Advice by an Informed Intermediary: Can You Trust Your Broker?

Anton Suvorov
Natalia Tsybuleva

Working Paper No 121

CEFIR / NES Working Paper series
Advice by an Informed Intermediary: Can You Trust Your Broker?

Anton Suvorov and Natalia Tsybuleva

October 7, 2008

Abstract

The paper investigates credibility of the intermediary’s advice in a bilateral trade model. A seller and a buyer with private and independent uniformly distributed valuations exchange a unit of good. Their trade is mediated by an intermediary, who at the pre-bargaining stage observes a coarse signal about the buyer’s valuation and reveals some information to the seller. We first show that if the broker gets a fixed fee for each executed transaction, he can transmit his information credibly via cheap talk. Full information revelation can be sustained even when the intermediary’s information about the buyer becomes arbitrarily precise. The transmission of information by the broker increases ex ante welfare of the seller and the broker, but has ambiguous impact on the buyer. If the intermediary observes signals about valuations of both participants, the fully revealing equilibrium exists only under certain restrictions on parameters of the model. Another limit to efficient communication can be imposed by competition between intermediaries. We then consider the mechanism design problem for an informed intermediary, and prove that choosing an appropriate system of two-part tariffs allows the intermediary to secure the same payoff as in the optimal direct mechanism.
1 Introduction

Bilateral bargaining often takes place under asymmetric information. The parties may seek better information about the other side trying to improve their bargaining positions. Natural sources of such information are intermediaries that in many cases implement the transactions. Indeed, a large fraction of real estate transactions are mediated by realtors, most IPOs are mediated by investment banks and art sales are often mediated by specialized dealers, etc. These intermediaries often have superior information about the demand for (or the supply of) the item being sold, and, sometimes, about the private valuation of a particular buyer or seller.

The informed intermediaries may and do affect the outcomes of the bargaining if they reveal their information in the form of advice. In case of art auctions, auction house experts help sellers to set reserve prices and at the same time provide in the pre-auction catalogues a low and a high price estimate for each item, which may signal the seller’s secret reserve price\(^1\). In the real estate sector the Federal Trade Commission survey reports that 20.9% of buyers and 30.5% of sellers use the real estate agent’s advice as the single most influential source of information to determine their first price offers and listing prices, respectively. Despite an apparent conflict of interests, the intermediaries may even advise both sides in the bargaining.

While the consumers welcome the intermediaries’ advice, they are also concerned about the abuse of the private information by the broker. For example, there was a serious debate on whether the financial institutions should be allowed to act as realtors. In December 2000 the Federal Reserve and Treasury Department issued a joint proposal which would allow financial holding companies (FHCs) and financial subsidiaries of national banks to engage in the real estate brokerage. This proposal met a large discontent of the realtors, and one of the concerns was the endangered consumer privacy. The National Association of Realtors (NAR) reports that ”81% of Americans

\(^1\)The seller’s secret reserve price, by convention, lies below the low estimate (Ashenfelter and Graddy (2002))
are worried that their bank could use their private information to sell real estate services to them”.\footnote{The National Association of Realtors (2001).}

NAR’s report further argues that ”FHC-operated real estate brokerage operations could have access to seller-client financial records and use that private credit information to the detriment of a home-seller...”.

Whether intermediaries’ advice about other party of potential transaction can be trusted, who gains and who loses from the intermediary’s ability to consult his clients, what is its overall effect on the welfare – these questions, by no means obvious, will be addressed in this paper. While the role of intermediaries in facilitating bargaining is vastly discussed in the literature,\footnote{Typically, intermediaries have access to some information technology that helps bringing two parties together and thus raises the efficiency of the market organization (Rubinstein and Wolinsky (1987), Yavas (1994), Gehrig (1993)). For instance, Gehrig (1993) considers a search model in which a continuum of buyers and sellers privately informed about their valuations engage in bilateral trade once they meet. In this context an intermediary can reduce the trading frictions in two ways: provide more efficient matching and, by committing to fixed buy and sell prices, replace the inefficient bilateral bargaining.} there are relatively few papers that investigate information transmission by intermediaries.

Biglaiser (1993) shows that the presence of a middleman, who detects a good’s true quality, increases efficiency in a market plagued by adverse selection. One important idea is that a middleman, being a long-lived player, attaches high value to his reputation which prevents him from cheating his customers. Another idea is that being a large player, a middleman has higher incentives to invest in appraising skills. In a similar vein, Dixit (2003) examines provision of information by intermediaries in a dynamic context; the credibility in his model is also reputation-based. Dixit’s model deals with inefficiencies due to moral hazard, rather than adverse selection: the intermediary informs its clients about past behavior of their trading partners.

Another strand of related literature investigates intermediaries who provide certification services. Lizzeri (1999) shows that a monopolistic certifica-
tion intermediary discloses only minimal information consistent with efficient trade, while appropriating a large share of total surplus. However, Lizzeri does not explore incentives of the intermediary to provide the information truthfully — he assumes that the certifier can commit to any disclosure rule. Strausz (2005) investigates the possibility of capture of the certifier by producers whose products he must appraise. Honest certification is sustainable only if the discount factor which determines relative weights of short run and long run profits is high enough. In this case the certifier, willing to stay for a long time at the market, cares about his reputation and does not falsify his reports. The possibility of collusion of intermediary with a seller whose products he certifies is also investigated in Peyrache and Quesada (2007).

All the papers mentioned above are dynamic, while we investigate whether an intermediary’s credibility can be ensured in a static context. There are circumstances in which a static framework may be more appropriate. First, interactions between an intermediary and his clients can be one-shot. Then, since advice given to previous clients is usually not observed by a new client, and its quality is difficult to estimate, reputation-based mechanisms of ensuring credibility may not be very effective.

Information transmission by intermediaries is studied from a somewhat different angle in Baron and Holmstrom (1980) and Baron (1982), who investigate the optimal contracts between an issuer and an investment bank which provides advising and distribution services, assuming that the investment banker is better informed about the capital market than the issuer is. Although these papers investigate an intermediary’s information services in a static context as we do, the mechanisms ensuring credibility are quite different. First, the issuer, who purchases the information services from the investment banker, designs the contract which specifies intermediary’s compensation as a function of his report. Second, the intermediary is directly engaged in selling the issue. Therefore, there is a moral hazard dimension which is not discussed in our paper: the banker may underprice the securities to reduce his efforts necessary for distribution of the issue. Third, Baron
and Holmstrom (1980) and Baron (1982) assume one-sided asymmetric information: only the intermediary has private information about demand for the issue. In our paper the intermediary’s client also possesses private information and this fact plays a key role in sustaining truthful information revelation by the intermediary. This assumption is often realistic. In many circumstances, the seller (issuer) has some important characteristics unobservable by the intermediary but affecting the sellers reaction to the intermediary’s advice: while a low-cost seller may decrease a price upon learning about unfavorable demand, a high-cost seller may simply withdraw from the market. In this paper we show, in particular, that inability to perfectly predict the seller’s behavior creates for the intermediary incentives to truthfully transmit information.

More precisely, we look at how the intermediary can improve efficiency of bilateral trade. As it is well known from Myerson and Satterthwaite (1983), bilateral trade with two-sided asymmetric information is inherently inefficient. We introduce in a basic double-auction model of bilateral trade an intermediary, who has access to some coarse information about one of the parties (say, the buyer). The intermediary can facilitate trade by transmitting this information to the other party at the pre-bargaining stage, thus reducing information asymmetry. First, in Section 3, we assume that the contract between the intermediary and the traders is fixed exogenously: it consists of a fixed per-transaction fee. We show that in this setting the intermediary is able to credibly transmit the information he observes by cheap talk; at the double-auction stage the traders play (piecewise) linear strate-

\footnote{Farrell and Gibbons (1989) were the first to show that cheap talk can matter in bargaining. Similar to our setup, in their model parties may announce at the pre-bargaining stage whether they are "keen" or "not keen" to trade. If saying that one is "keen" makes one’s partner more likely to negotiate, then it is the keenest types (high-value buyers, low-value sellers) who are most willing to say so, and hence cheap talk conveys some meaningful information. Compared to the case without cheap talk, the equilibrium of the modified bargaining game involves more trade when one of the parties has a keen type, and less trade when both parties have intermediate types. See also Mathews and Postlewaite 5}
gies that generalize those derived in Chatterjee and Samuelson (1983). In our set-up the intermediary may be tempted to understate his signal about the buyer’s willingness to trade: given that his messages are expected to be truthful, this lie would induce the seller to bid less aggressively thus increasing the probability of trade. However, if participation in bargaining is associated with some (infinitesimal) costs, sellers with high enough valuation would be dissuaded from participation in bargaining by a pessimistic message about the buyer’s valuation. It is this decrease in participation that deters the intermediary from lying. This mechanism of ensuring credibility plays a key role in our paper and differentiates it from other literature about information transmission by intermediaries.

Moreover, we show that truthful communication can be achieved even if the intermediary’s information is arbitrarily precise – in a seeming contradiction to Crawford and Sobel (1982) – who derive limits for the amount of information that can be transmitted given a certain degree of conflict of interests. The fact that the advisee – the seller – possesses in our model private information plays crucial role in explaining this disparity between our results and those of Crawford and Sobel. Relatedly, Seidmann (1990) shows, in a different context, that the receiver’s possession of private information may ensure arbitrarily precise communication.

We explore welfare consequences of the intermediary’s access to coarse information about the buyer’s willingness to trade and ability to transmit this information to the seller. We show that the probability of trade increases, as well as the aggregate welfare; so does intermediary’s own profit as well as the seller expected utility. The impact on the buyer is two-fold: buyers who are not eager to trade gain from increased probability of trade, while buyers with high willingness to trade lose since the seller starts playing more aggressively against them. The ex ante welfare of the buyer can either increase or decrease, depending on the parameters.

In Section 4 we explore some limits to effective communication. One limit
is posed by the intermediary’s access to some information about both traders. Then, his better knowledge of the seller’s reaction on his report about the buyer’s willingness to trade may give the intermediary incentives to understate the buyer’s eagerness to trade. Truth-telling equilibrium exists now only under some restrictions on parameters. Another limit to effective communication may be imposed by competition between intermediaries. We consider a stylized model in which two intermediaries, each having one "captured" buyer compete for one seller. Now, in contrast to the previous analysis, the intermediary can be tempted to overstate his buyer’s willingness to trade in order to attract the seller. Again, we show that fully revealing equilibria are possible, but only under some restrictions on parameters.

In Section 5 we study a more general problem of designing an optimal trading mechanism by a (partially) informed intermediary (in the same set-up). We first show that in an optimal direct mechanism the intermediary gets the same payoff he would get were his information about the buyer public. We then get back to the decentralized model. The intermediary now offers two-part tariffs to the buyer and the seller, consisting of a participation and a per-transaction fees. The traders then bargain via a double auction mechanism, in which trade happens if the spread between the bids exceeds the per-transaction fee set up by the broker. We show that there exists a separating equilibrium in which the broker selects different contracts for different signals he observes, and at the double-auction stage the traders play (piecewise) linear strategies. The intermediary’s expected profit achieves the upper bound derived via the optimal direct mechanism. The optimal contract specifies a higher transaction fee when the buyer is eager to trade, as well as a (weakly) higher participation fee. Again, comparing the parties ex ante welfare in the models with informed and uninformed intermediaries, we conclude that the intermediary and the seller always gain from the more precise information and thus improved efficiency, while the impact on the

---

5Our paper is thus also related to a recent literature on signaling via the choice of mechanism. See, in particular, papers by Cai et al. (2007), Jullien and Mariotti (2006).
buyer depends on the nature of the intermediary’s signal. Most proofs are given in the Appendix.

2 The Double-Auction Mechanism with an Intermediary

2.1 The Model

We consider a simple model of bilateral trade in the presence of an intermediary. As in a standard model (e.g. Myerson and Satterthwaite (1983)), there is a seller who can produce a unit of good at cost $v_S$, and a buyer who contemplates buying it and has valuation $v_B$. It is common knowledge that the agents’ valuations $v_S$ and $v_B$ are drawn independently from a uniform distribution on $[0, 1]$, but the valuations themselves are the agents’ private information. There is also an intermediary (or a broker) who has no valuation for the good and only implements a transaction between the buyer and the seller. All the parties are risk-neutral and their utility in the absence of trade is normalized to 0.

We model the bargaining game between the buyer and the seller as a double auction (see, e.g. Chatterjee and Samuelson (1983)), and assume that the intermediary charges the traders a commission $\delta \geq 0$ which can depend only on the fact of trade.\footnote{In reality, e.g. for real estate or art brokers, the intermediary’s fee usually depends on the trading price; often, it is a certain percent of the price. However, the simple mechanism, to which we restrict attention for tractability reasons, allows to illustrate a number of interesting effects which would be also observed in a more general setting.} Since the parties are risk-neutral and care only about expected payments, we can assume (without loss)\footnote{It can be easily shown that the division of the transaction fee between the buyer and the seller is irrelevant, i.e. the market is not two-sided (see Rochet and Tirole (2004) for an overview of two-sided markets).} that the intermediary charges a fee $\delta/2$ from each trader if trade happens and nothing otherwise. The (modified) double auction game proceeds as follows. The
seller submits a bid $p_S$ and the buyer submits $p_B$; trade occurs if $p_B \geq p_S + \delta$ at price $p^* = \frac{p_B + p_S}{2}$. Taking into account the intermediary’s commission, the seller gets $p^*_S = \frac{p_B + p_S - \delta}{2}$ and the buyer pays $p^*_B = \frac{p_B + p_S + \delta}{2}$.

An important feature of the model that distinguishes it from the standard bilateral trade setting is that the intermediary is partially informed about one trader’s valuation and can communicate this information to the uninformed party thus affecting her bargaining strategy. More specifically, we assume that the broker learns whether the buyer is "eager" or "not eager" to trade: $v_B \in [w, 1]$ or $v_B \in [0, w]$ for some fixed value $w \in (0, 1)$. This signal is exogenous, no truth-telling incentive constraints are to be satisfied for the intermediary to get this information. There are several justifications that can be given for this assumption. One is that a professional intermediary has more experience than a seller in interpreting the buyer’s observable characteristics or aspects of pre-play behavior, so his knowledge of the buyer’s valuation is more precise. Another interpretation is that (for similar reasons) the intermediary has better information about the demand for the seller’s good, or at least about the distribution of the types of buyers that he can match with the seller.\footnote{We want to avoid confusion with Farrell and Gibbons (1989), who use "keen" and "not keen" in a similar double auction setting (but without intermediary). In their model, where participants of the double auction can exchange cheap talk messages prior to the bargaining stage, the subsets of "keen" and "not keen" types are determined endogenously in such a way that the information on the player’s belonging to one of the subsets could be credibly transmitted in pre-play cheap talk. In our model, the subsets of "eager" and "not eager" types are defined exogenously.}

\footnote{This interpretation seems appropriate in many situations. For example, an art dealer may know better the demand for a particular piece of art than an accidental owner. Similar argument often applies to other types of intermediaries, such as real estate agents, recruiting agencies, etc.}
2.2 The Piecewise-Linear Equilibrium

Let us characterize the traders’ behavior at the bargaining stage. There are many equilibria in the standard double auction game (see Leininger et al. (1989))\(^{10}\), and this is of course true for our modified game as well. In this paper, however, we shall restrict attention to equilibria in piecewise-linear strategies, characterized in Lemma 1 below.\(^{11}\) In particular, we do not allow for a direct pre-play communication between the parties as in Farrell and Gibbons (1989) or Mathews and Postlewaite (1989).

**Lemma 1** Assume it is common knowledge that the buyer’s and the seller’s valuations are independent and distributed uniformly on \([v_B, \bar{v}_B]\) and \([v_S, \bar{v}_S]\) respectively. Let

\[
\begin{align*}
\hat{p}_S(v_S) & : = \frac{2}{3}v_S + \frac{1}{4}\bar{v}_B + \frac{1}{12}v_S - \frac{\delta}{4}, \\
\hat{p}_B(v_B) & : = \frac{2}{3}v_B + \frac{1}{12}\bar{v}_B + \frac{1}{4}v_S + \frac{\delta}{4}.
\end{align*}
\]

(i) Assume \(\hat{p}_B(v_B) < \hat{p}_S(\bar{v}_S) + \delta\). Then at the bargaining stage there exists an equilibrium in piecewise-linear strategies in which

\[
\begin{align*}
\hat{p}_S(v_S) & = \max\{\hat{p}_S(v_S), \hat{p}_B(v_B) - \delta\}, \\
\hat{p}_B(v_B) & = \min\{\hat{p}_B(v_B), \hat{p}_S(\bar{v}_S) + \delta\}.
\end{align*}
\]

Moreover, sets \(\{v_S | \hat{p}_S(v_S) = \hat{p}_S(v_S)\}\) and \(\{v_B | \hat{p}_B(v_B) = \hat{p}_B(v_B)\}\) have positive measure.

(ii) Assume \(\hat{p}_B(v_B) \geq \hat{p}_S(\bar{v}_S) + \delta\). Then at the bargaining stage there exists an equilibrium in which \(\hat{p}_S(v_S) = (v_B + \bar{v}_S - \delta)/2\), and \(\hat{p}_B(v_B) = (v_B + \bar{v}_S + \delta)/2\).

\(^{10}\)Even more equilibria exist if the parties are allowed to engage in cheap talk before submitting the bids (see Farrell and Gibbons (1989) and Mathews and Postlewaite (1989)).

\(^{11}\)Lemma 1 extends to the case of trade with a broker the results of Chatterjee and Samuelson (1983), where the equilibrium in quasi-linear strategies is derived for the standard double-auction game with uniformly distributed valuations.
Proof. See the Appendix. ■

Condition $\tilde{p}_B(\underline{v}_B) < \tilde{p}_S(\hat{v}_S) + \delta$ from the first part of Lemma 1 means that not all types of both parties are sure to trade if they are playing strategies defined in (1)-(2). If this condition is satisfied, there exists an equilibrium in piecewise-linear strategies: those types of seller that trade with probability less than 1 in equilibrium (i.e. $v_S \geq \hat{v}_S$ for some threshold $\hat{v}_S$) use (1), while those that are sure to trade submit the maximum bid that makes trade happen for sure. The buyer behaves in a similar way. The parties trade in this equilibrium if and only if $v_B \geq v_S + \frac{1}{4}v_B - \frac{1}{4}v_S + \frac{3}{4}\delta$.

When all types of both parties are sure to trade given strategies (1)-(2), i.e. $\tilde{p}_B(\underline{v}_B) \geq \tilde{p}_S(\hat{v}_S) + \delta$, the specified piecewise-linear equilibrium breaks down. Then, as the second part of the Lemma shows, there is an equilibrium in which trade happens with probability 1 and the price is constant, $p^* = \frac{v_S + v_B}{2}$. This equilibrium seems the most natural one when the distributions of the buyer’s and the seller’s types are symmetric with respect to each other (i.e. $\underline{v}_S = \underline{v}_B$).

3 Cheap Talk Communication

We assume in this section that the tariff $\delta$ is fixed exogenously.\textsuperscript{12} To make things interesting, we assume throughout the paper that $w > \delta$ (otherwise, the broker’s private information would have no impact). The broker gets the signal about the buyer’s eagerness to trade prior to the double-auction stage. Before the bids are submitted, he can send a message $m \in M$ about the buyer’s eagerness to trade to the seller. The message has no intrinsic cost – it is cheap talk. We assume that the intermediary’s message is secret, so the buyer’s strategy depends on the message(s) that is expected to be sent in equilibrium and does not change if the broker deviates from his equilibrium.

\textsuperscript{12}We do not solve for the optimal fee $\delta$ in this section (which could be done in a straightforward way). However, in Section 5 – in a more general setting – we derive the optimal menu of two-part tariffs.
announcement strategy.\footnote{In the case of public messages results are similar – see footnote 16.}

For a cheap-talk message to have some credibility, the interests of those who send it and of those who attend to it must not be too far apart. In our setting the interests of the intermediary and the seller have an important common element – both want feasible trade to happen. However, the seller faces a trade-off between the probability of exchange and the profit she expects from it, whereas the intermediary wants simply to maximize the probability of trade. Thus, it is not a priori clear whether the intermediary is able to communicate the information he observes to the seller.

We suppose that traders have an infinitesimal cost of submitting a bid. We do not model it explicitly, but assume that if the trader is indifferent between submitting a bid or not (which can happen in equilibrium only if the trader perceives the probability to perform a profitable transaction to be 0), she abstains from bidding.\footnote{This behavior would be rational even if there were a small probability that the broker makes mistakes in determining the buyer’s eagerness – it suffices to require that the probability of mistakes be sufficiently small (for a given cost of submitting a bid).} As we shall see, this abstention of the discouraged types of seller from trade plays a crucial role in disciplining the broker.

### 3.1 Fully revealing equilibria

We shall call an equilibrium \textit{fully revealing} if the intermediary can credibly communicate all his information to the seller, i.e. the seller’s beliefs about the buyer’s valuation induced by the intermediary’s message coincide with the intermediary’s own beliefs. We now want to explore the existence of fully revealing equilibria.\footnote{There are many equilibria in the game. For example, as in any cheap talk game, there exists a babbling equilibrium in which the intermediary transmits a message uncorrelated with his signal and the seller gives it no credibility. Thus, a babbling equilibrium at the first stage followed by some equilibrium of the double auction (with the intermediary) without communication is an equilibrium of the whole game.} Note that since the intermediary’s information is
binary, a binary message space is sufficient to fully transmit the intermediary’s information. Since we are interested in the existence of fully revealing equilibria, we shall assume without loss that there are just two messages, \( M = \{ "eager", "not eager" \} \).

**Proposition 1** There exists a fully revealing equilibrium in which the intermediary truthfully reports the buyer’s eagerness to trade to the seller, and the traders follow quasi-linear bidding strategies specified in Lemma 1.

**Proof.** We only need to check that the intermediary has incentives to tell the truth if the parties expect that he will do so.

If the buyer is not eager to trade, the intermediary can only lose from lying – this would induce the seller to increase the price from \( \bar{p}_S^{\text{not eager}}(v_S) = \frac{2}{3}v_S + \frac{1}{4}w - \frac{\delta}{4} \) to \( \bar{p}_S^{\text{eager}}(v_S) = \frac{2}{3}v_S + \frac{1}{4}w - \frac{\delta}{4} \), which would only reduce the probability of trade.

If the buyer is eager to trade, lying induces the seller to reduce the price from \( \bar{p}_S^{\text{eager}}(v_S) = \frac{2}{3}v_S + \frac{1}{4}w - \frac{\delta}{4} \) to \( \bar{p}_S^{\text{not eager}}(v_S) = \frac{2}{3}v_S + \frac{1}{4}w - \frac{\delta}{4} \) when \( v_S \leq v_S^* = \frac{3}{4}(w - \delta) \), but stop bidding when \( v_S > v_S^* \), i.e. when there is no chance to trade with the non-eager buyer.
If the buyer is eager, $v_B \geq w$, under truth-telling trade happens when $v_B \geq v_S + \frac{1}{4} + \frac{3}{4} \delta$. If the intermediary lies, trade happens when $v_B \geq v_S + \frac{3}{8} w - \frac{1}{8} + \frac{3}{4} \delta$ and $v_S \leq \frac{3}{4} (w - \delta)$. The second condition implies the first, so the gain from lying is represented by the area of $ABC$ triangle equal to $\frac{1}{32} (1 - w)^2$, and the loss by the area of $CDE$ equal to $\frac{9}{32} (1 - w)^2$ (see Figure 1): the loss exceeds the gain for any value of $w \in (0, 1)$.

The proof of Proposition 1 illustrates the intermediary’s fundamental trade-off between a higher probability of participation of the seller and his less aggressive pricing. When it is common knowledge that the buyer is not eager to trade, the sellers with low valuation ($v_S < v_S^*$) charge a lower price than in the equilibrium where the buyer is eager to trade. However, for sellers with higher valuations ($v_S > v_S^*$) probability to trade with a "bad" buyer is zero, so, they lose nothing if they abstain from bidding (abstention would be a dominant strategy for them if the participation cost were modelled explicitly). Since the intermediary is interested in maximizing the volume of trade, he might seem to be tempted to deviate from the equilibrium behavior and always report that the buyer is not eager to trade because such a message makes the seller reduce the price when $v_S < v_S^*$. However, as Proposition 1 shows, the potential gain from such a deviation is more than offset by the loss coming from the abstention of the sellers with $v_S > v_S^*$.\(^{16}\)

The mechanism that ensures credibility of the intermediary’s advise is different from the reputation-based mechanisms studied in the literature (e.g. Biglaiser (1993), Dixit (2003)). While reputation-based models are inherently dynamic, we show that the intermediary’s credibility can be achieved in a

\(^{16}\)If the messages are public, a fully revealing equilibrium also exists, where the traders play the same equilibrium strategies as in the equilibrium with private messages described in Proposition 1.

Once again, the broker can be only tempted to understate the buyer’s eagerness. Assume that he does so. The difference now is that the buyer can detect the broker’s deviation and thus he plays a best response to the seller’s strategy $p_{S, \text{not eager}}(v_S) = \frac{2}{3} v_S + \frac{4}{7} w - \frac{4}{9}$, which can be easily shown to be $p_B = \frac{3}{4} w + \frac{1}{4} \delta$. The probability of trade is the same as after a deviation in the case of private messages, so the deviation is not profitable.
static context. This static nature makes our model related to an extensive literature on expert advise, which largely elaborates on Crawford and Sobel’s (1982) model. The main difference is that we consider a specific cost of false advice that an intermediary is facing – the discouragement of his clients from bargaining – that is not studied in that literature.

### 3.2 Welfare

In our analysis of welfare implications of the intermediary’s access to information about the buyer we restrict ourselves to fully revealing equilibria in which traders play piecewise-linear strategies from Lemma 1. The previous analysis shows that, when the principal possesses information about the buyer that he then truthfully communicates to the seller, gains from trade are realized on a larger set of valuations than in the case of an uninformed intermediary. Hence, the intermediary’s ability to observe and communicate information increases the aggregate welfare. However, as the following Proposition shows, it is not necessarily true that all participants gain from the intermediary’s being informed.

**Proposition 2**  
(i) The intermediary’s expected profit is higher if he is able to observe the buyer’s predisposition to trade and communicate it to the seller.

(ii) The seller with any valuation \( v_S \in [0, 1] \) either strictly gains from the intermediary’s ability to observe the buyer’s predisposition to trade and communicate it or gets the same utility as with uninformed intermediary.

(iii) The buyer with valuation \( v_B \in [0, w] \) either strictly gains from the intermediary’s ability to observe her predisposition to trade and communicate it or gets the same utility as with uninformed intermediary; the

\[17\text{See, e.g., Grossman and Helpman (2001) for a review.}\]
buyer with valuation $v_B \in [w, 1]$ either strictly loses or gets the same utility.

**Proof.** See the Appendix.

The fact that the intermediary gains from communicating his information is quite intuitive: the probability of trade increases and the intermediary’s profit in our setup is proportional to it. It is also quite clear that the seller should gain: she is able to adapt her bidding strategy – bid less aggressively if the buyer is known not to be eager to trade ($v_B \in [0, w]$), or, in contrast, submit higher bids if the buyer is known to be interested in trade ($v_B \in [0, w]$), thus increasing the gains without compromising the probability of trade. Finally, if the buyer is not eager to trade ($v_B \in [0, w]$), she gains from the seller’s learning this: the seller adapts her strategy and submits lower bids. In contrast, if the buyer is eager to trade, she suffers from the seller’s learning this, since the seller starts submitting higher bids.

The following proposition evaluates the impact of the observability of the buyer’s predisposition to trade on the ex ante welfare of the traders. A direct consequence of Proposition 2 is that the seller unambiguously gains. However, the buyer’s welfare can increase or decrease, depending on the characteristics of the information structure.

**Proposition 3**

(i) If $\frac{7 - \sqrt{13}}{18} + \frac{11 + \sqrt{13}}{18} \delta < w < \frac{7 + \sqrt{13}}{18} + \frac{11 - \sqrt{13}}{18} \delta$, the buyer gains from the intermediary’s ability to observe her predisposition to trade and communicate it.

(ii) If $w < \frac{7 - \sqrt{13}}{18} + \frac{11 + \sqrt{13}}{18} \delta$ or $w > \frac{7 + \sqrt{13}}{18} + \frac{11 - \sqrt{13}}{18} \delta$ the buyer loses from the intermediary’s ability to observe her predisposition to trade and communicate it.

**Proof.** See the Appendix.
3.3 Fine Information Structures

To check the robustness of the fully revealing communication, let us consider an alternative information structure: assume now that the intermediary observes to which element $\left[\frac{i}{n}, \frac{i+1}{n}\right], i \in 0, ..., n - 1$ of a uniform $n$-element partition of the unit interval belongs the buyer’s type. When $n$ tends to infinity, the intermediary’s information about the buyer becomes arbitrarily precise. As the following proposition shows, the full revelation result continues to hold in this setting for any $n$.\(^{18}\)

**Proposition 4** Assume that the intermediary observes to which interval $\left[\frac{i}{n}, \frac{i+1}{n}\right], i \in 0, ..., n - 1$ belongs the buyer’s type ($n \in \mathbb{N}$). Then there exists a fully revealing equilibrium in which the intermediary truthfully reports the buyer’s eagerness to trade to the seller.

**Proof.** See the Appendix. \(\blacksquare\)

The result is in sharp contrast with the standard intuition of cheap talk communication models à la Crawford and Sobel (1982), where the sender can communicate his information to the receiver only with some noise, the amount of noise being increasing as the interests of the parties become more diverged. In our model there is an additional complication: the sender (i.e. the broker) is uncertain about the receiver’s (i.e. the seller’s) reaction to his announcement since he does not observe the seller’s type. Despite an apparent conflict of interests (the seller is concerned with both the probability of trade and the expected surplus in case of trade, whereas the broker cares only about the probability of trade), the broker can communicate arbitrarily fine information to the seller. The reason is that the broker’s inability to predict the seller’s reaction prevents him from understating the buyer’s valuation. As Section 4.1 shows, when the broker becomes better informed\(^{18}\) this result can be pushed even further: in the limit, when the broker knows with certainty the buyer’s type, there exists a fully revealing equilibrium in which the broker reveals truthfully the buyer’s valuation, the buyer bids $p_B = \frac{3}{4}v_B + \frac{1}{4}\delta$ and the seller bids $p_S = \frac{3}{4}(v_B - \delta)$ if $v_S < \frac{4}{5}(v_B - \delta)$ and any bid above $\frac{3}{4}(v_B - \delta)$ otherwise.

\(^{18}\)This result can be pushed even further: in the limit, when the broker knows with certainty the buyer’s type, there exists a fully revealing equilibrium in which the broker reveals truthfully the buyer’s valuation, the buyer bids $p_B = \frac{3}{4}v_B + \frac{1}{4}\delta$ and the seller bids $p_S = \frac{3}{4}(v_B - \delta)$ if $v_S < \frac{4}{5}(v_B - \delta)$ and any bid above $\frac{3}{4}(v_B - \delta)$ otherwise.
about the seller’s own valuation and his ability to predict the seller’s reaction is thus improved, no fully revealing equilibrium can exist for some values of parameters.\textsuperscript{19}

4 Limits to the Effective Communication

4.1 Two-sided advise

We have seen in the previous section that when the intermediary gets a signal about the demand for the seller’s good, he can credibly transmit this information to the seller and thus increase the efficiency of trade. We now explore what happens when the intermediary is able to observe signals about the valuations of both participants. Is he still able to share this information with the parties? We shall see that, in contrast to Proposition 1, the answer depends now on the parameters of the problem.

Assume the intermediary observes two signals: the first signal, as before, reveals whether the buyer is eager to trade ($v_B \in [w, 1]$) or not ($v_B \in [0, w]$); the second signal reveals whether the seller is eager to trade ($v_S \in [0, 1-w]$) or not ($v_S \in [1-w, 1]$). In particular, we assume for simplicity that the signals about both traders are symmetric with respect to each other and thus have equal ex ante informativeness. As before, the intermediary’s reports to the parties are assumed to be secret.

**Proposition 5** A fully revealing equilibrium, in which the intermediary truthfully reports each trader’s eagerness to her counterpart and the traders follow quasi-linear bidding strategies specified in Lemma 1. exists if and only if $w \leq \frac{3\sqrt{2}-2}{4} + \frac{3(2-\sqrt{2})}{4}\delta$ or $w \geq \frac{5}{8} + \frac{3}{8}\delta$.\textsuperscript{19}

\textsuperscript{19}The idea that the receiver’s private information can guarantee effective cheap-talk communication even in a situation of sharp conflict of interests dates back to Seidmann (1990). The receiver’s possession of private information may have important implications in more general signaling models. For example, Feltovich et al. (2002) show that it can lead to "countersignaling", that is, the sender’s action becoming non-monotonic in his type.
Proof. See the Appendix. ■

The proof of Proposition 5 shows that when both traders are eager to trade truthtelling constraints may not be satisfied under some parameters. More precisely, when \( w \geq \frac{5}{8} + \frac{3}{8} \delta \), the parties are sure to trade when it is common knowledge that both are eager, so the intermediary has no interest in falsifying this information. When \( w \leq \frac{3\sqrt{2}-2}{4} + \frac{3(2-\sqrt{2})}{4} \delta \), falsification of the report is too costly for the intermediary – too many types who have no chance to trade with non-eager partner refuse to trade after getting a false pessimistic report. In the intermediate case, however, truthtelling cannot be induced.

**Corollary 1** Assume that the intermediary observes the type of both parties but gives advice to only one of them (say, the seller). Then, truthtelling can be induced only if \( w \leq \frac{3\sqrt{2}-2}{4} + \frac{3(2-\sqrt{2})}{4} \delta \) or \( w \geq \frac{5}{8} + \frac{3}{8} \delta \).

**Proof.** Assume that \( \frac{3\sqrt{2}-2}{4} + \frac{3(2-\sqrt{2})}{4} \delta \leq w \leq \frac{5}{8} + \frac{3}{8} \delta \) and both traders are eager to trade. If truthtelling equilibrium did exist, then the players would follow the same strategies as in the case of two-sided advice covered in Proposition 5 (when the buyer knows that the seller is eager plays the same strategy as when she is uninformed about the seller’s type – see Lemma 1 for the specification of equilibrium strategies). Then, the intermediaries incentives to mislead the seller would be the same as in the case of two-sided advice. Hence, as the proof of Proposition 5 shows, he would like to lie to the seller breaking the truthtelling equilibrium. ■

Corollary 1 once again emphasizes that even with one-sided advice, the intermediary must not be too well informed about the advisee in order to have incentives for truthful revelation of information: his ability to predict the seller’s reaction tempts him to understate the buyer’s eagerness to trade.

### 4.2 Competition between Intermediaries

In this section we consider a simple model of competition between intermediaries. Assume there are two intermediaries, \( I_1 \) and \( I_2 \), and each intermediary
has one (captured) buyer, \( B_i, i = 1, 2 \), ready to perform a transaction via the intermediary. The buyers’ valuations \( v_{B_i}, i = 1, 2 \), are independent and uniformly distributed on \([0, 1]\). There is one seller (with valuation \( v_S \), uniformly distributed on \([0, 1]\)) and intermediaries compete for this seller. Once the seller chooses the intermediary, e.g. \( I_1 \), the game proceeds as described in Section 2: the seller and the buyer \( B_1 \) play a double auction game and transaction happens if the bid/ask spread exceeds the per transaction price charged by the intermediary, \( \delta_1 \).

As before, we assume that each intermediary observes whether his buyer is eager to trade or not, \( v_{B_i} \in [w, 1] \) or \([0, w]\). This signal is the intermediary’s private information, neither the seller nor the competing intermediary have any information about the intermediary’s captured buyer (except prior distribution of her types). Before the seller makes her choice, each intermediary announces to the seller whether his buyer is eager to trade or not; these announcements are made simultaneously. Importantly, we assume that these announcements are the only means of competition between the intermediaries: per transaction prices \( \delta_1, \delta_2 \) are assumed to be exogenously fixed at some level \( \delta \). While this assumption is not very realistic, it greatly simplifies the analysis and still allows to illustrate important trade-offs the intermediaries are facing. Prices for intermediaries’ services may be sticky since it may be too costly to adjust them for each transaction. This argument justifies the assumption that prices are fixed for each given transaction and do not depend on the willingness to pay of the particular buyer. Moreover, the assumption that they are equal, \( \delta_1 = \delta_2 \), can be explained by competition between ex ante identical intermediaries (formal modeling of this price competition is beyond the scope of this paper).

Let us check under which conditions there is a fully revealing equilibrium in this game, in which the intermediaries report truthfully and the traders play piecewise-linear strategies from Lemma 1 at the bargaining stage. We assume that if the seller obtains the same messages from two intermediaries, she chooses each of them with equal probability. If one of the buyers is eager,
and the other is not, then it is in the interests of the seller to choose the eager buyer.

**Proposition 6** Fully revealing equilibrium exists in a model with competing intermediaries if and only if \( w \in [0, \bar{w}(\delta)] \) for some threshold \( \bar{w}(\delta) \).

**Proof.** See the Appendix. ■

In the proof we show that an intermediary may be only tempted to overstate his buyer’s willingness to trade in order to attract the seller: the incentive constraint preventing understatement of the buyer’s willingness to trade is never binding. This is in sharp contrast to the case of monopolistic intermediary, who cannot gain from overstatement of the buyer’s eagerness to trade but may be tempted to understate it in order to soften the seller’s bidding strategy.

Given that only "no overstatement" constraint may be binding, the results of Proposition 6 are quite intuitive. Indeed, when \( w \) is small, an intermediary will not want to attract the seller by overstating his buyer’s willingness to trade since the probability of trade of a non-eager buyer with overoptimistic seller is too low in this case.

### 5 Signaling via Two-Part Tariffs

In this section we endogenize the intermediary’s choice of contract. We assume that the intermediary charges the traders two-part tariffs consisting of fixed fees for participation (\( \gamma_B \) for the buyer and \( \gamma_S \) for the seller) and a commission which can depend only on the fact of trade, \( \delta \).

Since the parties are risk-neutral and care only about expected payments, we can assume without loss that the intermediary charges a fee \( \delta/2 \) from each trader if trade

\[20\]We will show that in our setting an appropriately chosen menu of two-part tariffs allows the broker to obtain the same profit as he would get in the optimal complete mechanism, so there is no loss in restricting attention to this class of mechanisms (of course, this claim need not be true under more general assumptions).
happens and nothing otherwise\textsuperscript{21}.

\textbf{5.1 The Optimal Direct Mechanism}

In the absence of information observed by the broker, Myerson and Satterthwaite (1983) have characterized a direct mechanism maximizing the broker’s ex ante profit. They have shown that it is optimal for the broker to implement trade if and only if the buyer’s ”virtual valuation” (i.e. his true valuation minus the information rent required to induce truthtelling) exceeds that of the seller, which for the case of uniform distributions on \([0, 1]\) means that \(v_B \geq v_S + \frac{1}{2}\), and to leave no surplus to the worst buyer \((v_B = 0)\) and to the worst seller \((v_S = 1)\).\textsuperscript{22}

The broker can use the informative signal he gets in designing the optimal mechanism. For example, as Mylovanov (2005) shows, in quasi-linear environments with independent private valuations the informed principal implements an ex ante optimal allocation.\textsuperscript{23} Skreta (2007) shows a similar result in the context of an informed seller problem, where the seller’s private information is correlated with the buyers’ valuations: he shows that the optimal mechanism gives the seller the same expected surplus as he would get after full disclosure of his private information. A similar result obtains in our model. Indeed, by the inscrutability principle,\textsuperscript{24} the upper bound of the broker’s expected profit can be found through the optimal direct truthful mechanism, which specifies the probability of trade and the expected payments from the buyer to the broker and from the broker to the seller as

\textsuperscript{21}We assume that the broker’s commission is divided equally between the parties, but it can be easily shown that the division of the transaction fee between the buyer and the seller is irrelevant, i.e. the market is not two-sided (see Rochet and Tirole (2004) for an overview of two-sided markets).

\textsuperscript{22}These conditions uniquely determine the expected transfers from the buyer to the broker and from the broker to the seller.

\textsuperscript{23}A similar result in somewhat more restrictive context was obtained in Maskin and Tirole (1990) and (1992).

\textsuperscript{24}Inscrutability principle is a generalization of the revelation principle to the informed principal setting – see Myerson (1983).
functions of the three parties’ reports, subject to the incentive compatibility and interim participation constraints. The optimal mechanism\textsuperscript{25} is described in Proposition 7.

**Proposition 7** The mechanism maximizing the broker’s ex ante profit implements trade if \( v_B \geq v_S + \frac{1}{2} \) when the buyer is eager \((v_B \in [w, 1])\) and if \( v_B \geq v_S + \frac{w}{2} \) when the buyer is not eager \((v_B \in [0, w])\). The buyer with valuation \( v_B = 0 \) or \( v_B = w \) gets no ex ante surplus, as well as the seller with valuation \( v_S = 1 \).

**Proof.** See the Appendix. ■

Proposition 7 shows that the broker obtains the same expected profit as he would get if he shared his signal about the buyer’s valuation with the seller and then used an optimal direct mechanism to which only the buyer and the seller would report their valuations. Note that in our setting where the broker’s own announcement can be verified ex post (assuming that the buyer truthfully reveals his type), it is costless for the broker to induce his own truth-telling once the buyer’s truth-telling is secured.

### 5.2 Optimal Two-Part Tariffs

We shall show now that by choosing an appropriate menu of two-part tariffs the intermediary can achieve the same profit as in the optimal direct mechanism. We assume the following timing. First, the broker specifies a set \( \mathcal{T} \) of two-part tariffs \((\gamma_B, \gamma_S, \delta)\). Then, the traders observe their private valuations and the broker learns whether \( v_B \in [0, w] \) or \( v_B \in [w, 1] \). The broker publicly announces a two-part tariff from the set \( \mathcal{T} \).\textsuperscript{26} Next, the parties choose

\textsuperscript{25}The mechanism is uniquely defined by the probability of trade and the expected utility of the buyers and the sellers with valuations \( v_B = 1 \), \( v_B = w \) and \( v_S = 0 \). Transfer functions can be specified in multiple ways, but the expected payment of each type of agent is the same (see Myerson and Satterthwaite (1983)).

\textsuperscript{26}We assume that the broker specifies the menu of tariffs, \( \mathcal{T} \), prior to observing any information mainly for tractability reasons: this assumption allows to avoid the analysis of out-of-equilibrium beliefs that would be necessary if the broker offered an "unexpected"
whether they want to participate in the double auction and if they choose to participate, they pay the participation fees and submit their bids. Finally, if the bids $p_S$ and $p_B$ are submitted and the spread $p_B - p_S$ exceeds $\delta$, the broker implements a transaction at price $p^* = \frac{p_B + p_S}{2}$. We keep assuming that traders have an infinitesimal cost of submitting a bid, and that at the double auction stage they play quasi-linear equilibrium strategies specified in Lemma 1.

As the following Proposition shows, the intermediary can achieve the highest profit characterized in Proposition 7 through an appropriately chosen system of two-part tariffs.

**Proposition 8** There exists a menu of two-part tariffs $\mathcal{T} = \{(\gamma_f^E, \gamma_s^E, \delta^E), (\gamma_b^NE, \gamma_s^NE, \delta^NE)\}$ which gives the broker the same expected profit as the optimal direct mechanism.

- When the buyer is eager ($v_B \in [w, 1]$), the optimal tariff is $\delta^E = \frac{1}{3}$, $\gamma_s^E = 0$, $\gamma_b^E = \left\{\begin{array}{ll} \frac{1}{3}(w - \frac{1}{2})^2 & \text{if } w > \frac{1}{2}, \\ 0 & \text{if } w \leq \frac{1}{2}. \end{array}\right.$

- When the buyer is not eager ($v_B \in [0, w]$), the optimal tariff is $\delta^NE = \frac{w}{3}$, $\gamma_s^NE = 0$, $\gamma_b^NE = 0$.

**Proof.** Assume that the broker sets $\mathcal{T} = \{(\gamma_f^E, \gamma_s^E, \delta^E), (\gamma_b^NE, \gamma_s^NE, \delta^NE)\}$, where the two tariffs are as specified in the Proposition. We need to show that the broker’s choice of tariff once he observes the signal is incentive compatible and that these tariffs lead to the same expected profit as the optimal direct mechanism. Then, the choice of $\mathcal{T}$ is an optimal one.

A) Assume that the buyer is eager, $v_B \in [w, 1]$ and that $w \leq \frac{1}{2}$. If the broker chooses $(\gamma_f^E, \gamma_s^E, \delta^E)$, the seller infers that the buyer is eager (and it is common knowledge between the traders), and trade happens iff $v_B \geq v_S + \frac{1}{2}$ (EDF triangle on Figure 1). The probability of trade conditional on the tariff.
Figure 2:

The buyer being eager is $S_{EDF} = \frac{1}{8(1-w)}$. Thus, the broker’s expected revenue is $ER = \frac{1}{24(1-w)}$.

Assume that the intermediary deviates and chooses $(\gamma^N_B, \gamma^N_S, \delta^N)$. The seller then infers that the buyer is not eager (and this is again common knowledge between the traders), and trade happens iff $v_S \leq \frac{w}{2}$ (ABCD rectangle on Figure 2): the seller’s strategy is to bid $p_S(v_S) = \frac{2}{3}v_S + \frac{w}{6}$ if he believes there is a positive probability of trade and to abstain from bidding otherwise. The seller expects the buyer to bid $p_B(v_B) = \frac{2}{3}v_B + \frac{w}{6}$ and thus no trade to happen iff $v_S > \frac{w}{2}$, so only sellers with $v_S \leq \frac{w}{2}$ submit bids. The buyer’s best response to this strategy is to bid $\hat{p}_B(v_B) = \frac{5w}{7}$, so that all traders who submit bids trade with certainty. The broker’s expected revenue in case of deviation is $\tilde{ER} = \frac{w^2}{6}$ which can be easily checked to be lower than $\frac{1}{24(1-w)}$ for any $w \in [0, \frac{1}{2}]$.

B) Assume that the buyer is eager, $v_B \in [w, 1]$ and that $w > \frac{1}{2}$. If the broker chooses $(\gamma^E_B, \gamma^E_S, \delta^E)$, the seller infers that the buyer is eager (and it is common knowledge between the traders), and trade happens iff $v_B \geq v_S + \frac{1}{2}$. Thus, the broker’s expected revenue is $ER = \gamma^E_B + \gamma^E_S + \delta^E \Pr\{\text{Trade}\} =$
\[ \frac{1}{3}(w - \frac{1}{2})^2 + \frac{w^2}{6}. \]

If the broker deviates and chooses \((\gamma_{BE}^{NE}, \gamma_{SE}^{NE}, \delta^{NE})\), the seller infers that the buyer is not eager, trade happens iff \(v_S \leq \frac{w}{2}\) (the same reasoning as in the previous case) resulting in the broker’s expected revenue \(\overline{ER} = \frac{w^2}{6}\) which is lower than \(ER\).

C) Assume that the buyer is not eager, \(v_B \in [0, w]\), and that \(w \leq \frac{1}{2}\). If the intermediary chooses \((\gamma_{BE}^{NE}, \gamma_{SE}^{NE}, \delta^{NE})\), the seller infers that the buyer is not eager, and trade happens iff \(v_B \geq v_S + \frac{w}{2}\). Thus, the broker’s expected revenue is \(ER = \frac{w^2}{24}\). If he deviates and elects \((\gamma_{BE}^{E}, \gamma_{SE}^{E}, \delta^{E})\), there is no trade and \(\overline{ER} = 0\).

D) Assume that the buyer is not eager, \(v_B \in [0, w]\), and that \(w > \frac{1}{2}\). If the intermediary chooses \((\gamma_{BE}^{NE}, \gamma_{SE}^{NE}, \delta^{NE})\), again trade happens iff \(v_B \geq v_S + \frac{w}{2}\) and the intermediary’s expected revenue is \(ER = \frac{w^2}{24}\). If he deviates and elects \((\gamma_{BE}^{E}, \gamma_{SE}^{E}, \delta^{E})\), there is trade iff \(v_B \geq v_S + \frac{1}{2}\) and \(v_B \geq v_B^*\), where \(v_B^*\) is the type of buyer for whom the expected gain from trade is just sufficient to compensate her for the fee \(\gamma_{BE}^{E}\):

\[ U_B(v_B^*) = \frac{1}{2}(v_B^* - \frac{1}{2})^2 - \frac{1}{3}(w - \frac{1}{2})^2 = 0. \]

Thus

\[ \overline{ER} = \frac{1}{18w}(w - \frac{1}{2})^2 + \frac{1}{3w}(1 - \sqrt{\frac{2}{3}})(w - \frac{1}{2})^3, \]

which can be shown to be smaller than \(ER\) for all \(w \in [\frac{1}{2}, 1]\).

The seller’s expected return can be easily checked to be the same as in the optimal direct mechanism.

5.3 Welfare

It is natural that the intermediary gains from learning a signal about the buyer’s valuation. Looking at how the optimal mechanism takes this information into account one should expect that the seller also gains: the intermediary charges a lower fee when the buyer is not eager and trade happens more often. As for the buyer, the impact of the loss of privacy on her welfare...
is ambiguous: on the one hand, when the intermediary learns that the buyer is not eager to trade, he charges a lower transaction fee than he would in the absence of the knowledge on the buyer’s predisposition to trade (on top of direct benefit this also makes the seller’s bidding less aggressive). On the other hand, the buyer pays a participation fee if she is discovered to be eager to trade, the fee she does not pay if the intermediary is completely agnostic about her willingness to trade. Besides, as in the cheap-talk setting, the seller bids more aggressively. As Proposition 9 shows, which effect dominates is determined by the quality of information: if the signal that the buyer is "eager" is relatively weak (i.e. $w$ is not too high), the first effect dominates and the buyer benefits from the finer information together with the other players; if the signal that the buyer is "eager" is relatively strong ($w$ is high enough), the second effect dominates and the buyer loses.

**Proposition 9**

(i) The observability of the buyer’s predisposition increases the probability of trade; the ex ante welfare of the seller and of the broker is higher than in the benchmark no-signal case.

(ii) There exists $\bar{w} \in (\frac{1}{2}, 1)$ such that the buyer gains from her predisposition being discovered and communicated if $w < \bar{w}$ and loses if $w > \bar{w}$.

**6 Conclusions**

Our analysis shows that the intermediary can credibly transmit the information he possesses about one of the bargaining parties (the buyer) to the other (the seller). The mechanism ensuring credibility is different from traditional reputation-based mechanisms: rather, it relies on a trade-off the intermediary is facing between "softer" bargaining by the advisee and her willingness to participate in the bargaining at all. More precisely, although the intermediary is tempted to understate the buyer’s willingness to trade and thus encourage the seller to reduce price, he prefers not to do so since the withdrawal of the pessimistic sellers from the bargaining process after
such a report outweighs the gains from price reduction by those sellers that keep trading. An important element of our analysis is two-sided asymmetric information: the intermediary does not know the willingness to trade of the advisee (the seller) and thus cannot predict her reaction to a report about the willingness to trade of the buyer. We reveal two factors that can undermine the credibility of the intermediary’s advice. First, his ability to better predict the reaction of the seller in the case when the intermediary has information about both parties may undermine incentives for truthful information revelation. Second, when intermediaries (endowed with "captured" buyers) compete for attracting the seller, they may be tempted to overstate their buyer’s willingness to trade in order to attract the seller even at a cost of more aggressive pricing on the part of the seller. Finally, concerns about losses that some consumers may incur from the lack of privacy vis-à-vis an intermediary (as in the case of real estate services provided by a consumer’s bank) are shown to be justified: although the aggregate welfare increases when an intermediary becomes partially informed, some consumers (the buyers in our model) may get worse off.

In future work we would like to examine competition between intermediaries in more detail. For another interesting extension, assume that the sellers can either apply to an intermediary to be matched with a buyer or go directly to the search market (as in Gehrig (1993)), and the only difference is that the intermediary can give them some useful information about the buyer (like in our model). Then, what types of sellers will choose to trade through the intermediary? If the buyers also have choice, what buyers will choose to be matched through the intermediary?

It is also important to investigate the intermediary’s incentives to collect information. In our analysis we have assumed that the intermediary always gets an exogenous signal, but in some circumstances it may be more natural to presume that the intermediary has to exert effort to get a signal.
7 Appendix

The Proof of Lemma 1. (i) Assume the seller plays the specified strategy. Since \( \tilde{p}_S(v_S) \) is increasing, it means that \( \exists \hat{v}_S \in [v^S, \tilde{v}_S] \) such that \( p_S(v_S) = \tilde{p}_S(v_S) \) if \( v_S \geq \hat{v}_S \) and \( p_S(v_S) = \tilde{p}_B(u_B) - \delta \) otherwise. Assumption \( \tilde{p}_B(u_B) < \tilde{p}_S(\hat{v}_S) + \delta \) guarantees that \( \hat{v}_S < v^S \). Then, the buyer with valuation \( v_B \) solves

\[
\max_{p_B} \left( v_B - \frac{E[\tilde{p}_S(v_S) | \tilde{p}_S(v_S) \leq p_B - \delta, v_S \geq \hat{v}_S] + p_B + \delta}{2} \right) \times \Pr\{\tilde{p}_S(v_S) \leq p_B - \delta, v_S \geq \hat{v}_S\}
\]
\[
+ \left( v_B - \frac{\tilde{p}_B(u_B) - \delta + p_B + \delta}{2} \right) \Pr\{\tilde{p}_B(u_B) \leq p_B, v_S < \hat{v}_S\}
\]

Taking first-order condition we get, after some simplifications, \( p_B(v_B) = \tilde{p}_B(v_B) \) unless \( \tilde{p}_B(v_B) > \tilde{p}_S(\hat{v}_S) + \delta \) in which case \( \Pr\{\tilde{p}_S(v_S) \leq \tilde{p}_B(u_B) - \delta, v_S \geq \hat{v}_S\} = 1 \). In the latter case it is optimal to set \( p_B(v_B) = \tilde{p}_S(v_S) + \delta \) (i.e. the minimal \( p_B \) such that \( \Pr\{\tilde{p}_S(v_S) \leq p_B - \delta, v_S \geq \hat{v}_S\} = 1 \)).

Again, since \( \tilde{p}_B(u_B) \) is increasing, there exists \( \hat{v}_B \) such that the buyer plays \( p_B(v_B) = \tilde{p}_B(v_B) \) for \( v_B \leq \hat{v}_B \) and \( p_B(v_B) = \tilde{p}_S(\hat{v}_S) + \delta \) otherwise; assumption \( \tilde{p}_B(u_B) < \tilde{p}_S(\hat{v}_S) + \delta \) guarantees that \( \hat{v}_B > u_B \). Similar reasoning shows optimality of the piecewise-linear seller’s strategy specified in the Lemma if the buyer plays the indicated equilibrium strategy.

(ii) Given that the seller plays \( p_S(v_S) = (u_B + \hat{v}_S - \delta)/2 \) it is indeed optimal for the buyer to play \( p_B(v_B) = (u_B + \hat{v}_S + \delta)/2 \) provided that \( v_B \geq (u_B + \hat{v}_S + \delta)/2 \). Assumption \( \tilde{p}_B(u_B) \geq \tilde{p}_S(\hat{v}_S) - \delta \) guarantees that \( v_B \geq (u_B + \hat{v}_S + \delta)/2 \) is indeed verified for all values \( v_B \in [u_B, \tilde{v}_B] \).

Proof of Proposition 2. Consider first the benchmark case where the intermediary has no access to information about the buyer. Given equilibrium strategies, specified in Lemma 1, one easily finds the expected utilities.
of the parties:

\[
U_B(v_B) = \begin{cases} 
\frac{1}{2}(v_B - \frac{3}{4}\delta - \frac{1}{4})^2 & \text{if } v_B > \frac{1}{4} + \frac{3}{4}\delta, \\
0 & \text{if } v_B \leq \frac{1}{4} + \frac{3}{4}\delta;
\end{cases}
\]

\[
U_S(v_S) = \begin{cases} 
\frac{1}{2}(\frac{3}{4}(1 - \delta) - v_S)^2 & \text{if } v_S < \frac{3}{4}(1 - \delta), \\
0 & \text{if } v_S \geq \frac{3}{4}(1 - \delta);
\end{cases}
\]

\[
U_I = \frac{9(1 - \delta)^2\delta}{32}.
\]

Assume now that the intermediary learns the buyer’s predisposition to trade and fully revealing equilibrium is played.

Consider, first, the case when the buyer is not eager to trade, \(v_B \in [0, w]\).

The probability of trade now is \(\Pr\{\text{trade} \mid NE\} = \frac{9(w - \delta)^2}{32w}\), so the intermediary’s expected profit is \(U_I^{NE} = \frac{9(w - \delta)^2}{32w}\). The traders’ expected utility now is

\[
U_B^{NE}(v_B) = \begin{cases} 
\frac{1}{2}(v_B - \frac{3}{4}\delta - \frac{1}{4}w)^2 & \text{if } v_B > \frac{1}{4}w + \frac{3}{4}\delta, \\
0 & \text{if } v_B \leq \frac{1}{4}w + \frac{3}{4}\delta;
\end{cases}
\]

\[
U_S^{NE}(v_S) = \begin{cases} 
\frac{1}{2w}(\frac{3}{4}(w - \delta) - v_S)^2 & \text{if } v_S < \frac{3}{4}(w - \delta), \\
0 & \text{if } v_S \geq \frac{3}{4}(w - \delta);
\end{cases}
\]

Consider now the other case, when the buyer is eager, \(v_B \in [w, 1]\). Assume first that \(w \leq \frac{1}{4} + \frac{3}{4}\delta\). Then the probability of trade is \(\Pr\{\text{trade} \mid E\} = \frac{9(1 - \delta)^2}{32(1 - w)}\) and the parties’ expected payoffs are

\[
U_B^E(v_B) = \begin{cases} 
\frac{1}{2}(v_B - \frac{3}{4}\delta - \frac{1}{4})^2 & \text{if } v_B > \frac{1}{4} + \frac{3}{4}\delta, \\
0 & \text{if } v_B \leq \frac{1}{4} + \frac{3}{4}\delta;
\end{cases}
\]

\[
U_S^E(v_S) = \begin{cases} 
\frac{1}{2}(\frac{3}{4}(1 - \delta) - v_S)^2 \frac{1}{1-w} & \text{if } v_S < \frac{3}{4}(1 - \delta), \\
0 & \text{if } v_S \geq \frac{3}{4}(1 - \delta);
\end{cases}
\]

\[
U_I^E = \frac{9(1 - \delta)^2\delta}{32(1 - w)}.
\]

Otherwise, if \(w > \frac{1}{4} + \frac{3}{4}\delta\), the probability of trade is \(\Pr\{\text{trade} \mid E\} = \frac{9(w - \delta)^2}{32w}\).
\( \frac{1}{2}w + \frac{1}{4} - \frac{3}{4}\delta \), and the parties’ expected payoffs are

\[
U_E^B(v_B) = \begin{cases} 
\frac{5}{3}(v_B - \frac{3}{4}\delta - \frac{1}{4})^2 - \frac{1}{6}(v_B - w)^2 & \text{if } v_B > \frac{1}{4} + \frac{3}{4}\delta, \\
+\frac{1}{3}(v_B - \frac{3}{4}\delta - \frac{1}{4})(\frac{3}{4}\delta + \frac{1}{4} - w) & \text{if } v_B \leq \frac{1}{4} + \frac{3}{4}\delta; \\
0 & \text{if } v_B \leq \frac{1}{4} + \frac{3}{4}\delta.
\end{cases}
\]

\[
U_E^S(v_S) = \begin{cases} 
\frac{1}{2}(w + \frac{1}{4} - \frac{3}{4}\delta - v_S) & \text{if } v_S < w - \frac{1}{4} - \frac{3}{4}\delta, \\
\frac{1}{2}(1-w)(\frac{3}{4}(1-\delta) - v_S)^2 & \text{if } w - \frac{1}{4} - \frac{3}{4}\delta \leq v_S < \frac{3}{4}(1-\delta), \\
0 & \text{if } v_S \geq \frac{3}{4}(1-\delta);
\end{cases}
\]

\[
U_I^E = \frac{1}{2}w + \frac{1}{4} - \frac{3}{4}\delta\delta.
\]

Note, first, that the expected probability of trade with informed intermediary, \( \Pr\{\text{trade} \mid E\} (1-w) + \Pr\{\text{trade} \mid NE\} w \), is greater than the expected probability of trade with the uninformed one, \( \frac{9(1-\delta)^2}{32} \), which proves that the intermediary gets better off from access to the signal about the buyer.

The result concerning the seller follows from comparison of \( U_E^S(v_S)(1-w) + U_E^{NE}(v_S)w \) with \( U_S(v_S) \). In fact, it follows that sellers with valuations \( v_S < v'_S \) strictly gain, while sellers with valuations \( v_S \geq v'_S \) are indifferent, where \( v'_S = \frac{3}{4}(w - \delta) \).

Finally, the result concerning the buyer follows from comparing \( U_E^B(v_B) \) with \( U_B(v_B) \) if \( v_B > w \) and comparing \( U_E^{NE}(v_B) \) with \( U_B(v_B) \) if \( v_B < w \). It follows that buyers that are not eager (\( v_B < w \)) to trade strictly gain if \( \frac{1}{2}w + \frac{3}{4}\delta < v_B < w \) and are indifferent if \( v_B \leq \frac{1}{4}w + \frac{3}{4}\delta \). As for the buyers who are eager (\( v_B > w \)), they all strictly lose if \( w > \frac{1}{4} + \frac{3}{4}\delta \) and are indifferent otherwise.

**Proof of Proposition 3.** Part (i) follows immediately from Proposition 2. For parts (ii) and (iii), note that in the benchmark case, when the intermediary is not able to observe the buyer’s predisposition to trade, the buyer’s ex ante expected utility is \( EU_B = \frac{9}{128}(1-\delta)^3 \) (it is derived by taking expectation of \( U_B(v_B) \) from the proof of Proposition 2). When the intermediary is informed, the buyer’s expected utility is

\[
EU_{B}^{inf} = \frac{9}{128}(w - \delta)^3 + (1-w)(\frac{1}{8}w - \frac{1}{4} - \frac{1}{8}w\delta + \frac{3}{16}\delta^2 + \frac{1}{16}).
\]
After simplifications one gets that the buyer does not gain from the intermediary’s ability to observe her predisposition to trade, \( EU_B - EU_B^{inf} \geq 0 \), if and only if

\[ 9w^2 - (7 + 11\delta)w + 1 + 5\delta + 3\delta^2 \geq 0. \]

The latter inequality is satisfied if \( w < \frac{7 - \sqrt{13}}{18} + \frac{11 + \sqrt{13}}{18} \delta \) or \( w > \frac{7 + \sqrt{13}}{18} + \frac{11 - \sqrt{13}}{18} \delta \).

**Proof of Proposition 4.**

Assume \( v_B \in \left[ \frac{i}{n}, \frac{i+1}{n} \right] \). Then, trade happens if \( v_B \geq v_S + \frac{i+1}{4n} + \frac{3}{4}\delta \), so that sellers with valuations \( v_S \leq \hat{v}_S^{(i)} = \frac{i+1}{n} - (\frac{i+1}{4n} + \frac{3}{4}\delta) = \frac{3}{4}(\frac{i+1}{n} - \delta) \) participate in the bargaining. Assume the intermediary reports truthfully the buyer’s eagerness, i.e. sends message \( i \) if \( v_B \in \left[ \frac{i}{n}, \frac{i+1}{n} \right] \).

First, note the intermediary cannot gain from overstating the buyer’s eagerness. Indeed, if the intermediary deviates and reports \( j > i \), each type of seller submits a higher bid (see equilibrium strategies in Lemma 1) thus reducing the probability of trade. There is a second effect, potentially positive: some types of seller that abstained from bargaining under truth telling, start participating if the buyer’s eagerness is overstated. However, since the seller’s strategy is increasing in her type, none of these types of seller actually trades with a positive probability, so the effect of overstating the buyer’s eagerness is unambiguously negative for the intermediary.

Assume now that the intermediary understates the buyer’s eagerness to trade and reports \( j < i \). It is easy to see from the specification of equilibrium strategies in Lemma 1 that trade happens if \( v_B \geq v_S + \frac{1}{4n} + \frac{1}{5n}(3j - i) + \frac{3}{4}\delta \) and \( v_S \leq \hat{v}_S^{(j)} = \frac{3}{4}(\frac{j+1}{n} - \delta) \) (the latter condition ensures that seller of type \( v_S \) finds it worthwhile to participate in bargaining). It is easy to check that the second inequality implies the first.

A positive gain from downward deviation can occur only if \( \hat{v}_S^{(j)} > \frac{3}{4}(\frac{i}{n} - \delta) - \frac{1}{4n} \): the right-hand side of this inequality gives the value of \( v_S \) for which the probability of trade equals 1 under truth telling; if this condition is not satisfied, downward deviation brings no gain from less aggressive pricing but only losses from reduces participation. This inequality, when simplified,
reduces to $j + 1 - i > -\frac{1}{3}$, which implies that $j = i - 1$ is a necessary condition for a downward deviation to be profitable.

Like in the proof of Proposition 1, it is easy to check that the deviation to $j = i - 1$ brings gain from less aggressive bidding that is equal to $\frac{1}{32}$ and a loss from reduced participation equal to $\frac{9}{32}$, so it is clearly unprofitable.

**Proof of Proposition 5.** To prove the "if" part, we, as in Proposition 1, only need to check that the intermediary wants to say the truth if he expects the parties to believe his messages. It is clear that the intermediary never wants to tell that a party is eager to trade if this is not true: it would only decrease the probability of trade (this follows immediately from the specification of equilibrium bidding strategies in Lemma 1).

Let us verify that the intermediary tells the truth if both parties are eager to trade. Recall that the parties trade in this equilibrium if and only if $v_B \geq v_S + \frac{1}{4}v_B - \frac{1}{4}v_S + \frac{3}{4}\delta$. Substituting $v_B = w, \bar{v}_B = 1, v_S = 1 - w, \bar{v}_S = 0$ into this inequality, we see that both parties are sure to trade if $w \geq \frac{5}{8} + \frac{3}{8}\delta$. According to Lemma 1, in this case they trade with probability $1$ at price $\frac{1}{2}$, and the intermediary has no incentive to lie.

Assume that $\frac{1}{4} + \frac{3}{4}\delta \leq w \leq \frac{4}{7} + \frac{3}{7}\delta$ (i.e. $w \geq \frac{1}{4} + \frac{3}{4}\delta$ and $1 - w \geq \frac{3}{4}(w - \delta)$). Then, the situation is represented on Figure 3. The gain from lying to the seller about the buyer’s type while telling the truth to the buyer is the area of $ABC$ triangle, while the loss is the area of $CDEF$ trapezoid (after a false report to the seller and a truthful report to the buyer trade occurs when $v_S \leq v_S' = \frac{3}{4}(w - \delta)$ — all types of the seller that submit a bid are sure to trade). Given that $\frac{1}{4} + \frac{3}{4}\delta \leq w \leq \frac{4}{7} + \frac{3}{7}\delta$, the area of $ABC$ triangle does not exceed the area of $CDEF$ trapezoid if and only if $w \leq \frac{3\sqrt{2} - 2}{4} + \frac{3\sqrt{2} - 2}{4}\delta$. When this inequality is not satisfied, truth-telling cannot be induced if the parties play the strategies specified in Lemma 1 at the double auction stage, so the fully revealing equilibrium does not exist.

Assume now that the intermediary lies to both traders. Then, the buyer increases her bid but some buyers abstain from participation: trade happens
if \( v_S \leq v_S^* = \frac{3}{4}(w - \delta) \) and \( v_B \leq v_B^* = 1 - \frac{3}{4}(w - \delta) \). Now, if \( w \leq \frac{1}{2} + \frac{3}{4}\delta \), the gain from lying is the area of triangle with the same vertex C, but the horizontal edge now being given by \( v_B = 1 - \frac{3}{4}(w - \delta) \) line instead of \( v_B = w \); the loss is still given by \( CDEF \) trapezoid. Since \( 1 - \frac{3}{4}(w - \delta) \geq w \), the gain is lower than from lying to just one party (e.g. the seller), while the loss is the same. If \( \frac{1}{2} + \frac{3}{4}\delta \leq w \leq \frac{3}{8} + \frac{3}{4}\delta \), lying to both parties is unambiguously unprofitable (geometrically, the triangle, representing the gain, vanishes).

When \( \frac{3}{4} + \frac{3}{4}\delta \leq w \leq \frac{5}{8} + \frac{3}{4}\delta \), there is no cost of lying (to either one party or both) and a positive gain: since \( v_S^* = \frac{3}{4}(w - \delta) > 1 - w \), lying would result in trade with probability one were the parties to believe the intermediary’s reports. A fully revealing equilibrium does not exist for this range of \( w \).

Along similar lines it is easy to check that lying (to one or both parties) is not beneficial if \( w < \frac{1}{4} + \frac{3}{4}\delta \).

When just one trader is eager to trade the analysis proceeds analogously and shows that the intermediary never gains from lying in this case. ■

**Proof of Proposition 6.** Assume that an intermediary, say \( I_1 \), faces
an eager buyer. If he reports this truthfully, the seller will choose him with probability \( \frac{1}{2}(1 - w) + w \). Indeed, with probability \( w \) the other buyer, \( B_2 \), is not eager, so the seller chooses \( I_1 \), with probability \( 1 - w \) buyer \( B_2 \) is also eager, then \( I_1 \) gets the seller with probability \( 1/2 \). The intermediary’s expected profit if she reports truthfully and gets the seller is

\[
E[U^E_{I_1} \mid S \text{ chooses } I_1] = \begin{cases} 
\frac{9}{32(1-w)}(1-\delta)^2\delta & \text{if } \delta < w \leq \frac{1}{4} + \frac{3}{4}\delta, \\
\frac{1}{4}(1-3\delta+2w)\delta & \text{if } w > \frac{1}{4} + \frac{3}{4}\delta,
\end{cases}
\]

hence, the his expected utility from truthful reporting is

\[
EU^E_{I_1} = \begin{cases} 
\frac{9(1+w)(1-\delta)^2\delta}{64(1-w)} & \text{if } \delta < w \leq \frac{1}{4} + \frac{3}{4}\delta, \\
\frac{(1+w)(1-3\delta+2w)\delta}{8} & \text{if } w > \frac{1}{4} + \frac{3}{4}\delta.
\end{cases}
\]

Assume now that the intermediary deviates and reports that \( B_1 \) is not eager to trade. Then, the probability to attract the seller goes down to \( \frac{w}{2} \): the seller chooses \( I_1 \) (with probability \( 1/2 \)) only if the other buyer is not eager. The expected profit \( I_1 \) gets conditionally on the seller choosing \( I_1 \) is

\[
E[\hat{U}^E_{I_1} \mid S \text{ chooses } I_1] = \frac{3}{4}(w - \delta)\delta,
\]

so

\[
E\hat{U}^E_{I_1} = \frac{3}{8}w(w - \delta)\delta.
\]

It is easy to check that \( EU^E_{I_1} \) is always greater than \( E\hat{U}^E_{I_1} \), so an intermediary never has an incentive to understate his buyer’s willingness to trade in a competitive situation.

Assume now that \( B_1 \) is not eager to trade. If the intermediary reports the truth he attracts the seller with probability \( w/2 \) and his expected profit is

\[
EU^NE_{I_1} = \frac{9\delta(w - \delta)^2}{64}.
\]

If \( I_1 \) overstates his buyer’s eagerness to trade, he gets the seller with a higher probability, \( \frac{1}{2}(1 - w) + w \). However, the seller will compete more aggressively, so the probability of trade conditional on attracting the seller is now lower,
Figure 4:

\[
\frac{9}{128w}(3w - 1 - 2\delta)^2 \text{ if } w > \frac{1}{3} + \frac{2}{3}\delta \text{ and } 0 \text{ otherwise. }
\]

The expected profit in case of deviation is

\[
E\tilde{U}^{NE}_{1} = \begin{cases} 
\frac{9\delta(1+w)(3w-1-2\delta)^2}{256w} & w > \frac{1}{3} + \frac{2}{3}\delta, \\
0 & \text{if } \delta < w \leq \frac{1}{3} + \frac{2}{3}\delta.
\end{cases}
\]

Hence, truth-telling is optimal if \( w \leq \frac{1}{3} + \frac{2}{3}\delta \). Otherwise, truth-telling constraint is equivalent to a polynomial inequality (cubic in \( w \)):

\[
5w^3 + 3w^2 - 4w^2\delta - 5w - 8w\delta + 1 + 4\delta + 4\delta^2 \leq 0. \tag{5}
\]

Analytical solution of this inequality is too cumbersome; Figure 4 gives a plot of polynomial (5) in blue color; it also represents a plot of plane \( w - (\frac{1}{3} + \frac{2}{3}\delta) \) in green and a plot of zero-level surface in yellow.

The figure shows that truth-telling constraint is satisfied if \( w \in [\frac{1}{3} + \frac{2}{3}\delta, \bar{w}(\delta)] \) and not satisfied if \( w \in [\bar{w}(\delta), 1] \) for some threshold \( \bar{w}(\delta) \).

**Proof of Proposition 7.** Along the lines of Myerson and Satterthwaite
(1983) one easily shows that the broker’s expected profit

\[ U_I = w \int_0^1 \int_0^w [(v_B - \frac{1 - F_B(v_B)}{f_B(v_B)})
\]

\[ -(v_S + \frac{F_S(v_S)}{f_S(v_S)})p_L(v_S, v_B)f_B(v_S)dv_Bdv_S
\]

\[ +(1 - w) \int_0^1 \int_0^1 [(v_B - \frac{1 - F_B(v_B)}{f_B(v_B)})
\]

\[ -(v_S + \frac{F_S(v_S)}{f_S(v_S)})p_H(v_S, v_B)f_B(v_B)dv_Bdv_S
\]

\[ -wU_B(0) - (1 - w)U_B(w) - U_S(1),
\]

where \( F_B(v_B) = \frac{v_B}{w} \) and \( F_B(v_B) = \frac{v_B - w}{1 - w} \) for our case of uniform distributions, \( p_L(v_S, v_B) \) and \( p_H(v_S, v_B) \) are the probabilities of trade for the case of a non-eager and eager buyer. It is optimal to set

\[
    p_L(v_S, v_B) = \begin{cases} 
    1 & \text{if } v_B \geq v_S + \frac{w}{2}, \\
    0 & \text{otherwise}
    \end{cases}
\]

\[
    p_H(v_S, v_B) = \begin{cases} 
    1 & \text{if } v_B \geq v_S + \frac{1}{2}, \\
    0 & \text{otherwise}
    \end{cases}
\]

and \( U_B(0) = U_B(w) = U_S(1) = 0. \)

**Proof of Proposition 9.** One can check that under the optimal two-part tariff the ex ante expected utility of the buyer, the seller and the broker in the absence of information about the buyer’s eagerness to trade is the following:

\[ U_B^0 = U_S^0 = \frac{3^2}{2^7} (1 - \delta)^3 = \frac{1}{48}, \]

\[ U_I^0 = \frac{3^2(1 - \delta)^2 \delta}{2^5} = \frac{1}{24}. \]

When the buyer is discovered to be not eager to trade the expected utilities
are:

\[

\begin{align*}
U_{BE}^{NE} &= \frac{3^2(w - \delta)^3}{2^7w} = \frac{w^2}{48}; \\
U_{SE}^{NE} &= \frac{3^2(w - \delta)^3}{2^7w} = \frac{w^2}{48}; \\
U_{IE}^{NE} &= \frac{3^2(w - \delta)^2\delta}{2^5w} = \frac{w^2}{24}.
\end{align*}
\]

If the buyer is eager and \(w < \frac{1}{2}\)

\[

\begin{align*}
U_{BE}^E &= U_{SE}^E = \frac{3^2(1 - \delta)^3}{2^7(1 - w)} = \frac{1}{48(1 - w)}; \\
U_{IE}^E &= \frac{3^2(1 - \delta)^2\delta}{2^5(1 - w)} = \frac{1}{24(1 - w)}.
\end{align*}
\]

If the buyer is eager and \(w \geq \frac{1}{2}\)

\[

\begin{align*}
U_{BE}^E &= \frac{1}{1-w}(32(1 - \delta)^3 - \frac{3}{18}(w - \frac{1}{4} - \frac{3}{4}\delta)^3 - \frac{1}{18}(1 - w)^3 - \frac{3}{32}(1 - \delta)^2(w - \frac{1}{4} - \frac{3}{4}\delta) - \frac{1}{3}(w - \frac{1}{2})^2) \\
&= \frac{w}{12} - \frac{1}{3}(w - \frac{1}{2})^2; \\
U_{SE}^E &= \frac{1}{6}(1 - w)^2 + \frac{1}{4}(w - \frac{1}{2}); \\
U_{IE}^E &= \frac{w}{6}.
\end{align*}
\]

Thus, when the intermediary observes the buyer’s eagerness to trade, the ex ante expected utilities for the case \(w < \frac{1}{2}\) are

\[

\begin{align*}
\bar{U}_B &= \bar{U}_S = \frac{1}{48} + \frac{1}{48}w^3; \\
\bar{U}_I &= \frac{1}{24} + \frac{1}{24}w^3
\end{align*}
\]

and for the case \(w \geq \frac{1}{2}\) they are

\[

\begin{align*}
\bar{U}_B &= \frac{w(1 - w)}{12} + \frac{1}{48}w^3 - \frac{1}{3}(w - \frac{1}{2})^2; \\
\bar{U}_S &= \frac{1}{24} - \frac{w}{8} + \frac{w^2}{4} - \frac{7w^3}{48}; \\
\bar{U}_I &= \frac{w(1 - w)}{6} + \frac{w^3}{24}.
\end{align*}
\]

38
It is easy to check that $\bar{U}_S > U^0_S$ and $\bar{U}_I > U^0_I$ for all $w$ and $\bar{U}_B > U^0_B$ for $w < \bar{w}$, while $\bar{U}_B < U^0_B$ for $w > \bar{w}$, where $\bar{w}$ is a root of $\frac{5}{12}w - \frac{5}{12}w^2 + \frac{1}{48}w^3 - \frac{5}{48} = 0$, $\bar{w} \approx 0.61$. ■

8 References


Jullien, B. and T. Mariotti (2006) "Auction and Informed Seller Prob-


The National Association of Realtors (2001) "The Consequences of Mixing Banking and Commerce".


