Cooperation through Imitation and Exclusion in Networks

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Abstract

We develop a simple model to study the coevolution of interaction structures and action choices in prisoners' dilemma games. Agents are boundedly rational and choose both actions and interaction partners via payoff-biased imitation. The dynamics of imitation and exclusion yields polymorphic outcomes under a wide range of parameters. Whenever agents hold some information beyond their interaction neighbors defectors and cooperators always coexist in disconnected components. Otherwise polymorphic networks can emerge with a center of cooperators and a periphery of defectors. Any stochastically stable state has at most two disconnected components. Simulations confirm our analytical results and show that the share of cooperators increases with the speed at which the network evolves, increases with the radius of interaction and decreases with the radius of information.

JEL Classification: C70, C73, D85.

Keywords: Game Theory, Cooperation, Imitation Learning, Network Formation.

Comments welcome !!!!!!!!

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1 Introduction

1.1 Motivation

In this paper we aim to study the implications of the freedom to choose one's interaction partners for the emergence of cooperation in social dilemma situations. The paradigmatic model to analyze those situations is the prisoners' dilemma. In this game there are two actions, cooperation and defection. Defection is a dominant strategy but cooperation yields the highest benefit to the community. We consider agents interacting in a 2×2 prisoners' dilemma game with their neighbors through an (endogenous) network and study the *coevolution* of interaction structure and behavior, i.e. try to explain how the dynamics of linking choices influences action choices and vice versa.

A novelty of our model is that we introduce acute bounded rationality into a model of endogenous network formation. More specifically we focus on imitation. Imitation is widely recognized to be an important form of learning in humans.[1] Existing models of imitation in networks focus exclusively on imitation of actions, assuming either a fixed interaction structure or a different learning rule for link revision.[2] As a distinct and natural feature, we assume that agents learn about *both* actions and links through (payoff-biased) imitation. More precisely imitation learning is modeled as follows.

- Agents choose the action with the highest average payoff in their information neighborhood.
- They search new interaction partners *locally* using information from the agents in their information neighborhood. Link creation occurs with focus on the payoff of the interaction neighbors of the node in question.
- Agents face a fixed capacity constraint and link destruction depends on the opportunity cost of maintaining a link.

An important aspect of such a model of local search is the amount of information that agents have. Therefore we parametrize the radius of interaction and information of the agents. This allows us to cover a wide range of applications. A large information radius (relative to the interaction radius) can reflect situations where relevant information travels easily through the network. Think for example about information about one's friend's friends or the gossip in a village about the interaction of geographical neighbors. Situations where relevant information is hard to obtain are reflected in a small information radius (relative to the interaction radius). An example might be the interactions of buyers and sellers in a supply chain. The smaller both the radius of interaction and information, the more important is of course the network for the outcome of the game and the learning process.

[1] For an experiment on imitation learning see Apesteguía, Huck and Öchssler (2007).
[2] The related literature is described in detail in Section 1.2.

Our main analytical result shows that polymorphic states evolve under reasonable assumptions on the payoff parameters. Exclusion of defectors from beneficial interactions with cooperators forces them to interact with other defectors thereby stabilizing polymorphic outcomes under these conditions. A high degree of clustering (that is endogenously produced) favors the emergence of such polymorphic states. Furthermore we show that whenever agents hold some information beyond their interaction radius defectors and cooperators never interact in stochastically stable states, i.e. they are found in disconnected components.[3] On the other hand if agents only interact with and hold information about their first-order neighbors graphs in stochastically stable states can display a center of cooperators and a periphery of defectors. We also find that polymorphic stochastically stable states will consist of at most two disconnected components and monomorphic stochastically stable states will be connected (i.e. consist of one component).

We then simulate the model to gain insight into the importance of different parameters of the model, as well as into the topology of stochastically stable graphs. Confirming our analytical results, we find that polymorphic states tend to emerge. The share of cooperators in such states increases with the relative speed at which the network evolves (relative to actions). It increases with the radius of interaction and decreases with the radius of information. Maybe somewhat counter-intuitively thus more anonymity helps cooperation.

Consistently with empirical findings on social networks, and as a natural consequence of our assumptions, graphs display high clustering coefficients and moderately short average distances. More precisely, we reproduce the trade off between these two characteristics present in small world models (Watts and Strogatz (1998)). As in their model, the larger the radius of local search (i.e. the larger the information and interaction radius of the agents) the smaller is the clustering coefficient and the shorter is the average distance. Note though that as we always assume the search radius to be small relative to the size of the network the clustering coefficient never vanishes and average distances are higher than in random networks.

The paper is organized as follows. In Section 1.2 we relate our paper to the existing literature. In Section 2 we describe in detail the model, the learning dynamics and the analytical tools used. In Section 3 we present our main analytical results. In Section 4 we illustrate some of them through simulations and derive additional results describing equilibrium action choices and network topologies. In Section 5 we discuss several extensions of the model. Section 6 concludes. The proofs are relegated to an appendix.

1.2 Literature

Eshel, Samuelson and Shaked (1998) have analyzed imitation of behavior when agents are located on a circle. They found that cooperation in the prisoners'

[3] This contrasts with static models (like Eshel, Samuelson and Shaked (1998) or Mengel (2007)), where full defection prevails whenever agents hold some information beyond their interaction radius.

dilemma can survive but that states where all agents cooperate are never stable.[4] The intuition is that - as agents can only imitate their interaction neighbors - defectors will end up interacting with defectors and cooperators with other cooperators. This reveals the social benefit of cooperation and prevents that cooperators imitate defection. Mengel (2007) has shown though that this result is not robust. Firstly it does not hold if agents are allowed to hold some information beyond their interaction neighbors, secondly it does not extend to general networks and thirdly it is sensitive to minor changes in the imitation rule.

In recent years the coevolution of network structure and action choice in games has received increasing attention. Goyal and Vega-Redondo (2005) as well as Jackson and Watts (2002) study the coevolution of linking and action choices in Coordination Games. Both rely on myopic best responses as learning dynamics. The difference is that in Goyal and Vega-Redondo (2005) linking choice is unilateral (directed graph) and in Jackson and Watts (2002) bilateral (undirected graph). Goyal and Vega-Redondo (2005) find that for high linking costs the efficient action emerges and for low costs the risk-dominant action. The results in Jackson and Watts (2002) are more ambiguous. Skyrms and Pemantle (2000) investigate the dynamics of imitation in a stag hunt game, relying on simulation techniques.

To our knowledge the coevolution of interaction structure and behavior in the prisoners' dilemma has not been studied analytically.[5] One reason is of course that if best response dynamics are used all outcomes will involve full defection, as defection is a dominant strategy in this game. A way to model a non-trivial situation is to study more bounded rational learning dynamics, like for example imitation. There are several simulation studies studying cooperation in endogenous networks. All these rely on relatively complicated and partly arbitrary assumptions though. Biely, Dragosits and Thurner (2005) for example assume that agents find new partners through recommendation and that only cooperators can form new links. Hanaki et al. (2005) assume that while agents imitate action decisions, linking decisions are made rationally through myopic cost-benefit comparisons. There seems a priori no reason for us to assume that agents display a different degree of rationality in their linking and action decisions. Zimmermann and San Miguel (2005) assume that only links between two defectors are cut. As an immediate consequence of this assumption cooperators have a higher degree in their model and thus a higher total payoff. For other simulation studies see Zimmermann, Eguíluz and San Miguel (2004), Abramson and Kuperman (2001) or Ebel and Bornhold (2002). Ule (2005) simulates an interesting model of repeated interaction in which agents are forward-looking to some degree.

Also related are models of local search like Jackson and Rogers (2007) or Vázquez (2003) as well as models of preferential attachment (Barabási and Al-

[4]Previously also Nowak, Bonhoeffer and May (1994) have investigated cooperation in local interaction models through simulations.

[5]There are a few works pertaining to the literature on complex networks where some analytical results are obtained, for example Zimmermann and San Miguel (2005).

bert (1999)), in which link imitation occurs without taking into account payoffs explicitly. In the latter class of models agents simply (unilaterally) link to the node with the highest degree. The coevolution of cooperation and network structure has been studied experimentally by for example Riedl and Ule (2002).

2 The Model

2.1 The Network

There are $i = 1, ...n$ agents playing a 2×2 prisoners' dilemma game through a network. The network is endogenous, i.e. players decide who to form links with. Denote $l = (l_{i1}, ...l_{in})$ the vector of linking decisions of player i, where $l_{ij} \in \{0, 1\}$. A link ij is formed whenever $l_{ij}l_{ji} = 1$, i.e. if and only if both players "wish" to have the link. The set of first-order neighbors (or links) of any agent i is denoted $N_i^1 = \{j \neq i | l_{ij}l_{ji} = 1\}$ with cardinality η_i. Let it be a convention that $l_{ii} = 0, \forall i = 1, ..n$. The set of all linking decisions $L = \{l_1, ...l_L\}$ and the set of players (nodes) $N = \{1, ...n\}$ jointly define the network $G = (N, L)$. Denote $\chi \subseteq G$ a connected component of the graph, i.e. a maximal subset of nodes s.th. $\forall i, j \in \chi$ there is a path joining them.[6] No agent can be an element of two different components. Consequently the components $\chi \subseteq G$ define a partition of the graph. Finally denote $\chi(i)$ the component that contains agent i and let $\rho \in \{[1, n] \cap \mathbb{N}\}$ be the number of components of a graph.

Interactions are not necessarily restricted to an agent's first-order neighbors. Denote N_i^Z the set of agents agent i interacts with or the "interaction neighborhood" of player i. Furthermore the set of agents i interacts with N_i^Z will in general not coincide with the set of agents i has information about. Denote the latter set - the information neighborhood of agent i - by N_i^I. When we say that i has information about j we mean that i knows j's average payoff, degree and the identity of the other players that j interacts with. Let it be a convention that N_i^Z does not contain the player i herself while N_i^I does - i.e. while players do not interact with themselves they have information about themselves. As an illustration consider agents on a circle with interaction radius $Z = 1$ and information radius $I = 2$.

$$....\overbrace{(i-2) - \underbrace{(i-1)}_{N_i^Z} - i - \underbrace{(i+1)}_{N_i^Z} - (i+2)}^{N_i^I} - (i+3)\,...$$

Of course both N_i^I and N_i^Z vary endogenously with changes in the linking decisions of the agents. Denote $n_i^I(t)$ ($n_i^Z(t)$) the cardinality of the set N_i^I (N_i^Z) at time t. As mentioned before the smaller Z and I the more important is the network for the outcome of the game and the learning process. As Z and I approach the diameter of the network (defined as the largest distance between any two nodes) we approach a global interaction setting. Finally note that the

[6] A path between i and j is a finite set of links connecting i and j.

relation "j is an element of N_i^I (N_i^Z)" is symmetric, i.e. $j \in N_i^I$ (N_i^Z) $\Leftrightarrow i \in N_j^I$ (N_j^Z).

2.2 The Game

Individuals play a one-shot symmetric 2×2 game with their interaction neighbors. The set of actions is given by $\{C, D\}$ for all players. For each pair of actions z_i,$z_j \in \{C, D\}$ the payoff $\pi_i(z_i, z_j)$ that player i earns when playing action z_i against an opponent who plays z_j is given by the following matrix.

	C	D
C	a	b
D	c	d

(1)

We are interested in the case $c > a > d > b > 0$, i.e. the case where matrix (1) represents a prisoners' dilemma.[7] Assume also that $a > \frac{b+c}{2}$, i.e. that cooperation (C) is efficient. The payoffs at time t for player i from playing action z_i when the graph is G are given by[8]

$$\Pi_i^t(z_i^t, z_j^t, G^t) = \sum_{j \in N_i^Z(t)} \pi_i(z_i^t, z_j^t). \tag{2}$$

When choosing an action through the imitation learning process specified below agents are interested in the *average (per interaction)* payoff an action yields (in their information neighborhood). This seems the appropriate measure as we assume that agents are myopic and thus choose actions not foreseeing possible changes in the network. Consequently they are interested in whether an action performs good in a given interaction irrespective of whether players choosing this action have many interaction partners or not. Average payoffs (per interaction) for player i at time t are given by

$$\overline{\Pi}_i^t(z_i^t, z_j^t, G^t) = \frac{\Pi_i^t(z_i^t, z_j^t, G^t)}{n_i^Z(t)}. \tag{3}$$

In practice there are a large variety of factors (such as time and resource constraints) that limit the "linking capacity" of agents. We summarize such restrictions through the following simple assumption.

Assumption 1: No agent can have more than $\overline{\eta} \in \{[2, n) \cap \mathbb{N}\}$ links.

We assume that $\overline{\eta} \geq 2$ to allow a connected graph to form. Assumption 1 can be rationalized through a strictly convex cost-function for maintaining links. In the existing literature mostly constant marginal costs for forming links have been

[7] The assumption that all payoffs are strictly positive is made to simplify the exposition of the model and the main results.

[8] In equation (2) agents get the same payoff from all their interaction partners. One could also imagine a situation where - as in the connections model from Jackson and Wolinsky (1996) - payoffs are discounted in proportion to the geodesic distance between the two interaction partners.

assumed with the consequence that equilibrium graphs were either complete or emtpy.[9] Our equilibrium networks will be more realistic than these, but of course still quite stylized. Before starting to describe the learning dynamics let us introduce some notation.

Sample Payoffs

Denote $\overline{\Pi}^t(N_i^I) = \left(n_i^I(t)\right)^{-1} \sum_{k \in N_i^I(t)} \overline{\Pi}_k^t(\cdot)$ the average per interaction payoff of all agents contained in N_i^I at time t. Analogously denote $\overline{\Pi}^t(N_j^Z \cap N_i^I)$ the average per interaction payoff of all agents in the set $\left\{N_j^Z(t) \cap N_i^I(t)\right\}$ at time t and $\overline{\Pi}^t(N_i^I(z))$ the average per interaction payoff of all agents in $N_i^I(t)$ that choose action z. Let it be a convention that $\overline{\Pi}^t(N_i^I(z)) = 0$ if $\text{card}\{j \in N_i^I(t) | z_j = z\} = 0$. Furthermore denote $\Pi_{\min}^t(N_i^1) = \min_{j \in N_i^{1t}} \pi(z_i, z_j)$.the minimum payoff that player i obtains from any of her first-order neighbors.

2.3 Learning Dynamics

At each point in time $t = 1, 2, 3....$ the state of the system is given by the vectors of actions and linking decisions of all agents $s(t) = ((z_i^t, l_i^t))_{i=1}^n$. Denote S the state space. Agents learn about optimal behavior through imitation. More precisely in each period t the following happens.

1. α agents are randomly selected to revise their action choice. Each agent i compares the average per interaction payoff in her information neighborhood of the two actions. If and only if $\overline{\Pi}^{t-1}(N_i^I(\neg z_i)) > \overline{\Pi}^{t-1}(N_i^I(z_i))$ she changes her action.[10] With small probability ε_z she reverses her choice.[11]

2. β links ij with $j \in N_i^{I+Z}(t-1)$ are randomly selected for revision. If the link ij does not exist ($ij \notin G^{t-1}$) i and j are given the possibility to add it. If $\eta_i(t-1) < \overline{\eta}$ agent i chooses $l_{ij} = 1$. If $\eta_i(t-1) = \overline{\eta}$ agent i compares the average payoff of the agents interacting with j that she knows about, $\overline{\Pi}^{t-1}(N_j^Z \cap N_i^I)$, to the payoff she derives from her "worst" link, $\Pi_{\min}^{t-1}(N_i^1)$. If and only if $\Pi_{\min}^{t-1}(N_i^1) < \overline{\Pi}^{t-1}(N_j^Z \cap N_i^I)$, she chooses $l_{ij} = 1$.

   Agent j goes through the symmetric process. If and only if $l_{ij} l_{ji} = 1$ the link ij is added. If $\eta_i(t-1) = \overline{\eta}$ agent i destroys the links with her "worst" neighbors. With small probably ε_l their decisions are reversed and a randomly chosen link is added or destroyed. Finally any node exceeding the linking constraint randomly severs one of her links.

[9] See Goyal and Vega-Redondo (2005) or Jackson and Watts (2002). Jackson and Watts (2002) also consider a capacity constraint in their model of coevolving network and action choices in a coordination game. Whereas in their model a player that has reached the constraint is simply assumed not to want to form links anymore, he can in our model by severing other links.

[10] The notation $\neg z_i$ is used to indicate the action *not* chosen by i.

[11] This is the "imitate the best average" rule often used in the literature (Eshel, Samuelson, Shaked (1998), Schlag (1998), Apesteguía, Huck and Öchssler (2006)).

3. The game (1) is played and agents receive the payoffs.

If they are given the opportunity to revise their action choice, agents choose the action that has obtained the highest average payoff among all agents within their information radius I. Revising their linking choices agents search for new partners within their search radius $I+Z$. Note that these are all the agents they know of, i.e. the agents they have information about (within radius I) as well as the interaction partners of these agents (within $I+Z$).[12] They then choose to link to a randomly chosen agent from this set whenever they are either not linking constrained or whenever they expect to obtain higher payoffs from this agent then from their currently "worst" neighbor.

Note that if $Z > 1$ the set N_i^{I+Z} can contain agents that i is already interacting with. Why would she want to form links with these agents at all ? The reason of course is that (if $Z > 1$) any such agent can give i access to other agents. The payoff that other agents linked to j obtain $(\overline{\Pi}^{t-1}(N_j^Z \cap N_i^I))$ is a proxy for the payoff that i can expect from being linked to j.Of course this is not the most sophisticated decision rule, as (depending) on the node in question agents might or might not have more and better information to evaluate whether a link is worthwhile. We chose to stick to the simple formulation here. In section 5.2 we will discuss how robust our link formation process is.

To finish this subsection we want to discuss how I and Z affect the two dimensions of the learning dynamics. Of course the larger I the more information agents have. If $I-Z$ is large the information about the payoffs of the two actions will be of a more "global" nature as $N_i^I(z)$ will reflect less the local topology i faces. Under this condition it is also likely though that the two sets N_j^Z and $\left\{N_j^Z \cap N_i^I\right\}$ coincide i.e. that the information agents have about potential new partners is more precise. If $I - Z$ is small (maybe even negative) information about action payoffs will strongly reflect the local topology but information about new partners will be less precise.

2.4 Techniques used in the Analysis

The learning process described in subsection 2.3 gives rise to a finite Markov chain, for which the standard techniques apply. Denote $P^0(s,s')$ the transition probability for a transition from state s to s' whenever $\varepsilon_z = \varepsilon_l = 0$ and $P^\varepsilon(s,s')$ the transition probability of the perturbed Markov process with strictly positive trembles $\overrightarrow{\varepsilon} = (\varepsilon_z, \varepsilon_l)$. We make the following assumption on noise.

Assumption 2: $\varepsilon_z = \xi\varepsilon_l$ for some constant $\xi > 0$.[13]

An absorbing set under P^0 is a minimal subset of states which, once entered is never left. An absorbing state is a singleton absorbing set, or in other words

Definition 1 State s is absorbing $\Leftrightarrow P^0(s,s) = 1$.

[12] Remember that having information about an agent means knowing her average payoffs, degree and the identity of her interaction partners.

[13] We assume thus (as e.g. Jackson (2002)) that ε_z and ε_l tend to zero at the same rate. This assumption is relaxed in Section 5.1.

As (given that $\overrightarrow{\varepsilon} > 0$) trembles make transitions between any two states possible, the perturbed Markov process is irreducible and hence ergodic, i.e. it has a unique stationary distribution denoted μ^{ε}. This distribution summarizes both the long-run behavior of the process and the time-average of the sample path independently of the initial conditions.[14] The limit invariant distribution $\mu^* = \lim_{\varepsilon \to 0} \mu^{\varepsilon}$ exists and its support $\{s \in S | \lim_{\varepsilon_z \to 0} \mu^{\varepsilon_z}(s) > 0\}$ is a union of some absorbing sets of the unperturbed process. The limit invariant distribution singles out a stable prediction of the unperturbed dynamics ($\varepsilon_z = 0$) in the sense that for any $\varepsilon > 0$ small enough the play approximates that described by μ^* in the long run. The states in the support of μ^* are called stochastically stable states.

Definition 2 State s is stochastically stable $\Leftrightarrow \mu^*(s) > 0$.

Denote ω the union of one or more absorbing sets and Ω the set of all absorbing sets. Define $X(\omega, \omega')$ the minimal number of mutations (simultaneous trembles) necessary to reach ω' from ω.[15] The stochastic potential $\psi(s)$ of a state $s \in \Omega$ is defined as the sum of minimal mutations necessary to induce a (possibly indirect) transition to s from any alternative state $s' \in \Omega$, i.e. $\psi(s) = \sum_{s' \in \Omega} X(s', s)$.

Result (Young 1993) State s^* is stochastically stable if it has minimal stochastic potential, i.e. if $s^* \in \arg\min_{s \in \Omega} \psi(s)$.

The intuition behind Young's result is simple. In the long run the process will spend most of the time in one of its absorbing states. The stochastic potential of any state s is a measure of how easy it is to jump from the basin of attraction of other absorbing states to the basin of attraction of state s by perturbing the process a little. Ellison (2000) has shown that the time needed to converge to a stochastically stable state s is bound by $O\left(\varepsilon^{-\max_{s' \in \Omega} X(s', s)}\right)$ where $\max_{s' \in \Omega} X(s', s)$ is the maximum over all states of the smallest number of mutations needed to reach state s. The resulting wait time can be quite long, which is a criticism often brought forward to this type of models. Note though that - as in our model both action imitation and the search for new partners occur on a purely local level - the speed of convergence is independent of the size of the population.

3 Analysis

We first characterize the set of absorbing states of the dynamic process. We then provide a characterization of the set of stochastically stable outcomes.

[14] See for example the classical textbook by Karlin and Taylor (1975).

[15] It is important to note that these transitions need not be direct (i.e. they can pass through another absorbing set).

3.1 Absorbing States

Before we start our characterization of absorbing states let us remind the reader of some definitions.

Definition (Eccentricity) The *eccentricity* of a node in a graph (or component) is the largest distance from it to any other node in the graph (component).

Definition (Periphery) The *periphery* of a graph (or component) is the set of nodes that have maximal eccentricity.

Definition (Center) The *center* of a graph (or component) is the set of nodes that have minimal eccentricity.

Our first proposition has three parts. The first part places restrictions on the topology of networks that can arise in an absorbing state. Due to our assumption on linking constraints these restrictions will be weaker than those obtained in previous works on the coevolution of behavior and interaction structure.[16] On the other hand we will observe richer and more interesting network topologies. The second and third part of the proposition characterize action choices.

Proposition 1 (Absorbing States)

(i) In any absorbing state $\eta_i \leq \overline{\eta}, \forall i \in G$. *If for some* $i : \eta_i < \overline{\eta} \Rightarrow \forall j \in \left\{N_i^{I+Z} \cap \backslash N_i^1\right\} : \eta_j = \overline{\eta}$.

(ii) All states where $z_i = z_j, \forall i, j$ *s.t.* $\chi(i) = \chi(j)$ *and (i) holds are absorbing.*

(iii) *There exists* $\widehat{Z}(I)$ *s.t.* $\forall Z \leq \widehat{Z}(I)$ *polymorphic components with defectors in the periphery and cooperators in the center can be part of an absorbing state whenever payoffs are contained in a non-empty set* $\Psi(I, Z) \subset \left\{(d, a) \in [b, c] \times \left[\max\left\{\frac{b+c}{2}, d\right\}, c\right]\right\}$.

Proof. Appendix. ■

Obviously states where some agents exceeding the linking constraint ($\eta_i > \overline{\eta}$) are not absorbing. Furthermore if an agent i is not linking constrained either all her potential partners must be so or her search set (N_i^{I+Z}) must be empty. This is essentially what condition (i) is saying. If these conditions hold it is also quite obvious that states where all agents choose the same action are absorbing, as well as states where agents that choose different actions are in different components of the graph.

Part (iii) of Proposition 1 is the most interesting one. It shows that "truly" polymorphic absorbing states exist, in which cooperators and defectors are in the same component and interact with each other. In all such components the center always consists of cooperators, while defectors are found at the periphery.

[16] The topology most often observed in this literature is the complete graph. See Goyal and Vega-Redondo (2005) or Jackson and Watts (2002).

(But not all such components are part of an absorbing state). Loosely speaking one could say that the defectors act as "parasites" on a largely cooperative component. The conditions on the payoff parameters ensure that no agent is willing to imitate the other action. If defection is "too profitable" cooperators will want to imitate the defectors. Naturally there must also exist an upper bound on the interaction radius Z for which such states can be absorbing. If Z is "too" large relative to I peripheral defectors will interact with "too many" cooperators, increasing their average payoff (and making defection an attractive action to imitate). A special case is given whenever $Z = 1$ or $I \leq 2$. In these cases (as we show in the appendix) neighboring defectors must form a clique, i.e. they must all be linked to each other). Figure 1 illustrates such a polymorphic component (the darker red nodes are defectors).

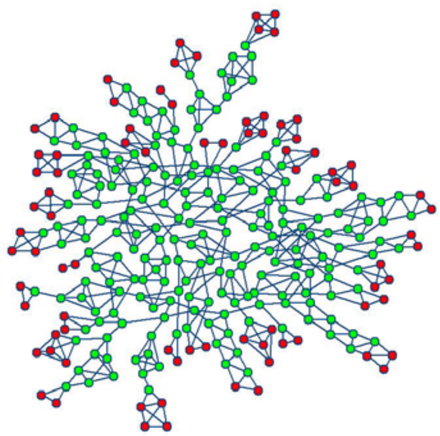

Fig. 1: Polymorphic Absorbing State

Why do polymorphic components need to have this particular structure? This is basically a consequence of local search. First note that any cooperator i linked to a defector k is always willing to substitute this link for a link with one of k's interaction neighbors (irrespective of the action that agent is taking).[17] If such a neighbor $j \in N_k^1$ is herself defecting she will want to link with i, if (except for i) she observes only defectors. In this case $\overline{\Pi}^t(N_i^Z \cap N_j^I) > \Pi_{\min}(N_j^1)$. The link ji will be established and the links ik and jk will be severed. Repeating this argument it can be shown that any two cooperators connected through a path of defectors will at some point find each other and form a link. But then defectors must lie in the periphery of the component or must be directly linked to another defector in the periphery. Note also that because of the local search process, where agents meet each other explicitly through common neighbors, all graphs will display a high degree of clustering.

[17] Note that the payoff she obtains from the defector $\Pi_{\min}(N_i^1) = b < \overline{\Pi}^t(N_j^Z \cap N_i^I)$ no matter what action j is taking, as j is linked to at least one defector i knows about (namely her own first-order neighbor).

3.2 Stochastically Stable States

Proposition 1 delimits the set of states that can potentially be stochastically stable, since (as explained in subsection 2.4) every such state must be absorbing for the unperturbed dynamics. In the following we will denote ω_ρ^z the set of absorbing states where all agents play action z and where the graph consists of ρ disconnected components. Denote $\cup_{\rho\in\{1,..n\}}\omega_\rho^z = \omega^z$. Analogously ω^{CD} is the set of polymorphic absorbing states. Of course we are ultimately interested in the set of stochastically stable states. Our main result is Proposition 2.

Proposition 2 (Stochastically Stable States) *All stochastically stable states are contained in $\omega_1^D \cup \omega_\rho^{CD}$, where $\rho \leq 2$. There exists a threshold level $a^*(Z, I, \overline{\eta}) \in (d, c)$ s.t. whenever $a \geq a^*(\cdot)$ all stochastically stable states are in $\omega_\rho^{CD}, \rho \leq 2$.*

Proof. Appendix. ■

Stochastically stable states are either polymorphic consisting of at most two disconnected components or characterized by full defection and connected. A sufficient condition for polymorphic states to emerge is that the payoff for joint cooperation be high enough. How high that depends on the number of links $\overline{\eta}$ each node can maintain and on the information (I) and interaction radius (Z).

What is the intuition for this result ? The tension in the prisoners' dilemma arises from the fact that while defection is a dominant strategy, cooperation provides the highest benefit to a community (is efficient). This is all the more so the higher the payoff parameter $a \in (d, c)$. Cooperation then will emerge as a stable outcome of the imitation learning process if cooperators interact with increased probability among themselves. This reveals the social benefit of cooperation and induces other agents to imitate cooperators. The most extreme situation is a state where cooperators and defectors coexist in two different components of the graph. Two forces in our model facilitate that the process arrives at such a situation. Firstly as action imitation occurs among one's information neighbors only, defection will spread locally. Secondly as new links are searched locally (at a radius of $I + Z$), cooperators can avoid the interaction with defectors in their interaction neighborhood by cutting these links and linking up with other cooperators. Of course if the defector payoffs are "too high" cooperators will easily tend to imitate defectors and cooperative components can easily be destabilized. But what does "high" mean exactly ? This depends of course on the relative size of the interaction and information radius (Z, I) as well as on the number of links $\overline{\eta}$ each node can maintain.

The relative size of the information radius I (relative to Z) has a double effect on the dynamic process. A smaller information radius I (relative to Z) forces defection to spread more "locally" and thus helps cooperation by forcing defectors to interact among each other. On the other hand a higher information radius I (relative to Z) improves the information agents have about potential partners inside their search radius $(I + Z)$ making it more easy for them to exclude defectors from beneficial interactions with cooperators. The density of the network (i.e. the number of nodes each agent can maintain $\overline{\eta}$) affects the size of the agent's sample (given Z and I) and consequently tends to exacerbate the effects described before.[18]

Proposition 2 is proved through a series of Lemmata. We will now state these Lemmata in turn to get a deeper intuition for our main result. The first Lemma relates to the topology of graphs at any stochastically stable state.

Lemma 1 (Topology) *If $\overline{\eta} > 2$ all polymorphic stochastically stable states will consist of at most two disconnected components and all monomorphic stochastically stable states will be connected.*

Proof. Appendix. ■

There is a tendency in the process that leads to large components in stochastically stable states. Note that one linking tremble suffices to connect any two disconnected components in which agents choose the same actions. On the other hand more than one tremble is generally needed for the reverse transition. It should be quite obvious that from some connected components strictly more than one tremble is needed to separate them. In addition any connected (monomorphic) component can be obtained from any other through a sequence of "one-trembles". It is a standard result, that if a state s is reached from another state s' via one tremble then s cannot have higher stochastic potential than s'. It then is a small step to show that - as some connected components are very unlikely to "break apart" (if $\overline{\eta} > 2$) - all stochastically stable states must have graphs with few components.[19]

What happens if $\overline{\eta} = 2$? In this case all connected graphs are circles or lines. Furthermore note that any rewiring of the graph will quickly lead to the creation of triangles, because of the local nature of the search process. Consequently the case where $\overline{\eta} = 2$ is not very interesting and we will focus in the following on the case $\overline{\eta} > 2$.

Assumption 3: $\overline{\eta} > 2$

Now we turn our attention to action choices. The first outcome is negative showing that fully cooperative states are never stochastically stable.

Lemma 2 (Instability of Full Cooperation) *States $s \in \omega^C$, where all agents cooperate, are not stochastically stable.*

Proof. Appendix. ■

The intuition for Lemma 2 is relatively simple. Starting from a cooperative state $s \in \omega^C$ assume one player trembles and switches to action D. This player will have the highest possible payoff and will be imitated by some other agents. The unperturbed process converges to either a polymorphic absorbing state or a state characterized by full defection. Fully cooperative states are thus easy to destabilize. On the other hand to reach a state of full cooperation from a polymorphic state or a state of full defection always at least two

[18] The effect of these parameters will be illustrated further in our simulations in Section 4.

[19] Note that isolated agents can form new links only through mistakes. Hence if there are many such agents (initially or during a transition) convergence to a stochastically stable state might be very slow.

trembles are needed. (One to induce the transition and one to induce the last defector remaining to adopt the cooperative action). While fully cooperative states are easy to destabilize, the next Lemma shows that this is not the case for polymorphic states.

Lemma 3 (Polymorphic States - I) *$\forall s \in \omega_1^D, \exists \widehat{a}(s) \in (d,c)$ s.t. whenever $a > \widehat{a}(s) : \exists s' \in \omega_\rho^{CD}, \rho \leq 2$ with $X(s,s') < X(s',s)$.*

Proof. Appendix. ■

Lemma 3 shows that (under some conditions on the payoff parameters) for any state s characterized by full defection there exists a polymorphic state s' such that s' is more easily reached from s than vice versa. The intuition is as follows. Starting from a state of full defection $s \in \omega_1^D$ simultaneous trembles of a small number of neighboring nodes can infect part of a component with cooperation and induce a transition to $s' \in \omega_2^{CD}$, as all cooperators have incentives to sever their links with defectors and form links among each other. The reverse transition now is more difficult to achieve, because the linking dynamics makes it difficult for defectors to find new partners. In particular there have to be either a large number of linking trembles for such a transition to occur or else a large enough number of action trembles s.t. cooperators might have incentives to form links with defectors. Denoting $a^*(\cdot) = \max_{s \in \omega^D} \widehat{a}(s)$ these Lemmata suffice to show Proposition 2. Note that the "reverse" to Lemma 3 is not true. In particular $\forall s' \in \omega_2^{CD}$ there exists a value $\widehat{a}'(s) \in (d,c)$ s.t. whenever $a > \widehat{a}'(s)$ one cannot find a state $s \in \omega^D$ s.t. $X(s',s) < X(s,s')$. At least two trembles (possibly many more) are needed for the transition $s' \to s$ (one action and one linking tremble). But for high enough a the reverse transition can always also be achieved after two trembles of neighboring agents and subsequent rewiring of the graph. If $s' \in \omega_1^{CD}$ and $I + Z = 2$ then for high enough a there is no way to induce a transition in which the defective action is imitated.

On the other hand it is shown in Lemma 4 below that states $s' \in \omega_1^{CD}$ are only stochastically stable if $I + Z = 2$. This Lemma is not necessary to prove Proposition 2. We mention it here, because we think it is of independent interest.

Lemma 4 (Polymorphic States - II) *If a polymorphic state $s \in \omega_1^{CD}$ is stochastically stable, then there also exists a stochastically stable state $s' \in \omega_2^{CD}$. If $I + Z > 2$ states in $s \in \omega_1^{CD}$ are not stochastically stable.*

Proof. Appendix. ■

Polymorphic states where cooperators and defectors are in disconnected components are "at least as stable" as states where they are in the same component. The intuition is as follows. Starting from any state $s \in \omega_1^{CD}$ one linking tremble can cut off a subcomponent of defectors. The stochastic potential of the resulting absorbing state is not higher than that of s. Cutting off subcomponents of defectors in this way and subsequently linking these components together leads to a state $s' \in \omega_2^{CD}$. This state (reached via a sequence of single trembles) cannot have higher stochastic potential than s. If $I + Z > 2$ this conclusion together with Lemma 1 imply that cooperators and defectors cannot be linked in a stochastically stable state. Note though that Lemma 4 does not say anything about the probability the limiting distribution places on the polymorphic states in the case where $I + Z = 2$. In fact - as we illustrate in the next section - we almost always observe polymorphic states $s \in \omega_1^{CD}$.

We have seen that while fully cooperative states will not be observed polymorphic states can often occur. The condition needed is that the payoff for joint cooperation is high enough, where the last qualification depends on many parameters of the model. The aim of the next section is thus twofold. Relying on simulation techniques we illustrate on the one hand how likely outcomes of the learning process look like, i.e. what the topology of networks and the distribution of actions will be. On the other hand we develop a better intuition of how our different model parameters influence these outcomes.

4 Simulation Results

In this section we illustrate and complement the analytical results through simulations. We explore essentially two aspects. First (under payoff parameters were polymorphic structures are "likely" to emerge) we show the effect of (β/α), I and Z on the fraction of cooperators denoted by φ_c. We address this question separately for $I + Z > 2$ and $I + Z = 2$. The difference between both cases is that when $I + Z > 2$, stochastically stable polymorphic states are always composed of two separate components. If $I + Z = 2$, there can be stochastically stable states with polymorphic components, like those illustrated in Figure 1. Second, we measure the effect of the search radius $(I + Z)$ on the topology of the graph, in particular with respect to average clustering and average distance within components.

In all simulations there are $n = 500$ nodes, the initial network is random with $\eta_i \leq \overline{\eta}$, $\overline{\eta} = 4$, and the initial number of cooperators is $0.5 * n$ (randomly placed on the graph). Payoff parameters are chosen such that for any $I, Z, (\beta/\alpha)$ polymorphic structures are "very likely" to emerge $(c = 1, a = 0.9, d = 0.01, b = 0)$. We choose $\alpha = 1$ and $\beta \in \{[1, 10] \cap \mathbb{N}\}$. The combinations of (I, Z) analyzed are $\{(1,1), (1,2), (1,3), (2,1), (3,1)\}$. For each case, we perform 100 realizations of the dynamic process.

Result 1 *If $I + Z = 2$, all realizations converge to a graph where the largest component consists of a core of cooperators with cliques of defectors lying on the periphery.*[20] *The parameter β has almost no effect on the fraction of cooperators (see the table below).*[21]

[20] In Figure 1 we showed a typical example.

[21] Intervals are asymptotic, with $\varphi_c \in \left[\widehat{\varphi}_c - 1.96\sqrt{\frac{\widehat{\varphi}_c(1-\widehat{\varphi}_c)}{100}}, \widehat{\rho}_c + 1.96\sqrt{\frac{\widehat{\varphi}_{cc}(1-\widehat{\varphi}_c)}{100}}\right]$.

β	Interval for φ_c (95%)
1	$[0.42, 0.54]$
5	$[0.41, 0.53]$
10	$[0.43, 0.55]$

The intuition for this result is as follows. If $I + Z = 2$ imitation of the defective action will necessarily lead to defectors interacting with each other reducing their average payoff. The action imitation dynamics itself is able to limit the spread of defection. Irrespective of the value of β defection never be able to infect more than a small group of agents. The linking dynamics then "locates" these defectors at the periphery of the graph, but naturally exclusion (β) is not necessary in maintaining higher levels of cooperation.

Result 2 *If $I + Z > 2$ the fraction of cooperators increases with β (the effect being more important if $I > Z$). If β is not too small φ_c increases with Z and decreases with I.*

To illustrate this result, we show in the next table the intervals for φ_c and in Figure 2 the observed distribution of φ_c for each sample. Panels (a) - (d) show the effect of β, while (e) and (f), the effect of Z and I, respectively.

	Interval for φ_c (95%)			
β	$I = 1; Z = 2$	$I = 1; Z = 3$	$I = 2; Z = 1$	$I = 3; Z = 1$
1	$[0.39, 0.59]$	$[0.32, 0.52]$	$[0.02, 0.13]$	$[0.04, 0.16]$
5	$[0.43, 0.63]$	$[0.48, 0.68]$	$[0.15, 0.32]$	$[0.11, 0.27]$
10	$[0.43, 0.62]$	$[0.50, 0.70]$	$[0.23, 0.42]$	$[0.19, 0.37]$

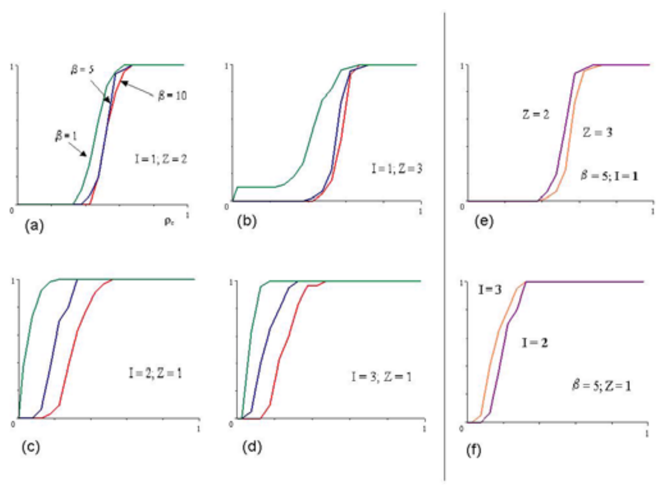

Fig.2: Fraction of Cooperators.

What is the intuition for this result? If $I + Z > 2$ higher values of β increase the fraction of cooperators. Since action imitation in these cases allows for the infection of “many” agents, exclusion (β) is very effective in raising the number of cooperators. Consider first the cases $I = 1$ and $Z > I$ (panels (a), (b), (e)). Cooperation has good chances, as the small information radius forces defectors to interact with each other as a consequence of the action imitation process.[22] On the other hand though (as Z (and thus $Z + I$) is “large” relative to I) the quality of information about potential new links is relatively bad and the linking dynamics leads to more “erroneous” new links reducing the effectiveness of the exclusion mechanism. This is why the effect of β is less important in the case $Z > I$ compared to the case where $I > Z$. This becomes more visible for higher values of β where exclusion plays a more important role. (Note that the “marginal” effect of β is decreasing after some value, compare $\beta = 5$ and $\beta = 10$). Now consider the case where $Z = 1$ and $I > Z$ (panels (c), (d), (f)). Clearly, being informed is not *per se* good for cooperation. Indeed, since agents imitate average behavior in this radius, the higher is I the more probable is that a cooperator imitates defection. On the other hand if the exclusion mechanism works (high β), the linking dynamics is more accurate due to the higher quality of information and less ”erroneous” choices are made. Inspecting overall cooperation rates, it can be seen clearly that the negative effect of I on the action imitation process dominates the positive effect of I on cooperation through the linking dynamics. The latter effect though explains that β has a higher “marginal” effect in the cases where $I > Z$ (compared with $I < Z$). Next we want to show some results on topology.

Result 3 *Graphs obtained display an average clustering coefficient and average distances that are both decreasing with* $I + Z$.[23]

$I+Z$	$\overline{c(i)}$	$\overline{d(i)}$
2	0.531	7.6
3	0.237	5.8
4	0.088	4.1

Result 3 confirms our intuition about some of the topological features of the components created by the dynamics. Of course, given the homogenous capacity constraint, the degree distribution is approximately degenerate.[24] Average clustering ($\overline{c(i)}$) and average distance ($\overline{d(i)}$) are both decreasing with the search radius $I + Z$. The search radius represents the extent of the locality in linking dynamics. When $I + Z$ is low, the probability that two first neighbors of any agent i are connected themselves is very high, but since links are concentrated within a small radius, the average distance between two nodes is large. When $I + Z$ is high, since each agent has more possible partners, the probability of

[22] Panel (b) shows that sometimes fully defective states have emerged in the case $\beta = 1$ and $Z = 3$. This might be due to the random initial configuration or it might be the case that for this range of parameters both full defection and polymorphic outcomes are stochastically stable with the limiting distribution placing a far higher weight on polymorphic outcomes.

[23] We measure these characteristics on the larger component.

[24] See Subsection 5.2. for a brief discussion related to this assumption.

choosing a second neighbor decreases (and so does the average clustering). But on the other hand links with nodes that are relative far away are shortcuts that reduce average distances. Note that these features are independent of β and on the particular combination of I and Z.

5 Extensions and Discussion of Assumptions

5.1 Heterogenous Noise

In this subsection we will relax the assumption of homogenous noise (A2) and consider two alternative assumptions.

A2′ : $\varepsilon_z = \xi \left(\varepsilon_l^{\xi'}\right)$ for some constants $\xi > 0, \xi' < 1$.

A2″ : $\varepsilon_z = \xi \left(\varepsilon_l^{\xi''}\right)$ for some constants $\xi > 0, \xi'' > 1$.

In particular Assumption 2′ seems to us very worthwhile investigating, as it is a case that is intuitively relevant in many applications. Note also that whereas an action tremble always is equivalent to one player making a mistake, a linking tremble will often require two players to *simultaneously* make a mistake. So even if each individual player is equally likely to make either mistake, a linking tremble is still (as noise tends to zero) infinitely less likely than an action tremble.[25] Our results show that the conclusions from section 3 continue to hold if and only if the probabilities of linking and action trembles are not too different.

Proposition 3 (Rigid Links) *Under A2′ there exists a value $\underline{\xi} \in (0,1)$ s.t. whenever $\xi' < \underline{\xi}$ all stochastically stable states are contained in ω_2^{CD}.*

Proof. Appendix. ■

If links are (sufficiently) more rigid than actions polymorphic states will always emerge irrespective of the payoff parameters. The intuition is as follows. First note that a change in the assumptions on noise naturally does not affect the set of absorbing states which is still given by Proposition 1. But if action choices are a lot more noisy then link choices polymorphic states tend to emerge as action experimentation will lead to a higher variation in behavior across agents. The unperturbed dynamics then stabilizes polymorphic states in which cooperators and defectors are not linked, because cooperators will always desire to link with each other. As linking trembles are rare these states - while they are relatively likely to be reached - are very hard to destabilize. In a sense the assumption of rigid links reinforces the importance of the network in shaping long-run outcomes. As linking decisions are subject to relatively less error the endogenous network can sanction defectors more effectively.

[25] Jackson and Watts (2002) maintain the assumption of homogenous noise throughout the paper in a context (where as in the present paper) links are bilaterally formed. It would be interesting to see how (if at all) their results change under the alternative assumptions.

In other applications, for example when interactions are relatively anonymous, linking choice might be more noisy than action choice. In this case we can state the following proposition.

Proposition 4 (Rigid Actions) *Under A2″ there exists a value* $\overline{\xi} \in (1, \infty)$ *s.t. whenever* $\xi'' > \overline{\xi}$ *all stochastically stable states are contained in* ω_1^D.

Proof. Appendix. ■

If actions are (sufficiently) more rigid than links, full defection will always emerge irrespective of the payoff parameters. If link choices are very noisy agents will relatively often connect to another agents they have no information about. Of course in this context it is harder for cooperators to protect themselves from exploitation. Note that in a sense Assumption 2″ is closer to a setting in which links are formed globally without information about the potential interaction partners. It is quite intuitive that in such a setting defection stands the best chances for survival.

We have seen that the outcomes of our model can change if alternative assumptions on the relative importance of noise are used. The assumption of homogeneous noise is thus not always innocuous. In fact Jackson and Watts (2002) also conjecture that the results obtained in their model of coevolution of interaction structure and action choices in a coordination game are sensitive to these kind of changes.[26] In the next subsection we discuss several other aspects of the model that we think deserve further attention.

5.2 Alternative Assumptions

In this subsection we address in turn a number of variations of the basic model to illustrate which assumptions are crucial and which could be relaxed.

Learning about Actions

Let us start with our action imitation rule. One could think of several alternative ways to formulate payoff-biased imitation. Firstly agents could copy the most successful agent in their information radius (instead of focusing on the average payoffs of each action).[27] Secondly they could compare the average payoff of the alternative action against their *own* payoff. We think that our rule is more intuitive than especially the first rule, as with this rule agents throw away some information that is a priori just as relevant as the information considered.[28] In any case our intuition is that results should not change much under either rule. To see this we consider the conditions under which the rules differ. Under

[26] Bergin and Lipman (1996) show that stochastic stability is often sensitive to the perturbation technology.

[27] This assumption is often used in simulations. See Abramson and Kuperman (2001), Zimmerman, Eguíluz and San Miguel (2004), Zimmerman and Eguíluz (2005) or Hanaki et. al (2007) among others.

[28] With the second rule this is not necessarily the case as my own payoff can give me better information than my neighbors payoff, as we face different environments. Note also though that our equilibrium networks are very homogenous. This argument is thus not very strong in our case. Apesteguía, Huck and Öchssler (2006) provide experimental support for our rule.

the first alternative rule an agent i that would not change her action under the basic rule would do so whenever there is an agent $k \in N_i^I | z_k \neq z_i$ s.t. $\overline{\Pi}_k^{t-1} > \overline{\Pi}^{t-1}(N_i^I(z_i)) \geq \overline{\Pi}^{t-1}(N_i^I(\neg z_i))$. Under the second alternative rule i would act differently whenever $\overline{\Pi}^{t-1}(N_i^I(z_i)) \geq \overline{\Pi}^{t-1}(N_i^I(\neg z_i)) > \overline{\Pi}_i^{t-1}$. These conditions are unlikely to occur in our model, as in both cases one agent (either k or i) have to face very different conditions from the other agents in N_i^I. Since the local nature of our model implies relatively homogenous local topologies with high levels of clustering (or cycles with respect to I, Z) this is unlikely to happen especially whenever $|I - Z|$ is not too large.[29]

A third possibility is that agents consider total instead of average payoffs when deciding to choose an action. Such an assumption would tend to favor cooperative outcomes. As cooperators always want to form links with each other severing links with defectors, there is a tendency during any transition for defectors to have less links (and thus a smaller total payoff). We do not choose such an assumption though, as it seems to imply a degree of forward-looking behavior that is absent in our model of myopic agents. In particular when choosing an action agents take as given the cardinality of their interaction neighborhood and thus should be interested in the average *per interaction* payoff of an action.[30]

Learning about Links

Next consider alternative link imitation rules. One possibility is that agents search for new links globally. Note that as in this case the sets $\left\{N_j^Z \cap N_i^I\right\}$ can be empty an additional rule is needed to evaluate potential new links. Irrespective of the specific form of such an additional rule, the results with global search could change and more defection would be observed. The intuition is similar to that of Proposition 4, as increasing the noise in link formation implies increasing the probability of the formation of global links. Local search is a crucial element of our model.

Other alternative assumptions pertain to how individuals evaluate potential new links. One could imagine that any agent i evaluates a link to j through the average per interaction payoff of all agents $h \in \left\{\left\{N_j^Z \cap N_i^I\right\} | a_h = a_i\right\}$, i.e. takes into account only agents that are playing the same action as herself. Note that again as $\left\{\left\{N_j^Z \cap N_i^I\right\} | a_h = a_i\right\}$ can be empty an additional rule is needed for this case. We conjecture that this rule will work towards more cooperative outcomes. To see this, consider a linking constrained cooperator i with $\Pi_{\min}^{t-1}(N_i^1) = a$. If for any potential new link cooperator i only considers those agents in $\left\{N_j^Z \cap N_i^I\right\}$ that are currently cooperating, she will never cut a mutually cooperative link in order to form another new link. With the current rule though this is possible if j is linked to many successful defectors.

At last, we want to address the problem of how to evaluate "worst links." In our model, $\Pi_{\min}^{t-1}(N_i^1)$ corresponds to the minimum payoff that player i obtains from any of her first-order neighbors. If $Z = 1$ this seems the only reasonable

[29]Note though that cliques of defectors at the periphery of a cooperative component are (generically) not absorbing under the second alternative rule.

[30]Hanaki et al. (2007) for example use total payoffs as a criterium.

rule. Yet if $Z > 1$ other rules could be possible. Indeed for some Z, I an agent can have additional information she might want to use. For example i could evaluate a link with j through the overall contribution of the link to the payoff of i (i.e. through both the interaction with j and the interaction with agents i uniquely interacts with because she is linked with j). Under this alternative assumption two links $ij, ih : a_j = a_h$ will typically have different values for player i. Any such rule has severe drawbacks. In fact convergence to an absorbing state can be almost impossible. Because of high clustering the contribution of any link to the total payoff will be small and a preferred link will always be found. Consequently agents will continuously rewire the network until it consists of many complete components.

Not all links are worthwhile

Next we want to discuss the possibility that not all links are worthwhile. This kind of assumption has been studied for example in Goyal and Vega-Redondo (2005) or Jackson and Watts (2002). Suppose that the costs of maintaining links are so high that links with a defector are not worthwhile for a cooperator. Then of course we would not observe such links and absorbing states would either be monomorphic or consist of separate components. If no links with a defector are worthwhile at all (neither for a defector nor for a cooperator) non-empty absorbing states will of course have to involve full cooperation.[31] In our model thus this extension does not seem particularly interesting.

Heterogenous capacity constraint

A more interesting variation seems to allow for less degenerate degree distributions. The homogeneous linking constraint allows us to obtain analytical results while maintaining the spirit of the network analysis. Alternatively one could for example assume that the capacity constraint of agent i ($\overline{\eta}_i$) is a random variable with discrete uniform distribution of support $[1, \overline{\overline{\eta}}] \cap \mathbb{N}$. The higher $\overline{\overline{\eta}}$ the more heterogeneity with respect to final degree distribution is possible. If $E(\overline{\eta}_i) \approx \overline{\eta}$ (being $\overline{\eta}$ the homogenous capacity constraint of the present version) absorbing states would essentially not change. The reason is that as agents focus on average (per interaction) payoffs when deciding on new links or actions this heterogeneity would be approximately neutralized.

6 Conclusions

We develop a simple model to study the coevolution of interaction structures and action choices in prisoners' dilemma games. Agents are bounded rational and choose both actions and interaction partners through payoff-biased imitation. We find that polymorphic states evolve under a wide range of parameters. Whenever agents hold some information beyond their interaction partners defectors and cooperators will never interact in stochastically stable states, i.e. they are found in disconnected components. Otherwise graphs in stochastically

[31] Zimmermann, Eguíluz and San Miguel (2004) assume throughout their model that links between a cooperator and defector can survive but not links between two defectors. This assumption seems rationalizable only in the context of unilateral link formation.

stable states can consist of a core of cooperators with cliques of defectors lying on the periphery of the component. Any stochastically stable state will consist of at most two disconnected components. Simulating the model confirms our analytical result that polymorphic states tend to emerge. The share of cooperators in such states increases with the speed at which the network evolves, decreases with the radius of information and increases with the radius of interaction. Consistently with empirical findings on social networks, the networks we obtain display high clustering coefficients and short average distances. Two directions of further research seem promising to us. On the one hand it would be interesting to incorporate more realistic degree distributions in analytical models, that study the coevolution of interaction structures and behavior. Yet it seems a difficult task to obtain analytical results in such settings. Also of some interest is how (if at all) predictions of existing models that have analyzed coordination games with best response dynamics change when more bounded rational learning rules (like our imitation rule) are used.

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A Appendix: Proofs.

Proof of Proposition 1:
Proof. (i) Of course $\eta_i \leq \overline{\eta}$ has to hold. Isolation ($\eta_i = 0$) can be part of any absorbing state, since whenever $\eta_i = 0 \implies \operatorname{card} N_i^{I+Z} = 0$. If $\eta_i \in (0, \overline{\eta}]$ a necessary condition is that if $\eta_i < \overline{\eta}$ there cannot be potential partners for i, i.e. $\forall j \in \left\{N_i^{I+Z} \backslash N_i^1\right\} : \eta_j = \overline{\eta}$.

(ii) States where for any $i, j : \chi(i) = \chi(j) \implies z_i = z_j$ are absorbing, as no agent has possibilities to imitate because $\operatorname{card} N_i^I(\neg z_i) = 0$. Any monomorphic state is of course absorbing.

(iii) First we show that defectors lie in the periphery of the component or are directly linked to an agent in the periphery and that each defector j is at a distance $d(i, j) \leq I + Z$ from some cooperator. Consider any mixed link ij where $z_i = C$. Since the cooperator i is obtaining the minimum possible payoff from link ij, this can be absorbing only if either $\nexists h \in N_i^{I+Z} : l_{hi} = 1$ or if $\exists h \in N_i^{I+Z} : l_{hi} = 1$ and hi is added, this change does not modify the structure of the subgraph up to a permutation of the identity of the nodes connected to i. It follows that all cooperators $k \neq i \in \left\{N_i^{I+Z} \cap \backslash N_i^1\right\}$ need to have $\eta_k = \overline{\eta}$ and $\operatorname{card} N_k^1(D) = 0$. On the other hand any defector j' with $d(i, j') = I + Z$ that is connected to i through some path of defectors is willing to form a link with i. This is because $\Pi_{\min}^{t-1}\left(N_{j'}^1\right) = d < \overline{\Pi}^{t-1}\left(N_i^Z \cap N_{j'}^I\right)$, since $\left\{N_i^Z \cap N_{j'}^I\right\}$ contains only defectors some of which are interacting with i. Consequently defectors must be in the periphery. Suppose not. Then there exist two cooperators i and i' separated by a path of defectors. Cooperator i will form a link with defector j' at distance $d(i, j') = I + Z$ until finally $i' \in N_i^{I+Z}$. But then i and i' will link and all mixed links will finally be cut. It follows analogously that any defector j must lie at a distance $d(i, j) \leq I + Z$ from cooperator i.

Next we prove that if defectors form a *clique* such states are absorbing and if either $I \leq 2$ or $Z = 1$ this is also a necessary condition. Assume that either $I = 1$ or $Z = 1$ and that there is only one cooperator i linked to some of a set of defectors. Any defector at a distance $2 \leq d(i, j) \leq I + Z$ has incentives to link to cooperator i. If $I = 1$, defectors observe only defectors interacting with cooperator i. But then $\Pi_{\min}^{t-1}\left(N_j^1\right) = d < \overline{\Pi}^{t-1}\left(N_i^Z \cap N_j^I\right)$. If $I > 1$, any defector may observe in addition other cooperators, but since $Z = 1$ these cooperators interact only with cooperators and again $\Pi_{\min}^{t-1}\left(N_j^1\right) = d < \overline{\Pi}^{t-1}\left(N_i^Z \cap N_j^I\right)$. Thus some new links might be formed. This rewiring can be part of a recurrent set if and only if N_i^{I+Z} remains unchanged. It follows that the set of defectors must form a clique.[32] Now assume $I > 1$ and $Z > 1$. Again cooperator i has incentives to sever any of her mixed links. The incentives of i's potential partners depend on how many cooperators interact with the defectors they observe. To characterize all structures also in this case is impossible without further assumptions.

Now consider agents' incentives to change actions. Assume that x defectors form a clique and that there is only one cooperator i linked with them.[33] Under these conditions there exists a threshold for the interaction radius, $\widehat{Z}(I)$ such that if $Z < \widehat{Z}(I)$ there always exists some set of values in the space (d, a) ,s.t. such an action profile is absorbing. To simplify the exposition we assume $c = 1$ and $b = 0$. It should be clear that if i does not want to change action, then no other cooperator h has incentives to do so. For cooperator i and any defector j in the clique,

$$\overline{\Pi}_i \left(N_i^I \left(D \right) \right) = \overline{\Pi}_j \left(N_j^I \left(D \right) \right) = \overline{\Pi}_j^t = \frac{n_j^Z - (x-1) + (x-1)\, d}{n_j^Z}.$$

Action choices are absorbing if and only if $\overline{\Pi}_i \left(N_i^I \left(C \right) \right) \geq \overline{\Pi}_j^t \geq \overline{\Pi}_j \left(N_j^I \left(C \right) \right)$. Now we show that for each I, there exists $\widehat{Z}$ s.t. if $Z \leq \widehat{Z}$, it is always possible to find some values of $(d, a) \in [0, 1] \times \left[\max \left\{ \frac{1}{2}, d \right\}, 1 \right]$ s.t. this is true. First of all note that $\overline{\Pi}_j^t = \frac{n_j^Z - (x-1) + (x-1)d}{n_j^Z}$ is monotonously increasing in both Z and d. The sample payoffs $\overline{\Pi}_i \left(N_i^I \left(C \right) \right)$ and $\overline{\Pi}_j \left(N_j^I \left(C \right) \right)$are increasing in a. If $Z < I$, an increase in Z has two effects. On the one hand each cooperator in the sets N_i^I and N_j^I interacts with more cooperators increasing the sample payoff. But on the other hand, more cooperators interact with defectors lowering the sample payoffs. The net effect depends on the precise structure of the component. Consider first the case where Z is small, in particular where $Z = 1$. Then $\lim_{d \to 0} \Pi_j^t = \frac{1}{x} \leq \frac{1}{2}$. On the other hand, for any $a > \frac{1}{2}$: $\overline{\Pi}_i \left(N_i^I \left(C \right) \right) = \frac{\left(\frac{\overline{\eta} - x}{\overline{\eta}} \right) a + \varphi_i(I) a}{\varphi_i(I) + 1}$ and $\overline{\Pi}_j \left(N_j^I \left(C \right) \right) = \frac{\left(\frac{\overline{\eta} - x}{\overline{\eta}} \right) a + \varphi_j(I) a}{\varphi_j(I) + 1}$, where $\varphi_i \left(I \right)$ and $\varphi_j \left(I \right)$ are, respectively, the number of cooperators $h \neq i$ contained in N_i^I and N_j^I. (Recall that $\varphi_i \left(I \right) > \varphi_j \left(I \right)$.) Then whenever $a > \frac{1}{x} \frac{\overline{\eta}}{\overline{\eta} - x}$,

$$\overline{\Pi}_i \left(N_i^I \left(C \right) \right) > \overline{\Pi}_j \left(N_j^I \left(C \right) \right) > \overline{\Pi}_j^t \left(d \to 0 \right) \approx \frac{1}{x}.$$

On the other hand for Z very large

$$\overline{\Pi}_j^t \left(d \to 0 \right) \to 1 > \overline{\Pi}_i \left(N_i^I \left(C \right) \right) \approx \overline{\Pi}_j \left(N_j^I \left(C \right) \right) \to a.$$

Consequently there exists a threshold value $\widehat{Z}(I)$, such that if $Z < \widehat{Z}$ there always exists some set of values in the space (d, a) for which there are no incen-

[32] If there are more than one cooperator bridging the same subgraph of defectors, *any* defector in the subgraph must be directly connected to all cooperators and *cycles are not possible* since, when a cooperator changes a link from one defector to another, there exists always the possibility that the set N_i^{I+Z} of another cooperator changes.

[33] Of course $x \geq 2$ has to hold.

tives to imitate actions.[34] Finally, the existence of absorbing states with linking cycles can be shown analogously.[35] ■

s−trees

For most of the following proofs we will rely on the graph-theoretic techniques developed by Freidlin and Wentzell (1984).[36] They can be summarized as follows. For any state s an $s-$tree is a directed graph on the set of absorbing states Ω, whose root is s and such that there is a unique directed path joining any other $s' \in \Omega$ to s. For each arrow $s' \to s''$ in any given $s-$tree the "cost" of the arrow is defined as the minimum number of simultaneous trembles necessary to reach s'' from s'. The cost of the tree is obtained by adding up the costs of all its arrows and the stochastic potential of a state s is defined as the minimum cost across all $s-$trees.

Proof of Lemma 1:

Proof. Let $\mathcal{G}^0$ denote the set of graphs consisting of at most two disconnected components. Let $\mathcal{G}^1$ be the set of graphs one tremble away from some graph in $\mathcal{G}^0$. Define $\mathcal{G}^2$ to be graphs not in $\mathcal{G}^0 \cup \mathcal{G}^1$ that are one tremble away from $\mathcal{G}^1$. For $\tau > 2$ let $\mathcal{G}^\tau$ denote graphs not in $\mathcal{G}^j$ for any $j < \tau$, that are one tremble from $\mathcal{G}^{\tau-1}$. Note that these exhaust all graphs that could be part of absorbing states. Consider an absorbing state graph $G \in \mathcal{G}^\tau, \tau > 0$. Transitions from G to some $G' \in \mathcal{G}^{\tau-1}$can occur after just one tremble, as it is always possible that two players i and h, with $\chi(i) \neq \chi(h)$ and $z_i = z_h$ form a link by mistake. This implies that for any s with $G \in \mathcal{G}^\tau$, there exists s' with $G' \in \mathcal{G}^{\tau-1}$ s.t. $\psi(s') \leq \psi(s)$. (Starting from an $s-$tree one can always redirect an arrow from s to a state s' which is one tremble away). Thus to complete the proof we show (i) that the stochastic potential of states with a graph in $\mathcal{G}^0$ is smaller than that of states with a graph in $\mathcal{G}^1$ and (ii) that the stochastic potential of connected monomorphic states is smaller than that of monomorphic states where graphs consist of two disconnected components. Start with an absorbing state s with $G \in \mathcal{G}^1$ and find a state s' with graph $G' \in \mathcal{G}^0$. We know that $X(s, s') = 1$ and of course $X(s', s) \geq 1$. We will now see in which cases strict inequality obtains. Consider first the transition through which s' is reached from s. For this transition a link ih is formed by mistake between i and h s.t. $\chi(i) \neq \chi(h)$ and $z_i = z_h$. If now either i or h is not linking constraint and in addition h or i have a neighbor, say k, that is not linking constrained, then (at least) the link ik (or hk) will be formed before an absorbing state is reached. But then at least two trembles are needed for the transition $s' \to s$ and consequently $X(s', s) > 1$. Note that such two states s' and s can always be found. What happens if for two states s with $G \in \mathcal{G}^1$ and s' with graph $G' \in \mathcal{G}^0$ we have that $X(s', s) = 1$? First note that for any s' a state s'' with $G'' \in \mathcal{G}^0$ can be found such that a) $X(s'', s) > 1$ and b) s'' can be reached from s' via a series of "one-trembles". But then we have that $\psi(s'') \leq \psi(s')$. Consider thus states

[34]Note also that $\overline{\Pi}_j^t(d \to a) = \frac{(x-1)a+1}{x} > a > \overline{\Pi}_j\left(N_j^I(C)\right)$ (no intersection).

[35]We have characterized the sets $\widehat{\Psi}$ and the particular form of the cliques exactly in the case $I = Z = 1$.

[36]See also Young (1993, 1998).

s' where $X(s', s) > 1$. Then starting from a minimal $s-$tree and add an arrow $s \to s'$. Then consider the old path $s' \to s$ and take the first s''' on that path (this could be s') such that the arrow pointing away from s''' involves at least two trembles. Such a state s''' must exist as at some point (at least) two links have to be severed to separate the component of players and s''' must have a graph in $\mathcal{G}^0$. Note that if starting from s' the component is separated at least two trembles are needed and thus $s' = s'''$. Thus s''' will have a graph in $\mathcal{G}^0$ and to separate the component at least two trembles will be needed (as any two agents i and h (such that in $s : \chi(i) \neq \chi(h)$) who cut a link starting from s' will be in each other's search radius and thus for s''' to be absorbing either have to form a link (but then again $s' = s'''$) or either of them has to form a link with another agent (but then at least two trembles are needed to reach s)). We have shown that for any s with $G \in \mathcal{G}^1$ there exists a state s''' with graph $G''' \in \mathcal{G}^0$ s.t. $\psi(s''') < \psi(s)$. The argument can be repeated starting from a monomorphic state s with two disconnected components. This completes the proof. ■

Proof of Lemma 2:

Proof. It follows from Lemma 1 that if stochastically stable states that involve full cooperation exist at least one of them has to be connected i.e. has to be contained in the set ω_1^C. We will now show that for any $s \in \omega_1^C$ there exists an alternative state in ω^{CD} that has strictly less stochastic potential. For any $s \in \omega^D$ consider the state $s' \in \omega^{CD}$ reached via one tremble from s in the following way. Assume one player i trembles and switches to action D. Then for all agents $j \in N_i^I$ the average payoff of action D will exceed that of action C. Assume α agents selected from that set switch to action D and that the subgraph containing these agents is cut off (through rewiring of cooperating neighbors who prefer being linked to a cooperator) only after $\kappa_D > \overline{\eta}$ agents in total (including the mutant) have switched to D. Note that irrespective of the payoff parameters and of I and Z this is always possible. State s' contains thus two disconnected components, one consisting of $\kappa_D > \overline{\eta}$ defectors and one of $n - \kappa_D$ cooperators. The reverse transition ($s' \to s$) will need at least 2 trembles, as one link tremble has to occur to merge the two components and in addition at least one of the defectors has to tremble to switch to cooperation. (Note again that any single (non-isolated) defector will have a higher per interaction payoff than cooperators). Next take a minimal $s-$ tree and add the arrow $s \to s'$ at a cost of $X(s, s') = 1$. Then consider the path $s' \to s$. If there is no other state on this path, cut the arrow $s' \to s$. This yields an $s'-$tree with $\psi(s') < \psi(s)$. If there is a state $s'' \in \omega^C$ on this path, then we know that $X(s', s'') \geq 2$ (as a single cooperator in a component of defectors will never be imitated). We can cut the arrow $s' \to s''$ and have constructed again an $s'-$tree with $\psi(s') < \psi(s)$. If $s'' \in \omega^{CD}$ then we know that $X(s'', s) \geq 2$ by the same argument as above. Cutting the arrow $s'' \to s$ leaves us with a $s''-$tree that has $\psi(s'') < \psi(s)$. This completes the proof. ■

Distance between graphs

Before stating the next Lemma and its proof let us introduce the following metric. Define $y(G, G') = \sum_{ij} \frac{\left|(l_{ij} l_{ji}) - (l'_{ij} l'_{ji})\right|}{2}$ to be the distance between the graphs G and G' associated with states s and s' respectively. The distance $y(G, G')$ between two graphs simply measures the number of links that differ between the two graphs.[37] Furthermore denote $\zeta_i^Z(t)$ the share of agents $j, k \in N_i^Z$ at time t that are Z-th order neighbors themselves. $\zeta_i^Z(t)$ is a measure of local clustering in i's interaction neighborhood.

Proof of Lemma 3:

Proof. (i) Starting from a state $s \in \omega_1^D$ we construct a state $s' \in \omega_2^{CD}$ as follows. Assume that $\lceil \kappa_C \rceil$ agents (where $\kappa_C \in \mathbb{R}$) tremble and switch to action C at time t. We want to consider the action choice of a defector k linked with a cooperating agent i. Assume that all other cooperators are $(1st-, 2nd- \ldots .Zth-$order) neighbors of i, i.e. are all interacting with each other. The average cooperator payoff $\overline{\Pi}^t(N_k^I(C))$ that agent k observes is given by $\overline{\Pi}^t(N_k^I(C)) = b + (a-b)h(\kappa_C, n^Z, \zeta_i^Z(t))$ where $h(\cdot)$ is an increasing function of clustering and of κ_C.[38] On the other hand the average defector payoff $\overline{\Pi}^t(N_k^I(D))$ that agent k observes is given by $\overline{\Pi}^t(N_k^I(D)) = d + (c-d)g(\kappa_C, n^Z, \zeta_i^Z(t))$ where $g(\cdot)$ is a decreasing function of clustering and of κ_C. Denote the value of κ_C that solves $\overline{\Pi}^t(N_k^I(C)) = \overline{\Pi}^t(N_k^I(D))$ by κ_C^*. This value is in general a complicated expression but note that $(\partial \kappa_C^* / \partial a) < 0$. Now whenever agent k has incentives to switch to cooperation (i.e. whenever $\overline{\Pi}^t(N_k^I(C)) > \overline{\Pi}^t(N_k^I(D))$) then $x_C \geq n^Z + 1 - \kappa_C$ agents can be infected through the ensuing operation of the unperturbed action dynamics alone.

Through the linking dynamics then all cooperators will sever their remaining links with defectors and form links among each other. (Note that this is possible because $x_C + \kappa_C \geq \overline{\eta} + 1$ so these agents can always at least form the complete component. Furthermore they have incentives to do so, as $\Pi_{\min}^t(N_h^1) = b < \overline{\Pi}^t(N_j^Z \cap N_h^I)$ for any pair of cooperating agents j, h. Note also that by construction all these agents are in each other's search set). The resulting distance $y(G, G')$ between the graphs G and G' associated with states s and s' respectively will satisfy $y(G, G') \geq \frac{3}{2}(1-\zeta)n^Z(\overline{\eta}-1)$. (The new component in s' will consist of at least $n^Z + 1$ agents, at least $(1-\zeta)n^Z(\overline{\eta}-1)$ links will be severed and $\frac{(1-\zeta)n^Z(\overline{\eta}-1)}{2}$ new links will be added, where $\zeta \in [0, \frac{1}{2})$ is a parameter taking care of clustering.)

(ii) Consider the reverse transition from $s' \in \omega_2^{CD}$ to $s \in \omega_1^D$. Essentially such a transition can occur in two ways. Either the cooperative component $\chi^C(s')$ is first infected by defection and then the graph is rewired to obtain state s. (In this case the transition is indirect, i.e. passes through other absorbing states among which at least one is in ω_2^D.) Or first a sufficient number of linking

[37] This metric has been used previously by Goyal and Vega-Redondo (2005).

[38] This expression can be approximated by

$$\overline{\Pi}^t(N_k^I(C)) = b + \frac{(a-b)(\kappa_C - 1)\left(1 + \zeta_i^Z(t) + \left[\zeta_i^Z(t)\right]^2(\kappa_C - 2)\right)}{n^Z\left(1 + \zeta_i^Z(t)(\kappa_C - 1)\right)}$$

where we have made the assumption that $n_h^Z = n^Z$, $\forall h \in N_k^I$. This assumption is not restrictive, as more favorable transitions can always be found.

trembles has to occur s.t. the ensuing operation of the unperturbed dynamics permits infecting all agents with defection while rewiring the graph. (In this case the transition is direct).

Consider the first type of transition. For this transition $\kappa_D^{Act.}$ action trembles are needed to infect the cooperative component and then $\kappa_D^{Link} \geq \frac{y(G,G')}{5}$ linking trembles are needed to rewire the graph. The latter inequality holds because each linking tremble can at most induce a rewiring of five links through the unperturbed dynamics (Assume one link is randomly added and two links severed. This leaves two agents below the linking constraint who can form a link with two other agents below the linking constraint (if such agents exist in their search set). This implies a total change of at most five links). Consequently $X(s', s) \geq \frac{3(1-\zeta)n^Z(\overline{\eta}-1)}{10} + \kappa_D^{Act.}$ with transitions of the first type. Now note that while $X(s', s)$ is strictly increasing with the payoff parameter $a \in (d, c)$, $X(s, s')$ is decreasing in a. Consequently there exists $\widehat{a}_1(s)$ s.t. $X(s', s) > X(s, s')$ holds whenever $a > \widehat{a}_1(s)$. Now consider the second type of transition. First note that a cooperating agent $i \in \chi^C(s')$ linked to a defector $j \in \chi^D(s')$ (after a linking tremble) has incentives to switch to defection if and only if

$$a < \frac{n_i^I(\frac{C}{D})[z_D^i d + (1 - z_D^i)c] - z_C^i b}{1 - z_C^i}, \tag{4}$$

where the factor $n_i^I(\frac{C}{D})$ gives the ratio of cooperators and defectors in the set N_i^I and z_D^i (z_C^i) is the share of defectors these defectors (cooperators) interact with on average. Note also that whenever (4) fails no links will be formed between neighbors h of i and neighbors k of j, unless h has a neighbor who is playing defection. (If h does not have a defector neighbor, then $\Pi_{\min}^t(N_h^1) = a > \overline{\Pi}^t(N_k^Z \cap N_h^I)$ if either $j \notin \{N_k^Z \cap N_h^I\}$ or $i \in \{N_k^Z \cap N_h^I\}$. But if $i \notin \{N_k^Z \cap N_h^I\}$ i.e. if $N_k^Z \cap N_h^I \cap \chi^C(s') = \varnothing$ then a failure of (4) implies $\Pi_{\min}^t(N_h^1) = a > \overline{\Pi}^t(N_k^Z \cap N_h^I)$). Then any transition needs to satisfy four conditions. There have to be incentives to imitate, each defector in $\chi^C(s')$ should be linked with a cooperator, in total $\frac{y(G,G')}{5} - 1$ such pairs have to exist and any linking trembles should reduce the distance $y(G, G')$. The number of trembles needed to induce such a transition is thus strictly increasing with the payoff parameter a. Consequently there exists a threshold level $\widehat{a}_2(s)$ such that whenever $a > \widehat{a}_2(s)$, $X(s', s) > X(s, s')$. Summing up whenever $a > \widehat{a}(s) = \max\{\widehat{a}_1(\cdot), \widehat{a}_2(\cdot)\}$ we have that $X(s, s') < X(s', s)$. This completes the proof. ■

Proof of Lemma 4:

Proof. Starting from any polymorphic absorbing state $s \in \omega_1^{CD}$ with ρ subgraphs of defectors one linking tremble suffices to reach the absorbing state s'' where one component contains $\rho - 1$ subgraphs and there is a second component of defectors. (Simply cut the "bridge" between the cooperating core and the cooperator that sustains a clique of defectors). But then $\psi(s'') \leq \psi(s)$. (Starting from a minimal $s-$tree simply add the arrow $s \to s''$ and cut the first arrow leaving s'' on the path $(s'', ..., s)$). Repeating this argument it should be clear that there exists a state s''' consisting of a cooperator-component and ρ defector components with $\psi(s''') \leq \psi(s)$. But then any two of these defector components can be linked via one tremble, implying that there exists a state $s' \in \omega_2^{CD}$ such that $\psi(s') \leq \psi(s''') \leq \psi(s)$. Now note that a component ρ is either complete with all agents below the linking constraint or all agents except one are linking constraint. But then whenever $I + Z > 2$ a transition from any state $s^{\#}$ with two defector components to a state $s' \in \omega_2^{CD}$ can always be constructed such that after one linking tremble two more agents that are under the linking constraint observe each other and want form a link. But then $y(G(s^{\#}), G(s')) \geq 2$. Now starting from a minimal $s^{\#}-$tree adding the arrow $s^{\#} \to s'$ and cutting the arrow from the last state on the path $(s', ...s^{\#})$ yields an $s'-$tree with $\psi(s') < \psi(s^{\#})$. ■

Proof of Proposition 2:

Proof. The first part follows immediately from Lemma 1 and 2. Focus thus on the second part. Take any two states $s \in \omega_1^D$ and $s' \in \omega_2^{CD}$ with $X(s, s') < X(s', s)$ (such states always exist if $a > \widehat{a}$ as we have seen in Lemma 3). Starting from a minimal $s-$tree consider the path from s' to s. Denote this path by $(s', ..., s)$. We know from the proof of Lemma 3 that no state on this path will be contained in ω^C or ω^{CD} (with the exception of the state s'). a) If $(s', ..., s) = (s', s)$ i.e. if the transition from s' to s is direct we can infer immediately that $\psi(s') < \psi(s)$. (Just redirect the arrow $s' \to s$. This yields an $s'-$tree with $\psi(s') = \psi(s) + [X(s, s') - X(s', s)] < \psi(s)$). b) Next assume that there exists a state $s'' \in (s', ..., s)$ with $s'' \in \omega_2^D$. Note that $X(s'', s) = \kappa_D^{Link} > X(s, s')$ always holds under the assumption that $a > a^*$. But if $X(s'', s) > X(s, s')$ we can find an $s''-$tree with $\psi(s'') < \psi(s)$ simply adding the arrow $s \to s'$ and deleting the arrow $s'' \to s$, and thus s cannot be stochastically stable. On the other hand it follows from Lemma 1 that states in ω_ρ^D where $\rho > 1$ cannot be stochastically stable either. (c) Furthermore it follows from the proof of Lemma 3 that whenever the path $(s', ..., s)$ in a minimal $s-$tree contains a state $s^{\#} \in \omega_1^D$, it also contains a state $s'' \in \omega_2^D$. But we have already seen that in this case s is not stochastically stable. Consequently all stochastically stable states are contained in ω_ρ^{CD} where $\rho \leq 2$. ■

Proof of Proposition 3:

Proof. First note that Lemma 1 still holds and thus all monomorphic stochastically stable states have to have one component. Now starting from any state $s \in \omega_1^D$ construct an alternative state $s' \in \omega_2^{CD}$ as follows. Assume that starting from s a tremble by κ_C^* agents occurs that is imitated by x agents s.t. subsequently $\kappa_C^* + x \geq \overline{\eta} + 1$ cooperating agents exist that are all in each other's search sets. These agents will prefer to form links with each other and to sever their links with defectors. The unperturbed dynamics converges to a polymorphic state. Now if $\xi' < \underline{\xi} \leq (\kappa_C^*)^{-1} \in (0, 1)$, this is infinitely more likely to occur (in the limit as $\varepsilon_z \to 0$) as a single linking tremble. Now take any minimal $s-$tree and add the arrow $s \to s'$. On the path from s' to s (in the old tree) there has to be a state s'' from which a linking tremble has to occur to reach s. Cut the arrow leaving from s''. The resulting tree is an $s''-$tree where s''

has less stochastic potential then s.[39] Now s'' can either be polymorphic what completes the proof (in fact s'' can coincide with s') or it can be monomorphic (with $s'' \in \omega_2^D$) but then neither s nor s'' can be stochastically stable because of Lemma 1. Now together with Lemma 2 this implies that all stochastically stable states have to be in ω_2^{CD}. ■

Proof of Proposition 4:

Proof. Again observe that Lemma 1 still holds. Starting from any polymorphic state $s \in \omega_2^{CD}$ - where no defectors and cooperators are linked - construct an alternative state $s' \in \omega_1^D$ as follows. Assume that starting from s a tremble by $2\kappa_L^*$ agents occurs (κ_L^* from each component) that form a link with each other. Take κ_L^* to be big enough s.t. the unperturbed dynamics afterwards converges to a monomorphic state. Now if $\xi'' > \overline{\xi} \geq 2\kappa_L^* > 1$ this is infinitely more likely to occur (in the limit as $\varepsilon_l \to 0$) as a single action tremble. Now take any minimal $s-$tree and add the arrow $s \to s'$. On the path from s' to s (in the old tree) there has to be a monomorphic state s'' from which an action tremble has to occur to reach s (it can well be that s'' coincides with s'). Cut the arrow leaving from s''. The resulting tree is an $s''-$tree where s'' has less stochastic potential then s. Now together with Lemma 1, Lemma 2 and Lemma 4 this implies that all stochastically stable states have to be in ω_1^D. ■

[39] Of course now as the probabilities of the two kinds of trembles are not of the same order, one cannot just sum the number of trembles to obtain the stochastic potential but one has to weight them with their respective probabilities (where less likely trembles have a higher weight).