A dynamic model to estimate the long-run trends in potential GDP

Lucian-Liviu Albu

Institute for Economic Forecasting, Bucharest

2006

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Abstract

To estimate long-run growth based on the so-called potential GDP became a constant preoccupation among economists. However, one remaining problem in every long-run growth model is to estimate a persistent trend in labour productivity outside of it, in order to avoid the implicit circular relationship between actual productivity growth and potential level of production. Coming from recent literature on natural rate of unemployment estimation we used a specific methodology in order to estimate NAIRU in case of post-communist economies and based on it to evaluate the potential GDP. Taking into account that the "classic" Hodrick-Prescott method is in fact equivalent to an interpolation procedure, we used in our experiment other three filters demonstrating very similar output. Moreover, we conceived a simple autonomous model in order to estimate the growth of a so-called "pure" productivity independently from the actual level of employment and to compare its dynamics with that of natural rate of unemployment.

1. Introduction

To estimate the size of the potential GDP and its long-run dynamics has been last years an issue of concern to policy makers. Recent evidence suggests that the problem is especially serious today, taking also into account the efforts needed to ensure both nominal and real convergence criteria in case of the new members of EU after their accession.

There are various methods trying to estimate economic cycles during last decades based on natural unemployment rate or NAIRU ("Non-Accelerating Inflation Rate of Unemployment"). Taking into account already accumulated experience on more than fifteen years of transition and data today available we try to estimate the level of natural rate in case of Romanian economy. Our study is coming from a general standard model recently used in order to estimate changes in NAIRU during last decades in USA and to investigate causes of such evolution (Ball and Mankiw, 2002). Moreover, in case of application on the Romanian economy in transition period, in order to verify what the type of correlation between natural rate and change in productivity is, we used an outside independent model to estimate the dynamics of so-called pure productivity.

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As in standard literature is asserted there is an implicit circular relationship between productivity growth and potential level of production (and consequently the estimation of natural rate of unemployment is also altered). In order to avoid such emerging impediment in any estimating macroeconomic model, an autonomous dynamic model to estimate the trend of productivity growth must be used. Moreover, taking into account that current level of productivity is implicitly influenced by the actual unemployment rate, usually it is recommended as a more accurate solution to try to obtain firstly an estimate for the "pure" productivity. This must be neutral relating to short-run changes in employment, but in long-run it is affected by factors such as: general technological progress, rising of education level, growth of R&D system, extending of the "new economy", etc. In this article we use a relative simple dynamic model to estimate the growth of pure productivity independently from the actual level of employment and implicitly of unemployment rate. Then estimated changes in pure productivity level are compared with potential GDP trend in case of Romanian economy during transition period but also in a longer period. Finally, some ways to further continue the research are deduced.

2. European experience: significant changes in inflation-unemployment relationship

Following some old preoccupations (Daianu and Albu, 1996; Albu, 1997 and 2001), here we present only few conclusions based on an empirical analysis of the inflation-unemployment relationship evolution in European area after 1970. So, during last three decades, on the background of business cycles, empirical studies demonstrate certain major changes of trends in Western countries. Among these could be noted: a significant lower inflation, a continuing growth in unemployment rate, and a general diminution of growth rate of production (GDP). An important fact is that a smaller volume in 3D map (estimated by including the variation of the three macroeconomic indicators) represents a greater economic stability and consequently less strain in economic system.

In Western European countries, evolution was from a period in which high inflation predominated toward one in which unemployment plays this role. This evolution could mean that on the unemployment-side occurred a relaxation, higher levels of unemployment being viewed as normal but is not the case on the inflation-side. Deeper analysis demonstrates occurrence of some persistent trends and long-run attractors. On the other hand, in Eastern European countries there was an opposite situation at least during the first years of transition after 1989; open inflation rose rapidly in the region whereas unemployment did also rise but at a relative smaller pace. There are evidences demonstrating that the long-run trends will be probably similar to those registered in Western countries. In case of each individual Eastern economy still most important question is when the convergence process will finish. Despite of relatively short period since 1989, in case of Eastern countries it seems to emerge a convergence relating to the natural rate of unemployment. The main problem continues to be a relative higher inflation in context of EU standard (for instance, in case of Romania the annual inflation decreased below 10% only after 2004).

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In case of Western countries in Europe, it would seem that long period of experiencing high unemployment might have infused a larger acceptance of public. This could be viewed in direct connection with a continuous development of social security programs, but also with other variables such as the budget deficit and public debt, their sustainability, power of trade unions, etc. In Eastern countries it seems that the acceptance was smaller at least during the incipient stages of transition period. Moreover, in these countries the development of social security programs is only in various re-building stages. Additionally, it should be noted that their relative larger informal sector could alter the level of official reported macroeconomic indicators currently used in analyses.

From such empirical evidence, we can conclude that transition in Eastern countries represented only a stage within a long-run wave on a general development scale. Generally, when the income per capita rises to higher level, it was demonstrated a specific evolution, as in many Western countries, namely that to higher natural rate of unemployment and to a period in which unemployment become more autonomous relating to the GDP growth. Important for new members of EU is that, in actual period of extending "new economy" and globalisation process, the convergence does not suppose necessarily a repetition of the Western evolution coming from the '60s and consequently its achieving period could be substantially reduced.

3. Estimation model of natural rate of unemployment and applications

Many times the natural unemployment rate or NAIRU ("Non-Accelerating Inflation Rate of Unemployment") is used to estimate economic cycles. Following the above mentioned study of Ball and Mankiw, in which they demonstrated that NAIRU is in fact very similar to the natural rate, in order to estimate its value in case of Romanian economy, we rewrite the Phillips curve equation as follows:

$$\Delta \pi = aU^* - aU + v \tag{1}$$

where a and U^* are parameters, $\Delta \pi$ is the deviation of actual inflation, π, from expected inflation, πe, and v is shock on supply side. U^* is named natural rate of unemployment. In case of accepting adaptive expectations, the expected inflation is a weighted average of the past inflation rates. The simplest solution is to consider expected inflation to be equal to the registered inflation in previous period, $\pi e = \pi_{-1}$. Supposing that U^* is constant and U is uncorrelated with v, then the value of U^* can be estimated by regressing the change in inflation, $\Delta \pi$, on a constant and unemployment U. So, the ratio of constant term, noted as $m=aU^*$, to the absolute value of the unemployment coefficient, noted as a, is an estimate of U^*.

Applying this exercise for annual US data in period 1960-2000, measuring inflation with the consumer price index, Ball and Mankiw reported a constant term of 3.8 and an unemployment coefficient of -0.63. The resulted NAIRU estimate in case of American economy was 6.1%. Applying the same procedure in case of Romanian economy on quarterly data for the period 1994-2005 (QI1994 – QIV2005) we obtained a value of 5.4

for the constant term m and an unemployment coefficient of -0.64. These values correspond to a NAIRU estimate of about 8.4%. We note the sensibility of the two parameters (m and respectively a) to changes in frequency of data (on rule annual, quarterly or monthly data) and period used. For instance, in case Romanian economy, using annual data for the period 1991-2005, we obtained a value for the constant term (m) of 111.8 and an unemployment coefficient of -14.3. Based on these values resulted a NAIRU estimate of around 7.8% (in case monthly data for the period December 1991 – December 2003, we estimated a value for the constant term of 0.124 and an unemployment coefficient of -0.023; the implicit computed value for NAIRU being around 5.4%).

Many economists contest however the assumption of a constant NAIRU and a growing literature tries to estimate persistent movements in natural rate. The main hypotheses are based on idea that changes in U* are long-term shifts in the unemployment-inflation relation, but the shock v captures short-run fluctuations. Following again the Ball and Mankiw's methodology, we used for application the following equation obtained by rearranging terms:

$$U^* + (v / a) = U + (\Delta \pi / a) \tag{2}$$

Its right-hand side can be computed from statistic data, generating in this way an estimate of U* + (v / a), which in fact measures the shifts in the Phillips curve. The authors noted that U* represents the longer-term trends and v/a is proportional to the shorter-term shocks. Consequently we can try to extract U* from U* + v/a using a standard approach to estimating the trend in a series. On the rule, in literature it is used the Hodrick-Prescott filter (Hodrick and Prescott, 1997), noted below as HP. In case of HP filter, we must choose two parameters: the Phillips curve slope, a, and respectively the smoothing parameter λ (this makes the trend, U*, to be smoothed and not with large oscillations, by replacing the banal procedure of fitting every movement in U* + (v / a)). We note that the selection of a value for parameter λ is very arbitrary.

In case of our experiment on the Romanian economy, we used in case of annual series 14.3 for coefficient a, value already obtained previously by regressing Δπ function of one constant and the actual rate of unemployment, U. This value can be also interpreted in relation with the disinflation cost (so, it means for the transition period, characterized generally by high level of inflation, that the inflation decrease by 10 percentage points generated in average 10/14.3 = 0.70 percentage points of unemployment per year). Regarding the selection of HP parameter λ, in literature there are reported numerous experiments. However, certain conclusions were outlined, but they derived only based on empirical analysis. So, in specialized literature there are recommended a number of values for parameter λ, as follows: 100 in case of annual series (other authors suggest value 1000 in order to obtain a more smoothed trend); 1600 in case of quarterly series; and 14400 in that of monthly series.

In fact, HP filter is equivalent to an interpolation method. Therefore, given a time series, it is natural to consider as candidate every other method permitting to estimate a smooth trend. In our exercise on Romanian economy during transition period, we used three procedures. They can be found within sources-packages in *MathCAD* referring to the

classes "Polynomial Regression" and respectively "Smoothing Data". Then we used them in order to estimate the trend of U*. The concrete estimation functions we chosen are:

- *regress* (vx, vy, k) returns a vector which *interp* uses to find the *k*th order polynomial that best fits the *x* and *y* data values in **vx** and **vy**; it generates a vector permitting interpolation, finally expressed by function *interp* (vs, vx, vy, x)); *k* is a positive integer specifying the order of the polynomial we want to use to fit the data (usually it is recommended to choose $k < 5$);

- *loess* (vx, vy, span) returns a vector which *interp* uses to find a set of second order polynomials that best fit a neighborhood of the *x* and *y* data values in **vx** and **vy**; it generates a vector permitting interpolation, finally expressed by function *interp* (vs, vx, vy, x)); *span* is a positive real number for specifying how big a neighborhood we want to use (usually it is recommended to select larger values of *span* when the data behaves very differently over different ranges of *x*; a good default value is *span*=0.75);

- *ksmooth* (vx, vy, b) returns an *m*-element vector created by smoothing using a Gaussian kernel to return weighted averages of the elements in **vy**; *b* is the bandwidth of the smoothing window (it should be set to a few times the spacing between *x* data points). In case of the first two procedures **vx** is a vector of real data values in ascending order. These correspond to the *x* values. **vy** is a vector of real data values and they correspond to the *y* values. The number of elements is the same as **vx**. **vs** is a vector generated by *regress* function and respectively by *loess* function. *x* is the value of the independent variable at which we want to evaluate the regression curve. In case of the third procedure **vx** is an *m*-element vector of real numbers and **vy** is an *m*-element vector of real numbers.

For applications in Romanian economy case, we used the following values for parameters: k = 3, span = 1 and respectively b = 5 (indeed, in case of quarterly or monthly series other values must be attributed to parameters). In case of HP filter we used λ=100. Some results of our exercise on Romanian economy using annual series are synthetically reported in Table of Appendix 1. The natural rate of unemployment estimated by simple regression in case of annual data, relation (1), is noted Un in order to not be confused with U* (Un has an unique value of 7.8% for the 1992-2005 period, but U* means the trend in long run of natural rate estimated conforming to all filters used.

Thus, following some existing studies in literature (Staiger, Stock and Watson, 2001; Ball and Moffitt, 2001; Ball and Mankiw, 2002), in order to estimate natural rate of employment we used aside the simple linear trend (Ye) other four trends based on the corresponding filters (see for details Albu, 2005): *regress* (Y_TR), *loess* (Y_TL), *ksmooth* (Y_TK), and *Hodrick-Prescott* (Y_HP). On the base of simulations, we can see in Figure 1 the unfavourable impact of a positive difference between the effective unemployment rate and its natural rate on inflation dynamics (Δπ). In case of linear trend the unemployment gap is ΔU=U-Ye, but in case of the four selected filters it is noted as ΔUR=U-Y_TR, ΔUL=U-Y_TL, ΔUK=U-Y_TK, and respectively ΔUH=U-Y_HP. As we can see from Figure 2, the points in 2D space, ΔU- Δπ, are generally distributed within the second and the fourth sectors (in trigonometric sense) over the right line transcending the origin of coordination axes. Eventual differences (the evading from two mentioned sectors) can be attributed to short run supply shocks.

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Moreover, corresponding to the four filters, we computed the potential (or natural) level of GDP, the output gap, and respectively the correlation coefficient between it and inflation variation, as it is shown in Table of Appendix 2. The level of correlation coefficient between output gap and variance of inflation ($\Delta\pi$), for the whole period 1992-2005, was positive (between +0.588 and +0.638). From Figure 3, we can see that in the first part of transition (before 1998) the inflation is accentuated procyclical relating to output gap (correlation coefficient between +0.669 in case of TL filter and +0.714 in case of HP filter). However, after 1998 it is countercyclical (correlation coefficient between -0.420 in case of HP filter and -0.836 in case of TR filter), that could mean a temporary favourable situation when a growth in output may be accompanied by a negative change in inflation. Indeed, after the accession to EU this favourable correlation will probably change, as is the standard situation in a consolidated market functioning economy. Related to the past evolution more explanation could be extracted in case of considering the dynamic process of real reforming and restructuring of the national economy: a prolonged and hesitant restructuring process of economy in first part of transition (before 1998); and a more determinate and accelerated process of it during last years.
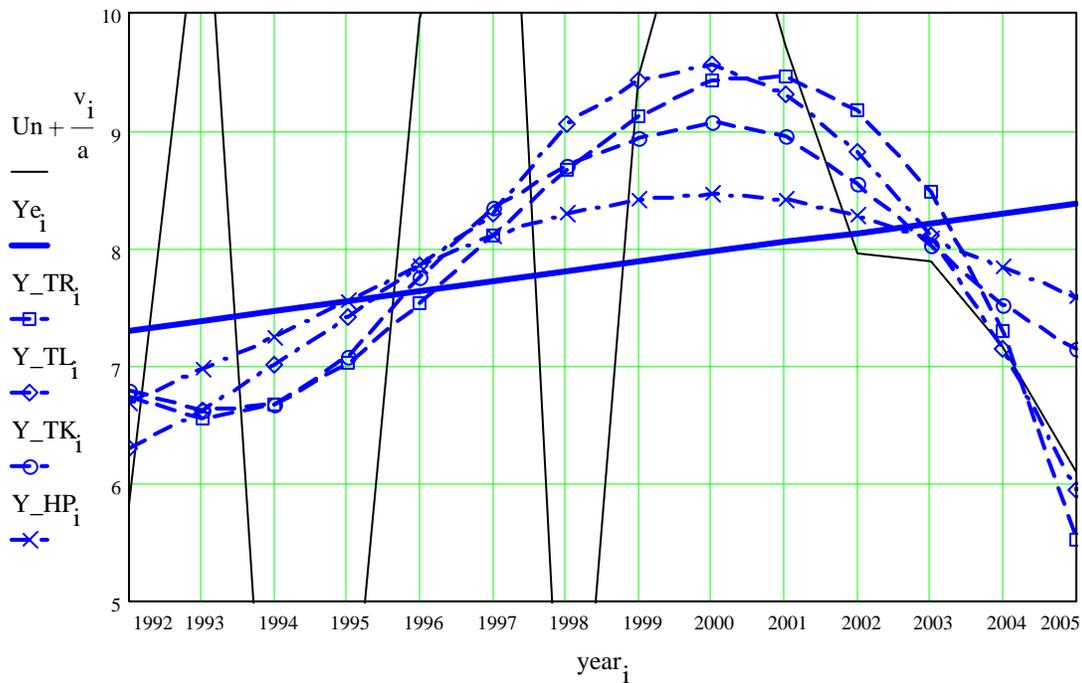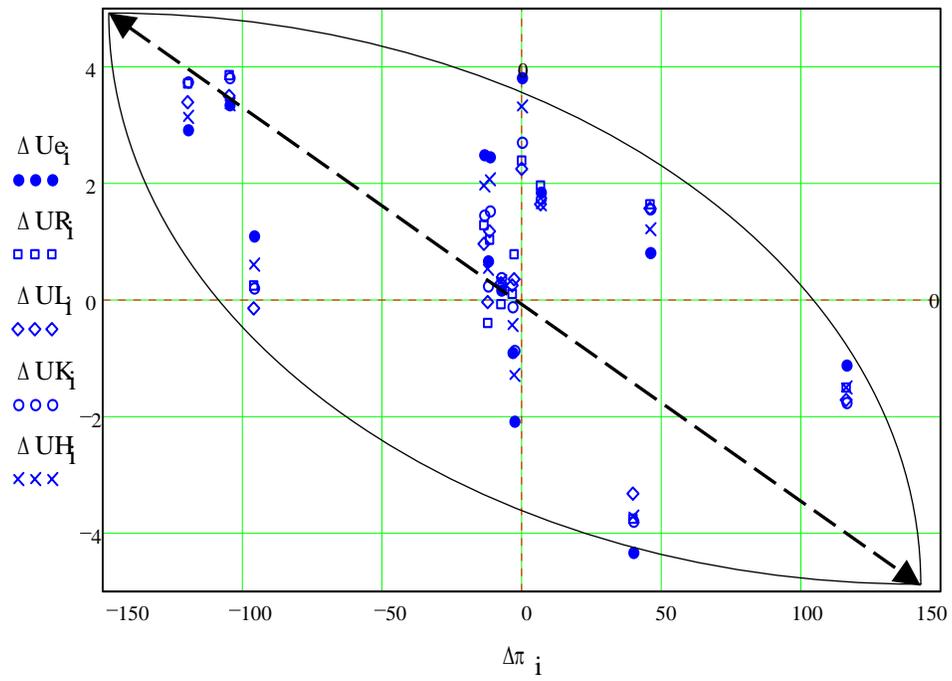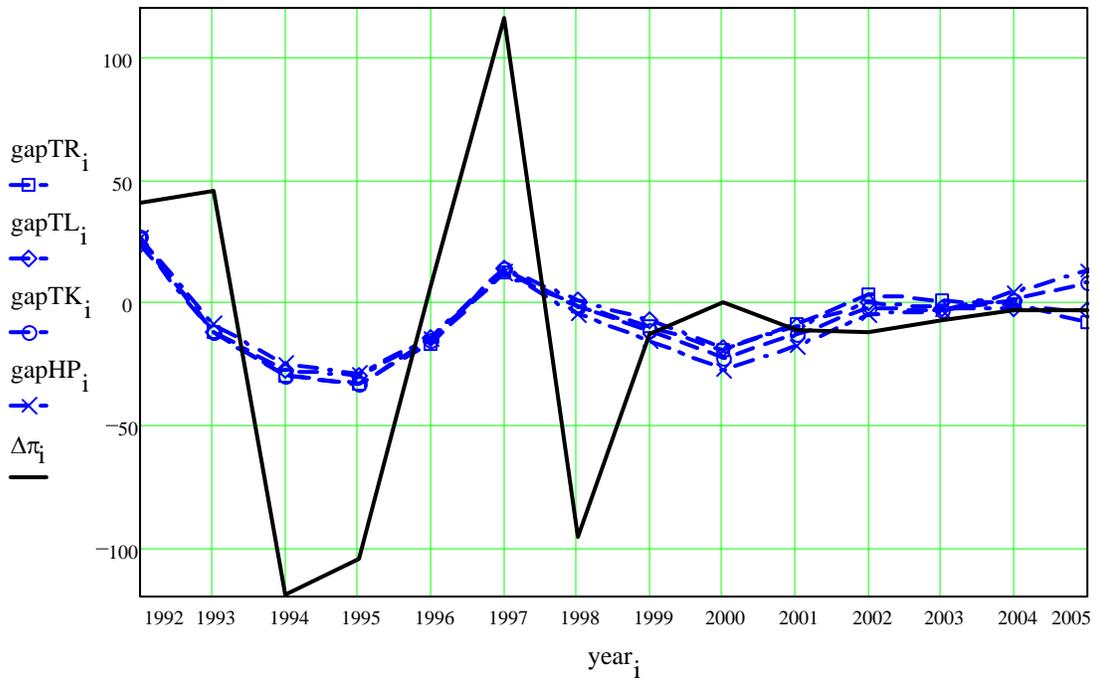

Figure 1.

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Figure 2.

Figure 3.

4. Estimation model of pure productivity

In order to estimate pure productivity in case of Romanian economy, we conceived a particular model having as hypotheses the following two equations (the time subscript, i, being omitted):

$$q = A\,La^{\alpha} = A\,L^{\alpha}\,\mu^{\alpha} = q_{max}\,\mu^{\alpha} \qquad (3)$$
$$s = s0\,La \qquad (4)$$

where A is a constant; q and s are production (GDP) and respectively all costs implied by it (taking into account that the production function has an alone factor, respectively labour); La and L are employment and respectively labour force; q_{max} and s0 are production under the hypothesis of an integral utilization of labour force (La=L) and unitary cost (indeed including also salary) per person employed, La, respectively; α is a positive and sub-unitary coefficient, which determinates how looks the trajectory curvature of production function of employment share, μ, in total labour force, L (μ=La/L).

Let consider for the moment that all variables are evaluated in real terms, therefore under the hypothesis of constant prices (of one year selected as base). The difference between q and s can be interpreted as being the profit or net accumulation, therefore the quantity that stimulates entrepreneurs to make future investments and to develop their affaires. It mainly depends on two factors: employment degree, μ, and respectively coefficient α. Since the evaluation of the employment share in total available labour force is not a problem, to estimate α is an extremely difficult issue, as well as its economic interpretation. Usually economists accept the sub-unitary restriction, as it ensures the concavity of production function. The explanation is: as employment share growths, tending to value one, the average level of labour productivity tends to decrease (as well as the adapting possibilities of entrepreneurs to some permanent changing markets). In order to solve the problem of estimating the production function curvature, we took into account also the long-run price evolution. The hypothesis that we adopted, however very restrictive, is referring to the absence of some pertinent information on the future evolution of prices, as it is the case of an economic system functioning in high inflation (as the Romanian economy in transition period). The remained solution is to compute maximization of the future profit by reporting to actual level of unitary costs (although knowing that in reality this is not the case for the future period). It would be reasonable that even such decision (founded on a highly restrictive hypothesis, like that of basing the maximization of the future profit on maintaining unchanged the specific costs) could yield sweet fruit in the future, in any way larger than in case of no evaluation calculus. The real adjustment to be operated (indeed instantaneously conforming to the "new wave" theory of rational expectations) then when the pressures on cost (such as for instance the trade unions' pressures) will not confirm the effective pre-evaluation. The implicit hypothesis of this "backward dynamics" mode of interpretation is that the effective change of unemployment rate in current period from precedent period corresponds even to the solution of profit maximization under the hypothesis of maintaining unchanged cost between the two consecutive periods, but also to the modification of total price of production exactly at the value effectively registered. So,

the actual level of unemployment rate means even its optimal level, however computed previously on the base of total cost in precedent period together with the index of prices in current period. Since we accept this interpretation, the maximization function will be:

$$Be(\mu) = Q - s = q\,p - s \tag{5}$$

where Be is the anticipated profit (despite of knowing that the planed benefit will not be integrally obtained), Q is value of production in current prices, p. This function admits a maximum given by the solution of the following equation:

$$p = (\mu^{1-\alpha})/\alpha \tag{6}$$

The restriction imposed by this equation allowed us to estimate, only by using a special numeric procedure, the values of α coefficient for the period 1990-2005. Moreover, the model permitted to estimate other synthetic indicators characterizing the evolution of Romanian economy in transition period, as follows: coefficient of using capacity (or the degree of using potential GDP, noted here as qmax), $k=\mu^{\alpha}$; share of profit, $b=1-\mu^{1-\alpha}$, etc.

In order to identify the type of relation between unemployment and productivity, we examined the estimated together data supplied by the above two sub-models (the model of natural rate of unemployment and respectively the "pure" productivity model). Many times the economists are using for the productivity growth an inverted scale to reflect better the two supposed inverse movements: the long-run unemployment trend and productivity growth trend. In case of our application, we maintained the original scales, but used a calibrating procedure to force the two trends to come in a closer region of their co-joint space. Figure 4 demonstrates a strong inverse correlation between the trend of natural rate and the growth rate of "pure" productivity (noted as y_wL90).
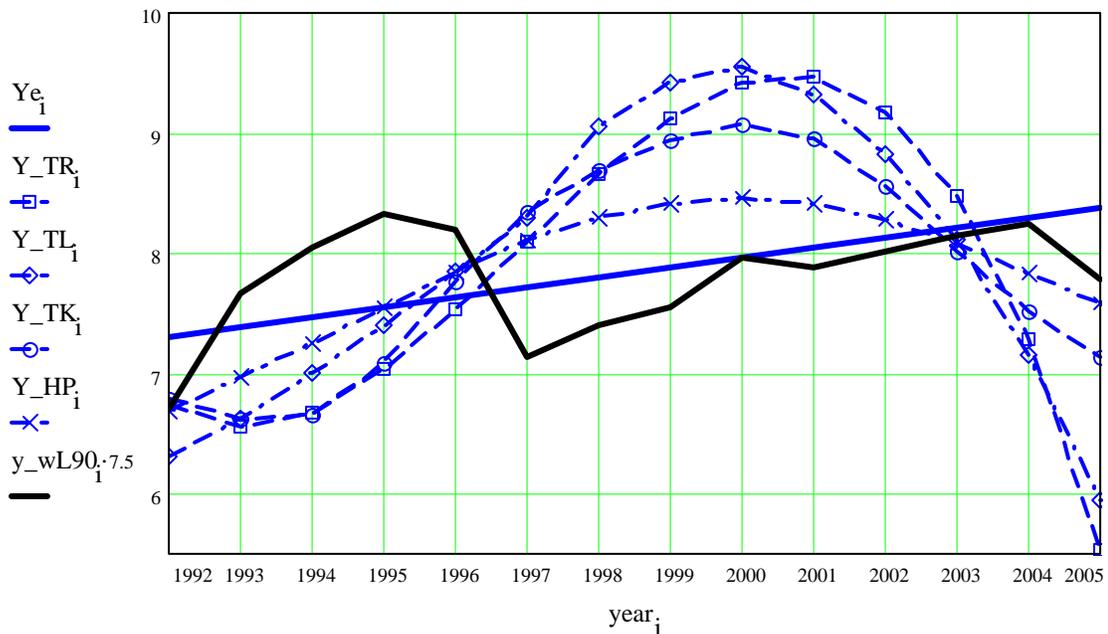

Figure 4.

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Estimated long run trend of NAIRU (in %)

Year	U (1 Jan. of year)	Un + (v / a)	Estimation Function			
			Y_TR_i [1]	Y_TL_i [2]	Y_TK_i [3]	Y_HP_i [4]
1992	3.0	5.8	6.7	6.3	6.8	6.7
1993	8.2	11.4	6.6	6.6	6.6	7.0
1994	10.4	2.0	6.7	7.0	6.7	7.2
1995	10.9	3.6	7.0	7.4	7.1	7.5
1996	9.5	10.0	7.5	7.9	7.8	7.9
1997	6.6	14.7	8.1	8.3	8.3	8.1
1998	8.9	2.2	8.7	9.1	8.7	8.3
1999	10.4	9.5	9.1	9.4	8.9	8.4
2000	11.8	11.8	9.4	9.6	9.1	8.5
2001	10.5	9.7	9.5	9.3	9.0	8.4
2002	8.8	8.0	9.2	8.8	8.6	8.3
2003	8.4	7.9	8.5	8.1	8.0	8.1
2004	7.4	7.2	7.3	7.1	7.5	7.8
2005	6.3	6.1	5.5	5.9	7.1	7.6
U* minim			5.5	5.9	6.6	6.7
U* maxim			9.5	9.6	9.1	8.5
U* average			7.8	7.9	7.9	7.8

[1] $Y\_TR_i = interp$ (RY, t, Y, t_i), where RY = $regress$ (t, Y, 3), Y = U + ($\Delta\pi$ / a)

[2] $Y\_TL_i = interp$ (LY, t, Y, t_i), where LY = $loess$ (t, Y, 1), Y = U + ($\Delta\pi$ / a)

[3] $Y\_TK_i = ksmooth$ (t, Y, 5), Y = U + ($\Delta\pi$ / a)

[4] Y_HP_i = Hodrick-Prescott filter (λ = 100)

Estimated level of output gap in case of the four filters, in 1990 prices (10^9 ROL)

	GapTR	GapTL	GapTK	GapHP
1992	26.3	23.2	26.6	26.0
1993	-12.4	-11.9	-11.9	-9.2
1994	-29.9	-27.3	-30.0	-25.3
1995	-33.5	-30.2	-33.0	-29.0
1996	-17.4	-14.5	-15.4	-14.6
1997	12.1	13.7	14.0	12.2
1998	-1.9	1.2	-1.6	-4.8
1999	-10.0	-7.7	-11.6	-15.6
2000	-19.5	-18.4	-22.3	-27.3
2001	-8.8	-10.1	-13.1	-17.8
2002	3.3	0.2	-2.2	-4.6
2003	0.7	-2.7	-3.5	-3.0
2004	-1.1	-2.5	1.2	4.3
2005	-7.8	-3.6	8.5	13.1
Correlation coefficient ($\Delta\pi$, output gap)	0.63560	0.63779	0.63614	0.58753