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Bootstrapping Fuzzy-GARCH Regressions on the Day of the Week Effect in Stock Returns: Applications in MATLAB

Eleftherios Giovanis

Abstract

This paper examines the well known day of the week effect on stock returns. Various approaches have been developed and applied in order to examine calendar effects in stock returns and to formulate appropriate financial and risk portfolios. We propose an alternative approach in the estimation of the day of the week effect. More specifically we apply fuzzy regressions with triangular membership function in four major stock market index returns. We expect that if the day of the week is valid, then the Monday returns should be negative or lower than the other days of the week and in addition Friday returns should be the highest. The main findings and results are mixed and based on the fuzzy regression we conclude that there isn't the day of the week or the Monday effect. Specifically, we find a reverse Monday effect in S&P 500, a negative Friday effect in FTSE-100, a positive Tuesday effect in NIKKEI-225 and no effects in DAX index. The specific approach is appropriate as fuzzy logic regression is appropriate and able to capture the impressions and nonlinearities in finance and human behaviour, which are main characteristics in financial industry. Furthermore fuzzy regression avoids the classification of dummy variables to values of one and zero, as we do in the traditional statistical and econometric methodology

Keywords: stock returns, day of the week effect, calendar effects/anomalies, GARCH regression, fuzzy logic, fuzzy rules, fuzzy regression, bootstrapping regression, MATLAB

JEL Classification: C15, C45, G11, G14, G15, G32

1. Introduction

Many researches and papers have been written in order to examine and to test the day of the week effect. The findings among these researches are mixed. Aggarwal and Tandon (1994) found that negative returns are presented in thirteen out of sixteen countries, but these are statistically significant in only seven countries. Lakonishok and Smidt (1988) found that there is a tendency for higher returns in the last trading day of the week and negative statistically significant returns are presented on Monday. Draper and Paudyal (2002) find that Monday returns are negative and generally the returns of the other four days of the week are significantly higher. Onyuma (2009) examined the NSE 20 Index of Kenyan Stock Market and he found that Monday and Friday present

the lowest negative and highest positive returns respectively. Alagidede (2008) rejects the day of the week effect in Egypt, Kenya, Morocco and Tunisia, but he finds higher positive returns on Friday in Zimbabwe. Some other studies present the same conclusions (Mills and Coutts, 1995; Arsal and Coutts, 1997)

On the other hand the finding of other studies is that there is a Tuesday effect, rather Monday effect (Aggarwal and Rivoli, 1989; Mills *et al.* 2000; Marquering, *et al.*, 2006). On the contrary in other research studies, as those of Sullivan *et al.* (2001), who applied bootstrap snooper and Giovanis (2009) who applied bootstrapping simulated t-statistics hypothesis tests, the day of the week effect is rejected among other calendar effects. Furthermore Monday effect probably is eliminated, because the reductions in the transaction costs allowed institutional investors to gain profit from the Monday anomaly (Kamara, 1997). More recent studies found a shift in the weekday pattern, where average returns on Monday were not longer negative, but researchers found positive and significantly different average returns on Monday than the other weekday returns (Mehdian and Perry, 2001; Pettengill, 2003).

All the studies we mentioned as also the most researches which have been written in the subject of the calendar anomalies use as main tools statistical and econometrical approaches, from parametric and non parametric test hypotheses to regression models, as ordinary least squares and GARCH estimations. Since 1990 new approaches entered the research field of economics and finance, which is the artificial intelligence, as neural networks and fuzzy logic, among others, which is more appropriate for data mining techniques. The last mentioned techniques have been applied in stock prediction, but very rare in calendar anomalies. In this paper we apply fuzzy regression in order to capture the imprecision, a phenomenon which is strong in the finance investing behavior and to examine if actually there is the day of the week effect in the

stock returns we examine. On the other hand there are academicians still working with probabilities and most of them which are not experts in artificial intelligence reject fuzzy logic, before they even test it or examine it, because they adopt the belief that finance and economics deal only with probabilities. Most of them have never traded according to the models they teach, as the financial traders who work in real and everyday applications use technical analysis or artificial intelligence. Moreover GARCH or OLS models are failed especially in forecasting. Additionally Sharpe symmetric or asymmetric beta models, Capital Asset Pricing Model (CAPM), Arbitrage Pricing Theory (APT) and financial derivatives are not useful and not practicable in real finance and have been failed for two reasons. Firstly, the formulation of this model might not be correct and secondly and most significant is the econometric estimation methods followed.

The structure of this paper has as follows. In the section 2 we present the methodology for GARCH, fuzzy-GARCH regressions and the bootstrapping regression simulation procedure. In section 3 we provide the data structure and the stock market indices we examine. In section 4 the estimation results are provided. Finally, section 5 incorporates brief conclusions of our findings.

2. Methodology

2.1 Regression specification for the day of the week effect

The stock returns are defined from the following equation.

$$R_t = \log(P_t - P_{t-1}) \quad (1)$$

For the day-of-the week effect we apply the following model:

$$R_t = \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + \beta_4 D_{4t} + \beta_5 D_{5t} + \gamma R_{t-1} + \varepsilon_t \quad (2)$$

,where R_t is defined as in the relation (1), dummy variable D_{1t} takes value 1 if returns are on Mondays and 0 otherwise , continuing at the last dummy variable D_{5t} which takes value 1 if returns are on Fridays and 0 otherwise and ε_t is the disturbance term. We obtain the autoregressive term R_{t-1} in (2) to correct for possible noncyhronous trading.

2.2 GARCH models

Because the data are daily and so are of high frequency we expect that ARCH effects exist. Applying ordinary least squares we confirm this assumption. We don't present the results as the estimations are biased and not reliable. Furthermore we must decide if we should estimate with symmetric or asymmetric GARCH models. Based on asymmetric test (Engle and Ng, 1993) as also based on *Log-Likelihood* statistic and the information criteria of *Akaike*, *Schwarz* and *Hannan-Quinn* we conclude that asymmetric GARCH models are more appropriate. We estimate two models for each case the GJR-GARCH and E-GARCH models (Glosten *et al.*, 1993; Nelson, 1991). We present only one of the two above asymmetric GARCH models and the choice is done based again on *Log-Likelihood* statistic and the information criteria we mentioned previously. The mean equation for the asymmetric GARCH model estimations remains the same as in (2). But the variance equations vary between them. The variance equations for GJR-GARCH (1,1) and E-GARCH (1,1) models are presented in relations (3) and (4) respectively.

$$\sigma_t^2 = \omega + a_0 u_{t-1}^2 + a_1 \sigma_{t-1}^2 + \delta u_{t-1}^2 I_{t-1} \quad (3)$$

$$\log(\sigma_t^2) = \omega + \log a_0 (\sigma_{t-1}^2) + \alpha_1 \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \delta \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \quad (4)$$

2.3 Fuzzy regression

On the contrary with equation (2), which is actually a crisp regression, the fuzzy regression, which allows for inexact relationships between input and output, the deterministic equation (2) is modified as (Papadopoulos and Sirpi, 1999):

$$R_t = A_1 L_{1t} + A_2 L_{2t} + A_3 L_{3t} + A_4 L_{4t} + A_5 L_{5t} + A_6 R_{t-1} + \varepsilon_t \quad (5)$$

, where A_1 - A_6 are the fuzzy parameters or numbers, L_i are the fuzzy variables, and R_t , R_{t-1} are crisp variables defined as previously. On the contrary with other researches where the dependent variable is fuzzy and so we have fuzzy interval estimations, our analysis is based to the fuzzification of the dummy variables representing the trading weeks to show the weakness of the classification of *one* and *zero* dummy variables which leads to misclassification errors. First we have to define the fuzzy rules. Based on the theory or empirical researches Monday presents negative or lower returns than the other days of the week, while Tuesday, Wednesday and Thursday present higher returns than Monday but lower than Friday and finally Friday presents the highest returns. Based on these assumptions we define the following linguistic fuzzy rules.

If stock returns are on Monday, then we expect that the returns are negative or the lowest than the other trading weekdays.

If stock returns are on Tuesday, Wednesday and Thursday, then we expect that the returns are higher than Monday returns and lower than Friday returns.

If stock returns are on Friday, then we expect that the returns are positive and highest than the other trading weekdays.

The triangular functions for dummy variables D_i are defined by relation (6)

$$f(L_i; \alpha, b, c) = \left\{ \begin{array}{l} 0, D_i \leq \alpha \\ \frac{D_i - \alpha}{b - \alpha}, \alpha \leq D_i \leq b \\ \frac{c - D_i}{c - b}, b \leq D_i \leq c \\ 0, c \leq D_i \end{array} \right\} \quad (6)$$

, where parameters a and c denote the spread vales and b denotes the center value. The values for each dummy, based on the assumptions we mentioned above, are set up as following:

$[\alpha_1, b_1, c_1] = [-0.5, -0.2, 0.1]$ for dummy variable representing Monday

$[\alpha_2, b_2, c_2] = [-0.4, -0.1, 0.2]$ for dummy variable representing Tuesday

$[\alpha_3, b_3, c_3] = [-0.3, 0.0, 0.3]$ for dummy variable representing Wednesday

$[\alpha_4, b_4, c_4] = [-0.2, 0.1, 0.4]$ for dummy variable representing Thursday

$[\alpha_5, b_5, c_5] = [-0.1, 0.2, 0.5]$ for dummy variable representing Friday

The final estimated regression is equation (3), where the fuzzy dummy variables are defined by L_i . We estimate asymmetric fuzzy GARCH regressions and the choice between EGARCH and GJR-GARCH, is based on the same criteria as with the crisp GARCH estimations.

2.4 Bootstrapping Regressions

In order to get more reliable results we apply bootstrapping simulated regressions in both methodology approaches we examine. The steps for bootstrapping regressions are:

a) We draw a random resample of t observations, which t is equal with the data set, from the estimated residuals ε_t of regression (2) and for example with EGARCH estimation. We call this sample ε_t^*

b) We compute the new values of R_t using sample of the residuals. It will be:

$$R_t^* = \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + \beta_4 D_{4t} + \beta_5 D_{5t} + \gamma R_{t-1} + \varepsilon_t^* \quad (7)$$

c) We fit the new equation

$$R_t^* = \beta_1^* D_{1t} + \beta_2^* D_{2t} + \beta_3^* D_{3t} + \beta_4^* D_{4t} + \beta_5^* D_{5t} + \gamma^* R_{t-1} + \varepsilon_t^{**} \quad (8)$$

d) We repeat steps 1-3 B times, where B indicates the bootstrap replications. More specifically we estimate with 1,000 bootstrap replications

3. Data

We examine four major stock market indices. These are the S&P 500 for U.S.A., FTSE-100 index for UK, DAX index for Germany and NIKKEI-225 for Japan. The estimation starting period is 3, January 1950 for S&P 500, 2, April 1984 for FTSE-100, 26, November 1990 for DAX and 4, January 1984 for NIKKEI-225. The ending time period is common for all estimations and it is 30, October 2009. The data are in daily frequency.

4. Empirical Results

In tables 1 and 2 the estimated regressions results with asymmetric GARCH and asymmetric Fuzzy-GARCH are respectively reported. Based on EGARCH(1,1) we observe that the day of the week effect exist in S&P 500 index, as Monday returns are negative and the highest and positive returns are presented on Friday, as well as on Wednesday. The respective Fuzzy-EGARCH estimation shows that the highest and

positive returns are reported on Monday followed by Tuesday and the highest negative returns are presented on Thursday followed by Friday and then on Wednesday. However it should be noticed that Thursday returns are statistically insignificant. This shows that actually we found a reverse Monday effect.

For the FTSE-100 stock index we found also the Monday effect based on asymmetric GJR-GARCH estimation, where returns only on Monday and Friday are statistically significant with Monday returns to be lower than those of Friday. On the other hand based on the estimations of asymmetric fuzzy GJR-GARCH model, we conclude that only Friday returns are statistically significant, with negative average returns. Additionally in both estimations ARCH effects are not eliminate as also we reject the null hypothesis of no-autocorrelation in GARCH estimation. In Fuzzy-GARCH regression we accept the null hypothesis of no-autocorrelation in $\alpha=0.01$.

The next stock index we examine is DAX. Based on the bootstrapping asymmetric GJR-GARCH model, statistically significant returns are reported on Monday, Wednesday and Thursday, with positive returns presented in all days, and the highest returns are reported on Monday. So, a first conclusion is that Monday or the day of the week effect is rejected in the case of DAX stock index, where we found actually a reverse Monday effect. On the other hand in fuzzy regression and table 2 we observe that all coefficients are statistically insignificant indicating that there is not pattern in DAX index.

The last stock index we examine is NIKKEI-225 index, where based on asymmetric EGARCH bootstrapping estimation statistically significant returns are presented only on Wednesday, which are positive while according to fuzzy EGARCH bootstrapping regression, only Tuesday presents significant and positive returns. So in both cases we reject the day of the week effect. Also we should mention that ARCH effects and

autocorrelation are not eliminated in the case of S&P 500 with EGARCH estimation and we reject autocorrelation only in $\alpha=0.01$, while these problems are rejected in $\alpha=0.01$ and $\alpha=0.05$ with fuzzy regression.. A quite similar situation is presented in FTSE-100, where the specific problems are not eliminated in any statistical significance level, while with fuzzy regression we accept the null hypothesis of no autocorrelation in $\alpha=0.01$ level. So based on these results, fuzzy regressions are able to eliminate ARCH effects and to solve for autocorrelation, so for this reason we prefer fuzzy than the crisp regressions.

The main conclusion is that according to fuzzy estimations, we reject the Monday effect in all cases. More specifically the DAX market is an efficient market, based always on the Monday or the day of the week effect pattern only, where there isn't a specific day in which the returns are statistically significant. Moreover there is a reverse pattern in S&P 500 index, where you can sell on closed Friday prices and buy on Monday, while there is a pattern in FTSE-100, where you can buy on Friday and sell any other weekday. Finally, the pattern in NIKKEI-225 is to buy on Tuesday.

Table 1. GARCH bootstrapping estimations for the day-of-the-Week effect

<i>Mean Equation Coefficients</i>						
<i>Index</i>	β_1	β_2	β_3	β_4	β_5	γ
S&P 500	-0.0002	0.0008	0.0014	0.0011	0.0014	0.0586
EGARCH	(0.0003)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0113)
	[-2.635]**	[3.478]*	[5.559]*	[4.317]*	[5.664]*	[5.167]*
FTSE-100	0.0001	0.0003	0.0002	-0.0001	0.0011	0.0205
GJR-GARCH	(0.0004)	(0.0004)	(0.0004)	(0.00039)	(0.00039)	(0.0171)
	[2.276]**	[0.616]	[0.470]	[-0.312]	[2.547]**	[1.198]
DAX	0.00093	0.00034	0.00068	0.00079	-0.00012	0.0142
GJR-GARCH	(0.00039)	(0.00038)	(0.00038)	(0.00038)	(0.00038)	(0.0122)
	[2.370]**	[0.902]	[1.801]***	[2.081]**	[-0.321]	[1.158]
NIKKEI-225	0.00036	5.67e-05	0.00077	-4.15-e05	0.00043	0.00794
EGARCH	(0.00034)	(0.00033)	(0.00032)	(0.00033)	(0.00033)	(0.0101)
	[1.071]	[0.168]	[2.330]**	[-0.124]	[1.311]	[0.782]
<i>Variance Equation Coefficients</i>						
<i>Index</i>	ω	α_0	α_1	δ		
S&P 500	-0.0850	0.0576	0.9903	-0.0257		
EGARCH	(0.0151)	(0.0056)	(0.0017)	(0.0036)		
	[-5.640]*	[10.380]*	[574.448]*	[-7.112]*		
FTSE-100	1.68e-05	0.0180	0.9334	0.0417		
GJR-GARCH	(1.82e-05)	(0.0063)	(0.0095)	(0.0095)		
	[0.920]	[2.877]**	[98.678]*	[4.384]*		
DAX	2.41e-05	0.198	0.9613	0.0231		
GJR-GARCH	(8.90e-06)	(0.0059)	(0.0063)	(0.0083)		
	[2.707]**	[3.373]*	[153.023]*	[2.766]**		
NIKKEI-225	-0.1656	0.0791	0.9789	-0.0383		
EGARCH	(0.0369)	(0.0112)	(0.0047)	(0.0073)		
	[-4.494]*	[7.054]*	[209.188]*	[-5.244]*		
<i>Diagnostic Tests</i>						
<i>Index</i>	<i>LL</i>	<i>AIC</i>	<i>SBC</i>	<i>ARCH-LM</i> (5)	<i>LBQ²</i> (5)	<i>F-statistic</i>
S&P 500	51658.21	-6.851	-6.847	665.473	25.1963	13.162
EGARCH				{0.000}	{0.0139}	{0.000}
FTSE-100	17764.15	-6.331	-6.326	483.376	30.8916	3.125
GJR-GARCH				{0.000}	{0.0020}	{0.0084}
DAX	11654.50	-5.710	-5.703	0.2993	1.3344	4.293
GJR-GARCH				{0.930}	{0.931}	{0.008}
NIKKEI-225	15716.10	-5.783	-5.778	0.388	16.843	2.063
EGARCH				{0.857}	{0.1556}	{0.0828}

*denotes significance in 0.01 level, **denotes significance in 0.05 level *** denotes significance in 0.10 level - standard errors, in parentheses, z-statistics in brackets., p-values in {}, LL is the Log Likelihood, AIC and SBC refer to Akaike and Schwarz information criteria, ARCH-LM is the Lagrange multiplier test for ARCH effects with 5 lags, LBQ² is the Ljung-Box test on squared standardized residuals with 5 lags

Table 2. Fuzzy-GARCH bootstrapping estimations for the day-of-the-Week effect

<i>Mean Equation Coefficients</i>						
<i>Index</i>	β_1	β_2	β_3	β_4	β_5	γ
S&P 500	0.00303	0.000158	-0.00033	-0.000156	-0.00124	0.1094
EGARCH	(0.0003)	(9.27e-05)	(0.000105)	(9.87e-05)	(0.00031)	(0.0080)
	[9.917]*	[1.707]***	[-3.191]*	[-1.578]	[-4.006]*	[13.636]*
FTSE-100	0.000348	-4.73e-05	0.000143	0.0003	-0.0020	0.0205
GJR-GARCH	(0.000636)	(0.0002)	(0.000199)	(0.0006)	(0.0012)	(0.0171)
	[0.546]	[-0.236]	[0.718]	[0.426]	[-1.666]***	[1.198]
DAX	0.000114	0.000210	0.00020	-5.34e-06	-2.13e-05	-0.00027
GJR-GARCH	(0.000923)	(0.000332)	(0.00032)	(0.00033)	(0.00092)	(0.0166)
	[0.123]	[0.631]	[0.642]	[-0.016]	[-0.023]	[-0.016]
NIKKEI-225	0.000233	0.000574	-0.000406	-0.000108	0.000488	0.0176
EGARCH	(0.00063)	(0.00025)	(0.00025)	(0.00025)	(0.00076)	(0.0137)
	[0.371]	[2.263]**	[-1.577]	[-0.430]	[0.642]	[1.281]
<i>Variance Equation Coefficients</i>						
<i>Index</i>	ω	α_0	α_1	δ		
S&P 500	-0.2507	0.1447	0.9853	-0.0646		
EGARCH	(0.0096)	(0.0031)	(0.0090)	(0.00320)		
	[-25.873]*	[45.434]*	[109.477]*	[-32.195]*		
FTSE-100	1.71e-06	0.0352	0.9058	0.0833		
GJR-GARCH	(2.07e-07)	(0.0064)	(0.0060)	(0.0075)		
	[8.260]*	[5.481]*	[149.51]*	[10.985]*		
DAX	3.30e-06	0.0214	0.9104	0.0964		
GJR-GARCH	(3.59e-07)	(0.0051)	(0.0064)	(0.0085)		
	[9.211]*	[4.212]*	[141.577]*	[11.303]*		
NIKKEI-225	-0.4546	0.2255	0.9678	-0.1111		
EGARCH	(0.0208)	(0.0071)	(0.0022)	(0.0052)		
	[-21.792]*	[31.400]*	[435.987]*	[-21.077]*		
<i>Index</i>	<i>Diagnostic Tests</i>					
	<i>LL</i>	<i>AIC</i>	<i>SBC</i>	<i>ARCH-LM</i> (5)	<i>LBQ²</i> (5)	<i>F-statistic</i>
S&P 500	51668.61	-6.855	-6.850	1.996	9.854	12.092
EGARCH				{0.0772 }	{0.079}	{0.000}
FTSE-100	17563.24	-8.269	-8.263	386.47	22.663	2.6285
GJR-GARCH				{0.000}	{0.0307}	{0.0164}
DAX	14303.17	-5.960	-5.946	0.432	2.160	1.022
GJR-GARCH				{0.826}	{0.827}	{0.3939}
NIKKEI-225	16982.14	-5.960	-5.949	0.378	1.858	0.304
EGARCH				{0.863}	{0.868}	{0.785}

*denotes significance in 0.01 level, **denotes significance in 0.05 level *** denotes significance in 0.10 level - standard errors in parentheses, z-statistics in brackets., p-values in {}, LL is the Log Likelihood, AIC and SBC refer to Akaike and Schwarz information criteria, ARCH-LM is the Lagrange multiplier test for ARCH effects with 5 lags, LBQ² is the Ljung-Box test on squared standardized residuals with 5 lags

5. Conclusions

We examined the well known day of the week effect with two methodologies. Asymmetric GARCH estimations with crisp dummy variables and asymmetric fuzzy GARCH models with fuzzy dummy variables. We applied bootstrapping simulations in both procedures in order to solve and improve the data mining problem. According to fuzzy regressions we found a reverse Monday effect in the case of the S&P 500 index, while we found only negative returns on Friday for FTSE-100 and positive returns on Tuesday for NIKKEI-225. Finally we found that the returns in DAX index are all insignificant. Generally a fuzzy regression with bootstrapping simulation exceeds the problems of the traditional procedure of crisp categorization of one and zero and we are able to set up an efficient data mining technique, as also fuzzy procedure is able to solve for ARCH effects and autocorrelation. Furthermore finance and economics are behavioral sciences so fuzzy and artificial intelligence are more able to capture nonlinearities and imprecision. Additionally we propose to combine GARCH and fuzzy logic, which means that we combine probabilities with possibilities. Of course before anyone draw rush conclusions it is necessary to understand the meaning of possibility and how the combination of possibilities and probabilities outperform the estimating procedures where we obtain only one of them and to fully understand the fuzzy procedure or neural networks and genetic algorithms before he or she make any attack or insult.

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Appendix

Programming routines for GARCH and Fuzzy-GARCH with standard bootstrap in the residuals.

```
clear all;
load file.mat

% There are two matrices. The first matrix, which is actually vector,
is y and contains the stock returns. The second matrix is x and
includes the dummy variables representing the respective days

fuzzy=1 % 0 for GARCH regressions and 1 for Fuzzy-GARCH regressions

model=1 % ! for GARCH, 2 for EGARCH and 3 for GJR-GARCH
Dist=' Gaussian ' % Gaussian or T
P=1 %Order for ARCH component
Q=1 %Order for GARCH component

if fuzzy==0
lag_y=lagmatrix(y,1)
x=[x lag_y]
x=x(2:end,:)
y=y(2:end,:)

elseif fuzzy==1
a1=-0.5
b1=-0.2
c1=0.1

a2=-0.4
b2=-0.1
c2=0.2

a3=-0.3
b3=0
c3=0.3

a4=-0.2
b4=0.1
c4=0.4

a5=-0.1
b5=0.2
c5=0.5

W1= trimf(x(:,1),[a1 b1 c1])
W2= trimf(x(:,2),[a2 b2 c2])
W3= trimf(x(:,3),[a3 b3 c3])
W4= trimf(x(:,4),[a4 b4 c4])
W5= trimf(x(:,5),[a5 b5 c5])
x=[W1 W2 W3 W4 W5]
lag_y=lagmatrix(y,1)
x=[x lag_y]
x=x(2:end,:)
y=y(2:end,:)
end
```

```

if model==1
spec = garchset('VarianceModel','GARCH','Distribution',Dist,'P', P,
'Q', Q,...
'C', NaN,'Display','off')

kk=3
elseif model==2

spec = garchset('VarianceModel','EGARCH','Distribution',Dist,'P', P,
'Q', Q,...
'C', NaN,'Display','off')
kk=4
elseif model==3
spec = garchset()
spec = garchset('VarianceModel','GJR','Distribution',Dist,'P', P,
'Q', Q,...
'C', NaN,'Display','off')
kk=4
end

[Coeff,Errors,LLF,Innovations,Sigmas,Summary] =garchfit(spec,y,x)

e= Innovations % Get the estimated residuals
bols=Coeff.Regress'
e=y-x*bols
b=10000; % Set up the bootstrapping replications
N=length(e) % Set the size of bootstrapping samples
for B = 0:b
iboot = ceil(N*rand(N,1));
yboot = e(iboot(1:N),:);
end
y=y-yboot
[Coeff,Errors,LLF,Innovations,Sigmas,Summary] =garchfit(spec,y,x)
bols=Coeff.Regress'

if model==1
K=Coeff.K
ARCH=Coeff.ARCH
GARCH=Coeff.GARCH
K_se=Errors.K
ARCH_se=Errors.ARCH
GARCH_se=Errors.GARCH
b_GARCH=[K;ARCH;GARCH]
se_GARCH=[K_se;ARCH_se;GARCH_se]
tstudent_GARCH=b_GARCH./se_GARCH
%bols=[C;bols]
%e=Innovations
e=y-x*bols
xsquare=x'*x;
k=5
s2 = (y-x*bols)'*(y-x*bols)/(N-k);
Vb=s2*inv(x'*x); % Get the variance-covariance matrix
se=sqrt(diag(Vb)); % get coefficient standard errors
tstudent_b=bols./se;
tstudent=[tstudent_b;tstudent_GARCH]
bols=[bols;b_GARCH]
se=[se;se_GARCH]

```

```

elseif model==2
K=Coeff.K
ARCH=Coeff.ARCH
GARCH=Coeff.GARCH
Leverage=Coeff.Leverage
K_se=Errors.K
ARCH_se=Errors.ARCH
GARCH_se=Errors.GARCH
Leverage_se=Errors.Leverage

b_GARCH=[K;ARCH;GARCH;Leverage]
se_GARCH=[K_se;ARCH_se;GARCH_se;Leverage_se]
tstudent_GARCH=b_GARCH./se_GARCH
%bols=[C;bols]
e=y-x*bols
xsquare=x'*x;
k=5
s2 = (y-x*bols)'*(y-x*bols)/(N-k);
Vb=s2*inv(x'*x); % Get the variance-covariance matrix
se=sqrt(diag(Vb)); % get coefficient standard errors
tstudent_b=bols./se;
tstudent=[tstudent_b;tstudent_GARCH]
bols=[bols;b_GARCH]
se=[se;se_GARCH]

elseif model==3
K=Coeff.K
ARCH=Coeff.ARCH
GARCH=Coeff.GARCH
Leverage=Coeff.Leverage
K_se=Errors.K
ARCH_se=Errors.ARCH
GARCH_se=Errors.GARCH
Leverage_se=Errors.Leverage
b_GARCH=[K;ARCH;GARCH;Leverage]
se_GARCH=[K_se;ARCH_se;GARCH_se;Leverage_se]
tstudent_GARCH=b_GARCH./se_GARCH
%bols=[C;bols]
e=y-x*bols
xsquare=x'*x;
k=5
s2 = (y-x*bols)'*(y-x*bols)/(N-k);
Vb=s2*inv(x'*x); % Get the variance-covariance matrix
se=sqrt(diag(Vb)); % get coefficient standard errors
tstudent_b=bols./se;
tstudent=[tstudent_b;tstudent_GARCH]
bols=[bols;b_GARCH]
se=[se;se_GARCH]
end

t=N
tss = (t-1) * std(y)^2; % total sum of squares
rss = e'*e; % residual sum of squares
ess = tss - rss; % explained (regression) sum of
squares
dfe=t-k; % degrees of freedom
r2 = ess / tss; % R-squared
rb2 = 1 - rss/tss * (t - 1)/dfe; % R-bar-squared
pt = 1-tcdf(abs(tstudent), dfe); % P-Values for t-student
see = sqrt(rss/dfe); % standard error of estimate
fs = ess/rss * dfe/(k-1); % F-statistic

```

```

ps =1- fcdf(fs, k-1, dfe);           % p-value for F-statistic
stdev=sqrt((y'*y)/t)
AIC=log(stdev^2)+2*k/t;              % Akaike Criterion
BIC=log(stdev^2)+k*log(t)/t;        % Schwarz criterion
HQ=log(stdev^2)+2*k*log(log(t))/t;  % Hanna-Quinn Criterion
LL=- (t/2)*(1+log(2*pi))+log((e'*e)/t) % Log-Likelihood statistic

% ARCH-LM statistic
[H,pValue,ARCHstat,CriticalValue] = archtest(e,5,0.05)

% Heteroskedasticity test
e2=e.*e;
x2=x.*x;
v=e2-x2*(e2'/x2)';
e2=e2-mean(e2)';
te=length(e(:,1))*(1-(v'*v)/(e2'*e2));
ht=1-chi2cdf(te,length(x(1,:)));

%ljung_box statistic

[H,p_Jung,Qstat,CriticalValue] =lbqtest(e,12,0.05)

ccc=0.1
re = '=====';
sp = '          ';

% print the results
disp([re ' Regression Results ' re])
disp([sprintf('Number of Observations :      %14.0f', t) ...
      sp 'Date:                ' datestr(today, 2)])
disp([sprintf('Residual Sum of Squares:      %14.4f', rss) ...
      sp sprintf('Rbar-squared :           %10.4f', rb2)])
disp([sprintf('Std Error of Estimate:       %14.4f', see)...
      sp sprintf('F-statistic:           %10.4f', fs)])
disp([sprintf('P-Value for Heteroskedasticity: %14.4f', ht)...
      sp sprintf('P-Value for F-statistic:%10.4f',ps)])
disp([sprintf('ARCH-LM statistic:           %14.4f', ARCHstat)...
      sp sprintf('Probability for ARCH-LM:%10.4f',pValue)])
disp([sprintf('Q-Stat for autocorrelation:    %14.4f', Qstat)...
      sp sprintf('P-value for Q-Stat :      %10.4f',p_Jung)])
disp([sprintf('Akaike criterion:            %14.4f', AIC)...
      sp sprintf('Hanna-Quinn criterion: %10.4f', HQ)])
disp([sprintf('Schwarz criterion:           %14.4f', BIC)...
      sp sprintf('Log-Likelihood statistic: %10.3f', LL)])
disp(blanks(1)')
disp(['VARIABLE      COEFFICIENT      STD ERROR      T-STATISTICS      P-
VALUE'])
disp(['-----',...
      '-----'])

for iii=1:k+1+kk
disp([sprintf('y%1.0f', iii-ccc)...
      blanks(4 - fix(log10(iii-.5*ccc)))...
      sprintf(' %14.4f', bols(iii))...
      sprintf(' %14.4f', se(iii))...
      sprintf(' %14.3f', tstudent(iii))...
      sprintf(' %14.3f', pt(iii))]);
end

```