Fiscal Policy, Home Production and Growth Dynamics

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Abstract

Using an endogenous growth model with physical and human capital, we explore short-run as well as long-run effects of fiscal policy in the presence of households' production activities. We first show that our model has a unique balanced-growth path that satisfies saddlepoint stability. We then conduct fiscal policy experiments both in and out of the balanced-growth equilibrium. The main focus of the paper is to study the dynamic behavior of the model economy and the effects of fiscal actions analytically. In so doing, we examine how the presence of home production yields the policy implications that are different from those obtained in the standard setting that does not consider home production.

JEL classification: H31, D13, O41

Keywords: fiscal policy, home production, multi-sector endogenous growth model.

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1 Introduction

Production activities within the households are substantial. Time and resources devoted to home production share considerable portions even in advanced countries. For example, inspecting the US data, Eisner (1988) concludes that an estimate of home-produced output relative to measured gross national product is in the range from 20 to 50 percent. Wrase (2001) reports that a married couple in the United States, on average, devotes 25 percent of discretionary time to unpaid home work and 33 percent of it to work in the market place for pay.

The idea that home production may play a relevant role in macroeconomics has generated a bulk of the recent studies focusing on how households' production activities affect business cycles, macroeconomic policy performances and long-term economic growth. Most of this literature has tried to reveal that introducing a home production sector into the otherwise standard macroeconomic models improves the models' ability in explaining observed data. For example, Benhabib et al. (1991) and Greenwood and Hercowitz (1991) show that the introduction of home production into the standard real business cycle theory significantly improves the performances of the calibrated models. The intuition behind such a good fitness is that the incorporation of a home sector in the standard one-sector real business cycle model brings about possibility of substitution between market and nonmarket production over time. Therefore, relative productivity differentials between the two sectors may enhance volatility in market activity. Furthermore, the substitution between home and market commodities at a given date, not just at different dates, affects the size of fluctuations induced by productivity shocks.[1] As for explanation of the observed economic development facts, Parente et al. (2000) illustrate that, by adding a home production sector to the neoclassical growth model, international income differences can be accounted well under relatively small differences in policies. This is because, in the presence of household production, fiscal policy affects not only capital accumulation but also the shares between market and nonmarket activities.

Along the line of recent research on macroeconomic analysis of household production, this paper explores the effects of fiscal policy in a growing economy with home production.

[1] The empirical work of McGattan, Rogerson and Wright (1997) claims that the elasticity of substitution between home and market goods is considerably high.

We construct a three-sector endogenous growth model with Beckerian home production and examine the effects of fiscal policy in and out of the balanced-growth equilibrium. More specifically, we introduce a household production sector into Lucas' (1988) model of endogenous growth in which continuing economic growth is sustained by accumulations of both physical and human capital. Since home production activities are tax-free, nonmarket ones, we can predict that introducing a home production sector into the Lucas model may yield the fiscal policy effects that are different from those obtained in the standard setting. To see this, we examine the effects of taxation on labor and capital income as well as subsidy to investment on human capital. We demonstrate that in the presence of a tax-free home production sector, fiscal policy affects resource allocation between market and nonmarket sectors, which generates new policy impacts that are not observed in the original Lucas' framework. We compare the derived results with those obtained in the model without home production.

The main contributions of this paper are twofold. First, we present an analytical discussion on fiscal policy effects in a Lucas-type endogenous growth model with home production. In the context of human-capital-based endogenous growth models without home production, short-run as well as long-run impacts of capital income taxation have been explored thoroughly[2]. In contrast, the number of existing studies on the role of fiscal policy in endogenously growing economies with home production is relatively small. In addition, the majority of this literature such as Einarsson and Marquis (2001) rely entirely on numerical experiments in considering policy impacts. Milesi-Ferretti and Roubini (1998) present an analytical discussion on the relation between income as well as consumption taxes and long-term economic growth. Their analysis, however, is restricted to the balanced-growth equilibrium and the short-run effects of policy changes are out of touch. In this paper, we examine both short-run and long-run impacts of policy changes analytically. Furthermore, in addition to the growth effect of fiscal policy, we study policy effects on other key variables such as human capital allocation to home production, factor intensities in the market and home goods sectors, and the rates of returns to physical and human capital.

The second contribution of this paper is to show the existence and stability of the balanced-growth path with home production and tax distortions. Using models without home production, Mino (1996) and Ortigueira (1998, 2000) confirm the existence of unique

[2] See, for example, Bond, Wang and Yip (1996), Mino (1996), and Ortigueira (1998).

stable path that converges to the balanced-growth equilibrium in the presence of fiscal policy distortions. On the other hand, by use of a Rebelo-type two-sector model, Bond, Wang and Yip (1996) find that under an alternative tax scheme, the balanced-growth equilibrium may be locally indeterminate (i.e. there is a continuum of converging paths near the balanced-growth path).[3] Furthermore, Ortigueira and Santos (2002) point out that in the Lucas' setting the existence of the interior equilibrium may be disturbed in the presence of tax distortions. These different results remind us of the necessity to conduct stability analysis for the home production model with fiscal policy distortions. The analysis in Section 3 demonstrates that unlike the finding of Bond, Wang and Yip (1996), the balanced-growth equilibrium satisfies local saddle-path stability even though the government carries out factor specific income taxation.[4]

The rest of the paper is arranged as follows. Section 2 constructs the base model. The existence and stability of the balanced-growth path are reported in Section 3. Section 4 conducts the long-run and short-run fiscal policy experiments. Section 5 concludes.

2 The Model

2.1 Production

There are three production sectors in the economy: market goods sector, home goods sector and education sector. The market goods sector employs human as well as physical capital to produce a homogenous output that can be used for consumption and investment. We specify the production function of the market goods as a Cobb-Douglas one:

$$Y_m = A(sK)^{\beta_1}(uH)^{1-\beta_1},\ A > 0, \quad 0 < \beta_1 < 1, \tag{1}$$

where Y_m, K and H are output of the market goods, stocks of physical and human capital, respectively. In addition, s and u respectively denote the ratios of physical and human capital devoted to the market goods production.

[3] In Rebelo (1991), the education sector uses physical as well as human capital under a constant-returns-to-scale technology.

[4] Perli (1998) discusses indeterminacy of equilibrium in a real business cycle model with production externalities and home production.

The production technology of the home goods sector is specified in the similar manner. Production activities within the household also need both physical and human capital. The home sector produces a pure consumption good and its production function is given by

$$Y_n = [(1-s)K]^{\beta_2}(lH)^{1-\beta_2}, \quad 0 < \beta_2 < 1, \tag{2}$$

where Y_n is output of the home goods and l is the ratio of human capital used for home production. For notational simplicity, the total factor productivity of home goods sector is normalized to one. If $\beta_2 = 0$ in (2), then the home goods are produced by human capital alone. This case corresponds to the model examined by Ortigueira (2000) who calls such a specification the 'quality leisure' model.

As for education activities, we follow Lucas' (1988) formulation: new human capital is produced by a linear technology that employes human capital alone. The production function of the education sector is

$$Y_e = B(1-u-l)H, \quad B > 0 \tag{3}$$

where Y_e denotes education services. Since human capital is also used for market and home production activities, the rate of human capital employed by the education sector is $1-u-l$. In this paper we assume that the education services are produced by an education industry, so that households purchase Y_e in the education service market.[5]

The market goods and education sectors are competitive. Letting r and w be the before-tax rates of return to physical and human capital, profit maximization of the firms in the final good sector yields:

$$r = \frac{\partial Y_m}{\partial (sK)} = A\beta_1 k_m^{\beta_1 - 1}, \qquad w = \frac{\partial Y_m}{\partial (uH)} = A(1-\beta_1)k_m^{\beta_1}. \tag{4}$$

Similarly, in the education sector it holds that

$$w = p\frac{\partial Y_e}{\partial (1-u-l)H} = pB, \tag{5}$$

where p is the price of education services in units of the final good. Note that the rate of return to human capital, w, can be considered the real wage rate in terms of the market good.

[5] In the absence of market distortions, whether or not education services are market goods does not affect resource allocation. If there are policy distortions, the equilibrium conditions may differ from those established in the model where education is a home activity so that it is free from taxation.

2.2 Households

There is a continuum of households whose number is normalized to one. The representative household's objective is to maximize a discounted sum of utilities over an infinite time horizon. The objective functional of the household is

$$U = \int_0^\infty \frac{C^{1-\sigma}}{1-\sigma} e^{-\rho t} dt, \quad \sigma > 0, \quad \sigma \neq 1, \quad \rho > 0.$$

In the above, C denotes a composite of consumption goods defined by

$$C = C_m^\gamma C_n{}^{1-\gamma}, \quad 0 < \gamma < 1,$$

where C_m and C_n are consumption levels of market goods and home-made goods, respectively.

The households purchase the market goods and education services, while they produce goods and services by using physical as well as human capitals. The flow budget constraint the representative household faces is

$$\dot{K} = (1 - \tau_k)\, rsK + (1 - \tau_h) w\, (1 - l)\, H - C_m - (1 - \tau_e)\, ph + T - \delta K, \tag{6}$$

where h is spending for education, and τ_k and τ_h respectively denote the rates of income tax on physical and human capital. In addition, τ_e is the rate of education subsidy (an investment tax credit for human capital)[6], T is a lump-sum transfer (a lump-sum tax if it has a negative value) from the government, and δ denotes the depreciation rate of physical capital. Income of the household consists of the after-tax revenue from physical capital holding which is used for market production, $(1 - \tau_k)\, rsK$, the after-tax revenue created by human capital that participates market activities, $(1 - \tau_h) w\, (1 - l)\, H$, and the transfer from the government, T. Notice that since we have assumed that there is an education service market, the human capital employed for market activities is $(1 - l)\, H$. Expenditures of the household are: gross investment for physical capital, $\dot{K} + \delta K$, gross investment for human capital, ph, and consumption expenditure for market goods, C_m. In addition to the budget constraint, the optimizing household takes the following human capital accumulation process into account:

$$\dot{H} = h - \eta H, \tag{7}$$

[6] An alternative implication is that τ_e expresses the rate of public education and $1 - \tau_e$ denote the ratio of private education.

where η denotes the depreciation rate of human capital.

The representative household maximizes U subject to (6), (7) and the home production technology (2), together with the initial holdings of K and H. Since all of the home goods are consumed within the household, it holds that $C_n = Y_n$. Therefore, we may set up the current value Hamiltonian in such a way that

$$\begin{aligned}\mathcal{H} &= \frac{1}{1-\sigma}\left\{C_m{}^{\gamma(1-\sigma)}[(1-s)^{\beta_2}K^{\beta_2}(lH)^{1-\beta_2}]^{(1-\gamma)(1-\sigma)}-1\right\}+p_k\Big[(1-\tau_k)r(sK) \\ &\quad +(1-\tau_h)w\,(1-l)\,H - C_m(t)-(1-\tau_e)\,ph-\delta K(t)+T\Big]+p_h(h-\eta H),\end{aligned}$$

where p_k and p_h respectively express the shadow values of physical and human capital in terms of utility. The household's control variables in this problem are C_m, s, l and h, where $s,\, l,\, h \in [0,1]$ and $l+h \in [0,1]$.

In what follows, we denote the factor intensities in the market goods and home production sectors by the following:

$$k_m \equiv \frac{sK}{uH}, \qquad k_n \equiv \frac{(1-s)K}{lH}.$$

We find that the first-order conditions for an interior optimum are given by:

$$\frac{\partial \mathcal{H}}{\partial C_m} = \gamma C_m^{\gamma(1-\sigma)-1}C_n^{(1-\gamma)(1-\sigma)} - p_k = 0, \tag{8}$$

$$\frac{\partial \mathcal{H}}{\partial s} = (1-\gamma)\beta_2 C_m^{\gamma(1-\sigma)}C_n^{(1-\gamma)(1-\sigma)-1}k_n^{\beta_2-1}K - p_k(1-\tau_k)rK = 0, \tag{9}$$

$$\frac{\partial \mathcal{H}}{\partial l} = (1-\gamma)(1-\beta_2)C_m^{\gamma(1-\sigma)}C_n^{(1-\gamma)(1-\sigma)-1}k_n^{\beta_2}H - p_k\,(1-\tau_h)\,wH = 0, \tag{10}$$

$$\frac{\partial \mathcal{H}}{\partial h} = -\,(1-\tau_e)\,p_k p + p_h = 0 \tag{11}$$

Condition (11) gives

$$\frac{p_h}{p_k} = (1-\tau_e)\,p = (1-\tau_e)\,\frac{w}{B}, \tag{12}$$

which shows the relation between the relative implicit price, p_h/p_k, the market price of education services, p and the real wage rate, w.

By use of (9), we see that the shadow value of physical capital follows

$$\dot{p_k} = p_k[\rho+\delta-(1-\tau_k)r]. \tag{13}$$

Similarly, in view of (10), the shadow value of human capital changes according to

$$\dot{p_h} = p_h\,(\rho+\eta) - p_k\,(1-\tau_h)\,w. \tag{14}$$

Additionally, these shadow values should satisfy the transversility conditions:

$$\lim_{t\to\infty} e^{-\rho t} p_k K = 0, \qquad \lim_{t\to\infty} e^{-\rho t} p_h H = 0. \tag{15}$$

2.3 The Government

As assumed above, the government imposes flat-rate income taxes on physical and human capital that are used for market production activities, while it subsidies to investment for human capital. Thus the flow budget constraint for the government is

$$\tau_k r(sK) + \tau_h w\,(1-l)\,H = \tau_e ph + T. \tag{16}$$

We assume that in each moment the government balances its budget by adjusting the lump-sum transfer, T, under fixed levels of τ_k, τ_h and τ_e.

2.4 Market Equilibrium Conditions

The equilibrium conditions for the market and home goods are respectively given by

$$Y_m = C_m + \dot{K} + \delta K, \tag{17}$$

$$Y_e = h. \tag{18}$$

In view of (3), (7) and (18), we obtain the equilibrium condition for the education service sector:

$$\dot{H} = B(1-u-l)H - \eta H. \tag{19}$$

3 Balanced-Growth and Equilibrium Dynamics

3.1 Dynamic System

In this subsection, we will summarize the model constructed in the previous section as a three-dimensional dynamic system. First, from (4) and (12), we see that (13) and (14) are respectively written as

$$\dot{p_k} = p_k \left[\rho + \delta - (1-\tau_k) A \beta_1 k_m^{\beta_1 - 1}\right], \tag{20}$$

$$\dot{p_h} = p_h \left(\rho + \eta - B\frac{1-\tau_h}{1-\tau_e}\right). \tag{21}$$

Therefore, keeping (12) in mind, from (20) and (21) we obtain the dynamic equation of the price of new human capital as follows:

$$\frac{\dot{p}}{p} = (1-\tau_k)A\beta_1 k_m^{\beta_1 - 1} - B\frac{1-\tau_h}{1-\tau_e} + \eta - \delta. \tag{22}$$

Since (5) and (12) mean that $w = Bp = A(1-\beta_1)k_m^{\beta_1}$, it holds that $\dot{k}_m/k_m = (1/\beta_1)\,\dot{p}/p$. As a result, combining (22) with the above, we obtain the dynamic equation of factor intensity in the market goods sector:

$$\dot{k}_m = k_m\Big\{(1-\tau_k)Ak_m^{\beta_1-1} - \frac{1}{\beta_1}\Big[B\frac{1-\tau_h}{1-\tau_e} - \eta + \delta\Big]\Big\}. \tag{23}$$

Next, observe that (8) and (9) give

$$\frac{C_m}{H} = A\Big(\frac{\beta_1}{\beta_2}\Big)\Big(\frac{\gamma}{1-\gamma}\Big)(1-\tau_k)lk_n k_m^{\beta_1-1}, \tag{24}$$

By definition, s and u satisfy

$$s = 1 - \frac{l}{k}k_n, \quad u = \frac{k}{k_m} - \frac{k_n}{k_m}l, \tag{25}$$

where $k \equiv K/H$ is the physical-human capital ratio of the economy at large. Hence, substitution of (24) and (25) into (17) and (19) presents:

$$\dot{k} = k\Big\{Ak_m^{\beta_1-1}\Big[1 - \frac{l}{k}k_n - \frac{l}{k}k_n\Big(\frac{\gamma(1-\tau_k)}{1-\gamma}\Big)\frac{\beta_1}{\beta_2}\Big] + (\eta-\delta) - B\Big[\frac{k_m - k}{k_m} + \frac{k_n - k_m}{k_m}l\Big]\Big\}, \tag{26}$$

It is to be noted that (9), (10) and (4) yield

$$k_n = \phi\frac{1-\tau_h}{1-\tau_k}k_m, \tag{27}$$

where

$$\phi = \Big(\frac{\beta_2}{1-\beta_2}\Big)\Big(\frac{1-\beta_1}{\beta_1}\Big)$$

Namely, the relative factor intensity depends not only on the technological parameters but also on the tax rates on physical and human capital. Substituting (27) into (26), we find that the dynamic behavior of k depends on k, k_m and l.

Finally, in order to derive the dynamic equation of l, substitute $C_n = [(1-s)\,K]^{\beta_2}\,(lH)^{1-\beta_2}$ into (8), (9) and (10). Then we obtain

$$J\begin{pmatrix} \log(lH) \\ \log C_m \\ \log(1-s)K \end{pmatrix} = \begin{pmatrix} \log p_k + \mathit{const.1} \\ \log p_k + (\beta_1 - 1)\log k_m + \mathit{const.2} \\ \log p_h + \mathit{const.3} \end{pmatrix},$$

where

$$J = \begin{pmatrix} (1-\beta_2)(1-\gamma)(1-\sigma) & \gamma(1-\sigma)-1 & \beta_2(1-\gamma)(1-\sigma) \\ (1-\beta_2)(1-\gamma)(1-\sigma) & \gamma(1-\sigma) & \beta_2(1-\gamma)(1-\sigma)-1 \\ (1-\beta_2)(1-\gamma)(1-\sigma)-1 & \gamma(1-\sigma) & \beta_2(1-\gamma)(1-\sigma) \end{pmatrix}.$$

Solving this with respect to $\log lH$ presents

$$\begin{aligned} \log lH \;\; &= \;\; \Big(\frac{1-\sigma}{\sigma}\Big)\Big\{(1-\gamma)\beta_2(1-\beta_1)\log k_m - [\gamma+\beta_2(1-\gamma)]\log p_k \\ &\quad -\Big(\frac{1}{1-\sigma} - [\gamma+\beta_2(1-\gamma)]\Big)\log p_h\Big\} + \text{a constant}, \end{aligned}$$

which yields:

$$\frac{\dot{l}}{l} = \Big(\frac{1-\sigma}{\sigma}\Big)\Big[\gamma + \frac{\beta_2}{\beta_1}(1-\gamma)\Big]\frac{\dot{p}}{p} - \frac{1}{\sigma}\frac{\dot{p}_h}{p_h} - \frac{\dot{H}}{H}.$$

Using (19), (21), (22) and (25), we obtain

$$\begin{aligned} \dot{l} \;\; &= \;\; l\Big\{\Big(\frac{1-\sigma}{\sigma}\Big)\Big[\gamma + \frac{\beta_2}{\beta_1}(1-\gamma)\Big]\Big[(1-\tau_k)A\beta_1 k_m^{\beta_1-1} - B\frac{1-\tau_h}{1-\tau_e} + \eta - \delta\Big] \\ &\quad -\Big(\frac{1}{\sigma}\Big)[\rho+\eta - B\frac{1-\tau_h}{1-\tau_e}] + \eta - B\Big[\frac{k_m - k}{k_m} + \frac{k_n - k_m}{k_m}l\Big]\Big\}, \end{aligned} \tag{28}$$

where from (27) k_n is proportional to k_m.

Consequently, a complete dynamic system can be expressed by (23) , (26) and (28) , which describe the motions of $k_m\,(= sK/uH)$, $k\,(= K/H)$ and l.

3.2 The Balanced-Growth Equilibrium

On the balanced-growth path, the state variables in the dynamic system derived above stay constant over time. First, $\dot{k} = 0$ means that both physical and human capitals grow at a common, constant rate. Second, $\dot{l} = 0$ shows that the human capital allocation rate to the home goods sector does not change on the balanced-growth path, implying that other ratio variables, u and s are also constant over time. Hence, $k_m\,(= sK/uH)$ and $k_n\,(= (1-s)\,K/lH)$ stay constant as well. In addition, since production technology of each sector satisfies constant returns to scale, Y_m, C_m and $C_n\,(= Y_n)$ also grow at the same rate as K and H.

Denote the balanced-growth rate of income, capital and consumption by g. From (8), (12) and (21), we obtain

$$g = \left(-\frac{1}{\sigma}\right)\frac{\dot{p}_k}{p_k} = \left(-\frac{1}{\sigma}\right)\frac{\dot{p}_h}{p_h} = \frac{1}{\sigma}\left[B\frac{1-\tau_h}{1-\tau_e} - (\rho+\eta)\right]. \tag{29}$$

In order to hold a positive growth rate, we assume that

$$B\frac{1-\tau_h}{1-\tau_e} > \rho + \eta. \tag{30}$$

Moreover, in the balanced-growth equilibrium the transversality conditions in (15) require that $(1-\sigma)\,g < \rho$, so that

$$\frac{1-\sigma}{\sigma}\left[B\frac{1-\tau_h}{1-\tau_e} - (\rho+\eta)\right] < \rho. \tag{31}$$

We assume that conditions (30) and (31) are fulfilled in the following analysis.

The steady-state value of k_m (denoted by k_m^*) is uniquely given by $\dot{k}_m = 0$ condition in (23):

$$(1-\tau_k)\,\beta_1 A k_m^{*\beta_1 - 1} - \delta = B\frac{1-\tau_h}{1-\tau_e} - \eta. \tag{32}$$

This is the steady-state expression of non-arbitrage condition between holding physical and human capital. It is easy to see that from (30) equation (32) uniquely determines a positive value of k_m^*. Denote

$$R = \frac{1}{\beta_1\,(1-\tau_k)}\left[B\frac{1-\tau_h}{1-\tau_e} - (\eta-\delta)\right]. \tag{33}$$

then the balanced-growth value of the pre-tax rental rate is $\beta_1 R$,

Next, $\dot{l} = 0$ condition in (28) presents

$$B\Big[1 - \frac{k}{k_m^*} + \Big(\frac{k_n^* - k_m^*}{k_m^*}\Big)l\Big] - \eta = g,$$

which yields

$$\frac{k}{k_m^*} = 1 - \frac{\eta+g}{B} + \Big(\frac{k_n^* - k_m^*}{k_m^*}\Big)l. \tag{34}$$

Using conditions $\dot{l} = 0$ and $\dot{k} = 0$ in (28) and (26), we obtain

$$\frac{k}{k_m^*}\Big[1 - \frac{g+\delta}{Ak_m^{*\beta_1-1}}\Big] = l\frac{k_n^*}{k_m^*}\Big[1 + (1-\tau_k)\Big(\frac{\gamma}{1-\gamma}\Big)\frac{\beta_1}{\beta_2}\Big].$$

Substituting (34) into the above equation, we obtain the following:

$$\begin{aligned} l^* &= \frac{[R-(g+\delta)]\left(1-\frac{\eta+g}{B}\right)}{\phi\Big(\frac{1-\tau_h}{1-\tau_k}\Big)\Big[1+(1-\tau_k)\Big(\frac{\gamma}{1-\gamma}\Big)\frac{\beta_1}{\beta_2}\Big]R - \Big[\phi\Big(\frac{1-\tau_h}{1-\tau_k}\Big)-1\Big]\Big[R-(g+\delta)\Big]} \\ &= \frac{1}{B}\frac{\Delta_1\Delta_2}{\Delta_3}, \end{aligned} \tag{35}$$

and hence

$$\begin{aligned}\frac{k^*}{k_m^*} &= \Big[1-\frac{\eta+g}{B}\Big]+\Big[\phi\Big(\frac{1-\tau_h}{1-\tau_k}\Big)-1\Big]l^* \\ &= \frac{1}{B}\frac{\Delta_2\Delta_4}{\Delta_3}.\end{aligned} \tag{36}$$

In the above, each Δ_i $(i=1,2,3,4)$ is defined as

$$\Delta_1 \equiv R-(g+\delta),$$

$$\Delta_2 \equiv (B-\eta)-g,$$

$$\Delta_3 \equiv \phi\Big(\frac{1-\tau_h}{1-\tau_k}\Big)\Big[1+(1-\tau_k)\Big(\frac{\gamma}{1-\gamma}\Big)\frac{\beta_1}{\beta_2}\Big]R-\Big[\phi\Big(\frac{1-\tau_h}{1-\tau_k}\Big)-1\Big]\Big[R-(g+\delta)\Big],$$

$$\Delta_4 \equiv R\phi(1-\tau_h)\Big[\frac{1}{1-\tau_k}+\frac{\beta_1}{\beta_2}\Big(\frac{\gamma}{1-\gamma}\Big)\Big]>0.$$

It is to be noted that the following holds:

$$\Delta_3 = R\phi\psi(1-\tau_h)+(g+\delta)\phi\Big(\frac{1-\tau_h}{1-\tau_k}\Big)+\Delta_1 = \Delta_4-\left[\phi\left(\frac{1-\tau_h}{1-\tau_k}\right)-1\right]\Delta_1.$$

The parameter values displayed above satisfy the following conditions:

Lemma 1 *It holds that* $\Delta_i>0$ *(*$i=1,2,3$*),* $\Delta_1>\Delta_2$*, and* $\Delta_1\Delta_2-B\Delta_3<0$*.*

Proof. The balanced-growth condition means that

$$\frac{Y_m}{K}=sAk_m^{*\beta_1-1}=sR<R.$$

Therefore, from (17) on the balanced-growth path we obtain:

$$\frac{C_m}{K}=\frac{Y_m}{K}-\delta-\frac{\dot{K}}{K}=sR-\delta-g<R-\delta-g=\Delta_1,$$

which shows that $\Delta_1>0$. Since the maximum growth rate of H is $B-\eta$, so that $\Delta_2=B-\eta-g>0$. In addition, it is easy to see that $\Delta_3>0$, because $\Delta_1>0$. Furthermore, we find the following relations:

$$\Delta_2-\Delta_1=B-\eta-(R-\delta)=(B-\eta+\delta)-R<\left[B\frac{1-\tau_h}{1-\tau_e}-\eta+\delta\right]-R<0,$$

$$\Delta_1\Delta_2-B\Delta_3=\Delta_1(\Delta_2-B)-B\Big[R\phi\frac{\beta_1}{\beta_2}\Big(\frac{\gamma}{1-\gamma}\Big)(1-\tau_h)+(g+\delta)\phi\Big(\frac{1-\tau_h}{1-\tau_k}\Big)\Big]<0.$$

■

This lemma shows that $k^*/k_m^*>0$ (so that $k^*>0$) and $0<l^*<1$. That is, the dynamic system has a feasible and unique stationary point. In sum, we have shown:

Proposition 1 *Suppose that* (30) *and* (31) *are satisfied. Then, there is a unique, feasible balanced-growth equilibrium with a positive growth rate.*

3.3 Local Stability

As for dynamic behavior of the system, first note that (23) is a complete system of k_m. Since $0 < \beta_1 < 1$, this system is globally stable. Due to the recursive nature of the system, local behavior of k and l around the balanced-growth equilibrium can be examined by the following two-dimensional, approximated system:

$$\begin{bmatrix} \dot{k} \\ \dot{l} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial l} \\ \frac{\partial \dot{l}}{\partial k} & \frac{\partial \dot{l}}{\partial l} \end{bmatrix} \begin{bmatrix} k - k^* \\ l - l^* \end{bmatrix}, \tag{37}$$

where elements in the coefficient matrix evaluated at the steady state are:

$$\frac{\partial \dot{k}}{\partial k} = \frac{k_n}{k_m}\frac{k_m^*}{k^*} l^* R[1 + (1-\tau_k)\psi] + B\frac{k^*}{k_m^*} = \Delta_4 \frac{k_m^*}{k^*} l^* + B\frac{k^*}{k_m^*} > 0, \tag{38}$$

$$\frac{\partial \dot{k}}{\partial l} = k^*\Big[-\Delta_4 \frac{k_m^*}{k^*} - B\frac{k_n - k_m}{k_m}\Big] < 0, \tag{39}$$

$$\frac{\partial \dot{l}}{\partial k} = B\frac{l^*}{k_m^*} > 0, \tag{40}$$

$$\frac{\partial \dot{l}}{\partial l} = l^*\Big[-B\Big(\frac{k_n - k_m}{k_m}\Big)\Big]. \tag{41}$$

For the detail of derivation of (39) , see Appendix 1 of the paper.

Equations (35) and (36) give $\Delta_4 \frac{k_m^*}{k^*} l^* = \Delta_1$. Thus the determinant of the coefficient matrix in (37) is:

$$\begin{aligned} & \frac{\partial \dot{k}}{\partial k}\frac{\partial \dot{l}}{\partial l} - \frac{\partial \dot{k}}{\partial l}\frac{\partial \dot{l}}{\partial k} \\ = \ & \Big[\Delta_1 + B\frac{k^*}{k_m^*}\Big] l^* \Big[-B\Big(\frac{k_n - k_m}{k_m}\Big)\Big] - B\frac{k^*}{k_m^*} l^* \Big[-\Delta_4 \frac{k_m^*}{k^*} - B\Big(\frac{k_n - k_m}{k_m}\Big)\Big] \\ = \ & B\Delta_1 \frac{k^*}{k_m^*}\Big[1 - \Big(\phi\frac{1-\tau_h}{1-\tau_k} - 1\Big)\frac{k_m^*}{k^*} l^*\Big] = B\Delta_1 \frac{k^*}{k_m^*}\Delta_3/\Delta_4 > 0. \end{aligned} \tag{42}$$

In view of (34) , the trace of the matrix is written as

$$\begin{aligned} \frac{\partial \dot{k}}{\partial k} + \frac{\partial \dot{l}}{\partial l} &= \Delta_1 + B\frac{k^*}{k_m^*} - B\Big[\phi\Big(\frac{1-\tau_h}{1-\tau_k}\Big) - 1\Big] l^* \\ &= \Delta_1 + B\Big[1 - \frac{\eta + g}{B}\Big] = \Delta_1 + \Delta_2 > 0. \end{aligned} \tag{43}$$

Inequalities (42) and (43) show that the coefficient matrix of the sub-dynamic system (37) has positive determinant and trace. This means that the subsystem (37) has two unstable roots. As mentioned above, (23) is stable, and therefore the original dynamic system of (k_m, k, l) contains one stable and two unstable roots. Since only $k\,(= K/H)$ is a non-jumpable variable in our system (so that the initial values of k_m and l should be endogenously specified), the presence of one stable root demonstrates that there locally exists a unique trajectory that converges to the balanced-growth equilibrium.[7] The following proposition summarizes our finding:

Proposition 2 *Under a given initial level of k, there locally exists a unique equilibrium path that converges to the balanced-growth equilibrium.*

4 Policy Implications

We are now ready to examine the long-run and transitional effects of fiscal policy. We start with the analysis of long-term impacts of policy changes, which will be the basis for the analysis of their transitional impacts.

4.1 Long-run Effects of Fiscal Policy

(i) *Balanced-growth rate*

As shown by (29), the long-term growth rate of capital and income is

$$g = \frac{1}{\sigma}\left[B\frac{1-\tau_h}{1-\tau_e} - (\rho+\eta)\right].$$

Since the home sector produces a pure consumption good, its technology has no effect on the determination of the balanced-growth rate. A rise in the rate of education subsidy, τ_e,

[7] By use of an endogenous growth model with physical and human capital, Bond, Wang and Yip (1996) demonstrate that asymmetric tax treatment of physical and human capital may yield indeterminacy of equilibrium. The key assumption in their analysis is that the education sector uses physical as well as human capital. The indeterminacy result, thus, comes from the fact that the relative factor ranking between the final goods and education sectors from the social perspective may differ from that in view of the private perspective. Since the Lucas model employed in our paper assumes that the education sector uses human capital alone, the education sector is always more human capital intensive than other sectors both from the social and private perspectives. Therefore, the source of multiple converging paths in the model of Bond, Wang and Yip (1996) cannot hold in our setting.

enhances long-term growth, while a higher rate of income tax on human capital, τ_h, depresses growth. Since the education sector does not employ physical capital and since the key to determine the balance growth rate in the Lucas' modelling is accumulation of human capital, the rate of income tax on physical capital fails to affect the long-term growth performance of the economy. It is to be noted that, unlike the original Lucas model, income tax on human capital, τ_h, has a long-run growth effect. This is because we have assumed that education services are supplied in the market. This means that, all else being equal, an increase in τ_e or a decrease in τ_h encourages households to spend more income for purchasing education services. As a consequence, human capital formation is accelerated, and hence the economy realizes a higher growth rate in the long-run. Note that the balanced-growth rate given above is independent of β_2. Therefore, even if the home goods production uses human capital alone ($\beta_2 = 0$), the growth effect of fiscal policy in the long-run equilibrium is the same as that obtained under the general home production technology.

In the standard Lucas model where education services are provided within the household, taxation on human capital is applied only to the wage income earned by the human capital employed in the final goods sector. Thus, in the absence of home production and educational subsidy, the flow-budget constraint for the household is $\dot{K} = (1 - \tau_k) \, rsK + (1 - \tau_h)wuH - C_m + T - \delta K$ and human capital formation follows $\dot{h} = B \, (1 - u) \, h$. Given these conditions, the optimal choice of the allocation rate, u, gives $p_k \, (1 - \tau_h) \, w = p_h B$. Since the behavior of p_h is given by (14), we obtain

$$\frac{\dot{p}_h}{p_h} = \rho + \eta - \frac{p_k}{p_h} \, (1 - \tau_h) \, w = \rho + \eta - B = -\sigma g.$$

Therefore, as is well known, taxation on human capital fails to affect the balanced-growth rate in the standard Lucas' setting.

To sum up, we have shown:

Proposition 3 *The balanced-growth rate increases with the rate of education subsidy, while it decreases with the rate of income tax on human capital.*

(ii) *Rates of return and price of education*

From (32) the steady-state value of the before-tax rate of return to physical capital,

$r = \beta_1 R$, is given by

$$r = \frac{1}{1-\tau_k}\left(B\frac{1-\tau_h}{1-\tau_e} + \delta - \eta\right).$$

The rate of return to physical capital in the steady state is thus independent of the production technology of home goods sector. A rise in τ_k or τ_e increases r, while a rise in τ_h lowers r. For example, a higher rate of income taxation on human capital promotes physical capital accumulation, which raises the physical and human capital ratio, k_m, in the market goods sector. Since the rate of return to physical capital satisfies $r = A\beta_1 k_m^{\beta_1 - 1}$, a higher k_m depresses the rate of return to capital. On the other hand, increases in τ_k and τ_e have the opposite effects.

Due to the Cobb-Douglas specification, the relation between pre-tax rates of returns to physical and human capital satisfies $w = (1-\beta_1)\, r k_m/\beta_1$. Thus in the steady state it holds that

$$w = \frac{1-\beta_1}{\beta_1}\left[\frac{1}{\beta_1 A\,(1-\tau_k)}\left(B\frac{1-\tau_h}{1-\tau_e} + \delta - \eta\right)\right]^{\frac{\beta_1}{\beta_1 - 1}}.$$

Again, the rate of return to human capital (the real wage rate) does not depend on β_2 that characterizes the home goods production technology. The effects of changes in τ_k, τ_h and τ_e on w are opposite to those effects on r: a rise either in τ_k or in τ_e depresses w, while a higher τ_h increases w. The price of education service, p, is proportional to w (see (5)), so that the effects of fiscal policy are the same as those on w. To sum up, we have found:

Proposition 4 *On the balanced-growth path, the pre-tax rate of return to physical capital increases with the rate of income tax on physical capital and the education subsidy rate, while it decreases with the rate of tax on human capital. Both the pre-tax rate of return to human capital and price of education* (*in terns of the market good*) *decrease with the rate of income tax on physical capital and with the education subsidy rate, while it increases with the tax rate on human capital.*

(iii) *Human capital allocation to home production*

To see the policy effects on factor allocation between the market and home sectors, we focus on the human capital allocation rate to the home goods sector, l. The stationary level of l is given by (35). Although this expression is rather complex, we can show the following results:

Proposition 5 *An increase in the rate of tax on physical or human capital raises working time in home, while an increase in education subsidy rate lowers it:*

$$\frac{\partial l^*}{\partial \tau_k} > 0, \quad \frac{\partial l^*}{\partial \tau_h} > 0, \quad \frac{\partial l^*}{\partial \tau_e} < 0. \tag{44}$$

Proof. See Appendix 2. ■

Intuitive implications of the above proposition are as follows. A higher taxation either on physical or human capital discourages the market production activities, because the after-tax rates of return to capital realized both in the market goods and education sector are lowered. Hence, production factors shift from the market goods sector to the tax-free home production sector to meet a higher distortion in the market production and education sector. In contrast, a higher investment tax credit (i.e. a rise in τ_e) accelerates human capital accumulation and enhances the education sector's activity. This reallocates human capital from the home goods sector to the market sectors.

(iv) *Factor intensities*

From (32), the steady-state level of factor intensity in the market goods sector is given by

$$k_m^* = \left[\frac{1}{\beta_1 A (1-\tau_k)} \left(B\frac{1-\tau_h}{1-\tau_e} + \delta - \eta\right)\right]^{\frac{1}{\beta_1 - 1}}.$$

Thus we find:

$$\frac{\partial k_m^*}{\partial \tau_k} < 0, \quad \frac{\partial k_m^*}{\partial \tau_h} > 0, \quad \frac{\partial k_m^*}{\partial \tau_e} < 0. \tag{45}$$

Economic intuition of those results are obvious. For example, a rise in τ_e promotes human capital formation and the resulting technologies used both by the market and home goods sectors become more human-capital intensive, and thus k_m and k_n decrease in the long-run equilibrium.

Remembering that k_m and k_n satisfies (27), in the steady state we obtain the following:

$$k_n^* = \phi\frac{1-\tau_h}{1-\tau_k} \left[\frac{1}{\beta_1 A (1-\tau_k)} \left(B\frac{1-\tau_h}{1-\tau_e} + \delta - \eta\right)\right]^{\frac{1}{\beta_1 - 1}},$$

where $\phi = \beta_2 (1-\beta_1) / \beta_1 (1-\beta_2) > 0$. Thus we see that

$$\frac{\partial k_n^*}{\partial \tau_k} < 0 \quad \frac{\partial k_n^*}{\partial \tau_h} > 0 \ \ (\text{if } \delta \cong \eta), \quad \frac{\partial k_n^*}{\partial \tau_e} < 0. \tag{46}$$

To understand (46), we should note that (27) gives:

$$\frac{\partial k_n^*}{\partial \tau_i} = \phi k_m^* \frac{\partial}{\partial \tau_i}\left(\frac{1-\tau_h}{1-\tau_k}\right) + \phi \frac{1-\tau_h}{1-\tau_k}\frac{\partial k_m^*}{\partial \tau_i}, \qquad i = k, h, e.$$

The first term in the right hand side of the above expresses the asymmetric taxation effect on physical and human capital. If capital income taxation is also applied to the physical and human capital used by the home goods sector, then this asymmetric taxation effect disappears.[8] The second term represents the factor substitution effect of a change in τ_i. Given the Cobb-Douglas technology specification, the effect of fiscal policy on the factor intensity in the market goods sector is directly linked to the factor intensity employed by the home goods sector. As for changes in τ_k and τ_e, the factor substitution effect dominates the asymmetric taxation effect, and therefore, both k_m and k_n move toward the same direction. The effect of a change in τ_h on k_n^* is ambiguous. However, if we assume that physical and human capital depreciate at the same rate, then we see that $\partial k_n^* / \partial \tau_h > 0$. If this is the case, a rise in τ_h depresses human capital formation, which make the home sector choose a less human capital-intensive technology.

Finally, let us consider the policy effects on the steady-state level of aggregate factor intensity, k^*. From (36) we obtain

$$\frac{\partial k^*}{\partial \tau_i} = \frac{k^*}{k_m^*}\frac{\partial k_m^*}{\partial \tau_i} + \frac{k_m^*}{B}\left(-\frac{\partial g}{\partial \tau_i}\right) + k_m^* \frac{\partial}{\partial \tau_i}\left[\phi\left(\frac{1-\tau_h}{1-\tau_k}\right) - 1\right] l^*, \qquad i = h, k, e. \tag{47}$$

The above shows that the effect of a change in τ_i on k^* can be separated into three parts. The first term in the right-hand side of (47) represents the allocation effect on the market goods sector, the second term is the growth effect, and the third one shows the allocation effect on the home goods sector. It is the third effect that distinguishes the present model from the original Lucas model. In the following, we will take a change in τ_e as an example for seeing the details of this fact.

Using $\phi\left(\frac{1-\tau_h}{1-\tau_k}\right) - 1 = \frac{k_n - k_m}{k_m}$ and (44), we find that (47) can be rewritten as

$$\frac{\partial k^*}{\partial \tau_e} = \frac{k^*}{k_m^*}\frac{\partial k_m^*}{\partial \tau_e} + \frac{k_m^*}{B}\left(-\frac{\partial g}{\partial \tau_e}\right) + \left(k_n^* - k_m^*\right)\frac{\partial l^*}{\partial \tau_e}. \tag{48}$$

[8] From (27) we have

$$\frac{\partial \left(k_n^*/k_m^*\right)}{\partial \tau_i} = \phi \frac{\partial\left(\frac{1-\tau_h}{1-\tau_k}\right)}{\partial \tau_i}, \quad i = k, h.$$

This expression shows the direct effects of capital income taxation on the relative factor intensity between the market and home goods secotrs.

As mentioned above, the steady-state effect of policy change on the aggregate factor intensity consists of three components. First, note that from (45) $\frac{\partial k_m^*}{\partial \tau_e} < 0$. Proposition 3 states that an increase in τ_e raises the balanced-growth rate of human capital, which lowers $k\,(= K/H)$ in the steady state. At the same time, Proposition 5 means that a higher τ_e decreases the human capital allocation to the home goods sector. Hence, if the household production uses a less human-capital intensive technology than the market goods sector ($k_n > k_m$), then a decreases in l shifts physical capital from the market to home production sector. This depresses physical capital accumulation, implying that the steady-state level of k declines. Consequently, if $k_n > k_m$, we obtain

$$\frac{\partial k^*}{\partial \tau_e} < 0. \tag{49}$$

To see whether it holds that $\frac{\partial k^*}{\partial \tau_e} > 0$ when $k_n < k_m$, we consider the possibility that the allocation effect on the home goods sector dominates the sum of the growth effect and the allocation effect on the market goods sector. Such a domination is most likely to happen when γ is close to one so that k_n is extremely small. In this case, simple calculation reveals that

$$\frac{\partial k^*}{\partial \tau_e} = \frac{k^*}{k_m^*}\frac{\partial k_m^*}{\partial \tau_e} < 0.$$

This implies that, if the utility share of home goods is sufficiently small, regardless of the relative factor intensities, (49) always holds.

It should be noted that, as well as in the Lucas model, an increase in τ_e affects human capital allocation as well as the long-term growth rate. Yet unlike the Lucas model, our model has an additional allocation effect on the home goods sector that affects the steady-state value of k. When $k_n > k_m$, comparing with the standard model, a rise in τ_e has a larger negative effect on k^*. This is the *magnification effect* of the home production model. When $k_n < k_m$, on the other hand, a higher τ_e generated a smaller negative effect on k^* than in the case of the standard model. This is the *reduction effect* of the present model. Similarly, we find:

$$\frac{1}{k_m^*}\frac{\partial k^*}{\partial \tau_k} = \phi\frac{1-\tau_h}{(1-\tau_k)^2}l^* - \frac{\Delta_2}{B(1-\tau_k)(1-\beta_1)} - \frac{k_n^*-k_m^*}{k_m^*}\left[\frac{l^*}{(1-\tau_k)(1-\beta_1)} - \frac{\partial l^*}{\partial \tau_k}\right], \tag{50}$$

$$\frac{1}{k_m^*}\frac{\partial k^*}{\partial \tau_h} = \frac{k^*}{k_m^*}\frac{B}{\beta_1(1-\beta_1)(1-\tau_k)(1-\tau_e)R} - \frac{1}{B}\frac{\partial g}{\partial \tau_h} - \frac{\phi}{1-\tau_k}l^* + \frac{k_n^*-k_m^*}{k_m^*}\frac{\partial l^*}{\partial \tau_h}. \tag{51}$$

These expressions show that even if we specify the relative factor ranking condition, the effects of changes in capital income taxes on k^* are ambiguous without imposing further restrictions on the parameter values involved in the model. In the original Lucas model where there is neither home production nor labor-leisure choice, we may set $l^* = 0$, $k_m = 0$ and $\beta_2 = 0$ (so that $\phi = 0$) in (50) and (51). Therefore, we always see that $\partial k^*/\partial \tau_k < 0$ and $\partial k^*/\partial \tau_h > 0$. In the model with quality leisure in which the home production sector does not employ physical capital $\beta_2 = 0$ and $k_n = 0$), (50) and (51) respectively become:

$$\frac{1}{k_m^*}\frac{\partial k^*}{\partial \tau_k} = -\frac{\Delta_2}{B(1-\tau_k)(1-\beta_1)} + \frac{l^*}{(1-\tau_k)(1-\beta_1)} - \frac{\partial l^*}{\partial \tau_k}.$$

$$\frac{1}{k_m^*}\frac{\partial k^*}{\partial \tau_h} = \frac{k^*}{k_m^*}\frac{B}{\beta_1(1-\beta_1)(1-\tau_k)(1-\tau_e)R} - \frac{1}{B}\frac{\partial g}{\partial \tau_h} - \frac{\partial l^*}{\partial \tau_h}.$$

Remembering that $\partial l^*/\partial \tau_k > 0$, $\partial l^*/\partial \tau_h > 0$ and $\partial g/\partial \tau_h < 0$, the above expressions demonstrate that the presence of the human capital allocation effect, $\partial l^*/\partial \tau_i$ $(i = k$ or $h)$, is still the source of ambiguity of the sings of $\partial k^*/\partial \tau_i$ $(i = k, h)$. However, compared with the model with the general home production technology, in the case of $\beta_2 = 0$, the possibilities that $\partial k^*/\partial \tau_k < 0$ and $\partial k^*/\partial \tau_h > 0$ seem to be relatively high.

To sum up, we have shown:

Proposition 6 *On the balanced-growth path, a rise in physical capital taxation lowers both k_m and k_n, while a rise in human capital taxation depress both k_m and k_n. An increase in education subsidy lowers k_m and k_n. In addition, a higher education subsidy lowers the aggregate factor intensity, k, regardless of the relative factor intensity ranking between the market and home goods sectors.*

(v) *The share of home sector*

In our setting, the magnitude of $1 - \gamma$ in the utility function represents the consumption share of the home-made goods and services. To investigate the economic implications of the presence of home production, suppose that γ decreases. It is easy to see that a fall in γ has no effects on the steady-state levels of rate of return to capital, the real wage rate, the factor intensities, k_m and k_n, and the balanced-growth rate unaffected. In addition, from (35), a lower γ increases the home work time, l, in the steady state. As for the effect of a change in γ on the steady-state value of the aggregate factor intensity, k, we should recall (36) :

$$\frac{k^*}{k_m^*} = \Big[1 - \frac{\eta + g}{B}\Big] + \Big[\phi\Big(\frac{1-\tau_h}{1-\tau_k}\Big) - 1\Big] l^*.$$

This equation means that if $k_m < k_n$ $\left(\text{so that } \phi\left(\frac{1-\tau_h}{1-\tau_k}\right) - 1 < 0\right)$, a decrease in γ, which yields a higher l^*, lowers k^*. In contrast, if $k_m > k_n$ $\left(\text{so that } \phi\left(\frac{1-\tau_h}{1-\tau_k}\right) - 1 > 0\right)$, a lower γ raises k^*. Therefore, a larger utility share of home goods (i.e. a lower value of γ) produces less distorting effects of capital income taxation on resource allocation. In other words, the existence of a nonmarket sector absorbs a part of effects caused by distorting taxation.

4.2 Transitional Effects of Fiscal Policy

Based on the long-run impacts of fiscal policy derived above, we can examine the effects of fiscal policy on the dynamic behaviors of key variables in the transitional process towards the new balanced-growth path.

(i) *Dynamics of* k_m

Inspecting (23), we obtain the phase diagram of k_m in Figure 1. As the figure shows, under our specification, k_m is globally stable. Since an increase either in τ_k or τ_h yields a lower steady-state value of k_m, it must cause a leftward shift of the converging path of k_m (to the the broken curve in the figure). Suppose that the economy initially stays on the balanced-growth path. We find that

$$\frac{\partial \dot{k}_m}{\partial \tau_k}\Big|_{(k_m^*,k^*,l^*)} = -A{k_m^*}^{\beta_1} < 0, \qquad \frac{\partial \dot{k}_m}{\partial \tau_e}\Big|_{(k_m^*,k^*,l^*)} < 0$$

Therefore, an unanticipated rise either in τ_k or τ_e makes $\dot{k}_m$ jump down. As a result, k_m starts decreasing and finally reaches the new steady state value, $k_m^{**}.(< k_m^*)$. In contrast, an unanticipated, permanent rise in τ_h increases k_m^* and

$$\frac{\partial \dot{k}_m}{\partial \tau_h}\Big|_{(k_m^*,k^*,l^*)} > 0.$$

This means that, as depicted by the figure, $\dot{k}_m$ first jumps up and k_m starts moving towards a higher new steady-state value of k_m.

(ii) *Transitional dynamics on the* k-l *plane*

In order to examine the dynamic behaviors of k and l graphically, we project the stable saddlepath onto the k-l plane. In so doing, the following result is useful.[9]

[9] Caballe and Santos (1993) and Ladron-de-Guevara et al. (1997) employ the following value function approach to discuss the global stability of the Lucas model.

Lemma 2 *On the converging equilibrium path, k_m can be expressed as a monotonically increasing function of k.*

Proof. Define the value function for the household's optimization problem in such a way that

$$V(K_t, H_t) \equiv \max \int_t^{\infty} \frac{C_v^{1-\sigma}}{1-\sigma} e^{-\rho(v-t)} dv.$$

Then it is easy to confirm that $V(K, H)$ is homogenous of degree $1 - \sigma$ in K and H. Differentiability of the value function ensures that

$$p_k = \frac{\partial V(K, H)}{\partial K}, \qquad p_h = \frac{\partial V(K, H)}{\partial H} \quad \text{for all } t \geq 0.$$

Thus the relative implicit price satisfies

$$\frac{p_h}{p_k} = \frac{V_K(k, 1) k^{1-\sigma}}{V_H(k, 1)} \equiv \psi(k), \tag{52}$$

where $V_j(k, 1) = \partial V(K, H)/\partial j \ \ (j = K, H)$. Noting that (5) and (12) give $Bp\,(= Bp_h/p_k) = A(1-\beta_1)k_m^{\beta_1}$, we see that k_m monotonically increases with p_h/p_k. Thus on the stable path k_m can be expressed as $k_m = k_m(k)$ with $k_m'(k) > 0$. Since all production technologies satisfy constant-return-to-scale and the momentary utility function exhibits strict concavity, $V(K, H)$ is also concave in K and H. Homogeneity and concavity of $V(K, H)$ ensure that $\psi(k)$ in (52) is monotonically increasing in k, and thus k_m also monotonically increases with k. ■

Relying on the above argument, we can show that the projected dynamic system on the k-l plane that is linealized at the steady state is expressed as

$$\begin{pmatrix} \dot{k} \\ \dot{l} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} k - k^* \\ l - l^* \end{pmatrix},$$

where

$$\begin{aligned} b_{11} &= \frac{\partial \dot{k}}{\partial k} + \frac{\partial \dot{k}}{\partial k_m} k_m'(k), \quad b_{12} = \frac{\partial \dot{k}}{\partial l} < 0, \\ b_{21} &= \frac{\partial \dot{l}}{\partial k} + \frac{\partial \dot{l}}{\partial k_m} k_m'(k), \quad b_{22} = \frac{\partial \dot{l}}{\partial l}, \end{aligned}$$

and $\text{sign}[b_{22}] = \text{sign}[k_m - k_n]$. Here, all the derivatives are evaluated at (k_m^*, k^*, l^*).

(iii) *The effects of education subsidy*

We first consider the effects of a change in the rate of education subsidy, τ_e, which yields unambiguous impacts on the transitional as well as long-run behavior of the economy. As shown in the previous subsection, the long-run effects of changes in τ_k and τ_h on k^* are ambiguous, which prevent us from obtaining clear results about the transitional impacts of changes in τ_k and τ_h. Hence, in what follows we restrict our attention to the transitional impacts of a change in τ_e. We should consider the following two cases:

Case 1: $k_n > k_m$

From (44) it holds that $\partial l^*/\partial \tau_e < 0$ and $\partial k^*/\partial \tau_e < 0$. Additionally, around the steady state where $(k_m, k, l) = (k_m^*, k^*, l^*)$, we obtain

$$\frac{\partial \dot{k}}{\partial \tau_e} = 0, \quad \frac{\partial \dot{l}}{\partial \tau_e} > 0. \tag{53}$$

The saddle-point property requires that $b_{11}b_{22} - b_{12}b_{21} < 0$. Since $b_{12} < 0$ and $b_{22} < 0$ for $k_n > k_m$, the condition for the saddlepoint property is written as $-b_{21}/b_{22} < b_{11}/b_{12}$. Observe that

$$\left.\frac{dl}{dk}\right|_{\dot{k}=0} = -\frac{b_{11}}{b_{12}}, \quad \left.\frac{dl}{dk}\right|_{\dot{l}=0} = -\frac{b_{21}}{b_{22}}.$$

Thus the slope of $\dot{k} = 0$ locus is larger than that of $\dot{l} = 0$ locus. Thus the possible patterns of phase diagram for the case of $k_n > k_m$ can be depicted by Figures 2, 3 and 4, which respectively correspond to the following conditions:

$$b_{21} > 0, \quad b_{11} > 0, \tag{I-a}$$

$$b_{21} < 0, \quad b_{11} > 0, \tag{I-b}$$

$$b_{21} < 0, \quad b_{11} < 0. \tag{I-c}$$

Since in Figures 2 and 3, $\dot{k} = 0$ locus has a positive slope, which means that $b_{11} > 0$. In Figure 4 $\dot{k} = 0$ locus has a negative slope so that $b_{11} < 0$.

In view of Figures 2, 3 and 4, we find that in all cases the stable saddle path in the k-l plane has a positive slope. Keeping (44) and (53) in mind, we have found that the transitional effects of a rise in τ_e can be depicted as Figure 5. If the economy is initially on the balanced-growth path, then an unanticipated rise in τ_e first increases l instantaneously. Along the transitional path towards the new steady state, l and k continue decreasing, and the resulting new steady state levels of both l and k are lower than those original levels. In the phase diagram we also depict a sample path that is obtained when a rise in τ_e is anticipated one. If the policy shift is announced in advance, then the transition process should be on the path with arrows before arriving to the new saddlepath. On this trajectory, k decreases continuously; while l still moves up. Once arriving at the new saddlepath, both l and k will move along this path down to the new balanced-growth equilibrium. How long is the increasing process of l depends on how large is the first jump in l caused by an increase in τ_e. This depends on the lag between the announcement and the execution of the new policy.

Case II: $k_n < k_m$

In this case, $b_{22} > 0$. Thus the saddlepoint stability means $-b_{21}/b_{22} > -b_{11}.b_{12}$, and hence

$$\frac{dl}{dk}\Big|_{\dot{k}=0} = -\frac{b_{11}}{b_{12}} < \frac{dl}{dk}\Big|_{\dot{l}=0} = -\frac{b_{21}}{b_{22}}.$$

As shown above, Figure 6-8 show all the possible patterns of phase diagram or the case of $k_n < k_m$, which correspond the following cases:

$$b_{21} > 0, \quad b_{11} < 0, \tag{II-a}$$

$$b_{21} < 0, \quad b_{11} < 0, \tag{II-b}$$

$$b_{21} < 0, \quad b_{11} > 0. \tag{II-c}$$

From these figures, we see that case (II-a) yields a negatively sloped, stable saddle path, while in cases (II-b) and (II-c) the stable saddle path has a positive slope. However, we can show that case (II-a) cannot satisfy $\partial l^*/\partial\tau_e < 0$ and $\partial k^*/\partial\tau_e < 0$ established in Proposition 3. Therefore, only cases (II-b) and (II-c) are feasible, so that the stable saddle path always has a positive slope. Consequently, the transitional impacts of a change in τ_e under $k_n < k_m$ are basically the same as those obtained for the case of $k_n > k_m$.

(iv) *Effects of Capital Income Taxation*

As a typical example of effects of capital income taxation, we consider the effect of a change in the rate of tax on the physical capital income, τ_k. As shown in the previous subsection, changes in τ_k yields an ambiguous impact on the steady-state value of k. We thus examine alternative cases: $\partial k^*/\partial\tau_k < 0$ and $\partial k^*/\partial\tau_k > 0$. We first notice that, using $\Delta_2 = B - \eta - g$ and $(k_n^* - k_m^*)/k_m^* = \phi\frac{1-\tau_k}{1-\tau_k} - 1$, equation (50) may be rewritten as

$$\frac{1}{k_m^*}\frac{\partial k^*}{\partial\tau_k} = \frac{1-\tau_h}{(1-\tau_k)^2}\phi l^* + \frac{l^*}{(1-\tau_k)(1-\beta_1)}\left(l^* + \frac{\eta+g}{B} - 1\right) + \left(\phi\frac{1-\tau_k}{1-\tau_k} - 1\right)\frac{\partial l^*}{\partial\tau_k}$$

Although the second term in the right-hand side of the above has an ambiguous sign, if k_n^* is sufficiently smaller than k_m^* (so that ϕ is sufficiently small), then we tend to have the normal result, $\partial k^*/\partial \tau_k < 0$: a higher taxation on physical capital income depresses the physical-human capital ratio of the entire economy. In this case, the transitional effects of a rise in τ_k are similar to those of a rise in τ_e discussed above (except that, unlike a change in τ_e, a change in τ_k has no growth effect). If the economy is initially on the balanced-growth path and there is a permanent increase in τ_k, then the steady-state value of k decreases and the economy starts moving along the new saddle path down to the lower level of k^{**}. During the transition, k and l continue declining. Since the price of education service, $p\,(= p_h/p_k)$ is positively related to k on the converging saddle path, p, k_m and k_n also monotonically decrease in the transition. As a result, the rate of return to physical capital will rise, while the real wage rate will fall. Those are the natural consequences, when a higher taxation on physical capital income discourage physical capital investment.

In contrast, if the households' production technology is more physical capital intensive than the market goods production technology, then it may holds that $\partial k^*/\partial \tau_k > 0$. In this case, since home production uses a physical-capital intensive technology, a rise in taxation on physical capital employed for the market activities produces a large shift of physical capital form the market goods sector to the home goods sector. At the same time, the higher income tax on physical capital raises l so that human capital shifts from the market to the home goods sector as well. As a consequence, the relative enhancement of the home goods sector that employes a physical-capital intensive technology increases the aggregate level of physical-human capital ratio in the long run. If this is the case, the transition process is exactly the opposite to the case of $\partial k^*/\partial \tau_k < 0$. After the initial impact of a rise in τ_k, both k and l continue rising toward their new, higher steady-state levels. In this process, p, k_m and k_n also increase, and thus the rate of return to physical capital continues falling and the real wage rate continues increasing until the economy reaches the new steady state.

5 Conclusion

This paper has analyzed the short-run as well as long-run impacts of fiscal policy in an endogenous growth model with home production. We characterize the balanced-growth equilibrium and analyze transitional dynamics. We show that even in the presence of policy distortions, our multisector growth model has well-behaved properties: under weak restrictions, the balanced-growth path is uniquely given and it satisfies saddlepoint stability. Based on this result, we have explored the effects of fiscal policy both in and out of the balanced-growth equilibrium. The key assumption in our discussion is that the household production is a nonmarket activity and thus it is free from direct taxation. Because of this asymmetric tax treatment, capital income taxation and education subsidy generate the effects on the behaviors of key economic variables that are different from those obtained in the standard model without household production activities.

Our study can be extended in several ways. Among others, the welfare evaluation of alternative fiscal policies and the optimal capital income taxation in the presence of home production would be interesting and relevant topics. Examining open-economy versions of our model is also an interesting topic that deserves further investigation.

Appendix 1 Proof of $\partial \dot{k}/\partial l > 0$

Equation (39) can be rewritten as

$$\begin{aligned}\frac{\partial \dot{k}}{\partial l} &= k^{*}\Big[-\Delta_4\frac{k_m^{*}}{k^{*}} - B\Big(\frac{k_n - k_m}{k_m}\Big)\Big] \\ &= k^{*}\Big[-\Delta_4\frac{B\Delta_3}{\Delta_2\Delta_4} - -B\Big(\frac{k_n - k_m}{k_m}\Big)\Big] \\ &= -\frac{Bk^{*}}{\Delta_2}\Big[\Delta_4 + (\Delta_2 - \Delta_1)\Big(\frac{k_n - k_m}{k_m}\Big)\Big],\end{aligned}$$

where we have used the relation of $\Delta_3 = \Delta_4 - \Delta_1\left(\frac{k_n - k_m}{k_m}\right)$ in deriving the last equation. Notice the relation in (27), we have

$$\begin{aligned}&\Delta_4 + (\Delta_2 - \Delta_1)\Big(\frac{k_n - k_m}{k_m}\Big) \\ =\ & R\phi\Big(\frac{1-\tau_l}{1-\tau_k}\Big)[1 + (1-\tau_k)\psi] + [B - \eta - R + \delta]\Big[\phi\Big(\frac{1-\tau_l}{1-\tau_k}\Big) - 1\Big] \\ =\ & R\phi\psi(1-\tau_l) + (B - \eta + \delta)\phi\Big(\frac{1-\tau_l}{1-\tau_k}\Big) - (\Delta_2 - \Delta_1) > 0,\end{aligned}$$

because $B - \eta > 0$ and $\Delta_2 - \Delta_1 < 0$. This completes the proof. □

Appendix 2 Proof of Proposition 5

First, note that following equations:

$$\begin{aligned}\frac{\partial\Delta_1}{\partial\tau_h} &= B(1-\tau_e)^{-1}\left[\frac{1}{\sigma}-\frac{1}{\beta_1(1-\tau_k)}\right], \qquad \frac{\partial\Delta_1}{\partial\tau_k}=\frac{\partial R}{\partial\tau_k}=\frac{R}{1-\tau_k}>0,\\ \frac{\partial\Delta_1}{\partial\tau_e} &= \frac{B(1-\tau_h)}{(1-\tau_e)^2}\left[\frac{1}{\beta_1(1-\tau_k)}-\frac{1}{\sigma}\right],\end{aligned}$$

$$\frac{\partial\Delta_2}{\partial\tau_h}=\frac{B}{\sigma(1-\tau_e)}>0, \qquad \frac{\partial\Delta_2}{\partial\tau_k}=0, \qquad \frac{\partial\Delta_2}{\partial\tau_e}=-\frac{B}{\sigma}\frac{(1-\tau_h)}{(1-\tau_e)^2}<0,$$

$$\begin{aligned}\frac{\partial\Delta_3}{\partial\tau_h} &= -(g+\delta)\left(\frac{\phi}{1-\tau_k}\right)-\phi\psi R+\left(\frac{B}{1-\tau_e}\right)\left\{\left[\frac{1}{\sigma}-\frac{1}{\beta_1(1-\tau_k)}\right]-\phi\frac{1-\tau_h}{1-\tau_k}\left(\frac{1}{\sigma}+\frac{\psi}{\beta_1}\right)\right\},\\ \frac{\partial\Delta_3}{\partial\tau_k} &= [\phi\psi(1-\tau_l)+1]\frac{R}{1-\tau_k}+(g+\delta)\phi\frac{1-\tau_l}{(1-\tau_k)^2}>0,\\ \frac{\partial\Delta_3}{\partial\tau_e} &= \frac{B(1-\tau_h)}{(1-\tau_e)^2}\left\{\phi\left(\frac{1-\tau_h}{1-\tau_k}\right)\left(\frac{1}{\sigma}+\frac{\psi}{\beta_1}\right)+\left[\frac{1}{\beta_1(1-\tau_k)}-\frac{1}{\sigma}\right]\right\},\end{aligned}$$

$$\begin{aligned}\frac{\partial\Delta_4}{\partial\tau_h} &= -\phi\left(\frac{1}{1-\tau_k}+\psi\right)\left[R+\frac{B(1-\tau_h)}{(1-\tau_k)(1-\tau_e)}\right]<0,\\ \frac{\partial\Delta_4}{\partial\tau_k} &= R\phi\left(\frac{1-\tau_h}{1-\tau_k}\right)\left(\frac{2}{1-\tau_k}+\psi\right)>0,\\ \frac{\partial\Delta_4}{\partial\tau_h} &= \phi\left(\frac{1}{1-\tau_k}+\psi\right)\frac{B(1-\tau_h)^2}{\beta_1(1-\tau_k)(1-\tau_e)^2}>0,\end{aligned}$$

where Δ_i $(i=1,2,3,4)$ are given in Section 3.2.

Using the above, we obtain:

$$\begin{aligned}\frac{\partial l^*}{\partial\tau_h} &= \frac{1}{B\Delta_3^2}\left[\frac{\partial(\Delta_1\Delta_2)}{\partial\tau_h}\Delta_3-\Delta_1\Delta_2\frac{\partial\Delta_3}{\partial\tau_h}\right]\\ &= \frac{1}{B\Delta_3^2}\left[\Delta_1\Delta_3\frac{\partial\Delta_2}{\partial\tau_h}+\Delta_2\left(\Delta_3\frac{\partial\Delta_1}{\partial\tau_h}-\Delta_1\frac{\partial\Delta_3}{\partial\tau_h}\right)\right],\end{aligned}$$

where

$$\begin{aligned}&\Delta_3\frac{\partial\Delta_1}{\partial\tau_h}-\Delta_1\frac{\partial\Delta_3}{\partial\tau_h}\\ =\ &\Delta_1(g+\delta)\left(\frac{\phi}{1-\tau_k}\right)+\Delta_1\phi\psi R\\ &+\left(\frac{B}{1-\tau_e}\right)\phi\left(\frac{1-\tau_h}{1-\tau_k}\right)[\psi(1-\tau_k)+1]\left[\frac{R}{\sigma}-\frac{g+\delta}{\beta_1(1-\tau_k)}\right]\end{aligned}$$

$$\frac{R}{\sigma}-\frac{g+\delta}{\beta_1(1-\tau_k)}=\frac{1}{\beta_1(1-\tau_k)}\left(\frac{\rho+\delta}{\sigma}-\delta\right)>0 \ \text{ for a small } \delta.$$

Thus if δ is small, see that $\Delta_3 \frac{\partial \Delta_1}{\partial \tau_h} - \Delta_1 \frac{\partial \Delta_3}{\partial \tau_h} > 0$, $\frac{\partial \Delta_2}{\partial \tau_h} > 0$ and $\frac{\partial l^*}{\partial \tau_h} > 0$. In addition, it holds that

$$\begin{aligned}
\frac{\partial l^*}{\partial \tau_k} &= \frac{1}{B\Delta_3^2}\Big\{\Delta_3\Big[\Delta_2 \frac{\partial \Delta_1}{\partial \tau_k} + \Delta_1 \frac{\partial \Delta_2}{\partial \tau_k}\Big] - \Delta_1 \Delta_2 \frac{\partial \Delta_3}{\partial \tau_k}\Big\} \\
&= \frac{\Delta_2}{B\Delta_3^2}\Big\{\Delta_3 \frac{\partial \Delta_1}{\partial \tau_k} - \Delta_1 \frac{\partial \Delta_3}{\partial \tau_k}\Big\} \\
&= \frac{\Delta_2}{B\Delta_3^2}\Big\{\Delta_3 \frac{\partial R}{\partial \tau_k} - \Delta_1[\phi\psi(1-\tau_l)+1]\frac{\partial R}{\partial \tau_k} - \Delta_1(g+\delta)\phi\frac{(1-\tau_l)}{(1-\tau_k)^2}\Big\} \\
&= \frac{\Delta_2}{B\Delta_3^2}\Big\{\Big(\frac{R}{1-\tau_k}\Big)\Big[(g+\delta)\phi\Big(\frac{1-\tau_l}{1-\tau_k}\Big) + (g+\delta)\phi\psi(1-\tau_l)\Big] - \Delta_1(g+\delta)\phi\frac{(1-\tau_l)}{(1-\tau_k)^2}\Big\} \\
&= \frac{\Delta_2}{B\Delta_3^2}\Big\{\Big(\frac{R}{1-\tau_k}\Big)(g+\delta)\phi\psi(1-\tau_l) + (g+\delta)^2\phi\frac{1-\tau_l}{(1-\tau_k)^2}\Big\} \\
&> 0.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial l^*}{\partial \tau_e} &= \frac{1}{B\Delta_3^2}\Big\{\Delta_3\Big[\Delta_2 \frac{\partial \Delta_1}{\partial \tau_e} + \Delta_1 \frac{\partial \Delta_2}{\partial \tau_e}\Big] - \Delta_1 \Delta_2 \frac{\partial \Delta_3}{\partial \tau_e}\Big\} \\
&= \frac{1}{B\Delta_3^2}\left[\Delta_2\left(\Delta_3 \frac{\partial \Delta_1}{\partial \tau_e} - \Delta_1 \frac{\partial \Delta_3}{\partial \tau_e}\right) + \Delta_1 \Delta_3 \frac{\partial \Delta_2}{\partial \tau_e}\right],
\end{aligned}$$

where

$$\begin{aligned}
&\Delta_3 \frac{\partial \Delta_1}{\partial \tau_e} - \Delta_1 \frac{\partial \Delta_3}{\partial \tau_e} \\
&= \frac{B(1-\tau_h)}{(1-\tau_e)^2}\phi\left(\frac{1-\tau_h}{1-\tau_k}\right)\left[\frac{g+\delta}{\beta_1(1-\tau_k)} - \frac{R}{\sigma}\right][\psi(1-\tau_k)+1] < 0 \text{ for a small } \delta.
\end{aligned}$$

Therefore, we find that $\frac{\partial l^*}{\partial \tau_e} < 0$. □

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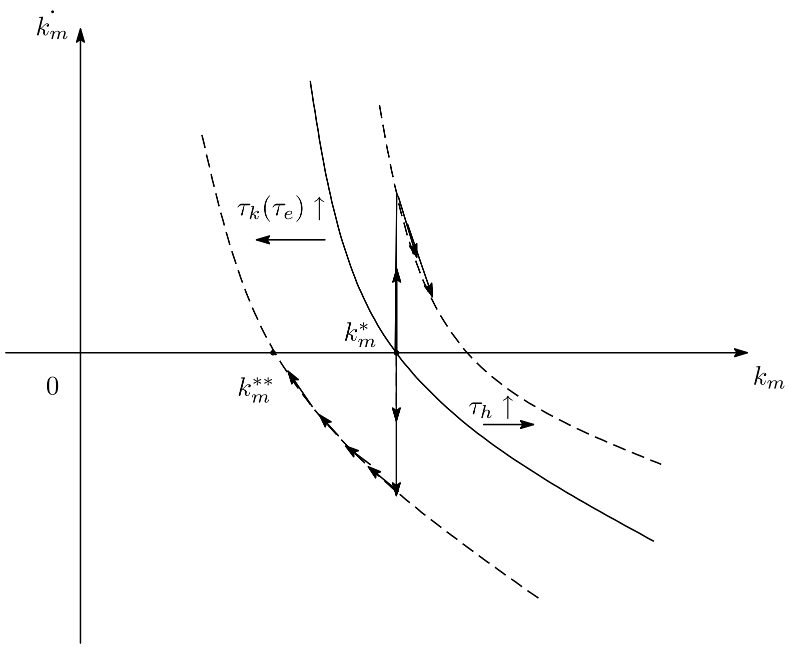

Figure 1: Phase diagram of k_m

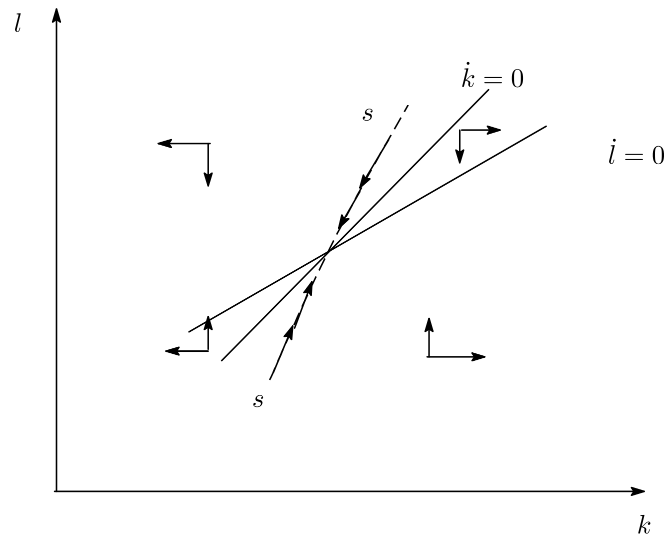

Figure 2: τ_e's increase: the case of $k_m < k_n$ (I-a)

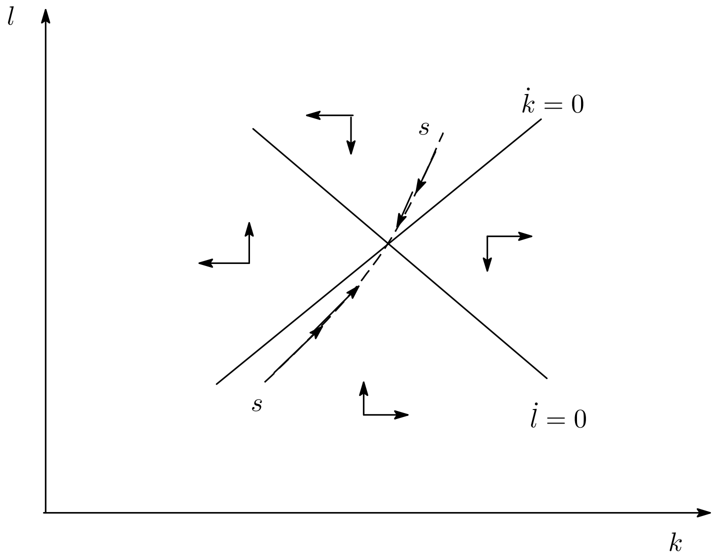

Figure 3: τ_e's increase: the case of $k_m < k_n$ (I-b)

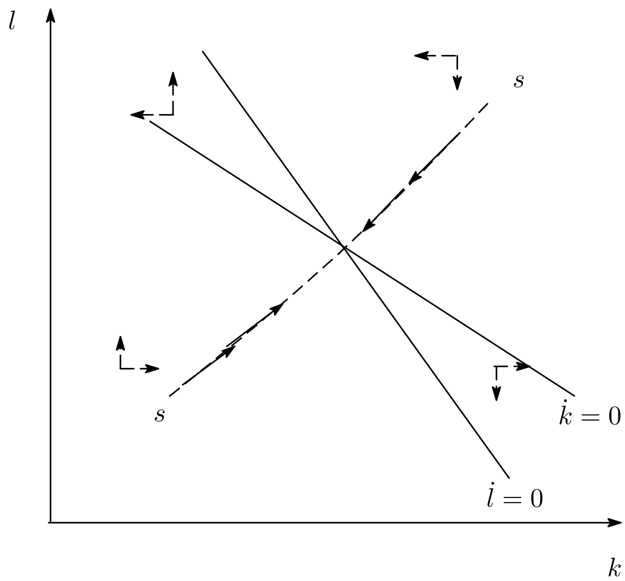

Figure 4: τ_e's increase: the case of $k_m < k_n$ (I-c)

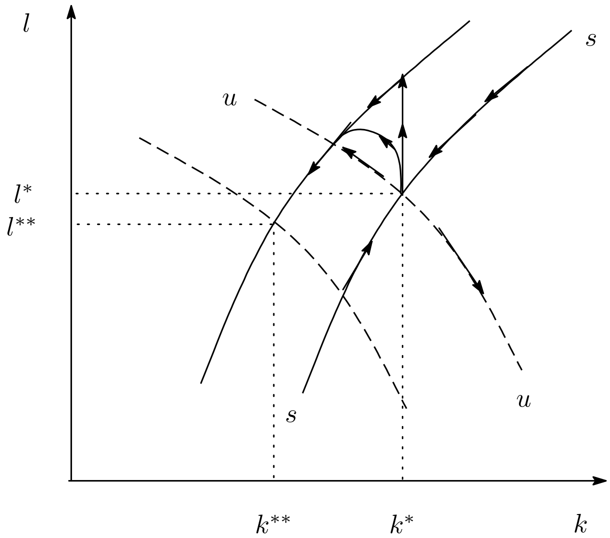

Figure 5: τ_e's increase: the case of $k_m < k_n$

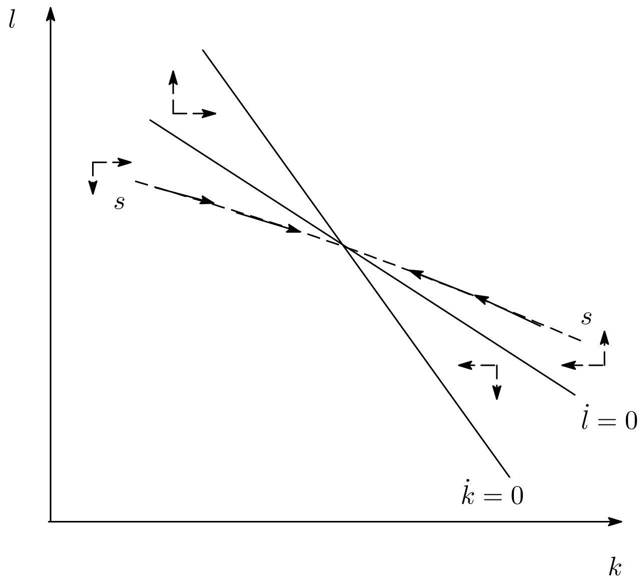

Figure 6: τ_e's increase: the case of $k_m > k_n$ (II-a)

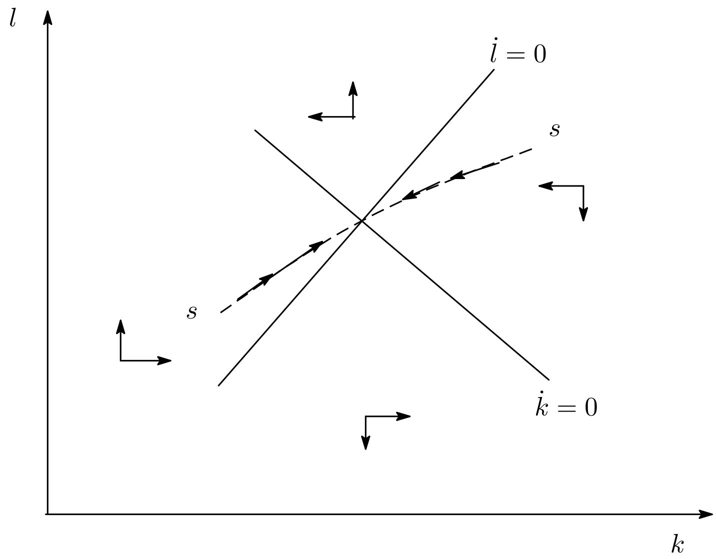

Figure 7: τ_e's increase: the case of $k_m > k_n$ (II-b)

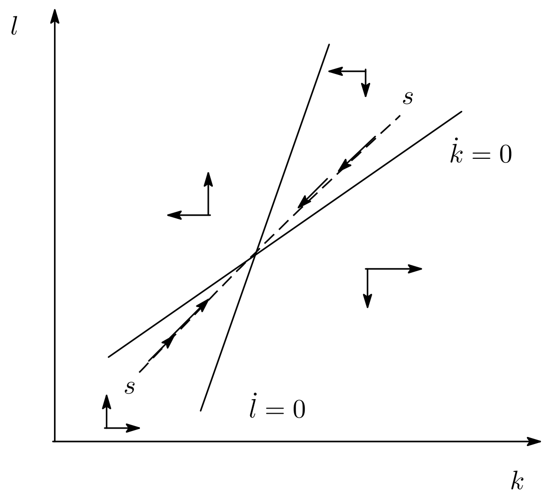

Figure 8: τ_e's increase: the case of $k_m > k_n$ (II-c)