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Abstract

Uniform price auctions admit a continuum of collusive seeming equilibria due to bidders' market power. In this paper, I modify the auction rules in allowing the seller to ration strategic bidders in order to ensure small bidders' participation. I show that many of these "bad" equilibria disappear when strategic bidders do not know small bidders' willingness to pay. Moreover, when the seller is unconstrained in the quantity she can allocate to small bidders, the unique equilibrium price is the highest that the seller could get.

Keywords: Uniform price Auctions, Treasury Auctions, IPO, Rationing.

JEL Classification: D44, G32

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Strategic Price Discounting and Rationing in Uniform Price Auctions

1 Introduction

Uniform price auctions are often criticized because they give rise to underpricing equilibria due to bidders' market power. This paper analyzes the existence of such equilibria when the seller is allowed to ration strategic bidders in order to ensure small bidders' participation. I show that uncertainty about the small bidders' willingness to pay induces strategic bidders to bid more aggressively, mitigating their market power. The main insight arising from this finding is that price discounting associated with rationing prevents bidders from manipulating uniform price auctions. This may explain why auctioneers in financial auctions usually set the offer price below the market clearing price without giving explicit rules.

The uniform price auction format is commonly used in financial and electricity markets to sell a divisible good. Following Milton Friedman's advice, the US Department of the Treasury has even moved from a discriminatory to a uniform price auction to sell government bonds. UK treasury also sells index-linked bonds using the same auction format. Uniform price auctions are also used in California to buy electricity in the power exchange. They are said to be superior to discriminatory auctions to allocate sulfur dioxide emissions permits. More recently, Open IPO, a new, web-based underwriter has proposed to sell shares using a uniform price auction[1]. The extensive use of this mechanism is due to the backing of leading economists and policy makers, who have asserted that uniform price auctions are the most efficient multi-unit auction format. Their conclusions are based on the generalization of the single-unit auctions literature to multi-unit auctions. However, as Ausubel and Cramton (1998a) have shown[2], the uniform price auction is not the generalization of the second price auction to multi-unit auctions. Consequently, truthful bidding is not the optimal strategy,

[1] Google IPO is a recent example of a uniform price aution IPO underwrited by Open IPO.
[2] See also Ausubel and Cramton (1998b).

and the bidders have an incentive to shade their bids[3].

In a uniform price auction, bidders strategically submit demand schedules for a divisible good and the price is set to equate supply and demand. Within this framework, Wilson (1979)[4] shows that there exist arbitrarily low equilibrium prices that may be sustained by the bidders. The intuition for this finding is the following. As a bidder is only concerned with his demand schedule at the market clearing price, the slope is not determined by the equilibrium condition. This indetermination allows them to submit a rather inelastic demand schedule which offsets other bidders' incentives to bid more aggressively. They cannot a get a larger share with only a modest price increase. Consequently, underpricing equilibria emerge.

Wilson (1979) was the first to point that the indivisibility of the good is a critical assumption in the revenue equivalence theorem for multi-unit auctions. Multi-unit auctions for an indivisible and homogenous good have the same nice properties as single-unit auctions, while share auctions (multi-unit auctions for a perfectly divisible good) have a multiplicity of equilibria, some of which are inefficient.

In practice, the rules of the uniform price auction are often modified. For instance, in the Mise en Vente, an auction-like IPO used in France, the price does not clear the market and pro-rata rationing is used. McDonald and Jacquillat (1974) analyze pricing in this process. After all bids are submitted, the seller and the underwriter meet and set the offering price subject to the seller's preferred ownership structure which usually involves pro-rata rationing. They provide empirical evidence that secondary market liquidity and ownership dispersion are fundamental in determining the offering price. This is also a rule used by Open IPO. One can read on their classical prospectus, "*Depending on the outcome of negotiations between the underwriters and us, the public offering price may be lower, but will not be higher, than the clearing price. The bids received in the auction and the resulting clearing price are the principal factors used to determine the public offering price of the stock that will be sold in this offering. The public offering price may be lower than the clearing price depending on a*

[3] See Binmore and Swierzbinski (2001) for a critical review about treasury auctions, and Martimort (2002) for a survey about multi-unit auctions.

[4] The Wilson's (1979) result has been generalized by Back and Zender (1993), Biais and Faugeron-Crouzet (2002) and Wang and Zender (2002).

number of additional factors, including general market trends or conditions, the underwriters' assessment of our management, operating results, capital structure and business potential and the demand and price of similar securities of comparable companies. The underwriters and us may also agree to a public offering price that is lower than the clearing price in order to facilitate a wider distribution of the stock to be sold in the offering." In a recent Open IPO (RedEnvelope, Inc.), the pro-rata percentage for the entire offering was approximately 56%[5].

This is also the case in Treasury Auctions. Keloharju, Nyborg and Rydqvist (2004) document that the Finnish Treasury never chooses the supply that maximizes revenue given the bids in an auction. They show that the quantity demanded but not awarded at the offering price averages 38.5% of the quantity awarded.

These discounts may be explained by regulatory constraints faced by the seller. Indeed, she may have to sell a proportion of the shares to retail, uninformed, and non-strategic bidders. For instance, when modelling Treasury Auctions, Back and Zender (1993) or Wang and Zender (2002), among others, introduce a noncompetitive demand which has priority over the competitive bids. Retail investors are also favoured in Initial Public Offerings in Singapore[6] and in France[7]. In IPOs, the seller may also want to favour those small investors in order to reduce the adverse selection problem characterized by Rock (1986)[8]. Another interesting justification of rationing is given by Brennan and Franks (1997). They argue that pre-IPO shareholders, who have private benefits of control, have incentives to ration in order to ensure oversubscription. Rationing raises the number of winning bidders in the IPO, and thus reduces the block size of new shareholders. According to this "control theory", rationing is used to give shares to small investors who would not have access to the IPO otherwise. This makes it more difficult for new, larger investors to obtain control of the firm. Rationing also creates a more diffuse investor base, making it more costly for investors to assemble large

[5] Another outstanding example is the Andover.net's IPO. When Andover.net decided to go public, the lead underwriter, OpenIPO, found a market clearing price of $24 using the uniform price auction process and decided to set an offer price of $18 in the registration statement. On the first day Andover.net was traded on the market, the shares closed at $77.50. See Boehmer and Fishe (2001) for more extensive comments about this case.

[6] See Lee, Taylor and Walter (1999).

[7] See Ljungqvist and Wilhelm (2002).

[8] See Chowdhry and Sherman (1996) or Chen, Mak and Chan (2002).

blocks of shares after the IPO. Indeed, even if they can acquire these shares on the secondary market, this is not profitable because if the market anticipates a change in ownership and control, the stock price rises. This reduces the probability of owners being subject to the monitoring of a larger shareholder or to a hostile takeover. Brennan and Franks (1997) find strong evidence that rationing tends to prevent the formation of large blocks of shares in the hands of outside shareholders. Moreover, few hostile takeovers have taken place after rationed IPOs, which is consistent with owners' willingness to prevent a change in ownership and control. Amihud, Hauser and Kirsh (2001) also support the hypothesis that rationing is a way to increase ownership dispersion[9].

To analyze the effects of rationing against the bidders' market power in such an environment, I modify the uniform price auction rules in allowing the seller to favour small, non-strategic, bidders in allocating them some quantity of the good even though she has to discount the offer price to do so. Consequently, the seller strategically rations strategic bidders in allocating them only a part of the quantity demanded. Moreover, small bidders demand some quantity if and only if the final price is low enough. However, the strategic bidders do not know the small bidders' willingness to pay for one unit of the good[10]. The degree of rationing is chosen by the seller in order to maximize the auction profits subject to small bidders' participation.

The main finding of this paper is that bidders have an incentive to bid more aggressively. Equilibrium underpricing can be considerably reduced by allowing the seller to strategically ration the quantity awarded to strategic bidders. This may help shed light on the common use of this strategy in IPOs or treasury auctions without giving explicit rules.

The main idea behind this finding is that, due to the possibility of rationing, bidders are

[9] Another well-documented rationale for rationing is the willingness to sustain the liquidity of the secondary market trading. Boehmer and Fishe (2001) and Booth and Chua (1996) state that rationing ensures oversubscription in IPOs and results in a greater dispersion of ownership which may improve liquidity in the secondary market and therefore higher trading profits and brokerage commissions. See also Castellenos and Oviedo (2003) and Roseboro (2002) for Treasury auctions or Cramton and Kerr (1999) for tradeable carbon permit auctions.

[10] For instance, there is no explicit rule for discounting in the prospectus of IPOs underwrited by OpenIPO or within the Mise en Vente process in France. Keloharju, Nyborg and Rydqvist (2004) also note that the Finnish Treasury did not have an explicit policy regarding neither the choice of quantity and stop-out price, nor the quantity awarded to noncompetitive bidders.

facing an uncertain residual supply. As they do not know the small bidders' willingness to pay with certainty, the bidders are not only concerned with their demand at the market clearing price but also with any price that could result from an ex-post rationing. This prevents them from sustaining equilibrium prices as low as they would like. The bidder's ability to inhibit competition from their rivals is reduced. They cannot manipulate bidding as they did in the Wilson (1979) model.

When small bidders can absorb the whole quantity, the previous effects are enhanced. The only resulting equilibrium price is the highest price that the seller could get. In this case, bidders have no way to manipulate the uniform price auction.

These findings may reconcile the uniform price auction theory with empirical evidence. Indeed, natural experiments about the Treasury auctions are not in line with theoretical predictions. Umlauf (1993) analyses the Mexican US Treasury auction and estimates that the switch from discriminatory to uniform price auction has enhanced competition and has reduced bidders' profits. Nyborg and Sundaresan (1996) find that the change in revenue, due to the same switch in the US, ranges from relatively small losses to moderate gains. Malvey and Archibald (1998) observe that this switch produces small gains in revenue. Keloharju, Nyborg and Rydqvist (2004) study Finnish Treasury auctions and show that underpricing is low, and is not due, as suggested by the theory, to the exercise of monopsonist market power by the bidders. The same evidence comes from the IPO literature. Indeed, Derrien and Womack (2000) prove that the auctioning mechanism leads to less underpricing than book building or fixed price methods. Kandel, Sarig and Wohl (1999) indicate that underpricing in Israeli uniform price IPO auctions is significant but small compared to what happens with other mechanisms. This gap is due to the fact that, in theory, there is no strategic role for the seller in uniform price auctions. However, in the real world, the seller is not passive and may take action to mitigate bidders' ability to exercise their market power, as in this analysis.

The seminal contribution by Klemperer and Meyer (1989) is one of the first papers to select among the continuum of equilibria of the uniform price auction. Klemperer and Meyer (1989) introduce uncertainty about the supplied quantity and search for ex-post optimal

supply function equilibria. They prove that the set of equilibria is significantly reduced when uncertainty is unbounded. The intuition is that the bidders' demand schedules have to be defined not only at the market clearing price, but also around it. Despite this promising result, Back and Zender (1993) and Wang and Zender (2002) in the risk averse case, show that arbitrarily low equilibrium prices always exist in the case of uniform price treasury auctions when supply is perfectly inelastic but subject to a horizontal shock. In this paper, I show that allowing the seller to ration bidders ex-post reduces the set of equilibria even though small bidders cannot absorb the whole issue (this is equivalent to a bounded uncertainty in their papers). Moreover, within a perfectly inelastic supply framework, I prove that there is a unique equilibrium in which the realized price is as high as possible when small bidders can absorb the whole quantity (uncertainty is unbounded).

This paper is also related to a growing literature on the ways to mitigate bidders' market power in uniform price auctions. For instance, McAdams (2002) and Back and Zender (2001) show that, when sellers have the possibility to adjust the quantity put for sale ex-post, bidders cannot sustain all equilibrium prices as before, and then underpricing is significantly reduced. The economic sense of this finding is that when the seller can adjust the supply ex-post, bidders are not only concerned with the market clearing price that would occur without possibility of adjustment, but also with any price that could result from an ex-post adjustment. McAdams (2002) also proves that "collusive seeming" equilibria disappear when some amount of cash is shared between rationed bidders. This encourages aggressive bidding for marginal units and thus raises the equilibrium price. LiCalzi and Pavan (2005) study uniform price auctions when the seller is strategic and can precommit to an increasing supply schedule. They prove that even if underpricing is still present, it is significantly reduced. In this setting of increasing supply, with a little price increase the residual supply increases a lot and bidders get a greater quantity. This reduces each bidder's market power. Consequently, bidders bid more aggressively and low price equilibria are eliminated. My paper deviates from theirs in several respects. First, instead of adjustable supply my model considers price discounting associated with rationing as a way to reduce the ability of bidders to manipulate

uniform price auctions. As discounting is a widely used strategy in financial auctions, it is interesting to understand what happens within this framework. Second, I make in this paper a complete characterization of equilibria without assuming that small bidders can absorb the whole quantity[11]. This makes the model closer to real-life situations in which one cannot adjust the quantity put for sale optimally. I also show that the McAdams' (2002) results can be found as a limiting case of mine when the bidders' capacity goes to the total quantity. Finally, under these modified rules bidders still pay the same per-unit price, whereas this is not the case when the seller divides a cash prize among bidders. As the seller first decides to sell the good using a uniform price auction, this may be a real concern.

However, adjusting the supply schedule is not the only way to boost competition in uniform price auctions. Kremer and Nyborg (2004a) and (2004b) show that allowing a finite number of bids is also a good way to eliminate bad equilibria. Their results, however, are proved in a framework in which bidders are allowed to submit discrete bids, whereas I consider a share auction.

Rationing in IPOs has also been studied by Parlour and Rajan (2005) in a model of book building. Following Sherman (2001), they model the Book Building process as a multi-unit common value auction and use the symmetric equilibrium characterized by Milgrom (1981). They show that rationing mitigates winner's curse, i.e., bidders bid more aggressively[12]. They also determine the optimal degree of rationing in the class of credible mechanism[13]. However, a critical assumption in their model is that the bidders have unit demand. I, in contrast, consider bidders who have multi-unit demand, an assumption which is necessary in most of the real-life applications of uniform price auction.

The paper is organized as follows. Section 2 describes the model. In Section 3, I present

[11] This is equivalent to relax the assumption that the seller can infinitely adjust the quantity put for sale in their models.

[12] See Bulow and Klemperer (2002) to understand how rationing may raise the expected prices when common value objects are sold. This model is more complex than the Bulow and Klemperer's (2002) one, but the intuition is the same.

[13] A credible mechanism is a mechanism in which the seller use the announced pricing rule. Here, it means that the seller uses the rationing rule and don't always set the IPO price equal to the market clearing strategy. Such a deviation from the seller is not credible in the sense that bidders would predict it and would bid according to it.

the Wilson (1979) and Back and Zender's (1993) model and results. In section 4, I solve the model with rationing and compare the results with the McAdams' (2002) ones. Finally, section 5 concludes and all proofs are provided in section 6.

2 The model

In this section, I develop a model of the uniform price auction, along the lines of Wilson (1979) or Back and Zender (1993).

A seller wants to auction a fixed quantity, Q, of a homogenous and perfectly divisible good using a uniform price auction. The market is formed by N strategic, rational and risk neutral agents (bidders) and by small, retail and non-strategic bidders. The maximum quantity small bidders can collectively get is $\overline{Q} \leq Q$, i.e. they cannot absorb the whole issue. Moreover, their willingness to pay for one unit of the good is p_0. All strategic bidders share the same information about the per unit common value of the good, v. However, they only know that the small bidders' willingness to pay for the good, p_0, is distributed on the interval $[0, v]$ with cdf $F(.)$[14].

Bidders compete in the auction in simultaneously submitting piecewise continuously differentiable demand schedules for the divisible good. Let $X(p) = \sum_{i=1}^{N} x_i(p)$ be the aggregate demand for the shares at price p. $X(p)$ is also piecewise continuously differentiable.

I modify Wilson's (1979) basic model of the uniform price auction in allowing the seller to favour small, non-strategic, bidders in allocating them some quantity even though she has to discount the offer price to do so. Consequently, the seller strategically rations strategic bidders in allocating them only a part of the quantity demanded. This degree of rationing is chosen by the seller in order to maximize the auction profits subject to small bidders' participation.

[14] For instance, there is no explicit rule for discounting in the prospectus of IPOs underwrited by OpenIPO or within the Mise en Vente process in France. Keloharju, Nyborg and Rydqvist (2004) also note that the Finnish Treasury did not have an explicit policy regarding neither the choice of quantity and stop-out price, nor the quantity awarded to noncompetitive bidders.

The seller selects an offer price $p^*(\alpha)$, lower than the market clearing price, at which strategic bidders only receive a fraction α of the total quantity of good. Here, $\alpha = 1$ corresponds to a case in which strategic bidders get all the quantity. When she selects a degree of rationing $\alpha < 1$, only a part of the total quantity is awarded to strategic bidders.

Moreover, it is assumed that bidders are perfectly symmetric[15].

According to these assumptions, the objective of the seller, when using degree of rationing α, is

$$W(\alpha) = p^*(\alpha)Q$$
$$s.t. \begin{cases} p(\alpha) \leq p_0 \\ (1-\alpha)Q \leq \overline{Q} \end{cases}$$

where Q is the number of shares put up for sale, $p^*(\alpha)$ is the market clearing price when the degree of rationing is α, $\overline{Q}$ is the maximum quantity small bidders can collectively get and p_0 is the small bidders' willingness to pay for one unit of the good, distributed on $[0, v]$ with c.d.f. $F(.)$.

The constraints in the seller's objective corresponds respectively to the small bidders' participation and capacity constraints.

The equilibrium price is now defined as[16]:

$$p^*(\alpha) = \max\{p \mid X(p) = \alpha Q,\ p \geq 0\} \tag{1}$$

where $\alpha \in [0, 1]$ is the degree of rationing chosen by the seller after having observed the aggregate demand.

The strategic bidders being risk neutral, their program is to choose a demand schedule which maximizes their expected profit under the market clearing condition, the expectation

[15] The model is developed only to handle the perfectly symmetric case. However, Bourjade (2007) has extended my results to the asymmetric case. Bourjade (2007) shows that my results in the perfectly symmetric case are still true and even enhanced when there are m types of bidders characterized by their (low enough) private costs of acquisition of the shares.

[16] This specification assumes that the seller's reserve price is zero.

being with respect to α.

I focus on symmetric ex-post optimal Nash equilibria in pure strategies, i.e. if a strategic bidder expects other bidders to submit their equilibrium strategy, his best response is also the equilibrium strategy. Moreover, given other bidders' bid schedules, for each realization of the degree of rationing, α, strategic bidders would not change their bids even though they were allowed to do so ex-post.

The timing of the game is the following

1. The seller announces that, depending on the small bidders' willingness to pay, she can ration the strategic bidders.

2. Each strategic bidder i simultaneously submit a piecewise continuously differentiable demand schedule.

3. After observing the aggregate demand, the seller chooses a degree of rationing, $\alpha \in [0, 1]$ according to his objective.

4. The seller sets the equilibrium price p^* which can be lower than the market clearing price without rationing.

5. Shares are allocated.

3 Equilibria of the basic uniform price auction

I first study the equilibria of the uniform price auction without rationing. This framework **was** first examined by Wilson (1979) and generalized by Back and Zender (1993).

In this basic model of the uniform price auction, the price is set to equate supply and demand. This comes down to taking $\alpha = 1$ in (1).

Wilson (1979) and Back and Zender (1993) show that the bidders can sustain arbitrarily low equilibrium prices. The main idea behind this result is that a bidder only cares about his

demand schedule at the market clearing price. As the slope of this demand schedule is not determined by the equilibrium condition, he may submit a rather inelastic demand schedule. The value of the slope acts as an implicit veto if a bidder tries to increase his market share. This offsets other bidders' incentives to bid more aggressively. A substantial price increase is required to get a larger share. Consequently, underpricing equilibria emerge.

These results can be summarized in the following proposition from Back and Zender (1993) extended by Kremer and Nyborg (2004b):

Proposition 1 (Back and Zender (1993) modified) *Bidders can sustain any equilibrium price $\widehat{p}$ in the range $[0, v]$ in submitting the following bidding strategy,*

$$d(p) = \left[\frac{v-p}{v-\widehat{p}}\right]^{\frac{1}{N-1}},$$

and the stop out price is $\widehat{p}$.

There is a multiplicity of equilibria in the uniform price auction, indexed by the slope of the demand schedule. Only the Bertrand equilibrium price v could achieve the first best.

Selling a divisible good using a uniform price auction is therefore risky. This is not in line with the conclusions of Friedman, who predicted that uniform price auctions have the same "nice properties" as Dutch auctions, i.e. truthful bidding is the best strategy.

To cope with the multiplicity of equilibria, Back and Zender (1993), followed by Kremer and Nyborg (2004b), introduce supply uncertainty[17]. They show that, under supply uncertainty, the previous equilibria are the only ones in the class of ex-post optimal equilibria[18], but arbitrary low equilibrium prices can still be sustained by the bidders.

The main conclusion of their studies is that supply uncertainty does not allow one to select among these equilibria as in Klemperer and Meyer (1989) when the supply is perfectly inelastic.

[17] This methodology to select among all equilibria is due to Klemperer and Meyer (1989).

[18] It means that bidders would not change their bids even though they were allowed to do it ex post.

4 Equilibria of the uniform price auction with rationing

In this section, due to the seller's objective, strategic bidders anticipate that the equilibrium is lower than p_0, the small bidders' willingness to pay for the good. However as they do not know the distribution of p_0, they are not able to determine the degree of rationing that maximizes the profits from the auction. However, they know that for each value of p_0, there is an optimal degree of rationing that maximizes the seller's objective. Indeed, if $p^*(\alpha)$ is strictly decreasing[19], this optimal degree of rationing is unique for each value of p_0.

The bidders being risk neutral, their program is to choose a demand schedule, $x_i(p)$, which maximizes their profit under the market clearing condition.

Let us remark that, within this framework, low equilibrium prices do not necessarily result from bidders' market power, because the seller is constrained to allocate a positive quantity to small bidders. A low equilibrium price may thus be due to a low willingness to pay of small bidders.

An ex-post optimal symmetric Nash equilibrium of this game is characterized by demand schedules $\{x_i(.)\}_{i=1,\ldots,N}$ and an equilibrium price $p^*(\alpha)$ that equalize the aggregate demand for shares and the supplied quantity when the degree of rationing is α.

The equilibrium price is defined by (1) where α is the degree of rationing maximizing the seller's objective under the small bidders' participation and capacity constraints, i.e. the offer price $p^*(\alpha)$ has to be lower than p_0 and small bidders have to be able to absorb the quanty awarded by the seller.

Let us remark that, given α, this price is uniquely defined when the aggregate demand is non increasing. The quantity $x_i(p^*(\alpha))$ is awarded to bidder i.

Let us denote X_{-i} the aggregate demand of all bidders but i[20].

Now, consider the bidder $i's$ program. Ex-post optimality implies that the quantity $x_i(p^*(\alpha)) = \alpha Q - X_{-i}(p^*(\alpha))$ and the price $p^*(\alpha)$ that emerges when the degree of rationing is α maximize the revenue of bidder i given $X_{-i}(.)$ and α. This amounts to $p^*(\alpha)$ being the

[19] This is proved in the Appendix.

[20] As bidders are perfectly symmetric, in equilibrium I have $X_{-i}(.) = (N-1)x(p)$.

solution of the program

$$p^*(\alpha) \in \arg\max_p \{(v-p)\left[\alpha Q - X_{-i}(p)\right]\}$$

The first order condition of this program is

$$-\alpha Q + X_{-i}(p) - (v-p)X_{-i}^{'}(p) = 0 \tag{2}$$

If second order conditions are satisfied (this is checked in the appendix), then (2) implicitly determines $p^*(\alpha)$ for each realization of α. The quantity awarded to bidder i is $\left[\alpha Q - X_{-i}(p^*(\alpha))\right]$.

Moreover, if $p^*(\alpha)$ is strictly decreasing, the quantity awarded to bidder i can be written as a function from price to quantity. This implies that $x_i(p)$ intersects $i's$ residual supply once and only once for each α at $p^*(\alpha)$. So, by submitting $x_i(p)$, bidder i can achieve ex-post optimal adjustment to the degree of rationing. Thus, bidder $i's$ best response to $X_{-i}(p)$ consists in submitting the demand schedule $x_i(p)$.

The following proposition characterizes the pure strategy ex-post optimal symmetric equilibrium of this game.

Proposition 2 *Assume that the seller can ration the quantity awarded to strategic bidders after having observed bids. Then, the equilibrium demand schedule of a strategic bidder is*

$$x(p) = \frac{Q}{N}\left[\frac{v-p}{v-p^*(1)}\right]^{\frac{1}{N-1}}$$

where $p^*(1) \in [0, p_0]$ is the equilibrium price when there is no rationing ex-post. Moreover, the stop-out price is:

$$p^* = \min\left\{p_0; v - \left[1 - \frac{\overline{Q}}{Q}\right]^{N-1}\left[v - p^*(1)\right]\right\}$$

When the number of bidders, N, goes to infinity, there is a unique equilibrium, $p^* = p_0$.

Finally, the degree of rationing selected by the seller is

$$\alpha(p_0) = \max\left\{\left[\frac{v-p_0}{v-p^*(1)}\right]^{\frac{1}{N-1}}; \left[1-\frac{\overline{Q}}{Q}\right]\right\}$$

Let us first note that $p^*(1)$ completely determines the equilibrium price. For each value of $p^*(1)$, there is an equilibrium of the uniform price auction. The equilibrium price that can be sustained by the strategic bidders is therefore not unique. However, the equilibrium demand schedules characterized in Proposition 2 are unique in the class of the pure strategy ex-post optimal symmetric equilibria.

Due to the previous proposition the equilibrium price is

$$p^* = \begin{cases} p_0 & \text{if} \quad p_0 \leq v - \left[1-\frac{\overline{Q}}{Q}\right]^{N-1}[v-p^*(1)] \\ v - \left[1-\frac{\overline{Q}}{Q}\right]^{N-1}[v-p^*(1)] & \text{if} \quad p_0 > v - \left[1-\frac{\overline{Q}}{Q}\right]^{N-1}[v-p^*(1)] \end{cases}$$

More specifically, if:

$$p^*(1) \geq v - \frac{[v-p_0]}{\left[1-\frac{\overline{Q}}{Q}\right]^{N-1}},$$

the equilibrium price of the uniform price auction is p_0, the willingness to pay for one unit of the good of small bidders.

However, when:

$$p^*(1) < v - \frac{[v-p_0]}{\left[1-\frac{\overline{Q}}{Q}\right]^{N-1}},$$

the equilibrium price is $v - \left[1-\frac{\overline{Q}}{Q}\right]^{N-1}[v-p^*(1)]$.

In this case, strategic bidders are able to exercise their market power and the small bidders' participation constraint is not bounded as the equilibrium price is lower than p_0. This is due to the fact that small bidders are capacity constrained. Indeed, there exists some p_0 for which the optimal degree of rationing would require a capacity higher than $\overline{Q}$. The

seller is therefore constrained to set a lower stop-out price in order to sell the total quantity.

The worst equilibrium price for the seller is then $\underline{p}^* = v\left(1 - \left[1 - \frac{\overline{Q}}{Q}\right]^{N-1}\right)$, i.e. with $p^*(1) = 0$. Assuming that the willingness to pay for one unit of the good of small bidders, p_0, is high enough, i.e. $p_0 \geq v\left(1 - \left[1 - \frac{\overline{Q}}{Q}\right]^{N-1}\right)$, $\underline{p}^*$ is thus the lowest equilibrium price that could be sustained by strategic bidders in exercising their market power.

When the seller strategically uses rationing, strategic bidders cannot sustain arbitrary low equilibrium prices. The "collusive seeming" equilibria, introduced by Wilson (1979) and Back and Zender (1993), are therefore very sensitive to the possibility of ex-post rationing.

Moreover, when the seller is not constrained in the quantity she can allocate to small bidders ($\overline{Q}$), the lower bound for the equilibrium prices is raised and the set of equilibrium prices is restricted.

Let us also note that there is a benefit in raising the number of bidders in this modified uniform price auction, whereas this was not the case under the conditions of Proposition 1.

This model with rationing is in the same vein as that of Back and Zender (2001) and McAdams (2002). However, it is designed to better correspond to the real world. Rationing is used instead of random supply as an alternative way to fight collusive-like equilibria in uniform price auctions. I also relax the assumption that the seller can infinitely increase, or decrease, the supply of shares in order to more accurately reflect many real-life situations, in which quantity put for sale cannot be adjusted optimally. Indeed, in IPOs there is a maximum amount of proceeds a seller can obtain which is determined before the IPO begins, and in treasury or electricity auctions, there are feasibility constraints on the supply.

However, it is interesting to study what happens when the equivalent assumption is added in my model, i.e. small bidders can absorb the whole issue, i.e. $\overline{Q} = Q$. Even though this corollary has a limited practical relevance, it allows one to compare the results with those of McAdams' (2002).

Corollary 3 *If small bidders can absorb the whole quantity, the only equilibrium price is* $p^* = p_0$.

This result is in line with McAdams' (2002) analysis. McAdams shows that when the seller can infinitely increase the supply[21] after having observed the bids schedules, no collusive seeming equilibria exist. This McAdams' (2002) result is therefore a limiting result of ours when $\overline{Q}$ goes to Q[22]. Intuitively, in McAdams' framework, if the seller infinitely increases the supply, bidders' ability to exercise their market power is reduced and their advantage of being strategic disappears. This is similar of what happens in my model with $\overline{Q} = Q$, as the seller can then allocate all the quantity to small bidders when the price sustained by strategic bidders is too low. This prevents them to exercise their market power. The equilibrium price is therefore the highest price the seller could get, i.e. the willingness to pay for one unit of the good of small bidders, p_0. Rationing is therefore as efficient as variable-supply to reduce bidders' ability to exercise their market power.

These findings may explain why the auctioneer in financial auctions usually sets the offer price below the market clearing price without giving explicit rules. Indeed, as she has incentives to ration, she must ration in an uncertain enough way[23] in order to reduce bidders' ability to exercise their market power. This allows her to select the optimal price discount, not subject to manipulation from bidders. However, let us remark that even though these new rules are not completely transparent, the distribution of shares is not chosen by the seller. The winners are the bidders who make the highest bids. This makes the modified uniform price auction a fairer IPO process than book building, as it maintains the uniform auction pricing rule that all winners pay the same price.

Moreover, this contribution may reconcile economists with uniform price auctions. Indeed, the unique ex-post optimal equilibrium of this modified uniform price auction is the efficient one. Each bidder bids for the maximum quantity he could absorb at any price below the price at which he makes zero profit and the resulting equilibrium price is the true value of the shares. This is exactly the same equilibrium that Back and Zender (1993) identify for

[21] In fact, the auctionner can increase the demand in McAdams (2002), because this is a model about electricity procurement.

[22] In this paper, McAdams also shows that "collusive-seeming" equilibria disappear when some amount of cash is shared between rationed bidders.

[23] The seller should not reveal the willingness to pay for one unit of the good of small bidders to strategic bidders.

discriminatory auctions with uncertainty. Therefore, these two auctioning processes result in the same equilibrium and yield the same revenue to the seller.

5 Conclusion

A large part of the literature supports the use of discriminatory auctions when selling a divisible good because of the "collusive seeming" equilibria of the uniform price auctions. Recent contributions have shown that adjusting supply after observing bids eliminates some of these equilibria. This analysis introduces a new way for the seller to reduce bidders' market power, commonly used in IPOs: rationing shares. This reduces the bidders' market power and thus eliminates many of these "bad equilibria".

When rationing is allowed, the only surviving equilibria are close to the Bertrand-type one. Moreover, when the seller can allocate all the quantity to small bidders, the previous effects are enhanced. This results in a unique surviving equilibrium price which is the highest price the seller could obtain.

These findings may shed light on the reasons why the auctioneer in financial auctions usually sets the offer price below the market clearing price without giving explicit rules. She is induced to ration in an uncertain enough way in order to reduce bidders' ability to exercise their market power. Moreover, this may attract bidders, with high entry costs enhancing the competition from current bidders[24]. Even though these rules lack transparency, all winners pay the same price and are the highest bidders, whereas this is not the case in many other IPO processes and auction formats.

Rationing is thus a response to the main criticism of uniform price auctions, namely the exercise of market power by strategic bidders. Small modifications to the rules eliminate "collusive seeming" equilibria. Rationing helps the seller to reduce the bidders' market power and consequently their ability to manipulate the uniform price auction. Moreover, the equilibria of the Wilson continuous share auction model with rationing and the equilibria

[24] See Bourjade (2007).

of the discrete model are almost identical. Using these modifications may thus make this more tractable model a good approximation of Treasury or electricity markets, thus enabling better comparisons with discriminatory auctions.

An interesting extension could be to solve a dynamic version of this model. Indeed, one way a seller can make her rationing strategy credible is to develop a reputation for rationing. An important feature of IPO, treasury, electricity or pollution permit auctions is that they are repeated frequently. The seller can use the previous auctions to make rationing credible. If the seller randomly rations shares in an auction, bidders may believe that she will play in the same way in the next auction. This makes them build prior beliefs about the rationing scheme used by the seller, anticipating that the degree of rationing used by the seller is distributed over some non-empty support. Bidders would therefore be induced to use the same strategy that we present in this paper. Moreover, if the seller does not ration at all, bidders anticipate it and use the same "collusive seeming" strategy as in Back and Zender (1993). Then, even though the seller is only profit maximizing but is nevertheless interested in his long-term profits, rationing should be optimal in order to build a "rationing" reputation and then make profits in the subsequent auctions. However, the viability of sustaining a reputation to play a mixed strategy is still an open question.

Another extension would be to introduce a private component to the value of the good for each bidder. This would introduce private information into the model. However, the characterization of the equilibria of the uniform price auction with private information is not an easy task. These issues are left for future research.

6 Appendix

Proof of Proposition 2. Let $\overline{Q}$ be the maximum quantity small bidders can absorb, Q the quantity of shares put up for sale, N the total number of risk neutral strategic bidders, $x_i(p)$ an equilibrium strategy (piecewise continuously differentiable), α the rationing scheme and p_0 the small bidders' willingness to pay for one unit of the good.

As the seller maximizes the profits from the auction, she prefers to set α such that the corresponding equilibrium price is as high as possible. Thus, as long as it is feasible[25], she will set the stop out price equals to p_0.

As strategic bidders do not know the distribution of p_0, they are not able to determine the degree of rationing, α, that maximizes the profits from the auction. However, they know that for each value of p_0, there is an optimal degree of rationing that maximizes those profits. Indeed, if $p^*(\alpha)$ is strictly decreasing[26], this optimal degree of rationing is unique for each value of p_0.

Let us denote X_{-i} the aggregate demand of all strategic bidders but i. Given the demand X_{-i}, and the degree of rationing α, the price $p^*(\alpha)$ maximizes strategic bidder i's profit.

I solve for the Nash equilibrium of this game. Let us first consider the bidder $i's$ program.

As I characterize the pure strategy symmetric equilibrium of this game, all strategic bidders use the same equilibrium strategy, i.e., $x_i(p) = x(p)$, and $X_{-i}(p) = (N-1)x(p)$, for $i = 1, ..., N$.

The market clearing price is then chosen such that $Nx(p) = \alpha(p_0)Q$. Where $\alpha(p_0)$ is the degree of rationing selected by the seller if the small bidders' willingness to pay for one unit of the good is p_0. In order to avoid a multiplicity of notations, let me call this degree of rationing α.

Bidder i wants to sustain a price $p^*(\alpha)$ when the degree of rationing is α, in submitting

[25] First, the market clearing condition has to be satisfied. Then, when setting the stop-out price, the seller has to take into account small bidders' capacity. Indeed, it may exists some p_0 for which the optimal degree af rationing would require a capacity higher than $\overline{Q}$. In this case, the seller is constrained to set a lower stop-out price in order to sell the total quantity.

[26] This is proved later.

a demand schedule $x_i(p)$. Ex-post optimality gives the following program for bidder i

$$p^*(\alpha) \in \arg\max_p \{(v-p)\left[\alpha Q - (N-1)x(p)\right]\}$$

where $[\alpha Q - (N-1)x(p)]$ is the residual supply that bidder i faces.

Bidder i's objective is to maximize his profits with respect to the price p. I obtain the following first order condition[27]

$$-\alpha Q + (N-1)x(p^*(\alpha)) - (N-1)(v - p^*(\alpha))x'(p^*(\alpha)) = 0 \tag{3}$$

However, in a symmetric equilibrium, the market clearing condition must be satisfied

$$Nx(p^*(\alpha)) = \alpha Q \tag{4}$$

The derivative of the market clearing condition with respect to α gives

$$N\frac{dp^*(\alpha)}{d\alpha}x'(p^*(\alpha)) = Q \tag{5}$$

Putting this into the first order condition (3) gives

$$-\alpha Q + \frac{(N-1)}{N}\alpha Q - \frac{(N-1)}{N}\frac{(v-p^*(\alpha))}{\frac{dp^*(\alpha)}{d\alpha}}Q = 0$$
$$\frac{dp^*(\alpha)}{d\alpha} - (N-1)\frac{p^*(\alpha)}{\alpha} + (N-1)\frac{v}{\alpha} = 0$$

A particular solution of this differential equation is $p(\alpha) = v$.

We have to solve the homogenous equation which is

$$\frac{\frac{dp^*(\alpha)}{d\alpha}}{p^*(\alpha)} = \frac{N-1}{\alpha}$$
$$\Longleftrightarrow p^*(\alpha) = k\alpha^{N-1}$$

[27] Second order conditions are satisfied, see the Lemma in the Appendix.

Hence, the general solution of this differential equation is

$$p^*(\alpha) = k\alpha^{N-1} + v \tag{6}$$

where $k = p^*(1) - v$, with $p^*(1) \in [0, p_0]$ is the equilibrium price with no rationing[28].

If the small bidders can absorb the quantity awarded by the seller, the equilibrium price is p_0 and the optimal degree of rationing selected by the seller is

$$\alpha(p_0) = \left[\frac{v - p_0}{v - p^*(1)}\right]^{\frac{1}{N-1}}$$

Moreover, one can remark that $p^*(.)$ is decreasing in α which implies that the equilibrium is ex-post optimal.

This remark together with the small bidders' capacity constraint $(1-\alpha) \leq \overline{Q}$, or equivalently $\alpha \leq \left[1 - \frac{\overline{Q}}{Q}\right]$, implies that this equilibrium only holds when

$$p_0 \leq v - \left[1 - \frac{\overline{Q}}{Q}\right]^{N-1} [v - p^*(1)]$$

Otherwise, the optimal degree of rationing is $\alpha = \left[1 - \frac{\overline{Q}}{Q}\right]$ and the equilibrium price $p^* = v - \left[1 - \frac{\overline{Q}}{Q}\right]^{N-1} [v - p^*(1)]$. This means that the equilibrium price is completely determined when the value of $p^*(1)$ is given.

Moreover,in this case, i.e. if the willingness to pay for one unit of the good of small bidders, p_0, is high enough, the lowest equilibrium price that could be sustained by strategic bidders in exercising their market power is the one with $p^*(1) = 0$, i.e.

$$\underline{p^*} = v\left(1 - \left[1 - \frac{\overline{Q}}{Q}\right]^{N-1}\right).$$

This condition significantly reduces the set of equilibrium prices, even though it is not unique. For each value of $p^*(1) \in [0, p_0]$, we can associate a price schedule $p^*(\alpha)$, and

[28] $p^*(1)$ has to be lower than p_0 in order to ensure small bidders' participation.

consequently the demand schedule, $x(p)$.

Indeed, using (6), we obtain

$$\frac{dp^*(\alpha)}{d\alpha} = (p^*(1) - v)\,(N-1)\alpha^{N-2}$$

Putting this in (5) gives

$$x'(p^*(\alpha)) = \frac{Q}{(p^*(1) - v)\,N(N-1)\alpha^{N-2}} \tag{7}$$

One could also rewrite (3), together with (4), as

$$x(p) = -(N-1)(v-p)x'(p)$$

This gives, with (7), for all $p \in]0, p^*(1)[$

$$x(p) = \frac{Q}{(v - p^*(1))\,N\alpha^{N-2}}\,(v-p) \tag{8}$$

Moreover, inverting $p(.)$ in (6)[29], together with (8) gives:

$$x(p) = \frac{Q}{N\,(v - p^*(1))}\left[\frac{v - p^*(1)}{v-p}\right]^{\frac{N-2}{N-1}}(v-p)$$

Finally, we obtain the equilibrium demand schedule for all $p \in]0, p^*(1)[$

$$x(p) = \frac{Q}{N}\left[\frac{v-p}{v - p^*(1)}\right]^{\frac{1}{N-1}} \tag{9}$$

Moreover,when N goes to infinity, the lower bound of the equilibrium prices goes to v. However, let me remind that the seller has to ensure small bidders' participation. This implies that whatever the degree of rationing in equilibrium, the equilibrium price goes to p_0 when N goes to infinity.

[29] $p(.)$ is strictly increasing and, thus, invertible.

It means that when the number of strategic bidders is large enough, the equilibrium price is the highest price that the seller could get. ■

Lemma 4 *The demand schedule $x(.)$ forms a symmetric "Demand" Function Equilibrium tracing through ex-post optimal points if and only if for all $p \geq 0$, $x(.)$ satisfies the first order condition of the problem* (3) *together with the Market Clearing condition* (4) *and is non increasing.*

Proof.

- Sufficiency:

Since $x(.)$ is non increasing, the aggregate demand intersects the fixed supply at a unique point for each α.

Moreover, the demand schedules satisfy the first order condition for ex-post profit maximization when the other firms choose the/an equilibrium strategy.

Both conditions together imply that $\Pi_i^{''} < 0$ for all $p^*(\alpha)$.

Indeed, the first order condition of bidder i's problem is

$$-\alpha Q + (N-1)x(p^*(\alpha))$$
$$-(N-1)(v - p^*(\alpha))x'(p^*(\alpha)) = 0$$

Putting the market clearing condition in the previous equation gives

$$x(p^*(\alpha)) = -(v - p^*(\alpha))(N-1)x'(p^*(\alpha))$$

We now make the derivative of this equation with respect to the price

$$x^{'}(p^*(\alpha)) = (N-1)x'(p^*(\alpha)) - (v - p^*(\alpha))(N-1)x^{''}(p^*(\alpha))$$

Or equivalently

$$(N-2)x'(p^*(\alpha)) = (v - p^*(\alpha))(N-1)x^{''}(p^*(\alpha))$$

Let us state the second order condition of bidder i

$$\begin{aligned}\Pi_i^{''}(p^*(\alpha)) &= (N-1)x^{'}(p^*(\alpha)) + (N-1)x'(p^*(\alpha)) \\ &\quad -(v - p^*(\alpha))(N-1)x^{''}(p^*(\alpha))\end{aligned}$$

Those conditions together give

$$\begin{aligned}\Pi_i^{''}(p^*(\alpha)) &= 2(N-1)x^{'}(p^*(\alpha)) - (N-2)x'(p^*(\alpha)) \\ &= Nx^{'}(p^*(\alpha))\end{aligned} \tag{10}$$

This is clearly non positive when the demand schedule $x(.)$ is non increasing.

Global second order conditions for ex-post profit maximization are satisfied everywhere. Thus, the demand schedules form a symmetric "Demand" Function Equilibrium tracing through ex-post optimal points.

- Necessity:

Satisfaction of the first order condition of the problem together with the market Clearing condition is a necessary condition for a supply function to trace through ex-post optimal points.

Moreover, if for some $p^*(\alpha)$, $x^{'}(p^*(\alpha)) \geq 0$, then $\Pi^{''}(p^*(\alpha)) \geq 0$, see (10).

Therefore, the demand schedules $x(.)$ cannot be a symmetric "Demand" Function Equilibrium. ■

Proof of Corollary 3. In the proof of Proposition 2, we obtain the following equilibrium

demand schedule for all $p \in]0, p^*(1)[$

$$x(p) = \frac{Q}{N}\left[\frac{v-p}{v-p^*(1)}\right]^{\frac{1}{N-1}}, \text{ with } 0 \leq p^*(1) \leq p_0.$$

Moreover, the equilibrium price is

$$p^* = \min\left\{p_0; v - \left[1 - \frac{\overline{Q}}{Q}\right]^{N-1}[v - p^*(1)]\right\},$$

I have shown that the minimum value for the right hand side is $v\left(1 - \left[1 - \frac{\overline{Q}}{Q}\right]^{N-1}\right)$. When $\overline{Q}$ goes to Q, this value goes to v and the equilibrium price becomes:

$$p^* = p_0,$$

which is the highest price the seller could get. ■

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