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# THE TRADE-OFF BETWEEN MONEY AND TRAVEL TIME: 

# A TEST OF THE THEORY OF REFERENCE-DEPENDENT PREFERENCES ${ }^{1}$ 

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#### Abstract

We formulate a model of reference-dependent preferences based on the marginal rate of substitution at the reference-point of a reference-free utility function. Using binary choices on the trade-off between money and travel time, reference-dependence is captured by value functions that are centered at the reference. The model predicts a directly testable relationship among four commonly used valuation measures (willingness to pay (WTP), willingness to accept (WTA), equivalent gain (EG) and equivalent loss (EL)). Moreover, we show that the model allows recovering the underlying 'reference-free' value of time. Based on a large survey data set, we estimate an econometric version of the model, allowing for both observed and unobserved heterogeneity. In a series of tests of high statistical power, we find that the relationship among the four valuation measures conforms to our model and that the constraints on the parameters implied by the model are met. The gap between WTP and WTA is found to be a factor of four. Loss aversion plays an important role in explaining responses; moreover, participants are more loss averse in the time dimension than the cost dimension. We further find evidence of asymmetrically diminishing sensitivity. Finally, we show that the fraction of 'mistakes' (in the sense that participants are observed to sometimes select dominated options), varies systematically in a way consistent with the model of referencedependence. The results of the paper have important implications for the evaluation of infrastructure investment and pricing reforms in the transport sector.


Keywords: Reference-dependence, loss aversion, WTP-WTA gap, value of time JEL codes: D01, C25

[^0]
## 1 Introduction

The value of travel time plays an important role in the appraisal of transport projects. For example, it is an essential ingredient in cost-benefit analyses of infrastructure investments, where a large fraction of the benefits of the project often consist of time savings. ${ }^{3}$ The value of time is also an essential input in the welfare economic evaluation of proposals for transport pricing reforms (Calfee \& Winston 1998;De Borger \& Proost 2001). Moreover, time costs have been crucial in shaping urban spatial structure, and they are a key element in understanding the economics of urban sprawl (Brueckner 2005). Not surprisingly, therefore, economists have intensively studied the determinants of consumers' valuation of time, both theoretically and empirically. Recent references include, among many others, Hensher ( 2001), Wardman ( 2001), Jiang \& Morikawa ( 2004) and Small, Winston and Yan ( 2005).

This paper contributes to this evolving literature on the value of travel time and more generally to the literature on valuation of non-market goods. Its main purpose is to study the implications of the theory of reference-dependent preferences for deriving estimates of consumers' time values in stated choice experiments. The theory of reference-dependent preferences originates in the seminal paper of Tversky and Kahneman (1991) in which they extend their earlier work on choice in risky situations (Kahneman \& Tversky 1979) to conditions of risk-less choice. A fundamental property of the theory is that preferences are defined in terms of value functions, which have four general features: (i) They are increasing; (ii) They capture reference-dependence: individuals interpret options in decision problems as gains or losses relative to a reference point; (iii) They exhibit loss aversion: losses relative to the reference are valued more heavily than gains; (iv) They incorporate diminishing sensitivity: the marginal values decrease with size, both for losses and for gains. Referencedependence and loss aversion jointly imply that the slope of the indifference curve through a point depends on the reference from which it is evaluated, and that kinks occur at the reference point.

[^1]Over the past decades, a substantial number of studies have documented a gap between willingness to pay ( $W T P$ ) and willingness-to-accept ( $W T A$ ). The gap has been noted in stated as well as revealed choice situations, and it appears in very different settings, including contingent valuation studies, laboratory experiments, public goods experiments, etc. ${ }^{4}$ It is well known that in a Hicksian preference setting, as long as goods are normal, it will be the case that $W T P<W T A$; the size of the difference depends on the magnitude of income effects (see, e.g., Randall \& Stoll 1980). However, the gap between WTP and WTA that is found in experiments is often so large that it is difficult to explain in a standard Hicksian setting (Horowitz \& McConnell 2003). Bateman et al. ( 1997) therefore use referencedependent preferences to study the systematic differences among the different concepts. They show that loss aversion immediately implies $W T P<W T A$. The authors then set up a series of experiments that allows testing the standard Hicksian theory versus a reference-dependent alternative, and they find strong evidence in favour of the latter.

We formulate a theoretical model of reference-dependence based on the marginal rates of substitution at a reference point of a reference-free utility function, defined over travel time and travel cost. The model has the feature that the marginal rate of substitution of the reference-free utility function may be recovered from observations of reference-dependent choices. In this we follow the program outlined by Bernheim \& Rangel (2007) that aims to distinguish between choices and preferences but still identify preferences empirically. This paper seems to be the first to tackle loss aversion in discrete choices in this context. It is the first paper to provide a detailed analysis of the implications of reference-dependent preferences for estimating consumers' valuation of time.

We use this theoretical setting to develop an econometric model that accounts for both observed and unobserved individual heterogeneity through the individual-specific reference-free value of time. The model is applied to a data set with observations from more than 2000 car drivers that were offered repeated choices between alternatives. These were defined in terms of time and cost changes relative to a recent trip, treated as the reference. ${ }^{5} \mathrm{~A}$

[^2]range of models is estimated, incrementally allowing a more general specification of the value functions. The most general version of the model allows for loss aversion as well as asymmetrically diminishing sensitivity.

Our experiment on the trade-off between travel time and money has advantages and disadvantages compared to earlier experiments in the literature (e.g., Bateman et al. 1997). Advantages are, first, that it involves the use of time; this is fundamentally important for everyone, and it can meaningfully be varied continuously, both up and down. Second, unlike some of the earlier experiments (involving mugs or chocolates), the choices that we ask subjects to make are similar to choices they make every day. Therefore, it seems less likely in our setting that, as argued in the literature (List 2004;Plott \& Zeiler 2005), that the size of the $W T P-W T A$ gap could be related to lack of training to deal with the choice environment, to lack of familiarity with the choice task, to lack of experience with the type of choices to be made, etc. Third, we have been able to gather a large database, so that our tests have considerable statistical power. Moreover, an advantage we share with Bateman et al. ( 1997) is that the experiment, discussed in more detail below, is likely to avoid large income effects; moreover, it is designed so as to be less susceptible to strategic behaviour by participants. ${ }^{6}$ Finally, time is a private good so that our experiment is not vulnerable to the criticism raised by Diamond and Hausman ( 1994) against contingent valuation. Disadvantages of our experiments are, first, that we employ data on hypothetical choices. This is necessary, since we are unable to endow subjects with time. For the same reason we cannot ensure incentive-compatibility, and we are unable to move the reference to control for income effects in the same way that Bateman et al. ( 1997) do. However, on the use of hypothetical data we do find support in Kahneman and Tversky ( 1979) who argue strongly in favour of this practice.

The contributions of this paper can be summarized as follows. First, we show the relevance of reference-dependent preferences for deriving estimates of consumers' valuation

[^3]of time in stated preference experiments. Using information on four types of binary choices between travel time and travel cost, we find that a model of reference-dependence cannot be rejected against more general alternatives. The results suggest that asymmetric loss aversion plays an important role in explaining responses. We find that, in absolute value, drivers attach more value to a time loss than a time gain, and that time values increase with the size of the time difference. We also confirm the very large gap between $W T P$ and $W T A$ found in other studies. Second, we show that under our model it is possible to obtain estimates of the underlying reference-free value of time. This is important, because the large gap between the willingness-to-pay and the willingness-to-accept has generated an ongoing debate on which value to use for policy evaluation, and even on the usefulness of contingent valuation methods in general (e.g., Diamond \& Hausman 1994;Horowitz \& McConnell 2003). The fact that the reference-free value of time can be recovered from the estimated models is highly relevant for the time values to be used in cost-benefit analysis and evaluations of pricing reforms or tax policies. Finally, we analyse data on choice situations involving 'mistakes', i.e., cases where subjects select an alternative that is dominated on both the cost and the time dimension. These choices also show a clear pattern that to a large extent can be explained by loss aversion.

The structure of the paper is the following. In Section 2 we introduce the model of reference-dependent preferences to analyze the trade offs between money and travel time. Section 3 describes the empirical application. We specify and estimate the empirical model, and we analyse the implications for the trade offs between money and travel time, emphasizing the role of loss aversion and asymmetries in the value functions. We further provide some supporting evidence for reference dependent preferences based on dominated choices. Section 4 contains some concluding remarks.

## 2 Methodology

### 2.1 Reference-dependent preferences

The stated preference choice experiment described below comprises choices between alternatives defined over two dimensions, travel cost and travel time. We denote, for any given alternative, deviations from a reference cost and time by ( $c, t$ ). Moreover, we assume an individual-specific 'reference-free’ value of travel time, denoted by $w$. We may
think of $w$ as the marginal rate of substitution at the reference between time and money for a reference-free utility function. We further capture reference-dependent gain-loss utility by a sum of value functions, as in Tversky and Kahneman ( 1991). For each dimension (cost, travel time), a value function $v(x)$ is defined, where $x$ is the deviation from the reference. The value function $v$ is monotonously increasing and satisfies $v(0)=0$. Moreover, it exhibits loss aversion (i.e. $v(x)<-v(-x)$ ) and diminishing sensitivity $\left(x v^{\prime \prime}(x) \leq 0\right)$. A cost or time increase leads to a utility loss so that, under the above assumptions, we can express referencedependent gain-loss utility by ${ }^{7}$

$$
\begin{equation*}
u((c, t) \mid 0)=v_{c}(-c)+v_{t}(-w t) . \tag{1}
\end{equation*}
$$

### 2.2. The choice framework

Consider binary choices between alternatives defined in terms of cost and time differences, relative to a reference. We assume that choices are made by maximising the reference-dependent gain-loss utility function (1). Four types of choice situations are depicted in Figure 1, where the axes pass through the reference situation. The quadrants of Figure 1 define four different measures of the trade-off between money and time.

First, suppose the individual has to choose between the reference and an alternative which is faster but more expensive than the reference. This is a 'willingness to pay' $W T P$-type of choice, presented as a choice between the origin and a point in the upper left quadrant of Figure 1. A second type of choice is of the WTA-type; it is the mirror image of the previous case, involving the reference and a slower but less expensive alternative (see lower right quadrant). Both these choices may be used to reveal compensating variations. Third, there is a choice between one alternative that is faster than the reference but with the same cost, and another alternative that is cheaper than the reference but with the same driving time. This is an equivalent gain ( $E G$ ) type choice (e.g., Bateman et al. 1997). Graphically, it is the choice between a point on the vertical axis and one on the horizontal axis, see the lower left quadrant. Finally, the fourth choice situation is again the mirror image of this (see the upper

[^4]right quadrant). It is an equivalent loss ( $E L$ ) type choice, involving the choice between either a time increase or a cost increase relative to the reference. The latter two choice types may be used to reveal equivalent variations.


Figure 1 The four quadrants

Define cost and time differences relative to the reference by $\left(c_{1}, t_{l}\right)$ and $\left(c_{2}, t_{2}\right)$ for alternatives 1 and 2 , respectively. We rearrange the alternatives freely such that the first alternative is the faster and more expensive alternative, i.e., $t_{1}<t_{2}$ and $c_{1}>c_{2}$. Using (1), indifference between the two alternatives occurs when:

$$
v_{c}\left(-c_{1}\right)+v_{t}\left(-w t_{1}\right)=v_{c}\left(-c_{2}\right)+v_{t}\left(-w t_{2}\right) .
$$

To define the four valuation measures $W T P, W T A, E G$ and $E L$ within this framework, first note that the type of choice has direct implications for the signs of $\left(c_{1}, t_{l}\right)$ and $\left(c_{2}, t_{2}\right)$, where times and costs are differences relative to the reference. Specifically, we can describe the four types of choices in the following way:

$$
\begin{align*}
& \text { WTP - type choice: } t_{1}<0<c_{1}, c_{2}=t_{2}=0 \\
& \text { WTA-type choice: } c_{1}=t_{1}=0, c_{2}<0<t_{2} \\
& \text { EG-type choice: } c_{2}<0, t_{1}<0, c_{1}=t_{2}=0  \tag{2}\\
& \text { EL-type choice: } c_{1}>0, t_{2}>0, c_{2}=t_{1}=0
\end{align*}
$$

Now consider a certain time change with absolute value $t=t_{2}-t_{1}$, where the signs of the $t_{i}$ depend on the type of choice, as indicated above. For each type of choice, we then implicitly define the corresponding valuation measure as a function of $t$ by considering the cost change that yields indifference. As an example, consider a $W T P$-type choice. In that case $c_{2}=t_{2}=0$ so that, using $v_{t}(0)=v_{c}(0)=0$, indifference is obtained for:

$$
v_{c}\left(-c_{1}\right)+v_{t}\left(-w t_{1}\right)=0 .
$$

Using the definition $t=t_{2}-t_{l}$ and noting that $t_{l}<0$ for a WTP-type of choice, we immediately obtain:

$$
\begin{equation*}
v_{c}(-W T P(t))+v_{t}(w t)=0 \tag{3a}
\end{equation*}
$$

In a similar way, we easily derive:

$$
\begin{align*}
& v_{c}(W T A(t))+v_{t}(-w t)=0 \\
& v_{c}(-E L(t))-v_{t}(-w t)=0  \tag{3b}\\
& v_{c}(E G(t))-v_{t}(w t)=0
\end{align*}
$$

These expressions give implicit equations in the four valuation measures that can be used to solve for $W T P, W T A, E G$ and $E L$. Using the properties of the value functions, it is then straightforward to derive the following inequalities (Bateman et al. 1997):

$$
\begin{equation*}
W T P(t)<\min [E G(t), E L(t)] \leq \max [E G(t), E L(t)]<W T A(t) \tag{4}
\end{equation*}
$$

### 2.3. Specification of the value functions

We specify a generic value function incorporating loss aversion, diminishing sensitivity and possible asymmetry between gains and losses in the degree of diminishing
sensitivity. We use a power function, which is a common formulation for empirical work. ${ }^{8}$ For a good $x$, the value function is specified by

$$
\begin{equation*}
v(x)=S(x) e^{-\eta S(x)}|x|^{1-\beta-\gamma S(x)} \tag{5}
\end{equation*}
$$

In this formulation, the notation $S(x)=x /|x|$ is the sign of $x$. Using (1), it immediately follows that, if $\eta=0, \gamma=0, \beta=0$ in both value functions, the marginal rate of substitution between time and money boils down to $w$.

An essential feature of this value function is that gains are under-weighted as much as losses are over-weighted. ${ }^{9}$ This is an essential assumption that will allow us to identify the reference-free value of time. Moreover, it serves as a substitute for the conditions necessary for using the argument of Munro and Sugden (2003). They show that, if value functions are smooth at the reference with a fixed common derivative, then the marginal rate of substitution at the reference of the reference-dependent utility is the same as that of the reference-free utility. Maximizing reference-dependent preferences in a series of trades, each time updating the reference, will under those conditions lead to the reference-point being equal to the reference-free optimum. The Munro-Sugden argument, however, hinges on smoothness of the value functions at zero. This is a very local property that is not susceptible to empirical verification in our framework. ${ }^{10}$ We therefore use the symmetry condition instead to provide the link from reference-dependent to reference-free utility.

Define the degree of loss aversion as $-v(-x) / v(x)=e^{2 \eta}|x|^{2 \gamma}$ for $\mathrm{x}>0$ (Tversky \& Kahneman 1991). Then to interpret (5) first note that, when $\beta=\gamma=0$, the model reduces to the case of constant loss aversion and non-diminishing sensitivity; the degree of loss aversion is captured by the parameter $\eta>0$. The parameter $\beta \geq 0$ introduces diminishing sensitivity,

[^5]whereas $\gamma$ allows diminishing sensitivity to be asymmetric for gains and losses and makes the degree of loss aversion variable. Second, for nonzero values of all parameters, it is clear that $\beta, \gamma$ and $\eta$ must satisfy certain restrictions if $v$ is to serve as a value function. We require the value function to be monotonically increasing, i.e., $v^{\prime}(x)>0$. This is easily shown to be equivalent to $\beta-1<\gamma<1-\beta$. Diminishing sensitivity requires $x v "(x) \leq 0$, which boils down to $\beta \leq \gamma \leq \beta$. Finally, when $\gamma$ is non-zero, we have loss aversion when $\exp (-\eta / \gamma)<|x|$, so that loss aversion does not hold generally for very small $x$.

Using value functions defined by (5) for both the cost and time dimension, it is easy to derive explicit expressions for the functions $W T P(t), W T A(t), E L(t)$ and $E G(t)$ that were discussed in section 2.2. Using (3a) and (5), solving for the willingness to pay yields:

$$
\begin{equation*}
W T P(t)=|w t|^{\frac{1-\beta_{t}-\gamma_{t}}{1-\beta_{c}+\gamma_{c}}} e^{-\frac{\eta_{c}+\eta_{t}}{1-\beta_{c}+\gamma_{c}}} . \tag{6}
\end{equation*}
$$

For the other quadrants we find the other measures, using (3b) and (5):

$$
\begin{align*}
& W T A(t)=|w t|^{\frac{1-\beta_{t}+\gamma_{t}}{1-\beta_{c}-\gamma_{c}}} e^{\frac{\eta_{c}+\eta_{t}}{1-\beta_{c}-\gamma_{c}}} \\
& E L(t)=|w t|^{\frac{1-\beta_{t}+\gamma_{t}}{1-\beta_{c}+\gamma_{c}} e^{\frac{-\eta_{c}+\eta_{t}}{1-\beta_{c}+\gamma_{c}}}}  \tag{7}\\
& E G(t)=|w t|^{\frac{1-\beta_{t}-\gamma_{t}}{1-\beta_{c}-\gamma_{c}} e^{\frac{\eta_{c}-\eta_{t}}{1-\beta_{c}-\gamma_{c}}} .}
\end{align*}
$$

These expressions have several important implications. First, whereas the four types of binary choices (one for each quadrant in Figure 1) result in four independent estimates of $W T P, W T A, E L$ and $E G$, equations (6) and (7) show that reference dependence imposes a particular relationship among the different measures. To see this most clearly, take the example where the $\gamma$ 's are zero; it then follows from (6)-(7) that the difference among the four measures is governed solely by the two loss aversion parameters, the $\eta$ 's. This implies a restriction on the parameters that can be used to test the model empirically, as will be explained in Section 3 below. Second, note that under some simplifying assumptions we obtain particularly simple relations among the four measures and the underlying referencefree value of time. For example, if the $\gamma$ 's are zero, (6) and (7) imply:

$$
\ln [W T P(t) W T A(t)]=\ln [E L(t) E G(t)]=2 \frac{1-\beta_{t}}{1-\beta_{c}} \ln [w t \mid]
$$

We find that the geometric average of the WTP and the WTA is equal to the geometric average of the EG and the EL. In the case when also the $\beta$ s s are zero (this case is considered by Tversky and Kahneman ( 1991, figure V), this in turn equals the reference-free underlying value of time.

Finally, note the implications of different degrees of diminishing sensitivity between the time and cost dimension, i.e., $\beta_{t} \neq \beta_{c}$. Even if the asymmetry parameters $\gamma$ are zero, the fact that the ratio $\frac{1-\beta_{t}}{1-\beta_{c}}$ is then different from 1 implies that there is a nonlinear relationship between the size of the time difference $t$ and the four valuation measures. In particular, when the ratio is greater than 1 (as it turns out to be in the empirical section), (6) and (7) show that the value of time per minute, as measured by the four valuation measures, increases with the size of the time difference, in spite of the fact that the model employs a constant reference-free value of time $w$. It is a common empirical finding that the marginal value of time increases with the size of the time difference (see, e.g., Bates \& Whelan 2001;Cantillo, Heydecker, \& de Dios Ortuzar 2006;Hultkrantz \& Mortazavi 2001). The analysis of this section shows that reference-dependence may provide an alternative explanation for this empirical regularity.

### 2.4. Econometric model specification

In this sub-section, we formulate the econometric model that will be used in the empirical section of the paper below. The empirical model can be considered a descendant of models pioneered by Beesley ( 1965) and Cameron and James ( 1987). To transform the theory of the previous section into an econometric specification, we proceed in several steps.

First, given the specification of reference-dependent gain-loss utility (1) and the definition of the generic value function in (5), the slow alternative 2 will be selected when

$$
\begin{align*}
& -S\left(c_{1}\right) e^{S\left(c_{1}\right) \eta_{c}}\left|c_{1}\right|^{1-\beta_{c}+\gamma_{c} S\left(c_{1}\right)}-S\left(t_{1}\right) e^{S\left(t_{1}\right) \eta_{t}}\left|w t_{1}\right|^{1-\beta_{t}+\gamma_{l} S\left(t_{1}\right)}  \tag{8}\\
& <-S\left(c_{2}\right) e^{S\left(c_{2}\right) \eta_{c}}\left|c_{2}\right|^{1-\beta_{c}+\gamma_{c} S\left(c_{2}\right)}-S\left(t_{2}\right) e^{S\left(t_{2}\right) \eta_{t}}\left|w t_{2}\right|^{1-\beta_{t}+\gamma_{l}\left(t_{2}\right)}
\end{align*}
$$

Now from (2) we know that in the binary choice situations considered, for each type of choice there is one cost variable and one time variable that equals zero. Defining $c=c_{1}+c_{2}, t=t_{1}+t_{2}$ and noting the fact that for each type of choice some terms are zero, we can simplify inequality (8). Using this information and some straightforward algebra then shows that for all four types of choice the slow alternative will be selected if:

$$
\begin{equation*}
w^{1-\beta_{t}+\gamma_{1} S(t)}<\frac{e^{S(c) \eta_{c}} \mid c c^{1-\beta_{c}+\gamma_{c} S(c)}}{e^{S(t) \eta_{l}}|t|^{1-\beta_{t}+\gamma_{l} S(t)}} . \tag{9}
\end{equation*}
$$

Taking logs, we get the condition:

$$
\begin{equation*}
\left[1-\beta_{t}+\gamma_{t} S(t)\right] \ln w<S(c) \eta_{c}-S(t) \eta_{t}+\left[1-\beta_{c}+\gamma_{c} S(c)\right] \ln |c|-\left[1-\beta_{t}+\gamma_{t} S(t)\right] \ln |t| \tag{10}
\end{equation*}
$$

The second step is to specify the reference-free value of time $w$. We use the following log-linear formulation:

$$
\begin{equation*}
\ln w=\delta_{0}+\delta z+\sigma u \tag{11}
\end{equation*}
$$

To ease on notation, we omit subscripts to denote that $\ln w$ is individual specific. In expression (11), $\delta_{0}$ is a constant, $\delta z$ captures the effect of observed heterogeneity while ou captures unobserved heterogeneity through a standard normal random variable $u$ and standard deviation $\sigma$. ${ }^{11}$

Third, we introduce random error terms $\mu \varepsilon_{i}$, where $\mu$ is the scale of the errors and the $\varepsilon_{\mathrm{i}}$ are iid. standard logistic error terms for a sequence of choices $i$. Substituting (11) in (10) and adding the error terms yields then the binary discrete choice model to be estimated; the dependent variable indicates that the slow alternative is chosen in choice situation $i$ when:

[^6]\[

$$
\begin{align*}
& {\left[1-\beta_{t}+\gamma_{t} S\left(t_{i}\right)\right]\left[\delta_{0}+\delta z+\sigma u\right]} \\
& <  \tag{12}\\
& S\left(c_{i}\right) \eta_{c}-S\left(t_{i}\right) \eta_{t}+\left[1-\beta_{c}+\gamma_{c} S\left(c_{i}\right)\right] \ln \left|c_{i}\right|-\left[1-\beta_{t}+\gamma_{t} S\left(t_{i}\right)\right] \ln \left|t_{i}\right|+\mu \varepsilon_{i}
\end{align*}
$$
\]

As a last step, we normalise this expression by dividing through by $1-\beta_{t}$. Then (12) can finally be written in the following compact form:

$$
\begin{align*}
& p_{0}+p_{1} z+p_{2} u+p_{3} S\left(t_{i}\right)+p_{4} S\left(t_{i}\right) z+p_{2} p_{5} S\left(t_{i}\right) u+\ln \left|t_{i}\right|+p_{5} S\left(t_{i}\right) \ln \left|t_{i}\right| \\
& <  \tag{13}\\
& p_{6} S\left(c_{i}\right)+p_{7} \ln \left|c_{i}\right|+p_{8} S\left(c_{i}\right) \ln \left|c_{i}\right|+p_{9} \varepsilon_{i}
\end{align*}
$$

where, under the assumptions of our model, the parameters are defined as follows:

$$
\begin{align*}
& p_{0}=\delta_{0}, p_{1}=\delta, p_{2}=\sigma \\
& p_{3}=\frac{\gamma_{t} \delta_{0}+\eta_{t}}{1-\beta_{t}}, p_{4}=\frac{\gamma_{t} \delta}{1-\beta_{t}}, p_{5}=\frac{\gamma_{t}}{1-\beta_{t}},  \tag{14}\\
& p_{6}=\frac{\eta_{c}}{1-\beta_{t}}, p_{7}=\frac{1-\beta_{c}}{1-\beta_{t}}, p_{8}=\frac{\gamma_{c}}{1-\beta_{t}}, p_{9}=\frac{\mu}{1-\beta_{t}}
\end{align*}
$$

We note first that the parameters $\delta_{0}, \delta, \sigma$ that describe the determinants of the reference-free value of time are in fact identified. Consequently, given reference-dependence, we can infer the reference-free value of time from the type of binary discrete choices studied here. Further observe that the parameters $\eta$ and $\gamma$ of the value functions are, given the normalisation that we employ, identified only relative to $\beta_{t}$ (defining the scale). Moreover, the $\beta_{c}, \beta_{t}$ are not separately identified. However, many of the economically interesting phenomena are identified. For example, we do identify $p_{7}$, the parameter capturing the ratio $\frac{1-\beta_{t}}{1-\beta_{c}}$; the latter governs the relationship between the size of the time difference $t$ and the valuation measures, see the discussion above. Information on loss aversion is also readily available. We have loss aversion when $p_{6}>0$ and $p_{3}-p_{5} p_{0}>0$. Moreover, the relative size of the $\gamma$ 's can be determined from the parameters $p_{5}$ and $p_{8}$. Finally, monotonicity of the value functions can
easily be checked empirically. Indeed, note that an estimate $p_{9}>0$ is equivalent to $1-\beta_{t}>0$. If in addition $\left|p_{5}\right|<1$ and $\left|p_{8} / p_{7}\right|<1$ we have monotonicity of the value functions.

## 3 Empirical application

### 3.1 Data

We employ data from a large-scale survey of car drivers (Fosgerau, Hjort, \& Vincent LykJensen 2006). Interviews were conducted over the internet or face-to-face in a computer assisted personal interview. All subjects in the experiment had to choose between two alternatives, described by travel time and travel cost. This is similar to choices that car drivers make routinely every day. So, in the transport literature, the validity of such data is generally thought to be high, even though choices are hypothetical. Moreover, having travel time as an attribute in the experiment is particularly useful since travel time meaningfully can be varied continuously both up and down.

All choices were designed relative to a recent actual trip subjects had made. ${ }^{12}$ We use observations with trip durations greater than 10 minutes, since for shorter durations it is hard to generate meaningful faster alternatives. We interpret the recent trip as the reference situation and generate choice situations by varying travel time and cost around the reference. Four types of choice situations were presented, as described in section 2.2. Each subject was presented with eight non-dominated choice situations. Our data contain 16,559 observations of such choices from 2,131 individuals. Subjects were furthermore presented with a dominated choice situation, where one alternative was both faster and cheaper than the other. The quadrant for this choice situation was random. The data contain 2,062 such observations. The data on dominated choices are analysed in section 3.3 below.

[^7]The eight choice situations were generated in the following way. First, eight choices were assigned to quadrants: two to each quadrant in random sequence. Second, two absolute travel time differences were drawn from a set, depending on the reference travel time, in such a way that respondents with short reference trips were only offered small time differences. Thus travel times vary symmetrically around the reference. Both travel time differences were applied to the two situations assigned to each of the four quadrants. Third, eight trade-off values of time were drawn at random from the interval [2:200] Danish Crowns ( $\mathrm{DKK}^{13}$ ) per hour, using stratification to ensure that all subjects were presented with both low and high values. The absolute cost difference was then found for each choice situation by multiplying the absolute time difference by the trade-off value of time. Fourth, the sign of the cost and time differences relative to the reference were determined from the quadrant. The differences were added to the reference to get the numbers that were presented to respondents on screen. Travel costs were rounded to the nearest 0.5 DKK. ${ }^{14}$

It should be noticed that alternatives differ only with respect to time and cost, so that issues such as heterogeneous preferences for various transport modes play no role. Some summary information regarding the data set is given in Tables 1 and 2 . Table 1 shows some descriptive statistics regarding trip characteristics and the time and cost differences presented in the experiment. In interpreting the mean trip duration of almost 50 minutes, it should be remembered (see above) that only observations with trip durations exceeding 10 minutes were selected ${ }^{15}$. Table 2 shows statistics regarding the socio-economic characteristics used in the models to control for observed heterogeneity. In the interview, subjects stated their personal gross annual income, grouped into intervals of 100,000 DKK up to 1 million DKK. We have computed the net annual income by applying national tax rates to interval midpoints. The progressive Danish tax system implies a difference in income elasticity with respect to gross

[^8]and net income of $26 \%$. Also note from Table 2 that our subjects tend to be richer and older than the national average.

Table 1. Summary statistics, trip characteristics

| Variable | Mean | Min | Max |
| :--- | :--- | :--- | :--- |
| Cost difference, DKK | 8.79 | 0.5 | 200 |
| Time difference, minutes | 9.27 | 3 | 60 |
| Reference cost, DKK | 58.4 | 1 | 850 |
| Reference time, minutes | 49.2 | 11 | 240 |
| Share of time due to congestion | 0.09 | 0 | 0.7 |

Table 2 Summary statistics, socio-economic characteristics

| Variable | Mean | Min | Max |
| :--- | :--- | :--- | :--- |
| Net annual income, DKK | 179,000 | 42,900 | 470,000 |
| Lowest income group, dummy | 0.07 | 0 | 1 |
| Missing income info, dummy | 0.07 | 0 | 1 |
| Female, dummy | 0.41 | 0 | 1 |
| Age | 50.4 | 16 | 89 |
| Greater Copenhagen area dummy | 0.19 | 0 | 1 |
| Employer pays, dummy | 0.06 | 0 | 1 |
| Student, dummy | 0.05 | 0 | 1 |
| Internet interview, dummy | 0.66 | 0 | 1 |
| Family type: couple w/ children, dummy | 0.33 | 0 | 1 |
| Family type: single w/ children, dummy | 0.04 | 0 | 1 |
| Family type: single no children, dummy | 0.15 | 0 | 1 |
| Family type: other, dummy | 0.01 | 0 | 1 |

### 3.2 Estimation results

In this section, we present the results of estimating a series of different models. First, we estimate a number of models directly based on the theory of reference-dependence, as developed in the previous section. In what follows, these will be denoted M1R, M2R, etc.; the ' R ' refers to the fact that these models impose the parameter constraints (and the resulting restrictions on the relations among $W T P, W T A, E G, E L$ ) implied by the theory of referencedependence. We not only estimate the general model (13), allowing for loss aversion and asymmetrically diminishing sensitivity, but also consider a series of simpler versions that result from setting particular parameters equal to zero; some of these models do not allow
asymmetries, or they impose the absence of diminishing sensitivity. Second, however, since we want to empirically test the proposed theory of reference-dependence, we also estimate for each 'restricted' model the corresponding 'unrestricted' version (which will be denoted M1, M2, etc., see below) that allows for a different constant term for each quadrant. This gives rise to four independent valuation measures $W T P, W T A, E G, E L$. Since each restricted model is nested within the corresponding unrestricted model, a standard likelihood ratio test can be used to test the restriction of reference-dependence.

More information on the models estimated is given in Table 3, which sketches the overall empirical strategy. It lists the different models estimated, indicating in each case the parameters to be estimated in the restricted version of the corresponding model. To save space, the unrestricted models with constants by quadrant are not included in the table. To clarify the difference between the restricted and unrestricted models, consider as an example the first restricted model M1R. It assumes the slow alternative is chosen if

$$
\ln w=\delta_{0}+\sigma u<S\left(c_{i}\right) \eta_{c}-S\left(t_{i}\right) \eta_{t}+\ln \left|c_{i}\right|-\ln \left|t_{i}\right|+\mu \varepsilon_{i}
$$

where the index $i$ refers to the quadrant of that particular choice. It assumes $\beta_{c}=\beta_{t}=\gamma_{c}=\gamma_{t}=0$, so that only potential loss aversion through the $\eta$ 's is captured. It further allows for unobserved heterogeneity in the reference-free value of time, but variables capturing observed heterogeneity are not included. The 'unrestricted' version M1 has the same structure, but instead of the loss aversion parameters that determine the relation among the valuation measures under reference-dependence, it estimates four separate constants, one for each quadrant.

The other models are then easily summarised. Models M2a and M2Ra include income variables to control for observed heterogeneity in the reference-free value of time, while models M2b and M2Rb use a much larger set of controls (see the descriptive statistics in Tables 1 and 2). For reasons discussed below, models M3 and M3R again drop the controls and introduce instead diminishing sensitivity through the $\beta$ parameters. Finally, M4 and M4R allow for asymmetry in curvature via the $\gamma$ s ${ }^{16}$

[^9]Table 3: Model plan

| Parameter | M1R | M2R | M3R | M4R |
| :--- | :--- | :--- | :--- | :--- |
| $p_{0}=\delta_{0}$ | $\delta_{0}$ | $\delta_{0}$ | $\delta_{0}$ | $\delta_{0}$ |
| $p_{1}=\delta$ |  | $\delta$ |  |  |
| $p_{2}=\sigma$ | $\sigma$ | $\sigma$ | $\sigma$ | $\sigma$ |
| $p_{3}=\frac{\gamma_{t} \delta_{0}+\eta_{t}}{1-\beta_{t}}$ | $\eta_{t}$ | $\eta_{t}$ | $\frac{\eta_{t}}{1-\beta_{t}}$ | $\frac{\gamma_{t} \delta_{0}+\eta_{t}}{1-\beta_{t}}$ |
| $p_{4}=\frac{\gamma_{t} \delta}{1-\beta_{t}}$ |  |  |  |  |
| $p_{5}=\frac{\gamma_{t}}{1-\beta_{t}}$ |  |  |  | $\frac{\gamma_{t}}{1-\beta_{t}}$ |
| $p_{6}=\frac{\eta_{c}}{1-\beta_{t}}$ | $\eta_{c}$ | $\eta_{c}$ | $\frac{\eta_{c}}{1-\beta_{t}}$ | $\frac{\eta_{c}}{1-\beta_{t}}$ |
| $p_{7}=\frac{1-\beta_{c}}{1-\beta_{t}}$ |  |  | $\frac{1-\beta_{c}}{1-\beta_{t}}$ | $\frac{1-\beta_{c}}{1-\beta_{t}}$ |
| $p_{8}=\frac{\gamma_{c}}{1-\beta_{t}}$ |  |  |  | $\frac{\gamma_{c}}{1-\beta_{t}}$ |
| $p_{9}=\frac{\mu}{1-\beta_{t}}$ | $\mu$ | $\mu$ | $\frac{\mu}{1-\beta_{t}}$ | $\frac{\mu}{1-\beta_{t}}$ |

All models are estimated in Biogeme (Bierlaire 2003;Bierlaire 2005). In Table 4 we present the estimation results for the different models. The table further reports information on the different valuation measures $W T P, W T A, E G$ and $E L$ implied by the estimates, and it provides the $W T P / W T A$ gap. Moreover, it gives the estimated median reference-free value of time for each model, as well as its mean, estimated at the sample mean.

The main findings are easily described. Consider model M1. A first observation is that the estimated unrestricted constants per quadrant (these are denoted by $p_{w t p}, p_{w t a}, p_{e l}, p_{e g}$ in Table 4, where the subscripts refer to the appropriate quadrant) are very significant and very different from one another. Importantly, they imply the relative sizes for the $W T P, W T A$, $E G$ and $E L$ that would be predicted by the theory of reference-dependence (Bateman et al. (1997); see also section 2 above). More in particular, the table indicates that we derived the following median values for the four measures from the estimated model, expressed in DKK
per hour ${ }^{17}: W T P=8.7, W T A=38.4, E G=14.6, E L=24.9$, such that $W T P<E G<E L<W T A$. The difference between willingness to pay and willingness to accept is large, and highly unlikely to be due to the experimental setup or the presence of income effects; it amounts to a factor of more than four. ${ }^{18}$

Model M1R imposes the restriction on the constants implied by the theory, as indicated in Table 3. The parameters $p_{3}$ and $p_{6}$ yield direct estimates of $\eta_{t}$ and $\eta_{c}$, respectively. Both are positive as expected and strongly significant, in this and all following models. This strongly indicates the presence of loss aversion. Since $\eta_{t}>\eta_{c}$, there is evidence that car drivers are more loss averse in the time dimension than in the cost dimension. Importantly, the likelihood indicates that the restriction implied by M1R, relative to M1, is easily accepted. This is not a light test, considering that the constants by quadrant in M1 are highly significant and very different.

[^10]Table 4 Estimation results (t-stats in parentheses)

| Model | M1 | M1R | M2a | M2Ra | M2b | M2Rb | M3 | M3R | M4 | M4R |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Loglike | $9097.5$ | $9098.0$ | $8998.0$ | $8998.5$ | $8840.1$ | $8840.6$ | $9014.2$ | $9014.7$ | $9001.9$ | $9002.4$ |
| $p_{0}$ |  | $\begin{array}{r} -1.17 \\ (-26.5) \\ \hline \end{array}$ |  | $\begin{array}{r} -2.54 \\ (-18.7) \\ \hline \end{array}$ |  | $\begin{gathered} -3.71 \\ (-6.5) \\ \hline \end{gathered}$ |  | $\begin{array}{r} -1.35 \\ (-46.0) \\ \hline \end{array}$ |  | $\begin{array}{r} -1.34 \\ (-45.9) \\ \hline \end{array}$ |
| $p_{2}$ | $\begin{array}{r} 1.66 \\ (31.2) \\ \hline \end{array}$ | $\begin{array}{r} 1.66 \\ (31.2) \\ \hline \end{array}$ | $\begin{array}{r} 1.56 \\ (30.8) \\ \hline \end{array}$ | $\begin{array}{r} 1.56 \\ (30.8) \end{array}$ | $\begin{array}{r} 1.40 \\ (29.9) \\ \hline \end{array}$ | $\begin{array}{r} 1.40 \\ (29.9) \\ \hline \end{array}$ | $\begin{array}{r} 1.01 \\ (25.1) \\ \hline \end{array}$ | $\begin{array}{r} 1.01 \\ (25.1) \end{array}$ | $\begin{array}{r} 1.00 \\ (25.2) \\ \hline \end{array}$ | $\begin{array}{r} 1.00 \\ (25.2) \\ \hline \end{array}$ |
| $p_{3}$ |  | $\begin{array}{r} 0.50 \\ (21.6) \\ \hline \end{array}$ |  | $\begin{array}{r} 0.50 \\ (21.6) \\ \hline \end{array}$ |  | $\begin{array}{r} 0.50 \\ (21.6) \\ \hline \end{array}$ |  | $\begin{array}{r} 0.31 \\ (19.4) \\ \hline \end{array}$ |  | $\begin{array}{r} \hline 0.24 \\ (7.0) \\ \hline \end{array}$ |
| $p_{5}$ |  |  |  |  |  |  |  |  | $\begin{array}{r} \hline 0.030 \\ (2.4) \end{array}$ | $\begin{array}{r} 0.035 \\ (2.4) \end{array}$ |
| $p_{6}$ |  | $\begin{array}{r} 0.24 \\ (11.1) \end{array}$ |  | $\begin{array}{r} 0.24 \\ (11.1) \end{array}$ |  | $\begin{array}{r} 0.24 \\ (11.2) \\ \hline \end{array}$ |  | $\begin{array}{r} 0.15 \\ (10.7) \\ \hline \end{array}$ |  | $\begin{array}{r} 0.09 \\ (4.7) \\ \hline \end{array}$ |
| $p_{7}$ |  |  |  |  |  |  | $\begin{array}{r} 0.70 \\ (45.5) \\ \hline \end{array}$ | $\begin{array}{r} 0.70 \\ (45.5) \\ \hline \end{array}$ | $\begin{array}{r} 0.70 \\ (45.6) \\ \hline \end{array}$ | $\begin{array}{r} 0.70 \\ (45.6) \\ \hline \end{array}$ |
| $p_{8}$ |  |  |  |  |  |  |  |  | $\begin{aligned} & \hline 0.04 \\ & (4.4) \\ & \hline \end{aligned}$ | $\begin{gathered} \hline 0.044 \\ (4.3) \end{gathered}$ |
| $p_{\text {eg }}$ | $\begin{array}{r} -1.41 \\ (-24.3) \\ \hline \end{array}$ |  | $\begin{array}{r} -2.79 \\ (-19.6) \\ \hline \end{array}$ |  | $\begin{array}{r} -3.95 \\ (-6.9) \\ \hline \end{array}$ |  | $\begin{array}{r} -1.50 \\ (-41.1) \\ \hline \end{array}$ |  | $\begin{array}{r} -1.48 \\ (-30.4) \\ \hline \end{array}$ |  |
| $p_{e l}$ | $\begin{array}{r} -0.88 \\ (-16.1) \\ \hline \end{array}$ |  | $\begin{array}{r} -2.26 \\ (-16.3) \\ \hline \end{array}$ |  | $\begin{array}{r} -3.42 \\ (-6.0) \\ \hline \end{array}$ |  | $\begin{array}{r} -1.17 \\ (-31.4) \\ \hline \end{array}$ |  | $\begin{array}{r} -1.18 \\ (-23.6) \\ \hline \end{array}$ |  |
| $p_{\text {wta }}$ | $\begin{gathered} -0.45 \\ (-8.3) \\ \hline \end{gathered}$ |  | $\begin{array}{r} -1.82 \\ (-13.5) \\ \hline \end{array}$ |  | $\begin{array}{r} -2.99 \\ (-5.2) \\ \hline \end{array}$ |  | $\begin{array}{r} -0.90 \\ (-22.5) \\ \hline \end{array}$ |  | $\begin{array}{r} -1.03 \\ (-20.5) \\ \hline \end{array}$ |  |
| $p_{\text {wtp }}$ | $\begin{array}{r} -1.93 \\ (-30.0) \\ \hline \end{array}$ |  | $\begin{array}{r} -3.30 \\ (-22.4) \\ \hline \end{array}$ |  | $\begin{aligned} & -4.47 \\ & (-7.8) \end{aligned}$ |  | $\begin{array}{r} -1.82 \\ (-46.4) \\ \hline \end{array}$ |  | $\begin{array}{r} -1.68 \\ (-32.5) \end{array}$ |  |
| $\mathrm{z}_{\text {inc }}$ |  |  | $\begin{array}{r} 1.37 \\ (11.9) \end{array}$ | $\begin{array}{r} 1.37 \\ (11.9) \\ \hline \end{array}$ | $\begin{array}{r} 0.59 \\ (4.9) \\ \hline \end{array}$ | $\begin{array}{r} 0.59 \\ (4.9) \\ \hline \end{array}$ |  |  |  |  |
| $\mathrm{z}_{\text {lowinc }}$ |  |  | $\begin{array}{r} 0.84 \\ (3.7) \\ \hline \end{array}$ | $\begin{array}{r} 0.84 \\ (3.7) \\ \hline \end{array}$ | $\begin{array}{r} 0.36 \\ (1.7) \\ \hline \end{array}$ | $\begin{array}{r} 0.36 \\ (1.7) \\ \hline \end{array}$ |  |  |  |  |
| $\mathrm{z}_{\text {missinc }}$ |  |  | $\begin{array}{r} 1.03 \\ (5.1) \\ \hline \end{array}$ | $\begin{aligned} & 1.03 \\ & (5.1) \end{aligned}$ | $\begin{array}{r} 0.65 \\ (3.5) \\ \hline \end{array}$ | $\begin{array}{r} 0.65 \\ (3.5) \\ \hline \end{array}$ |  |  |  |  |
| $\begin{array}{\|l} \hline \begin{array}{l} \text { P9 } \\ \text { (t-value wrt. 1) } \end{array} \\ \hline \end{array}$ | $\begin{array}{r} 0.98 \\ (0.8) \\ \hline \end{array}$ | $\begin{array}{r} 0.98 \\ (0.9) \\ \hline \end{array}$ | $\begin{array}{r} 0.98 \\ (0.9) \\ \hline \end{array}$ | $\begin{array}{r} 0.98 \\ (0.9) \\ \hline \end{array}$ | $\begin{array}{r} 0.98 \\ (0.9) \\ \hline \end{array}$ | $\begin{array}{r} 0.98 \\ (0.8) \\ \hline \end{array}$ | $\begin{array}{r} 1.58 \\ (10.6) \\ \hline \end{array}$ | $\begin{array}{r} 1.58 \\ (10.6) \\ \hline \end{array}$ | $\begin{array}{r} 1.58 \\ (10.7) \\ \hline \end{array}$ | $\begin{array}{r} 1.58 \\ (10.7) \\ \hline \end{array}$ |
| Median EG | 14.64 |  | 14.61 |  | 14.79 |  | 13.37 |  | 13.62 |  |
| Median EL | 24.86 |  | 24.80 |  | 25.08 |  | 18.62 |  | 18.51 |  |
| Median WTA | 38.36 |  | 38.32 |  | 38.73 |  | 24.36 |  | 21.48 |  |
| Median WTP | 8.74 |  | 8.72 |  | 8.80 |  | 9.72 |  | 11.13 |  |
| Median $w$ |  | 18.70 |  | 18.67 |  | 18.80 |  | 15.59 |  | 15.67 |
| Mean $w$ at sample means |  | 73.75 |  | 62.81 |  | 49.78 |  | 25.89 |  | 25.95 |
| $\begin{aligned} & \text { WTA/WTP } \\ & \text { gap } \end{aligned}$ | 4.39 | 4.37 | 4.39 | 4.38 | 4.40 | 4.38 | 2.51 | 2.50 | 1.93 | 1.92 |

The next models introduce observed heterogeneity. First, models M2a and M2Ra just incorporate income variables: log income is included together with dummies for low income and missing income information. As could be expected, accounting for this type of
heterogeneity reduces the estimated standard deviation of the unobserved heterogeneity captured in $p_{2}$. The income variables are generally very significant. Note that the parameter for log income equals 1.37 ; this can be interpreted as the estimated income elasticity of the reference-free value of time. The relative ranking of the four valuation measures is unaffected compared to model M1. Furthermore, the restricted model M2Ra is again easily accepted against the unrestricted alternative. Second, models M2b and M2Rb extend on the previous ones by including a more elaborate list of controls for observed heterogeneity. The parameter estimates for these are not shown (but are, of course, available on request). The estimated income elasticity is now lower, because some of the added independent variables correlate with income. The restriction implied by the theory of reference-dependence is again easily accepted. We moreover note that the inclusion of lots of observed heterogeneity reduces the estimated standard deviation of the unobserved heterogeneity even more.

We observe that the parameters $p_{3}$ and $p_{6}$, and also the relative size of the four constants for the quadrants, are hardly affected by the inclusion of both observed and unobserved heterogeneity. This could be expected, since individual heterogeneity has the same effect in all types of choice situations. Given this observation, and since observed heterogeneity is not our main concern in this paper, we drop the variables for observed heterogeneity from the remainder of the models below.

Models M3 and M3R allow for diminishing sensitivity. The parameter $p_{7}=\frac{1-\beta_{c}}{1-\beta_{t}}$ implicitly measures the ratio of diminishing sensitivities in the cost and time dimensions. The resulting improvement in likelihood relative to models M1 and M1R is large, given that just one extra parameter is included. Importantly, $p_{7}$ is greater than zero, as implied by the theory. Moreover, it is less than 1 , implying that $\beta_{\mathrm{c}}>\beta_{\mathrm{t}}$. This means that the value function for cost bends more than the value function for time. This is equivalent to all the valuation measures increasing with the size of the time difference, a common empirical finding (see, e.g., Bates \& Whelan 2001;Cantillo, Heydecker, \& de Dios Ortuzar 2006;Hultkrantz \& Mortazavi 2001). Again, the restriction from M3 to M3R is easily accepted. Finally, note that the standard deviation of the unobserved heterogeneity term now has decreased a lot, even though the variables for observed heterogeneity are omitted. This shows that the random coefficient also captured some of the nonlinearity now captured by $p_{7}$. The parameters $p_{3}$ and $p_{6}$ are also affected.

The last pair of models, M4 and M4R, introduces the parameters $p_{5}$ and $p_{8}$ to capture asymmetries in the curvature of the value functions. These parameters are significantly different from zero and positive as expected. We find that $p_{5}<p_{8}$, which implies that $\gamma_{t}<\gamma_{c}$. Introduction of the asymmetry parameters causes the loss aversion parameters to decrease. Moreover, we find that $p_{8}<p_{7}$, implying that $\gamma_{c}<1-\beta_{c}$, and $p_{5}<1$, implying that $\gamma_{t}<1-\beta_{t}$. As we have seen in section 2 , this is required for the value functions to be monotonous; our estimates thus confirm this requirement. Finally, $p_{6}<p_{3}-p_{0} p_{5}$, such that $\eta_{c}<\eta_{t}$. Thus there is still more loss aversion in the time dimension than in the cost dimension.

In summary, the estimation results are systematically consistent with referencedependence of the form predicted by the model. We found substantial evidence of loss aversion, and more so in the time than in the cost dimension. The parameter $p_{7}$ is less than one, indicating that $\beta_{c}>\beta_{t}$. Finally, the estimates regarding $\gamma_{t}$ and $\gamma_{c}$ conform to the theoretical requirements as well.

We now turn to the implications of the estimated models for the underlying reference-free value of time and the size of the WTA/WTP gap; these results are shown in the bottom rows of Table 4. Before describing the results, however, two important remarks are in order. First, while the estimates of medians must be regarded as quite robust, the same is not necessarily true for estimates of the mean, because the latter quite sensitive to the tail behaviour of the distribution. In this paper, we have assumed that the distribution of unobserved heterogeneity is lognormal. Although this is quite reasonable, recent estimation of more flexible models suggests that it is not very precise in the right tail (see, Fosgerau (2006) and Fosgerau \& Bierlaire (2007)). Hence, contrary to the estimated medians, the estimates of the means presented here should be interpreted with caution. Second, our empirical model implies that the difference between the mean and median reflects the scale of the unobserved heterogeneity in $u$, captured by the parameter $p_{2}$. Indeed, a property of the lognormal distribution is that the ratio of the mean and the median is a function of $p_{2}{ }^{19}$ The mean estimate is therefore highest in models that treat all heterogeneity as unobserved.

[^11]Models M1R, M2Ra and MR2b produce median estimates of reference-free time values of around 18-19 DKK/hour; the estimates are stable over the different models. For comparison, we note that the sample average net hourly wage is about $100 \mathrm{DKK} / \mathrm{hour} .{ }^{20}$ Note that the medians in the models M2Ra-M2Rb (that account for observed heterogeneity) are calculated at the sample means for the covariates. As shown above, the estimates of the mean are lower due to the introduction of observed heterogeneity: the mean for model M1R is estimated at 73.75 DKK compared to 49.78 DKK for the model with an extensive list of explanatory variables M2Rb. The WTA/WTP gap is stable around 4.4 across the models with linear value functions.

In models M3R and M4R, the median estimates are somewhat lower (15.6 DKK) compared to previous models, due to the introduction of nonlinearity into the value functions. Allowing for diminishing sensitivity of the value functions has the effect of reducing the estimated scale of unobserved heterogeneity quite a lot; hence, the difference between the median and the mean is also substantially smaller. It is worth noting that the fundamental issue here is that we treat $w$ as constant for each individual, so that all relations between choices and the time and cost differences between choice alternatives are captured into the value function. Further observe that the WTA/WTP gap is also reduced by the introduction of nonlinearity in the last two models.

To conclude this subsection, we briefly report on two additional modelling exercises that were executed. First, we performed a small series of split sample experiments, in which we split the sample according to sex, income above/below the median and age above/below the median. Model M4R was then estimated for each sub-sample. In all cases, the distribution of the value of time was found to differ significantly across splits. Splitting the sample according to sex did not produce a significant difference in the parameters representing the value functions. For income, the high income group appears to be more time loss averse. The largest differences are found when the sample is split according to age, where

[^12]the older group is significantly more loss averse for both time and cost. ${ }^{21}$ Interestingly, this finding agrees with those of Johnson, Gächter and Herrman (2006).

Second, as explained above, each subject was also presented with one dominated choice situation in which one alternative was both cheaper and faster than the other. Under standard rational preferences such dominated options should not be chosen. We have therefore also re-estimated all models over the sample of "rational" survey responses, i.e., retaining only those who gave the "correct" answer in the dominated choice situation. This left us with 14,303 observations from 1,835 individuals. Again, we do not report the full empirical results, but limit the discussion to the main insights. Importantly, in all cases we still accept the restriction from the model with constants by quadrant against the referencedependent model with two loss aversion parameters and a general constant. The median and the mean values of time are virtually unchanged. The most noticeable change is that the size of the gap is reduced. It falls by about 20 percent in the models with linear value functions and by somewhat less in the models with nonlinear value functions: about 15 percent in M3R with diminishing sensitivity and 10 percent in M4R with asymmetrically diminishing sensitivity. This suggests that people with a high degree of loss aversion have a higher probability of giving the "wrong" answer in the dominated choice situation. This agrees with the results in the next subsection where it is shown that the share of mistakes exhibits the pattern predicted by loss aversion.

### 3.3 Dominated choice situations

As argued above, under standard Hicksian reference-free preferences, selecting the dominated alternative is clearly irrational. This implies, therefore, that we would expect the share of "mistakes", i.e. choosing the dominated alternative, to be largest for small differences of $c$ and $t$. Moreover, we would not expect the share of mistakes to differ across the four types of choices. Under reference-dependence, we would similarly expect more mistakes for small differences of $c$ and $t$, but we would expect to find differences across quadrants. Observations

[^13]of such mistakes are rarely analysed; in our case, they do provide a useful outside check on the theory of reference-dependence.

The dominated choice situations are labelled as shown in Figure 2. Under the standard preference model, all subjects would be expected to choose the fast and cheap alternative (to the South-West in the figure). The only way the dominated alternative can be chosen is by mistake.


Figure 2: Labelling of dominated choice situations

As it turned out, $11.5 \%$ of subjects chose the dominated alternative. Table 5 summarises the data for the dominated choice situations. There are indeed large differences by quadrant. Independence is rejected in this table with overwhelming significance. Under reference-dependence, we would expect most mistakes in the $E G$-quadrant as the dominated alternative is equal to the reference in both the cost and the time dimension. Similarly, we would expect least mistakes in the $E L$-quadrant, since the dominating alternative is then equal to the reference. Both relationships are clearly evident from the table. For the WTP and WTAquadrants we expect the number of "wrong" choices to be in between, as both alternatives in these choice situations match the reference on one dimension. If the loss aversion parameter for time is greater than that for cost $\left(\eta_{t}>\eta_{c}\right)$, as found above (at least in the linear models where this inference can be made), we would expect to find more mistakes in the WTP-
quadrant than in the $W T A$-quadrant. These expectations are also matched by the data. So at a first glance, the predictions of reference-dependence are closely supported by the data, also for the dominated choice situations.

Table 5. Dominated choice situations

| No. choosing alternative | EL | EG | WTA | WTP | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Dominant | 444 | 365 | 547 | 469 | 1825 |
| Dominated | 20 | 75 | 51 | 91 | 237 |
| Share of mistakes | $4.3 \%$ | $17.0 \%$ | $8.5 \%$ | $16.3 \%$ | $11.5 \%$ |
| Reference-dependence | $-\eta_{t}-\eta_{c}$ | $+\eta_{t}+\eta_{c}$ | $-\eta_{t}+\eta_{c}$ | $+\eta_{t}-\eta_{c}$ |  |

As a further check, we estimate a series of binary logit models, letting the dependent variable be 1 if the dominated alternative is chosen and 0 otherwise. The estimation results are summarised in Table 6. The first model, denoted as D0, is specified just with a constant, such that the share of mistakes is predicted to be constant over quadrants. Model D1 specifies constants by quadrant to allow the share of mistakes to differ by quadrant; we find that the differences between quadrants are indeed strongly significant. Model D1R imposes the same restriction on the constants as in section 3.2. The loss aversion terms are positive as expected and time loss aversion is larger than cost loss aversion, as was also found for the non-dominated choice situations. The decrease in log-likelihood from model D1 to model D1R corresponds to a level of significance of 3.2 \%.

Models D2 and D2R are similar to models D1 and D1R, but now the differences in cost and time between alternatives are used as extra controls. These variables are jointly significant and negative, indicating that the share of mistakes decreases as the cost and time differences become larger. The restriction from model D2 to D2R is significant at the 4\% level. The loss aversion terms are unaffected. In conclusion, we find that the pattern of mistakes across the four quadrants largely matches the predictions from the referencedependence model.

Table 6. Model summary - dominated choices ( $t$-stats in parentheses)
$\left.\begin{array}{|l|l|l|l|l|l||}\hline \text { Model } & \text { D0 } & \text { D1 } & \text { D1R } & \text { D2 } & \text { D2R } \\ \hline \text { Log likelihood } & -735.5 & -706.2 & -708.5 & -702.5 & -704.6 \\ \hline \text { Constant } & -2.041 & & -2.132 & & -1.976 \\ & (-29.6) & & (-28.4) & & (-18.9) \\ \hline \text { Constant EG } & & -2.373 & & -2.239 & \\ & & (-16.2) & & (-13.8) & \\ \hline \text { Constant EL } & & -1.640 & & -1.475 & \\ & & -1.582 & & (-10.8) & \\ \hline \text { Constant WTA } & & (-12.5) & & -1.416 & (-9.5)\end{array}\right]$

## 4 Summary and concluding remarks

In this paper, we have specified a model of reference-dependent preferences to explain individuals' valuation of travel time. Using data from a large-scale choice experiment, where each choice concerned a simple trade-off between travel time and travel cost, we estimate four valuation measures: willingness to pay, willingness to accept, equivalent gain and equivalent loss. We confirm the large gap between willingness to pay and willingness to accept, observed in the literature in other contexts. The implications of the theory of reference-dependence are consistently accepted against more general alternatives in tests of considerable statistical power. The results suggest that loss aversion plays an important role in explaining responses to binary choice options. Finally, we analyse data on choice situations involving "mistakes", i.e., cases where subjects select an alternative that is dominated on both
the cost and the time dimension. These choices also show a clear pattern that to a large extent can be explained by loss aversion.

We further show that under our model it is possible to recover the underlying reference-free value of time. This is an important finding. Indeed, the large gap between the $W T P$ and the WTA has generated a debate on which value to use for policy evaluation, and on the usefulness of contingent valuation methods as such (e.g., Diamond \& Hausman 1994;Horowitz \& McConnell 2003). Our model implies that the trade-off of references-free preferences is the (geometric) average of the WTP and the WTA. This conclusion hinges crucially on the assumed specification of the value functions, whereby losses are overweighted relative to the reference-free marginal utility by the same factor as gains are underweighted. We are currently investigating how this assumption may be justified. Without it, all we can say is that the reference-free marginal rate of substitution lies somewhere between the $W T P$ and the $W T A$.

Our study differs from some other studies in the respect that the reference was clearly defined: Our experiment was based on a specific recent trip, identified to subjects as the specific trip hypothetical choices would concern. We are not able to say what will happen in situations where the reference is less clear. Maybe the degree of loss aversion diminishes, which would cause the four valuation measures to converge. It is a question if and how subjects form a reference and how reference-dependence can then be defined.

If one were to estimate models ignoring reference-dependence in situations where reference-dependence was present, then there would be bias. In our setup, the direction of the bias would depend on the distribution of choice situations over the four quadrants around the reference. Many WTP type choice situations would give a downward bias and so on. There would be bias even when choice situations were equally distributed over quadrants, since the resulting estimate would be some sort of average of $W T P, E G, E L$ and $W T A$ that would not in general be the same as the geometric average of $W T P$ and $W T A$ or (in our case equivalently) the geometric average of $E G$ and $E L$.

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[^1]:    ${ }^{3}$ For recent developments in cost-benefit analysis of transport projects see, among many others, Kidokoro ( 2006), Pilegaard \& Fosgerau (2007) and Venables (2007).

[^2]:    ${ }^{4}$ See, among many others, Cummings et al. ( 1986) and Andreoni (1995) in the context of public goods evaluation, Kahneman, Knetsch and Thaler ( 1990), Benartzi and Thaler (1995) and Bateman et al. ( 1997) in a market exchange environment.
    ${ }^{5}$ There is an ongoing discussion concerning the determination of the reference point. A recent reference on this issue is Köszegi and Rabin (2006). They argue that in some applications the reference is probably not the status

[^3]:    quo of the current situation, but rather recent expectations about the outcome. Given the setup of our survey, we believe the current trip is the most plausible candidate for the reference in our application, see below.
    ${ }^{6}$ As noted by a referee, strategic behaviour cannot be fully ruled out. Respondents may interpret the experiment as a precursor to a policy change (new investment, introduction of pricing, etc.) and skew their answers to their desired outcome.

[^4]:    ${ }^{7}$ Bateman et al. (2005) provide support for applying value functions to both money and time.

[^5]:    ${ }^{8}$ A recent survey of functional forms (Stott 2006) suggests the power function as the best empirical formulation for the value function.
    ${ }^{9}$ Formally, when $\beta=\gamma=0$ we have $v(x) / x=-x / v(-x)$, for $x \geq 0$.
    ${ }^{10}$ Another problem with using the Munro-Sugden argument in our setting is that it is not mathematically possible to formulate a value function that, at the same time, is smooth at the reference, exhibits diminishing sensitivity and loss aversion, and under-weighs gains and much as losses are over-weighted in an interval around zero. The reason is that equal over- and under-weighing in our setting means $v(x) v(-x)=-x^{2}$ in at least a small interval around zero; together with diminishing sensitivity this implies that $v$ is linear in this neighbourhood both in the positive and the negative domain. Then loss aversion and smoothness are inconsistent. Note that, if $v(x) v(-x)=-x^{2}$ only holds in the limit as x tends to zero, then no inconsistency occurs.

[^6]:    ${ }^{11}$ The choice to parametrise $w$ directly rather than parametrising marginal utilities, as it is common with discrete choice models, is supported by Fosgerau ( 2007) for very similar data. The use of the normal distribution for $u$ is supported by Fosgerau ( 2006), at least when the mean of $w$ is not the object of interest. We shall find below that the parameters of interest are not much affected by the representation of heterogeneity. For this reason, we did not try to relax the assumption regarding the distribution of $u$. (For more on relaxed distributional assumptions see Fosgerau \& Bierlaire 2007;Fosgerau \& Nielsen 2006;Honoré \& Lewbel 2002).

[^7]:    ${ }^{12}$ Subjects were asked to think about domestic home-based trips with duration more than five minutes during the last eight days. They were then asked to identify which transport modes they had used and the number and length of trips by each mode. A specific trip type was then selected by the interview software based on sampling quotas. The subject was asked to provide detailed information about the most recent trip of this type. Finally, subjects were asked to imagine the hypothetical choice situations as applying to this specific recent trip.

[^8]:    ${ }^{13} 1$ Euro $\approx 7.5$ DKK.
    ${ }^{14}$ In some cases, rounding caused the cost difference to be zero. These observations are omitted from the analysis.
    ${ }^{15}$ The argument for excluding travel times of less than 10 minutes has been given before. Of course, one can speculate whether this selection mechanism may not raise the possibility of some selectivity bias, in the sense that people with low values of time may be overrepresented. This is an interesting issue with potential implications for the measurement of the levels of time values. However, it does not affect the conclusions of the paper concerning reference-dependence or the difference between quadrants.

[^9]:    ${ }^{16}$ Note that the selection of models estimated implies that the parameter $p_{4}$ drops out.

[^10]:    ${ }^{17}$ These median values per hour have been computed as $W T P=60 * \exp \left(p_{w t p}\right)$ for the willingness-to-pay, and similarly for the other valuation measures. Note that log w has a symmetric distribution according to (11), such that the distribution of $w$ is skewed to the right and the mean is greater than the median.
    ${ }^{18}$ In general, these time valuations are on the low side of the range of recent estimates in the literature (see, e.g. Hensher (2001), Wardman (2001), Small et al. (2005)). Note that the recent study by Small et al ( 2005) combines revealed and stated preference data. They suggest that median values of time are substantially higher in real as opposed to hypothetical situations.

[^11]:    ${ }^{19}$ If $\mathrm{X} \sim N(\mu, \sigma)$, then $\mathrm{E}(\exp (\mathrm{X}))=\exp \left(\mu+1 / 2 \sigma^{2}\right)$. Note that $\sigma=p_{2}$

[^12]:    ${ }^{20}$ Small (2005) reviews a number of studies to find that the value of travel time for private journeys is often around 50 percent of the gross wage rate.

[^13]:    ${ }^{21}$ In principle these effects could be incorporated by interacting the loss aversion parameters with background variables. We have opted for the simpler model since our aim is to show that loss aversion explains the differences between quadrants.

