What is the "value" of value-at-risk in a simulated portfolio decision-making game?

Matjaz Steinbacher

2009

WHAT IS THE "VALUE" OF VALUE-AT-RISK IN A SIMULATED PORTFOLIO DECISION-MAKING GAME?

Matjaz Steinbacher
e-mail: matjaz.steinbacher@gmail.com

ABSTRACT
In the paper, I simulate the social network games of a portfolio selection where agents consider VaR when managing their portfolios. Such agents behave quite differently from the agents considering only the expected returns of the alternatives that are available to them in time. The level of omniscience of agents and the presence of liquidity agents are demonstrated to be significant factors for the portfolio management.

Keywords: social networks, portfolio decision-making, stochastic finance, Value-at-Risk.
JEL Classification: G11, G32, C73.

1 Introduction

A stochastic process is defined on the probability space $(\Omega, \Im, P)$, with Ω representing an outcome space of all possible outcomes of events $\omega \in \Omega$, $\Im$ is a $\sigma-$algebra, the collection ob subsets of a sample space Ω that occur with the probability P, with $P(\Im) \geq 0$ and $P(\Omega)=1$. Filtered probability space is a subspace of a probability space, whereas $\Im_t \subseteq \Im$. Prices on financial markets change in time stochastically, making it one such process with risk inherently being part of it. Namely, decisions create patterns and patterns create back the decisions, whereas in this intertwined process in time neither of the two develops in a predictable way nor can be foreseen. Portfolio theory provides a broad context of managing ones wealth, considering risks and returns (Markowitz 1952).

In a number of papers (Steinbacher 2008a, b; 2009a, b), I simulated games of a portfolio decision-making under uncertainty and under different circumstances using models of social network with agents capable of cooperation and learning as defined by Watts and Strogatz (1998). The main features of the papers were that under the social network environment expected return and the level of omniscience of agents affect the portfolio decision-making, whereas the volatility of assets per se does not. In all the papers, the level of loss-aversion was only implicitly introduced into the decision-making processes, as agents were interested in gains and not losses. This article considers this, introducing the Value-at-Risk (VaR) a measure of risk.

Unlike Markowitz's portfolio theory and its variants, using VaR in managing portfolio means to consider for an expected loss of a given portfolio due to changes in market prices of assets in the portfolio over a specified holding period with the given confidence level (Jorion 2006). As such, VaR represents a quantified measure of the riskiness of a given portfolio or its parts. In the model, I assume that when they need to make decisions under uncertainty agents try to avoid losses, which has been extensively documented (Kahneman and Tversky 1979; Thaler 1980; Tversky and Kahneman 1991; Hirshleifer 2001). I assume that agents do it by using the value of VaR as a reference, through which they minimize the probability of suffering the loss.

The remainder of the paper is organized as follows. Section 2 defines the model and the data, and Section 3 simulation results. A final section provides some conclusions.

2 The model

2.1 Computation of VaR

The value of VaR can be estimated in many ways, while the most commonly used are the following three. First is a historical simulation approach, which relies on the historical realizations of returns of securities under study and uses actual percentiles of the observation period to measure the value of VaR. The second method uses a variance-covariance matrix of the assets under study. I use this method in the sequel in two alternatives; first, I take the variance of an entire up-to-date data history, followed by a weighted approach with more distant the data the lesser the relevance. The third way to compute the value of VaR is to use Monte Carlo simulations from a given distribution of market changes. Detailed characteristics of different computational methods for the estimation the value of VaR are provided in Jorion (2006). In simulations below, I use 95% VaR and 99% VaR on a daily basis.

Based on a historical data of the stocks of CreditSuisse and Citigroup, I compute variance of market changes of both stocks in time. This means that as daily prices of the stocks change, so do also the mean value and variance of both stocks change daily. The calculation of portfolio standard deviations using a variance approach is defined as

$$\sigma_t = \left[\frac{1}{t-1} \sum_{t=2}^{T} \left(x_t - \mu_t \right)^2 \right]^{\frac{1}{2}} \tag{1}$$

where σ_t denotes the estimated standard deviation of a portfolio in time t, x_t indicates realizations of market changes of a portfolio in time and μ_t represents the mean value of outcomes of portfolio realized to the point in time.

2.2 The network

As in the previous papers, there are $n = 1000$ infinitely lived agents, populated on the ring lattice to form a small world network, on average each connected with six others (Watts and Strogatz 1998).

The network $g = (V, E)$ is a set of vertices $V = \{v_1, v_2, \ldots, v_{1000}\}$, representing agents, and edges $E = \{e_1, e_2, \ldots, e_n\}$, representing their pairwise relations. If two agents are connected, we denote $ij \in g$, whereas $ij \notin g$ represents two unconnected vertices. Using adjacency matrix, $ij = 1$ if $ij \in g$ and $ij = 0$ if $ij \notin g$. Games in the sequel are simulated on an undirected graph, with edges being unordered pairs of vertices, thus if $ij = 1 \Leftrightarrow ji = 1$. In a small world network, people have many local and some global connections with others, which we get by rewiring some of the connections. In the model, agents are rewired with the probability $p = 0.01$.

Agents are split in the two groups as per their initial preferences. Agents in the first group prefer stocks of CreditSuisse, their share is $0 \le u \le 1$, whereas the portion of $(1-u)$ agents in the second group prefer stocks of Citigroup. Again, agents in both groups are allowed either to opt for a pure strategy, making a portfolio of only the stocks that one prefer, or make a portfolio out of the two stocks available. Agents who prefer stocks of CreditSuisse and make a portfolio only of the CreditSuisse stocks are denoted CS, and CSp if they opt for a portfolio of the two. Contrary, with C are denoted agents that prefer stocks of Citigroup and opt for a pure Citigroup portfolio, whereas they are denoted Cp if they opt for the portfolio of the two. In either case, portfolio is selected from the part of stocks one prefers, $0 \le pi \le 1$, and the remainder, $(1-pi)$, that represents the part of stocks in a portfolio of a non-preferred company. Agents accumulate their wealth in time, but they decide upon the VaR of an alternative in time.

As in Steinbacher (2009b), so also do I here introduce liquidity agents, who never change their initial strategies, and assume that their portion in the network is $l = 0.1$. In the model, liquidity agents are numbered from $901 \le n \le 1000$ of $n = 1000$ agents that are present in the game. In the games, liquidity agents pursue all four alternatives, for each individual randomly defined in the beginning of the games. Agents accumulate their wealth in time according to the strategy they choose.

$$\begin{aligned}
&W_{t+1}(A_C) = W_t(A_C)\cdot[1+Cr] \\
&W_{t+1}(A_{Cp}) = W_t(A_{Cp})\cdot[1+Cr\cdot pi + CSr\cdot(1-pi)] \\
&W_{t+1}(A_{CSp}) = W_t(A_{CSp})\cdot[1+CSr\cdot pi + Cr\cdot(1-pi)] \\
&W_{t+1}(A_{CS}) = W_t(A_{CS})\cdot[1+CSr]
\end{aligned} \tag{2}$$

$W_{t+1}(\bullet)$ and $W_t(\bullet)$ represent wealth of an individual in time t and $t+1$, whereas $(\bullet)$ denotes the strategy played by an individual in time. Returns of stocks, denoted Cr and CSr, are exogenous to the individuals and they cannot foresee them, neither do they know the system how prices change in time. Despite agents accumulate the wealth in time that is increasing due the returns of their portfolios, they make decisions upon the value of *VaR* of the four alternatives in time as defined in (1). From the set of alternatives, they choose the strategy with the highest value of *VaR*, which corresponds to the lowest expected loss of the portfolio.

Finally, the level of omniscience of agents is introduced into the model through the probability function and the coefficient κ as (Szabó and Tőke 1998)

$$\wp = \left[1+\exp\left[\left(VaR(A_i)-VaR(A_j)\right)/\kappa\right]\right]^{-1} \tag{3}$$

In every time period t an agent A_i chooses one of the individuals to which he is directly connected, A_j, and compares his own value of *VaR*, $VaR(A_i)$, to the value of *VaR* of a selected individual, $VaR(A_j)$. This means that higher the value of κ, more non-omniscient the agents. In simulations, I use $\kappa = 1.0$ to define non-omniscient agents and $\kappa = 0.001$ for omniscient agents.

2.3 Data

Data is used from finance.yahoo.com portal. Data refer to adjusted closed prices of both stocks from 21.1.1999 until 19.11.2008. An adjusted closed price is a price adjusted for splits and dividends. In order to use the same time-period for both stocks, I omit adjusted closed prices for the stock for a time units if the other stock was inactive on that day.

Figures 1a and 1b below plot the returns ob both stocks in time, as well as the values of 95% and 99% VaRs of both stocks in time.

Figure 1a: Returns and VaRs of Citigroup stock

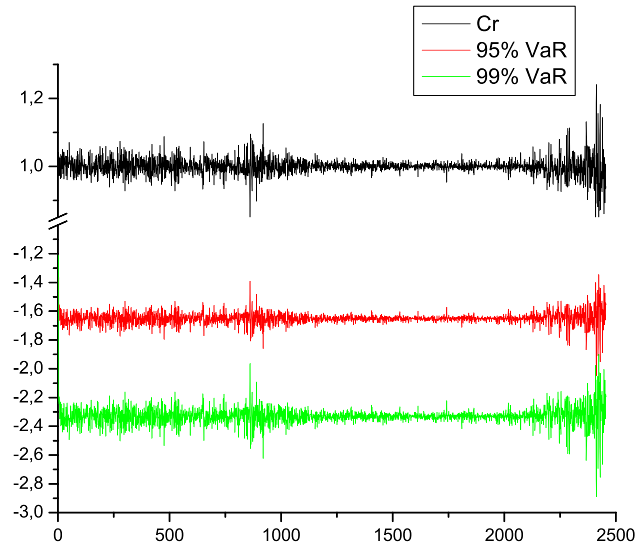
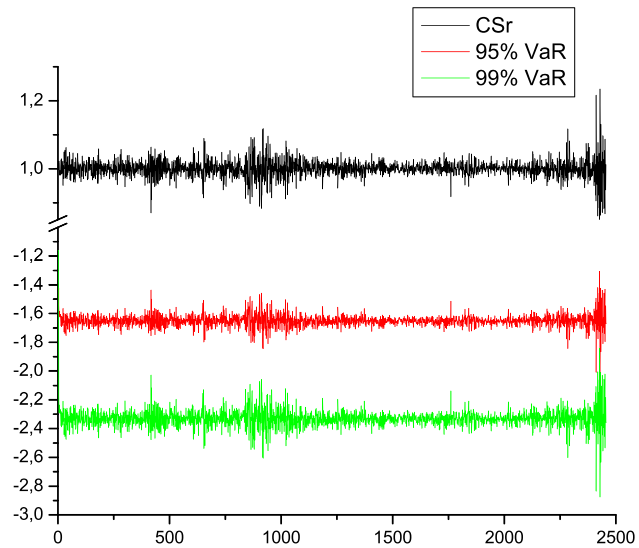


Figure 1b: Returns and VaRs of CreditSuisse stock



3 Simulation results

3.1 Omniscient agents

95% VaR

First are simulations of the games with omniscient agents with $\kappa = 0.001$ under different circumstances. I first take a 95% *VaR* rule and set $u = 0.5$, which means that $C[0] = Cp[0] = CSp[0] = CS[0] = 0.25$, and $pi = 0.3$. C, Cp, CSp and CS represent the portions of agents pursuing each alternative and is changing in time and pi is constant. I do two independent realizations of the game. Simulation results of the first realization are plotted in Figure 2a and of the second in Figure 2b. Colored lines in figures represent portions of agents that pursue each of the alternatives as defined in (2).

Figure 2a: Portions of agents playing each alternative in time

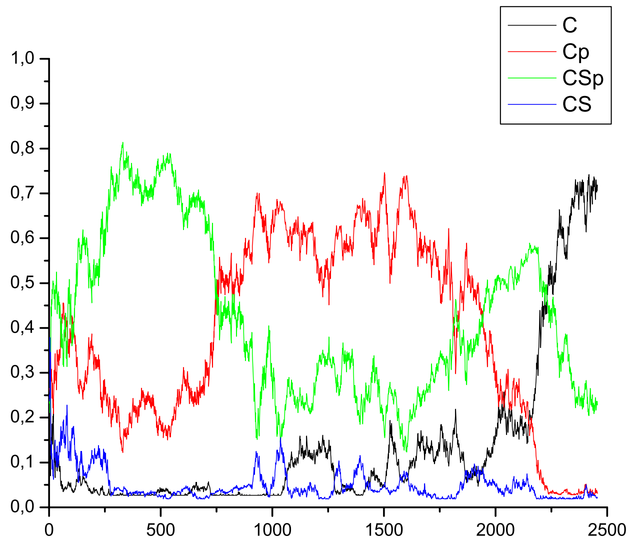
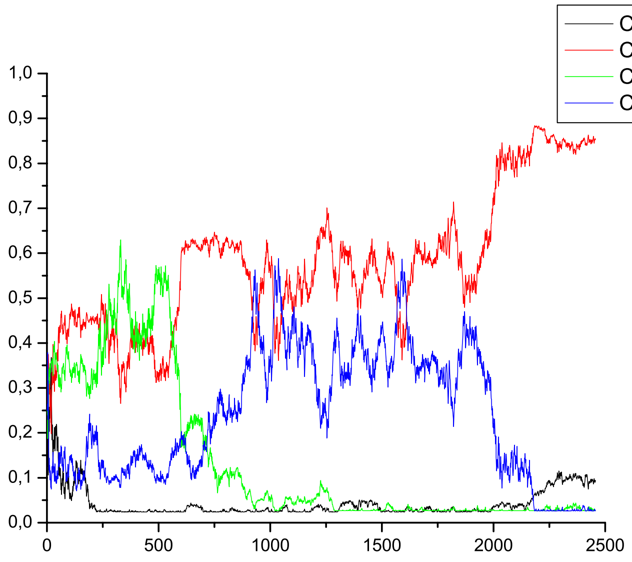


Figure 2b: Portions of agents playing each alternative in time



Both figures demonstrate that under VaR rule there is no clear unanimous decision as in Steinbacher (2009b) where herding induced a Citigroup stock to outperform other alternatives quite soon. Despite both simulations are simulated on equal data and other circumstances, Figure 2a and Figure 2b indicate of a strong butterfly-effect, meaning that the development of the game is determined on whom among friends one takes as a reference. Namely, before making the decision, an agent first picks one of his friends and compares both values of VaR. As expected, agents often opt for a portfolio of the two stocks, by which they reduce the variance and the risk. As in Steinbacher (2009b), so is the role of liquidity agents also very significant within the VaR rule.

99% VaR

I now switch to a 99% VaR rule and keep other conditions unchanged; thus $u = 0.5$ and $C[0] = Cp[0] = CSp[0] = CS[0] = 0.25$, which are changing in time, whereas $pi = 0.3$ and $\kappa = 0.001$ are constant in time. Figure 3a and Figure 3b represent the results of the two independent realizations of the game.

Figure 3a: Portions of agents playing each alternative in time

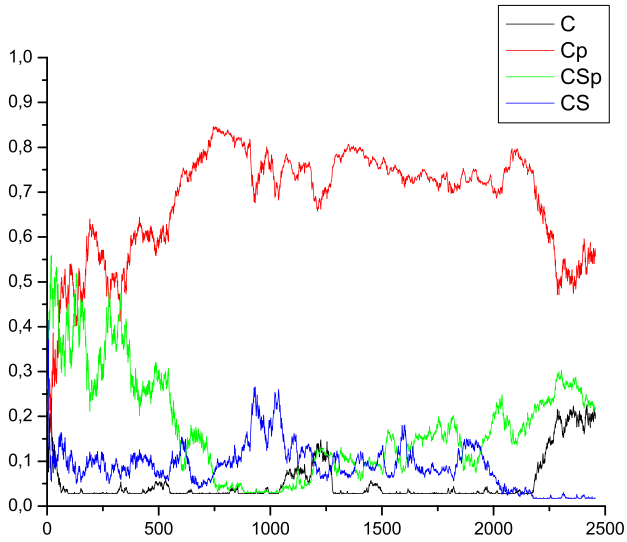
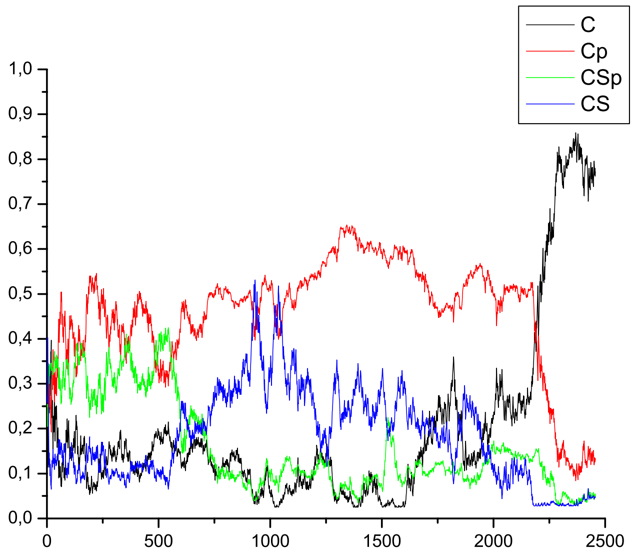


Figure 3b: Portions of agents playing each alternative in time



When switching to a 99% VaR rule, there is again no dominant strategy, whereas the figures pretty much resemble that of the 95% VaR rule. This means the strong butterfly-effect is present and the role of liquidity agents very significant.

3.2 Non-omniscient agents

95% VaR

I now move to non-omniscient agents with $\kappa = 1.0$ and again simulate games under different circumstances. Again, I first use a 95% VaR rule, set the initial value $u = 0.5$, which means that $C[0] = Cp[0] = CSp[0] = CS[0] = 0.25$, and set $pi = 0.3$. The shares of agents pursuing each alternative are changing in time, whereas pi is constant. I do two independent realizations of the game and plot the results of the first in Figure 4a and of the second in Figure 4b.

Figure 4a: Portions of agents playing each alternative in time

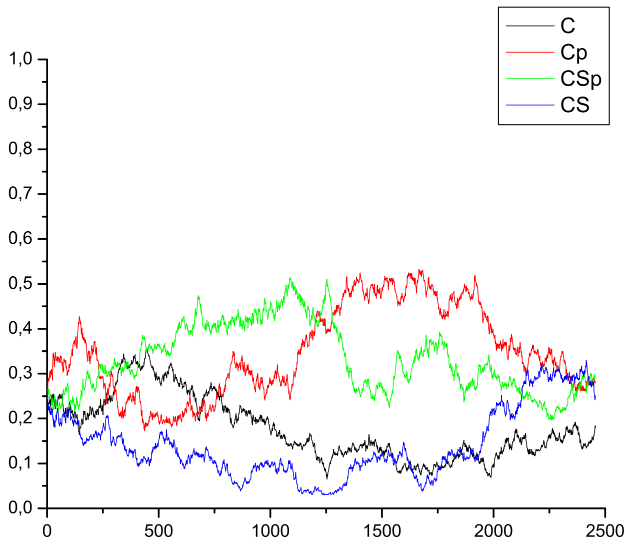
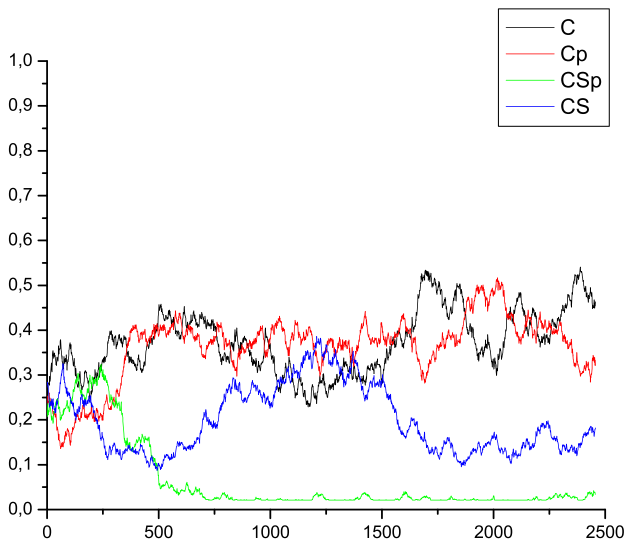


Figure 4b: Portions of agents playing each alternative in time



Figures demonstrate that the decisions of non-omniscient agents are quite different from the ones of omniscient agents. Figures also reveal that the decision-making of non-omniscient agents is much more constant in time than it is under omniscient agents and the presence of liquidity agents. This was also the case in Steinbacher (2008a, b; 2009a, b). The role of liquidity agents is again very significant.

99% VaR

Finally, I turn to non-omniscient agents who pursue a 99% VaR rule. Again, in the start of the game $u = 0.5$, meaning that $C[0] = Cp[0] = CSp[0] = CS[0] = 0.25$, and $pi = 0.3$. The shares of agents pursuing each alternative are changing in time, whereas pi is constant. I do two independent realizations of the game and plot the results of the first in Figure 5a and of the second in Figure 5b.

Figure 5a: Portions of agents playing each alternative in time

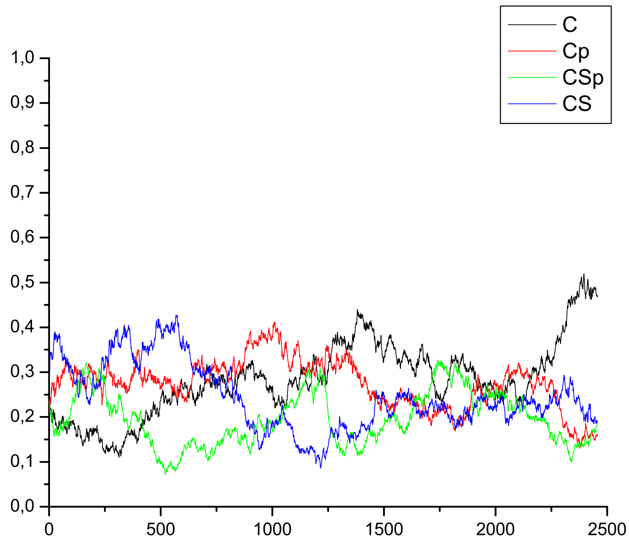
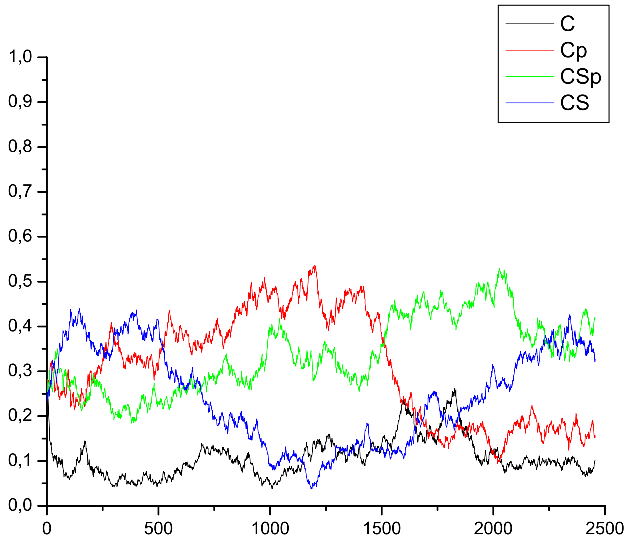


Figure 5b: Portions of agents playing each alternative in time



Figures again demonstrate that non-omniscient agents make decisions in a different way from omniscient agents, and that decisions of non-omniscient agents are much more constant in time, despite still variable. Again, the role of liquidity agents is much emphasized.

4 Concluding remarks

The article somehow supplements some of my previous simulation games in which agents managed their portfolios by comparing efficiencies of the alternatives in time and considering riskiness only implicitly. This is done by introducing VaR into the model as an explicit measure of risk, which agents pursue in their decision-making.

As in the previous simulations, the level of omniscience of agents and the presence of liquidity agents proved to be very significant for the developments of the games. These conclusions are in line with Simon (1997) who argued that new knowledge how to obtain data about beliefs, attitudes and expectations of people is needed in order to make better decisions.

REFERENCES

[1] Hirshleifer, Daniel (2001). “Investor Psychology and Asset Pricing.” *Journal of Finance* 56(4): 1533-1597.
[2] Jorion, Philippe (2006). *Value at Risk: The New Benchmark for Managing Financial Risk*. New York: McGraw-Hill.
[3] Kahneman, Daniel, and Amos Tversky (1979). “Prospect Theory: An Analysis of Decision under Risk.” *Econometrica* 47(2): 263-291.
[4] Markowitz, Harry (1952). “Portfolio Selection.” *Journal of Finance* 7(1): 77-91.
[5] Simon, Herbert (1997). *Models of Bounded Rationality*. Cambridge: MIT Press.

[6] Steinbacher, Matjaz (2008a). “Evolutionary Portfolios through the Small World Network.” *FSI Working Paper 5*. http://ssrn.com/abstract=1160412.
[7] Steinbacher, Matjaz (2008b). “Knowledge, Preferences, and Shocks in a Portfolio Analysis.” *Working Paper*. http://ssrn.com/abstract=1317319.
[8] Steinbacher, Matjaz (2009a). “Acceptable Risk in a Portfolio Analysis.” *Working Paper*. http://ssrn.com/abstract=1320993.
[9] Steinbacher, Matjaz (2009b). “The Role of Liquidity Individuals in the Decision-Making.” *Working Paper*. http://ssrn.com/abstract=1336151.
[10] Thaler, Richard (1980). “Toward a Positive Theory of Consumer Choice.” *Journal of Economic Behavior and Organization* 1(1): 39-60.
[11] Tversky, Amos, and Daniel Kahneman (1991). “Loss Aversion in Riskless Choice: A Reference-Dependent Model.” *Quarterly Journal of Economics* 106(4): 1039-1061.
[12] Watts, Duncan, and Steven Strogatz (1998). “Collective Dynamics of Small World Networks.” *Nature* 393(4): 440-442.