STUDY ON THERMAL MODEL FOR CALCULATING TRANSFORMER HOT SPOT TEMPERATURE

JAMAL ALI RAMADAN DOFAN

A thesis is submitted of the fulfillment of the requirements
for the award of the degree of Master of Electrical & Electronic
Engineering

Faculty of Electrical & Electronic Engineering
University Tun Hussein Onn Malaysia (UTHM)

may 2011

ABSTRACT

A power transformer is a static piece of apparatus with two or more windings which, by electromagnetic induction, transforms a system of alternating voltage and current into another system of voltage and current usually of different values and at same frequency for the purpose of transmitting electrical power.

The hot spot temperature depends on instantaneous load and ambient temperature, winding design and also cooling model. There are two possible methods for hotspot temperature determination. The first method is to measure the hot spot temperature using a fiber optic, and other is to calculation the hotspot temperature using transformer thermal models. It was noticed that the hot spot temperature rise over top oil temperature due to load changes is a function depending on time as well as the transformer loading (overshoot time dependent function). It has also been noticed that the top oil temperature time constant is shorter than the time constant suggested by the present IEC loading guide, especially in cases where the oil is guided through the windings in a zigzag pattern for the *ONAN* and *ONAF* cooling modes. This results in winding hottest spot temperatures higher than those predicted by the loading guides during transient states after the load current increases, before the corresponding steady states have been reached.

This thesis presents more accurate temperature calculation methods taking into account the findings mentioned above. The models are based on heat transfer theory, application of the lumped capacitance method, the thermal-electrical analogy and definition of nonlinear thermal resistances at different locations within a power transformer. The methods presented in this thesis take into account all oil physical parameters change and loss variation with temperature. In addition, the proposed equations are used to estimate the equivalent thermal capacitances of the transformer oil for different transformer designs and winding-oil circulations. The models are validated using experimental results, which have been obtained from the normal heat run test performed by the transformer manufacturer at varying load current on a 250-MVA-ONAF-cooled unit, a 400-MVA-ONAF-cooled unit and a 2500-KVA-ONAN-cooled unit. The results are also compared with the IEC 60076-7:2005 loading guide method.

Keywords: power transformers, hot spot temperature, top oil temperature, non-linear thermal

ABSTRAK

Transformator daya adalah sebagai apparatus yang tidak bergerak dengan dua atau lebih gulungan melalui aruhan electromagnetic mengubah viltan and arus yang ada dalam system kepada system yang lain dengan frekuansinya sama dengan voltan dan arus yang biasanya nilai-nilai berbeza, dengan ini bertujuan untuk penghantaran tenaga eletrik.

Suhu panas transfomator daya ini adalah bergantung pada beban sekitar dan suhu alam, reka bentuk dan juga model pendindingan. Dua kaedah yang mungkin untuk menentukan suhu panasnya. Kaedah pertama adalah mengukur suhu panasnya dengan menggunakan serat optic. Selain itu, mengira suhu panasnya dengan menggunakan terma model transfomator. Didapati bahawa suhu panasnya akan meningkat lebih daripada suhu minyak kerana perubahan beban merupakan fungsi yang bergantung pada waktu apabila pembebanan transfor (masa yang melebihi bergantung pada fungsi). Didapati juga bahawa masa malar yang diperlu oleh suhu minyak adalah lebih pendik daripada masa malar yang dicadangkan oleh panduan IEC, terutamannya dalam kes di mana minyak yang dipandu melalui gulungan dalam pola zigzag ONAN and ONAF mod pendindingan. Hal ini demikian bahawa suhu yang tertinggi kaji lilitan tempat adalah tinggi daripada semua yang diramalkan dalam panduan tersebut, manakala keadaan transiens selepas kenaikan bebean arus sebelum Negara-negara sepadan telah tercapai.

Tesis ini mengajikan cara pengiraan suhu yang lebih tepat dengan mempertimbang cara-cara yang disebut di atas. Model tersebut berdasarkan pada teori penukaran haba, aplikasa kaedah lumped capacitance, analogi terma-kuasa dan definisi pertahanan terma nonlinier di lokasi yang berbeza dalam sebuah transfomator kuasa. Kedah yang dikaji dalam tesis juga mengambil kira semua perubahan fizikal parameter miyak dan kehilangan veriasi dengan suhu. Selain itu, persamaan yanh dicadangkan diguna untuk penganggaran capacitance terma serta untuk desain transformator yang lain dan peredaran berliku minyak. Model diaktifkan menggunakan hasil eksperimen, yang telah diperolehi daripada uji coba panas biasa dilakukan oleh pengeluar transformer di pelbagai arus beban pada unit 250-MVA-ONAF pendingin, unit 400-MVA-ONAF-disejukkan dan 2500 -KVA-Onan-cooled unit. Hasilnya juga dibandingkan dengan kaedah 60076-7:2005 panduan loading IEC.

Kata Kunci: kuasa transformator, suhu panas, suhu minyak atas, non-linear terma

CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDIXES

List of symbols and abbreviations

A	Area
A1	A constant
A2	A constant
A3	A constant
A4	A constant
A5	A constant
A6	A constant
A7	A constant
A8	A constant
A9	A constant
Bp	Overshoot factor (maximum of the function $f_2(t)$
C	A constant
C_1	A constant
C_{oil}	Specific heat capacity of oil
g_r	Rated average winding to average oil temperature gradient
G_r	Grashof number
$f_2(t)$	Normalized time variation of hot-spot temperature rise above top-oil
H	Heat transfer coefficient
H	Hot spot factor
H_{HS}	Per unit winding height to hot spot
HV	High Voltage
i	Electrical current
I	Load current
IEC	The International Electrotechnical Commission
IEEE	The Institute of Electrical and Electronics Engineers
K	Oil thermal conductivity
Kg	Kilogram

K	Load factor
L	A characteristic dimension length, width or diameter
LV	Low voltage
MVA	Megavoltampere
KVA	Kilovoltampere
N	A constant
Nu	Nusselt number
AN	Air natural
ON	Oil natural
ONAN	Oil natural and air natural
ONAF	Oil natural and air forced
OFAN	Oil forced and air natural
OFAF	Oil forced and air forced
$P_{Dc,pu}$	DC losses per unit value
$P_{addy,pu}$	Eddy losses, per unit value
$P_{l,pu}$	Load losses, per unit value
P_O	No-load los
P_S	Stray losses, watts
P_W	DC losses, watts
$P_{wdn,pu}$	Winding losses, per unit value
$P_{wdn,pu}(hs)$	Winding losses dependence on temperature
P_E	Eddy losses, watts
P_{TH}	Thermal power
q	Heat generation
q_{fe}	Heat generated by no-load losses
$q_{fe,rated}$	Heat generated by rated no-load losses
q_l	Heat generated by load losses
$q_{l,rated}$	Heat generated by rated load losses
q_{st}	Heat generated by the stray losses
q_{tot}	Heat generated by total losses

$q_{tot,rated}$	Heat generated by rated total losses
q_{wdn}	Heat generated by winding losses
$q_{wdn,rated}$	Heat generated by rated winding losses
R	Ratio load losses at rated current to no-load losses
R_{el}	Electrical resistance
R_{th}	Thermal resistance
$R_{th,rated}$	Rated thermal resistance
$R_{th-fe-oil}$	Non-linear core to oil thermal resistance
$R_{th-hs-oil}$	Non-linear winding to oil thermal resistance
$R_{th-hs-oil,rated}$	Rated non-linear winding to oil thermal resistance
R_{th-oil}	Non-linear thermal resistance of the oil
$R_{th-oil-air}$	Non-linear oil to air thermal resistance
R_{th-wdn}	Winding thermal resistance
$R_{th-wdn-oil}$	Non-linear winding to oil thermal resistance
u	Electrical voltage
W	Watt
ρ_{oil}	Oil density
β	Coefficient of thermal cubic expansion of the oil
μ	Oil viscosity
μ_{pu}	Oil viscosity per unit value
μ_{rated}	Oil viscosity rated value
θ	Temperature
θ_{amb}	Ambient temperature
θ_{hs}	Hot spot temperature
$\theta_{hs,lv}$	Low voltage winding hot spot temperature
$\theta_{hs,hv}$	High voltage winding hot spot temperature
θ_{k}	Temperature factor for the loss correction
θ_{oil}	Top oil temperature
θ_{Hs}	Initial winding hottest spot temperature

θ_{To}	Initial top oil temperature
$\Delta\theta_{hs}$	Hot spot temperature rise over top-oil
$\Delta\theta_{hs,rated}$	Rated hot-spot temperature rise over top-oil
$\Delta\theta_{oil}$	Top oil temperature rise over ambient
$\Delta\theta_{oil,rated}$	Rated top oil temperature rise over ambient
$\Delta\theta_{H\backslash A,R}$	Tested or rated hot-spot rise over ambient
$\Delta\theta_{To,R}$	Tested or rated top-oil rise over ambient
$\tau_{oil,rated}$	Rated top-oil time constant
$\tau_{wdn,rated}$	Rated winding time constant
C^0	Degrees Celsius

CHAPTER 1

INTRODUCTION

1.1 Overview

Power transformers represent the largest portion of capital investment in transmission and distribution substations. In addition, power transformer outages have a considerable economic impact on the operation of an electrical network since the power transformers are one of the most expensive components in an electricity system (Susa, 2005). Therefore knowing their condition is essential to meet the goals of maximizing return on investment and lowering the total cost associated with transformer operation. One of the most important parameters governing a transformer's life expectancy is the hot-spot temperature value.

The transformer winding hot spot temperature is one of the most critical parameters in determining the life of transformer insulation, since the highest ageing rate occurs at the hottest point which experiences the maximum temperature. The hotspot temperature depends on instantaneous load and ambient temperature, winding design and also cooling model. The hot spot temperature is normally located at the top of the winding. The hotspot temperature has to be blow the allowable limit value in order for the transformer to have a normal life expectancy. This is why there are many interests to know the hotspot temperature of transformer especially during actual operation conditions in which the load and ambient temperature vary with time.

The classical approach has been to consider the hot-spot temperature as the sum of the ambient temperature, the top-oil temperature rise in tank, and the hot-spot-to-top-oil (in tank) temperature gradient. There are two possible methods for hotspot temperature determination. The first method is to measure the hotspot temperature using a fiber optic temperature sensors positioned at the predicted hotspot of the windings. The thermal sensors, attached to the end optical fiber, are usually placed between the insulated conductor and spacer, and their signals via optical fibre transmitted out of the tank. However due to the cost which may be difficult to justify in terms of cost for every new transformer. It is not practical for retro-fitting the existing transformers. The main difficulty with direct measurement technique is how to accurately locate the hotspot and possible the sensors.

Another method to identify the hotspot temperature is by using transformer thermal model or calculation method. The calculation of the internal transformer temperature (Hotspot temperature) is a very complicated and difficult task. However, engineers made simplifying assumptions in the generally accepted methods for calculating the temperature of power transformers as reported in the IEEE and IEC standards (IEEE Standard C57.19, 1995 and IEC Standard, 354: 1991). The thermal model of the power transformer is the more accurate methods to calculation the transformer hot spot temperature is using thermal electrical analogy based on heat transfer theory.

1.2 Statement of the Problem

The commonly used models for hotspot temperature calculations are described in the international standards; IEC 354:1991 loading guide (IEC, 1991) and IEEE Std. C57.91-1995 loading guide (IEEE, 1995).

These models are in the form of simple mathematical equations and were developed by simplifying the fundamental heat transfer equations and combining them with simplified transformer loss calculations. According to the loading guides, the hot spot temperature is calculated as the sum of the ambient temperature, the top oil temperature rise in tank, and the hot spot to top oil (in tank) temperature gradient. One assumption has been made in developing these thermal models are the variation in the oil viscosity with temperature can be neglected.

However, when fiber optic probes were taken into use to record local hot spots in windings and oil ducts, it was noticed that the hot-spot temperature rise over top oil temperature due to load changes is a function depending on time as well as the transformer loading (overshoot time dependent function). It has also been noticed that the top-oil temperature time constant is shorter than the time constant suggested by the present IEC loading guide, especially in cases where the oil is guided through the windings in a zigzag pattern for the ONAN and ONAF cooling modes. This results in winding hottest spot temperatures higher than those predicted by the loading guides during transient states after the load current increases, before the corresponding steady states have been reached. The foregoing thermal phenomena will directly cause the transient winding hottest spot temperatures to reach higher values than those predicted by the present IEC and IEEE loading guides for oil-immersed power transformers. Therefore it is important to have a thermal model to consider this transient state so that the hotspot temperature can be calculated with more accurate.

1.3 Objectives of the research

The main objectives of this research are listed as follows:

1. To study a transformer thermal model using thermal electric analogy method that improves the prediction of hotspot temperatures for the power transformer.
2. To introduce of all oil physical parameters change and loss variation with temperature.
3. To do a comparison between the derived thermal model and the international standard models.

1.4 Scope of the Research

The scope of the research work is to study the physical background for power transformer thermal model using thermal electrical analogy based on heat transfer theory, to allow capacity monitoring using data obtained from the normal heat run test performed by the transformer manufacturer. The mathematical model was developed using the MATLAB software package.

1.5 Outline of the research

The remainder of this is organized as follows:

- Chapter 2: literature review

In this chapter, an intensive literature review thermal model of the power transformer using thermal electrical analogy based on heat transfer theory. This chapter also presents the simple thermal diagram used to describe the thermal performance of a transformer and the hotspot temperature of winding where the severest ageing process would occur and also presents transformer losses. Different cooling modes available in are also discussed.

- Chapter 3: Methodology

In this chapter the mathematical equation derivation for transformer oil thermal characteristics and also for transformer thermal model using thermal-electric analogy are explained. The last part of this chapter the derived thermal model is established in Simulink/Matlab software.

- Chapter 4: Results and Analysis

In this chapter the hot spot temperature and top oil temperature have been simulated using SIMULINK model which consider all oil physical parameters change and loss variation with temperature, The simulations are done for three different transformer units and two different tank types (i.e., tanks with and without external cooling) during different load tests. The comparison of hot spot temperature and top oil temperature between the derived thermal model and the IEC thermal model are done in this chapter.

- Chapter 5: Conclusion and recommendation

This chapter gives conclusions of this thesis research and recommendations for future study.

CHAPTER 2

LITRATURE REVIEW

2.1 Introduction

The hotspot temperature represents the most important factor in determining the life of transformer insulation, since the highest ageing rate occurs at the hottest point which experiences the maximum temperature. The hotspot temperature has to be below the allowable limit value in order for the transformer to have a normal life expectancy. This is why there are many interests to know the hotspot temperature of transformer especially during actual operation conditions in which the load and ambient temperature vary with time.

The hotspot temperature depends on instantaneous load and ambient temperature, winding design and also cooling model. There are two possible methods for hotspot temperature determination. The first method is to measure the hotspot temperature using a fiber optic, and other is to calculation the hotspot temperature using transformer thermal models. This project presents the thermal electrical analogy to calculation hot spot temperature.

2.2 Transformer Thermal Diagram

A basic thermal model for power transformers is given in Figure 2.1, where it is assumed that the change in the oil temperature inside and along the winding is linearly increasing from bottom to top. The increase in the winding temperature from bottom to top is linear with a constant temperature difference (g). At the winding top the hot spot temperature is higher than the average temperature (g) rise of the winding. The difference in the temperature between the hot spot and the oil at the top of the winding is defined as (H.g), where H is a hot spot factor. It may be varied from 1.1 to 1.5, depending on short circuit impedance, winding design and transformer size. (Ahmed, 2009).

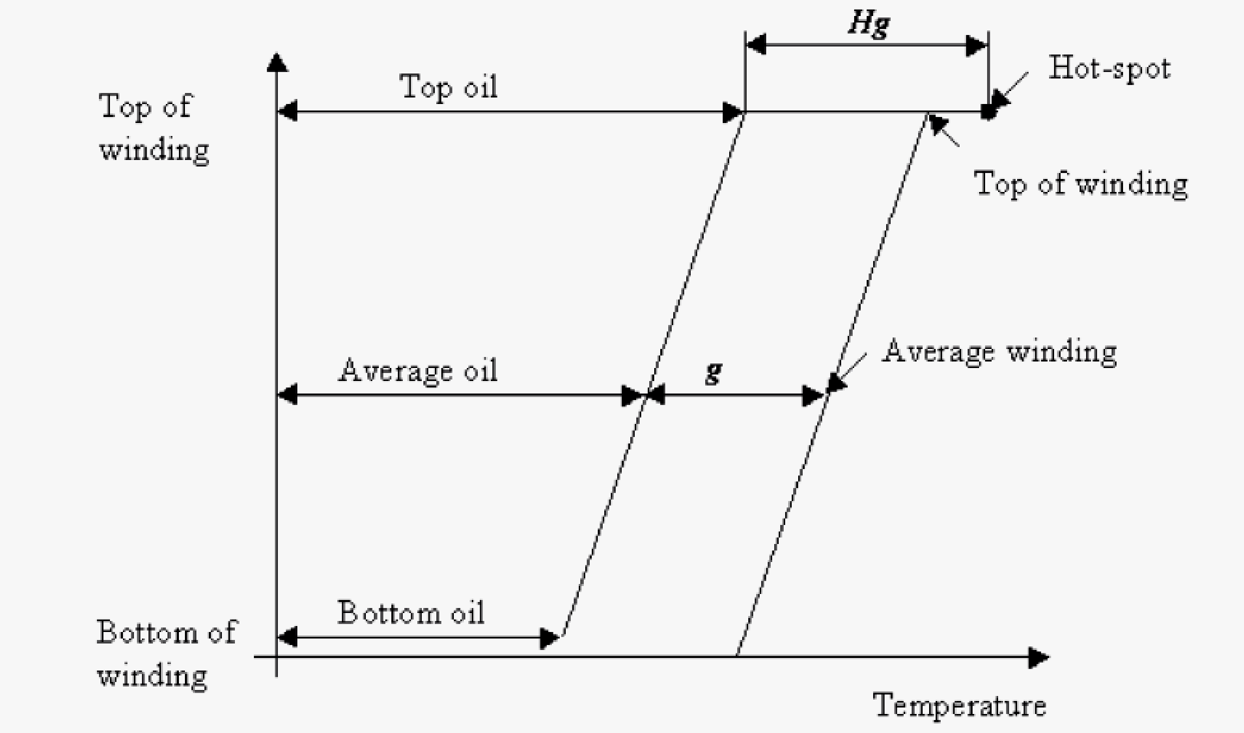

Figure 2.1: Transformer Thermal Diagram

2.3 Transformer Losses

During transformer operation losses are generated and these losses can be categorized into no load losses and load losses as shown in Figure 2.2.

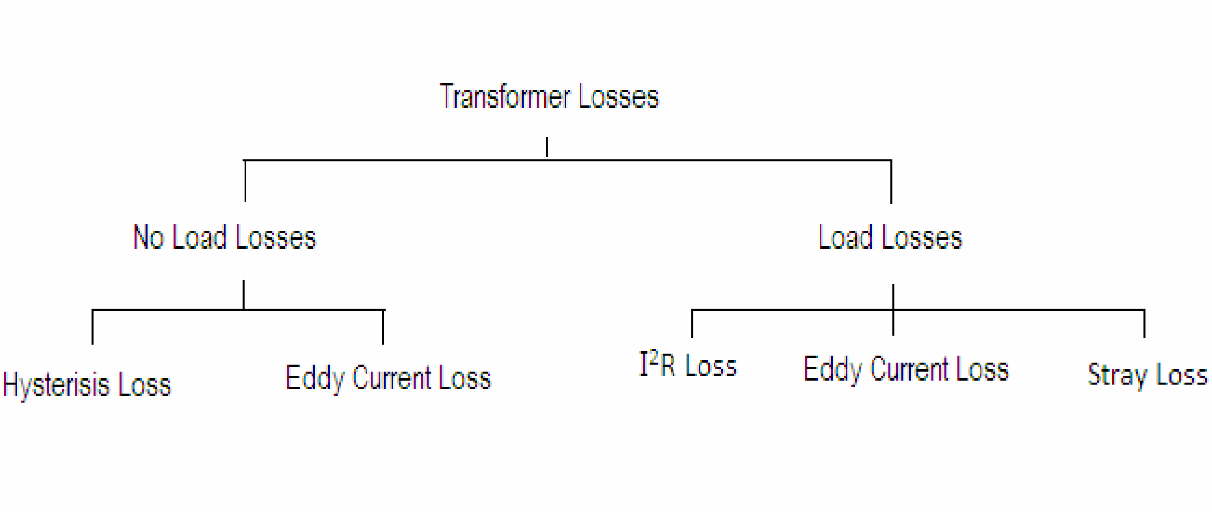

Figure 2.2: Transformer Losses (Kulkani, 2004)

The no load losses, sometimes referred as core loss or iron, are the losses that are caused by the variation of variation flux in core steel materials. This is related to contain two main components, hysteresis loss and eddy current loss. This is related to the magnetic induction and hence the applied voltage. No-load losses are roughly constant and exist whenever the transformer is energized.

The load losses, often called copper loss, consist of I^2R loss, stray loss and eddy current loss. The I^2R loss is due to load current in the winding conductors whereas the stray loss occurs in various transformer parts such as in the core clamps, metallic structural parts, connections, tap changers, tank walls and bushings due to eddy current induced by leakage fields. The eddy current loss is due to the currents induced by the alternating leakage flux impinging on the conductors. These losses are related to the current and hence they are roughly proportional to the load square.

All these losses cause heating in the corresponding parts of transformer and this heat must be taken away to avoid high temperature which will cause deterioration of insulation. The winding made of copper can hold their mechanical strength up to several hundred degrees Celsius without deterioration and the transformer oil dose not significantly degrade blow 140 C^o (Heathcote, 1998), however this is not case foe the paper insulation. The paper insulation deterioration rapidly if its temperature is more than 90 C^o (Heathcote, 1998). Therefore we can say that one of the most important components determining the transformer life is the paper.

Since the temperature distribution is not uniform in a transformer , the part that is operating at the highest temperature will usually be considered in estimating transformer insulation life since it will undergo the greatest deterioration . This temperature is referred as hotspot temperature adds it depends on the ambient temperature, loading condition, transformer winding design and also cooling system.

2 .4 Transformer cooling system

The heat produced in a transformer must be dissipated to an external cooling medium in order to keep the temperature in a specified limit. If transformer insulation is experienced higher temperature than the allowed value for a long time, it will cause rapid degradation of insulation and hence severely affect the transformer life.

In oil immersed transformer, the heat is transferred from the active parts (core, winding and structural components) to the external cooling medium by the oil. The heat from the active parts is transferred by the process of oil circulation. The process of transferring heat from involves three different heat transfer mechanisms which are conduction, convection and radiation (Kulkani, 2004). The conduction process involves the heat transfer between the solid parts, whereas the convection process involves the heat transfer between a solid surface to a liquid or vice versa. The heat transfer by radiation is between solid or liquid to the surrounding ambient temperature.

The most important heat transfer mechanism in an oil immersed transformer is through the convection. The convection process occurs between transformer winding and oil. It is always neglected in thermal calculation because of low surface temperature and small area available on a transformer for radiation process to occur. Four common types of cooling arrangement have been used in the industry and they will be explained in more details.

2.4.1 Cooling Arrangement

2.4.1.1 Natural Cooling of Oil and Air (ONAN)

The simple and most common cooling type used in the practice is ONAN. ONAN refers to Oil Natural Air Natural. The ONAN cooling is achieved when the oil flow through the transformer winding is driven by pressure difference between the tank oil and the cooler oil. This pressure difference is due to a temperature difference between the oil temperature in the tank and the oil temperature in the radiators. This natural circulation of oil sometimes has been referred as a “thermo siphon” effect. The ONAN design is shown in Figure 2.3.and arrows in the figure show the oil flow direction in the transformer.

The term siphon effect occurs when the heat generated in transformer core and winding are dissipated to surrounding oil mainly through the convection process. The density of the oil is inversely proportional to the temperature and is proportional to the pressure and height. As the oil temperature increases, its density reduces. The oil becomes light and due to buoyancy effect it moves upwards towards the top of the tank. Its place is taken by the cool oil from bottom which has a higher density. As the oil enters the cooler, the heat is dissipated along colder surfaces of the cooler, at the same time oil increases its density. The oil then flows downwards through the cooler and enters the bottom of transformer tank from the inlet thus the continuous oil circulation occurs.

The oil velocity in this natural circulation is relatively slow throughout the transformer and radiators. For this reason, ONAN transformers have large temperature difference between top oil and the bottom oil. They also have relatively large temperature difference between the winding temperature and the oil temperature.

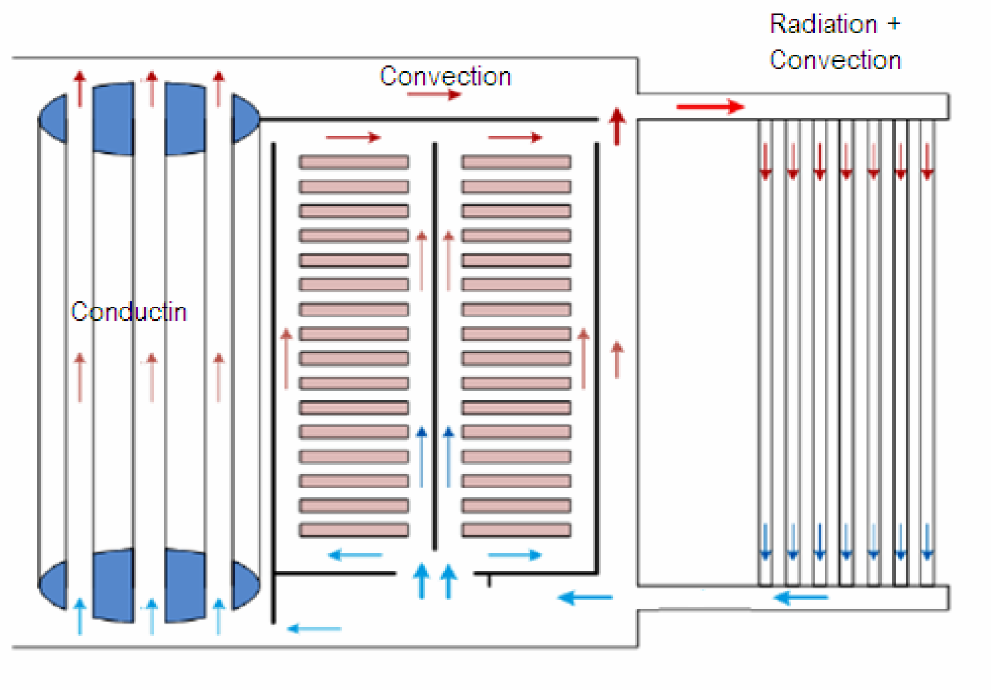

Figure 2.3: ONAN cooling diagram (Heathcote, 1998) and (Kulkani, 2004)

This ONAN cooling mode is normally used for smaller rating transformer (distribution transformer). The ONAN cooling mode has a few advantages. They are (Kulkani, 2004):

- It requires less maintenance and more reliable as no cooler controls are involved.
- It is useful when low noise transformers are needed. The low noise level is easier achieve when the transformer are without the fans.
- No cooler loss due to malfunction of the fans and pumps.

2.4.1.2 Natural cooling of oil and force air (ONAF)

One way to increase the oil circulation rate is by improving the efficiency of the external heat dissipation. This can be done by using the fans to blow air onto the cooling surfaces of radiators. The forced air from the fans takes away the heat from the radiators (cooling) at a faster rate than natural air hence gives a better cooling rate. This leads to a lower average oil temperature (MO) hence increases the capability of the transformer to operate at a higher load. This type of cooling is termed as ONAF (Oil Natural and Air Forced) as shown in figure 2.4. The introduction of the fans to the radiators improves the cooling characteristics of the radiators thereby reducing the number of radiators required to achieve the same amount of cooling. This also leads to smaller overall dimensions of the transformer/cooling design.

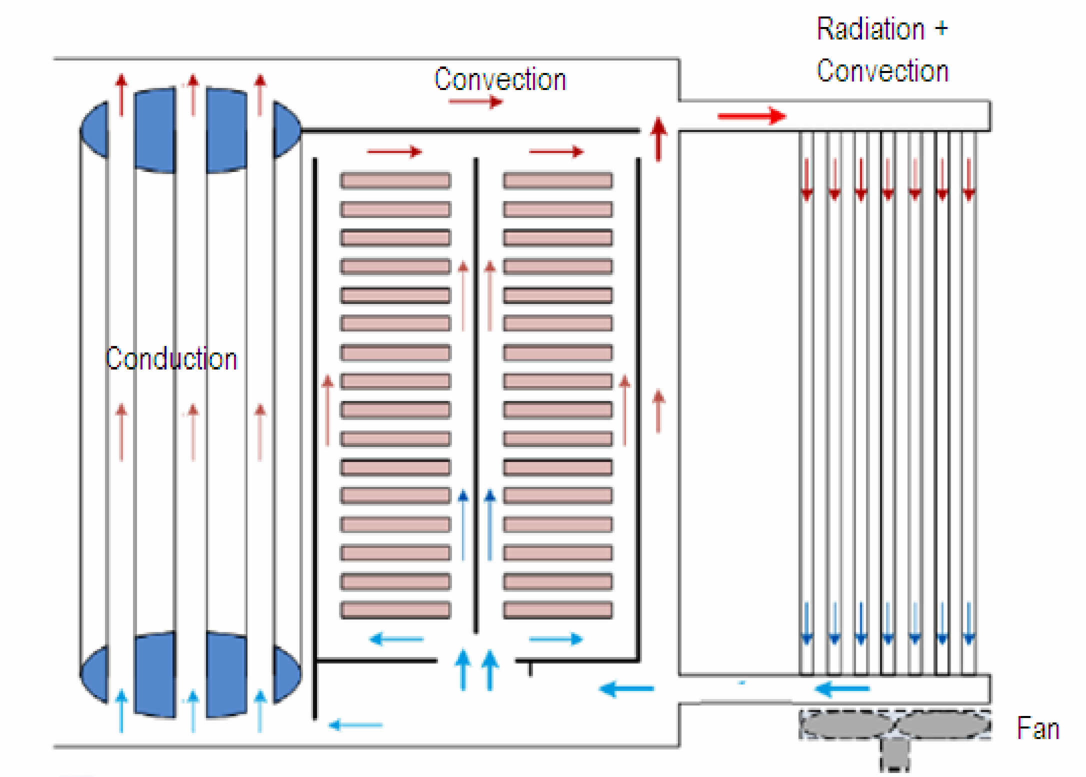

Figure 2.4: ONAF cooling diagram (Heathcote, 1998) and (Kulkani, 2004)

In the ONAF cooling mode the oil circulates through the core and winding as the same as in the ONAN cooling mode. The flow rate inside the winding under ONAN and ONAF cooling arrangement is controlled by the themosiphon effect. Normally this flow rate is relatively low. Because of this, the heat dissipating of oil is low. The heat capacity can be expressed as

$$Q = mC_P(T_{out} - T_{in}) \tag{2.1}$$

Where Q is heat flow in W, m is mass flow rate in Kg/s, Cp is specific heat in J/ (Kg C^{O}), and temperature T_{out} (top oil temperature) and T_{in} (bottom oil temperature) are in C^{o}.

2.4.1.3 Force Cooling of Oil and Force Air (OFAF)

One way to improve the heat dissipation capability is to increase the value of mass flow rate; m and this can be done by using a pump to circulate the oil. Moreover to increase heat transfer rate, fans have to be always operating at the radiators. This improves the heat transfer to the radiators (cooling) and reduces considerably the temperature difference between the top and bottom of the radiators hence lower the oil temperature rise in the top parts of the transformer. This type of cooling is called OFAF (Oil Forced and Air Forced) as shown in Figure 2.5.

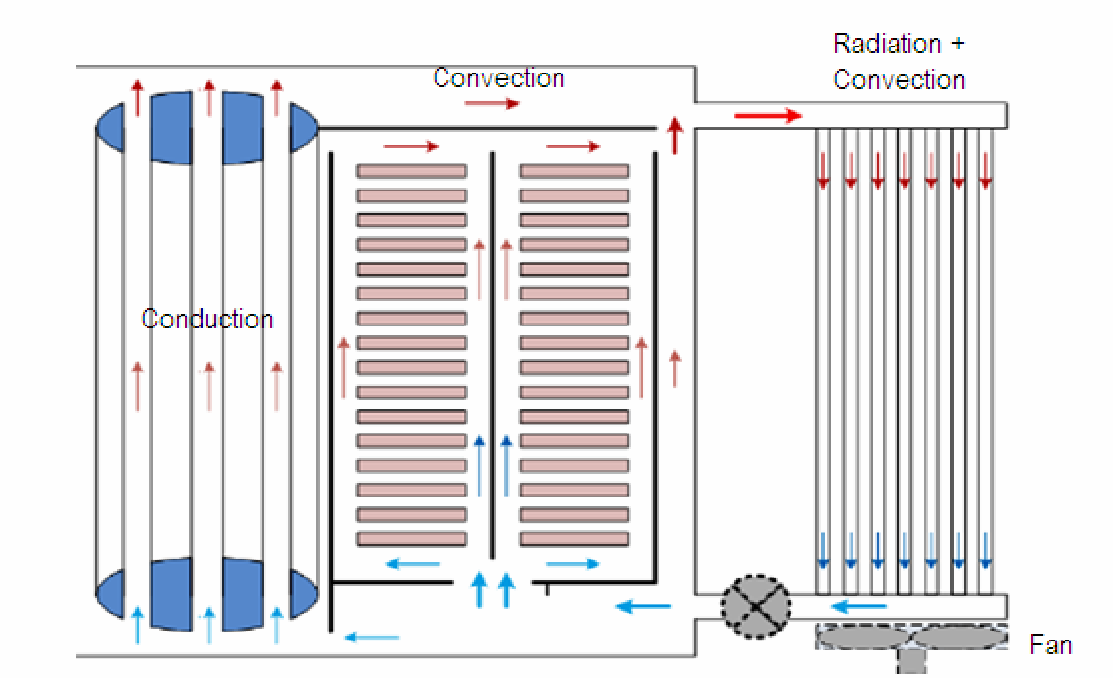

Figure 2.5: OFAF cooling diagram (Heathcote, 1998) and (Kulkani, 2004)

Even through the oil is pumped from the radiators to the transformer tank, the oil in the winding tends to circulate at a velocity closer to the natural oil circulation modes, since most of the oil circulation by the pumps flows in the tank outside of the winding, due to the fact that oil tends to flow in the least resistance path which is the bulk oil space between the winding barriers and to the tank. Therefore the oil temperature rise at the top of the winding may be higher than the measured top oil temperature rise.

2.4.1.4 Force and Directed Cooling of Oil and Force Air (ODAF)

Figure 2.6. Shows a group of conductors surrounded by vertical and horizontal cooling ducts. The heat generated in each conductor must be transferred to the oil to keep the temperature within the limits. The heat flow in the horizontal direction from a central conductor is limited by the similar temperature conductors on either side of it. Therefore the heat can transferred via vertical directions.

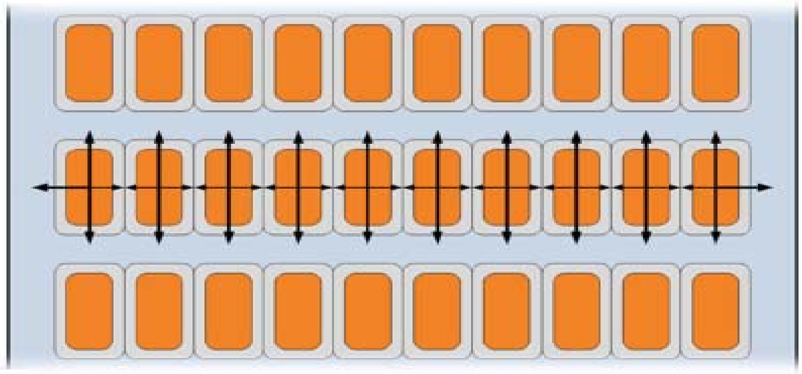

Figure 2.6: Cross section of a disc or helical winding showing heat flow paths (AREVA, 2008)

Naturally the oil tends to rise when it become hot. The vertical ducts provide a natural circulation path for this hot oil. This causes the oil flow through the horizontal ducts is much less than that in the vertical ducts and hence poor heat transfer between the conductors and the oil in the horizontal ducts. However the discs depend on the horizontal oil ducts for their cooling. This is the reason why directing the oil through the winding using block washer to occasionally block the vertical ducts is so important in achieving effective heat transfers from the conductors. The oil flow between the discs for a typical directed oil design is shown in Figure 2.7.

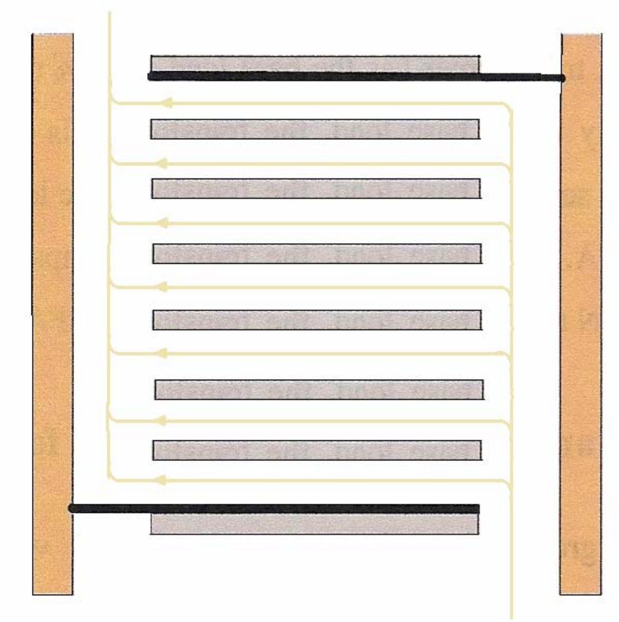

Figure 2.7: Oil flow in a directed flow winding (Heathcote, 1998)

The transformer with directed forced cooling is called ODAF (Oil Directed Air Forced). Atypical arrangement is detailed in Figure 2.8. Where the pumps are used to move the oil into the transformer and block washer are used to direct the oil flows inside the winding. The OD design will result in lower winding gradients than the ON and OF. It also reduces the top oil temperature rise of the winding and therefore the hotspot rise is much reduced compared to the ON and OF cooling mode.

As seen in Figure 2.8, block washer are often added alternately on the inner and outer diameters of the winding. The block washer will direct the oil to flow in horizontal ducts between the discs in order to improve conductor-oil heat transfer.

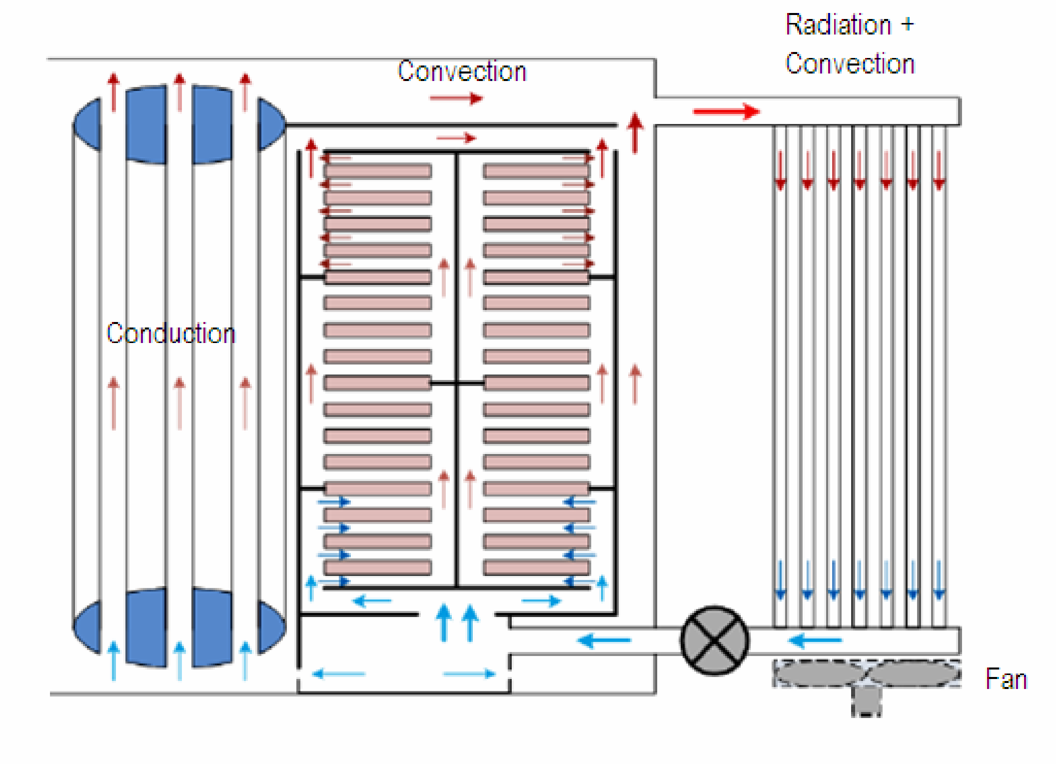

Figure 2.8: ODAF cooling diagram (Heathcote, 1998) and (Kulkani, 2004)

2.5 Transformer thermal models

A thermal model of a power transformer in the form of an equivalent circuit based on the fundamentals of heat transfer theory has been suggested by Swift in (Swift, 2001). The proposed thermal model was established to determine the hot spot temperature. The top oil temperature was calculated from the air-to-oil model. The top oil temperature becomes the ambient temperature for the winding to oil model.

Based on this approach a model which considers the non-linear thermal oil resistance has been introduced by Susa (Susa, 2005). The oil viscosity changes and loss variation with temperature were included in the method. The model was shown to be valid for different transformer units.

2.5.1 Background

In order to analyses the temperature conditions inside a transformer, the analogy between thermal and electrical processes is briefly reviewed below, (Susa, 2004) and (Swift, 2001).

A thermal process can be defined by the energy balance equation:

$$q \times dt = C_{th} \times d\theta + \frac{\theta - \theta_{amb}}{R_{th}} \times dt \qquad (2\text{-}2)$$

where:

q is the heat generation,

C_{th} is the thermal capacitance,

θ is temperature,

R_{th} is the thermal resistance,

θ_{amb} is the ambient temperature.

The equation may be rewritten as follows:

$$q = C_{th} \times \frac{d\theta}{dt} + \frac{\theta - \theta_{amb}}{R_{th}} \qquad (2\text{-}3)$$

Now, if we define a simple electrical RC circuit, as given by Figure.2.10, we can write a similar equation based on both the first Kirchoff's law and Ohm's law:

$$i = C_{el} \times \frac{du}{dt} + \frac{u}{R_{el}} \qquad (2.4)$$

where: i is the electrical current, C_{el} is the electrical capacitance, R_{el} is the electrical resistance and u is the electrical voltage.

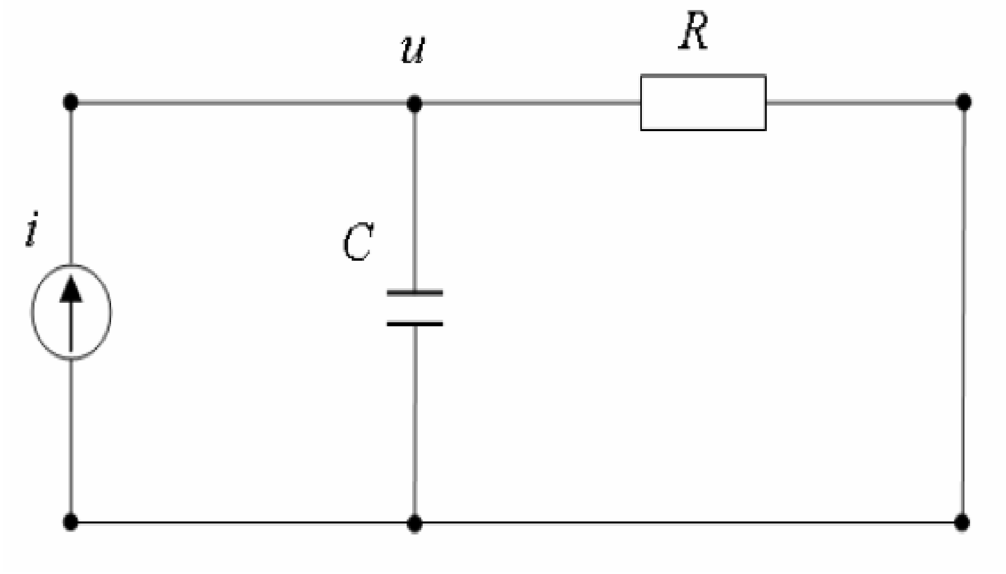

Figure 2.9: An electrical RC circuit

Simply, by comparing equations (2.3) and (2.4) we obtain the analogy between electrical and thermal processes, Table 2.1.

Table 2.1: Thermal-electrical analogy

Thermal		Electrical	
Generated heat	q	Current	i
Temperature	θ	Voltage	u
Resistance	R_{th}	Resistance	R_{el}
Capacitance	c_{th}	Capacitance	c_{el}

The analogous thermal circuit for the electrical circuit, Figure 2.9, is given in Figure 2.10.

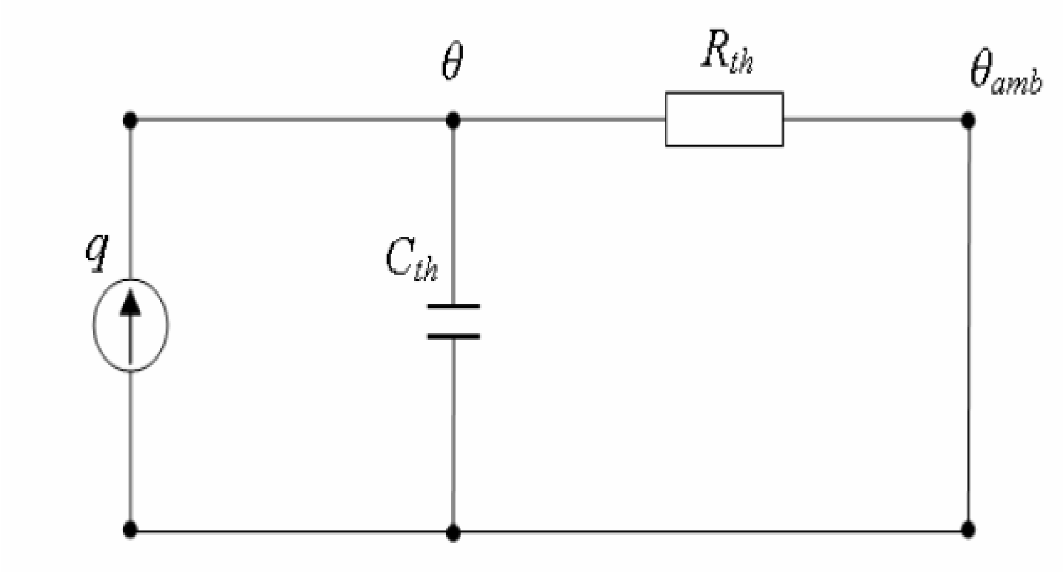

Figure 2.10: The analogous thermal circuit

2.5.2 The non-linear thermal resistance

The nonlinear oil thermal resistance, R_{th-oil} (m^2 K)/W, according to heat transfer theory [(Incropera, 1996) - (King, 1932)], [(Rice, 1923) - (Rice, 1931)] and [(Susa, 2004) - (Swift, 2001)] is given in the following equation:

$$R_{th-oil} = \frac{1}{h \times A} = \frac{\Delta\theta_{oil}}{q}$$
(2.5)

where:

h is the heat transfer coefficient,

A is the area,

$\Delta\theta_{oil}$ is the oil temperature gradient,

q is the heat generated by the corresponding losses.

Hence, the nonlinear thermal resistance is inversely proportional to the heat transfer coefficient, whose dependence on temperature is explained in the text to follow. Based on heat transfer theory, the natural convection oil flow around vertical, inclined and horizontal plates and cylinders can be described by the following empirical correlation,[(Incropera, 1996) - (King, 1932)], [(Rice, 1923) - (Rice, 1931)]:

$$N_u = C \times [G_r \times P_r]^n \tag{2.6}$$

where C and n are empirical constants dependent on whether the oil circulation is laminar or turbulent. The basic values are given in Table 2.2, (Incropera, 1996).

Table2.2: Empirical values for constants C and n

The oil circulation	C	N
laminar	0.59	0.25
turbulent	0.10	0.33

The Nusselt number (N_u) Prandtle number (P_r) and Grashof number (G_r) are described in the following equations, [(Incropera, 1996) - (King, 1932)], [(Rice, 1923) - (Rice, 1931)]:

$$N_u = \frac{h \times L}{k} \tag{2.7}$$

$$P_r = \frac{c_{oil} \times \mu}{k} \tag{2.8}$$

$$G_r = \frac{L^3 \times \rho_{oil}^2 \times g \times \beta \times (\Delta\theta_{oil})}{\mu^2}$$

(2.9)

where:

L is the characteristic dimension, length, width or diameter,

g is the gravitational constant ,

k is the oil thermal conductivity,

ρ_{oil} is the oil density,

β is the oil thermal expansion coefficient,

c_{oil} is the specific heat of oil,

μ is the oil viscosity,

$\Delta\theta_{oil}$ is the oil temperature gradient, (K).

The transformer oil has thermal characteristics strongly dependent on temperature as presented in Table 2.3, where oil viscosity dependency on temperature is most pronounced, (Grubb, 1981) and (Pierce, 1992).

Table 2.3: Thermal characteristics of transformer oil

Temperature θ, C^0	Density ρ, kg/m^3	Specific heat c_{oil} Ws/(kgC^0)	Thermal conductivity k, W/(mC^0)	C0efficient thermal cubic expansion β,1/C^0	Viscosity μ, kg/(ms)
-15	896.885	1900	0.1262	$8.6\text{x}10^{-4}$	0.0694
-5	890.295	1940	0.1247	$8.6\text{x}10^{-4}$	0.0463
5	883.705	1980	0.1232	$8.6\text{x}10^{-4}$	0.0318
15	877.115	2020	0.1217	$8.6\text{x}10^{-4}$	0.0224
25	870.525	2060	0.1201	$8.6\text{x}10^{-4}$	0.0162
35	863.935	2100	0.1186	$8.6\text{x}10^{-4}$	0.0119
45	857.345	2140	0.1171	$8.6\text{x}10^{-4}$	0.0089
55	850.755	2180	0.1156	$8.6\text{x}10^{-4}$	0.0068
65	844.165	2220	0.1140	$8.6\text{x}10^{-4}$	0.0053
75	837.575	2260	0.1125	$8.6\text{x}10^{-4}$	0.0042
85	830.985	2300	0.1110	$8.6\text{x}10^{-4}$	0.0033
100	821.100	2360	0.1087	$8.6\text{x}10^{-4}$	0.0024

By substituting (2.7), (2.8) and (2.9) in (2.6) the following expression is obtained:

$$\frac{h \times L}{k} = C \times \left[\left(\frac{c_{oil} \times \mu}{k} \right) \times \left(\frac{L^3 \times \rho_{oil}^2 \times g \times \beta \times (\Delta\theta_{oil})}{\mu^2} \right) \right]^n$$

(2.10)

The variation of viscosity with temperature is much higher than the variation of other transformer oil physical parameters, Table 2.3, (Blume, 1951) ,(Blume, 1938) ,(Grubb, 1981),(Karsai, 1987) and (Pierce, 1992) Therefore, all oil physical parameters except the viscosity in (2.10) will be replaced by a constant and (2.10) will be solved for the heat transfer coefficient, *h*, as follows:

$$h = C_1 \times \left(\frac{\Delta\theta_{oil}}{\mu} \right)^n \qquad (2.11)$$

where C_1 is assumed to be a constant, expressed as:

$$C_1 = C \times \left[\rho_{oil}^2 \times g \times \beta \times k^{\left(\frac{1-n}{n}\right)} \times L^{\left(\frac{3n-1}{n}\right)} \times c_{oil} \right]^{(n)} \qquad (2.12)$$

and μ is the viscosity, *kg(ms)* The viscosity dependence on temperature is given by the following equation, :

$$\mu = A_1 \times e^{\left[\frac{A_2}{\theta_{oil} + 273}\right]}$$

(2.13)

An example of oil viscosity variation with temperature, compared with the other physical properties of transformer oil, is shown in Figure 2.11.

RFFRERENCES

Ahmed Al-Nadabi and Hisham Al-Riyami (2009) “*Design A Simulation Model to Estimate the Hot Spot Temperature in Power Transformers*” Sixth IEEE International Multi-conference on systems, signals & devices (SSD-09) March 23-26, 2009.

Blume L. F., Boyajian A., Camilli G., Lennox T. C., Minneci S., and Montsinger V. M.,(1951) "*Transformer Engineering – 1st edition*", John Wiley & Sons, Inc., New York, 1938.

Blume L. F., Boyajian A., Camilli G., Lennox T. C., Minneci S., and Montsinger V. M., (1938)"*Transformer Engineering – 2nd edition*", John Wiley & Sons, Inc., New York, 1951.

Grubb R.L., Hudis M., and Traut A. R., (1981)"*A Transformer Thermal Duct Study of Various Insulating Fluids*", Power Apparatus and Systems, IEEE Transactions on, Vol. PAS-100 , No. 2 , February 1981, pp. 466 - 473.

IEC (1991)"*IEC 354-1991 loading Guide for Oil-immersed Power Transformers*"

IEC (1994)"*IEC 72-2 Power Transformers; Part 2: Temperature rise*", 1994-04 second edition

IEEE (1995)"IEEE Std C57.91-1995 IEEE guide for loading mineral-oil immersed transformers"

Incropera F. P., and DeWitt D. P., (1996) "*Fundamentals of Heat and Mass Transfer*", 4th ed., John Wiley & Sons, 1996, pp. 886.

Karsai K., Kerenyi D., and Kiss L., (1987)"*Large Power Transformers,*” Elsevier Science Publishers, 1987, pp 607.

King W.J., (1932)"*The basic Laws and Data of Heat Transmission*", Mechanical Engineering, March-August, 1932, pp.191.

Montsinger V. M., (1916) "Effect *of Barometric Pressure on Temperature Rise of Self-Cooled Stationary Induction Apparatus*", A.I.E.E. Trans., 1916.

Montsinger V. M., and W.H. Cooney, (1924)"*Temperature Rise of Stationary Electrical apparatus as Influenced by Radiation, Convection, and Altitude*", A.I.E.E. Trans., Vol.43, 1924, p. 803.

Montsinger V. M., and Ketchum M.P., (1942) "*Emergency Overloading of Air-Cooled Transformers by Hot-Spot Temperature*", Supplemental Issue Elec. Eng, Decembar 1942.

M.J. Heathcote, J&P Transformer Book (1998). 12th ed. London, Reed Education and Professional Publishing Ltd, 1998.

Nordman H., and Lahtinen M., (2003) "*Thermal overload tests on a 400-MVA power transformer with a special 2.5-p.u. Short time loading capability*", IEEE Transactions on Power Delivery, Vol. 18 , Iss. 1, Jan 2003, pp.107 – 112.

Nordman H., Rafsback N., and Susa, D., (2003) "*Temperature responses to step changes in the load current of power transformers*", IEEE Transactions on Power Delivery, Vol. 18, Iss. 4 , Oct. 2003, pp.1110 – 1117.

Pierce L.W., (1992) "*An investigation of the thermal performance of an oil filled transformer winding*", IEEE Transactions on Power Delivery, Vol. 7, Iss. 3, July 1992, pp.1347-1358.

Pierce, L.W., (1994) "*Predicting liquid filled transformer loading capability*", IEEE Transactions on Industry Applications, Vol. 30, Iss.1, Jan.- Feb. 1994, pp.170-178.

Rice W. Chester, (1923)" Free *and Forced Convection of Heat in Gases and Liquides-II*", Physical Review, Vol. 21, April, 1923and A.I.E.E. Trans., 1923, p. 1288.

Rice W. Chester, (1931)" *Free and Forced Convection of Heat in Gases andn Liquides-II*", A.I.E.E. Trans., February 1924, pp. 131.

Räfsbäck N. T., (2001) “Short-*time emergency overloading of power transformers*, *Bachelor thesis*”, Power transformer company, ABB, Vaasa, 2001.

Susa D., Lehtonen M., and Nordman H., (2004)"*Dynamic Thermal Modelling of Distribution Transformers*", the paper has been approved for publication in the Transactions on Power Delivery

Susa D., Lehtonen M., and Nordman H., (2005)" *Dynamic Thermal Modelling of Power Transformers*", IEEE Transactions on Power Delivery, Vol. 20, Iss. 1, January 2005, pp. 197 – 204.

Swift G., Molinski T.S., and Lehn W., (2001) "*A fundamental approach to transformer thermal modelling- I. Theory and equivalent circuit*", IEEE Transactions on Power Delivery, Vol. 16, Iss. 2, April 2001, pp. 171 – 175.

S.V. Kulkani and S.A. Khaparde, Transformer Engineering (2004): *Design and Practice*. New York,Marcel Dekker,Inc,2004.