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# UNIVARIATE POTENTIAL OUTPUT ESTIMATIONS FOR HUNGARY

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**Abstract**

Potential output figures are important ingredients of many macroeconomic models and are routinely applied by policy makers and global agencies. Despite its widespread use, estimation of potential output is at best uncertain and depends heavily on the model. The task of estimating potential output is an even more dubious exercise for countries experiencing huge structural changes, such as transition countries. In this paper we apply univariate methods to estimate and evaluate Hungarian potential output, paying special attention to structural breaks. In addition to statistical evaluation, we also assess the appropriateness of various methods by expertise judgement of the results, since we argue that mechanical adoption of univariate techniques might led to erroneous interpretation of the business cycle. As all methods have strengths and weaknesses, we derive a single measure of potential output by weighting those methods that pass both the statistical and expertise criteria. As standard errors, which might be used for deriving weights, are not available for some of the methods, we base our weights on similar but computable statistics, namely on revisions of the output gap for all dates by recursively estimating the models. Finally, we compare our estimated gaps with the result of the only published Hungarian output gap measure of Darvas-Simon (2000b), which is based on an economic model.

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# I. Introduction

The potential or permanent component of output is an important unobserved variable for decision making, policy analysis, and macroeconomic modelling. For instance, sustainability of fiscal positions and monetary policy actions are frequently evaluated in the light of the cyclical position of the economy. Another example from the central bank point of view is the setting of monetary targets and analysis of money demand where the permanent component of the output should be considered instead of the actual one.

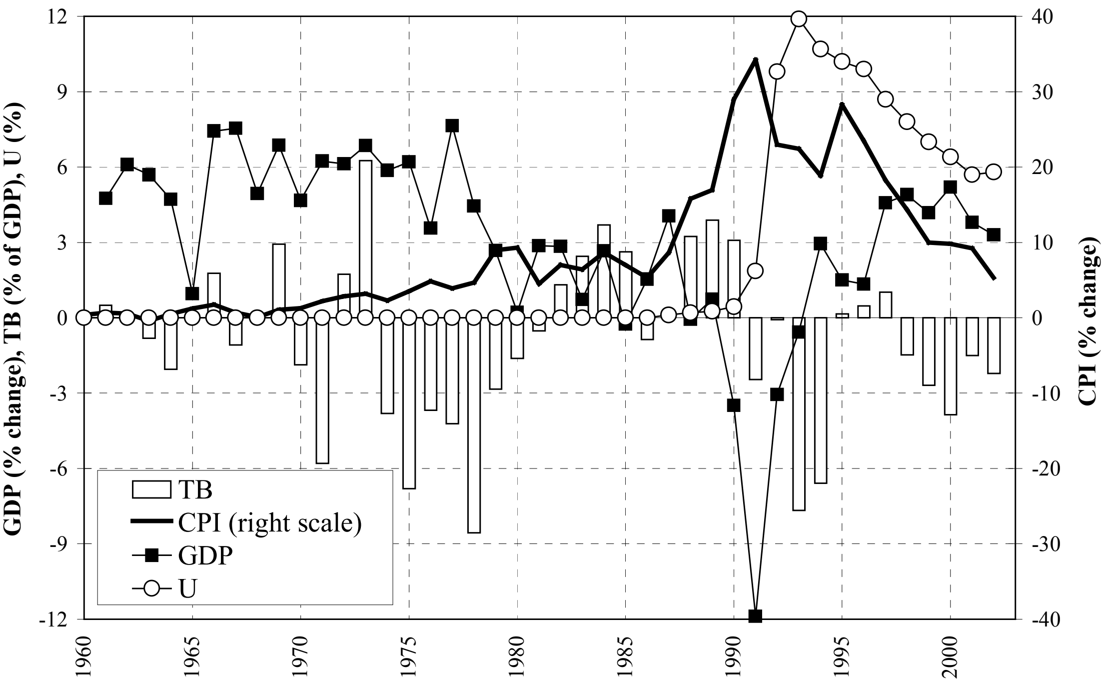
Estimation of potential output, though, is very difficult even for developed countries. The troubles are not mainly associated with the complexities of empirical methods, but rather with the mapping of an economic concept of potential output into a plotted time series. There are hundreds of methods and models for empirical estimation: some are intended to measure potential output and the output gap, others call their objectives trends and cycles, while the terms permanent and transitory components are also frequently used. The common idea behind these methods is to uncover a component that is likely to persist over the long-run.[1] Problems arise when we want to define what the long-run component is. For example, if we followed the tradition of the empirical real business cycle literature and defined the long-run component as the trend revealed by the Hodrick-Prescott filter, then this filter would be the best method to decompose an observed series into long-run and short-run components. However, due to several unfavourable characteristics, this estimator, or to word it properly, this concept of a long-run component has been strongly criticized. Critiques stemming from properties and economic implications of an estimator characterize all methods. Those who employ multivariate methods criticize univariate models on the grounds that such methods do not take into account all relevant information. Those who prefer non-structural methods (either univariate or multivariate) criticize structural multivariate approaches on the basis that they impose *a priori* structures for the economy that may be invalid. Univariate modellers usually favour their methods because of simplicity and the absence of a number of *ad hoc* assumptions; several univariate methods, however, make one very important *ad hoc* assumption.

Disagreement on the mapping of the concept into empirical estimates in mature economies renders estimates even more problematic for countries facing deep structural changes, such as transition economies. Although most transition countries had already introduced some market institutions by the end of the 1980s, processes such as democratisation, further decentralization, the collapse of COMECON, privatisation, comprehensive adoption of market institutions, opening and Western integration changed dramatically these economies. As a natural consequence, standard models might not work for the transition period that covers several years. For example, the considerable downturn in the early years of transition was accompanied by a massive rise in inflation, while inflation contracted during periods of rapid growth in several countries (see Hungarian data in Figure 1, for example). Issues of structural changes and non-applicability of standard concepts are coupled with poor quality, short databases.

[1] Throughout this paper the terms potential output/trend output/permanent component of output and output gap/cycle/transitory component are regarded as synonymous. Although there is a slight difference among the generally assumed ideas behind these concepts, the same methods were applied to recover all of them. Policy makers and global agencies use the wording potential output and output gap more frequently while academics mostly prefer the other expressions.

Emergence of thousands of enterprises and retail stores, quickly changing product and quality structures posed huge challenges for statistical offices. Many important figures are simply not available for several years and the available data necessary for potential output estimates are mostly at annual frequency. Quarterly national accounts figures started to be published only a few years ago and were frequently subject to major revisions. The aforementioned drawbacks leave us very cautious when it comes to estimating potential output figures for Hungary.

**Figure 1 GDP growth, trade balance, unemployment rate and inflation, 1960-2002**




In this paper we do not develop a model suitable for transition countries, but adopt and evaluate various univariate techniques. In order to derive a single measure we calculate the weighted average of some selected methods that we will call a 'consensus estimate'. We also compare our consensus estimate to the results of a structural estimate of the output gap by Darvas-Simon (2000b).

The paper is structured as follows. Section II reviews the concepts of potential output and underline its limitation in case of open economies. Section III highlights some issues of the estimation. As univarate methods are well known, we do not describe them in the main text, but provide detailed descriptions in Appendix 1. Section IV contains the topics related to our estimation of the Hungarian output gap. The first subsection summarizes data problems, while the following one gives an overview of the development of Hungarian GDP. The third subsection covers detailed empirical results of all methods, and the fourth part presents our consensus estimate and compares this with a structural method. Section V presents the conclusions.

# II. Concepts of potential output

## II.1 Mainstream concepts

Probably the most widely adopted concept of potential output is the level of output that represents a balanced state of the economy. This balanced state is frequently defined as stable inflation. Stable inflation corresponding to a certain level of unemployment is called NAIRU (non-accelerating inflation rate of unemployment) that relates to a certain level of output via the Okun-law. However, NAIRU is not easy to measure because of structural and hysteresis reasons. Partly due to this reason and partly due to theoretical Phillips-curve considerations, there are models that use inflation directly as information about the output gap.[2,3]

The so-called production function method analyses factor inputs to production. It is usually combined with the concept of NAIRU; thus the potential labour input is taken into account instead of the actual labour input.[4]

Another line of the literature defines potential output as the level of output free of the effect of demand shocks. Demand and supply shocks are frequently studied in the framework of SVARs pioneered by Blanchard–Quah (1989). They estimated a two-variable VAR model for output and unemployment and constrained the parameters the following way. There are two types of shocks: (1) Supply shocks have a transitory effect on unemployment and a permanent effect on output; (2) Demand shocks have a transitory effect on both variables. Behind these constraints there is a model where the permanent shocks can be interpreted as supply shocks while the transitory shocks as demand shocks.[5]

A further class of models assumes that potential output is driven by exogenous productivity shocks that determine long-run growth. Short-run movements in output are due to the behaviour of rational agents who react to unexpected productivity shocks by writing off old capacities and rearranging resources to new conditions. In fact, most of the univariate methods we study are equally applied by this literature as well.

We argue that, in addition to the highlighted conceptual weaknesses of NAIRU and the econometric weaknesses of the SVARs,[6] the assumptions behind these models are rather questionable for open economies in general and for transition countries in particular.

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[2] See, e.g. Kuttner (1994) and Gerlach–Smets (1999).

[3] There is new line of literature estimating the so-called New-Keynesian Phillips-curve with forward looking dimension (see, e.g Galí et al 2002). However, this literature discuss the issue whether the given values marginal cost and the output gap (estimated by univariate methods) should be included in the equation, and we are not aware of any paper that tries to estimate the output gap based on this specification. Another research project of ours investigates this issue.

[4] See Giorno *et al.* (1995) and McMorrow–Werner (2001). As described in Giorno *et al.* (1995), the OECD Secretariat used the split time trends (= segmented deterministic trends) method with *ad hoc* judgements till the first half or the 1980s, then switched to the production function (PF) approach incorporating a very simple NAWRU model. Principal drawbacks of the segmented trend approach were a key impetus for changing to the PF-approach.

[5] The data induced even Blanchard-Quah to adopt an atheoretical pre-filter: they detrended the unemployment rate with a broken deterministic trend.

[6] See, e.g. Faust–Leeper (1997) and Cooley–Dwyer (1998).

## II.2 The transition shock

We argue that standard concepts are incapable of describing transition shocks.

During transition the economy was shocked by enormous relative price changes,[7] which resulted in a structural change in demand. The structure of supply was unable to adjust to this change quickly. Therefore, capacities became redundant while new capacities were established only in a gradual evolutionary process. Meanwhile, excess demand and excess supply existed side by side, and aggregate output decreased because the short-side rule prevailed in each micro-market. Decrease in output brought about unemployment.

It is rather difficult to assign the nature of this shock either to supply or demand. The definition based on the concept of demand as aggregate planned purchases and supply as aggregate planned sales does not help much in finding the answer as it was the structure and not the aggregate amount that differed.

The definition of aggregate excess demand implies that a demand shock would create excess demand in the short run, but does not affect output in the long run. A supply shock related mostly to technical changes would have permanent effects. Generally, supply shocks may have transitory effects as well. A monetary squeeze affects both supply and demand temporarily, as supply is constrained through the credit channel. Similarly, a structural shock may have transitory effects. This has led to the reinterpretation of demand and supply shocks: in several contexts they do not mean changes in planned purchases or planned sales, but only the temporary or permanent nature of the shock.[8] The question may arise then, whether the recession of the early 1990s in the transition economies was the result of a temporary or a permanent negative shock. On one hand, the high rate of unemployment that has arisen would suggest that the shock was temporary. On the other hand, it is clear that the persistence of the crisis is longer than the usual excess-demand driven business cycle recessions. If output drops below equilibrium because of a lack of aggregate demand, then it is the speed of price adjustment that determines the length of the impact of the shock. However, if output drops because of a structural mismatch, not only prices have to adjust, but the structure of supply. This is presumably much slower than price adjustment, because it requires the establishment of whole new production cultures. The inertia in this process is too large to be explained by pure construction costs: uncertainties owing to limited information constrain the speed of adjustment decisively.[9]

How long does the effect of a transitory shock last? In practice in finite samples it is difficult to separate shocks which have an autoregressive representation with dominant (inverted) roots that are 1 from those which have roots less than 1. Sometimes it is useful to consider some roots to be 1 even though theory would tell that they are less than 1. In this manner, some shocks that are transitory in theory may be considered as permanent in some models. Although it is true that employment reverts to its natural rate and therefore its fluctuation gives a transitory element to output, the observation period of the transition countries did not render either the quality or the variability of data that would be required for using them as information on this effect. Therefore, in

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[7] Due to the transfer to a system of market pricing.

[8] For example, univariate trend-cycle decompositions that we study adopt this interpretation.

[9] See Stiglitz (1992) for a thorough development of this argument.

Hungary for example, even though unemployment rose from 0 to 13 percent after the system change in 1990 and then slowly declined to 6 percent (Figure 1), we do not consider the slow decline of unemployment as an important element of a transitory increase in output.[10]

The persistence of unemployment as a result of this structural shock is similar to the phenomenon of hysteresis. The difference is that in the literature on hysteresis the emphasis is on the fact that unemployment erodes the capabilities of the worker, while in the case of a structural shock it is the production environment (structure of capacity, geographic location, invisible business capital) that erodes and cannot be recovered. hide

### II.3 Open economy considerations

Renouncing unemployment as a source of information may be motivated from another aspect as well. We do not even use information given by inflation data, as the steady fall of inflation in the second half of the 1990s (Figure 1) should not be regarded as the result of a continuously negative output gap. Rather, it might have been the consequence of the larger weight of expectations than inertia in a standard neo-keynesian Phillips-curve.[11]

However, in an open economy excess demand may simply result in increased imports without any direct effect on inflation. The increased imports may have an impact on future inflation, but the effects may be variable both in lags and magnitude, depending on policies and on capricious business sentiment. In addition to that, during the sample period of Hungarian data, inflation was hit by so many other shocks,[12] that a decomposition of these shocks would face insurmountable difficulties.

This is the reason that Darvas–Simon (2000b) developed a model, which makes use of the information that is rendered by the openness of the economy. In this paper we compare our consensus univariate estimate to the result of this model as well.

## III. Empirical methods

A general weakness of univariate methods is that they are univariate: they take into consideration neither the consequences of non-zero output gaps, nor structural constraints and limitations of growth.

There are some surveys available on these methods. Results of these surveys do not simplify our task of selecting the most appropriate univariate method, as many papers have found that both quantitative and qualitative properties of estimated business cycles vary widely across detrending methods and all estimates are subject to considerable uncertainty. Therefore, there are no firm grounds for selecting a preferred univariate method as all of them are criticized from different aspects. Given disagreements on the

[10] The level of employment is not a suitable substitute variable for the unemployment rate either due to the full employment nature of the socialist system, which in fact covered hidden unemployment. The lay-off of hidden unemployment was a gradual process parallel with privatization.

[11] See Benczúr-Simon-Várpalotai (2002) for the case of costless disinflation in small macromodels.

[12] E.g. price control during the central planning period, hardly quantifiable expectations in the circumstances of a highly uncertain system change, full price liberalization in the early 1990s.

appropriate methods, we adopted the pragmatic approach to apply all methods to Hungarian data, contrast the results with our *a priori* thoughts on the business cycles in Hungary, and derive a final estimate by selecting the sensible estimates described below and weighting them. We will call the weighted estimate a 'consensus estimate'.

We considered various variants of the following univariate models[13]:

- Deterministic trend
- Hodrick-Prescott filter
- Band-pass filter
- Beveridge-Nelson decomposition
- Unobserved components
- Wavelet transformation

In the next subsection, we describe the method of evaluation and weighting. It is followed by two other general issues: structural breaks and the choice of starting values for maximum likelihood estimation.

## III.1 Method of calculating the consensus output gap and potential output

The most challenging task is the evaluation of results received by various methods. Our procedure is the following. First, we disregard those estimates that do not pass some econometric specification tests. Second, we define some priors on the sign of the output gap, which are widely accepted among experts. In each case we evaluate whether the output gaps provided by the models meet these expectations, and exclude those that run *strongly* counter to economic reasoning. Finally, we weight the remaining estimates with weights proportional to the inverse of revisions of the output gap for all dates estimated for recursive samples. Therefore, we prefer methods that lead to more stable inference for the end of the sample. Although the variance of the various output gap estimations should form the weights in an optimal-weighting framework, for several methods it is rather difficult to derive a confidence band.

Specifically, our weighting scheme considers those methods "better" that have smaller revisions. Equation (1) describe how we derive the size of revision at a certain date.

$$REV_t^{(m)} = \frac{1}{T-k}\sum_{s=k+1}^{T}\left|(q_{t,s}^{(m)*} - q_{t,s-1}^{(m)*})\right| = \frac{1}{T-k}\sum_{s=k+1}^{T}\left|(q_t - q_{t,s}^{(m)*}) - (q_t - q_{t,s-1}^{(m)*})\right| \tag{1}$$

[13] Recently smooth transition autoregressions (STAR) and models with continuous structural change have been suggested in the literature (see, e.g. Lin- Teräsvirta, 1994). Although they seem appealing for modelling the transition period, there are severe problems for their application for output gap caculations. We have three parametric methods that could be candidates for these methods. STARs could be used for the Beveridge-Nelson decomposition (BN) and for unobserved components (UC) models, which assumes autoregressive processes. However, the BN decompose the series into a random walk and a stationary component, which are rather difficult to interpret when the autoregressive parameter changes continuously. UC models should be linear so we can not incorporate a STAR specification. A continuous structural change model could be estimated for the parameter of the deterministic trend, but then the very nature of the trending process is lost.

where $REV_t^{(m)}$ is the revision of the $m^{th}$ method for observation $t$, $q_t$ is the logarithm of actual GDP, $q_{t,s}^{(m)*}$ is the logarithm of potential output revealed by the $m^{th}$ method for observation $t$ in the sample starting at the first observation and ending in $s$, $s \in [k,...,T]$ and $s \geq t$, where $k$ denotes the length of the shortest sample possibly taken into consideration and $T$ is the full sample size. Consequently, the number of sample periods used for estimation is $T-k+1$ for $t \leq k$, and $T-t+1$ for $k < t < T$. The average revision for the $m^{th}$ method is computed by the average of revision at certain date in the whole time span:

$$REV^{(m)} = \frac{1}{T}\sum_{t=1}^{T} REV_t^{(m)} \tag{2}$$

The final step is the calculation of the consensus output gap from various methods that fulfil statistical and expertise requirements. Since revision describes the stability of estimated output gap we use revision for weighting the different methods in a way that low weights belong to high revision values and vice versa. Thus the weights are computed by the following way:

$$w_m = \frac{1/REV^{(m)}}{\sum_{j=1}^{p} 1/REV^{(j)}} \tag{3}$$

where $w_m$ denotes the weight of $m^{th}$ method among $p$ selected ones. Thus, the consensus output gap is derived by:

$$\hat{c}_t = \sum_{j=1}^{p} w_j \hat{c}_{t,j} \tag{4}$$

where $\hat{c}_t$ is the consensus output gap measure and $\hat{c}_{t,m}$ is the output gap measure of the $m^{th}$ method.

Using the definition of the output gap, our consensus measure of potential output is given by:

$$\hat{q}_t^{POT} = q_t - \hat{c}_t \tag{5}$$

where $\hat{q}_t^{POT}$ denotes the level of potential output. Consequently, the potential growth rate is estimated by the growth rate of $\hat{q}_t^{POT}$.

## III.2 Structural breaks

We use either annual data covering the period 1960-2002 or quarterly data for 1991-2002. Both time spans are likely burdened with structural breaks. For several parametric methods, such as the deterministic trend, the Beveridge-Nelson decomposition, and unobserved components model, we should handle this issue.

Thus the identification of the number and location of possible breakpoints is crucial. Maddala and Kim (1996) suggest a Bayesian method, which makes the identification of a number of breakpoints endogenously. Nevertheless, we apply an another method to be able to control the number of breakpoints using expertise judgement, but at the same time allowing formal test to identify the likeliest places.

After ‘visual-inspection’ of time series we define the number of breakpoints and the possible interval around them. In the next step we compute the F-statistics of the Chow breakpoint test with all combinations and choose the likeliest one. Therefore, the steps are the following:

1. Based on the graph of time series and expert knowledge we identify the number of breakpoints.
2. We assign intervals around each breakpoint.
3. F-statistics and *p* values of the Chow breakpoint test are computed for every combination of dates between the intervals.

Finally, we sort the results and choose the highest F-statistic or equivalently the lowest *p* values.

## III.3 Starting values

We faced severe problems with maximum likelihood estimation of state space models, as they proved to be time consuming and sensitive on starting values.[14] Maximization of the likelihood function itself required considerable time and did not converge in several cases.

In stationary models the starting values are usually set to be equal to the unconditional means, but for non-stationary models they should be specified. When convergence was achieved, then estimates were reasonably close to each other irrespective of the starting values, but starting values strongly effected both the existence of convergence and its speed. And of course, convergence does not ensure reasonable parameter estimates. For example, there were several cases when a negative point estimate was achieved for a variance.

To handle this problem we applied various starting values and sorted the results according to certain criteria. We defined possible intervals of initial parameters and estimated the models for every combination of starting values. To avoid obtaining thousands of potential GDP estimations we applied some constraints. First, the estimation should converge. Second, the output gap should not be higher than 15%. At the first glance, this seems to be too high but the early years of transition requires such a high value. On the other hand, several specifications failed to fulfil this criterion, for example some provide more than 30% output gaps.

Even if we use this method several results might occur from one model. Fortunately, these results were generally quite close to each other. In a few cases of dissimilar results we chose the one that is closer to our expertise views (for the *a priori* assumption see the next section).

[14] In most cases we used Marquardt algorithm of Eviews for estimation. The so-called "structural time series" models were estimated with the TSM module of GAUSS.

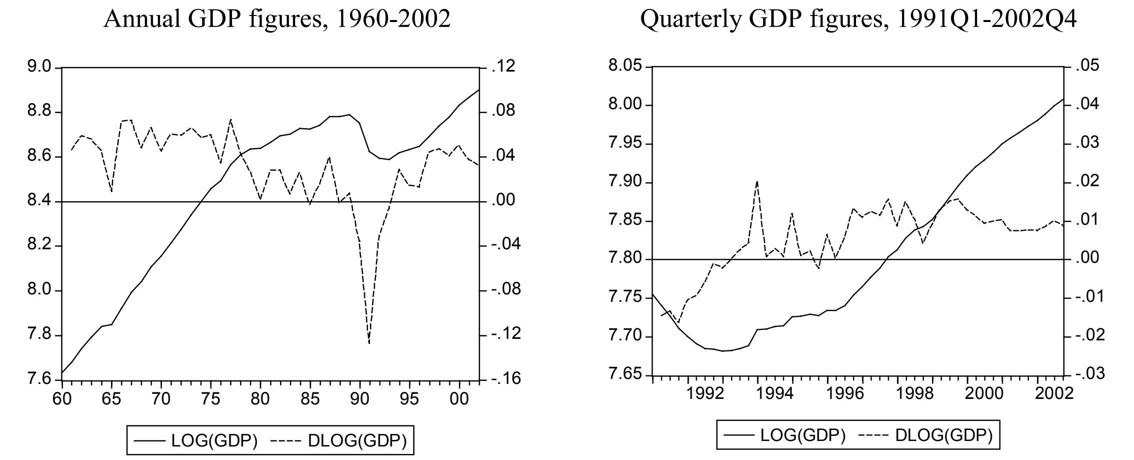
# IV. Results

## IV.1 Data

Before turning to specification issues we should clear the very first obstacle.

The Hungarian Central Statistical Office (CSO) has been publishing quarterly GDP figures since 1995. This data set provides 32 quarterly observations that are extremely short, not even much longer than what is considered a usual business cycle in the US. On the other hand, there are severe methodological problems with quarterly GDP figures that are manifested in frequent and substantial revisions of previously published figures. In addition to that, there are important methodological problems with available Hungarian quarterly national accounts data (see Várpalotai (2003)). Therefore, we are either obliged to use annual data or compelled to accept published quarterly national accounts figures and to approximate quarterly GDP prior to 1995 as well. We adopted both approaches. Quarterly data transformation was prepared by Várpalotai (2003), which is shown in the right panel of Figure 2. We tested the effects of the approximated quarterly GDP figures on our inference for the post-1995 period. The results indicate minor differences in most cases as it is shown in Appendix 3.[15]

**Figure 2 Log level and growth of annual and quarterly GDP**




## IV.2 Preliminary view on the output gap in Hungary

The evolution of Hungarian economy has been rather turbulent over the last 10-15 years (Figure 1). First, in the first few years after regime shift in 1990 Hungarian GDP dropped substantially. Despite this strong output loss we have no preconceived ideas as to how potential output and the output gap evolved during the early years of transition, as we emphasized in Section II.2.

[15] We could have used industrial production instead of GDP as the measure of economic activity, for which longer time series are available. However, we have just seen that the approximated quarterly GDP figures for 1991-94 have minor effects on the inference. Due to this result and to the fact that industrial production constituted less than one-third of the economy, we solely analyze GDP.

In 1993-1994, the external and government balance deteriorated sharply, finally inducing the stabilization measures of March 1995. As a consequence, in 1995-1996 GDP growth slowed considerably, real wages declined sharply mainly due to inflation, and external balances improved over the next few years. Later, in 1997-1999, growth picked up strongly and the external balance deteriorated to some extent. During this period, the Hungarian economy was influenced by the Russian crisis of 1998 that cut demand for Hungarian exports while production capacity remained unchanged. These developments suggests that the Hungarian economy might have been above its potential in the 1993-1994 period, below it in 1995-1996, and that the negative gap shrank or turned positive in 1997 and turned negative again in 1998 or 1999. After this period, Hungarian GDP grew steadily until 2001. Needless to say, this preliminary view is *not* a guide for model selection. This view only serves as a comparison of estimation results with a 'consensus' view of economists.

## IV.3 Estimation results

For each model we first present results for annual data and continue with quarterly data.

### IV.3.1 Segmented deterministic trend[16]

A visual inspection of Figure 2 might help to decide the possible number and location of breakpoints. There are three ranges where the breaking points can occur, namely: 1975-1980, 1985-1990, and 1993-1998. First interval refers to the effect of the oil price shock. The second one tries to capture the starting date of transition while the third one is connected to the end of the early years of transformation. Hence, we assume three breakpoints in annual GDP figures, while the possible locations of breakpoints are determined endogenously in each case by the method described in Section III.2. The result indicates that the most possible breakpoints are 1979, 1990 and 1993 (see Table 12 in the Appendix).

As the quarterly database starts at the first quarter of 1991, two of three annual breakpoints fall outside of the quarterly sample. Figure 2 strengthens the one-breakpoint hypothesis, which might be somewhere between 1993-1994.

Consequently, we estimated the segmented trend model based on the breaks identified above. All estimated parameters are significantly different from zero; however, residuals are autocorrelated, heteroskedastic and not normally distributed in case of annual data. Figure 11 and Figure 12 show the fitted values and the resulting output gaps, while Table 1 displays the numerical values of the output gap for 1989-2002.

Figure 11 reveals that the method can hardly capture the dynamics occurring during the regime switch. Visual inspection suggests that in the pre-1989 and post-1994 period the method might have some relevance; however, this method will break down whenever there is a structural change in the slope parameter, which is also expected to occur in the future. Still, recent results can be interpreted in economic terms, as the deviation from a deterministic trend fully reflects the growth cycles.

Figure 12 reveals that quarterly data provide more detailed information compared to annual data. Following the outbreak of the Russian crisis in August 1998, the output gap

[16] Description of this method can be found on page 34.

became more negative at first and then after some quarters actual GDP return to potential. This can be easily observed in quarterly data (see Figure 12), but is totally hidden in the yearly averages (see Table 1).

**Table 1 Output gap from segmented deterministic trend, 1989-2002**

| Output gap* | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
|---|---|---|---|---|---|---|---|
| Annual data | 2.3 | -2.3 | -7.4 | -2.9 | 4.0 | 3.0 | 0.5 |
| Quarterly data | - | - | 1.6 | -1.8 | -1.5 | 2.1 | 1.4 |
| | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| Annual data | -2.1 | -1.6 | -0.8 | -0.7 | 0.4 | 0.2 | -0.5 |
| Quarterly data | -1.3 | -1.0 | -0.4 | -0.2 | 0.9 | 0.5 | -0.2 |

*In percentage points

Quarterly and annual output gaps are close to each other (with the exception of 1993) and to our expert opinion of the Hungarian economy.

### IV.3.2 Hodrick-Prescott filter[17]

For both annual and quarterly data we tried two smoothing parameters: in addition to the US-data suggested parameters, we also imposed less smoothness on the data. Specifically, for annual data we tried $\lambda = 100$ and $\lambda = 10$, while for quarterly data $\lambda$=1600 and $\lambda$=100 was used. The fitted HP–trends and output gaps are shown in Figure 13 and Figure 14, while the numerical values of the inferred output gaps are shown in Table 2.

Figure 13 showing long-span annual data makes it clear that the HP-filter "over-smoothes" periods of large structural changes. This is an indication of the inappropriateness of the HP filter. Consequently, we cannot interpret the period of regime switch with the filter.

For quarterly data the over-smoothing result is not so obvious in Figure 14, mainly because the sample starts in 1991, so only part of the large fall and no part of the previous high level is included in the sample. Both the dynamics and the numerical values of output gaps seem to be acceptable for later years. Nonetheless, the evaluation of the Russian crisis depends greatly on the smoothing parameter. The suggested $\lambda$=1600 indicates that the economy return to potential following the Russian crisis, while the less smooth $\lambda$=100 indicates a substantially negative gap.

Comparing the revealed output gaps to other methods, the less smooth HP(100) is somehow closer to the other methods (see cross-correlation in Table 17 in the Appendix). However, we chose HP(1600) for conformability to the literature.

[17] Description of this method can be found on page 36.

**Table 2 Output gap from HP filters, 1989-2002**

| | Output gap* | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
|---|---|---|---|---|---|---|---|---|
| Annual data | HP(100) | 7.8 | 5.5 | -6.0 | -8.1 | -8.2 | -5.4 | -4.7 |
| | HP(10) | 5.4 | 4.8 | -4.4 | -4.6 | -3.5 | -0.4 | -0.1 |
| Quarterly data | HP(1600) | - | - | 2.9 | -1.1 | -1.8 | 0.0 | -0.1 |
| | HP(100) | - | - | 0.8 | -0.9 | -0.8 | 0.8 | 0.5 |
| | | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| Annual data | HP(100) | -4.8 | -2.3 | 0.0 | 1.3 | 3.3 | 3.9 | 4.0 |
| | HP(10) | -1.1 | 0.0 | 0.8 | 0.7 | 1.3 | 0.7 | -0.4 |
| Quarterly data | HP(1600) | -1.3 | -0.4 | 0.4 | 0.4 | 1.1 | 0.5 | -0.5 |
| | HP(100) | -0.7 | 0.0 | 0.2 | -0.2 | 0.4 | 0.1 | -0.2 |

*In percentage points

### IV.3.3 Band-pass filter[18]

Among several parameter combinations we find that the most acceptable output gap is obtained if the upper frequency period is 20 quarters and the lower one is 2 years or quarters in the case of annual or quarterly data, respectively. Figure 15 and Figure 16 show estimated potential outputs and gaps for annual and quarterly data, while Table 3 displays the numerical values of the gaps for 1989-2002.

The BP filter led to interpretable output gaps: both after the stabilization package of 1995 and after the Russian crisis of 1998 the output gaps are negative.

Since the HP filter is the subversion of the BP filter (see Baxter and King(1995)), it is worth comparing the relationship between HP, BP and other methods. Table 16 in the Appendix shows cross-correlation of output gaps derived by various methods. It is clear from this table that the BP provides more similar results to the other methods than the HP filter.

Comparing Figure 14 and Figure 16, the relationship between BP and HP filter is more obvious for quarterly data, which may be due to the fact that the quarterly data include a much shorter period of the transitional recession. The cross-correlation between the BP filter with a 20-quarter-upper limit and the HP filter with $\lambda$=100 is extremely high (0.96) thus these two methods provide almost the same result. Among the HP and BP filters, the estimated output gaps for 1991 favours the BP filter. Although we have no sign prior to the early years of the 1990s, it is more interpretable if output gaps had the same sign during this period. While the HP filter yields ‘sign-changing’ output gap from 1991 to 1992, the BP filter does not show this kind of property.

[18] Description of this method can be found on page 37.

**Table 3 Output gap from the BP(20) filter, 1989-2002**

| Output gap* | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
|---|---|---|---|---|---|---|---|
| Annual data | 1.0 | 2.8 | -2.8 | -0.6 | -0.1 | 1.0 | 0.4 |
| Quarterly data | - | - | 1.1 | -1.3 | -0.8 | 1.2 | 0.6 |
| | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| Annual data | -0.9 | 0.1 | 0.4 | -0.4 | 0.3 | 0.1 | -0.1 |
| Quarterly data | -1.0 | -0.1 | 0.5 | 0.0 | 0.2 | -0.3 | -0.2 |

*In percentage points

### IV.3.4 Beveridge-Nelson decomposition[19]

Four types of ARIMA models were estimated. Table 4 displays the estimation results for annual data:

**Table 4 Estimation result of ARIMA models for annual data**

| Method | Results |
|---|---|
| ARIMA(0,1,1) | $\Delta q_t = \underset{(0.007)}{0.030} + \varepsilon_t + \underset{(0.132)}{0.546}\varepsilon_t$ ; LM-SC(2): 0.031, RESET(1): 0.976 |
| ARIMA(1,1,1) | $\Delta q_t = \underset{(0.015)}{0.028} + \underset{(0.161)}{0.750}\Delta q_{t-1} + \varepsilon_t - \underset{(0.241)}{0.193}\varepsilon_t$ ; LM-SC(2): 0.804, RESET(1): 0.848 |
| ARIMA(1,1,0) | $\Delta q_t = \underset{(0.012)}{0.029} + \underset{(0.122)}{0.642}\Delta q_{t-1} + \varepsilon_t$ ; LM-SC(2): 0.740, RESET(1): 0.753 |
| ARIMA(2,1,0) | $\Delta q_t = \underset{(0.014)}{0.028} + \underset{(0.163)}{0.759}\Delta q_{t-1} + \underset{(0.162)}{0.089}\Delta q_{t-2} + \varepsilon_t$ ; LM-SC(2): 0.898, RESET(1): 0.840 |
| ARIMA(1,1,0)S | $\Delta q_t = \underset{(0.018)}{0.069} - \underset{(0.307)}{0.254}\Delta q_{t-1} + d_{79}(-\underset{(0.020)}{0.049} + \underset{(0.492)}{0.052}\Delta q_{t-1})$ $+ d_{90}(-\underset{(0.018)}{0.094} + \underset{(0.434)}{0.011}\Delta q_{t-1}) + d_{93}(\underset{(0.018)}{0.093} + \underset{(0.318)}{0.688}\Delta q_{t-1}) + \varepsilon_t$ LM-SC(2): 0.000, RESET(1): - |
| ARIMA(2,1,0)S | $\Delta q_t = \underset{(0.022)}{0.080} - \underset{(0.254)}{0.285}\Delta q_{t-1} - \underset{(0.264)}{0.180}\Delta q_{t-2}$ $+ d_{79}(-\underset{(0.024)}{0.059} + \underset{(0.402)}{0.091}\Delta q_{t-1} + \underset{(0.352)}{0.139}\Delta q_{t-2}) + d_{90}(-\underset{(0.016)}{0.068} + \underset{(0.462)}{1.415}\Delta q_{t-1}$ $- \underset{(1.048)}{4.77}\Delta q_{t-2}) + d_{93}(\underset{(0.016)}{0.071} - \underset{(0.541)}{0.993}\Delta q_{t-1} + \underset{(1.042)}{4.96}\Delta q_{t-2}) + \varepsilon_t$ LM-SC(2): 0.098, RESET(1): - |

Notes: Standard errors are in brackets. LM-SC(2): p-value of 2nd order serial correlation Lagrange multiplier test; RESET: p-value of Ramsey reset test.

Almost all parameters are significant and only the residuals of the ARIMA(0,1,1) specification are autocorrelated. However, there are structural breaks in each case (see Table 12 in the Appendix). We searched for the location of breakpoints with the method described in Section III.2. The tests identify almost the same points for all models. In

[19] Description of this method can be found on page 16.

the case of ARIMA(1,1,0) and ARIMA(0,1,1) the first break point is at 1979, while it is 1980 in ARIMA(2,1,0) and 1978 in ARIMA(1,1,1). The second breakpoint is at 1990 in all cases. With the exception of ARIMA(0,1,1) the last breakpoint is at 1993. These results are quite consistent with those of the segmented time trend. Table 5 displays the estimated output gaps from BN-decomposition:

**Table 5 Output gap from Beveridge-Nelson decomposition based on annual data, 1989-2002**

| Output gap* | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
|---|---|---|---|---|---|---|---|
| ARIMA(0,1,1) | -0.2 | -3.5 | -6.7 | 0.3 | -2.1 | 1.1 | -1.4 |
| ARIMA(1,1,0) | -3.9 | -11.6 | -28.0 | -10.8 | -6.3 | 0.0 | -2.6 |
| ARIMA(1,1,1) | -5.9 | -15.4 | -37.5 | -20.5 | -11.6 | -2.1 | -3.4 |
| ARIMA(2,1,0) | -4.8 | -13.3 | -32.8 | -16.0 | -8.3 | -0.6 | -2.6 |
| ARIMA(1,1,0) S | -10.6 | -11.4 | -7.8 | -5.4 | -7.6 | -2.9 | -2.5 |
| ARIMA(2,1,0) S | -25.3 | -25.5 | -22.6 | -10.6 | -8.4 | -3.7 | -2.5 |
| | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| ARIMA(0,1,1) | -0.1 | 0.9 | 0.5 | 0.3 | 0.9 | -0.1 | 0.2 |
| ARIMA(1,1,0) | -2.8 | 2.8 | 3.3 | 2.2 | 3.9 | 1.5 | 0.6 |
| ARIMA(1,1,1) | -4.0 | 2.8 | 4.8 | 3.8 | 5.7 | 3.2 | 1.5 |
| ARIMA(2,1,0) | -3.2 | 3.1 | 4.5 | 3.2 | 5.0 | 2.6 | 1.2 |
| ARIMA(1,1,0) S | -3.4 | -0.4 | 1.4 | 1.0 | 1.6 | 0.8 | -0.4 |
| ARIMA(2,1,0) S | -2.8 | -1.1 | 0.5 | 0.7 | 1.0 | 0.6 | -0.2 |

*In percentage points. Note: "S" indicates models with structural breaks.

Although the output gaps of ARIMA(1,1,0)S seem acceptable on the first perusal, we can rule out the ARIMA(1,1,0)S model since its residuals are auto-correlated. On the other hand the earlier output gaps are negative in the previous period, which is also true for ARIMA(2,1,0)S (see Figure 17). This phenomenon is not surprising because the three breakpoints distort the adaptability of Beverage-Nelson decomposition. Due to that we ignore these approximation in the case of annual data.

For quarterly data we apply the same specifications. According to Table 6 all estimated parameters are significant; however the residuals of ARIMA(0,1,1) and ARIMA(1,1,0) are autocorrelated. Moreover, the Ramsey test indicates specification problem with these models.

**Table 6 Estimation result of ARIMA models for quarterly data**

| Method | Results |
|---|---|
| ARIMA(0,1,1) | $\Delta q_t = \underset{(0.002)}{0.005} + \varepsilon_t + \underset{(0.127)}{0.483}\,\varepsilon_t$ ; LM-SC(2): 0.000, RESET(1): 0.001 |
| ARIMA(1,1,1) | $\Delta q_t = \underset{(0.001)}{0.010} + \underset{(0.019)}{0.874}\,\Delta q_{t-1} + \varepsilon_t - \underset{(0.024)}{0.984}\,\varepsilon_t$ ; LM-SC(2): 0.135, RESET(1): 0.187 |
| ARIMA(1,1,0) | $\Delta q_t = \underset{(0.003)}{0.007} + \underset{(0.092)}{0.721}\,\Delta q_{t-1} + \varepsilon_t$ ; LM-SC(2): 0.012, RESET(1): 0.013 |
| ARIMA(2,1,0) | $\Delta q_t = \underset{(0.003)}{0.009} + \underset{(0.141)}{0.378}\,\Delta q_{t-1} + \underset{(0.132)}{0.379}\,\Delta q_{t-2} + \varepsilon_t$ ; LM-SC(2): 0.19, RESET(1): 0.49 |
| ARIMA(1,1,0)S | $\Delta q_t = \underset{(0.004)}{0.004} + \underset{(0.198)}{1.264}\,\Delta q_{t-1} + d_{94.2}(\underset{(0.002)}{0.00089} - \underset{(0.249)}{0.959}\,\Delta q_{t-1}) + \varepsilon_t$<br>LM-SC(2): 0.313, RESET(1): - |
| ARIMA(2,1,0)S | $\Delta q_t = \underset{(0.002)}{0.006} + \underset{(0.584)}{0.638}\,\Delta q_{t-1} + \underset{(0.614)}{0.712}\,\Delta q_{t-2}$<br>$+ d_{94.2}(-\underset{(0.002)}{0.002} - \underset{(0.604)}{0.391}\,\Delta q_{t-1} - \underset{(0.632)}{0.427}\,\Delta q_{t-2}) + \varepsilon_t$<br>LM-SC(2): 0.057, RESET(1): - |

Notes: Standard errors are in brackets. LM-SC(2): 2nd order serial correlation Lagrange multiplier test; RESET: Ramsey reset test.

In contrast to the annual case the likeliest place of breakpoints in the different models does not coincide in each case (see Table 13 in the Appendix), but they are quite close to each other. Chow test indicates breakpoint at 1994:Q2 in the case of ARIMA(1,1,0) and ARIMA(2,1,0); while it is at 1994:Q1 and 1993:Q4 in the case of ARIMA(1,1,1) and ARIMA(0,1,1), respectively. Applying the necessary dummy and re-estimating the models the autocorrelation in ARIMA(1,1,0) turns to insignificant.

**Table 7 Output gap from Beveridge-Nelson decomposition based on quarterly data, 1989-2002**

| Output gap* | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
|---|---|---|---|---|---|---|---|
| ARIMA(0,1,1) | - | - | -0.51 | -0.34 | -0.12 | 0.01 | -0.08 |
| ARIMA(1,1,0) | - | - | - | -3.6 | -1.5 | -0.2 | -1.0 |
| ARIMA(1,1,1) | - | - | - | -11.8 | -6.5 | -3.3 | -3.2 |
| ARIMA(2,1,0) | - | - | - | -7.7 | -3.6 | -1.0 | -2.4 |
| ARIMA(1,1,0) S | - | - | - | -2.5 | -0.9 | -0.2 | -0.9 |
| ARIMA(2,1,0) S | - | - | - | -16.8 | -8.1 | -0.8 | -3.3 |
| | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| ARIMA(0,1,1) | 0.10 | 0.25 | 0.10 | 0.29 | 0.17 | 0.09 | 0.11 |
| ARIMA(1,1,0) | -0.1 | 1.4 | 0.7 | 1.6 | 1.0 | 0.3 | 0.4 |
| ARIMA(1,1,1) | -1.1 | 0.3 | -0.5 | 0.0 | -0.8 | -0.7 | 0.0 |
| ARIMA(2,1,0) | -1.6 | 1.8 | 0.9 | 1.7 | 1.3 | -0.1 | 0.0 |
| ARIMA(1,1,0) S | -0.3 | 1.0 | 0.4 | 1.1 | 0.6 | 0.1 | 0.2 |
| ARIMA(2,1,0) S | -1.6 | 5.6 | 3.7 | 5.4 | 4.5 | 1.6 | 1.8 |

*In percentage points. Note: "S" indicates models with structural breaks.

Due to the unfavourable statistics, the estimated output gaps of ARIMA(0,1,1) and ARIMA(1,1,0) models are not acceptable. All other models display negative output gap after the stabilization package and Russian crisis (see Table 7 and Figure 18); however, the ARIMA(1,1,1) model provides a positive output gap in 1994. A further argument in favour of this specification is that it seems to give smoother output gaps (see Figure 18).

### IV.3.5 Unobserved components[20]

Since state space models were sensitive to starting values we apply the searching method described in Section III.3 to find the most appropriate estimation result.

Although several UC models in the literature assume an I(1) process for potential output, it is clear that this process can not approximate Hungarian potential output very well, as there were prolonged periods of markedly different growth rates of actual output (Figure 1). Consequently, either an I(1) process with shifts in the drift or an I(2) process might be used as approximations. As an example, we estimated a UC model assuming a standard I(1) process:

UC1 model for annual data (6)

$$q_t = q_t^* + c_t^*$$

$$\Delta q_t^* = \underset{(0.0554)}{0.0303} + e_t \quad \hat{\sigma}_e^2 = \underset{(0.084885)}{0.001188}$$

$$c_t^* = \underset{(4.1947)}{0.9954} c_{t-1}^* + u_t \quad \hat{\sigma}_u^2 = \underset{(0.084774)}{0.000104}$$

$$\text{LLF} = 82.10$$

[20] Description of this method can be found on page 40.

where initial conditions were $q_0 = q_1 - \overline{\Delta q}$, $c_0 = 0$ and starting values were $\mu_q = \overline{\Delta q}$, $\hat{\sigma}_e = 0.03$, $\alpha_c = 0.1$ and $\hat{\sigma}_u = 0.01$. Standard errors are in parentheses and *LLF* is the value of the log-likelihood function. Although the estimated drift term nearly equals the mean growth rate of GDP per capita for 1961-2002, this parameter and the autoregressive coefficient of the cycle are insignificant. The flaws of this model are also reflected in the results: actual output is higher than potential output for the entire period, which is clearly unacceptable (see Figure 19a-b).

A possible solution for the changing drift case would be the assumption of autoregressivity of the growth rate. Although this model implies a certain long-run growth rate, it might allow different growth rates for certain sub-periods.

UC2 model for annual data (7)

$$q_t = q_t^* + c_t^*$$

$$\Delta q_t^* = \underset{(0.0089)}{0.0087} + \underset{(0.2720)}{0.7065}\,\Delta q_{t-1}^* + e_t \quad \hat{\sigma}_e^2 = \underset{(0.000453)}{0.000574}$$

$$c_t^* = \underset{(6.3743)}{0.8420}\,c_{t-1}^* + u_t \quad \hat{\sigma}_u^2 = \underset{(0.000797)}{0.000113}$$

$$\text{LLF} = 92.64$$

Initial conditions were $q_0 = q_1 - \overline{\Delta q}$, $c_0 = 0$ and starting values for the first state equation were derived from the $\Delta q_t = \alpha_0 + \alpha_1 \Delta q_{t-1}$ regression, $\hat{\sigma}_e = 0.01$, $\alpha_c = 0.2$ and $\hat{\sigma}_u = 0.01$.

In this case, we obtained better statistics, but the estimation results show rather strange values before the mid-1970s, as the output gap is negative until that time. After the regime change in 1989 the output gap displays similar dynamics to the other methods, but it is negative until 2000 (see Figure 19c-d). Consequently, this model can also be discarded.

Next we allow breaks in the drift to allow different growth rate of potential output in different periods.

UC1_S model for annual data (8)

$$q_t = q_t^* + c_t^*$$

$$\Delta q_t^* = \underset{(0.0012)}{0.0555} + d_{79}(\underset{(0.0040)}{-0.0458}) + d_{90}(\underset{(0.0114)}{-0.07869}) + d_{93}(\underset{(0.0114)}{0.1059}) + e_t$$

$$\hat{\sigma}_e^2 = \underset{(4.85\mathrm{E}-05)}{1.41\mathrm{E}-07}$$

$$c_t^* = \underset{(0.4153)}{0.5609}\,c_{t-1}^* + u_t \quad \hat{\sigma}_u^2 = \underset{(8.94\mathrm{E}-05)}{0.000349}$$

$$\text{LLF} = 109.20$$

Initial parameters of first state equation were derived from the growth rate of GDP and $\hat{\sigma}_e = 0.01$, $\alpha_c = 0.9$ and $\hat{\sigma}_u = 0.01$. As one can see, the estimated parameters are significant and the output gaps seem to be more acceptable (see Figure 19c-d). There are negative output gaps both after the beginning of the transition and after the stabilization package in 1995.

In the next we allow breaks in the autoregressive drift parameter:

UC2_S model for annual data (9)

$$q_t = q_t^* + c_t^*$$

$$\Delta q_t^* = \underset{(0.001)}{0.005} + \underset{(0.038)}{0.910}\,\Delta q_{t-1}^* + d_{79}(\underset{(0.009)}{0.003} - \underset{(0.538)}{0.751}\,\Delta q_{t-1}^*)$$
$$+ d_{90}(-\underset{(0.019)}{0.097} - \underset{(0.625)}{0.732}\,\Delta q_{t-1}^*) + d_{93}(\underset{(0.016)}{0.121} + \underset{(0.0427)}{0.737}\,\Delta q_{t-1}^*) + e_t$$

$$\hat{\sigma}_e^2 = \underset{(4.12\text{E-}06)}{1.79\text{E - }08}$$

$$c_t^* = \underset{(0.263)}{0.554}\,c_{t-1}^* + u_t \quad \hat{\sigma}_u^2 = \underset{(7.00\text{E-}05)}{0.000287}$$

LLF = 109.59

Initial conditions were derived from

$$\Delta q_t^{HP(10)*} = \alpha_1 + \alpha_2 \Delta q_{t-1}^{HP(10)*} + d_{79}(\alpha_3 + \alpha_4 \Delta q_{t-1}^{HP(10)*})$$
$$+ d_{90}(\alpha_5 + \alpha_6 \Delta q_{t-1}^{HP(10)*}) + d_{93}(\alpha_7 + \alpha_8 \Delta q_{t-1}^{HP(10)*}) + e_t$$

regression and $\alpha_c = 0.1$, $\hat{\sigma}_e = 0.03$, $\hat{\sigma}_u = 0.03$. Similarly to the other model containing structural breaks (UC1_S) the statistics of parameters become acceptable. The resulting output gaps are shown on Figure 19g-h.

Finally for annual data, we estimate the local linear trend plus cycle model of Harvey that assumes an I(2) process for potential output:[21]

UC_LLTCH model for annual data (10)

$$q_t = q_t^* + c_t^*$$

$$\Delta q_t^* = \Delta q_{t-1}^* + \Delta\Delta q_{t-1}^* + \eta_t$$

$$\Delta\Delta q_t^* = \Delta\Delta q_{t-1}^* + \zeta_t$$

$$\begin{bmatrix} c_t^* \\ c_t^+ \end{bmatrix} = \underset{(0.167)}{0.672}\begin{bmatrix} \cos\underset{(1.408)}{1.236} & \sin\underset{(1.408)}{1.236} \\ -\sin\underset{(1.408)}{1.236} & \cos\underset{(1.408)}{1.236} \end{bmatrix}\begin{bmatrix} c_{t-1}^* \\ c_{t-1}^+ \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^+ \end{bmatrix}$$

$$\hat{\sigma}_\eta = \underset{(0.1131)}{0.0000},\ \hat{\sigma}_\zeta = \underset{(0.0049)}{0.0175},\ \hat{\sigma}_\kappa = \hat{\sigma}_{\kappa^+} = \underset{(0.0068)}{0.0103}$$

LLF = 92.74

[21] Although we applied the searching approach to find suitable staring values, we could not obtain reasonable estimates in Eviews. Therefore, this model was estimated using the TSM module of GAUSS.

The result if zero variance for $\eta_t$ is in line with Harvey-Jaeger (1993) arguments and results, implying a "smooth" trend component. The plot of the revealed potential output and output gap is shown in Figure 19i-j.

Table 8 shows the inferred output gaps for 1989-2002 in comparison to quarterly estimates described later.

**Table 8 Output gap from unobserved component models based on annual data, 1989-2002**

| | Output gap* | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
|---|---|---|---|---|---|---|---|---|
| Annual data | UC1_S | 2.9 | -1.6 | -7.4 | -3.6 | 2.7 | 2.0 | -0.3 |
| | UC2 | 0.8 | 1.1 | -1.2 | -0.7 | -0.7 | -0.1 | -0.2 |
| | UC2_S | 2.5 | -2.0 | -5.2 | -4.8 | 1.5 | 2.4 | 0.3 |
| | UC_LLTCH | -0.2 | -0.6 | -1.6 | -1.6 | 0.2 | 1.7 | 1.9 |
| Quarterly data | UC1 | - | - | 0.14 | 0.10 | 0.07 | 0.05 | 0.04 |
| | UC2 | - | - | 1.49 | 0.23 | -0.19 | 0.30 | 0.18 |
| | UC_LLTCAR | - | - | 0.01 | 0.11 | 0.12 | 0.07 | -0.01 |
| | | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| Annual data | UC1_S | -2.6 | -1.8 | -0.8 | -0.3 | 1.0 | 1.1 | 0.7 |
| | UC2 | -0.6 | -0.2 | 0.0 | -0.2 | 0.1 | 0.1 | 0.0 |
| | UC2_S | -2.1 | -1.4 | -0.5 | -0.2 | 1.0 | 1.0 | 0.4 |
| | UC_LLTCH | 1.0 | 0.2 | 0.1 | 0.0 | 0.0 | 0.0 | -0.2 |
| Quarterly data | UC1 | 0.03 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 |
| | UC2 | -0.26 | 0.04 | 0.10 | -0.13 | 0.13 | 0.04 | -0.05 |
| | UC_LLTCAR | 0.03 | 0.08 | 0.00 | 0.02 | -0.01 | -0.03 | 0.00 |

*In percentage points

The dynamics of the output gaps derived from annual data differs substantially from each other. Among the four annual models presented above, the UC2 model seems the least acceptable as it does not indicate any demand push in the period 1993-1995 when the trade balance deteriorated sharply. But most importantly, the assumption of an I(1) process for the full period is highly unacceptable as the growth rate of GDP indicates sustained periods with markedly different average growth rates (Figure 1). Among the other three, the LLTC model that assumes an I(2) process seems to deliver more acceptable results than the two breaking I(1) models. First, it does not reveal substantial change in the output gap from 1989 to 1990. Second, the magnitude of the negative gap of the other models for 1991-1992 seems rather large, and the upturn in 1993 (that exceeds 6 percent of GDP) is also very difficult to explain in economic terms. Therefore, we will include LLTC in the calculation of the consensus estimate.

In the case of quarterly state space models we use the same specifications as we did for annual data and try to find the most suitable starting values. The estimated models are the following.

We start the presentation of results with the UC2 model as in case of the UC1 model the standard deviation of the cycles was estimated to be zero.

UC2 model for quarterly data (11)

$$q_t = q_t^* + c_t^*$$

$$\Delta q_t^* = \underset{(0.00049)}{0.00097} + \underset{(0.0716)}{0.8972}\,\Delta q_{t-1}^* + e_t \quad \hat{\sigma}_e^2 = \underset{(2.24E-06)}{4.31 * 10^{-6}}$$

$$c_t^* = \underset{(0.2131)}{0.7443}\,c_{t-1}^* + u_t \quad \hat{\sigma}_u^2 = \underset{(3.35E-06)}{9.94 * 10^{-6}}$$

$$\text{LLF} = 168.05$$

Initial conditions were $q_0 = q_1 - \overline{\Delta q}$, $c_0 = 0$ and stating values for the first state equation were derived from $\Delta q_t = \alpha_0 + \alpha_1 \Delta q_{t-1}$ regression, $\hat{\sigma}_e = 0.04$, $\alpha_c = 0.2$ and $\hat{\sigma}_u = 0.02$. The estimated parameters are significant and, according to Figure 20b, the estimated potential output captures the setback after the stabilization package and the Russian crisis. However, the magnitude of the gap is very small.

We also estimated the LLT model for quarterly data. We could not achieve statistically and economically significant estimate adopting Harvey's cycle, but an AR cycle specification leads to a reasonable estimate, if we constrained $\sigma_\eta = 0$. This constraint has theoretical foundations (see argument in the Appendix at page 46) and was the result for our estimation to annual data.

UC_LLTCAR model for quarterly data (12)

$$q_t = q_t^* + c_t^*$$

$$\Delta q_t^* = \Delta q_{t-1}^* + \Delta\Delta q_{t-1}^* + \eta_t$$

$$\Delta\Delta q_t^* = \Delta\Delta q_{t-1}^* + \zeta_t$$

$$c_t^* = \underset{(0.387)}{0.693}\,c_{t-1}^* + \kappa_t$$

$$\sigma_\eta = 0,\ \hat{\sigma}_\zeta = \underset{(0.0007)}{0.0028},\ \hat{\sigma}_\kappa = \underset{(0.0008)}{0.0029}$$

$$\text{LLF} = 183.69$$

The resulting potential output and the output gap are shown in Figure 20c-d. Although the dynamics are consistent with our preliminary view on the output gap, the magnitude of the gap is very small again.

### IV.3.6 Wavelet transformation[22]

There are several possible ways to compute wavelet transformations as is described in the Appendix. We test different type of wavelets with several parameterisations to find the most appropriate ones. Based on smoothness of estimated potential GDP we found that the Daubechies' wavelets with 4, 8 and 16 filter elements seem acceptable with a 2-scale multiresolution scheme. For quarterly data we also apply Daubechies' wavelets with 4, 8 and 16 filter elements, however, we use a 3-scale multiresolution scheme. The graphs of potential outputs and gaps are shown in Figure 21 and Figure 22, while the numerical values of the gaps for 1989-2002 are displayed in Table 9.

For annual data, the Daubechies filer with 4 filter elements (D42) seems to reveal too large a positive output gap for 1989. We have no firm grounds to select between B82 and D162, but as the output gap revealed by them is rather similar and highly correlated (0.94), the choice is of less importance. We have selected D162 for inclusion in the consensus estimate.

For quarterly data there is no significant difference between numerical values of output gaps, and the cross-correlation coefficients also show strong relationships among various estimates (see Table 17 in the Appendix). The only minor point that makes the D83 and D163 filter a bit favourable is the negative output gaps in 1995. We have selected D83 for inclusion in the consensus estimate.

**Table 9 Output gaps from Wavelet transformations, 1989-2002**

| | | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
|---|---|---|---|---|---|---|---|---|
| Annual data | D42 | 4.6 | 2.0 | -5.0 | -0.1 | -0.4 | 2.0 | -1.0 |
| | D82 | 1.4 | -1.2 | -7.9 | 2.1 | 3.1 | 4.4 | -0.6 |
| | D162 | 2.7 | 1.0 | -6.7 | 2.0 | 2.3 | 2.8 | -1.2 |
| Quarterly data | D43 | - | - | -0.3 | -0.4 | -0.2 | 0.4 | 0.0 |
| | D83 | - | - | -0.4 | -0.3 | -0.1 | 0.4 | -0.1 |
| | D163 | - | - | -0.6 | -0.3 | 0.1 | 0.3 | -0.2 |
| | | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| Annual data | D42 | -1.5 | 0.9 | 0.2 | -1.4 | 1.5 | 1.3 | 1.1 |
| | D82 | -2.8 | 0.1 | -0.1 | -0.5 | 0.8 | -0.2 | 0.5 |
| | D162 | -2.6 | 0.0 | 0.1 | -0.3 | 0.9 | 0.3 | 1.0 |
| Quarterly data | D43 | -0.1 | 0.1 | -0.2 | 0.1 | 0.0 | -0.1 | 0.3 |
| | D83 | -0.1 | 0.2 | -0.2 | 0.2 | 0.0 | -0.1 | 0.3 |
| | D163 | -0.1 | 0.3 | -0.2 | 0.1 | -0.1 | -0.1 | 0.3 |

*In percentage points. The first number after "D" denotes the length of the filter while the second one denotes the number of iteration in multi-scale analysis.

[22] Description of this method can be found on page 47.

## IV.4 The consensus estimate and its comparison to other methods

In the previous sections we presented detailed results for each model and selected some of them as candidates for the consensus estimates (see Table 15 for the overview of models). In this section we present the consensus estimate as described in Section III.1, that is, we derive weights based on revisions on recursive samples. The results of recursive sample estimation are graphed in Figure 23.

However, a further consideration arises regarding the measure of output gap for which we study the revision. Namely, one can expect smaller absolute revisions for those output gap series that have smaller standard deviations. That is, our method would generally prefer output gap measures having smaller standard deviations, although there are no theoretical arguments regarding the standard deviation of the gap.[23] Figure 24-Figure 25 in the Appendix shows the cross plot of the standard deviation of the gap and revisions. Indeed, there is a strong relationship between the two measures. Therefor, we also calculated the revisions for standardized output gap figures. Table 14 shows the ranking of the methods according to revisions both for the estimated the output gap and for its standardized values. We have highlighted the methods that were eventually selected for the consensus estimate. It can be seen that the ranking of the selected series is very similar in both cases (if we exclude the quarterly UC_LLTCAR model) and weights do not differ much from each other. The two weighting schemes led to almost identical results after plotting the resulting output gap estimates. Therefore, we report the results achieved by the non-transformed output gaps.

Table 10 shows the structure of the consensus output gap estimates.

**Table 10 Structure of consensus output gap estimates**

| Annual data | | Quarterly data | |
|---|---|---|---|
| method | weight | method | weight |
| Segmented deterministic trend | 24% | Segmented deterministic trend | 21% |
| HP filter with λ=10 | 20% | HP filter with λ=1600 | 15% |
| BP filter with 20 quarters upper limit | 12% | BP filter with 20 quarters upper limit | 17% |
| (Beveridge-Nelson decomposition are not included) | | Beveridge-Nelson decomposition based of ARIMA(1,1,0) | 16% |
| Unobserved component model with local linear trend plus cycle model of Harvey (UC_LLTCH) | 15% | (Unobserved component models are not included) | |
| Wavelet transformation with 16 filter elements Daubechies wavelet in 2-scale multiresolution | 29% | Wavelet transformation with 8 filter elements Daubechies wavelet in 3-scale multires0lution | 32% |

Note: Weights are derived from the recursive revisions of non-transformed output gaps.

[23] For example, among the quarterly estimates the UC models led to very small output gaps. Having the selected UC_LLTCAR model among the ingredients for the consensus, our method would give an 84% weight to this model. We did not include this model in the consensus estimate.

It is interesting to highlight that the HP filter, which is heavily criticized because of its instability at the end of the sample, ranked in the midfield among annual estimates. However, this result is partly due to the lower than usual smoothness parameter we adopted. Using the US-suggested data parameters, the HP perform worse (Table 14). In the case of quarterly data, the revision of the HP filter is similar to that of the BP and BN filters.

We compare our consensus result to that of the updated Darvas-Simon (2000b) model (hereafter DS), which is available at the annual frequency only. Figure 3 shows the results of the annual consensus, the DS-model, and the annualised quarterly consensus. Figure 4 shows the quarterly consensus estimate and also the results of the individual methods to have a better picture on the heterogeneity of results. Table 11 displays the numerical values of output gap estimates for 1989-2002 at the annual frequency. The growth rate of actual and potential GDP (compared to the previous quarter) is shown in Figure 5.

**Table 11 Consensus output gap estimates, 1989-2002**

| | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
|---|---|---|---|---|---|---|---|
| Consensus, annual data | 1.9 | 1.2 | -4.3 | -1.2 | 0.6 | 1.6 | 0.0 |
| Consensus, quarterly data | - | - | -0.1* | -1.2 | -0.7 | 0.8 | 0.3 |
| Darvas-Simon (2000b) | -0.7 | -1.1 | -2.4 | -1.0 | 1.6 | 3.2 | 2.5 |
| | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| Consensus, annual data | -1.4 | -0.1 | 0.3 | -0.1 | 0.7 | 0.3 | 0.0 |
| Consensus, quarterly data | -0.6 | 0.0 | 0.0 | 0.1 | 0.4 | 0.1 | 0.0 |
| Darvas-Simon (2000b) | 0.9 | 0.2 | 0.0 | -0.2 | -0.2 | 0.1 | 0.1 |

* The 1991Q1-Q2 values used for calculating the annual average is the average of 4 methods only, due to the shrink of the sample in case of ARIMA models because of lags.

**Figure 3 Output gap from consensus univariate and benchmark model: annual data**

**a) Full period** **b) 1989-2002**

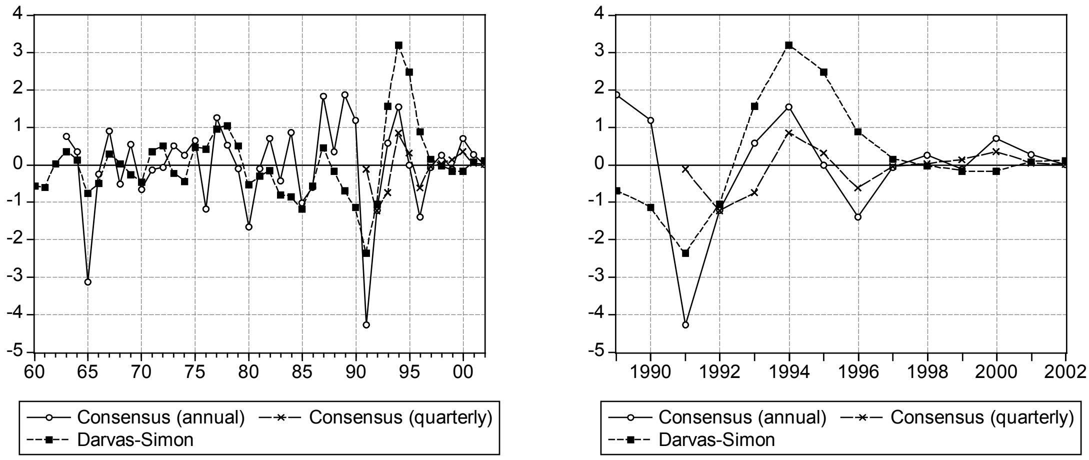


**Figure 4 Quarterly univariate consensus output gap and its components**

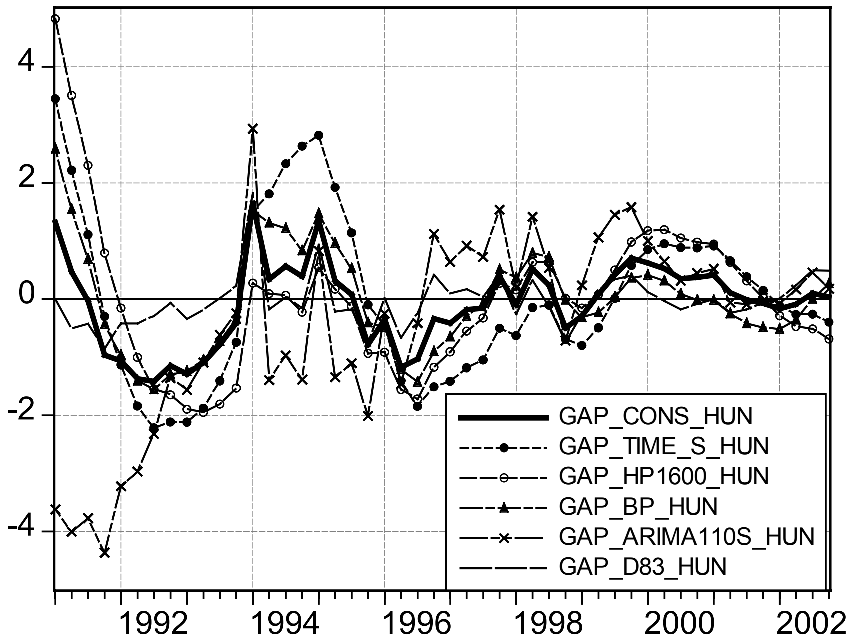
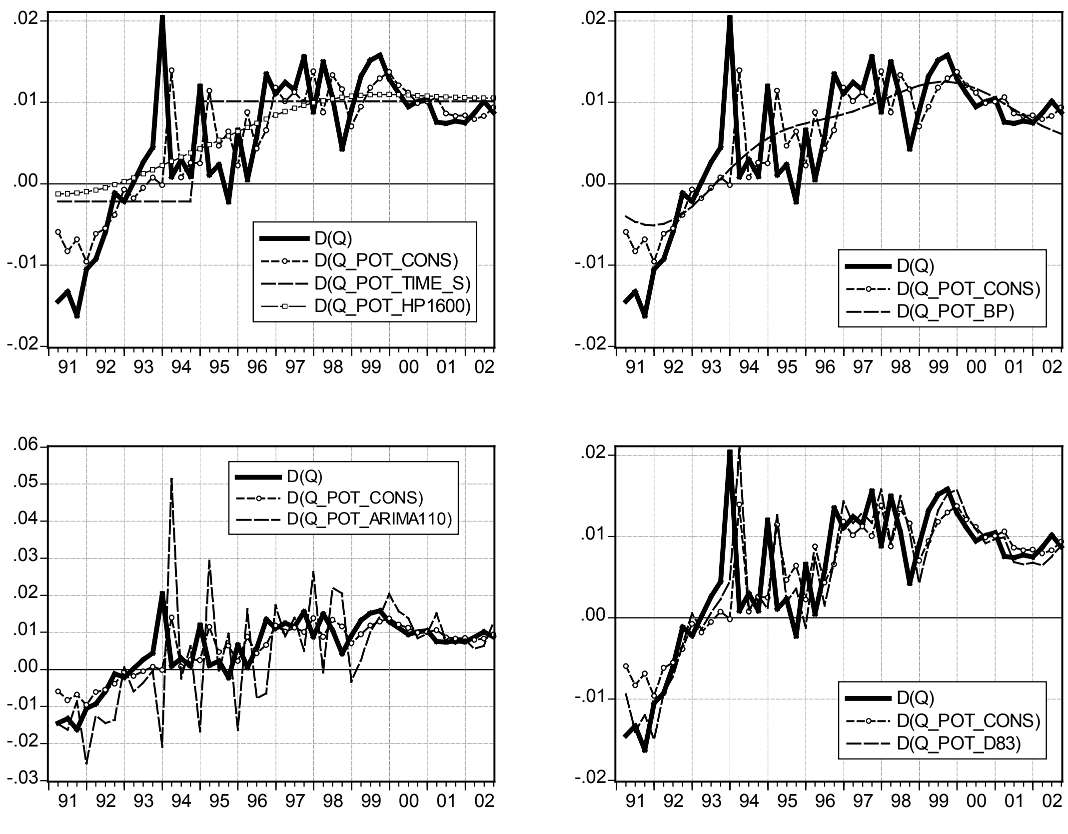



**Figure 5 Quarterly univariate consensus potential GDP growth and its components**



The univariate consensus estimates for the two frequencies are reasonably close to each other with the exception of 1991. For 1991 the quarterly estimates infer a minor positive gap, while annual estimates indicates a substantial shortage of demand. This might be the consequence of the fact that univariate models tend to "smooth" output fluctuations. Annual data are available since 1960, so data for the high output level before the large GDP fall in 1991 is included in the annual estimates, while the quarterly sample starts in 1991. For later years, the two frequencies lead to similar estimates although the variance of the annual gap is somehow larger.

The consensus univariate estimate and the DS model differ substantially in the interpretation of the transition period. The univariate estimate indicates an overheated economy before the transition and a large negative output gap (-4.3%) at the slump in economic activity. In contrast, the DS model indicates some shortage of demand before transition and a smaller (-2.4%) negative gap at the slump. Economic reasoning would give more credit to the DS model. The GDP growth was almost zero in 1989-1990 and the trade balance surplus was close to historical highs (Figure 1), conditions which are difficult to rationalize in an overheated economy, as implied by the univariate models.[24]

The tendency of output gap developments in the consensus univariate and the DS model for subsequent years is similar, although the magnitude of the 1993-1995 overheating and the effects of the stabilization package are different. The consensus univariate estimate indicates a 1.6% output gap peak for 1994 and a sharp fall in excess demand and even a considerable shortage of excess demand by 1996 (-1.4%). In contrast, the DS model indicates a much larger positive gap for 1994 (3.2%) and gradual return to potential by 1997, but not an undershooting. Other indicators do not help us in opting for any of the alternatives. The trade deficit disappeared so quickly which might indicate a quick reversal to potential output. On the other hand, inflation even increased in 1995 and declined by the same rate in subsequent years, and the unemployment rate continuously fell both before and after the introduction of the stabilization program.

From 1997 there are minor differences between the results of the univariate and DS models. Perhaps the only notable difference is in 2000, when the consensus univariate indicates a slight overheating (0.7%), while the DS model infers a minor slackness of the economy (-0.2%).

Quarterly estimates can describe within-year developments of the output gap as well (Figure 4). In fact, univariate methods reproduced our expectation about potential GDP and the output gap for most of the 1990s, with the exception of the early years of transition. The output gap was negative after stabilization package of March 1995 and the Russian financial crisis that started in the third quarter of 1998. After the recovery by early 1999, the Hungarian economy performed somehow above potential until early 2001. Since then the economy has been very close to potential, according to the quarterly consensus estimate. This also implies that the growth rate of potential output was close to actual growth, as can be seen on Figure 5. Thus the univariate methods considered here suggests that the recent slowdown of the Hungarian economy was not primarily due to cyclical reasons but mostly the consequence of decelerated potential growth.

[24] Note that the result of the UC_LLTCH model assuming an I(2) process for potential output differs markedly from other univariate models in the transition period and indicates an output gap development similar to the DS model (panel (j) of Figure 19 on page 63).

# V. Summary

In this paper we have reviewed some of the numerous univariate detrending methods, namely the segmented time trend, Hodrick-Prescott filter, band-pass filter, Beveridge-Nelson decomposition, unobserved component models and wavelet transformation, which can be applied in the estimation of potential output and output gaps.

Various variants of these methods were estimated for Hungarian data for the period 1960-2002 in respect of annual data and for 1991-2002 in respect of quarterly data. As these periods are likely burdened with structural breaks, we paid special attention to addressing this issue.

We evaluated the results of the various specifications by statistical tools on the one hand and by expertise judgement on the other, as we are critical of mechanical adoption of the methods.

As all methods have strengths and weaknesses we derived a single measure of potential output by weighting those methods that pass both the statistical and expertise criteria. We derived weights for the selected methods based on revisions of the output gap for all dates by recursively estimating the models. As a benchmark, we compared this estimate to the only other available Hungarian output gap estimate of Darvas-Simon (2000b) that is based on an economic model.

The results of our univariate and the economic model based approach differ substantially regarding the interpretation of the transition shock and somehow in the aftermath of the stabilization package of 1995, but are similar in other periods. Our general conclusion is that apart from the transition shock, the weighted average of univariate model results can provide a useful indicator for the stance of the economy.

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# VII. Appendix 1: Survey of measures of potential output

There are some surveys available on potential output/trend/permanent component measurement. The results of these surveys do not make our task of selecting the most appropriate method simple. The most wide-ranging survey can be found in Canova (1998), who compare the properties of the cyclical components of US data using seven univariate (Hodrick–Prescott filter, Beveridge–Nelson decomposition, linear trend, segmented trend, first order differencing, unobservable components model, frequency domain masking) and three multivariate (cointegration, common linear trend and multivariate frequency domain) detrending techniques. His conclusion is that, both quantitatively and qualitatively, properties of business cycles vary widely across detrending methods and that alternative detrending filters extract different types of information from the data.

Several other papers compare a few methods. Dupasquier–Guay–St-Amant (1997) focuses on three multivariate methodologies: structural vector autoregression, multivariate Beveridge–Nelson decomposition, and Cochrane's methodology. Due to the deficiencies of mechanical filters and other univariate approaches they apply some of them only for comparison. They arrive to the conclusion that statistical properties of cycles derived from different methods are dissimilar, conclusions regarding certain recessions in the US are different, and also highlight that the confidence intervals of different measures are generally wide.

Harvey–Jaeger (1993) compares some univariate models: the structural times series models of Harvey, the Hodrick–Prescott filter, ARIMA modelling (i.e. Beveridge–Nelson decomposition), and discuss segmented trend approach. They argue that all but the structural times series models suffer from significant deficiencies and make a case for their structural models that in fact encompasses all the other models.

OECD procedures are presented in Giorno *et.al.* (1995), who compare three methods: the previously used segmented trend method, the Hodrick-Prescott filter, and the production function approach. They conclude that the last one seems to be superior, therefore, production functions are used by the OECD in assessing the cyclical position of member countries. On the other hand, they also recognize that all output gaps measures are subject to considerable uncertainty.

As this is the first attempt in the National Bank of Hungary to measure potential output we have highlighted the main properties of the methods to a larger extent than what a "usual" survey would require.

## VII.1 Segmented deterministic trend

Deterministic time trends were among the first methods to estimate potential output. The slope of the trend might be constant through the full period or the trend line might break at several points in time. In this case, the model is called as segmented trend or split-time-trend method, that is,

$$q_t = \alpha + \sum_{i=1}^{k} \beta_i t I_{t \geq T_i} + \varepsilon_t, \qquad (13)$$

where $q_t$ is output, $\alpha$ and $\beta_i$ are parameters, $t$ is the time trend, $I_{t \geq T_i}$ is the indicator function which equals zero before $T_i$ and one afterwards, and $\varepsilon_t$ is the error term.

This approach was applied by the OECD up to the first half of the 1990s.

The estimates of potential output figures, $q_t^*$, are the fitted values of this regression. In OECD calculations $\sum \beta_i$ (the slope of the trend in a given point of time) was constant within a cycle but might have changed between cycles, therefore, in this case $k$ refers to the number of cycles in the estimation period. The cycle was defined as the period between peaks in economic growth, and the peaks were defined as the date when the estimated output gap is the largest. Therefore, the estimated trend determines the peak, but the peak determines the trend as well. Since it is indeed problematic to determine the peak, the timing of which is even uncertain during the most recent cycle, and structural breaks can occur only at the end of the cycle regardless of what kind of shocks hit the economy, the OECD Secretariat replaced this method by the production functions based approach. See Giorno *et.al.* (1995).

Academic literature concentrates on three issues: (1) whether the break point(s) should be set *a priori* or be endogenous, (2) how to test for unit root in both cases, (3) and how to determine the number of break points.

The first influential contribution to unit root tests with structural breaks was a paper by Perron in the second half of the 1980s. Perron showed that taking into account two "exogenous" breaks in the trend, one at the time of Great Depression and the other at the oil shock, most US data seem to be stationary. This paper was followed by a lively discussion on whether the date of the break should be imposed or treated as unknown, how many breaks should be allowed, and how to endogenously find the breaking date(s).

Testing for a single break point can be easily tackled, but why only one break? Endogenously testing for the number of breaks is much complicated. See, for example the survey in Maddala–Kim (1996).

When we adopt more break points exogenously, we might arrive at biased and misleading inference. For example, Perron and several economist assumed that a break point occurred at the first oil shock in 1973. However, Harvey–Jaeger (1993) reminds us that as early as 1972 Nordhaus published a paper titled as "The recent productivity slowdown".[25] Harvey–Jaeger's model (see later) assumes a slowly changing growth rate of potential output, and their estimates showed that the slowdown started in the late sixties. They warn against using endogenously determined breaks of the deterministic trend models, because of unnecessary complications and the sensitivity of results to the applied method.

[25] Nordhaus, W.D. (1972): *The Recent Productivity Slowdown*, Brookings papers on Economic Activity, 3, pp.493-546.

On the other hand, allowing a sufficient number of breaks renders each series trend-stationary around a broken trend line. For example, if we analyse the Dow Jones for a given year, allowing 10 breaks in the trend will likely make it stationary. Of course, this example is complete nonsense, but it serves to illustrate the problems with breaking trend models.

To avoid too many breakpoints we define exogenously the number of breaks and find the likeliest places endogenously.

## VII.2 Hodrick-Prescott filter

The Hodrick–Prescott (HP) filter is an extensively used method for extracting a "growth" and a "cycle" component of a time series and also the most widely criticized method. The HP filter extracts the growth component by minimizing:

$$\min_{\{q_t^*\}_{t=1}^T} \left\{ \sum_{t=1}^{T} \left(q_t - q_t^*\right)^2 + \lambda \sum_{t=1}^{T} \left[\left(q_t^* - q_{t-1}^*\right) - \left(q_{t-1}^* - q_{t-2}^*\right)\right]^2 \right\} \tag{14}$$

where $q_t$ is the logarithm of observed output, $q_t^*$ is the logarithm of growth component (i.e. potential output), $q_t - q_t^* = c_t^*$ is the cycle component, and $\lambda$ is a positive parameter that penalizes variability in the growth component series. The larger the value of $\lambda$, the smoother the solution series is. As $\lambda$ goes to infinity, the growth rate of $q_t^*$, $\left(q_t^* - q_{t-1}^*\right)$, goes to a constant and the solution is the least squares fit of the linear trend model, $q_t^* = \beta_0 + \beta_1 t$. In most empirical applications the value of $\lambda$ is set to 100 for annual data and to 1600 to quarterly data. These numerical values are hoped to remove shorter-run (higher frequency) cycles than the observed business cycles in the United States, that is, cycles with less than eight years periodicity.[26] See Hodrick–Prescott (1997), which is essentially the same as their 1980 working paper in which they proposed this filter.

Several major criticisms of this filter have been raised, some of them simply originate from the arbitrary choice of the smoothness parameter. Cogley–Nason (1995) shows that when applied to stationary time series (including trend-eliminated trend stationary series) the HP filter works as a high-pass filter, that is, suppress cycles with higher frequencies while letting low frequency cycles go through without change. However, for different stationary series the HP filter is not a high-pass filter but suppresses high and low frequency cycles and amplifies business cycle frequencies, therefore creating artificial business cycles. Similar criticism was voiced by Harvey–Jaeger (1993). They showed that the HP-filter creates spurious cycles in detrended random walks and I(2) processes, and that the danger of finding large sample cross-correlations between independent but spurious HP cycles is not negligible.

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[26] Kydland–Prescott (1990) provide the following rationale for the choice of λ: "We found that if the time series is quarterly, a value of λ=1600 is reasonable. With this value the implied trend path for the logarithm of real GNP is close to the one that students of business cycles and growth would draw through a plot of this series." (Citation selected by Laxton–Tetlow (1992)).

On the other hand, the HP–filter works as a symmetric two-sided filter in the middle of the sample, but becomes unstable at the end and the beginning of the sample. For example, in the case of $\lambda = 1600$ it is suggested that three years at both ends of the sample of the fitted trend should be disregarded.[27]

Another important weakness of the HP–filter is the treatment of structural breaks. Sudden changes in output, such as the early years of all transition countries or the downturn in the Finnish economy following the collapse of the Russian market in 1989, are smoothed away by the HP–filter: it moderates the decline when the change occurs but spreads it out over several years.

### VII.3 Band-pass filter

The moving average of time series has the following form:

$$y_t^* = \sum_{k=-K}^{K} a_k L^k y_{t-k} \tag{15}$$

where L denotes the lag operator, thus $a(L) = \sum_{k=-K}^{K} a_k L^k$ . If $a_k = a_{-k}$ and this symmetric moving average has weights that sum to zero, $\sum_{k=-K}^{K} a_k = 0$ then this moving average makes stationary the series which contain quadratic deterministic trend or stochastic trends.

Based on Cramer representation of the stationary time series $y_t$ is:

$$y_t = \int_{-\pi}^{\pi} \xi(\omega) d\omega \tag{16}$$

i.e. time series can be expressed as the integral of random periodic components, $\xi(\omega)$, where $E\xi(\omega_1)\xi(\omega_2)$ for $\omega_1 \neq \omega_2$, thus the filtered time series is:

$$y_t^* = \int_{-\pi}^{\pi} \alpha(\omega)\xi(\omega) d\omega \tag{17}$$

where $\alpha(\omega) = \sum_{k=-K}^{K} a_h e^{-i\omega h}$ is the frequency response function of linear filter.

The basic building block of filter design is the low-pass filter, which passes only frequencies $-\underline{\omega} \leq \omega \leq \underline{\omega}$. The frequency response function of the ideal low-pass filter gives $\beta(\omega) = 1$ for $|\omega| \leq \underline{\omega}$, and $\beta(\omega) = 0$ for $\underline{\omega} < |\omega|$. The time domain representation

[27]The end-point instability might cause this if an economy recovers from a recession relatively slowly, then the filter tends to underestimates potential GDP. OECD estimations, that use the HP–filter only for comparison, try to reduce the end-point problem by using projections for several years ahead. Generally, they use $\lambda = 25$ in most cases. See Giorno *et.al.* (1995).

of the ideal low-pass filter is $b(L)=\sum_{h=-\infty}^{\infty} b_h L^h$, where $b_h$ is the inverse Fourier transformation of frequency response function:

$$b_h = \int_{-\pi}^{\pi} \beta(\omega) e^{i\omega h} d\omega \tag{18}$$

Note that the infinite-order moving average is necessary to construct the ideal low-pass filter $LP_{\infty}(p)$. Hence, we approximate the ideal filter with a finite moving average $a(L)=\sum_{h=-K}^{K} a_h L^h$ that has a frequency response function $\alpha_K(\omega)=\sum_{h=-K}^{K} a_h e^{-i\omega h}$. To be able to choose an approximate filter, $\alpha_K(\omega)$, we have to approximate a specific filter $\beta(\omega)$. This method is described more detailed in Baxter and King (1995).

Now we can create high-pass and band-pass filters from the low-pass filter. The ideal high-pass filter, $HP_{\infty}(p)$, passes frequency components which are less than or equal to $p$. If the weights of the low-pass filter are $b_h$ for $h = 0$ and $h = \pm 1,2,...$, then the weights of high-pass filter are $1-b_0$ at $h = 0$ and $-b_h$ at $h = \pm 1,2,...$; thus the optimal approximate high-pass filter, $HP_K(p)$ is simply constructed by truncating the weights of $HP_{\infty}(p) = 1 - LP_K(p)$.

The ideal band-pass filter passes frequencies which are in the range $\underline{\omega} \le |\omega| \le \overline{\omega}$, which is constructed from two low-pass filters with cutoff frequencies $\underline{\omega}$ and $\overline{\omega}$. Now, it is easy now to derive the weights of band-pass filter. If $\underline{b_h}$ and $\overline{b_h}$ are the filter weights of low-pass filters with cutoff frequencies $\underline{\omega}$ and $\overline{\omega}$ then the weights of band-pass filter are $\overline{b_h} - \underline{b_h}$.

## VII.4 Beveridge-Nelson decomposition

Any first order integrated process whose first difference satisfies certain conditions can be written as the sum of a random walk (*permanent component* in the terminology of Beveridge–Nelson(1981)), initial conditions, and a stationary process (*transitory component*). Let the first difference of actual output $\Delta q_t \equiv u_t$ satisfy the following conditions:

$$u_t = \mu + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}, \tag{19}$$

$$\varepsilon_t \sim i.i.d.\left(0, \sigma^2\right) \tag{20}$$

$$\sum_{j=0}^{\infty} j\left|\psi_j\right| < \infty. \tag{21}$$

Then

$$q_t = q_0 + u_1 + u_2 + \cdots + u_t = \psi(1)(\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_t) + t\mu + \eta_t + (q_0 - \eta_0), \tag{22}$$

where

$$\psi(1) = \sum_{j=0}^{\infty} \psi_j \ , \ \eta_t = \sum_{j=0}^{\infty} \alpha_j \varepsilon_{t-j} \ , \ \alpha_j = -(\psi_{j+1} + \psi_{j+2} + \psi_{j+3} + \cdots) \ , \ \sum_{j=0}^{\infty} |\alpha_j| < \infty.$$

The condition in equation (21) is satisfied by any stationary *ARMA* process. By this decomposition, the permanent component defined as the random walk (with drift if $\mu \neq 0$) part can be regarded as potential output $\left(q_t^* = \psi(1)(\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_t) + t\mu\right)$ and the stationary part as cycle $\left(c_t^* = \eta_t\right)$.

Two examples might be useful in understanding this decomposition.

**Example 1.** BN-decomposition of a random walk with drift

Let $y_t = \mu + y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim i.i.d.(0, \sigma^2)$. Then $\Delta y_t \equiv u_t$, and $u_t = \mu + \varepsilon_t$, therefore $\psi_0 = 1$ and $\psi_j = 0 \ (j \neq 0)$, so the condition in equation (21) is trivially satisfied. Now $\psi(1) = 1$, $\alpha_j = 0 \ \forall j$, $\eta_t = 0$, so (supposing the initial condition, $y_0 = 0$) the permanent or trend component of the random walk is itself $\left(y_t^* = \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_t + t\mu = y_t\right)$ and the random walk has no cycle component.

**Example 2.** BN-decomposition of an ARIMA(1,1,1) process

Let $\Delta y_t \equiv u_t$, $(1 - \phi L)(u_t - \mu) = (1 + \theta L)\varepsilon_t$, $|\phi| < 1$, therefore, $u_t - \mu = (1 - \phi L)^{-1}(1 + \theta L)\varepsilon_t = (1 + \phi L + \phi^2 L^2 + \phi^3 L^3 + \cdots)(1 + \theta L)\varepsilon_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$ where $\psi_0 = 1$ and for $j \geq 1$, $\psi_j = \phi^j + \phi^{j-1}\theta = \phi^{j-1}(\phi + \theta)$. If, for example, $0 < \phi < 1$ and $\theta > 0$, then $\sum_{j=0}^{\infty} j|\psi_j| = \frac{\phi+\theta}{(\phi-1)^2} < \infty$ so the condition in equation (21) is satisfied. Now $\psi(1) = \sum_{j=0}^{\infty} \psi_j = 1 + \sum_{j=1}^{\infty} \phi^{j-1}(\phi + \theta) = 1 - \frac{\phi+\theta}{\phi-1}$, so the permanent component of an *ARIMA*(1,1,1) is $y_t^* = 1 + \frac{\phi+\theta}{1-\phi}(\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_t) + t\mu$, and the cycle component can be calculated from either as $y_t - y_t^* - (y_{-\infty} - \eta_{-\infty})$ or directly from the definition as $\alpha_j = -(\psi_{j+1} + \psi_{j+2} + \psi_{j+3} + \cdots) = -\sum_{i=j+1}^{\infty} \psi_i = -\sum_{i=j+1}^{\infty} \phi^{i-1}(\phi + \theta) = \phi^j \frac{\phi+\theta}{\phi-1}$ so $\eta_t = \sum_{j=0}^{\infty} \alpha_j \varepsilon_{t-j} = \sum_{j=0}^{\infty}\left(-\sum_{i=j+1}^{\infty} \phi^{i-1}(\phi + \theta)\right)\varepsilon_{t-j} = \sum_{j=0}^{\infty} \phi^j \frac{\phi+\theta}{\phi-1} \varepsilon_{t-j}$.

As noted by Beveridge–Nelson (1981), operationality requires that we be able to write the process in terms of a finite number of parameters. Assuming that $u_t = \Delta y_t$ follows a stationary $ARMA(p,q)$ process, we can write that

$$u_t - \mu = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j} \equiv \frac{\left(1+\theta_1 L + ... + \theta_q L^q\right)\varepsilon_t}{\left(1-\phi_1 L - ... - \phi_p L^p\right)}, \tag{23}$$

or equivalently

$$u_t = \mu\left(1-\phi_1 - ... - \phi_p\right) + \phi_1 u_{t-1} + ... + \phi_p u_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + ... + \theta_q \varepsilon_{t-q}. \tag{24}$$

The definition implies that the cycle is the sum of all forecastable future changes of the original $y_t$ series.[28] The forecasts of $\hat{u}_{t,t+s} = E_t(u_{t+1} | u_t, u_{t-1}, ...)$ can be calculated using estimated parameters of equation (22) and the cycle component should be $c_t^* = \sum_{s=1}^{\infty} (\hat{u}_{t,t+s} - \hat{\mu})$. Beveridge–Nelson (1981) suggest to estimate the cycle component by forecasting $\hat{u}_{t,t+s}$ only until $s \leq k$. $k$ should be large enough to ensure that the last forecast is very close to the mean, i.e. $\hat{u}_{t,t+s} \cong \mu$. Therefore, the cycle component is calculated as

$$c_t^* = \sum_{s=1}^{k} \left(\hat{u}_{t,t+s} - \hat{\mu}\right) \tag{25}$$

For quarterly data Beveridge–Nelson (1981) use $k = 100$.

Since the BN–decomposition identifies permanent output as a random walk, the critiques by Lippi–Reichlin (1994) and Dupasquier–Guay–St-Amant (1997) described on page 43 fully apply to this method.

Canova (1998) underlines that problems inherent to $ARIMA$ specifications are carried over to this method. He also states that results of his paper varied considerably with the choice of the lags both in terms of the magnitude of the fluctuations and of the path properties of the data.

## VII.5 Unobserved components models

Unobserved components ($UC$) or latent variable models are frequently applied to potential output estimates. These models can be conveniently represented in a state-space form. The *state-space representation* of the dynamics of an $(n \times 1)$ observed vector time series $y_t$ consists of a *state equation* (or *transition equation*) describing the

[28] Beveridge–Nelson (1981) indeed gave their definition in terms of forecasts while equations in (2-5) of this paper were adopted from Hamilton (1994).

dynamics of the unobserved $(r \times 1)$ state vector $\xi_t^*$, and an *observation equation (*or *measurement equation)* describing $y_t$ as a function of the state vector and possibly other exogenous variables $x_t$. See Hamilton (1994), Chapter 13 for an excellent discussion; we follow his notation here. Harvey (1989) gives an extensive and also splendid guide to the specification, estimation, and testing issues of state-space models.

$$\text{state equation: } \underset{(r\times1)}{\xi_{t+1}^*} = \underset{(r\times r)}{F}\,\underset{(r\times1)}{\xi_t^*} + \underset{(r\times1)}{v_{t+1}} \tag{26}$$

$$\text{observation equation: } \underset{(n\times1)}{y_t} = \underset{(n\times k)}{A'}\,\underset{(k\times1)}{x_t} + \underset{(n\times r)}{H'}\,\underset{(r\times1)}{\xi_t^*} + \underset{(n\times1)}{w_{t+1}} \tag{27}$$

where

$$E(v_t) = E(w_t) = 0 \text{ for all } t \tag{28}$$

$$E(v_t v_\tau') = \begin{cases} \underset{(r\times\tau)}{Q} & \text{for all } t = \tau \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

$$E(w_t w_\tau') = \begin{cases} \underset{(r\times\tau)}{R} & \text{for all } t = \tau \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

It is assumed that the disturbance vectors $v_t$ and $w_\tau$ are not correlated with each other and with the state and the observed variables contemporaneously and with all lead and lags as well. Exogeneity of $x_t$ in the observation equation (26) means that $x_t$ provides no information about $\xi_{t+s}^*$ and $w_{t+s}$ for $s = 0,1,2,...$ beyond that contained in $y_{t-1}, y_{t-2}, ..., y_1$.

This system can be generalized to a system in which there is contemporaneous correlation between $v_t$ and $w_\tau^{'}$, and in which the various parameters $(F, Q, A, H, R)$ are functions of time. Given parameters, the unobserved state vector and its variance-covariance matrix can be calculated by the Kalman–filter. Apart from the parameters, we might be interested in three types of inferences for the unobserved state vector: $\xi_{t|t-1}^*$, $\xi_{t|t}^*$, $\xi_{t|T}^*$, that is, the forecast from the previous period, inference for current period $t$ based on all information up to $t$, and inference for $t$ using the full sample. The last magnitude is called the *smoothed* series. Similar magnitudes can be calculated for the variance-covariance matrix of the state vector.

When the parameters are unknown, the Kalman–filter also allows for the evaluation of the likelihood function; therefore, it permits maximum likelihood (or quasi maximum likelihood) estimation of the parameters, regardless whether $y_t$ and $\xi_t^*$ are stationary or not.

It should also be mentioned that identification problems may appear; therefore, a completely unrestricted estimation of parameters is in general unfeasible.

Examples of unobserved components models for potential output are the following.

**Example 1.** Watson's models

Watson's (1986) models decomposes output into an integrated trend and a stationary cycle:

$$\Delta q_t^* = \mu_{q*} + e_t \tag{31}$$

$$c_t^* = \Psi(L) u_t \tag{32}$$

$$q_t = q_t^* + c_t^* \tag{33}$$

where $q_t^*$ is the logarithm of potential output with $\mu_{q^*}$ mean growth, $q_t$ is the logarithm of observed GDP, $c_t^*$ is the cyclical GDP that is assumed to follow a stationary process, $L$ is the lag operator and $\Psi(L)$ denotes a polynomial of the lag operator, $e_t$ and $u_t$ are white noises. According to assumptions on $u_t$ and $\Psi$, Watson (1986) presents three types of models:

$$c_t^* = \Psi^c(L) u_t \text{ where } E(u_t e_{t-k}) = 0 \quad \forall t, k, \tag{34}$$

$$c_t^* = \Psi^q(L) e_t, \tag{35}$$

$$c_t^* = \Psi^c(L) u_t + \Psi^q(L) e_t \text{ where } E(u_t e_{t-k}) = 0 \quad \forall t, k, \tag{36}$$

and shows that models (34) and (35) are identified while (36) is not. He also shows that only those processes can be represented by the first model (34) whose first differences' spectrum has a global minimum at zero frequency. This restriction rules out many common processes, such as *ARIMA*(1,1,0) with positive autoregressive coefficients. In his empirical section he found that many US macroeconomic data violate this restriction; therefore, theoretically this model is inappropriate for these series. Nonetheless, he found that for his data set *UC* models delivered economically better estimates than *ARIMA* models and in-sample forecasts were also more accurate. The term "economically better" indicates that these estimates were much more in line with NBER peak and trough business cycle calculations.[29]

[29]Watson compares *UC* models of this type to the Beveridge-Nelson decomposition (i.e. *ARIMA* models), and shows that theoretically both representations are identical. However, estimated *UC* and *ARIMA* models for US GNP figures yielded essentially identical values of the likelihood function and short-run forecasts, but their long-run implications differed remarkably: a unit innovation is estimated to increase the level of GNP by 1.68 units and 0.57 units according *ARIMA* and *UC* models, respectively. Watson underlines the dangers of using any of these methods for long-run inference.

The model in ((31)-(33)) when we adopt Watson's preferred (34) specification for (32) can be easily be written in terms of the general specification of equations (26) and (27). For example, supposing that the cycle can be approximated by an $AR(2)$ process, we have

$$\begin{bmatrix} q^*_{t+1} \\ 1 \\ c^*_{t+1} \\ c^*_t \end{bmatrix} = \begin{bmatrix} 1 & \mu_{q^*} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q^*_t \\ 1 \\ c^*_t \\ c^*_{t-1} \end{bmatrix} + \begin{bmatrix} e_{t+1} \\ 0 \\ u_{t+1} \\ 0 \end{bmatrix} \tag{37}$$

$$q_t = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q^*_t \\ 1 \\ c^*_t \\ c^*_{t-1} \end{bmatrix} + 0 \tag{38}$$

Lippi–Reichlin (1994) and Dupasquier–Guay–St-Amant (1997) criticize this model and Kuttner's (1998) model in the first multivariate example on the basis that the random walk model of potential output is inconsistent with the generally accepted view of productivity growth. They argue that technology shocks are likely to be absorbed gradually by the economy because of adjustment costs, learning, and the time-consuming process of investments, for example. An implication of random walk would be that, if the contemporary and the long-run effect of permanent shocks are different, then an output gap would show up. They argue that the inclusion of a diffusion process associated with permanent shocks is preferable since the economy is likely to remain on its production possibility frontier as adjustments unfold.

However, this critique applies to certain models but not to all unobserved components models. Although the state-space specification does not allow autocorrelation in the innovation of state equation ($v_{t+1}$ in equation (26) or $e_t$ in equation (31)), the structure of the state equation (26) allows unit root processes other than the random walk. For example, if potential output followed an $ARI(1,1)$ process,

$$(1-L)(1-\rho_2 L)q^*_t = \mu_{q^*} + e_t, \tag{39}$$

then

$$\begin{aligned} q^*_t &= q^*_{t-1} + \mu_{q^*} + e_t + \rho_2 e_{t-1} + \rho_2^2 e_{t-2} + \rho_2^3 e_{t-3} + \cdots = \\ &= q^*_0 + t\mu_{q^*} + e_t + (1+\rho_2)e_{t-1} + \left(1+\rho_2+\rho_2^2\right)e_{t-2} + \left(1+\rho_2+\rho_2^2+\rho_2^3\right)e_{t-3} + \cdots, \end{aligned} \tag{40}$$

so the short-run effect of a permanent shock, $\partial q_t^*/\partial e_t = 1$, and the long run effect, $\lim_{s\to\infty}(\partial q_{t+s}^* / \partial e_t) = \sum_{j=0}^{\infty} \rho_2^j = 1/(1-\rho_2)$, will differ, and there will be a diffusion process linking short-run and long-run effects. This dynamic structure can be easily incorporated into the state-space representation of (37)-(38) as

$$\begin{bmatrix} q_{t+1}^* \\ q_t^* \\ 1 \\ c_{t+1}^* \\ c_t^* \end{bmatrix} = \begin{bmatrix} 1+\rho_2 & -\rho_2 & \mu_{q^*} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_t^* \\ q_{t-1}^* \\ 1 \\ c_t^* \\ c_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_{t+1} \\ 0 \\ 0 \\ u_{t+1} \\ 0 \end{bmatrix} \tag{41}$$

$$q_t = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_t^* \\ q_{t-1}^* \\ 1 \\ c_t^* \\ c_{t-1}^* \end{bmatrix} + 0 \quad . \tag{42}$$

**Example 2.** Harvey' models

Models proposed by Harvey in several publications differ from that of Watson in three aspects. The first and most important is that the drift of the random walk component also follows a random walk, that is, the series is integrated of order 2. Other differences are that the observation equation is augmented with an error term, and cycles are modelled differently. Harvey calls his four proposed models "structural time series models" because all components have economic interpretation, although these models are all univariate and not structural in the usual econometric sense. The models are the following.

$$Local level (LL) model \tag{43}$$

$$y_t = \mu_t^* + \varepsilon_t$$

$$\mu_t^* = \mu_{t-1}^* + \eta_t$$

where $\varepsilon_t$ and $\eta_t$ are uncorrelated white noises with variances $\sigma_\varepsilon^2$ and $\sigma_\eta^2$ .

$$Local linear trend (LLT) model \tag{44}$$

$$y_t = \mu_t^* + \varepsilon_t$$

$$\mu_t^* = \mu_{t-1}^* + \beta_{t-1}^* + \eta_t$$

$$\beta_t^* = \beta_{t-1}^* + \zeta_t$$

where $\varepsilon_t$, $\eta_t$ and $\zeta_t$ are uncorrelated white noises with variances $\sigma_\varepsilon^2$, $\sigma_\eta^2$ and $\sigma_\zeta^2$.

$$Basicstructural(BS)model \tag{45}$$

$$y_t = \mu_t^* + \gamma_t^* + \varepsilon_t$$

$$\mu_t^* = \mu_{t-1}^* + \beta_{t-1}^* + \eta_t$$

$$\beta_t^* = \beta_{t-1}^* + \zeta_t$$

$$\sum\nolimits_{i=0}^{s-1} \gamma_{t-i}^* = \omega_t$$

where $\varepsilon_t$, $\eta_t$, $\zeta_t$ and $\omega_t$ are uncorrelated white noises with variances $\sigma_\varepsilon^2$, $\sigma_\eta^2$, $\sigma_\zeta^2$ and $\sigma_\omega^2$. The inclusion of $\gamma_t^*$ allows us to model a dummy-type seasonality, since it constrains that the expected value of the sum of seasonal effect over each $s$ consecutive periods is zero.

$$Cyclemodel(CM) \tag{46}$$

$$y_t = \psi_t^*$$

$$\begin{bmatrix} \psi_t^* \\ \psi_t^+ \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} \psi_{t-1}^* \\ \psi_{t-1}^+ \end{bmatrix} + \begin{bmatrix} \kappa_t^* \\ \kappa_t^+ \end{bmatrix}$$

where $\rho$ is a damping factor such that $0 \le \rho \le 1$, $\lambda$ is the frequency of the cycle in radians, $\kappa_t$ and $\kappa_t^+$ are uncorrelated white noises with variance $\sigma_\kappa^2$.

The cycle model (46) is the only one without an error term in the observation equation and is usually not estimated separately but added to *LL*, *LLT* or *BS*. For business cycle analysis, Harvey–Jaeger (1993) suggest the *LLT* plus cycle model. For further reference we will denote this model as *LLTC*. In this case the parameters to be estimated are $\sigma_\varepsilon^2$, $\sigma_\eta^2$, $\sigma_\zeta^2$, $\sigma_\kappa^2$, $\rho$, and $\lambda$ that can be carried out by maximum likelihood either in the time domain or the frequency domain.

Harvey–Jaeger shows that this model is also an encompassing model, which nests various other models that impose arbitrary restrictions on the properties of the data and the degree of smoothness of the trend component:

> "The model is equivalent to an ARIMA(0,2,1) process. However, if $\sigma_\zeta^2 = 0$, it reduces to a random walk with drift. If, furthermore, $\sigma_\eta^2 = 0$ it becomes deterministic, that is, $\mu_t = \mu_0 + \beta t$. When $\sigma_\eta^2 = 0$ but $\sigma_\zeta^2 > 0$, the trend is still a process integrated of order two, abbreviated *I(2)*, that is, stationary in second differences. A trend component with this feature tends to be relatively smooth. An important issue is therefore whether or not the constraint $\sigma_\eta^2 = 0$ should be imposed at the outset. We argue that there are series where it is unreasonable to assume a smooth trend a priori and therefore the question

whether or not $\sigma_\eta^2$ is set to zero is an empirical one. ... HP filtering is therefore equivalent to postulating model (1) and imposing restrictions $\sigma_\zeta^2/\sigma_\varepsilon^2 = \overline{q}_\zeta$, $\sigma_\eta^2 = 0$, and $\psi_t = 0$." (Harvey–Jaeger (1993) pp.232-33.)

In the cited model (1) refers to *LLT*, $\overline{q}_\zeta$ is the inverse of the smoothing parameter (that is $1/\lambda$ in the notation of equation (14)), and $\psi_t$ to $\psi_t^*$ in the above notation.

In justifying the $I(2)$ assumption they first note that both Box-Jenkins identification and formal unit root tests frequently found that US GNP is $I(1)$. However, they claim that since $\sigma_\zeta^2$ is relatively small, the $I(2)$ component may be difficult to detect by *ARIMA* methodology. A convincing demonstration is that they simulate the *LLTC* model using the estimated parameter for sample sizes 100 and 500 and calculate the autocorrelation function of the first differences and test for unit root in it. Both experiments indicates serious biases, for example, the size of the ADF test at $T = 100$ and $k = 8$ is 74 percent, that is, using the 5 percent critical value it rejects the true null hypothesis of unit root in the first differences in 74 percent of experiments.

Economic arguments for two unit roots are also persuasive:

"A trend plus cycle model of the form (1) with $\sigma_\eta^2 = 0$ has stationary components with no persistence and a smooth *I(2)* trend with infinite persistence. But since the trend reflects slow long-term changes in growth rates, perhaps arising from demographic changes, innovations in technology, changes in savings behavior, or increasing integration of capital and goods markets, the shock which drive the smooth trend may have no connection with short-term economic policy. Following the extensive literature on the productivity slowdown phenomenon, we may well argue that understanding the reasons for persistent changes in growth rates is one of the key problems in macroeconomics." (Harvey–Jaeger (1993) pp.242-43.)

Their conclusion regarding ARIMA modelling is the same as that of Watson (1986): "For purposes of short-term forecasting a parsimonious ARIMA model, such as ARIMA(1,1,0), may well be perfectly adequate compared with a trend plus cycle model. But as a descriptive device it may have little meaning and may even be misleading." (Harvey–Jaeger (1993) pp.242.)[30]

[30]Funke (1998) applies these models to measure German potential output and output gap.

## VII.6 Wavelet transformation

In addition to several sciences and disciplines,[31] the wavelet transformation becomes more and more familiar and popular in economics as well. Although wavelet transformation is a quite new concept, it has wide literature from the introductory level to advanced usage. An introductory description is presented by Vidakovic and Mueller (1994) and Polikar (1996). Schleicher (2002) provides a custom-tailored paper for economists. A detailed textbook was written by Percival and Walden (2000).

First, we give some examples for wavelet transformation in economics. Ramsey and Lampart (1998) use this method to examine the relationship between money supply and output. In the literature the results of Ganger-causality are ambiguous, which may be caused by structural breaks and non-linearities. They use wavelet transformation to decompose time series into different frequency levels and apply Granger-causality tests to decomposed series. The authors find that, at the lowest scales, income Granger causes money, but business-cycle periods, money Granger causes income. At highest scales, the Granger causality goes in both directions. Arino (1998) and Arino, Pedro and Vidakovic (1995) describe an approach for forecasting time series using wavelets. Ariano shows that these forecasts are preferable to standard Box and Jenkins approach. The most relevant usage regarding our topic is Conway and Frame (2000) and Scacciavillani–Swagel (2002), CF and SS hereafter. CF uses Fourier and wavelet techniques in analysing New Zealand output gaps in comparison to other methods: SVAR, HP, UC etc. As a special issue, they use FT and WT to compare the frequency component of different output gaps. According to the paper all of the output gap measures have common cyclical characteristics at particular frequencies. SS also compares several methods for Israeli potential output: PF, HP, RMS,[32] WT, and SVAR, and finds that, with the exception of PF, the other four methods resulted in qualitatively similar output gaps.

For better understanding of the wavelet transformation (WT) it is worthwhile to briefly survey its historical background. WT is relatively new concept in signal processing fields, however its roots go back to the Fourier transformation (FT) developed at beginning of the 19th century. Similarly to the Fourier analysis, wavelet transformation also converts the signal in time domain to frequency domain vice versa. However, there are several important differences. The well-known form of Fourier transformation is

$$x(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \tag{47}$$

and

$$x(t) = \int_{-\infty}^{\infty} x(\omega) e^{i\omega t} d\omega \tag{48}$$

[31] A geologist used wavelet transformation technique for earthquake prediction in California. The FBI realised the favourable data compression property of wavelets and reorganised their finger print database using wavelets in 1992. Finally, Coifman and his Yale team used wavelet transformation to de-noise sound recording, including old recordings of Brahms playing his First Hungarian Dance on the piano.

[32] Running median smoothing.

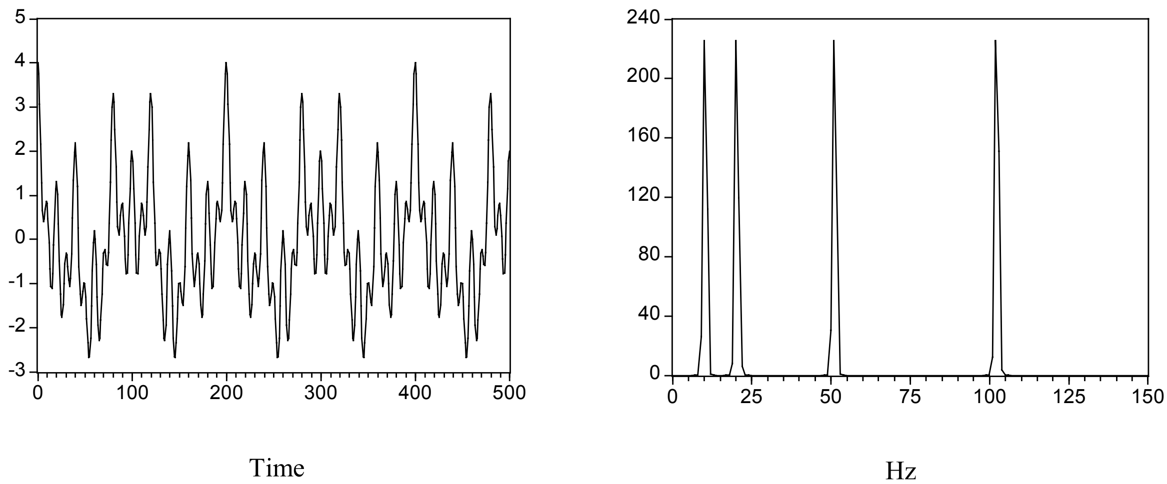
where $t$ and $\omega$ denotes the time and frequency respectively thus $x(t)$ is the signal at hand in time domain and $x(\omega)$ is the frequency domain. Equation (47) is called the Fourier transformation of $x(t)$, while equation (48) is called inverse Fourier transformation of $x(\omega)$ which reproduces $x(t)$.

The main drawback of Fourier transformation is that it can only handle stationary signals appropriately. Note that the definition of the expression stationary is not the same as in time series analysis even though they are related in certain cases. A signal is called stationary if its all frequency components exist at all times. To illustrate this problem consider the following signal:

$$x(t) = \cos(2\pi 10t) + \cos(2\pi 25t) + \cos(2\pi 50t) + \cos(2\pi 100t)$$

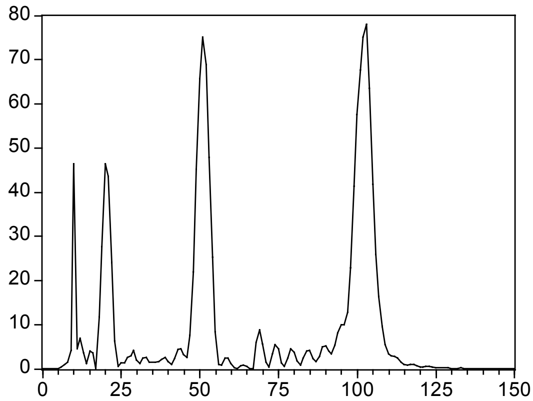
which is stationary because its frequency components (10, 25, 50 and 100Hz) exist at any given time. Figure 6 displays the original signal and the spectrum of it. One can see the peaks in spectrum displaying frequency components.

**Figure 6 A stationary signal and its spectrum**

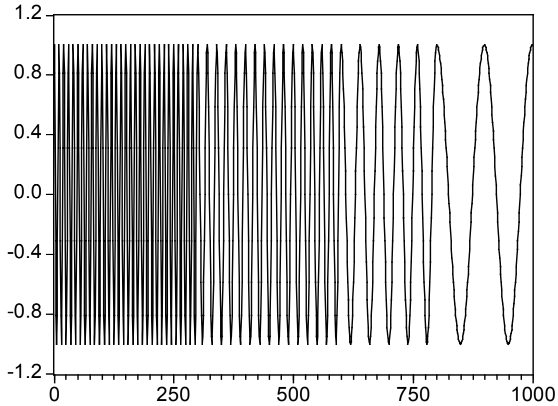



Let us turn now to a signal, which has the same frequencies but it is separated in time implying not stationary signal. 100 Hz exists between 0 and 300 ms, while 50 Hz, 25 Hz and 10 Hz between 300 and 600, 600 and 800, 800 and 1000, respectively.

**Figure 7 A non-stationary signal and its spectrum**




Hz




Time

As one can see, the spectrum of non-stationary signal is quite similar to the stationary. This indicates that all frequencies exist all the time. This means that one cannot reproduce the original signal from spectrum. In other words, the same spectral components belong to the stationary and non-stationary signal. We can conclude that Fourier transformation is not suitable if the signal has time varying frequency, i.e. the signal is non-stationary.

This phenomenon is apostrophised as Fourier transformation and has no time localization property. To handle this problem mathematicians developed the short time Fourier transformation (STFT). STFT approach is based on that assumption some portions of non-stationary signal are stationary. In STFT the original signal is divided into smaller parts, which are narrow enough to grab the stationary part of the signals. For this purpose we apply a window function with appropriate width. This window function is located at the beginning of the signal. The signal and the window function is multiplied and then a Fourier transformation of this segment is computed. In the next step window function is shifted and FT is computed again. This procedure is continued until the window function reaches the end of the signal.

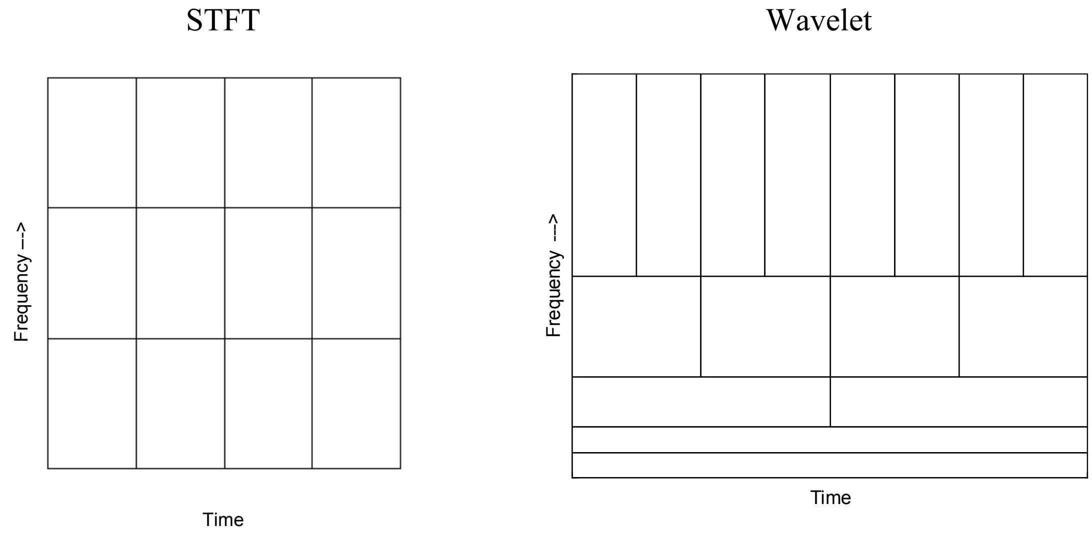
$$STFT_x^{\omega}(t^{'},\omega) = \int_{-\infty}^{\infty}\left[x(t)w(t-t^{'})\right]e^{-i\omega t}dt \qquad (49)$$

Applying STFT we get FTs at different times, thus we obtain a time-frequency representation of the signal. Although this appears to be an ultimate solution, there is a problem with STFT. The problem is based on the Heisenberg uncertainty principle, which states that the exact time-frequency representation of the signal cannot be known exactly, i.e. one cannot know what spectral components exist at what points in time. In other words, one can only know which the time intervals exist in certain band of frequencies. This is called the resolution problem.

The drawback of STFT is in the width of the window function. The time resolution in the FT, and the frequency resolution in the time domain is zero. The frequency resolution in the FT is perfect because the window used in its kernel, which lasts at all times from minus infinity to plus infinity. In the case of STFT the window function has finite length, thus it covers only a portion of the signal causing the frequency resolution to be poorer. All in all, due to the finite length of window we do not have perfect frequency resolution. The dilemma is what kind of window should be used? Narrow windows give good time resolution, but poor frequency resolution, while wide windows give good frequency resolution, but poor time resolution, and moreover wide windows may not be compatible with the condition of stationary. This is the stage where the wavelet transformation comes into the picture.

Let us see then how wavelet transformation can solve the dilemma of resolution to certain extent. Unlike the STFT, the width of window function in wavelet transformation is not fixed, implying that WT has good time and poor frequency resolution at high frequencies and good frequency and poor time resolution at low frequencies. Figure 8 displays the difference visually. While each box represents an equal portion of the time-frequency plane in both case, the widths and heights of the boxes change in WT. Note that at low frequencies the heights of boxes are shorter (which corresponds to better frequency resolution, since there is less ambiguity regarding the value of exact frequency), however, their widths are longer (which corresponds to poor time resolution, since there is more ambiguity regarding the value of exact time). At higher frequencies the width of the boxes decrease, i.e. the time resolution gets better while the heights of the boxes increase i.e. the frequency resolution gets poorer.

**Figure 8 Time-frequency plane for Short Time Fourier and Wavelet Transformation**




Note that the areas of boxes are same however different window functions in STFT or different mother wavelets in WT result different areas. Due to Heisenberg's uncertainty principle we cannot reduce the areas of the boxes as much as we want. On the other hand, for a given mother wavelet the dimensions of boxes can be changed, while keeping the area unchanged. This is exactly what wavelet transformation does.

After this intuitive introduction of wavelet transformation we can turn to the formal description. Unlike sinuses and cosines in Fourier transformation,[33] wavelets are used as a basis function in representing the other function. The wavelet term refers to a small wave. Small because the wavelet function is non-zero over a finite length and wave because the function oscillates. Several wavelet forms can be used in WT such as the 'ancient' Haar wavelet or Biorthogonal, Mexican hat, Morlet, etc. wavelets. Perhaps the most frequently used wavelets are the Daubechies wavelet family developed by Daubechies (1988). The denotation of family suggests that there are several types of "mother-wavelets". Daubechies wavelets have even number of filter elements, starting at 4. Wavelets within the family are usually denoted by the length of their filters. Increasing the number of filter elements makes the wavelet smoother.

The basic concept of WT is the mother wavelet. If the mother wavelet $\Psi(x)$ is given the space of square integrable function (wavelets) are constructed on the basis of location and scale parameters:

$$\Psi_{\tau,s} = \frac{1}{\sqrt{s}}\Psi\left(\frac{t-\tau}{s}\right) \tag{50}$$

where $\Psi(.)$ is mother wavelet which is an orthonormal basis $\{\Psi_{\tau,s}(x)\}$ in $L^2(R)$; $s$ and $\tau$ represent the scale and location parameters, respectively.

[33] Note that $e^{ix} = \cos x + i \sin x$.

According to aforementioned, one can see that:

$$\int \Psi_{\tau,s} \Psi_{\tau',s'} = 0 \,,\ (\tau \neq \tau') \vee (s \neq s') \ \text{ and}$$

$$\int \{\Psi_{\tau,s}(x)\}^2 dx = 1$$

The parameter scale in the wavelet analysis is similar to scale used in maps. As in the case of maps, high scales correspond to a non-detailed global view of the signal, and low scales correspond to a detailed view. In terms of frequency, low frequencies (high scales) correspond to a global information of a signal that usually spans the entire signal, while high frequencies (low scales) correspond to a detailed information of a hidden pattern of the signal that usually lasts a relatively short time. In other words scaling either dilates or compresses the signal. Larger scales correspond to dilated signal and small scales correspond to compressed signal.

To extract time information from the time series the set of wavelets at different scales are moved through the time series from the beginning to the end. The position of a particular wavelet function is determined by the location parameter, τ.

Using equation (50) we can generate the full set of wavelets from the mother wavelet and compute the wavelet transformation:

$$CWT_x^{\Psi}(\tau, s) = \int x(t) \Psi_{\tau,s}(t) dt \tag{51}$$

Wavelets constructed over short time scales will tend to detect high frequency volatility in the time series. Due to the short time scales, we get good time resolution, but poor scale (frequency) resolution. Relatively long-scale wavelets tends to capture low frequency volatility; thus we get poor time resolution but good scale (frequency) resolution.

Take special values for τ and s in defining wavelet basis: $s = 2^{-j}$ and $\tau = k 2^{-j}$ where *k* and *j* are integers, thus $\Psi_{j,k} = 2^{j/2} \Psi(2^j x - k)$ where *j* makes wavelet function finer, while *k* shifts it. Consider a data vector **y** with $2^n$ elements that can be associated with a piecewise constant function, *f*(x) on [0,1][34]. The wavelet transformation of *f*(x) has the form:

$$f(x) = c_0 \Phi(x) + \sum_{j=0}^{n-1} \sum_{k=0}^{2^j - 1} c_{jk} \Psi_{jk}(x) \tag{52}$$

[34] Any function can be transformed to function defined on the unit interval:

$$f(x) = \sum_{k=0}^{2^n - 1} y_k \cdot \mathbf{1}(k 2^{-n} \leq x < (k+1) 2^{-n})$$

where $\Phi(x)$ is called the scaling function (or father wavelet) associated with the wavelet basis $\Psi_{jk}$. While the father wavelet integrates to one, the mother wavelet integrates to zero.

The calculation of wavelet transformation it is quite time consuming. Mallat (1989) introduced a fast algorithm using quadrature mirror filtering. Multiscale analysis projects a function on a set of closed subspaces:

$$\ldots \subset V_{-1} \subset V_0 \subset V_1 \ldots$$

The spaces are nested. $\cup_{j \in Z} V_j$ is dense in $L^2(R)$. The intersection of all $V_j$ is empty. Furthermore:

$$f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1},\ j \in Z$$

The spaces $V_j$ and $V_{j+1}$ are 'similar'. If the space $V_j$ is spanned by $\Phi_{jk}(x)$, $k \in Z$ then the space $V_{j+1}$ is spanned by $\Phi_{j+1,k}(x)$, $k \in Z$. The space $V_{j+1}$ is generated by the functions $\Phi_{j+1,k}(x) = \sqrt{2}\Phi_{j,k}(x)$.

Because $V_0 \subset V_1$, any function in $V_0$ can be written as a linear combination of the basis function $\sqrt{2}\Phi(2x-k)$ from $V_1$:

$$\Phi(x) = \sum_{k \in Z} h(k)\sqrt{2}\Phi(2x-k) \tag{53}$$

Consider now the orthogonal complement $W_{j-1}$ of $V_{j-1}$ to $V_j$ (since the subspaces are nested $V_j$ can be represented as the direct sum of the coarsely approximation subspace $V_{j-1}$ and its orthogonal complement, $W_{j-1}$ i.e. $V_j = V_{j-1} \oplus W_{j-1}$ )

$$\Psi(x) = \sum_{k \in Z} g(h)\sqrt{2}\Phi(2x-k) \tag{54}$$

The sequences $\{h(k)\}$ and $\{g(h)\}$ for $k \in Z$ are quadrature mirror filters. The sequence $h(k)$ is called a low pass or low band filter, while $g(k)$ is known as a high pass or high band filter. The connection between $h(x)$ and $g(x)$ is the following:

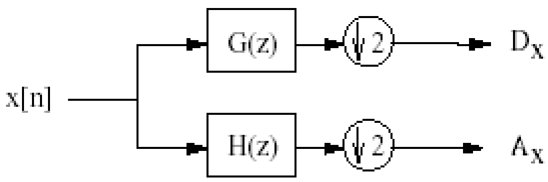
$$g(n) = (-1)^n h(1-n) \tag{55}$$

The multiscale analysis then uses filters to split up a function $f \in L^2(R)$. One can find N such that $F_N \in V_N$ approximates $f$ up to preassigned precision. If $g_i \in W_i$ and $f_i \in V_i$ then wavelet decomposition of $f$ is given by:

$$f_N = f_{N-1} + g_{N-1} = \sum_{i=1}^{M} g_{N-M} + f_{N-M} \tag{56}$$

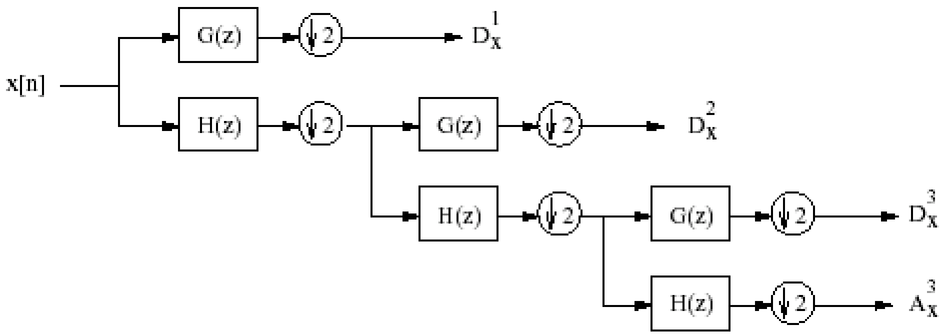
Hence, the decomposition of the signal into different frequency bands is obtained by successive highpass and lowpass filtering on the time domain signal. Figure 9 shows one-scale wavelet transformation where $A_x$ denotes the lowpass residue and $D_x$ denotes the highpass residue.

**Figure 9 Basic cell for the wavelet transformation**




Multi-scale scheme is obtained by iterating basic cells on the lowpass residue of each of the previous cells. Figure 10 shows the 3-scale case.

**Figure 10 Iterated filter scheme for the multi-scale transform**



# VIII. Appendix 2: Figures

## VIII.1 Segmented deterministic trend

**Figure 11 Annual potential GDP and output gap from segmented time trend**

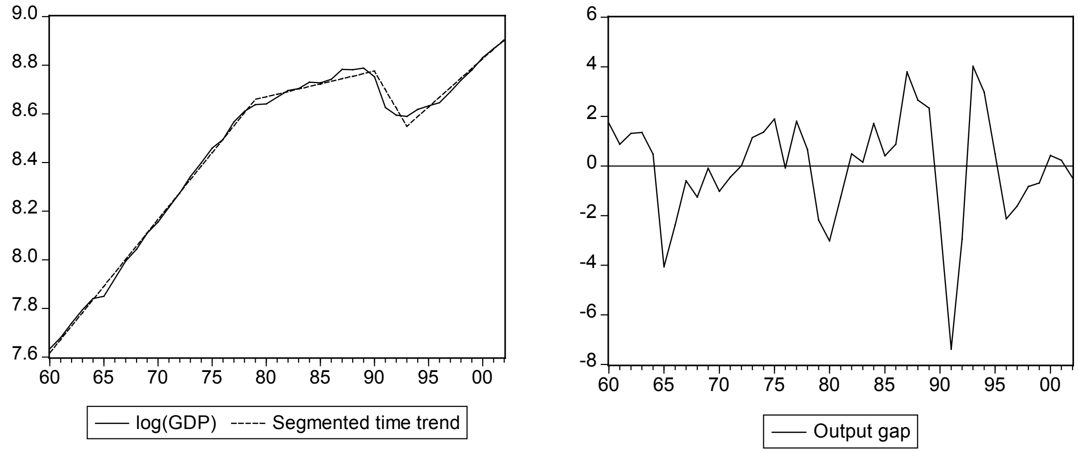
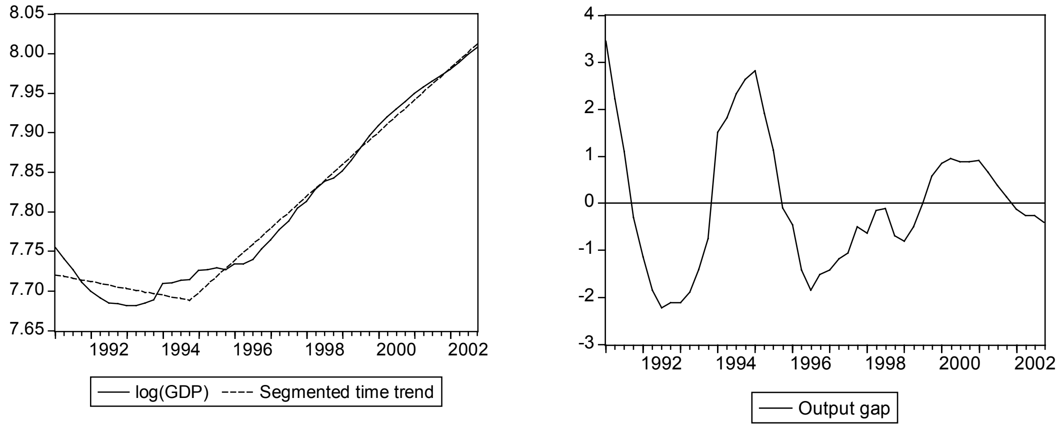



**Figure 12 Quarterly potential GDP and output gap from segmented time trend**



## VIII.2 Hodrick-Prescott filter

**Figure 13 Annual potential GDP and output gap from HP trends**

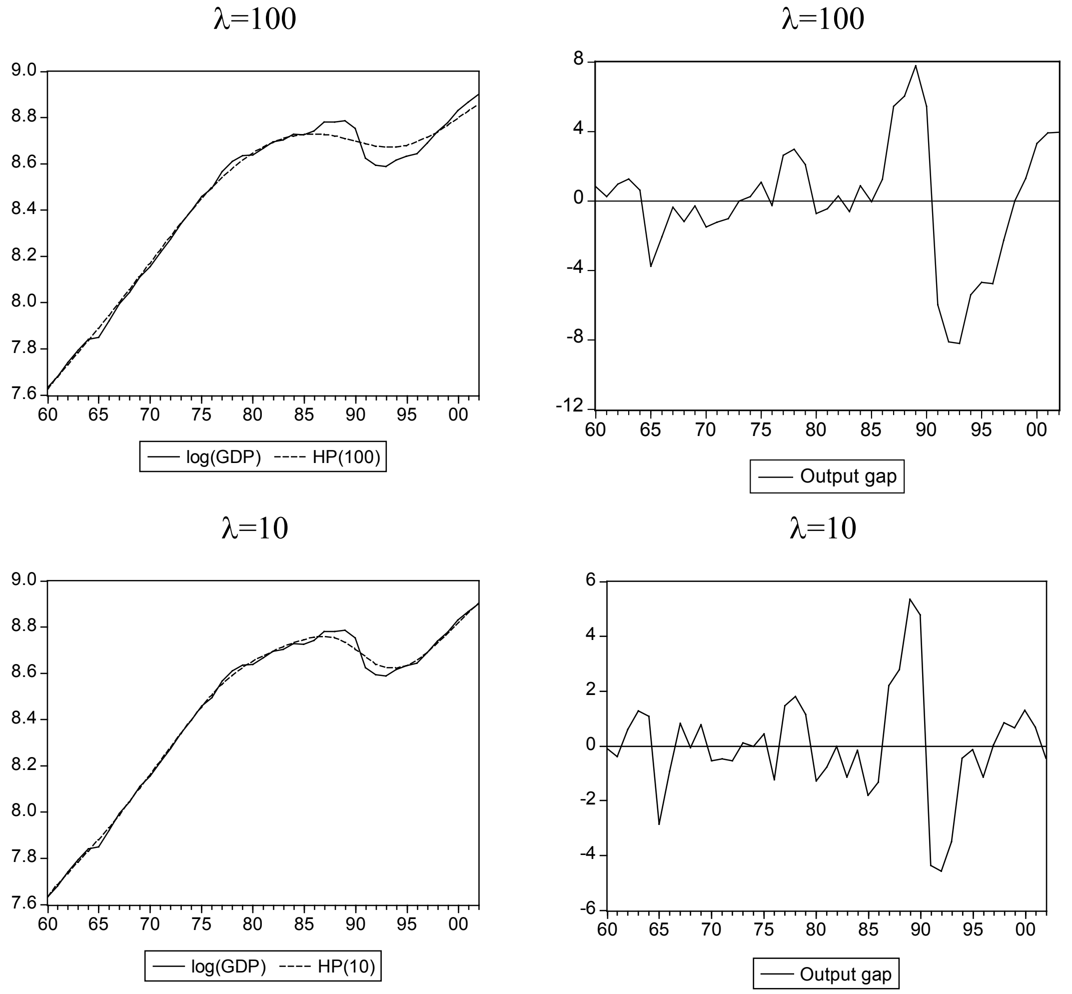
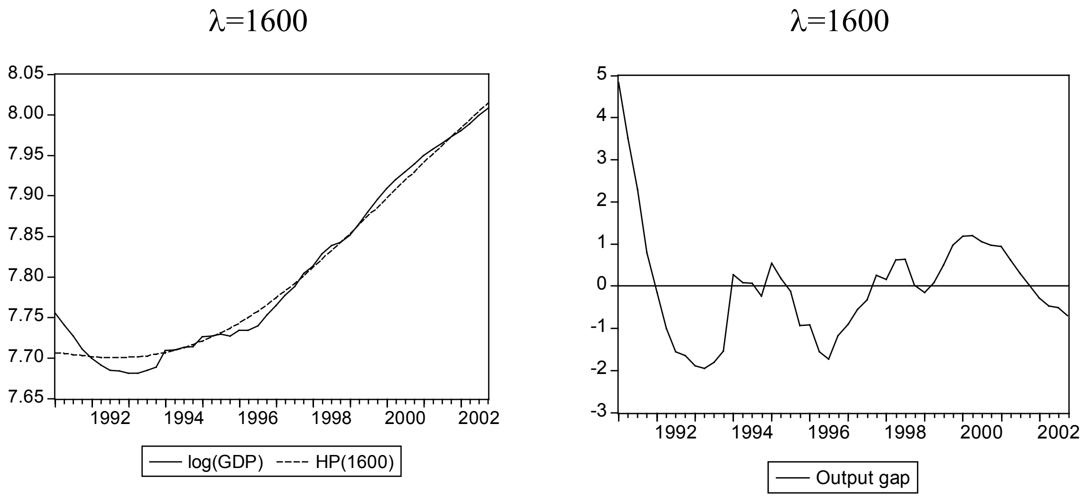
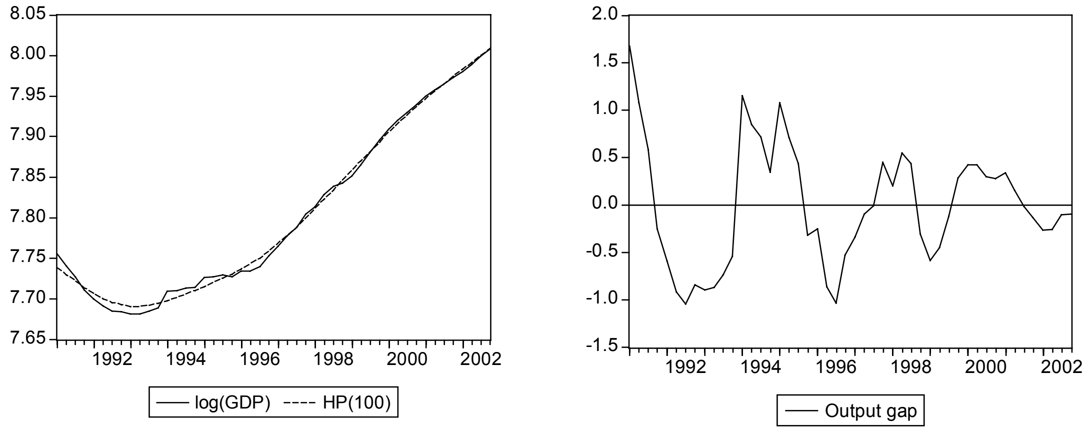



**Figure 14 Quarterly potential GDP and output gap from HP trends**



λ=100 λ=100




## VIII.3 Band-pass filter

**Figure 15 Annual potential GDP and output gap from BP(20) filter**

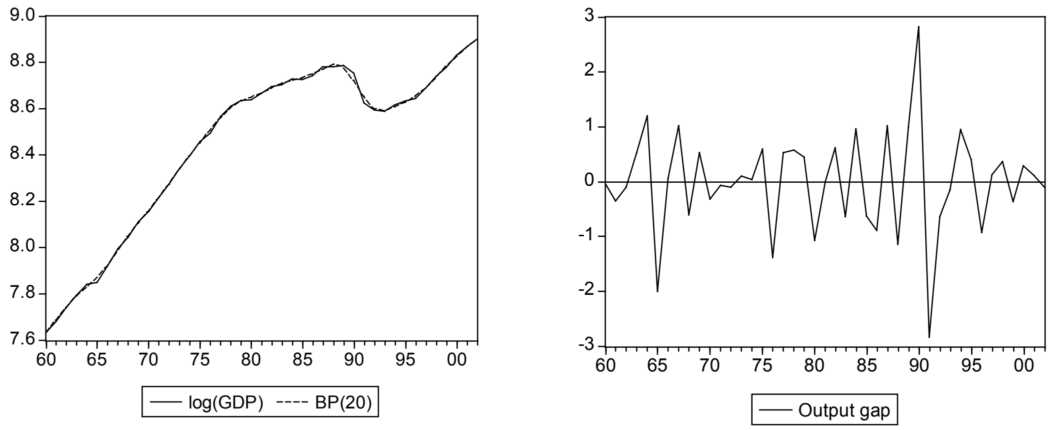
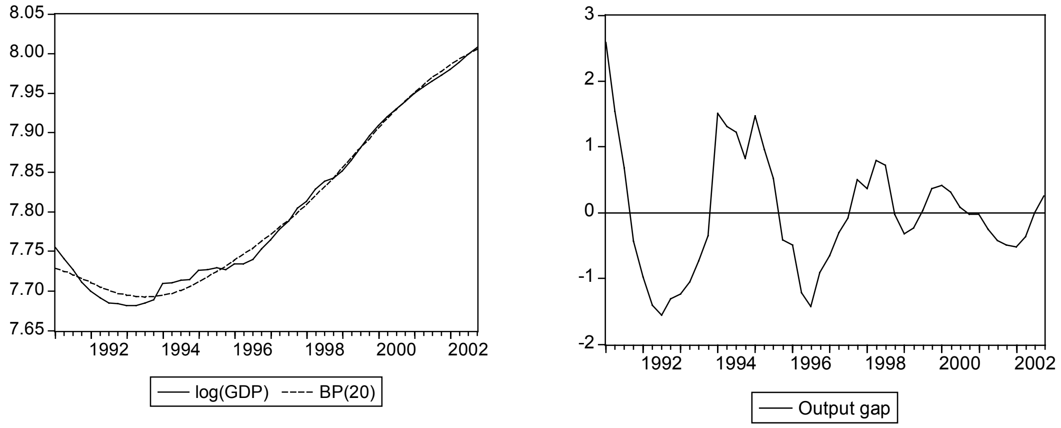
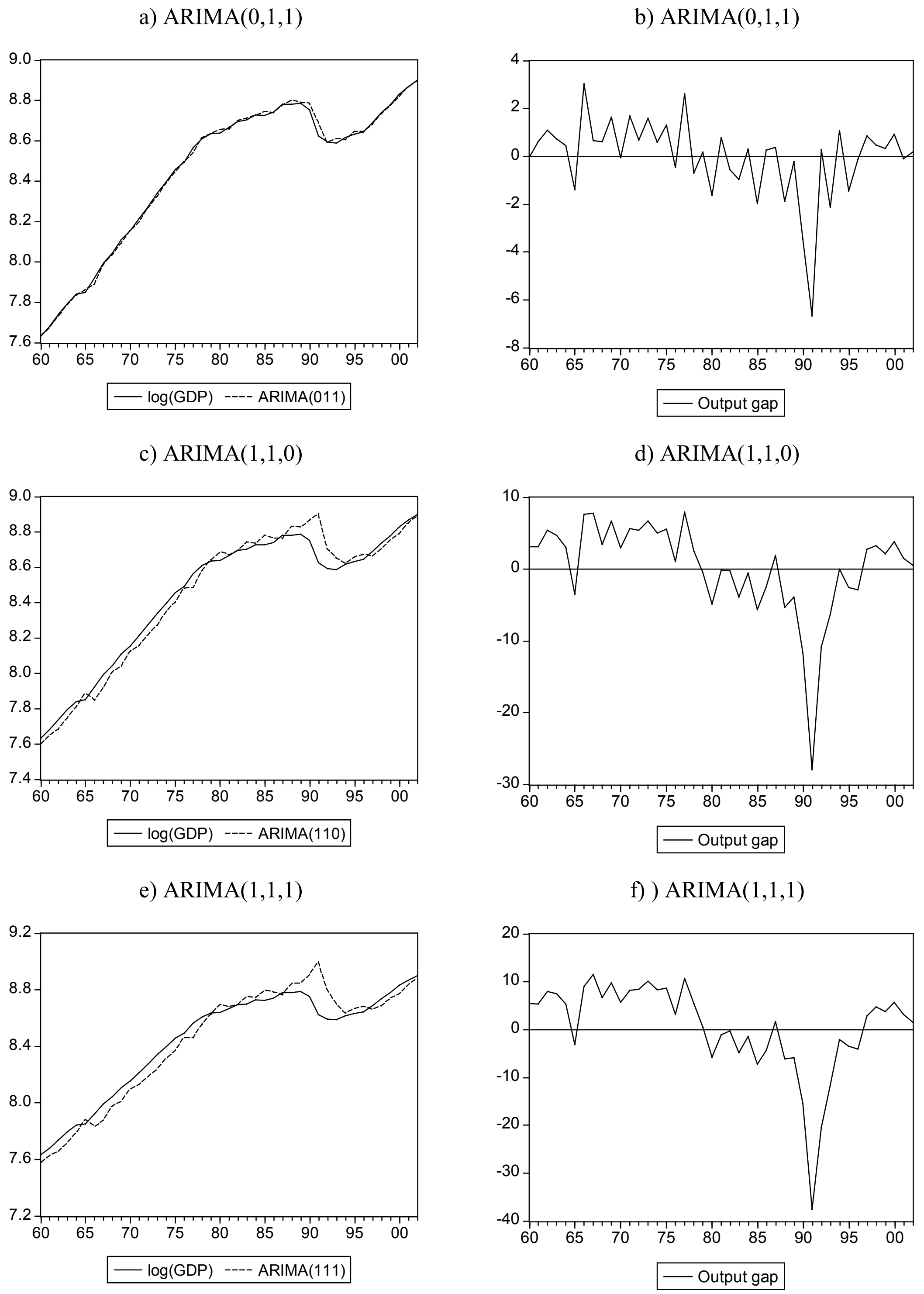



**Figure 16 Quarterly potential GDP and output gap from BP(20) filter**

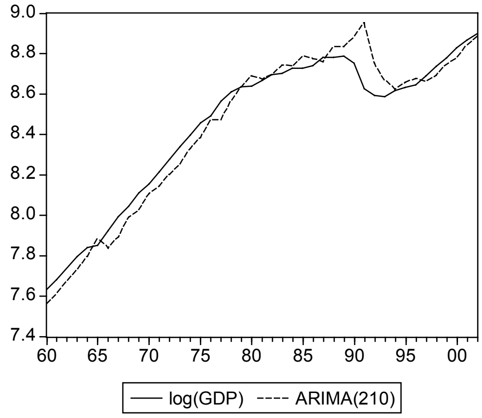


## VIII.4 Beveridge-Nelson decomposition

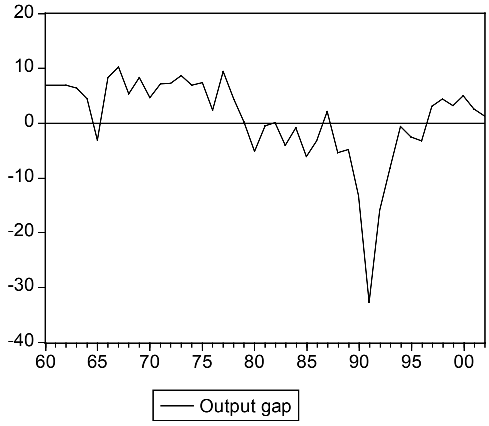
**Figure 17 Annual potential GDP and output gap from BN decomposition**



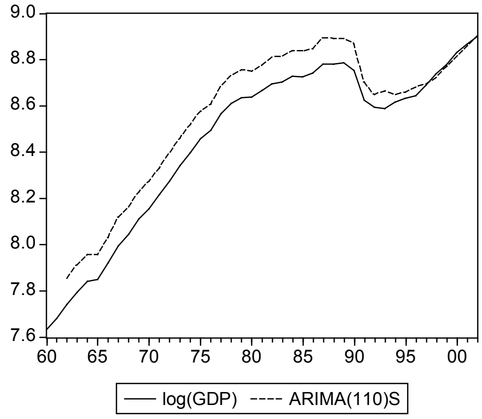
g) ARIMA(2,1,0)



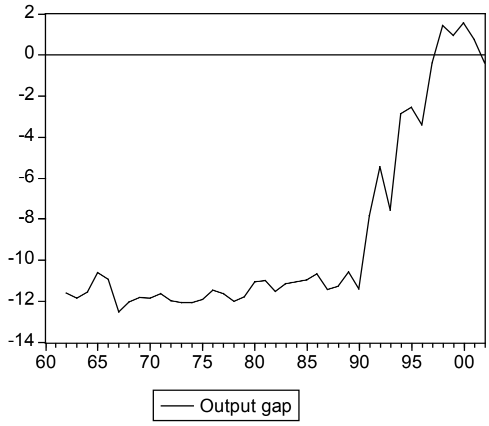

h) ARIMA(2,1,0)



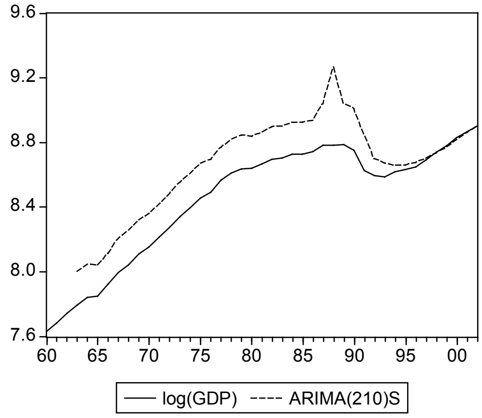

i) ARIMA(1,1,0)S




j) ARIMA(1,1,0)S




k) ARIMA(2,1,0)S




l) ARIMA(2,1,0)S

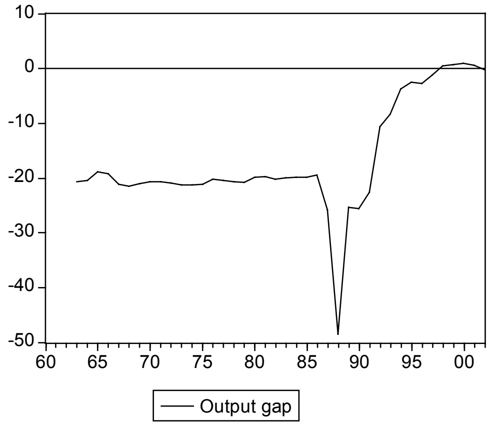
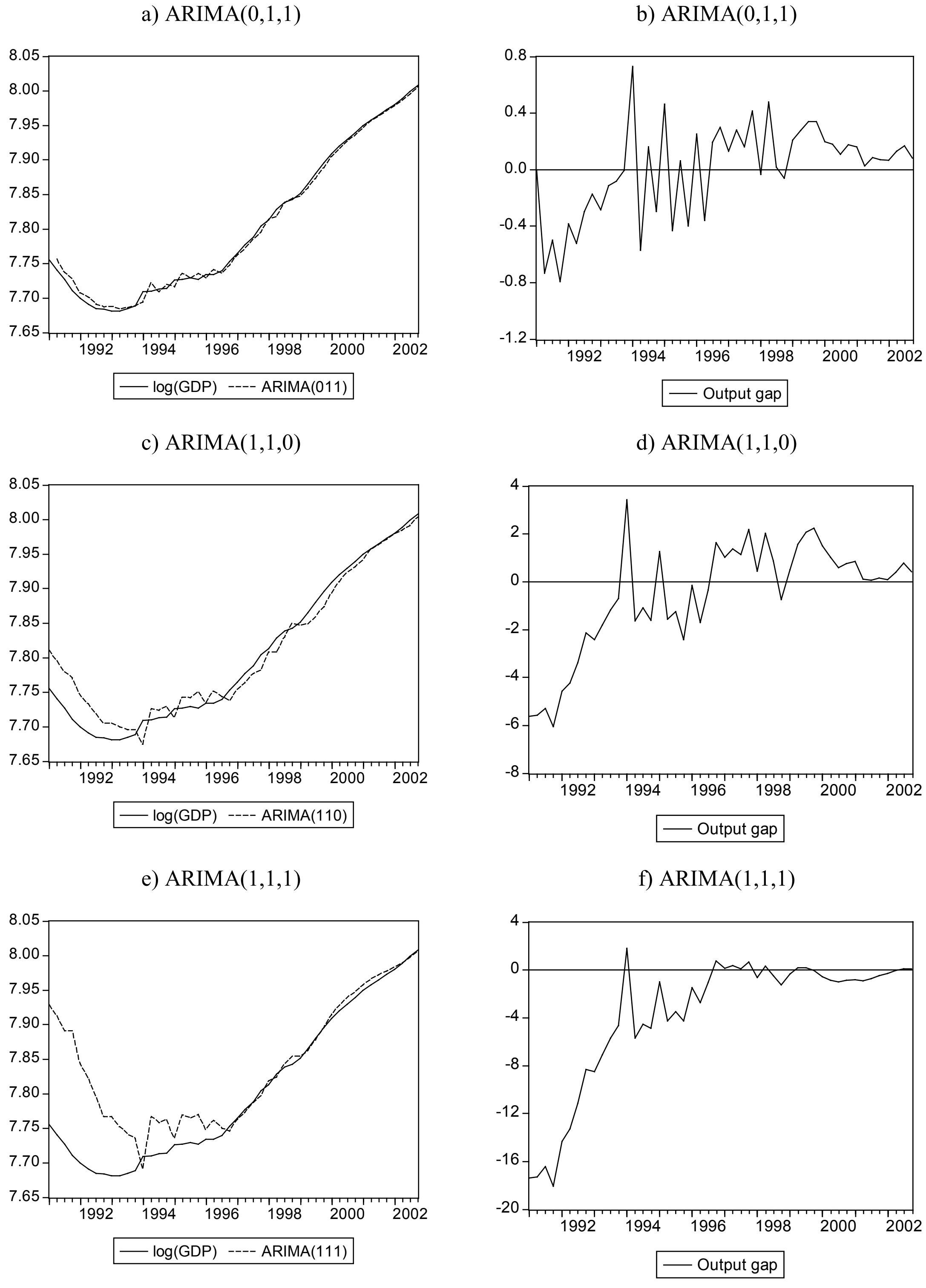
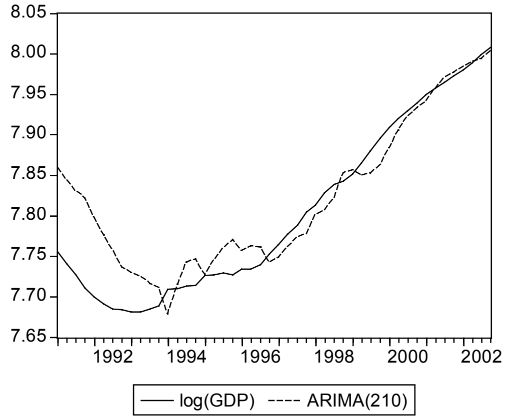


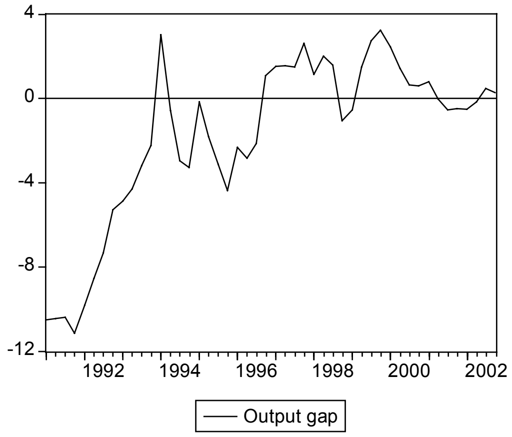
**Figure 18 Quarterly potential GDP and output gap from BN decomposition**



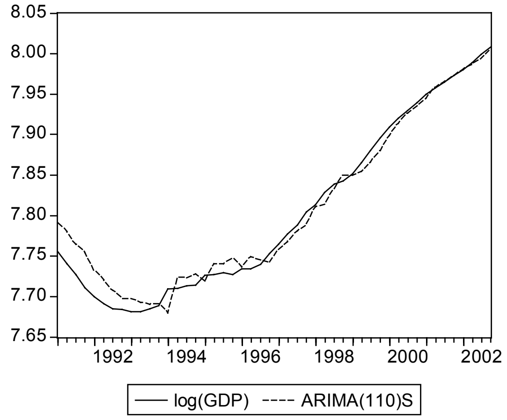
g) ARIMA(2,1,0)



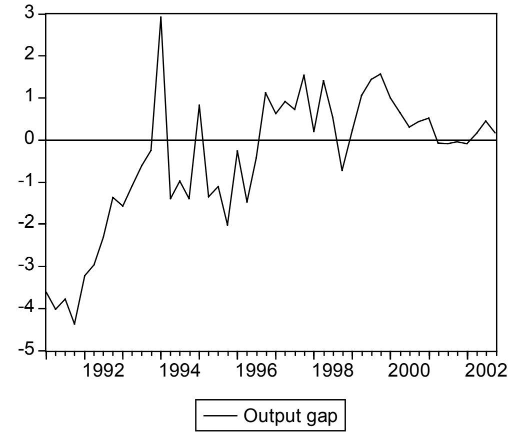

h) ARIMA(2,1,0)



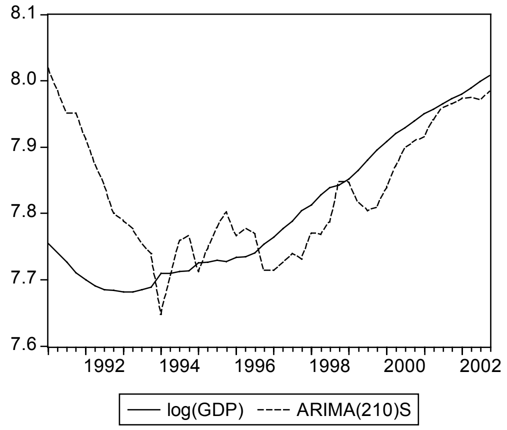

i) ARIMA(1,1,0)S



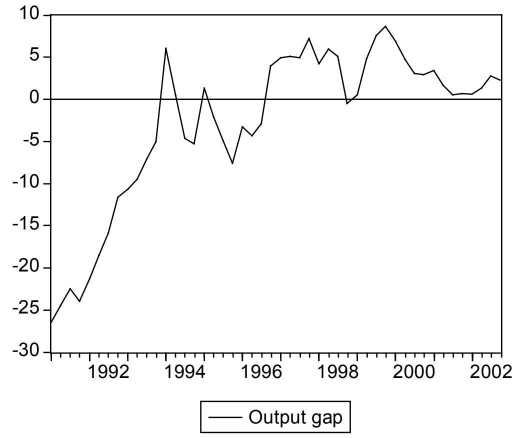

j) ARIMA(1,1,0)S




k) ARIMA(2,1,0)S



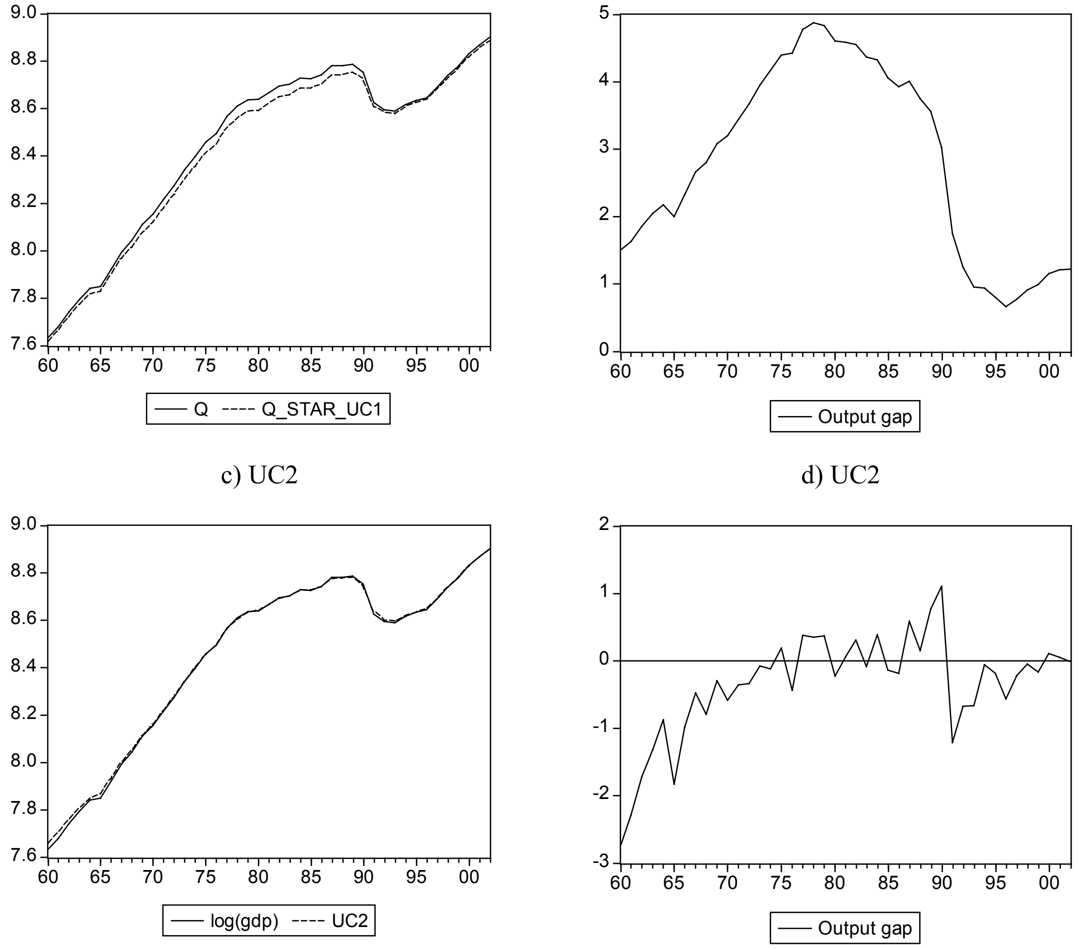

l) ) ARIMA(2,1,0)S



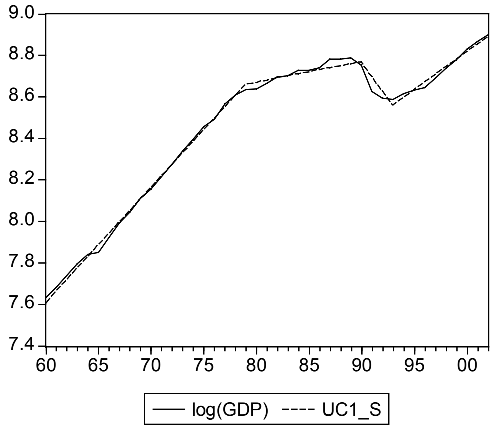
## VIII.5 Unobserved components

**Figure 19 Annual potential GDP and output gap from unobserved component models**

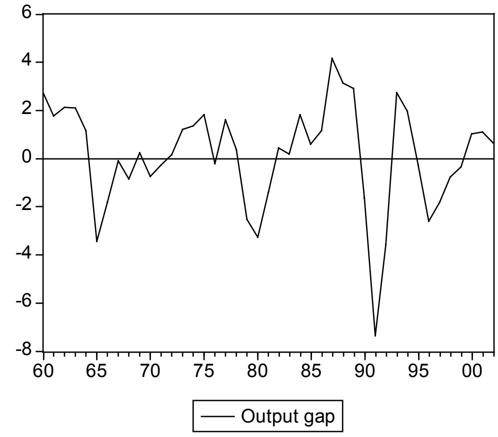
a) UC1 b) UC1



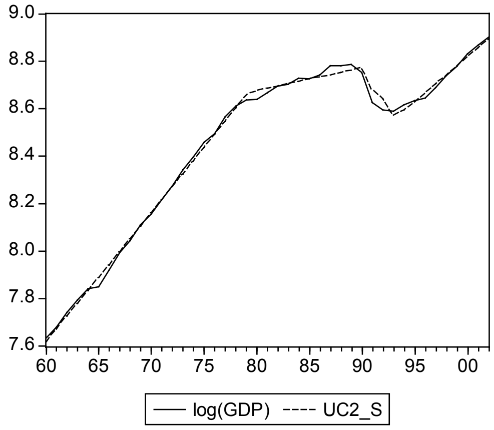
e) UC1_S



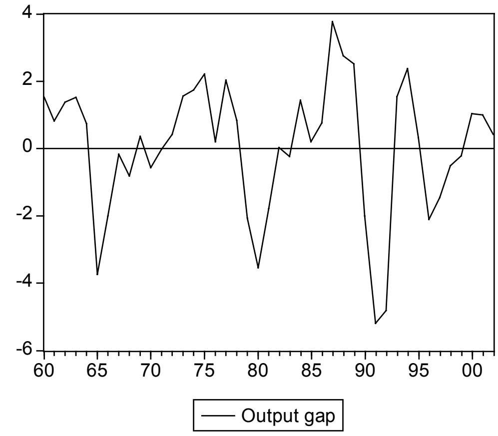

f) UC1_S



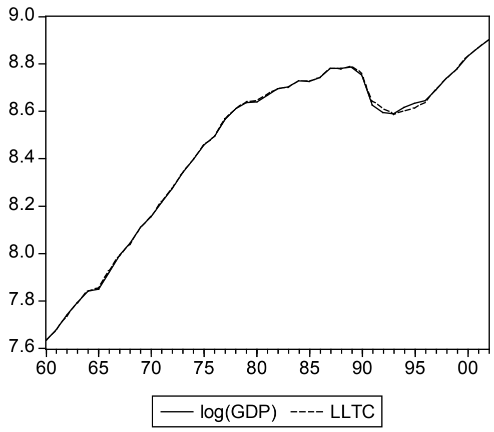

g) UC2_S




h) UC2_S




i) UC_LLTCH




j) UC_LLTCH

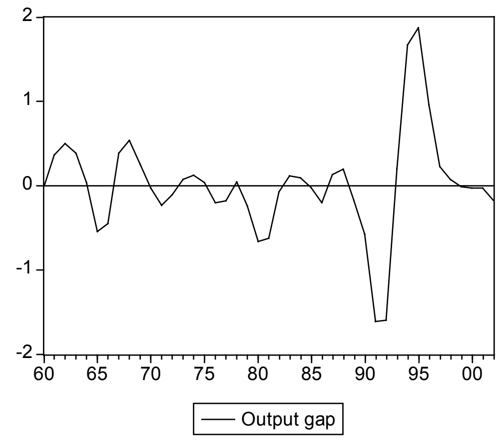
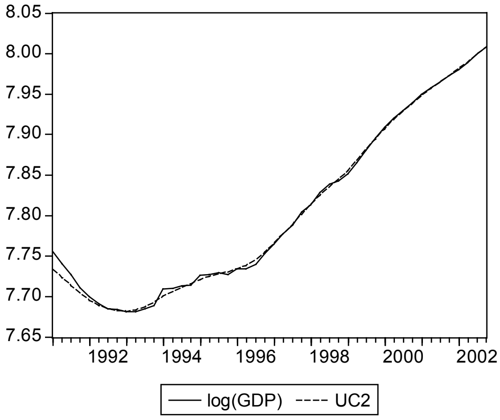
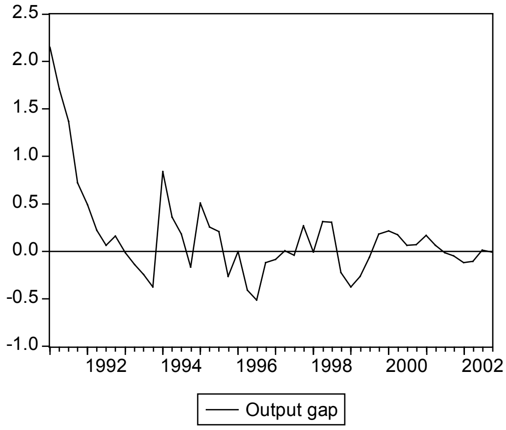


**Figure 20 Quarterly potential GDP and output gap from unobserved component models**

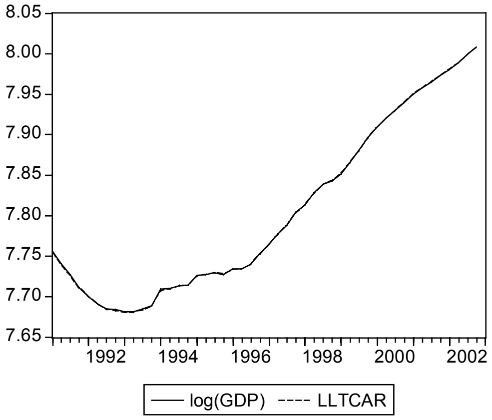
a) UC2



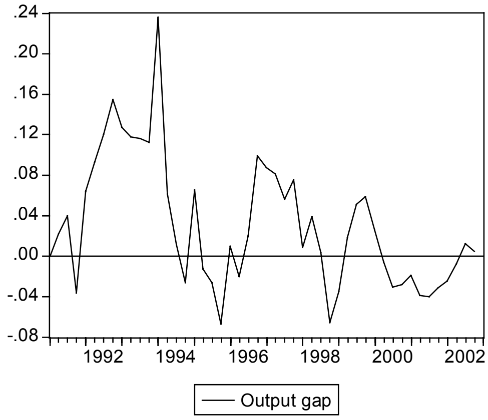

b) UC2




c) LLTCAR



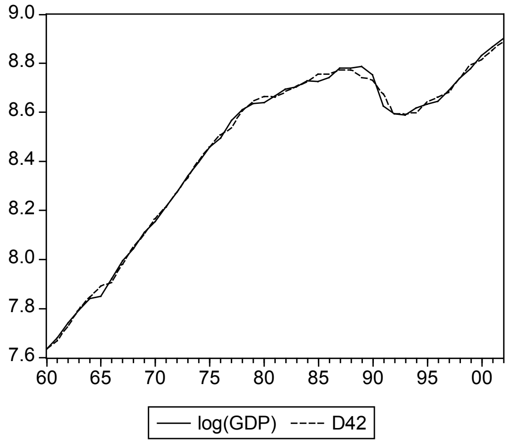

d) LLTCAR

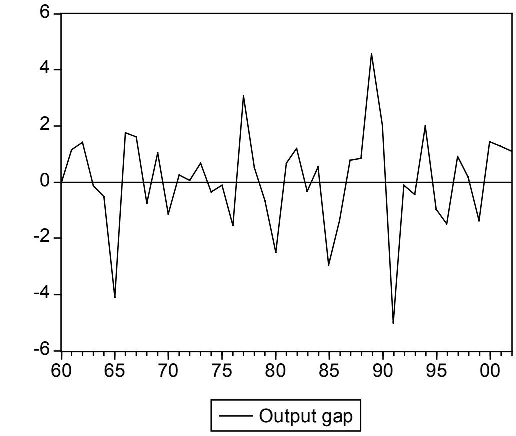


## VIII.6 Wavelet transformation

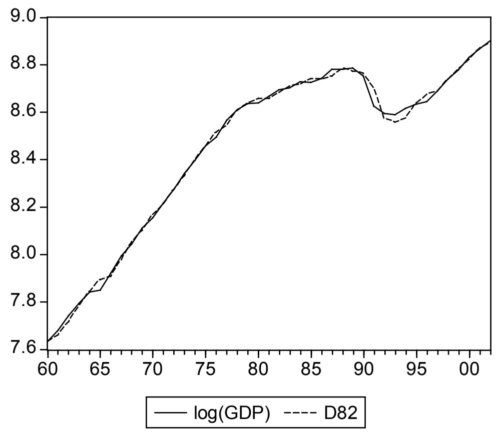
**Figure 21 Annual potential GDP and output gap from wavelets transformation**

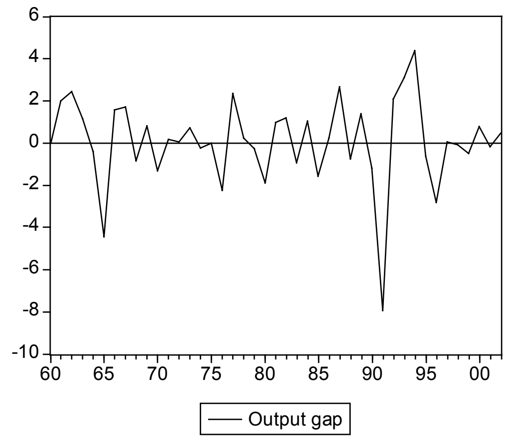
D-4-2



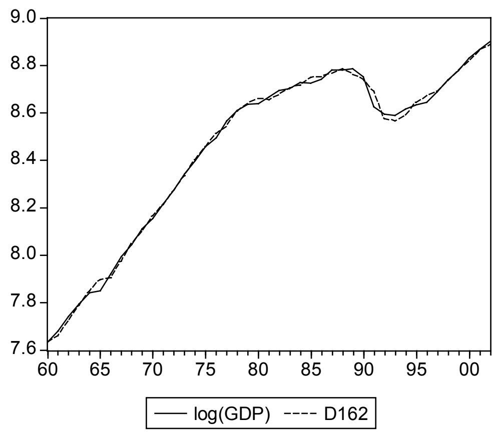




D-8-2




D-16-2



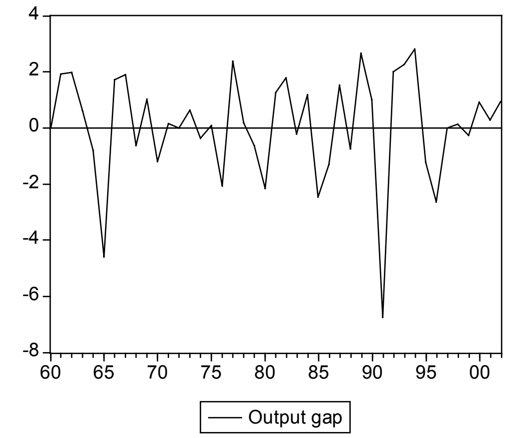
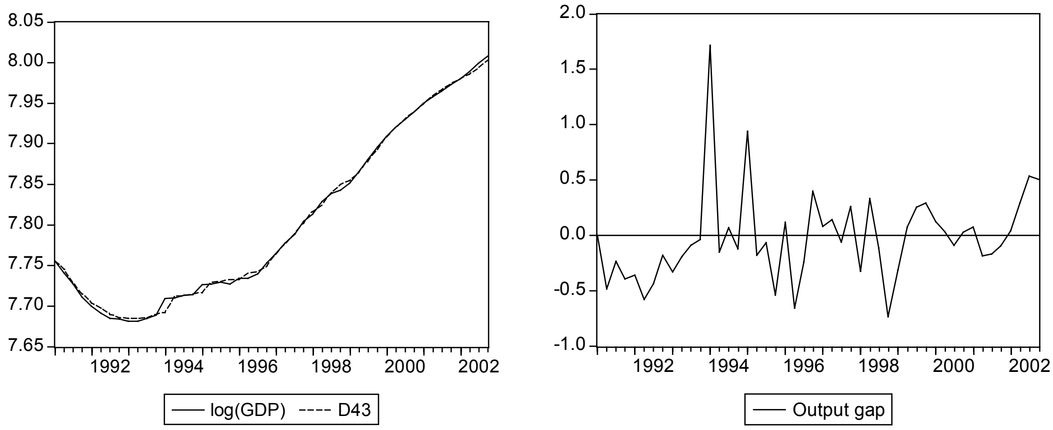
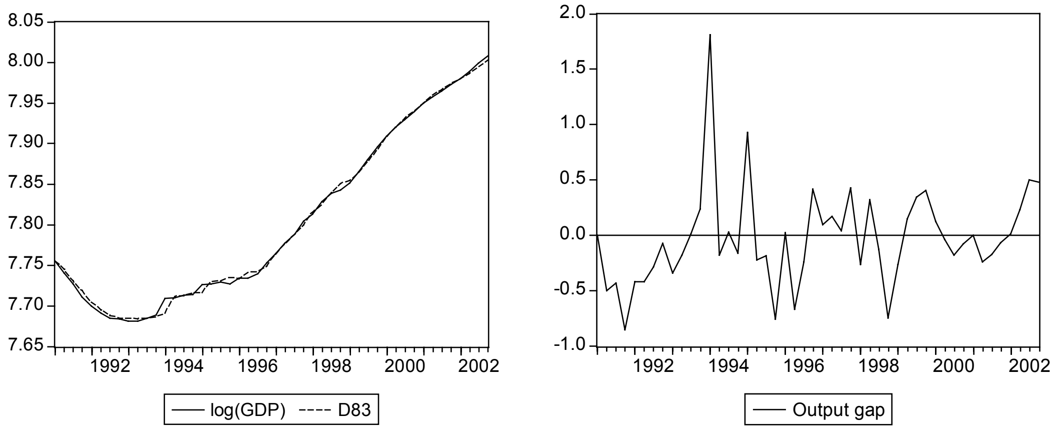



**Figure 22 Quarterly potential GDP and output gap from wavelets transformation**

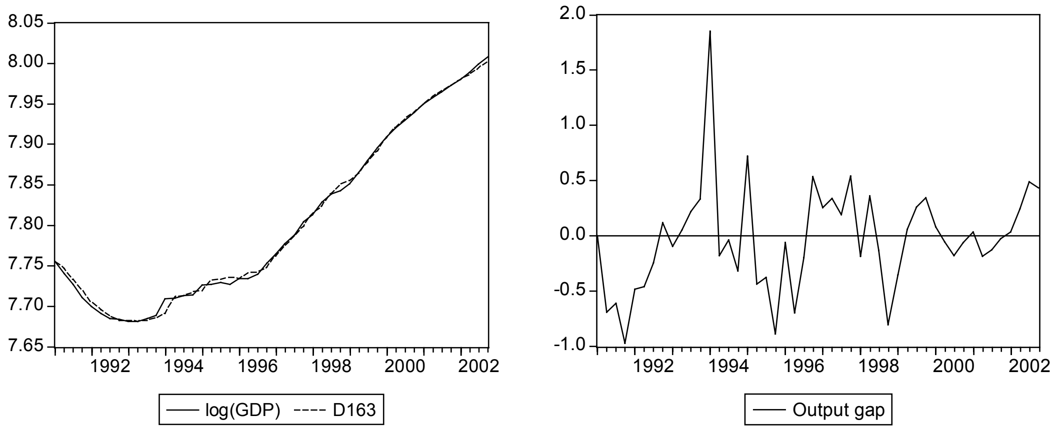
D-4-3




D-8-3



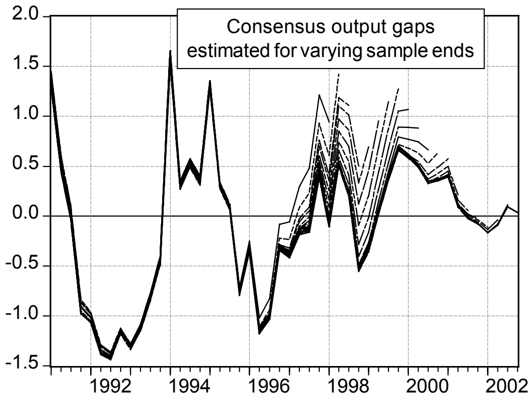

D-16-3



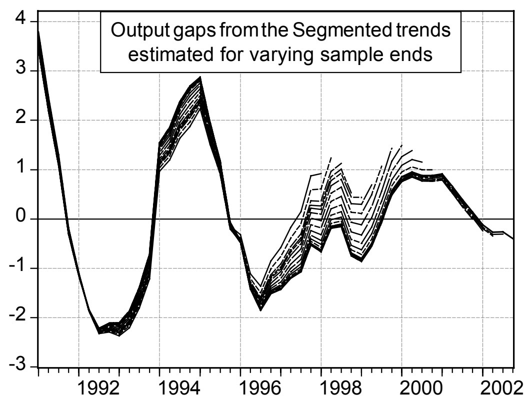
## VIII.7 Revision of selected methods

**Figure 23 Output gaps from selected models estimated for varying sample ends**

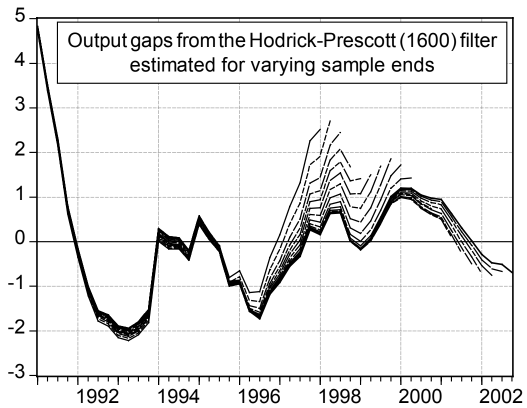
a) Consensus estimate (weighted average)



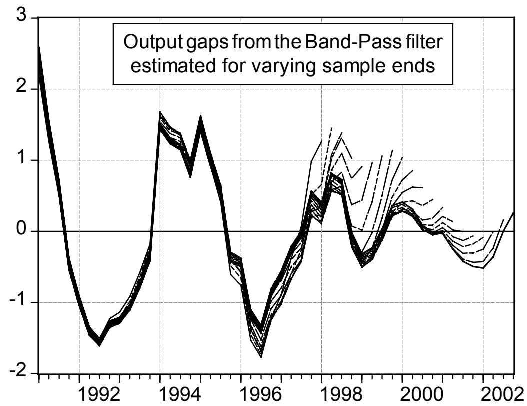

b) Segmented deterministic trend



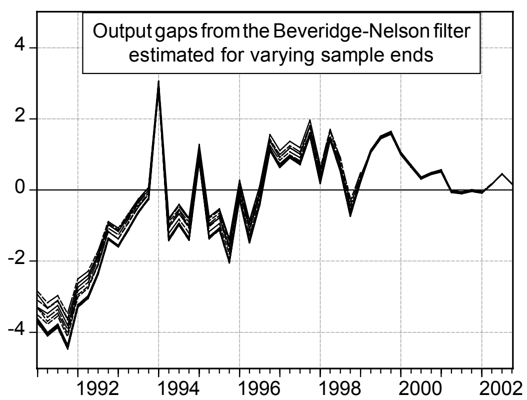

c) Hodrick-Prescott filter



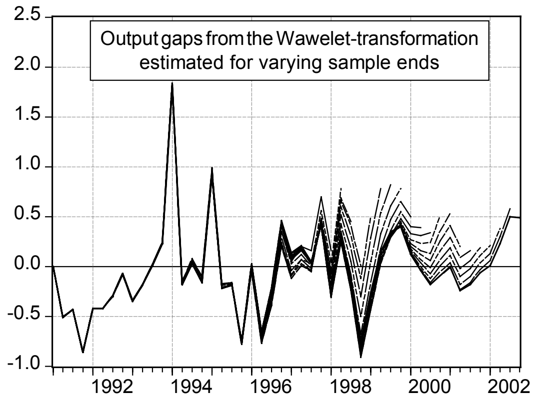

d) Band-Pass filter




e) Beveridge-Nelson decomposition




f) Wavelet transformation



## VIII.8 Revision vs. standard deviation

**Figure 24 Cross-plot of the standard deviation of output gap and revision (annual data)**

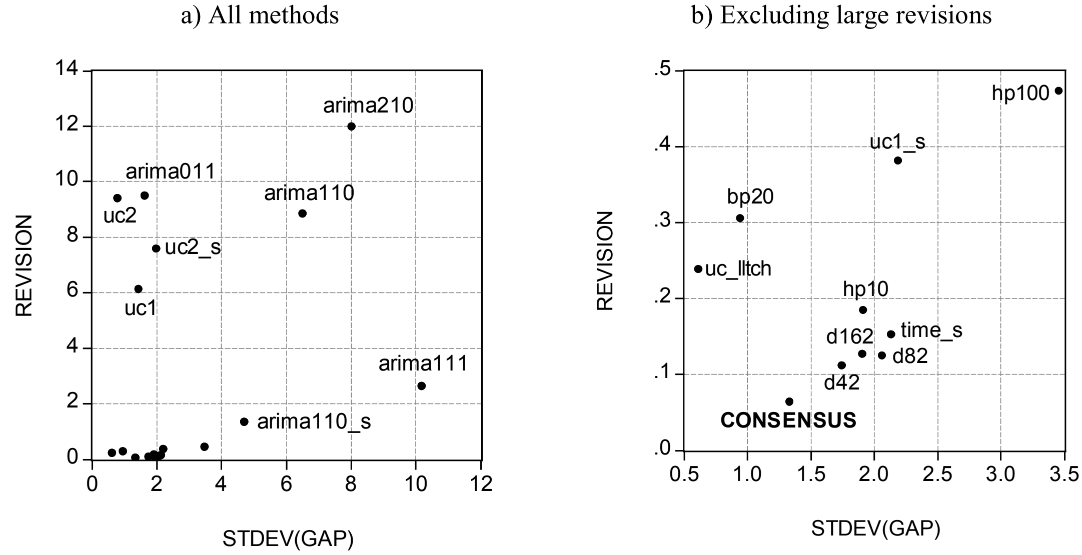
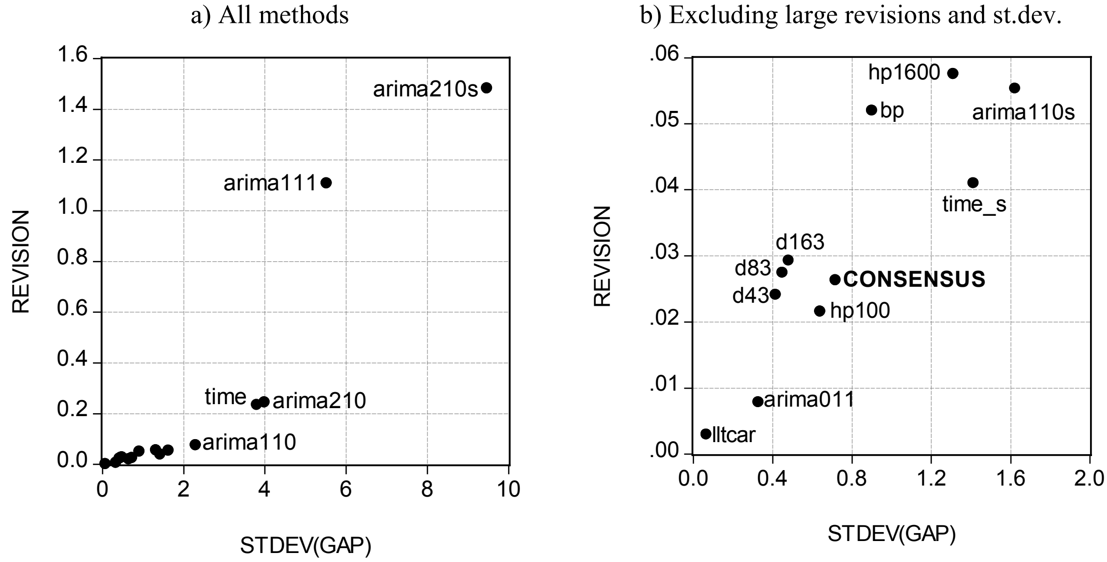



**Figure 25 Cross-plot of the standard deviation of output gap and revision (quarterly data)**




# IX. Appendix 3: Effects of the pre-1995 generated quarterly data on the estimates

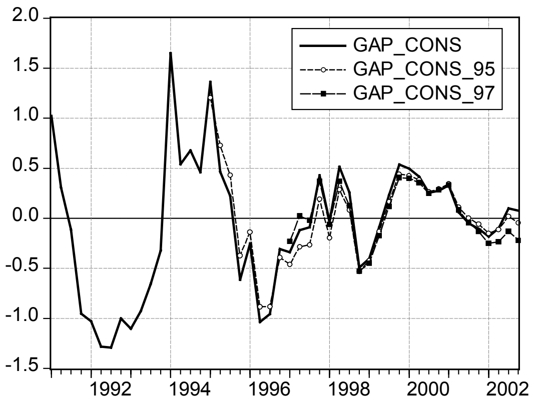
As we indicated in Section IV.1, quarterly GDP figures are available since 1995 and we used the data approximated by Várpalotai (2003) for 1991-94. To test the sensitivity of our results for these generated data, we estimated our final 5 models for sample starting in 1995 and also in 1997.

Panel (a) of Figure 26 shows the weighted average of the 5 methods. The similarity of the inference for the period starting in 1995 is very close to the full sample estimates.

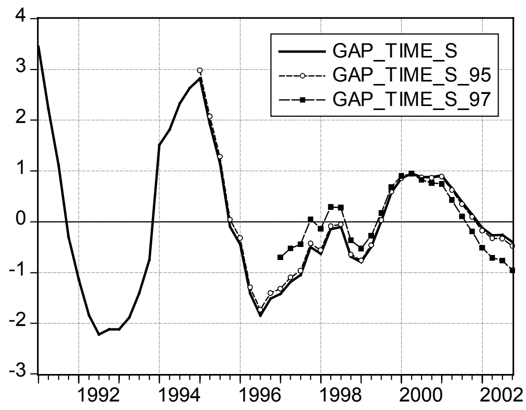
Among the individual methods, the 1995 results of the HP-filter and the wavelet transformation differs somehow from the full sample estimates. The former is due to the strong smoothing property of the HP filter, as the 1995 sample starts just before the slowdown that followed the March 1995 stabilisation program.

**Figure 26 Comparison of output gap estimates for the full sample and for samples starting in 1995Q1 and 1997Q1**

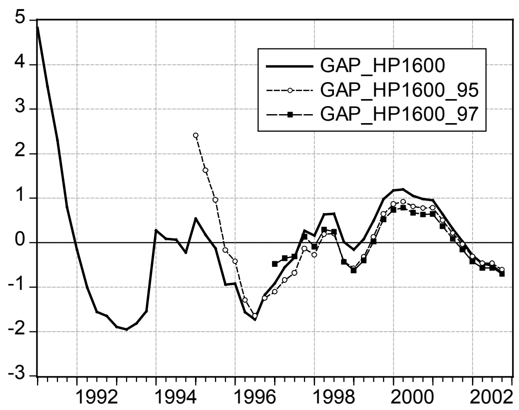
a) Consensus estimate



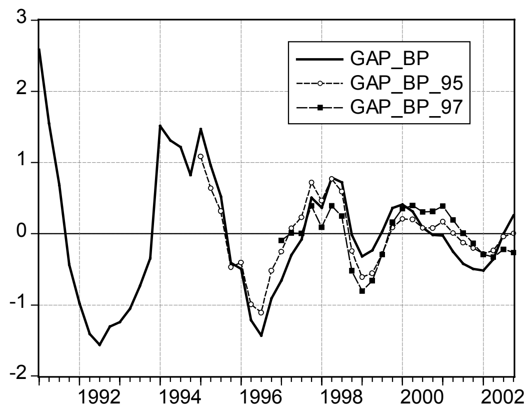

b) Segmented deterministic trend



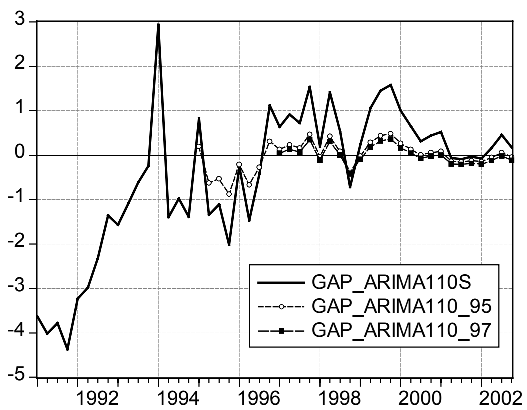

c) Hodrick-Prescott filter



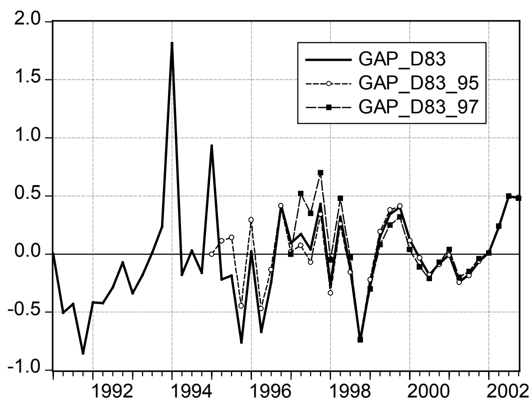

d) Band-Pass filter




e) Beveridge-Nelson decomposition




f) Wavelet transformation



# X. Appendix 4: Tables

**Table 12 The likeliest locations of break points (yearly data)**

| time trend | | arima(110) | | arima(210) | | arima(111) | | arima(011) | |
|---|---|---|---|---|---|---|---|---|---|
| Years | F stat. | Years | F stat. | Years | F stat. | Years | F stat. | Years | F stat. |
| 1979,1990,1993 | 754.3 | 1979,1990,1993 | 8.3 | 1980,1990,1993 | 11.7 | 1978,1990,1993 | 68.9 | 1979,1990,1994 | 32.3 |
| 1978,1990,1993 | 749.1 | 1980,1990,1993 | 7.7 | 1979,1990,1993 | 10.9 | 1976,1986,1993 | 56.7 | 1978,1990,1994 | 30.2 |
| 1979,1989,1993 | 747.8 | 1978,1990,1993 | 7.7 | 1978,1990,1993 | 9.9 | 1978,1986,1993 | 56.5 | 1980,1990,1994 | 27.5 |
| 1978,1989,1993 | 745.3 | 1976,1990,1993 | 6.0 | 1976,1990,1993 | 7.5 | 1976,1990,1993 | 55.1 | 1979,1989,1993 | 24.1 |
| 1977,1989,1993 | 709.9 | 1979,1990,1994 | 5.9 | 1975,1990,1993 | 7.1 | 1976,1986,1994 | 48.1 | 1977,1990,1994 | 23.9 |
| 1980,1990,1993 | 706.0 | 1977,1990,1993 | 5.7 | 1977,1990,1993 | 7.1 | 1978,1986,1994 | 48.0 | 1978,1989,1993 | 23.9 |
| 1977,1990,1993 | 702.7 | 1980,1990,1994 | 5.5 | 1980,1990,1994 | 5.9 | 1976,1989,1993 | 27.2 | 1979,1990,1993 | 23.5 |
| 1980,1989,1993 | 701.0 | 1975,1990,1993 | 5.5 | 1979,1990,1994 | 5.6 | 1975,1990,1993 | 25.1 | 1978,1990,1993 | 22.2 |
| 1979,1989,1994 | 643.3 | 1978,1990,1994 | 5.5 | 1978,1990,1994 | 5.2 | 1979,1986,1993 | 24.7 | 1980,1989,1993 | 20.9 |
| 1979,1990,1994 | 641.5 | 1979,1989,1993 | 5.0 | 1976,1990,1994 | 4.1 | 1975,1986,1993 | 24.6 | 1980,1990,1993 | 20.5 |
| 1978,1989,1994 | 641.5 | 1980,1989,1993 | 4.8 | 1975,1990,1994 | 4.0 | 1978,1989,1993 | 23.6 | 1977,1990,1993 | 18.3 |
| 1978,1990,1994 | 637.8 | 1979,1988,1993 | 4.6 | 1977,1990,1994 | 4.0 | 1979,1986,1994 | 22.7 | 1977,1989,1993 | 17.8 |
| 1977,1989,1994 | 615.0 | 1978,1989,1993 | 4.6 | 1980,1989,1993 | 3.1 | 1975,1986,1994 | 22.6 | 1976,1990,1994 | 17.3 |
| 1980,1989,1994 | 608.3 | 1979,1989,1994 | 4.4 | 1980,1990,1995 | 3.0 | 1979,1990,1993 | 21.3 | 1979,1989,1994 | 16.0 |
| 1980,1990,1994 | 606.1 | 1978,1988,1993 | 4.4 | 1979,1989,1993 | 3.0 | 1977,1990,1993 | 19.9 | 1978,1989,1994 | 15.9 |
| 1977,1990,1994 | 603.8 | 1980,1988,1993 | 4.3 | 1979,1990,1995 | 2.9 | 1977,1986,1993 | 18.6 | 1975,1990,1994 | 14.8 |
| 1979,1988,1994 | 564.7 | 1976,1990,1994 | 4.3 | 1980,1989,1994 | 2.8 | 1977,1986,1994 | 17.4 | 1980,1989,1994 | 14.2 |
| 1979,1988,1993 | 564.4 | 1979,1988,1994 | 4.2 | 1979,1989,1994 | 2.7 | 1980,1986,1993 | 17.2 | 1976,1990,1993 | 13.8 |
| 1978,1988,1994 | 564.0 | 1980,1989,1994 | 4.1 | 1978,1989,1993 | 2.7 | 1980,1990,1993 | 16.8 | 1976,1989,1993 | 13.7 |
| 1978,1988,1993 | 563.7 | 1977,1990,1994 | 4.1 | 1978,1990,1995 | 2.7 | 1980,1986,1994 | 16.1 | 1977,1989,1994 | 12.4 |
| 1977,1988,1994 | 547.4 | 1978,1988,1994 | 4.0 | 1980,1988,1993 | 2.6 | 1979,1989,1993 | 15.3 | 1975,1989,1993 | 11.9 |

In all cases F statistics indicate significant break points

**Table 13 The likeliest locations of break points (quarterly data)**

| time trend | | | arima(110) | | | arima(210) | | | arima(111) | | | arima(011) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | F stat. | p value | Quarter | F stat. | p value | Quarter | F stat. | p value | Quarter | F stat. | p value | Quarter | F stat. | p value |
| 1994:4 | 132.2 | 0.0 | 1994:2 | 7.0 | 0.0 | 1994:2 | 2.9 | 0.0 | 1994:1 | 5.6 | 0.0 | 1993:4 | 24.3 | 0.0 |
| 1995:1 | 132.1 | 0.0 | 1994:1 | 6.2 | 0.0 | 1994:3 | 2.7 | 0.1 | 1993:2 | 2.6 | 0.1 | 1994:1 | 23.6 | 0.0 |
| 1995:2 | 131.1 | 0.0 | 1993:4 | 6.0 | 0.0 | 1994:1 | 1.6 | 0.2 | 1993:4 | 2.5 | 0.1 | 1993:2 | 23.1 | 0.0 |
| 1994:3 | 130.9 | 0.0 | 1993:3 | 5.7 | 0.0 | 1993:4 | 1.5 | 0.2 | 1993:3 | 2.5 | 0.1 | 1993:3 | 22.8 | 0.0 |
| 1995:3 | 129.9 | 0.0 | 1993:2 | 5.1 | 0.0 | 1993:3 | 1.5 | 0.2 | 1993:1 | 2.5 | 0.1 | 1993:1 | 19.6 | 0.0 |
| 1994:1 | 129.8 | 0.0 | 1993:1 | 4.1 | 0.0 | 1993:2 | 1.3 | 0.3 | 1994:2 | 1.4 | 0.3 | 1994:2 | 18.0 | 0.0 |
| 1994:2 | 129.6 | 0.0 | 1995:1 | 2.4 | 0.1 | 1993:1 | 1.2 | 0.3 | 1995:4 | 1.2 | 0.3 | 1994:3 | 14.4 | 0.0 |
| 1993:4 | 128.9 | 0.0 | 1995:4 | 1.8 | 0.2 | 1995:4 | 0.6 | 0.6 | 1995:1 | 1.1 | 0.4 | 1995:1 | 12.4 | 0.0 |
| 1995:4 | 127.4 | 0.0 | 1994:3 | 1.8 | 0.2 | 1995:3 | 0.6 | 0.6 | 1994:4 | 1.1 | 0.4 | 1994:4 | 10.3 | 0.0 |
| 1993:3 | 124.1 | 0.0 | 1994:4 | 1.8 | 0.2 | 1995:1 | 0.5 | 0.7 | 1994:3 | 1.0 | 0.4 | 1995:4 | 9.7 | 0.0 |
| 1993:2 | 115.0 | 0.0 | 1995:3 | 1.7 | 0.2 | 1994:4 | 0.3 | 0.8 | 1995:3 | 1.0 | 0.4 | 1995:3 | 9.4 | 0.0 |
| 1993:1 | 102.1 | 0.0 | 1995:2 | 0.9 | 0.4 | 1995:2 | 0.1 | 1.0 | 1995:2 | 1.0 | 0.4 | 1995:2 | 8.7 | 0.0 |

**Table 14 Ranking of methods according to revision**

| Revision on yearly data | | Revision on standardized yearly data | | Revision on quarterly data(*) | | Revision on standardized quarterly data(**) | |
|---|---|---|---|---|---|---|---|
| <u>*CONSENSUS*</u> (***) | 0.064 | bn-arima(2,1,0) | 0.057 | uc-lltcar | 0.003 | **SDT** | **0.029** |
| wt-d42 | 0.112 | bn-arima(1,1,0) | 0.067 | bn-arima011 | 0.008 | hp100 | 0.034 |
| wt-82 | 0.125 | **SDT** | **0.084** | hp100 | 0.022 | bn-arima011 | 0.035 |
| **wt-d162** | **0.127** | wt-d42 | 0.085 | wt-d43 | 0.024 | <u>*CONSENSUS*</u> (***) | <u>*0.037*</u> |
| **SDT** | **0.153** | wt-d82 | 0.085 | <u>*CONSENSUS*</u> (***) | <u>*0.026*</u> | **hp1600** | **0.044** |
| **hp(10)** | **0.185** | **wt-d162** | **0.085** | **wt-d83** | **0.028** | uc-lltcar | 0.048 |
| **UC_LLTCH** | **0.239** | bn-arima(0,1,1) | 0.097 | wt-d163 | 0.029 | **bp** | **0.058** |
| **bp(20)** | **0.306** | <u>*CONSENSUS*</u> (***) | 0.098 | **SDT** | **0.041** | wt-d43 | 0.059 |
| UC1_S | 0.382 | **hp(10)** | **0.106** | **bp** | **0.052** | wt-d163 | 0.061 |
| hp(100) | 0.474 | **bp(20)** | **0.110** | **bn-arima110s** | **0.055** | **wt-d83** | **0.062** |
| bn-arima(1,1,0)S | 1.356 | bn-arima(1,1,1) | 0.153 | **hp1600** | **0.058** | time | 0.062 |
| bn-arima(2,1,0)S | 2.636 | hp(100) | 0.157 | bn-arima110 | 0.077 | bn-arima110 | 0.541 |
| UC1 | 6.141 | UC1_S | 0.195 | time | 0.236 | **bn-arima110s** | **0.613** |
| UC2_S | 7.598 | **UC_LLTCH** | **0.228** | bn-arima210 | 0.246 | bn-arima210s | 0.753 |
| bn-arima(1,1,0) | 8.872 | bn-arima(2,1,0)S | 0.331 | bn-arima111 | 1.109 | bn-arima210 | 0.984 |
| UC2 | 9.431 | bn-arima(1,1,0)S | 0.336 | bn-arima210s | 1.484 | bn-arima111 | 1.612 |
| bn-arima(0,1,1) | 9.513 | UC2 | 0.712 | | | | |
| bn-arima(2,1,0) | 12.000 | UC1 | 0.787 | | | | |
| bn-arima(1,1,1) | 4.06E+13 | UC2_S | 0.854 | | | | |

Notes: (*) In percentage points. (**) Multiplied by 100. (***) The consensus gap is the weighted average of gaps of highlighted methods.

**Table 15 Overview of methods**

| Annual models | Statistical requirements | | Expertise judgement | Selected | Quarterly models | Statistical requirements | | Expertise judgement | Selected |
|---|---|---|---|---|---|---|---|---|---|
| | Parameters* | Residuals | | | | Parameters* | Residuals | | |
| SDT | ✓ | × | ✓ | ✓ | SDT | ✓ | × | ✓ | ✓ |
| HP(100) | - | - | × | | HP(1600) | - | - | ✓ | ✓ |
| HP(10) | - | - | ✓ | ✓ | HP(100) | - | - | ✓ | |
| BP(20) | - | - | ✓ | ✓ | BP(20) | - | - | ✓ | ✓ |
| ARIMA(0,1,1) | ✓ | × | × | | ARIMA(0,1,1) | ✓ | × | × | |
| ARIMA(1,1,0) | ✓ | ✓ | × | | ARIMA(1,1,0) | ✓ | × | × | |
| ARIMA(1,1,1) | ✓/×(1/3) | ✓ | × | | ARIMA(1,1,1) | ✓ | ✓ | ✓ | |
| ARIMA(2,1,0) | ✓/×(1/3) | ✓ | × | | ARIMA(2,1,0) | ✓ | ✓ | ✓ | |
| ARIMA(1,1,0)S | ✓/×(3/8) | × | × | | ARIMA(1,1,0)S | ✓/×(1/4) | ✓ | ✓ | ✓ |
| ARIMA(2,1,0)S | ✓/×(4/12) | ✓ | × | | ARIMA(2,1,0)S | ✓/×(5/6) | × | × | |
| UC1 | × | | × | | UC1 | ✓/× | | × | |
| UC1S | ✓/×(2/7) | | ✓ | | UC2 | ✓ | | ✓ | |
| UC2 | ✓/×(4/5) | | × | | UC_LLTCAR | ✓ | | ✓ | |
| UC2S | ✓/×(4/11) | | ✓ | | D43 | - | - | ✓ | |
| UC_LLTCH | ✓/×(1/4) | | ✓ | ✓ | D83 | - | - | ✓ | ✓ |
| D42 | - | - | ✓ | | D163 | - | - | ✓ | |
| D82 | - | - | ✓ | | | | | | |
| D162 | - | - | ✓ | ✓ | | | | | |

* ✓ denotes if all parameters are significant at 10% level, ✓/×(1/3) indicates that one parameter is not significant out of three estimated ones at 10% level, while × shows none of estimated parameters is acceptable at 10% significance level.

**Table 16 Cross-correlation of output gap from yearly data**

| | Segmented time trend | HP(10) | HP(100) | BP(20) | arima (011) | arima (110) | arima (111) | arima (210) | arima (110)s | arima (210)s | UC1_S | UC2 | UC2_S | LLTC | D42 | D82 | D162 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Segmented time trend | 1 | | | | | | | | | | | | | | | | |
| HP10 | *0.44* | 1 | | | | | | | | | | | | | | | |
| HP100 | *0.38* | **0.84** | 1 | | | | | | | | | | | | | | |
| BP | *0.48* | **0.70** | *0.43* | 1 | | | | | | | | | | | | | |
| ARIMA011 | *0.39* | 0.21 | 0.14 | *0.37* | 1 | | | | | | | | | | | | |
| ARIMA110 | *0.44* | 0.10 | -0.02 | *0.42* | **0.92** | 1 | | | | | | | | | | | |
| ARIMA111 | *0.50* | 0.10 | -0.04 | *0.44* | **0.90** | **0.99** | 1 | | | | | | | | | | |
| ARIMA210 | *0.48* | 0.10 | -0.03 | *0.44* | **0.90** | **1.00** | **1.00** | 1 | | | | | | | | | |
| ARIMA110S | 0.35 | *0.45* | 0.28 | **0.77** | *0.55* | *0.63* | *0.64* | *0.64* | 1 | | | | | | | | |
| ARIMA210S | 0.23 | 0.25 | 0.14 | *0.52* | *0.43* | *0.47* | *0.48* | *0.48* | **0.75** | 1 | | | | | | | |
| UC1_S | **0.98** | *0.52* | *0.50* | *0.50* | *0.40* | *0.42* | *0.47* | *0.45* | 0.34 | 0.21 | 1 | | | | | | |
| UC2 | *0.46* | *0.66* | *0.65* | *0.61* | 0.07 | 0.10 | 0.10 | 0.10 | *0.56* | *0.40* | *0.44* | 1 | | | | | |
| UC2_S | **0.95** | *0.54* | *0.51* | *0.47* | *0.38* | 0.35 | *0.39* | *0.38* | 0.28 | 0.26 | **0.97** | *0.46* | 1 | | | | |
| LLTC | *0.55* | 0.28 | -0.03 | 0.33 | 0.28 | 0.23 | 0.30 | 0.27 | 0.14 | 0.04 | *0.49* | 0.11 | *0.55* | 1 | | | |
| D42 | *0.55* | *0.68* | *0.50* | **0.74** | *0.54* | *0.58* | *0.59* | *0.59* | *0.69* | *0.51* | *0.57* | *0.61* | *0.54* | 0.22 | 1 | | |
| D82 | **0.72** | 0.30 | 0.14 | *0.65* | *0.66* | **0.81** | **0.85** | **0.84** | **0.72** | *0.55* | *0.67* | *0.38* | *0.58* | 0.34 | **0.76** | 1 | |
| D162 | *0.62* | *0.43* | 0.26 | **0.74** | *0.60* | **0.74** | **0.77** | **0.76** | **0.78** | *0.58* | *0.60* | *0.49* | *0.51* | 0.22 | **0.89** | **0.94** | 1 |
| Average | *0.53* | *0.41* | 0.29 | *0.54* | *0.49* | *0.51* | *0.53* | *0.52* | *0.54* | *0.39* | *0.53* | *0.39* | *0.51* | 0.27 | *0.60* | *0.62* | *0.62* |

**Table 17 Cross-correlation of output gap from quarterly data**

| | Segmented time trend | HP(100) | HP(1600) | BP(20) | arima (011) | arima (110) | arima (111) | arima (210) | arima (110)s | arima (210)s | UC2 | LLTC | D43 | D83 | D163 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Segmented time trend | 1 | | | | | | | | | | | | | | |
| HP100 | **0.91** | 1 | | | | | | | | | | | | | |
| HP1600 | **0.75** | **0.81** | 1 | | | | | | | | | | | | |
| BP | **0.89** | **0.96** | **0.76** | 1 | | | | | | | | | | | |
| ARIMA011 | 0.04 | 0.18 | -0.02 | 0.16 | 1 | | | | | | | | | | |
| ARIMA110 | -0.04 | 0.09 | -0.18 | 0.06 | **0.87** | 1 | | | | | | | | | |
| ARIMA111 | -0.08 | -0.02 | -0.29 | -0.03 | **0.78** | **0.95** | 1 | | | | | | | | |
| ARIMA210 | 0.00 | 0.12 | -0.15 | 0.09 | **0.78** | **0.98** | **0.96** | 1 | | | | | | | |
| ARIMA110S | -0.05 | 0.09 | -0.17 | 0.06 | **0.89** | **1.00** | **0.93** | **0.96** | 1 | | | | | | |
| ARIMA210S | -0.01 | 0.09 | -0.18 | 0.06 | **0.75** | **0.97** | **0.97** | **1.00** | **0.95** | 1 | | | | | |
| UC2 | *0.57* | *0.69* | **0.81** | *0.65* | -0.24 | -0.49 | -0.64 | -0.51 | -0.45 | -0.55 | 1 | | | | |
| LLTC | -0.33 | -0.16 | -0.33 | -0.14 | 0.16 | 0.08 | -0.11 | -0.01 | 0.16 | -0.06 | 0.10 | 1 | | | |
| D43 | 0.30 | *0.43* | 0.09 | *0.41* | **0.74** | *0.63* | *0.47* | *0.53* | *0.66* | *0.49* | 0.12 | *0.41* | 1 | | |
| D83 | 0.21 | 0.35 | 0.02 | 0.35 | **0.77** | *0.67* | *0.50* | *0.57* | **0.72** | *0.53* | 0.06 | *0.52* | **0.97** | 1 | |
| D163 | 0.05 | 0.22 | -0.10 | 0.21 | **0.78** | *0.69* | *0.52* | *0.59* | **0.74** | *0.54* | -0.02 | *0.61* | **0.92** | **0.97** | 1 |
| Average | 0.23 | 0.34 | 0.13 | 0.32 | *0.47* | *0.45* | *0.35* | *0.42* | *0.46* | *0.40* | 0.01 | 0.06 | *0.51* | *0.51* | *0.48* |

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