firms in this model are bank dependent in the sense that they have no choice but to borrow funds from banks to cover labor wages that must be paid in advance of production. The results show that a cut in the policy rate enhances firm entry by mitigating the severity of credit rationing. This policy transmission is different from the conventional balance-sheet channel in that a change in the policy rate directly affects borrowers' credit availability. I also show that a sudden stop in the credit supply to new firms is most likely to occur shortly after a credit boom. This is because endogenous downward wage rigidity prohibits the credit risk of prospective firms from decreasing enough to re-equilibrate the loan market.

*JEL Classification:* E32, E44, E52

*Keywords:* credit channel, credit crunch, credit rationing, firm entry, monetary policy transmission.

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# 1 Introduction

In the face of the recent financial turmoil, central banks have paid great attention to the credit availability of firms, especially small and medium-sized enterprises (SMEs). In the traditional literature on the credit channel of monetary policy, it is widely recognized that SMEs are more susceptible to shifts in monetary policy compared to large firms (Gertler and Gilchrist, 1993, 1994, Boughéas et al., 2006). Proponents of the bank lending channel argue that this is because SMEs do not have access to alternative sources of external finance such as CP or corporate bonds, which are issued mostly by large firms. Advocates of the balance sheet channel, on the other hand, insist that SMEs bear the brunt of a tightening of monetary policy since SMEs' external finance premiums tend to be higher than large firms' (Bernanke, Gertler and Gilchrist, 1996). A crucial difference between the balance sheet channel and the bank lending channel is that, in the former, a rise in the policy rate *does not* affect credit availability itself while it reduces SMEs' demand for external funds.

In this paper I focus on the role of credit availability in monetary policy transmission and business cycles. Although credit availability has been deemed a key factor in the context of the bank lending channel, the practical validity of this channel has been questioned since most financial institutions in practice can usually raise external funds from the financial market in various ways.<sup>1</sup> In Bernanke, Gertler and Gilchrist's (1999) "financial accelerator" model, on the other hand, the possibility of credit rationing is precluded for the purpose of solving the model, so that the credit market is always equilibrated. However, various kinds of data and empirical studies strongly support the existence of credit rationing, and some of them report that monetary policy affects the severity of firms' credit availability (e.g., Atanasova and Wilson, 2004, Jiménez et al., 2010).

Based on Bergin and Corsetti (2005), Ghironi and Melitz (2005) and Bilbiie et al. (2007, 2008), I construct a dynamic general equilibrium model that incorporates endogenous firm entry.<sup>2</sup> I assume that each firm is required to raise funds to cover fixed costs that must be paid in advance of production. The most important departure from the previous studies is that firms have no choice but to obtain credit from financial intermediaries to finance the fixed costs. Other sources of financing, such as equity, commercial papers and corporate bonds, are unavailable.<sup>3</sup> In this environment, whether or not a potential firm can enter the market depends fully on the availability of credit.

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<sup>1</sup>See, for example, Bernanke (2007). Ashcraft and Campello (2007) point out that small banks that are affiliated with the same holding company can reallocate funds internally in response to monetary policy shocks. They conclude that borrowers' creditworthiness is crucial in determining the volume of bank loans.

<sup>2</sup>Devereux et al. (1996), Broda and Weinstein (2007) and Bernard, et al. (2006) show that net firm entry is procyclical and that a significant fraction of output fluctuations is attributable to the creation of new products and the destruction of existing products.

<sup>3</sup>As I show below, approximately 90% of SMEs in Japan exploit bank loans as a source of funding. Jaffee and Stiglitz (1990) provide some reasons why bank lending tends to be the only source of financing for small firms.

Following Williamson (1987), Bernanke et al. (1999) and others, financial intermediaries must incur auditing costs in order to reveal the state of defaulted firms (costly state verification). In the presence of auditing costs, there would exist a threshold of loan rates above which the lender can no longer extend credit.<sup>4</sup> If the lending rate is determined at the threshold value, then some applicants will necessarily be unable to obtain credit. In fact, the analysis shows that credit rationing arises in equilibrium since prospective entrants continue to enter the market until their credit risk becomes too high for financial intermediaries to extend credit. The point is that the creditworthiness of prospective firms is decreasing in the mass of firms because an increase in the amount of labor leads to higher wages. The mass of new firms is thus determined at a finite value even when the expected profits of entry are still positive.

Since firms with lower technology are more likely to go bankrupt, the distribution of firm-specific technology levels would shift over time. As a consequence, the credit spread tends to become smaller as the firm ages. Each firm's specific technology will be revealed only if a bank audit is made while the bank knows the distribution of each generation's technology. When solving this model, one needs to keep track of each generation's technology distribution and the corresponding mass.

I show that a change in the nominal interest rate has a significant impact on real output through its effect on loan supply. The intuition for this is as follows: suppose that the central bank cuts the nominal interest rate, which is the cost of funds for financial intermediaries. Since debt contracts are made in nominal terms, a reduction in the nominal interest rate will allow the financial intermediaries to take higher risks. This implies that a part of the prospective entrants who would otherwise be unable to obtain credit become able to get credit. Therefore, an expansionary monetary policy enhances new firm entry and stimulates aggregate output. This "credit rationing channel" originated by the extensive margin effect differs from the conventional bank lending channel in that rationing stems not from the lack of bank liquidity, but from the lack of borrowers' creditworthiness. In this sense, the credit rationing channel may be viewed as complementing the traditional balance sheet channel.

The influence of monetary policy on firm dynamics is also explored by Bergin and Corsetti (2005) and Bilbiie et al. (2008), who assume that new entrants can issue equity to cover entry costs. In their models, a policy shift can affect each firm's entry decision through its influence on the firm's current value as long as price stickiness exists. In contrast, the real effects of monetary policy considered in this paper do not rely on price stickiness. Other studies related to this paper are De Fiore and Tristani (2008) and Stebunovs (2008). De Fiore and Tristani (2008) considered the role of nominal financial contract in a model

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<sup>4</sup>Another strand of literature, such as the work by Stiglitz and Weiss (1981), focuses on adverse selection between lenders and borrowers as the source of credit rationing. See Jaffee and Stiglitz (1990), Tirole (2006) and Freixas and Rochet (2008) for a survey of the literature on credit rationing.

without endogenous entry based on Bernanke et al.'s (1999) model. Stebunovs (2008) introduced endogenous entry into a DSGE model with banks, but he considered a situation in which banks' bargaining power is so strong that they can collect all the firms' profits without incurring auditing costs.

Finally, I examine in what circumstances a "credit crunch" or "credit crisis" is most likely to occur. In this model, a "credit crunch" or "credit crisis" is defined as a situation in which bank lending to new entrants suddenly stops. The simulation shows that, in the midst of a credit boom, a relatively small shock can induce a credit crunch. More specifically, a credit crunch tends to follow a significant increase in the aggregate output, the mass of firms, the amount of credit supplied, aggregate labor, labor wages, TFP, and sharp declines in default probability and credit spreads. I discuss why these phenomena can be symptomatic of a credit crisis.

## 2 SMEs' credit constraints and the real economy: The case of Japan

Before proceeding to a formal analysis, this section takes a brief look at the Japanese data regarding the condition of SMEs' external finance and its correlation with the real economy.

### 2.1 Two types of credit rationing

In general, credit rationing arises for two reasons. One is a borrower's lack of creditworthiness. If a loan applicant is judged unable to yield a sufficient amount of profits in the future, then the expected net return of lending will be negative and the bank will not extend credit. Let us call this sort of rationing *type-D credit rationing* since it stems from the demand side. The other possibility is that banks cannot extend credit even when the net expected return of lending is nonnegative. This situation arises either when banks cannot collect enough funds to lend, as the theory of bank lending channel insists, or when banks are constrained by bank-capital requirements. Financial institutions that do not have enough capital are required to control the total amount of loans they make in order to meet bank-capital requirements.<sup>5</sup> If this is the case, the source of SMEs' difficulty in external finance stems from banks' balance sheet condition rather than applicants' creditworthiness. Let us call this type of rationing *type-S credit rationing* since it is caused by a supply-side factor.

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<sup>5</sup>Van den Heuvel (2006) investigates the effectiveness of monetary policy when a bank is required to maintain a certain level of capital.

## 2.2 Limited availability of credit for SMEs: Evidence from survey data

A necessary condition for the theoretical analysis of this paper to be valid is that the size of type-D rationing is economically significant. However, showing the existence of credit rationing is necessarily difficult, as it requires an identification of loan demand and loan supply. While some empirical studies propose various ways to identify loan demand and loan supply, I show more direct evidence of the presence of rationing using survey data concerning Japanese SMEs.<sup>6</sup>

First, I show data taken from the *Basic Survey of Small and Medium Enterprises*, which is published by the Ministry of Small and Medium Enterprise Agency every year. Table 1 reports the proportion of SMEs for which the application of a bank loan has been denied by their “main banks” in the past year.<sup>7</sup> It reveals that at least 6-9% of loan applicants had their loan application refused or the amount of it reduced. One might think that this fraction is not significant, but there are at least two reasons why this may be an underestimate. First, there is a strong possibility that there existed “discouraged borrowers”, who did not apply for loans because they thought they would be rejected even though they needed funds. Unfortunately, precise data on the number of discouraged borrowers are not available regarding Japanese SMEs, but the *Survey of Small Business Finances (SSBF) 2003* in the US reports that 17.9% of SMEs were discouraged borrowers.<sup>8</sup>

The second limitation of the above survey data is that it excludes young firms that were launched in the past few years due to the unavailability of real-time information. For example, the sample firms included in the *Basic Survey of Small and Medium Enterprises 2008* are taken from the *Establishment and Enterprise Census 2006* published by the Ministry of Internal Affairs and Communications.<sup>9</sup> It is natural to expect that younger and smaller firms are more financially constrained than older and larger SMEs. If this is the case, the extent of credit rationing will be more severe than the above data suggest. To examine this possibility, Table 2 shows how the availability of credit and the source of funding depend on firms’ developmental phases. It states that, among SMEs at the initial stage of firm growth, 38.7% answered that they have had some problems with external financing. The corresponding fraction for the most developed SMEs is 2.6%. This strongly suggests that the fraction of firms that have difficulty in obtaining funds shown in Table 1 is biased downward due to the fact that the data do not include the youngest SMEs, which are most likely to be financially constrained. Moreover, the table also reveals that the fraction of loans from financial institutions among all sources of finance is lower for less-developed

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<sup>6</sup>For empirical analyses of credit rationing, see Atanasova and Wilson (2004) and Ciccarelli et al. (2010).

<sup>7</sup>There can be various definitions of “main bank”, but the *Basic Survey of Small and Medium Enterprises* directly asks each SME who its main bank is.

<sup>8</sup>SSBF 2003 also reports that 85.1% of loan applicants have always been approved and 10.3% have always been denied.

<sup>9</sup>This type of information lag varies since the *Establishment and Enterprise Census* is not conducted every year. The maximum lag is 4 years.

SMEs than for developed SMEs. This reflects the fact that young firms are likely to rely on relatively less efficient sources of funding, such as borrowing from executives, business partners and relatives. This observation reinforces the view that smaller firms are more likely to be credit constrained than larger firms.

Next, let us look at some judgment surveys taken from TANKAN, which is published by the Bank of Japan on a quarterly basis. Figure 1 shows the judgment survey of SMEs on changes in the lending attitudes of financial institutions, lending rates and the ease with which SMEs get external funds (denoted as “financial position”). According to these data, there is a strong correlation between a financial institution’s lending attitude and the ease of external finance: an accommodative lending attitude makes it easier for firms to get external funds. This correlation could be viewed as supporting the model of balance sheet channel as long as the ease of external finance stems from changes in the cost of borrowing. However, the figure reveals that SMEs’ difficulty in external finance is not necessarily positively correlated to loan rates. This implies that SMEs do not regard the cost of borrowing as the only source of financing difficulty. There must be another reason why SMEs have problems obtaining external finance. Time-varying credit availability is one possible factor that prevents the difficulty of external finance from moving in tandem with the cost of borrowing.<sup>10</sup>

Decomposing the observed rationing into type-S and type-D is beyond the scope of this paper, but some data appear to support the presence of type-D rationing. Table 3 shows that about 50% of SMEs that feel financial institutions’ credit standards have risen attribute this rise to the deterioration of their own profitability, while 43.8% of SMEs reported that the increasing difficulty in obtaining external funds stems from the financial institutions’ problems. In addition, as is shown in Table 2, the fact that the fraction of SMEs that have some problems with external financing declines with firm age implies that the availability of credit depends largely on demand-side factors.

In the model presented below, I examine the relationship between firm entry and credit availability. As pointed out by Ghironi and Melitz (2005) among others, the terminology “firm entry” could also be interpreted as the establishment of new plants or investments. If a certain fraction of firms are credit constrained, then the ease of external finance is expected to have a positive correlation with the number of new firm entries and investments. Figure 2 shows this is indeed the case. There is a strong positive correlation between the number of newly launched firms, SMEs’ fixed investments and the ease of obtaining external finance.

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<sup>10</sup>Lown and Morgan (2002, 2006) stress the importance of credit standards, as opposed to loan rates, in explaining the behavior of bank loans, real GDP and inventory investment in the U.S. They show that credit standards, as a proxy for credit availability, are far more informative than loan rates about the total amount of loans.

### 3 The model

The model economy consists of a representative household, goods-producing firms, a financial intermediary, a central bank, and the government. The representative household consumes a variety of goods while supplying labor service in the goods sector. Since the mass of firms is allowed to vary over time, the goods firms can be classified into two groups: new entrants and incumbents. Firms have to employ a fixed amount of labor in order to make preparations for production at the beginning of each period. Accordingly, they need to raise funds in the financial market in advance of production to cover the fixed costs. I consider a situation in which bank lending is the only source of financing. Unlike the standard endogenous entry models, equity issuing is not allowed. Since renegotiation of debt contract is not permitted, a firm goes bankrupt if it fails to earn enough profits to repay the principal plus interest to the financial intermediary. The sequence of events is illustrated in Figure 4.

#### 3.1 Households

The one-period utility function of a representative household is given as

$$U_t = \log C_t - \eta \int_{i \in \Omega_t} \frac{L_t(i)^{1+\phi}}{1+\phi} di - \tilde{\eta} \int_{i \in \Omega_t} \frac{f_t(i)^{1+\tilde{\phi}}}{1+\tilde{\phi}} di \quad \phi, \tilde{\phi}, \eta, \tilde{\eta} \geq 0$$

where  $C_t \equiv \left[ \int_{i \in \Omega_t} C_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ , and  $C_t(i)$  and  $L_t(i)$  are the consumption of differentiated good  $i$  and the hours worked for the production of consumption good  $i$ , respectively.  $\theta (> 1)$  denotes the elasticity of substitution between the variety of goods. The optimization of the allocation of consumption goods yields the aggregate price index  $P_t \equiv \left[ \int_{i \in \Omega_t} P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$ , where  $P_t(i)$  denotes the price of good  $i$ .  $\Omega_t$  is the set of the total variety of goods available in period  $t$ .  $f_t(i)$  represents the hours worked for firm  $i$  at the beginning of period  $t$  in preparation for production. I assume that a certain amount of preparatory work needs to be done at the beginning of each period in order to begin production. The point is that the preparatory work is a requisite for production, so that  $f_t(i)$  is not necessarily positively correlated with the realized amount of products. Practical examples of this type of “non-productive” but requisite work would be personnel management, test marketing, negotiation with banks in obtaining credit, etc. The necessity of hiring those preparatory workers requires firms to incur fixed costs, which must be paid in advance of production.

The household needs to use cash to purchase consumption goods. At the beginning of period  $t$ , the amount of cash available for the purchase of consumption goods is  $M_t + \int_{i \in \Omega_t} \tilde{W}_t(i) f_t(i) di - S_t$ , where  $M_t$  is the nominal balance held from period  $t-1$  to  $t$ , and  $\int_{i \in \Omega_t} \tilde{W}_t(i) f_t(i) di$  represents the total nominal preparatory-labor wage paid at the



beginning of period  $t$ . The production-labor wage  $\int_{i \in \Omega_t} W_t(i)L_t(i)di$  is paid after production has started. The household also makes a one-period deposit  $S_t$  at the beginning of the period and the interest on it,  $R_t$ , is paid at the end of the period. It follows that the following cash-in-advance (CIA) constraint must be satisfied:<sup>11</sup>

$$P_t C_t \leq M_t + \int_{i \in \Omega_t} \tilde{W}_t(i) f_t(i) di - S_t.$$

The household's budget constraint is given by

$$M_{t+1} = M_t + \int_{i \in \Omega_t} \tilde{W}_t(i) f_t(i) di - S_t - P_t C_t + R_t S_t + \int_{i \in \Omega_t} W_t(i) L_t(i) di + \Pi_t - T_t$$

where  $\Pi_t$  denotes the sum of profits transferred from firms and the financial intermediary, and  $T_t$  is a lump-sum transfer from the government. Since the CIA constraint holds with equality in each period as long as there is an opportunity cost of holding cash, the budget constraint can be rewritten as:

$$P_t C_t + S_t = \int_{i \in \Omega_t} \tilde{W}_t(i) f_t(i) di + R_{t-1} S_{t-1} + \int_{i \in \Omega_{t-1}} W_{t-1}(i) L_{t-1}(i) di + \Pi_{t-1} - T_{t-1}.$$

Due to the CIA constraint, the production-labor wage income in period  $t$ ,  $\int_{i \in \Omega_t} W_t(i) L_t(i) di$ , cannot be used until period  $t+1$ , while the preparatory-labor wage income,  $\int_{i \in \Omega_t} \tilde{W}_t(i) f_t(i) di$ , can be spent in period  $t$ .

Assume for simplicity that the amount of labor required for the preparation of production depends solely on the status of the firm: new entrant or incumbent. New entrants and incumbents hire preparatory-labor  $f_{E,t}$  and  $f_{I,t}$ , respectively. The first-order conditions for the household's optimization problem are

$$R_t^{-1} = E_t \frac{\beta P_t C_t}{P_{t+1} C_{t+1}}, \quad (1)$$

$$\frac{\tilde{W}_{E,t}}{P_t} = \tilde{\eta} C_t f_{E,t}^{\tilde{\phi}}, \quad \frac{\tilde{W}_{I,t}}{P_t} = \tilde{\eta} C_t f_{I,t}^{\tilde{\phi}} \quad (2)$$

$$\beta E_t \frac{W_t(i)}{P_{t+1} C_{t+1}} = \eta L_t^{\phi}(i), \quad (3)$$

where  $\beta$  and  $E_t$  are the subjective discount factor and the expectations operator conditional on information in period  $t$ .  $\tilde{W}_{E,t}$  and  $\tilde{W}_{I,t}$  are the nominal preparatory-labor wages for new entrants and incumbents, respectively. Conditions (1) and (2) are fairly standard. Eq.(3) says that the current disutility of labor must equal the real wage evaluated in terms of future price level and future marginal utility of consumption. This is because the production-labor wages cannot be spent until the next period, whereas the household

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<sup>11</sup>It is assumed that financial markets open before the goods market.

incurs disutility of labor today. Using the Euler equation (1), this relation can be rewritten as

$$\frac{W_t(i)}{P_t} = \eta R_t C_t L_t^\phi(i). \quad (4)$$

Eq. (4) is a labor supply condition that equates the real wage to the marginal rate of substitution between consumption and leisure. However, unlike the standard one, the nominal interest rate  $R_t$  also appears in the relation. The presence of  $R_t$  represents the distortion stemming from the time lag between the earning and spending of wage income. As demonstrated below, the nominal interest rate has a real impact on the economy due to this time lag. An intuitive reason for the appearance of the nominal interest rate in (4) is as follows. In the optimal equilibrium, the current disutility of labor must equal the “future real wage”,  $W_t(i)/P_{t+1}$ , times the future marginal utility of consumption,  $C_{t+1}^{-1}$ . On the one hand, the difference between the current real wage,  $W_t(i)/P_t$ , and “future real wage” is given by the rate of inflation. On the other hand, the Euler equation (1) suggests that the difference between the current and future marginal utility of consumption is given by the real rate of interest. Therefore, when the optimality condition (3) is expressed exclusively by time- $t$  variables, the inflation terms offset each other and only the nominal interest rate term remains.

### 3.2 Goods sector

The production function of a goods-producing firm takes the form of a constant-returns-to-scale function:

$$Y_t(i) = (Z_t + z(i))L_t(i), \quad (5)$$

where  $Y_t(i)$ ,  $Z_t$  and  $z(i) \in (0, z_u)$  denote output, a common productivity shock and firm-specific productivity level, respectively. While the common productivity level is allowed to change over time, the firm-specific technology is not.  $Z_t$  takes a positive value for all  $t$ .

There are three economic agents who consume goods: the household, the government and the financial intermediary. The financial intermediary is a consumer of goods since auditing costs take the form of consumption spending. All agents have an identical elasticity of substitution between differentiated goods, which means that they will demand the same consumption basket. It follows that the goods demand function is simply given by

$$Y_t(i) = \rho_t^{-\theta}(i)Y_t, \quad (6)$$

where  $\rho_t(i) = P_t(i)/P_t$  and  $Y_t = C_t + G_t + \xi_t$ .  $G_t$  and  $\xi_t$  denote the consumption baskets purchased by the government and the financial intermediary, respectively. In the following, I assume that government spending is determined such that  $C_t = \gamma Y_t$  and hence  $G_t + \xi_t = (1 - \gamma)Y_t$ , where  $\gamma \in (0, 1)$ . This implies that fluctuations in  $\xi_t$  are completely

absorbed by the corresponding shift in government spending so as to keep the household's consumption a constant fraction of aggregate output. The reason for imposing this assumption is twofold: first, it is natural to consider that the contribution of auditing costs incurred by financial institutions to GDP is negligible compared to that of household consumption and government spending. Second, this simplification enables us to analytically prove the existence and uniqueness of financial market equilibrium. Without this assumption, there arise trivial second- and higher-order effects of a change in  $\xi_t$  on output. This is because  $\xi_t$  depends on firms' profits, which depend on aggregate output, which in turn depends on  $\xi_t$ , and so forth.

Goods prices are flexible. The relative price of good  $i$  is then given as

$$\rho_t(i) = \left( \frac{\theta}{\theta - 1} \right) \frac{W_t(i)}{P_t(Z_t + z(i))}. \quad (7)$$

Using (4) - (7) and the relation  $Y_t = \left[ \int_{i \in \Omega_t} Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ , the aggregate output can be expressed as

$$\begin{aligned} Y_t &= \left[ \left( \frac{\theta}{\theta - 1} \right) \gamma \eta R_t \right]^{\frac{-1}{1+\phi}} \left[ \int_{i \in \Omega_t} (Z_t + z(i))^\alpha di \right]^{\frac{1}{\alpha}} \\ &= \left[ \left( \frac{\theta}{\theta - 1} \right) \gamma \eta R_t \right]^{\frac{-1}{1+\phi}} \left[ \int_0^{z_u} J_t(z) (Z_t + z)^\alpha dz \right]^{\frac{1}{\alpha}}, \end{aligned} \quad (8)$$

where  $\alpha \equiv (\theta - 1)(1 + \phi)/(1 + \theta\phi)$ , and  $J_t(z)$  denotes the mass of firms whose firm-specific productivity is  $z$ . As is clear from the equation, an increase in the mass of firms will expand the aggregate output, and a reduction in the nominal interest rate will also have a positive impact on output. As I mentioned above, the latter effect stems simply from the time lag between the payment and the usage of production-labor wages.

Let  $d_t(i)$  denote the real profit prior to repayment to the financial intermediary.<sup>12</sup> The diversity in  $d_t(i)$  among firms can be expressed solely by the difference in firm-specific technology:

$$\begin{aligned} d_t(i) &= \frac{1}{\theta} Y_t(i)^{\frac{\theta-1}{\theta}} Y_t^{\frac{1}{\theta}} \\ &= \frac{1}{\theta} (Z_t + z(i))^\alpha \left[ \left( \frac{\theta}{\theta - 1} \right) \gamma \eta R_t \right]^{\frac{-1}{1+\phi}} \left[ \int_0^{z_u} J_t(z) (Z_t + z)^\alpha dz \right]^{\frac{1-\alpha}{\alpha}}, \end{aligned} \quad (9)$$

Notice that an increase in the mass of firms, which is reflected by a rise in  $\int_0^{z_u} J_t(z) (Z_t + z)^\alpha dz$ , may or may not improve the profitability of each firm, depending on the value of  $\alpha$ . This is because an increase in the mass of firms will raise the real wages while expanding aggregate demand. If the former effect dominates the latter, then profits will decrease as the mass of firms rises.

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<sup>12</sup>Firms pay the right amount of wages to production workers before paying back to the financial intermediary. Note that this is always possible since  $d_t(i)$  is nonnegative.

It is assumed that hours worked at the beginning of period  $t$  as a preparatory worker are  $f_{E,t} = f_E/Z_t$  for entrants and  $f_{I,t} = f_I/Z_t$  for incumbents.  $f_E$  and  $f_I$  are assumed to be constant. This specification implies that the fixed costs will decrease as common productivity improves.

### 3.3 Financial intermediary

Let us turn to the financial contract between a firm and a financial intermediary.<sup>13</sup> After a common productivity shock  $Z_t$  is realized, each firm attempts to borrow funds from a financial intermediary in order to pay fixed costs  $\tilde{W}_{E,t}f_{E,t}$  or  $\tilde{W}_{I,t}f_{I,t}$ . Firm-specific technology is not observable to the financial intermediary unless an audit is made after bankruptcy. Firms cannot raise capital in other ways, such as equity, trade credits, corporate bonds, or commercial paper.

#### 3.3.1 New entrants

First, let us consider a debt contract between a new entrant and a financial intermediary. Every debt contract is made in nominal terms. The gross lending rate,  $R_{E,t}^l$ , is determined according to the following no-arbitrage condition:

$$(1 - x_{E,t})R_{E,t}^l\tilde{W}_{E,t}f_{E,t} + (1 - \mu_E) \int_0^{\bar{z}_{E,t}} g(z)P_t d_t(z) dz = \chi_t R_t \tilde{W}_{E,t}f_{E,t}, \quad \mu_E \in (0, 1) \quad (10)$$

where  $\bar{z}_{E,t}$  denotes the threshold of firm-specific productivity below which firms go into bankruptcy.  $x_{E,t}$  is the probability of new entrants' default and  $g(z)$  is the density function of firm-specific technology levels.  $\chi_t \geq 1$  represents a shock that would affect the financial contract. A plausible interpretation of this shock would be a shift in the credit standard. For instance, a change in the bank's risk aversion and/or the expected profitability of borrowers will affect the bank's lending attitude. As in Williamson (1987) and Bernanke et al. (1999), the financial intermediary has to incur auditing costs in order to reveal the state of defaulted firms. As is well known, in the presence of such costs, there would arise a threshold of firm-specific technology above which the lender can no longer grant credit. This is because a rise in the threshold will increase the probability of default and thereby increase the expected cost of auditing while a rise in the threshold has a direct positive effect on the bank's revenue through a rise in the lending rate. Following Bernanke et al. (1999), I assume that the auditing costs take a form such that the financial intermediary spends a certain fraction of its revenue on final goods consumption. This leads to  $\xi_{E,t} = \mu_E \int_0^{\bar{z}_{E,t}} g(z) d_t(z) dz$ . An advantage of this specification of auditing costs is that the financial intermediary always has an incentive to audit since auditing is always beneficial. Since prospective entrants are ex ante identical, the financial intermediary assigns an identical loan rate to all entrants.

<sup>13</sup>I use the terms "financial intermediary" and "bank" interchangeably.

There exist at least two sorts of financial market imperfections here. One is the presence of auditing costs, whose role of have been occasionally discussed in the literature. The other is the unavailability of additional liquidity at the end of each period. If financial markets were perfect, then the financial intermediary would try to avoid having firms default by extending additional credit as long as the firms would be able to earn positive net profits in the future. In such a situation, firms could get the same amount of external funds as their discounted value of expected profits, and thereby the financial market structure would basically be the same as that of the standard endogenous entry models (Bilbiie et al., 2007, 2008) where equity issuing is allowed. However, this model does not allow for renegotiation, and thus the financial intermediary commits to the original contract by letting the defaulting firms exit from the market.<sup>14</sup> This unavailability of additional credit, as well as the presence of auditing costs, will keep the mass of new entrants below the frictionless level.

As a preparation for analyzing equilibrium dynamics, let us express eq.(10) in terms of threshold  $\bar{z}_{E,t}$  and the mass function  $J(\cdot)$ . The threshold of default is defined implicitly as

$$P_t d_t(\bar{z}_{E,t}) = R_{E,t}^l \tilde{W}_{E,t} f_{E,t}. \quad (11)$$

It follows that the probability of default,  $x_{E,t}$ , can be given as

$$\begin{aligned} x_{E,t} &= \text{Prob}(P_t d_t(z) < R_{E,t}^l \tilde{W}_{E,t} f_{E,t}) \\ &= \text{Prob}(z < \bar{z}_{E,t}) \\ &= G(\bar{z}_{E,t}), \end{aligned}$$

where  $G(\cdot)$  denotes the cumulative distribution function of idiosyncratic technology levels. Then, eq.(10) leads to

$$(1 - G(\bar{z}_{E,t}))d_t(\bar{z}_{E,t}) + (1 - \mu_E) \int_0^{\bar{z}_{E,t}} g(z)d_t(z)dz = \frac{\chi_t R_t \tilde{W}_{E,t} f_{E,t}}{P_t},$$

Or, owing to the absence of aggregate uncertainty, the financial contract can be expressed in a simpler form by eliminating terms that are common to both sides of the equation.

$$\begin{aligned} (1 - G(\bar{z}_{E,t}))(Z_t + \bar{z}_{E,t})^\alpha + (1 - \mu_E) \int_0^{\bar{z}_{E,t}} g(z)(Z_t + z)^\alpha dz \\ = \chi_t R_t \theta \tilde{\eta} \gamma f_{E,t}^{1+\tilde{\phi}} \int_0^{z_u} J_t(z)(Z_t + z)^\alpha dz. \end{aligned} \quad (12)$$

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<sup>14</sup>The introduction of renegotiation will change the static financial contract into a dynamic one. However, as will be made clear below, such forward-lookingness makes it quite difficult to solve the model since technology distribution changes over time. Although I do not specify a particular microfounded rationale for why the financial intermediary refuses to extend further credit to firms in default, various reasons were proposed in the previous studies. For instance, Stiglitz and Weiss (1983) insisted that a credible threat of cutting off credit will reduce the likelihood that the borrowers undertake risky projects. Jaffee and Stiglitz (1990) provide a brief review of studies on this topic.

Notice that the “time-lag effect” regarding wage income has no impact on this financial contract since the term  $R_t^{-\frac{1}{1+\phi}}$  appears in every term of eq.(10) and they cancel out each other. The LHS and RHS of (12) can be interpreted as the “quasi-expected revenue” and the “quasi-cost of funds”, respectively. For later use, let us define  $\Gamma(\bar{z}_{E,t}; Z_t)$  as follows:

$$\Gamma(\bar{z}_{E,t}; Z_t) \equiv (1 - G(\bar{z}_{E,t}))(Z_t + \bar{z}_{E,t})^\alpha + (1 - \mu_E) \int_0^{\bar{z}_{E,t}} g(z)(Z_t + z)^\alpha dz.$$

From the definition of the threshold  $\bar{z}_{E,t}$ , the loan rate can be written as

$$\begin{aligned} R_{E,t}^l &= \frac{d_t(\bar{z}_{E,t})}{(\tilde{W}_{E,t}/P_t)f_{E,t}} \\ &= \frac{(Z_t + \bar{z}_{E,t})^\alpha}{\theta\tilde{\eta}\gamma f_{E,t}^{1+\bar{\phi}} \int_0^{z_u} J_t(z)(Z_t + z)^\alpha dz} \\ &= \chi_t R_t \frac{(Z_t + \bar{z}_{E,t})^\alpha}{\Gamma(\bar{z}_{E,t}; Z_t)} \geq \chi_t R_t. \end{aligned} \quad (13)$$

Thus, the minimum value of the entrants’ loan rate is necessarily equal to or higher than the safe rate, depending on the realization of financial shock  $\chi_t$ .

An important point is that the equilibrium condition (12) holds only under “normal” circumstances. If aggregate shocks are moderate, then the mass  $J(\cdot)$  is determined such that condition (12) holds with equality through the adjustment of the mass of new entrants. However, if large unfavorable aggregate shocks hit the economy, then it becomes possible that the adjustment of  $J(\cdot)$  may fail to satisfy condition (12) because the mass of new entrants is nonnegative. In that circumstance, no prospective entrants can get credit since the expected revenue becomes smaller than the bank’s cost of funds. This possibility will be thoroughly examined in later sections.

### 3.3.2 Incumbents

By definition, incumbents survived the last period. This implies that the lower bound of the firm-specific technology distribution generally shifts rightward as the firm ages, since those firms that have inferior technologies are more likely to go bankrupt and exit from the market. The financial intermediary does not know each incumbent’s technology level. What is known to the financial intermediary is the technology distribution of each generation. Individual loans may be exposed to some risk while there is no aggregate uncertainty. The loan rates of older firms are likely to be lower than those of younger firms.

Let  $\bar{z}_{I,t-1}^q$  denote the lower bound of the firm-specific technology levels of generation- $q$ , which is a predetermined variable at  $t$ . By generation- $q$ , I mean the firms that entered the market  $q$  periods ago. A debt contract will be made if the following condition is met:

$$(1 - x_{I,t}^q)R_{I,t}^{l,q}\tilde{W}_{I,t}f_{I,t} + (1 - \mu_I) \int_{\bar{z}_{I,t-1}^q}^{\bar{z}_{I,t}^{1+q}} g_{t-1}^q(z)P_t d_t(z) dz \geq \chi_t R_t \tilde{W}_{I,t} f_{I,t}, \quad \mu_I \in (0, 1) \quad (14)$$

where  $x_{I,t}^q$  and  $g_{t-1}^q(\cdot)$  denote the probability of default and the density function of firm-specific technologies of generation- $q$ , respectively. Thus,  $x_{I,t}^q = \int_{\bar{z}_{I,t-1}^q}^{\bar{z}_{I,t}^{1+q}} g_{t-1}^q(z) dz \equiv G_{t-1}^q(\bar{z}_{I,t}^{1+q})$ . The financial contract will determine  $\bar{z}_{I,t}^{1+q} \geq \bar{z}_{I,t-1}^q$  so as to attain (14) with equality. If any value of  $\bar{z}_{I,t}^{1+q} \geq \bar{z}_{I,t-1}^q$  cannot attain equality, which occurs when the expected revenue always exceeds the cost of funds, then  $\bar{z}_{I,t}^{1+q} = \bar{z}_{I,t-1}^q$ . In this case, the generation- $q$  firms are no longer risky borrowers and that they can obtain credit at the lowest rate,  $\chi_t R_t$ .

As in the case of new entrants, the financial contract can be simplified as

$$\begin{aligned} \bar{z}_{I,t}^{1+q} &\geq \bar{z}_{I,t-1}^q && \text{if } \Gamma^q(\bar{z}_{I,t}^{1+q}; Z_t) = \chi_t R_t \theta \tilde{\eta} \gamma f_{I,t}^{1+\tilde{\phi}} \int_0^{z_u} J_t(z) (Z_t + z)^\alpha dz \\ \bar{z}_{I,t}^{1+q} &= \bar{z}_{I,t-1}^q && \text{if } \Gamma^q(\bar{z}_{I,t}^{1+q}; Z_t) > \chi_t R_t \theta \tilde{\eta} \gamma f_{I,t}^{1+\tilde{\phi}} \int_0^{z_u} J_t(z) (Z_t + z)^\alpha dz, \end{aligned} \quad (15)$$

where

$$\Gamma^q(\bar{z}_{I,t}^{1+q}; Z_t) \equiv (1 - G_{t-1}^q(\bar{z}_{I,t}^{1+q})) (Z_t + \bar{z}_{I,t}^{1+q})^\alpha + (1 - \mu_I) \int_{\bar{z}_{I,t-1}^q}^{\bar{z}_{I,t}^{1+q}} g_{t-1}^q(z) (Z_t + z)^\alpha dz.$$

Again, there is another possibility that  $\Gamma^q(\bar{z}_{I,t}^{1+q}; Z_t) < \chi_t R_t \theta \tilde{\eta} \gamma f_{I,t}^{1+\tilde{\phi}} \int_0^{z_u} J_t(z) (Z_t + z)^\alpha dz$ . If this is the case, some of the incumbents are forced to exit from the market since they cannot obtain credit required to begin production. As is discussed below, however, I do not treat this possibility because there is a technical difficulty in obtaining a unique solution.

### 3.3.3 Financial market equilibrium

Since the central bank injects money into the financial intermediary at the beginning of each period, the financial market equilibrium condition is given as

$$N_t^E \tilde{W}_{E,t} f_{E,t} + N_t^I \tilde{W}_{I,t} f_{I,t} = S_t + \Delta M_{t+1},$$

where  $\Delta M_{t+1} \equiv M_{t+1} - M_t$  and  $N_t^E$  and  $N_t^I$  denote the mass of entrants and incumbents, respectively. The profits of the financial intermediary are transferred to the household at the end of the period.

## 4 Firm dynamics and the role of credit rationing

### 4.1 Firm entry condition

There is an infinite mass of prospective entrants, and they will try to enter the market if and only if the discounted value of the expected profits is positive. In this model, however, the bank will not extend credit if the firm's one-period profits are negative. Thus, those firms that would be able to obtain credit automatically satisfy the entry condition. Clearly,

positive expected profits are not a sufficient condition for the prospective entrants to be able to obtain credit. Because the bank has to incur some auditing costs, the bank lending standards are more severe than the simple zero profit condition.

## 4.2 Equilibrium credit rationing

Since prospective entrants have nothing to lose, their net benefit is zero if they go bankrupt. This implies that firm entry continues as long as bank credit is available. The question is: how can the mass of firms be determined at a finite value?

The total demand for funds is basically infinite since an unbounded mass of prospective firms tries to enter the market. Thus, the determination of the mass of new entrants depends fully on credit availability. In other words, there is always room for an “effective supply” of funds as long as the mass of firms is finite.<sup>15</sup>

In order to see how the mass of firms is determined, we have to see eq. (12) in more detail. First of all, in order to exclude a trivial situation in which the probability of default is one, the LHS of (12),  $\Gamma(\bar{z}_{E,t}; Z_t)$ , must have a global maximum at  $\bar{z}_{E,t} = \bar{z}_{E,t}^* < z_u$ . If this is not the case, it is optimal for the financial intermediary to set  $\bar{z}_{E,t}$  at  $z_u$  so that no firm will be able to survive. In fact, whether this is the case or not depends on the distribution of idiosyncratic technologies.<sup>16</sup> In the following, I assume that new entrants’ firm-specific technology levels are uniformly distributed over  $[0, z_u]$ .

The first-order differentiation of  $\Gamma(\bar{z}_{E,t}; Z_t)$  with respect to  $\bar{z}_{E,t}$  leads to

$$\begin{aligned} \Gamma'(\bar{z}_{E,t}; Z_t) &= (Z_t + \bar{z}_{E,t})^\alpha \left[ \alpha(1 - G(\bar{z}_{E,t}))(Z_t + \bar{z}_{E,t})^{-1} - \mu_E g(\bar{z}_{E,t}) \right] \\ &= (Z_t + \bar{z}_{E,t})^\alpha \left[ \alpha \left(1 - \frac{\bar{z}_{E,t}}{z_u}\right) (Z_t + \bar{z}_{E,t})^{-1} - \frac{\mu_E}{z_u} \right]. \end{aligned} \quad (16)$$

It follows that  $\Gamma(\bar{z}_{E,t}; Z_t)$  attains the global maximum at

$$\hat{\bar{z}}_{E,t} = \frac{\alpha z_u - \mu_E Z_t}{\alpha + \mu_E}. \quad (17)$$

Recall that the bank will lend funds as long as the equilibrium condition (12) is satisfied. An increase in the mass of firms following the granting of a bank loan is reflected by an increase in the value of  $\int_0^{z_u} J_t(z)(Z_t + z)^\alpha dz$  since the mass  $J(\cdot)$  changes as new firms enter the market. More and more firms will be entering the market until a threshold is reached where a slight rise in the mass of firms would prohibit (12) from holding with equality.

The probability of default turns out to be less than one since  $\hat{\bar{z}}_{E,t} < z_u$  implies  $\mu_E(z_u + Z_t) > 0$ , which holds as long as there is a positive auditing cost. On the other hand,  $\hat{\bar{z}}_{E,t}$

<sup>15</sup>The practical importance of “effective supply” was stressed by Blinder (1987).

<sup>16</sup>Bernanke et al. (1999) assumed that the distribution of idiosyncratic technology levels has a decreasing-hazard property in order to eliminate the situation in which the probability of default is one in equilibrium.



must be greater than zero to ensure  $x_{E,t} > 0$ , which requires  $Z_t < \alpha z_u / \mu_E$ . Therefore, the equilibrium threshold is attained at  $\bar{z}_{E,t}^*$  defined as follows:

$$\bar{z}_{E,t}^* = \max \left\{ 0, \frac{\alpha z_u - \mu_E Z_t}{\alpha + \mu_E} \right\}.$$

Figure 4 illustrates the loan market equilibrium. Depending on the sizes of shocks, there are two possible cases prior to firm entry. The first situation is that the quasi-cost of funds, RHS of (12), lies below the maximum of quasi-expected revenue,  $\Gamma(\bar{z}_{E,t}^*; Z_t)$  (CC). In this case, the debt contract would be made at B provided that the mass of total firms was unchanged.<sup>17</sup> In fact, the mass of total firms will instantly increase until the quasi-cost line reaches  $\Gamma(\bar{z}_{E,t}^*; Z_t)$ . In equilibrium, the mass of new entrants equalizes the quasi-cost of funds with  $\Gamma(\bar{z}_{E,t}^*; Z_t)$ .

CC in figure 4 corresponds to the situation in which the financial intermediary can increase its credit supply due to either a decline in the cost of funds or an increase in the expected revenue (after subtracting auditing costs). In either case, the default risk will be reduced prior to the start of firm entry, so that the financial intermediary becomes able to extend more credit than before. Another possible situation is that the cost of funds is greater than the expected revenue (DD). No prospective entrants can get funds, so that the mass of total firms must decrease. This leads to a decline in the mass of new entrants.

The intuition for the determination of the mass of new firms is given as follows: prospective entrants continue to enter the market as long as they can obtain credit. On the other hand, the financial intermediary will extend credit only if the expected profits are nonnegative. At this point, it is important to notice that an increase in the mass of total firms will raise real wages through the increased disutility of labor. A rise in real wages itself makes the creditworthiness of borrowers worse since the amount of products is independent of the preparatory works. Therefore, the more new firms enter the market, the worse the creditworthiness of borrowers. This externality of bank lending makes it possible to uniquely pin down the mass of total firms as the bank becomes unable to extend credit at some level of the mass of firms.

In the literature, this type of credit rationing is called “*pure credit rationing*” (Jaffee and Stiglitz, 1990) or “*type-II credit rationing*” (Keeton, 1979), where some people can get the full amount of credit they demand while apparently identical people cannot.<sup>18</sup> In this model, the aggregate output will be stimulated by the *effective supply* of liquidity to those prospective entrants who failed to obtain credit. Notice that the source of rationing is not related to the bank’s financial position. This is a situation in which banks can obtain

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<sup>17</sup>Note that the right intersection does not make sense since the financial intermediary can earn positive profits by lowering the threshold.

<sup>18</sup>In contrast, the presence of a binding collateral constraint is an example of *type-I* rationing, where some agents cannot get the desired amount of credit.

external funds as much as they want, and there is no regulation on the volume of loans. Hence, this situation corresponds to the type-D credit rationing discussed in section 2.

### 4.3 Incumbents' default risk

Figures 5 and 6 illustrate how  $\bar{z}_{I,t}^{1+q}$  is determined. I assume that  $\mu_E > \mu_I$  and  $f_E < f_I$ . These two assumptions locate the incumbents' quasi-revenue and the quasi-cost curves above those of the entrants, and the maximizer of the incumbents' quasi-revenue curve becomes larger than that of the entrants. This makes it possible to obtain a time-invariant long-run distribution of firm-specific technology levels.

Figure 5 explains the case in which incumbents of generation- $q$  are risky borrowers. If the intersection of the hump-shaped curve and the horizontal line is located to the right of the generation's lowest technology level, where  $\bar{z}_{I,t}^{1+q} > \bar{z}_{I,t-1}^q$ , then the generation- $q$  firms will default with probability  $x_{I,t}^q = \frac{\bar{z}_{I,t}^{1+q} - \bar{z}_{I,t-1}^q}{z_u - \bar{z}_{I,t-1}^q}$ . On the other hand, if there is no left-hand intersection, then  $\bar{z}_{I,t}^{1+q} = \bar{z}_{I,t-1}^q$  (figure 6). The latter situation implies that even firms that have the lowest technology among the corresponding generation will be able to pay back the full amount they owe. Thus all the firms that belong to the generation can obtain credit at the lowest rate,  $\chi_t R_t$ .<sup>19</sup> The loan rate offered to the generation- $q$  firms can be summarized as

$$\begin{aligned} R_{I,t}^{l,q} &= \frac{d_t(\bar{z}_{I,t}^{1+q})}{(\bar{W}_{I,t}/P_t)f_{I,t}} > \chi_t R_t \quad \text{if } \bar{z}_{I,t}^{1+q} > \bar{z}_{I,t-1}^q \\ &= \chi_t R_t \quad \text{if } \bar{z}_{I,t}^{1+q} = \bar{z}_{I,t-1}^q. \end{aligned}$$

### 4.4 Firm dynamics and heterogeneity in technology

Since the threshold of default depends on firm-specific technology, the aggregate technology distribution generally changes over time. We need to keep track of each generation's mass and the lower bound of firm-specific technology. This section describes how the distributional dynamics of firm-specific technology are calculated. As Bartelsman and Doms (2000) indicate using industrial micro-data, in practice there exists a significant degree of technology heterogeneity among firms, and new firms are generally less productive than older firms.

Suppose that the economy is in the steady state at  $t = 0$ . At time  $t = 1$ , the integral

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<sup>19</sup>One might think that no-risk firms can borrow at the safe rate  $R_t$ . I assume that the lowest loan rate is  $\chi_t R_t$  in order to keep the continuity of loan rates around  $\bar{z}_{I,t}^{1+q} = \bar{z}_{I,t-1}^q$ . Notice that if  $\bar{z}_{I,t}^{1+q}$  is a little greater than  $\bar{z}_{I,t-1}^q$ , then  $R_{I,t}^{l,q}$  is approximately equal to  $\chi_t R_t$ .

of the technology level relevant to the aggregate variables is expressed as<sup>20</sup>

$$\int_0^{z_u} J_1(z)(Z_t + z)^\alpha dz = \int_{\hat{z}_1^I}^{z_u} H_1(z)(Z_t + z)^\alpha dz + N_1^E \int_0^{z_u} g(z)(Z_t + z)^\alpha dz,$$

where

$$\begin{aligned} \int_{\hat{z}_1^I}^{z_u} H_1(z)(Z_t + z)^\alpha dz &= (1 - x_{I,0})(1 - \delta)N_0^I \int_{\bar{z}_{I,0}}^{z_u} h_0(z)(Z_t + z)^\alpha dz \\ &\quad + (1 - x_{E,0})N_0^E \int_{\bar{z}_{E,0}}^{z_u} g_0^1(z)(Z_t + z)^\alpha dz. \end{aligned} \quad (18)$$

$H_t(z)$  denotes the mass of incumbent firms whose technology is  $z$ , and  $\hat{z}_t^I$  is the minimum value of firm-specific technology among all incumbents at  $t$ .  $h_0(z)$  is the density function of incumbents' technology in the steady state. In period  $t = 0$ , all incumbents are uniformly distributed on  $[\bar{z}_{I,0}, z_u]$ . The incumbents' technology distribution in period 1 is the same as that in period 0 if  $\bar{z}_{I,0} = \bar{z}_{E,0}$ . It follows that  $h_0(z) = g_0^1(z)$  for all  $z$ , and  $x_{I,0} = 0$ . As in previous studies, incumbents are assumed to suffer from a "death shock" at the end of each period with probability  $\delta$  (Ghironi and Melitz, 2005, Bilbiie et al., 2007, 2008). The mass of incumbents in period 1 leads to:

$$N_1^I = (1 - \delta)N_0^I + (1 - x_{E,0})N_0^E.$$

The corresponding equations in period 2 are as follows:

$$\int_0^{z_u} J_2(z)(Z_t + z)^\alpha dz = \int_{\hat{z}_2^I}^{z_u} H_2(z)(Z_t + z)^\alpha dz + N_2^E \int_0^{z_u} g(z)(Z_t + z)^\alpha dz, \quad (19)$$

where

$$\begin{aligned} \int_{\hat{z}_2^I}^{z_u} H_2(z)(Z_t + z)^\alpha dz &= (1 - x_{I,1}^1)(1 - \delta)^2 N_0^I \int_{\bar{z}_{I,1}^2}^{z_u} g_1^2(z)(Z_t + z)^\alpha dz \\ &\quad + (1 - x_{E,0})(1 - x_{I,1}^1)(1 - \delta)N_0^E \int_{\bar{z}_{I,1}^2}^{z_u} g_1^2(z)(Z_t + z)^\alpha dz \\ &\quad + (1 - x_{E,1})N_1^E \int_{\bar{z}_{E,1}}^{z_u} g_1^1(z)(Z_t + z)^\alpha dz. \end{aligned} \quad (20)$$

It follows that

$$N_2^I = (1 - x_{I,1}^1)(1 - \delta)^2 N_0^I + (1 - x_{E,0})(1 - x_{I,1}^1)(1 - \delta)N_0^E + (1 - x_{E,1})N_1^E.$$

In this way, we can keep track of aggregate technology distribution by following each generation's distribution and mass. Although most of the previous models that introduced

<sup>20</sup>Notice that the relevant aggregate productivity can be expressed as  $\int_{i \in \Omega_1} (Z_1 + z(i))^\alpha di = \int_{i \in I_1} (Z_1 + z(i))^\alpha di + \int_{i \in E_1} (Z_1 + z(i))^\alpha dz$ , where  $I_1$  and  $E_1$  denote the sets of indices for incumbents and entrants in period 1, respectively. Firms share the same technology distribution among the same generation.

productivity heterogeneity focus on the long-run equilibrium where technology distribution does not change, this model makes it possible to treat dynamic distributional shifts.<sup>21</sup>

As mentioned above, there exists a possibility that incumbents, as well as new entrants, will be credit rationed under certain circumstances. There are three possible phases in regard to the condition of bank lending: i) entrants are credit rationed while incumbents are not. ii) No prospective entrants can obtain credit, whereas incumbents can. iii) No prospective entrants can obtain credit and some of the incumbents are credit rationed. As for case iii), a difficult issue is which generation of incumbents should be excluded from the market. This decision is necessarily arbitrary, since re-equilibration of condition (15) only requires a reduction in  $\int_0^{z_u} J_t(z)(Z_t+z)^\alpha dz$ . This is made possible if a certain mass of incumbents exit as a whole, so that it does not matter which generations the exiting firms belong to. For this reason, I will only treat cases i) and ii).

## 5 Summary of the model

In the following, I solve the model focusing on the following variables:  $\bar{z}_{E,t}$ ,  $\{\bar{z}_{I,t}^{1+q}\}_{q=1}^t$ ,  $x_{E,t}$ ,  $\{x_{I,t}^q\}_{q=1}^t$ ,  $\int_0^{z_u} J_t(z)(Z_t+z)^\alpha dz$ ,  $\int_0^{z_u} H_t(z)(Z_t+z)^\alpha dz$ ,  $N_t^E$ ,  $N_t^I$ ,  $N_t$ ,  $Y_t$ ,  $d_t(\bar{z}_E)$ ,  $\bar{W}_{E,t}/P_t$ ,  $\bar{W}_{I,t}/P_t$ ,  $R_{E,t}^l$  and  $\{R_{I,t}^{q,l}\}_{q=1}^t$ . The rest of the endogenous aggregate variables, such as the real interest rate, can be determined residually.

As for the process of exogenous shocks, I employ the following simple autoregressive process:

$$\begin{aligned} Z_t &= \exp(\varepsilon_t^z) Z_{t-1}^{\rho_z} Z^{1-\rho_z}, \\ R_t &= \exp(\varepsilon_t^r) R_{t-1}^{\rho_r} R^{1-\rho_r}, \\ \chi_t &= \exp(\varepsilon_t^\chi) \chi_{t-1}^{\rho_\chi} \chi^{1-\rho_\chi}, \end{aligned}$$

where  $Z$ ,  $R$  and  $\chi$  denote the steady-state values of common productivity shock, the nominal interest rate and the financial shock, respectively.  $\varepsilon_t^i$ ,  $i = z, r, \chi$ , represent iid shocks to the corresponding variable.

If one focuses on the endogenous variables listed above, the model can be solved analytically in a recursive manner without relying on any linearization technique. The steady states of those variables are summarized in the Appendix.<sup>22</sup>

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<sup>21</sup>See, for example, Hopenhayn and Rogerson (1993) and Gomes (2001). Recall that the model's tractability depends largely on its backward-looking nature. If forward-lookingness is introduced, an approximation method will be needed in order to obtain aggregate dynamics (e.g., Krusell and Smith 1998).

<sup>22</sup>Since the process of nominal interest rate is exogenously given, the aggregate price level will be indeterminate as long as the public forms rational expectations (Sargent and Wallace, 1975). Notice that the aggregate price level is not necessarily indeterminate if the public's expectation formation is not rational. The public's expectation formation is left unspecified in this model.

## 6 Numerical experiments

### 6.1 Calibration

The model's frequency is quarterly. The baseline parameter values are as follows: for the utility parameter,  $\phi = 2$ ,  $\tilde{\phi} = 1$ ,  $\eta = \tilde{\eta} = 1$ , and  $\beta = .99$ .  $R$  is set at  $1/.99$ . The auditing cost parameters  $\mu_E$  and  $\mu_I$  are set at .12 and .08, respectively.<sup>23</sup> The consumption-GDP ratio,  $\gamma$ , is .6 and the mean of the common productivity shock is set at 1. Following Bilbiie et al. (2007, 2008), I employ  $\theta = 3.8$ . The steady-state preparatory labor,  $f_E$ , is determined such that it equals the "average" individual production labor, which corresponds to the hours worked under productivity of  $Z + z_u/2$ . This results in

$$f_E = \left\{ \frac{\Gamma(\bar{z}_E)}{R\theta\tilde{\eta}\gamma} \left[ A^{1+\phi} \left( Z + \frac{z_u}{2} \right)^{-\alpha} \right] \right\}^{1/(\tilde{\phi}-\phi)},$$

where  $A \equiv [\theta/(\theta - 1)\eta\gamma R]^{1/(1+\phi)}$ .  $f_I$  is set such that the steady-state threshold of new entrants' default is identical to the lower bound of the incumbents' technology distribution in the steady state. It follows that

$$\begin{aligned} \frac{(Z + \bar{z}_I)^\alpha}{\Gamma(\bar{z}_E)} &= \left( \frac{f_I}{f_E} \right)^{1+\tilde{\phi}} \\ \text{or } f_I &= f_E \left[ \frac{(Z + \bar{z}_I)^\alpha}{\Gamma(\bar{z}_E)} \right]^{\frac{1}{1+\tilde{\phi}}}. \end{aligned}$$

This implies that  $f_E < f_I$ .

Bilbiie et al. (2007, 2008) set  $\delta$  at .025 to match the U.S. data for job destruction, which is approximately 10% per year. Because there is no incumbents default in the steady state, I set the value of  $z_u$  such that  $x_E = \delta$ . This results in  $z_u = .126$ . The total exit rate in the steady state is equal to the total entry rate, thus  $N^E/N = \delta$ . With these parameter values, the steady state entrants' annual credit spread, defined as  $(R_E^l/R)^4 - 1$ , is 1.22%. The AR coefficients  $\rho^z$  and  $\rho^r$  are both set at .9. As for the financial shock, it is assumed that  $\chi = 1$  and  $\chi_t$  is a temporary shock whose AR coefficient is zero.

### 6.2 Impulse responses

As discussed above, the effects of economic shocks are highly asymmetric and dependent on the sizes of the shocks. Thus, I first show impulse responses to moderate shocks. After that, I examine the case of large shocks that will break down the equilibrium condition of debt contracts.

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<sup>23</sup>Bernanke, et al. (1999) set  $\mu$  at .12.

### 6.2.1 A common productivity shock

The direct effect of  $Z_t$  on the quasi-cost of funds is twofold. First, a rise in  $Z_t$  decreases the quantity of each firm's demand for funds by reducing the required preparatory labor. This is reflected by a downward shift of the horizontal line. Second, a rise in  $Z_t$  increases the term  $\int_0^{z_u} J_t(z)(Z_t + z)^\alpha dz$ . On the other hand, a positive productivity shock also moves the quasi-expected revenue,  $\Gamma(\cdot)$ , upward, which corresponds to an improvement in the profitability of borrowers.

Figure 7 illustrates impulse responses to a *positive* (annualized) one-percent deviation of the common productivity shock from the steady state. The path of each variable is shown in terms of the percentage deviation from the corresponding steady state.<sup>24</sup> The figures show that an improvement in the common productivity level enhances firm entry and that the entrants' probability of default is lowered. Graphically, this reflects a shift in the quasi-revenue curve in the upper-left direction. However, the average probability of incumbents' default significantly increases in period 2. This is because those firms that entered in period 1 have lower technologies on average than the incumbents in the steady state do. An improvement in the common technology allows the threshold of new entrants to decrease, so that the average incumbent productivity necessarily deteriorates in the following periods.

Figure 8 illustrates impulse responses to a *negative* common productivity shock. It is important to note that the effects of productivity shocks are highly asymmetric when it comes to the loan rates and the default risk. This is because a deterioration in the common productivity level moves the threshold rightward, which leads to an immediate reduction in the degree of uncertainty regarding firm-specific technology. After a negative shock to  $Z_t$ , the threshold takes the maximum value on impact,  $\bar{z}_{E,1}$ , and thereafter the financial intermediary can make debt contracts, knowing that the generation's lowest technology is  $\bar{z}_{E,1}$ . It follows that *intra*-generational technology distribution does not change from period 1 onward. If a *positive* productivity shock hits the economy, in contrast, the threshold takes the minimum value on impact and then gradually goes up over time. In the latter case, *intra*-generational distribution can change over time. This is why the movements of incumbents' default probability and loan premium are temporary when the productivity shock is negative and persistent when it is positive.

The figures also show the total factor productivity (TFP) and the average firm technology level. In this model these two variables may differ, for TFP is defined as

$$TFP_t = \frac{\int_{i \in \Omega_t} Y_t(i) di}{\int_{i \in \Omega_t} L_t(i) di + N_t^I f_{I,t} + N_t^E f_{E,t}},$$

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<sup>24</sup>The average credit spread for incumbents are raw values since it is zero in the steady state.

while the average productivity (henceforth, AP) is given as

$$AP_t = \frac{\int_{i \in \Omega_t} (Z_t + z(i)) di}{N_t}.$$

In response to a common productivity shock, these two measures of aggregate productivity move in tandem because  $Z_t$  is a major factor for both. However, this is not necessarily the case when it comes to other sorts of shocks. Roughly speaking, TFP defined as above moves in tandem with the aggregate output,  $Y_t$ , since the denominator of TFP, the total amount of labor, contains fixed components. That is, the total amount of labor is less flexible than the linear aggregate of outputs. In contrast, AP and the aggregate output may move in opposite directions. For example, if the mass of new entrants and the aggregate output are simultaneously increased without changing the threshold of default, then TFP will go up while AP will be lowered. This is because the average technology of entrants is lower than that of incumbents.

### 6.2.2 A monetary policy shock: the credit rationing channel

Next, let us examine the effects of a monetary policy shock. Since goods prices are flexible, the standard interest channel of monetary policy through the Euler equation does not work here. Instead, there are two alternative channels: first, a change in  $R_t$  influences the household's labor supply decision, (4), because there is a time lag between the payment and the use of production-labor wages. I refer to this as the intensive margin effect. Second, a shift in  $R_t$  affects the bank's loan supply by changing the cost of funds. Since financial contracts are made in nominal terms, a change in the nominal cost of funds can change the total amount of loans. This is an extensive margin effect, which is also called the credit rationing channel throughout the paper.

Figure 9 shows impulse responses to a (annualized) one-percent cut in the nominal interest rate. The figure reveals that the credit rationing channel of monetary policy may play a significant role in the policy transmission mechanism. It turns out that a shift in monetary policy can have significant real effects through its effect on credit supply. Although it is not clearly shown in the figure, I found that the intensive margin effect is relatively minor compared to the extensive margin effect.

A cut in the nominal interest rate will lead the bank to increase credit by reducing the cost of funds. The intuition is that a reduction in the cost of funds allows the expected (nominal) revenue of each loan to decrease, which enables the bank to extend credit to less creditworthy borrowers. This enlarges prospective entrants' opportunities to obtain credit and thus promotes firm entry. Recall that a rise in the mass of total firms exacerbates the creditworthiness of prospective entrants by increasing the labor wages to a larger extent than their expected profits.

Figure 9 also reveals that the probability of default and the credit spreads are kept

unchanged. This is because a shift in  $R_t$  will be offset by the opposite shift in the mass of new entrants. The quasi-expected revenue is not affected at all by a change in  $R_t$ . Thus, it can be said that the effect of monetary policy has a property of *intra*-generational distribution neutrality in the sense that the technology distribution within each generation does not change over time. In the aggregate, however, a cut in the nominal interest increases the share of low-productivity firms by increasing the fraction of new entrants. Since the threshold is not changed, this will lower AP while increasing TFP. Monetary policy does not ensure *inter*-generational distribution neutrality.

### 6.2.3 A financial shock

Figure 10 illustrates the impulse responses to the financial shock,  $\chi_t$ . Basically, a rise in  $\chi_t$  has similar effects as a rise in  $R_t$ , except for the intensive margin effect. One thing that is different from the policy shock is that  $\chi_t$  can have some influence on the credit spreads. This is simply because the minimum value of loan rates increases as  $\chi$  goes up. The *intra*-generational distribution neutrality still holds, but again AP fluctuates according to changes in the share of new entrants among all firms. Thus, *inter*-generational distribution neutrality does not hold.

## 6.3 Credit crunch

Next, let us consider a situation in which “large” shocks hit the economy. There is a possibility that the equilibrium condition (12) might not hold if a large shock hits the economy even when the economy was previously in the steady state. If the unfavorable shock is so large that the mass of firms cannot decline enough to attain equilibrium, then credit will not be supplied to prospective entrants. I refer to this situation as a “credit crunch” or “credit crisis”.

### 6.3.1 Impulse responses

Figure 11 shows impulse responses to a (annualized) 5% negative deviation of  $Z_t$  from the steady-state value.<sup>25</sup> In the face of a -5% common productivity shock, the mass of new entrants immediately takes the value of zero. Incumbents’ default risk and credit spreads rise immediately. Accordingly, the mass of total firms and the aggregate output decline. It is important to notice that these effects are not necessarily proportional to the -1% reduction in  $Z_t$ . The term  $\int_0^{z_u} J_t(z)(Z_t + z)^\alpha dz$  plays a balancer role in the loan-market equilibrium condition as long as the mass of new entrants,  $N_t^E$ , takes a positive value, but the equilibrium condition breaks down once  $N_t^E$  hits 0.

Figure 12 illustrates responses to an (annualized) 11% positive financial shock. What makes this figure qualitatively different from figure 10 is the behavior of default probability.

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<sup>25</sup>The shocks are kept as small as possible in absolute value.



As long as shifts in  $\chi_t$  are absorbed by a change in  $\int_0^{z_u} J_t(z)(Z_t + z)^\alpha dz$ , the financial shock will never affect default probability. However, the lack of a balancer in the loan-market equilibrium allows the quasi-cost of funds to rise, and the thresholds of incumbents default move rightward accordingly. This suggests that the impacts of a financial shock and thereby a policy shock can lead to intra-generational distribution non-neutrality when the shocks are sufficiently large.

### 6.3.2 Simulation

Thus far I have shown impulse responses of various variables given that the economy was in the steady state. This requires relatively large shocks in order to generate a credit crunch. In this section, I instead show long-period simulation where shocks can hit the economy in each period. Since state variables fluctuate over time, a relatively smaller shock could trigger a credit crisis. A similar exercise can be found in the recent work by Mendoza (2009), who solved a nonlinear problem with an occasionally binding constraint in the context of “sudden stops”.<sup>26</sup>

I now suppose that  $\varepsilon^z$  and  $\tilde{\varepsilon}^x$  follow a uniform distribution on  $[-(1.045^{1/4} - 1), 1.045^{1/4} - 1]$  and  $[-(1.01^{1/4} - 1), 1.01^{1/4} - 1]$ , respectively. Then,  $\varepsilon_t^x$  is defined as  $\varepsilon_t^x = \max(0, \tilde{\varepsilon}_t^x)$ . The nominal interest is kept constant. I run a 200-period simulation and discard the initial 100 periods.

Figure 13 illustrates a sample path of the simulation. At the time of a credit crunch, indicated by a vertical dotted line, the loan rates and the probability of incumbents default spike while the aggregate output, the mass of firms and the amount of credit decline sharply. This phenomenon is consistent with the impulse responses shown above, but the difference is that in this simulation relatively moderate shocks caused the crisis.

To see what happened shortly before and after the credit crunch, Figure 14 shows the behavior of various variables around the crisis date ( $t = 0$  indicates the period of the credit crunch). It reveals that on the eve of the crisis, some variables exhibit significant increases. Those variables are aggregate output, mass of firms, amount of credit supplied, aggregate labor, fixed-labor wages, AP and TFP. On the other hand, some other variables, such as new entrants’ risk premium and their default probability, are significantly decreased on the eve of the crisis.

The intuitive explanation of this result is as follows: suppose that the threshold of entrants’ default is significantly lowered in period  $t$ . This means that the next period’s total mass will be higher than usual since most of the entrants can survive into the next period. The increase in the mass of firms will also raise the real wages of preparatory workers in period  $t + 1$ , other things being equal. Due to the upward pressure on real

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<sup>26</sup>The importance of nonlinearity at the time of a credit crisis is also noted by He and Krishnamurthy (2008).

wages, the mass of new entrants needs to be reduced in period  $t + 1$  in order to maintain a certain degree of entrants' creditworthiness. However, if an unfavorable shock, such as a fall in the common technology level or a rise in the cost of funds, hits the economy in period  $t + 1$ , then the mass of new entrants must be further reduced. If no nonnegative mass of entrants can attain the loan-market equilibrium, then no entrants can enter the market in period  $t + 1$ . Hitting the lower bound of  $N_{t+1}^E$  implies that there arises a downward rigidity of real wages, and it is this downward rigidity that prohibits the creditworthiness of prospective entrants from improving.

It can be said that a sharp decline in the threshold has a negative externality in the sense that it will make the following period's firm entry more difficult by creating an upward pressure on real wages. An increase in the mass of firms itself expands the aggregate output, but at the same time it becomes an obstacle to an improvement in the creditworthiness of prospective firms. It should be noted that if the decrease in the threshold is significant, then a crisis would be caused by a very small shock that would never trigger a crisis in normal circumstances.

The symptoms of credit crises noted above are reconfirmed when annualized data are used. Figure 15 shows an annual version of Figure 14. To create the figure, I conducted a 500-period simulation 30 times and discarded the initial 100 periods in each simulation. In total, credit crises are observed 45 times, for a frequency of about 1.5 times per 100 years. The four-quarter means of each variable are illustrated.

On average, rises in the aggregate output, the mass of firms, the amount of credit, aggregate labor, real wages, AP and TFP precede a credit crisis, and those variables continue to decline in the aftermath of the crisis. In contrast, entrants' credit spread and the default probability significantly decline shortly before the crisis, whereas they recover rapidly after it. The result that a credit crunch is likely to follow a credit boom is consistent with empirical studies such as Keeton (1999), Jiménez and Saurina (2006) and Mendoza and Terrones (2008). Keeton (1999) and Jiménez and Saurina (2006) report that there is a positive, although lagged, relationship between credit growth and credit risk. Mendoza and Terrones (2008) also show that credit booms tend to be preceded by periods of high TFP in industrialized countries. Although they do not emphasize the role of real wages in credit boom-bust cycles, the above model suggests that downward wage rigidity may be a key to explaining how credit booms end and crises begin.

## 7 Concluding remarks

This paper explored a situation in which firms have to borrow funds from a financial intermediary in order to cover fixed costs. It is shown that credit rationing arises in equilibrium, and thus the mass of new entry firms depends on the volume of credit the financial intermediaries can supply.

It turns out that a cut in the nominal interest rate leads to an increase in the credit supply as long as debt contracts are made in nominal terms. The main differences in the transmission mechanism between the balance sheet channel, the bank lending channel and the credit rationing channel are summarized in Table 4. The credit rationing channel and the balance sheet channel are similar in that the key policy variable is an interest rate and borrower's creditworthiness plays a crucial role in policy transmission. On the other hand, the credit rationing channel and the bank lending channel share the property that the presence of rationing is a key to the policy transmission mechanism while the sources of supply shortage are different. The traditional balance sheet channel emphasizes that a shift in monetary policy has real effects through its influence on the borrowers' net worth. However, it only focuses on a demand-driven effect of monetary policy, neglecting the presence of financial market disequilibrium. If credit demand is greater than credit supply, which is more likely for younger firms, then monetary policy can have real effects through a supply-side channel as opposed to a demand-side channel.

Since the model can be solved without relying on any linearization technique, the condition for the loan market equilibrium is allowed to occasionally break down. The bank's loan supply to new firms suddenly stops when the equilibrium condition is not satisfied. I showed that a credit crisis is likely to occur after a sharp rise in the aggregate output, the mass of firms, the amount of credit supplied, the aggregate labor, fixed-labor wages, AP and TFP. On the other hand, both new entrants' premium and their default probability tend to be significantly decreased on the eve of a credit crunch. These findings suggest that a credit crisis would occur when the economy is in boom rather than in recession, a result confirmed by existing empirical studies.

The following topics should be addressed in future research: first, it is necessary to clarify the quantitative importance of the credit rationing channel. To do this, more empirical works will be needed to quantify the size of type-D credit rationing. Second, capital accumulation is absent in this paper. It would be useful to see how the introduction of capital affects the results. The challenge is that one needs to solve a forward-looking problem with an occasionally holding equilibrium in a model where the distribution of productivity levels changes over time. Such a nonlinear forward-looking problem with heterogeneous agents seems quite important in order to better understand credit crises.

## 8 Appendix: The steady state

The steady states of  $\bar{z}_{E,t}, \{\bar{z}_{I,t}^{1+q}\}_{q=1}^t, x_{E,t}, \{x_{I,t}^q\}_{q=1}^t, \int_0^{z_u} J_t(z)(Z_t + z)^\alpha dz, \int_0^{z_u} H_t(z)(Z_t + z)^\alpha dz, N_t^E, N_t^I, N_t, Y_t, d_t(\bar{z}_E), \tilde{W}_{E,t}/P_t, \tilde{W}_{I,t}/P_t, R_{E,t}^l$  and  $\{R_{I,t}^{q,l}\}_{q=1}^t$  are as follows:

$$\begin{aligned}
\text{Threshold for entrants:} & \quad \bar{z}_E = \frac{\alpha z_u - \mu_E Z}{\alpha + \mu_E}, \\
\text{Threshold for incumbents:} & \quad \bar{z}_I^{1+q} = \bar{z}_E \equiv \bar{z}_I, \\
\text{Prob. of entrants' default:} & \quad x_E = \frac{\bar{z}_E}{z_u}, \\
\text{Prob. of incumbents' default:} & \quad x_I^q = 0, \\
\text{Productivity (total):} & \quad \int_0^{z_u} J(z)(Z + z)^\alpha dz = \frac{\Gamma(\bar{z}_E; Z)}{R\theta\tilde{\eta}\gamma f_E^{1+\tilde{\phi}}}, \\
\text{Productivity (incumbents):} & \quad \int_{\bar{z}_I}^{z_u} H(z)(Z + z)^\alpha dz = N^I \int_{\bar{z}_I}^{z_u} \frac{g(z)}{1 - G(\bar{z}_I)} (Z + z)^\alpha dz, \\
\text{Mass of entrants:} & \quad N^E = \frac{\delta}{1 - \delta} N^I, \\
\text{Mass of incumbents:} & \quad N^I = \int_0^{z_u} J(z)(Z + z)^\alpha dz \\
& \quad \times \left( \int_{\bar{z}_I}^{z_u} \frac{g(z)}{1 - G(\bar{z}_I)} (Z + z)^\alpha dz + \frac{\delta}{1 - \delta} \int_0^{z_u} g(z)(Z + z)^\alpha dz \right)^{-1}, \\
\text{Total mass:} & \quad N = N^I + N^E, \\
\text{Aggregate output:} & \quad Y = \left[ \left( \frac{\theta}{\theta - 1} \right) \gamma \eta R \right]^{\frac{-1}{1+\tilde{\phi}}} \left[ \int_0^{z_u} J(z)(Z + z)^\alpha dz \right]^{\frac{1}{\alpha}}, \\
\text{Threshold profits:} & \quad d(\bar{z}_E) = \frac{1}{\theta} (Z + \bar{z}_E)^\alpha \left[ \left( \frac{\theta}{\theta - 1} \right) \gamma \eta R \right]^{\frac{-1}{1+\tilde{\phi}}} \left[ \int_0^{z_u} J(z)(Z + z)^\alpha dz \right]^{\frac{1-\alpha}{\alpha}}, \\
\text{Real wage (entrants):} & \quad \frac{\tilde{W}_E}{P} = \tilde{\eta}\gamma Y f_E^{\tilde{\phi}}, \\
\text{Real wage (incumbents):} & \quad \frac{\tilde{W}_I}{P} = \tilde{\eta}\gamma Y f_I^{\tilde{\phi}}, \\
\text{Loan rate (entrants):} & \quad R_E^l = \frac{d(\bar{z}_E)}{(\tilde{W}_E/P) f_E}, \\
\text{Loan rate (incumbents):} & \quad R_I^{q,l} = R.
\end{aligned}$$

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Table 1: Availability of credit from the main bank

	2004	2005	2006	2007	2008
Credit application was refused or its amount reduced (among applicants)	8.6	8.8	6	6.8	9.2
Credit application was accepted with tightened borrowing conditions (among applicants)	18.1	17.8	15.8	15.9	17.2
Did not submit credit application in the past year	43	50.9	46.7	49.7	48.7

Source: *Basic Survey of Small and Medium Enterprises*, The Ministry of Small and Medium Enterprises Agency.

Table 2: Availability of credit and the source of funds by developmental phase

	early growth	growth and expansion	stable	total
<i>Availability of credit</i>				
Obtained funds as desired	61.3	87.9	97.4	93.9
Did not obtain funds as desired	38.7	12.1	2.6	6.1
<i>Source of funds</i>				
Financial institutions	82.9	91.3	91.9	91.3
Representative/executives	68.6	28.7	19.2	24.3
Venture capital	45.7	16.2	10.3	13.7
Business partners/affiliated companies	28.6	13.0	11.9	13.1
Relatives/acquaintances	35.2	13.0	11.0	12.9
Company funds (cash flow)	17.1	22.0	27.4	25.5

Source: *White Paper on Small and Medium Enterprises in Japan 2009*, The Ministry of Small and Medium Enterprises Agency. The questionnaire was conducted by Tokyo Shoko Research Ltd.

Note: Only for SMEs that conduct R&D.



Table 3: What has made external finance difficult?

Deterioration in the business condition of the related industry	50.6
Deterioration in the company's profitability	46.4
Financial institutions' problems	43.8
Deterioration in the asset value of the company and/or the manager	14.2
Others	3.6

Source: *White Paper on Small and Medium Enterprises in Japan 2009*, The Ministry of Small and Medium Enterprises Agency. The questionnaire was conducted by Mizuho Research Institute.

Note: Based on the questionnaire to the SMEs that replied that loans from financial institutions have become tighter than before.

Table 4: Classification of the credit channels

	BS	BL	CR
key policy variable	interest rate	money supply	interest rate
credit rationing?	no	yes	yes
source of rationing	n.a.	bank liquidity	creditworthiness

Note: BS, BL and CR denote the balance-sheet channel, the bank-lending channel and the credit rationing channel, respectively.

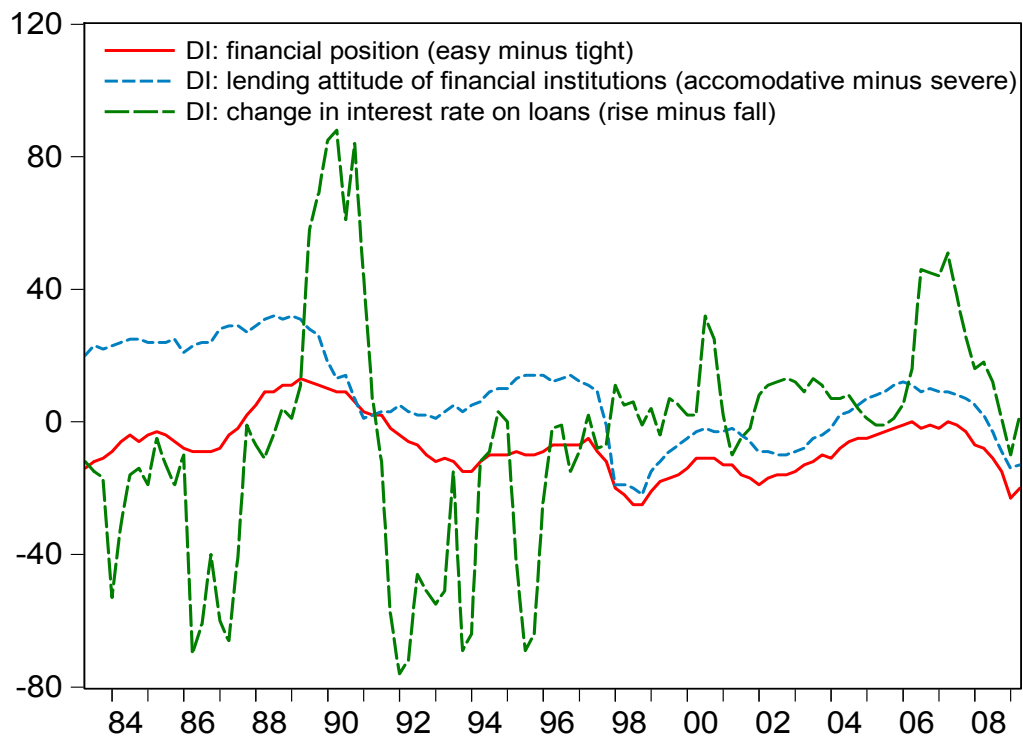


Figure 1: Diffusion indices regarding the external finance conditions of SMEs

Source: *TANKAN*, Bank of Japan.

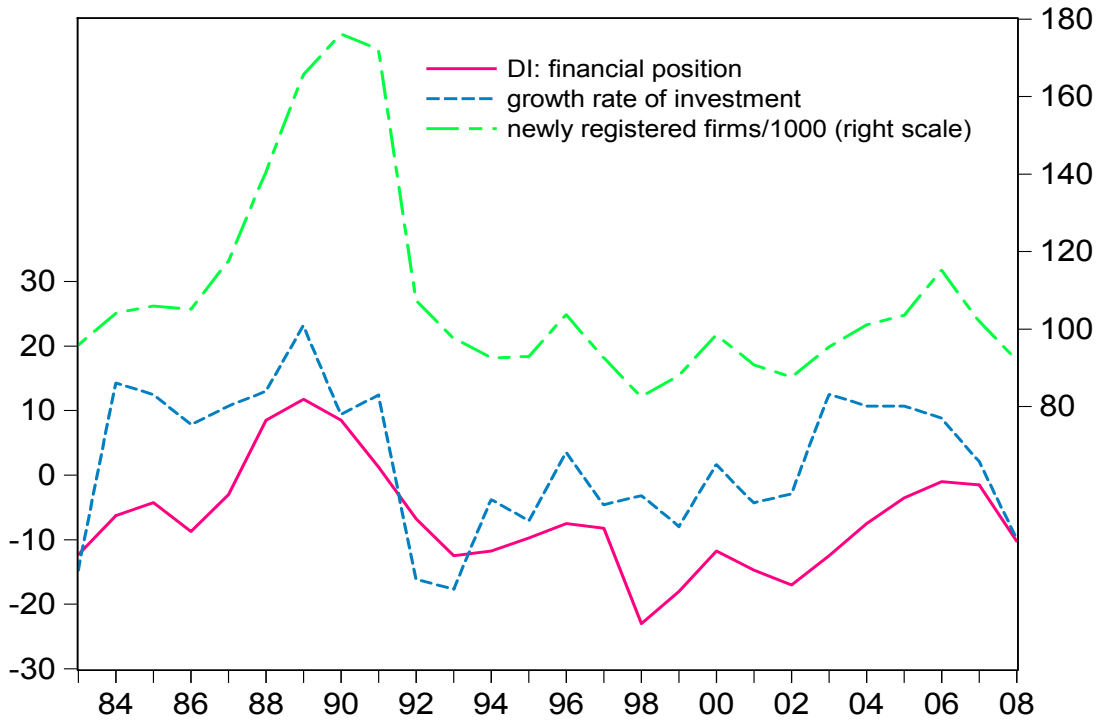


Figure 2: Easiness in external finance, firm entry and fixed investment

Source: DI and fixed investment: *TANKAN*, Bank of Japan. Number of newly registered firms: *White paper on Small and Medium Enterprises 2009*, The Ministry of Small and Medium Enterprise Agency.

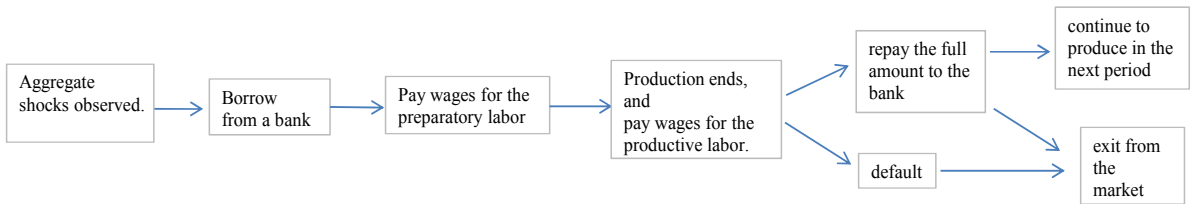


Figure 3: Sequence of events

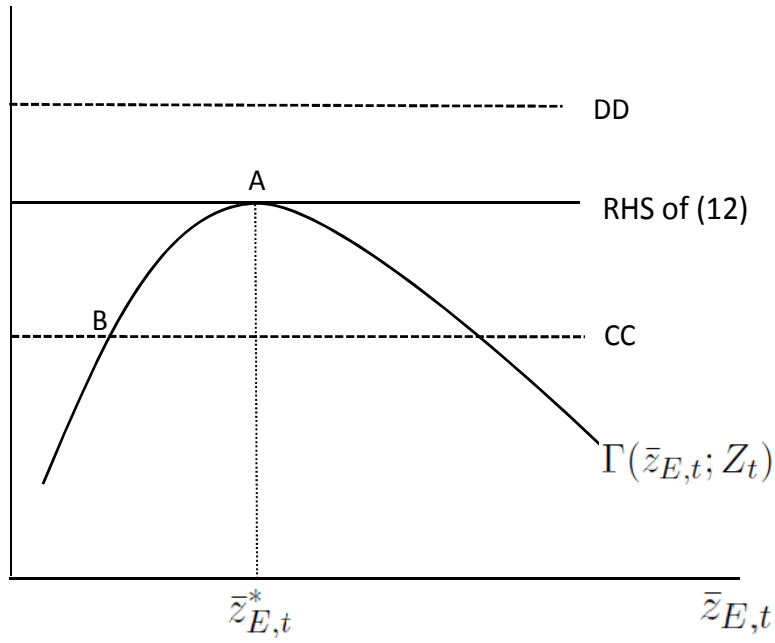


Figure 4: Graphical illustration of equilibrium credit rationing

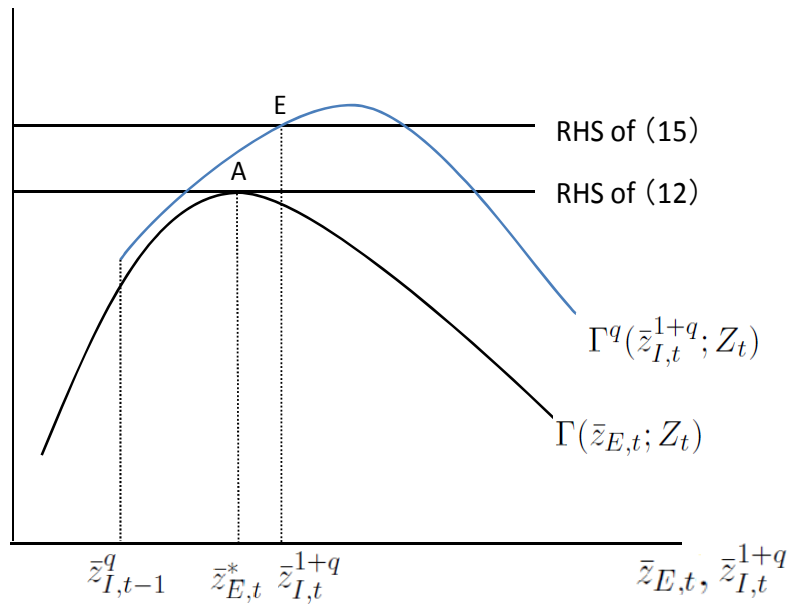


Figure 5: Determination of  $\bar{z}_{I,t+1}^{1+q}$  when incumbents are risky borrowers

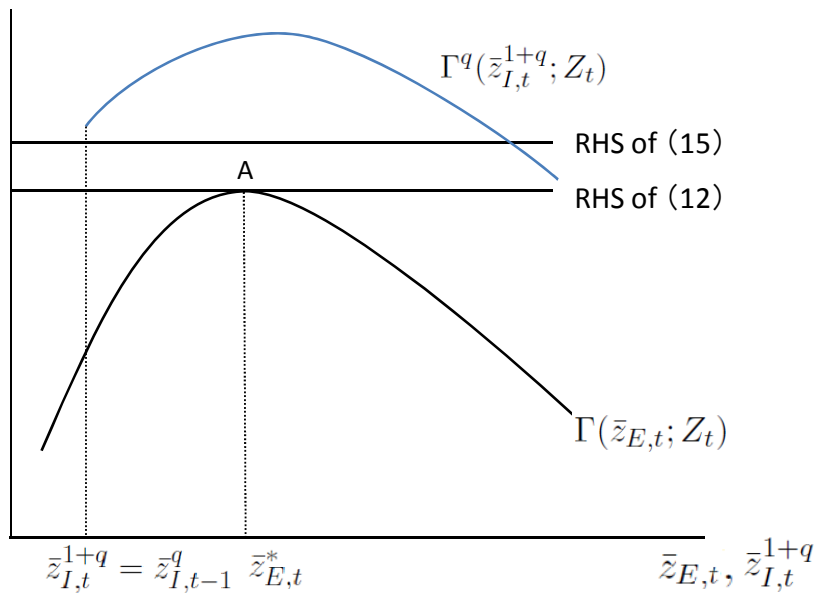


Figure 6: Determination of  $\bar{z}_{I,t+1}^{1+q}$  when incumbents are safe borrowers

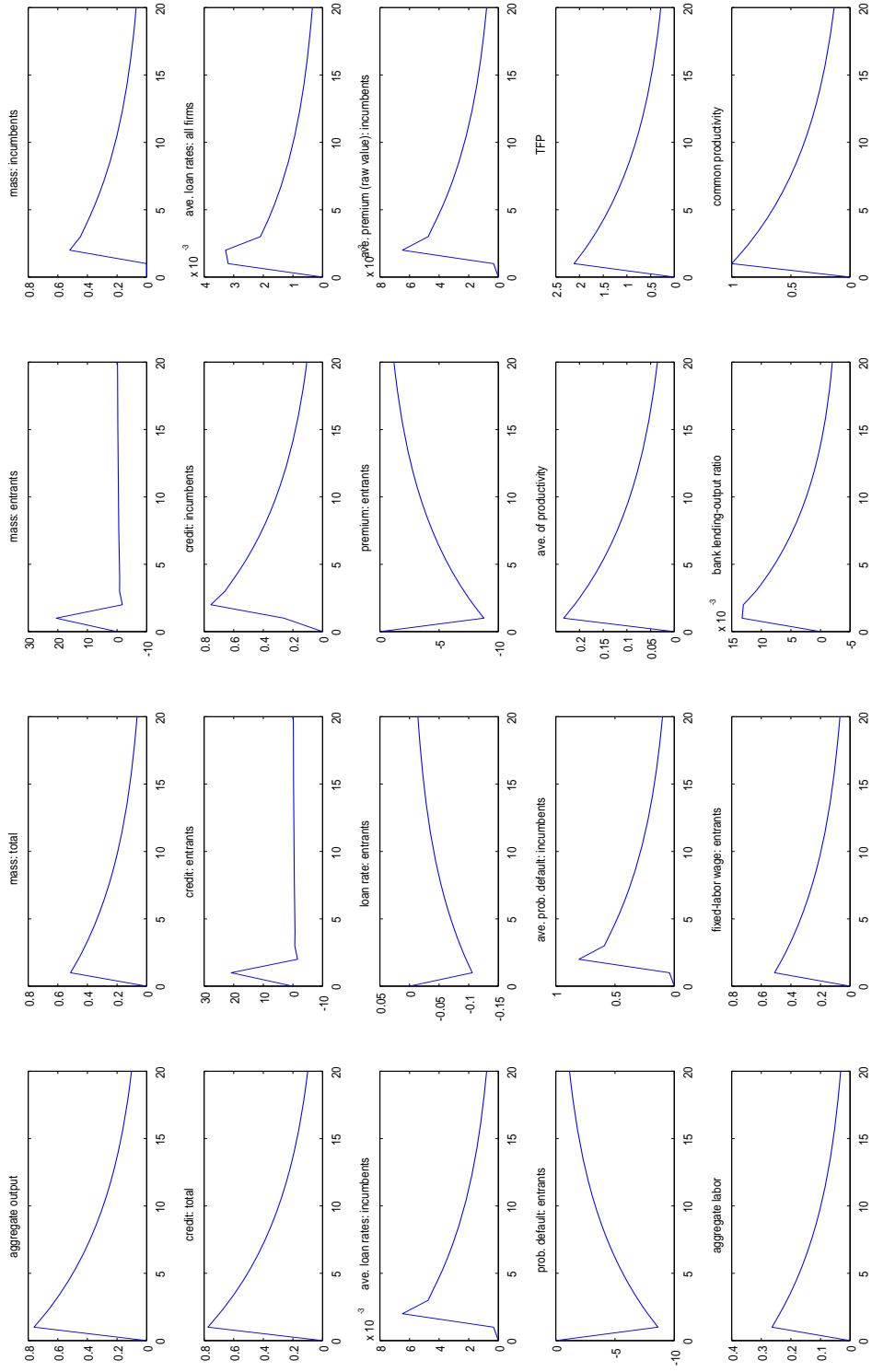


Figure 7: The impacts of a positive common productivity shock

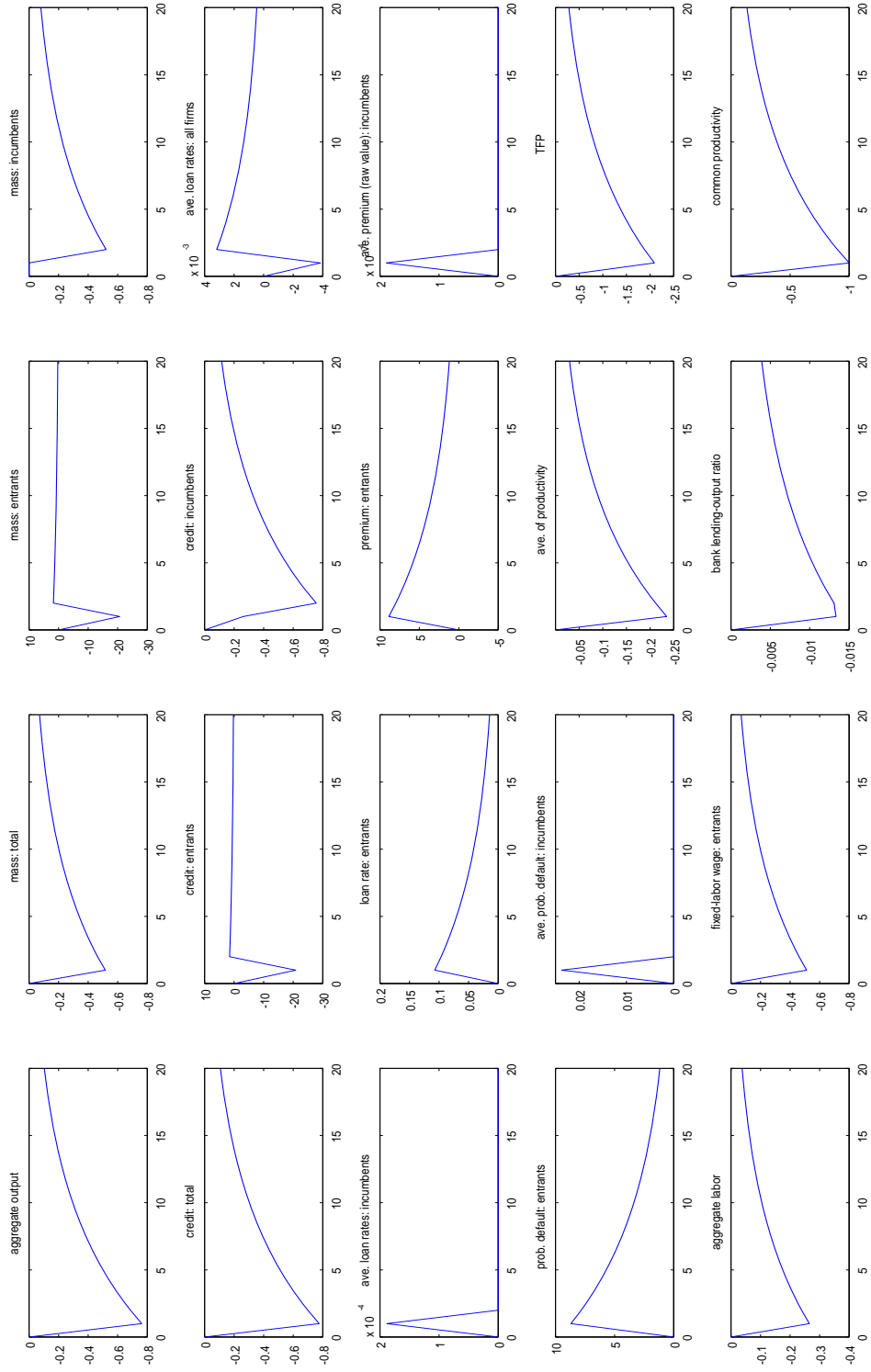


Figure 8: The impacts of a negative common productivity shock

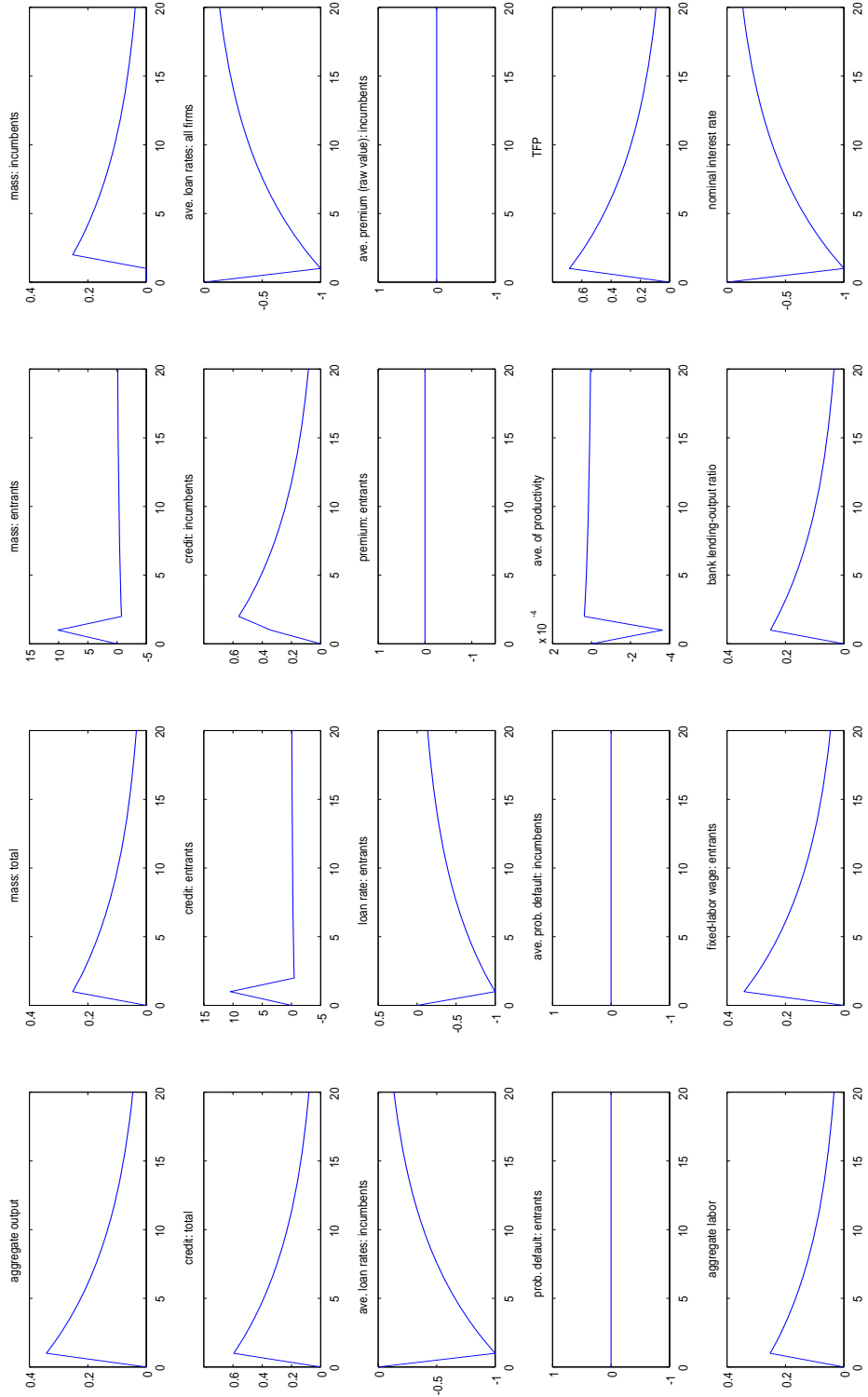


Figure 9: The impacts of a negative policy-rate shock



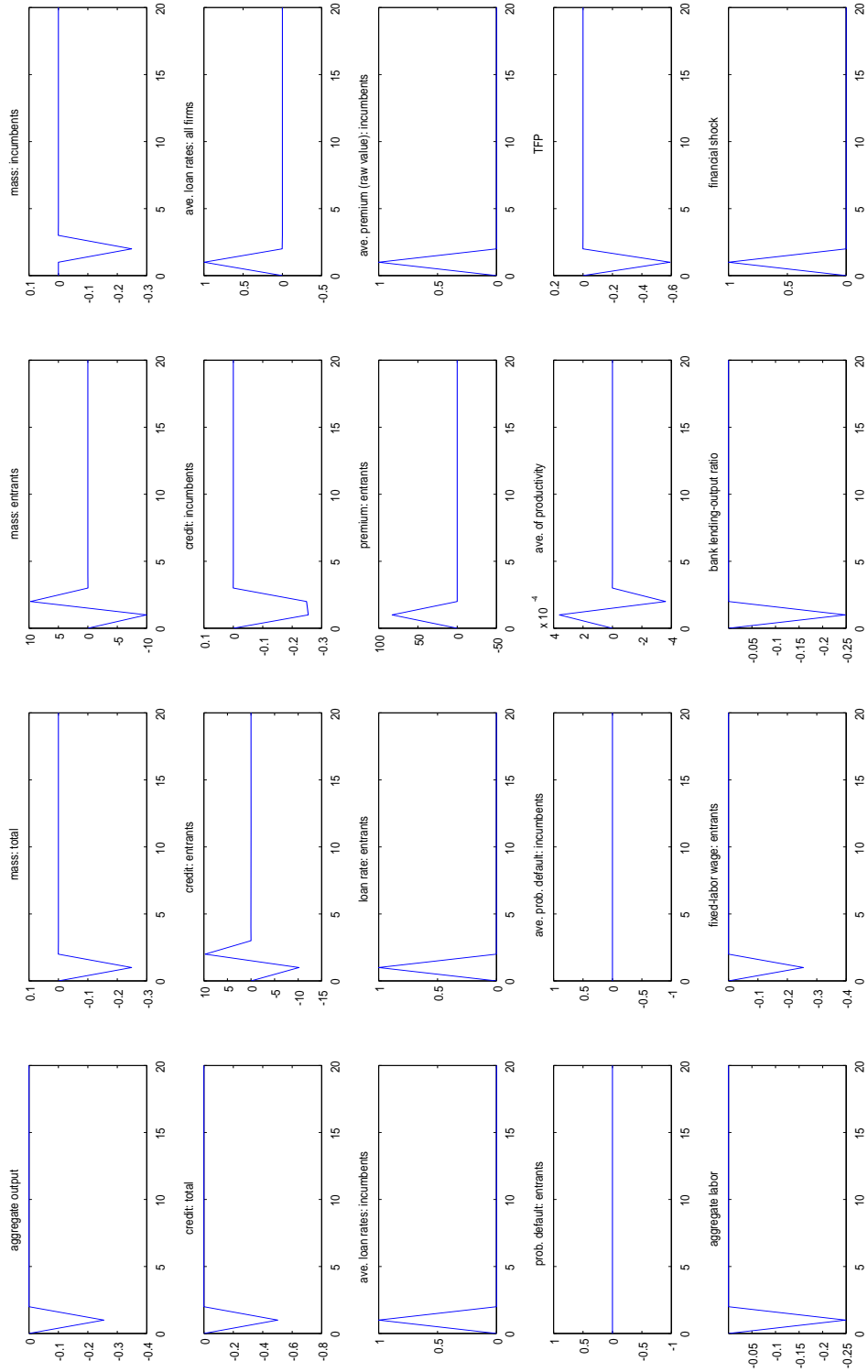


Figure 10: The impacts of a positive financial shock

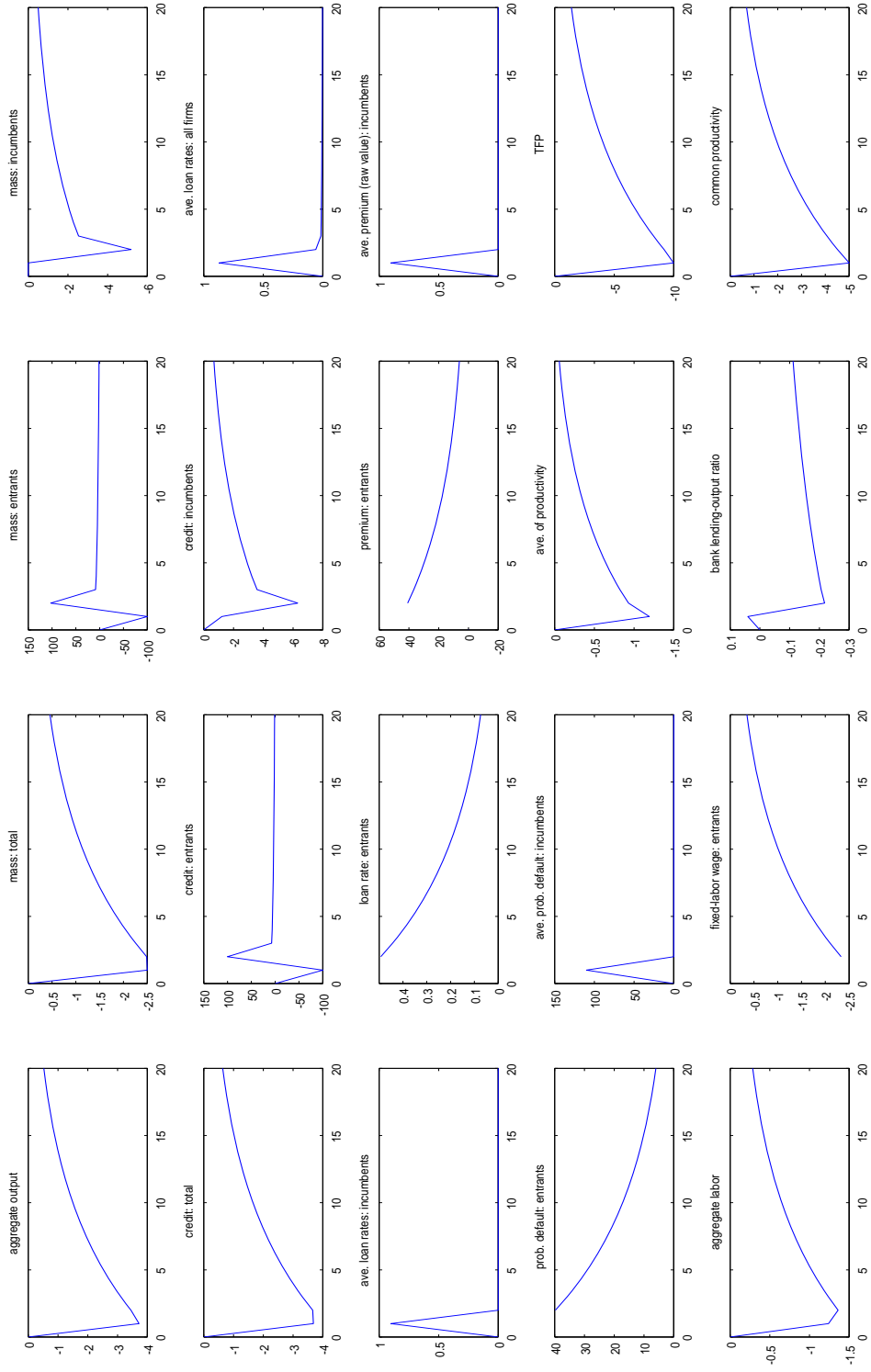


Figure 11: The impacts of a large negative productivity shock

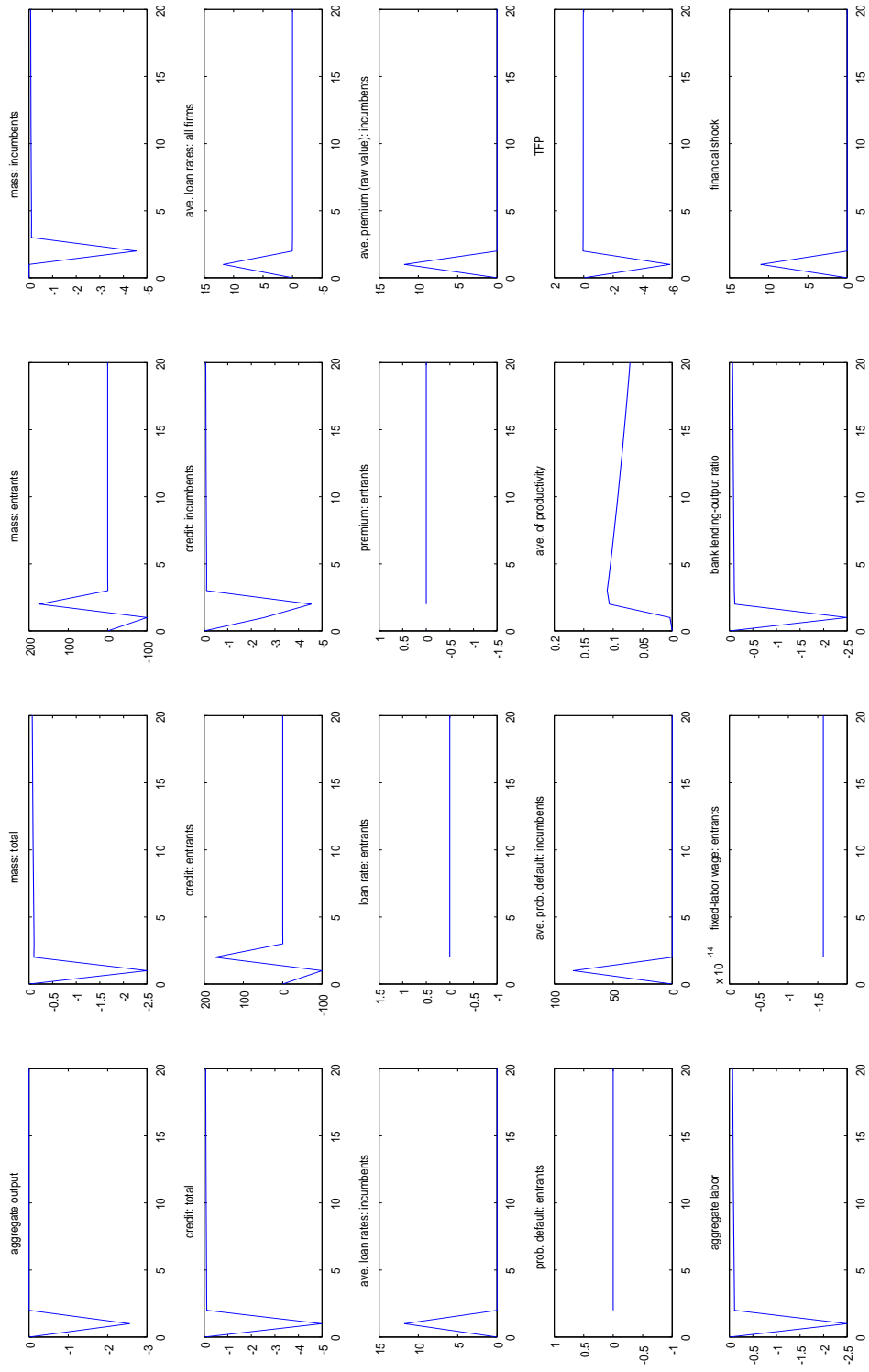


Figure 12: The impacts of a large positive financial shock

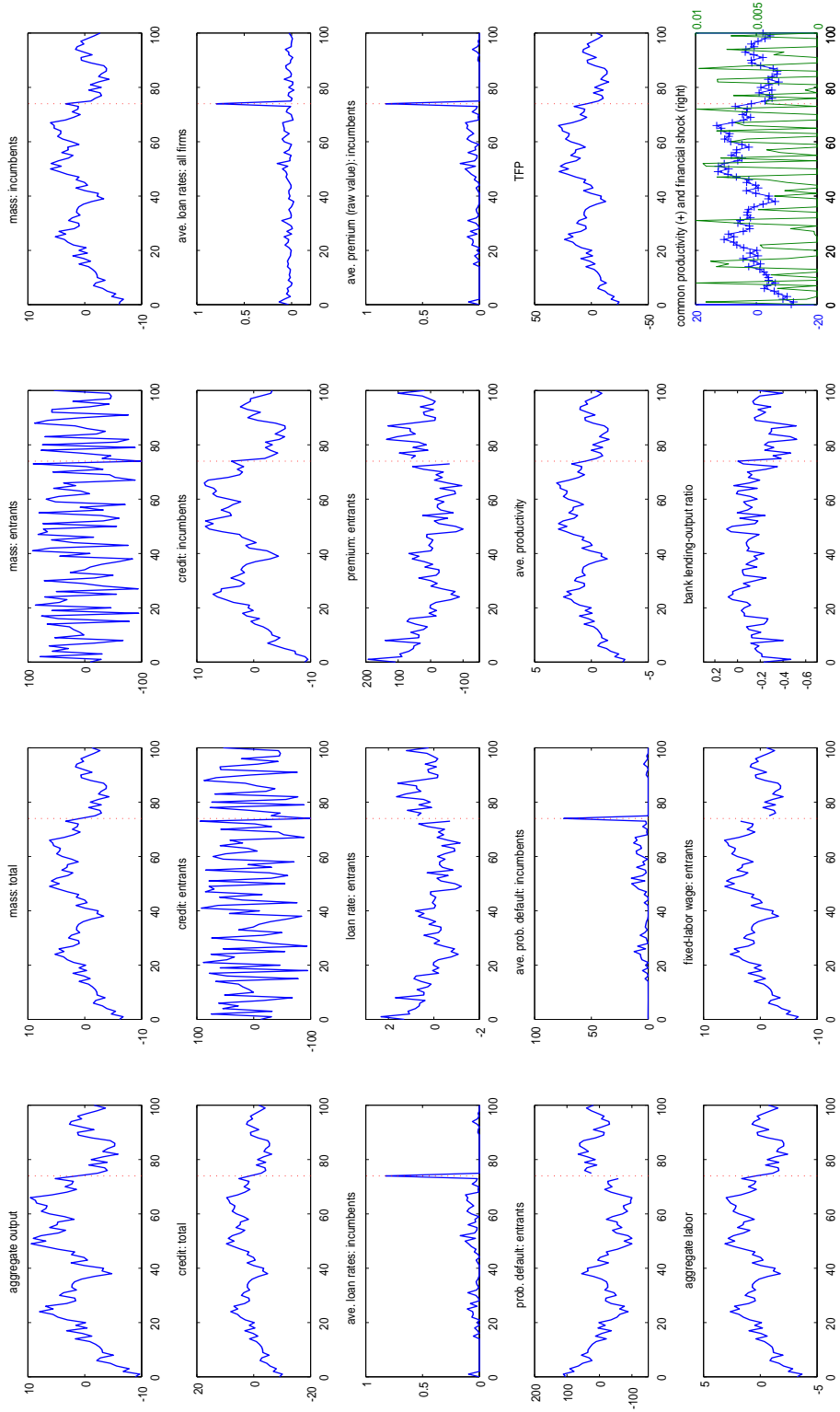


Figure 13: A simulated path: the common productivity and financial shocks

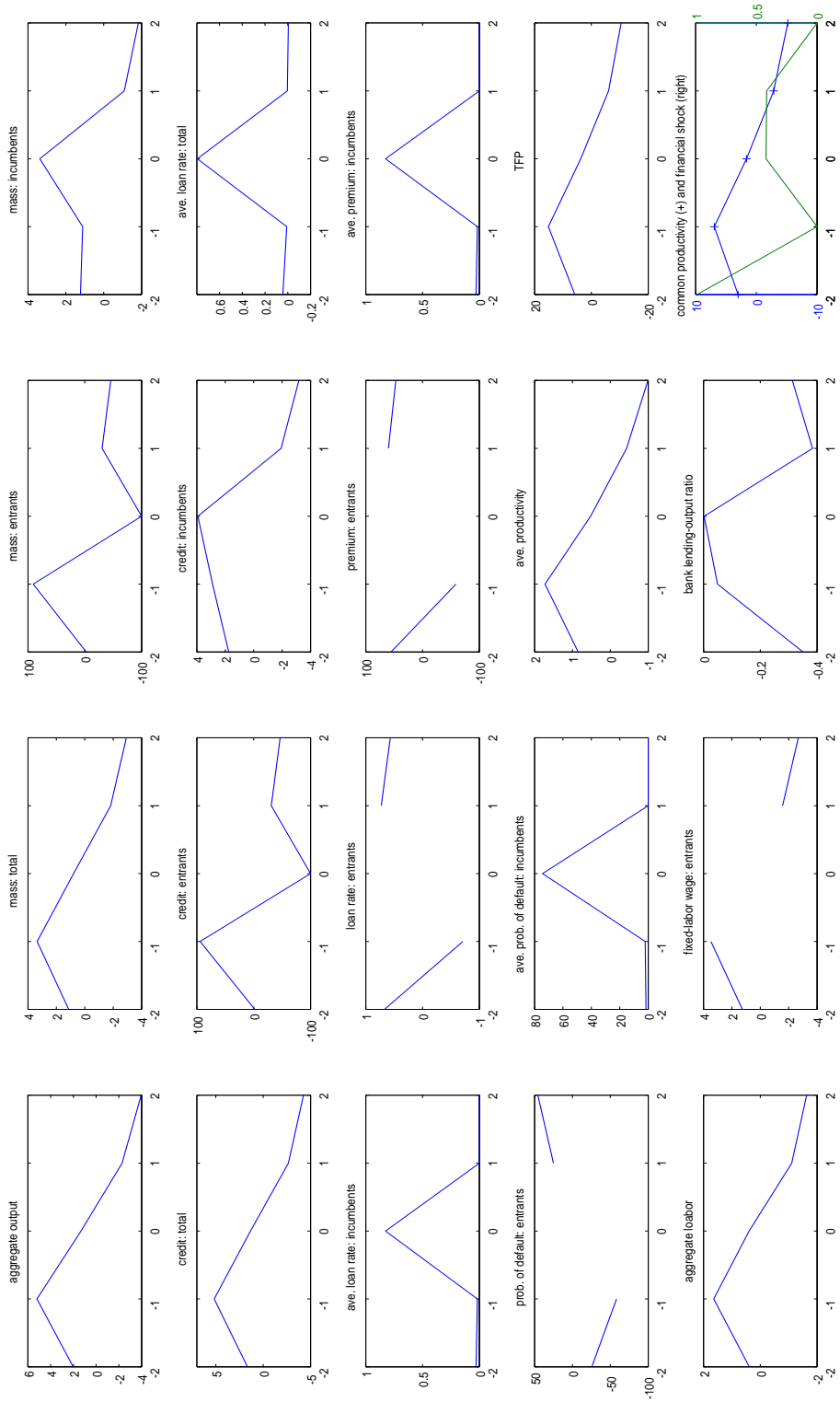


Figure 14: Around the crisis period: quarterly

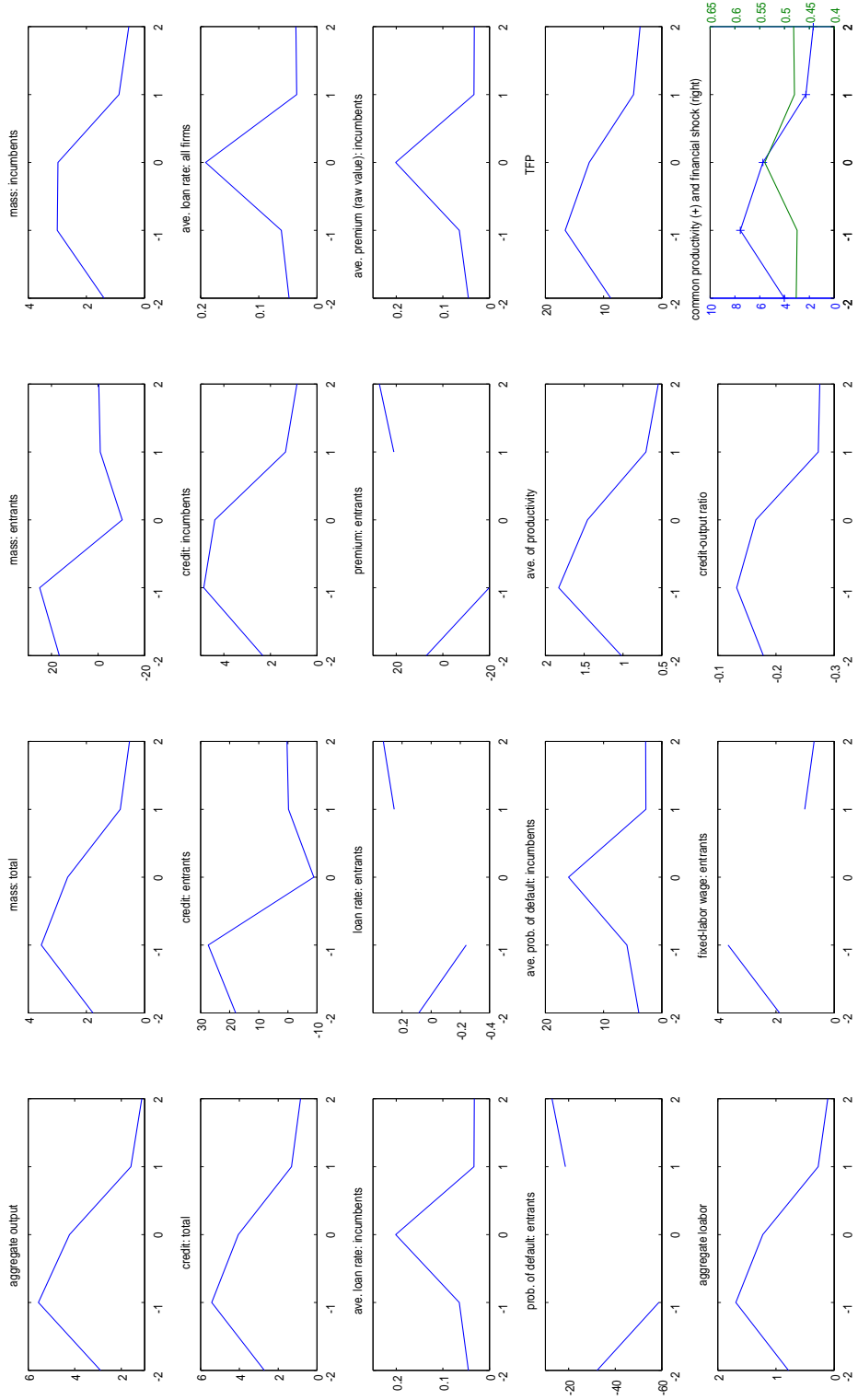


Figure 15: Around the crisis period: annual