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Density forecast evaluation and the effect of risk-neutral central moments on the currency risk premium: tests based on EUR/HUF option-implied densities

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Density forecast evaluation and the effect of risk-neutral central moments on the currency risk premium: tests based on EUR/HUF option-implied densities

(Eloszlások előrejelző képességének vizsgálata és a kockázatsemleges eloszlások momentumainak hatása a kockázati prémiumra: mit mutatnak az opciókból becsült forint/euro eloszlások?)

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Contents

Abstract

In this paper we estimate risk-neutral probability density functions from EUR/HUF currency options using the Malz (1997) method. First, we compare different option-based indicators. We present so-called 'shortcut' indicators, i.e. indicators that can be calculated directly, without the estimation of RNDs, but which show strong co-movement with the central moments of estimated densities. We also find that it is possible to construct probability-based indicators, which again exhibit strong correlation with the central moments.

We present evidence that risk-neutral densities do not provide accurate forecasts for the distribution of the historical EUR/HUF exchange rate. The higher moments of risk-neutral densities are responsible for the rejection of forecasting ability. Our interpretation is that the standard deviation, the skewness and the kurtosis of the risk-neutral densities are significantly higher than the central moments of subjective densities. Finally, we show that the higher moments of risk-neutral densities are able to explain a significant part of the variability in the estimated risk premium. These latter results suggest that risk-neutral standard deviation and skewness can be used as proxy variables for the respective central moments of subjective densities.

JEL: F31, G13, C53.
Keywords: Currency option, Implied risk-neutral density function, Density forecasting, Risk premium, GMM.

Összefoglalás

Tanulmányunkban kockázatmentes valószínűség-eloszlásokat becslünk forint/euro devizaopciókból a Malz-módszer (1997) alkalmazásával. Elsőként különböző, opciókból származtatott mutatókat hasonlítunk össze egymással. Definiáljuk az ún. short-cut mutatókat, amelyek jellemzője, hogy a kockázatsemleges eloszlások becslése nélkül számíthatóak, ennek ellenére szoros együttmozgást mutatnak a becsült eloszlások centrális momentumaival. Emellett bemutatunk olyan, valószínűségekből számolt mutatókat is, amelyek szintén szoros korrelációt mutatnak a centrális momentumokkal.

Vizsgálataink alapján a kockázatsemleges eloszlások nem adnak jó előrejelzést a forint/euro árfolyam historikus eloszlására. Az előrejelző képesség elutasítása a magasabb momentumok alakulásából ered. Értelmezésünk szerint ezt azt jelenti, hogy a kockázatsemleges eloszlások szórása, ferdesége és csúcsossága nagyobb, mint a szubjektív eloszlásoké. Végezetül megmutatjuk, hogy az eloszlások magasabb momentumai a kockázati prémium ingadozásainak egy nem elhanyagolható részét képesek megmagyarázni. Ez utóbbi eredmények arra utalnak, hogy a kockázatsemleges eloszlások szórása és ferdesége használható a szubjektív eloszlások megfelelő momentumainak közelítésére.

1 Introduction

Option prices can provide a rich source of information about investors' expectations related to the underlying asset. A straightforward use of option prices is the calculation of implied volatility, which is in connection with the uncertainty of future asset prices. The MNB also observes EUR/HUF option implied volatility as a market sentiment indicator. The key motivation of this paper is to examine ways of extracting some further information from option prices by estimating risk-neutral probability density functions (RNDs) and by calculating their central moments. The standard deviation reflects the general uncertainty attached to the RND, the skewness is connected to the asymmetry, while the kurtosis is related to the probability of extreme exchange rate movements.

The empirical literature on estimating risk-neutral densities from currency option prices has begun to develop, in parallel with the deepening of the currency option markets. Presently, there are numerous techniques for estimating risk-neutral densities. A large part of the literature takes the relationship derived by Breeden and Litzenberger (1978) as a starting point. Their formula establishes a direct link between option prices and implied risk-neutral densities.

In this paper, we apply the method proposed by Malz (1997a) to estimate risk-neutral probability density functions from EUR/HUF options. This technique allows the estimation of RNDs from option prices with only 3 different strike prices. The motivation behind the choice of this method was the limited availability of forint currency options data. Since this method is reported to be very sensitive to possible observation errors in the input data, we also perform sensitivity analysis.

The main objective of this paper – apart from estimating RNDs – is threefold. First, we present and compare different option-based indicators. Some of these indicators are based on quoted options data, while others are derived from the estimated risk-neutral densities. We evaluate the strength of the relationship amongst these indicators empirically. Second, we examine whether the estimated risk-neutral densities can forecast the distribution of the EUR/HUF exchange rate. Third, we test the hypothesis that the higher moments of RNDs (standard deviation and skewness) are able to explain the time series development of the estimated currency risk premium.

As to the first objective, we look at three different types of indicators extracted from option prices. These indicators describe the shape of the RNDs, but are calculated in different ways. First, we calculate the central moments of the estimated risk-neutral densities: standard deviation, skewness and kurtosis. Second, we compute indicators which can be calculated from option prices without the estimation of RNDs, despite being highly correlated with the estimated central moments (so-called '*shortcut*' indicators). Based on the empirical literature it is expected that the ATMF implied volatility and the standardised 25-delta risk reversal (i.e. the 25-delta risk reversal divided by the ATMF implied volatility) fulfil these criteria. The advantage of these indicators is the ease of availability, while the calculation of central moments is more cumbersome. The third proposed set of indicators is based on depreciation/appreciation probabilities. We are again seeking indicators which show strong co-movement with the central moments. These indicators are somewhat easier to interpret than the central moments, thus one can use them as proxies for the latter. We show that it is not irrelevant how exactly these probability indicators are defined. The results of the above exercises are rather similar to that of Lynch and Panigirtzoglou (2003), who estimated RNDs from stock index options: there is a strong relationship between the central moments, shortcut indicators and probability-based indicators. Since the results also hold for an emerging market currency as the underlying asset, this suggests that the association between different indicators can stem from a general characteristic of option-implied RNDs.

The remaining part of the paper intends to answer many interrelated questions. From a central bank's point of view, currency option implied RNDs are important because these are putatively related to market expectations about the future exchange rate. The first question is whether RNDs and the so-called subjective densities are identical. It should be noted that the theoretical relationship between the risk-neutral and subjective densities will not be addressed in this paper; for our analysis the following simplified definitions are satisfactory. Risk-neutral densities are those that are used to price the options and that can be estimated directly from option prices, while the subjective densities are those describing the expectations of the representative market participant. The second question is that if the equality of RNDs and subjective densities does not hold, is it true that there is difference only in the mean of the densities, while their shape is the same? (Rubinstein, 1994) Third, even if the shape of the two densities differs, do changes in risk-neutral higher central moments coincide with the changes in those of the subjective densities? All in all, the main question is how to interpret risk-neutral densities in situations where the subjective densities are what we are really interested in.

We wish to answer the above questions indirectly, without the estimation of subjective densities, using the EUR/HUF option market as a testing ground. Even though some authors have estimated subjective densities from currency options recently (e.g. Bliss and Panigirtzoglou, 2004), in this paper we do not intend to perform similar exercises because it would require rather strong assumptions (for example, in respect of the risk preferences of a representative investor). In our study, the key assumption upon which we rely is the rationality of investors.

With respect to the second aim, we test whether the estimated risk-neutral densities can forecast the historical distribution. Here the basic idea is the following: if rationality holds, subjective densities are expected to have forecasting power with respect to the future realizations of the exchange rate. However, if RNDs fail to forecast the realised distribution, this gives evidence against the equality of RNDs and subjective densities. Our tests will be based on the so-called *density forecast evaluation,* proposed by Berkowitz (2001). With this, it is possible to test the forecasting power of the whole density function, not only that of the central moments. To test the null hypothesis of the accurate forecasting power of RNDs a GMM estimation method is employed, similar to the one used by Christoffersen and Mazzotta (2004).

We found that 1-month risk-neutral densities did not provide accurate forecasts for the realised distribution of the EUR/HUF exchange rate in the period ranging from 2003 until mid-2007. This result is in line with the findings based on other currency pairs (see e.g. Christoffersen and Mazzotta, 2004). However, our results differ from that of Castrén (2005), who could not reject the forecasting ability of EUR/HUF RNDs for a shorter sample.

Our results suggest that RNDs and subjective densities are not identical, all the first four central moments are responsible for the rejection of the forecasting ability (except from the mean). Thus – in the case of EUR/HUF options – the aforementioned Rubinstein hypothesis does not hold, i.e. higher moments of RNDs can be different from those of the moments of subjective densities, even though the first central moments are not statistically different. A potential explanation behind this is that risk-neutral central moments contain a risk premium with respect to the subjective ones (see e.g. Breuer, 2003). This may stem from the behaviour of option market makers.

The third aim of this paper is to test whether risk-neutral standard deviation and skewness are able to explain the estimated risk premium. Similar empirical analyses were performed by Malz (1997b), and Gereben (2002). The underlying idea is that the higher moments of subjective densities are expected to affect the risk premium: higher standard deviation and/or higher skewness towards forint depreciation should be reflected in a rise of the risk premium. Thus, if the risk-neutral standard deviation and skewness have a positive and significant effect on the risk premium, it suggests that these can be used as proxy variables for the corresponding subjective moments. We estimate the risk premium by two different methods, one based on historical exchange rates and the other based on survey expectations.

We found that the risk-neutral standard deviation and the skewness are able to explain a significant part of the variability of the estimated risk premium in the examined period. These results suggest that the direction of the changes to risk-neutral central moments coincides with changes to central moments of subjective densities, i.e. these indicators, in general, move in the same direction.

With this paper we wish to contribute to the existing empirical literature about option-implied densities in three different aspects. With respect to the comparison of option-based indicators, to our knowledge, similar analysis was performed only for interest rate options and stock index options (Lynch and Panigirtzoglou, 2003), but not for currency options. However, we obtained similar results. Second, for testing the forecasting power of RNDs we modified the estimation method used by Christoffersen and Mazzotta (2004). Using this, it possible to decide which central moments of the risk-neutral densities are responsible for rejecting the forecasting ability. Third, regarding the relationship between risk-neutral central moments and the risk premium, the method is similar to that used in the literature (Malz, 1997b or Gereben, 2002). However, the cited authors used the risk-neutral central moments of RNDs implicitly assuming that these are equal to the subjective ones. In this study, this is actually the hypothesis we will test: thus what is new, is the interpretation.

The paper is organised as follows. In Section 2 we provide a brief review about the literature related to the methods of estimating risk-neutral densities. Section 3 discusses the EUR/HUF option data. Section 4 presents the comparison of different option-based indicators: the central moments, the shortcut indicators and the probability-based indicators. Section 5 provides a sensitivity analysis of the estimated central moments. In Section 6 we describe the methods for testing the forecasting ability of RNDs and present the empirical results. Section 7 examines the relationship between the RNDs' central moments and the risk premium.

2 Review of the literature on estimating RNDs from option prices

The literature has suggested several approaches on how to extract probability density functions from option prices. One class of these methods is based on a given stochastic option pricing model, while the other group relies on the risk-neutral valuation of European-style options. A common assumption of these two methods is that by inverting the pricing process, it is possible to obtain the risk-neutral density function which is consistent with market option prices. This is also called the terminal density function, i.e. the probability distribution related to the expiry date of the options. In this section we briefly review these two approaches.

The estimation techniques in the first group assume particular stochastic processes for the price of the underlying asset, the parameters of which can be estimated from observable option prices. One of the earliest of stochastic models is the Black-Scholes model, which assumes that the underlying asset evolves over time according to a geometric Brownian motion, thus the (logarithmic) changes to the asset price follow a constant trend with constant volatility. The terminal probability distribution of this process is a lognormal density.

Two other models commonly used to recover implied probabilities are the following. One example is the jump-diffusion model of Malz (1996) which allows for the asset price to have a jump over the life of option. Under the assumptions of his model, the terminal probability density function will be a mixture of two lognormal distributions. For more complex stochastic processes, in general, the terminal density function can not be expressed in closed-form formulas, but it can be estimated by numerical methods. Heston's (1993) stochastic volatility model allows for the volatility to also be a stochastic variable, with correlation between the asset price and the volatility. The terminal distribution related to this process can capture skewness and kurtosis different from that of the lognormal density.

The other broad class of estimation techniques takes a more general option pricing model as a starting point, relying on the so-called risk-neutral valuation. As the method we use in the empirical part is within this group, we present these techniques in more detail. According to Cox and Ross (1976), the value of a European-style call option is given as the expected payoff of the option at expiry, discounted by the risk-free interest rate:

$$c(X,r,\tau)=e^{-r\tau}\int_{-\infty}^{\infty}\max(S_T-X,0)f(S_T)dS_T=e^{-r\tau}\int_{X}^{\infty}(S_T-X)f(S_T)dS_T \quad (1)$$

where c represents the call value, X denotes the strike price, r is the risk-free interest rate, τ is the time to maturity, S_T is the price of the underlying asset at maturity, and $f(S_T)$ stands for the so-called risk-neutral density function *(RND)*. Here, the only assumption about the terminal probability density is that its mean (expected value) should be equal to the forward price, in the case of currencies to the forward exchange rate, which is necessary to exclude arbitrage opportunities. However, there are no restrictions about the shape of the distribution.[1]

If we take the second-order derivative of the call option value with respect to the strike price, it will be equal to the probability density function discounted by the risk-free interest rate (Breeden–Litzenberger, 1978):

$$\frac{\partial^2 c(X,r,\tau)}{\partial X^2}=e^{-r\tau}f(X)_{|X=S_T}{}^{2} \quad (2)$$

[1] Assumptions behind risk-neutral pricing are partly similar to that of the Black–Scholes model. Markets should be complete in order to the density function to be unique, and perfect markets are needed for the non-arbitrage condition to hold.

[2] The notation $X=S_T$ means that we take the second derivative by the strike price, but the *f(.)* we arrive at is the density in function of the underlying spot exchange rate.

Thus, given the continuous call price in function of the strike price, the implied probability density function of the underlying asset can be estimated. As only discrete option prices can be observed in the market, i.e. several option prices with the same maturity but with different strike prices, the continuous option price function should be estimated.

Based on the above equations, the call price function can be estimated directly or indirectly. There are three main methods to do so:

- assuming a parametric form for the RND function,
- interpolating call option prices directly,
- interpolating the volatility smile.

The first class of these techniques assumes a particular form for *f(X),* the parameters of which can be recovered by minimising the difference between the estimated and observed option prices. Generally, it is assumed that the density function is the weighted average of some lognormal densities.[3] One advantage of these methods is that these are also applicable when the available data set is limited; for example the estimation of the 2-lognormal method requires only at least 5 cross sectional data. However, the main drawback of this method is its instability; it often reported that 'spikes' are observable in the estimated distribution, which can reflect observation errors (see Cooper 1999).

Another approach is a direct interpolation of option prices; the resulting continuous function can be substituted into equation (2). For example, Bates (1991) fitted a cubic spline to the observed data. As the call option price should be monotonic and convex with respect to the strike price, the pricing function should have a rather complex form, which reduces the degree of freedom of the estimation and makes it data intensive. Another technical drawback is that small observation errors can have large effects on the estimated distribution, especially on the tails.

The third, most widely used group of methods interpolates the call pricing function indirectly, by estimating the volatility smile. By definition, the *volatility smile* is the implied volatility in function of the strike price or the delta[4] of the option, where implied volatilities are calculated by the Black–Scholes model back from observed option prices. The methods in this group differ from each other in the dependent variable used to fit the volatility smile (strike price or delta) and in the functional form of the fitted volatility smile.

One of the earliest of these methods was that of Shimko (1993), who interpolated the smile curve in implied volatility/strike price space using a quadratic function. Later, based on this idea, Malz (1997a) also fitted a quadratic function to implied volatilities but he interpolated across deltas instead of strike prices. The main advantage of these two methods is that it is also possible to estimate densities only from 3 cross sectional option data. Moreover, the Malz-method is said to be preferred because it permits a more flexible shape near the centre of the estimated density (Mandler, 2002). Furthermore, Malz showed that Shimko's technique can violate the non-arbitrage condition for deep out-of-the money options.

With more data available, there are many other methods to estimate. For example, Campa et al. (1998) fitted the volatility smile applying a cubic smoothing spline. The difficulty of all of these methods is related to the extrapolation: how to estimate the part of the probability distributions outside available strike prices. Shimko assumed that the tails of the distribution are from lognormals, Campa et al. let the volatility smile be flat in terms of strike prices beyond available deltas, while Malz allowed the quadratic curve to cover the entire range of deltas.

Because of the limited range of available data we have chosen the Malz (1997a) method to estimate implied density functions. Another reason for this choice was that his method is widely used; there are many central banks and also international institutions which apply it to estimate risk-neutral densities from currency options (e.g. ECB, Fed, Reserve Bank of New Zealand, Deutsche Bundesbank, BIS, etc.).[5]

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[3] For example, Bahra (1997) used the mixture of two lognormal densities for equity indices, or Melick and Thomas (1997) used three lognormal distributions.

[4] The delta of an option measures the sensitivity of option prices to changes in strike price and is also used to measure the moneyness of options, i.e. how far OTM or ITM an option is.

[5] See the related authors of central banks in: Christoffersen and Mazzotta (2004), Malz (1997a), Gereben (2002), Bundesbank (2001).

2.1 ABOUT THE MALZ-METHOD AND OPTION MARKET QUOTING CONVENTIONS

In the following, we provide a brief description about the main building blocks of the Malz (1997a) method and about the relevant quoting conventions of option markets.[6]

On OTC option markets, market makers quote the prices of option combinations, from which it is possible to calculate 3 implied volatilities with different strike prices and use them for the estimation. These commonly traded (or at least quoted) option combinations are: straddle, risk reversal and strangle.

An *ATMF straddle* is an option combination of a European-style call and a put option with the same strike price, equal to the forward exchange rate (see Appendix 1 for the payoff of option combinations). The buyer of the straddle bets on the expected volatility of the exchange rate. The implied volatility of the two options should be the same for the put-call parity to hold (at least theoretically, without the existence of bid/ask spreads). The implied volatility of these options is called *ATMF implied volatility* and we will refer to it in this way in the following.

A *risk reversal* (RR) is an option strategy where an investor simultaneously purchases an out-of-the-money call option and sells an out-of-the-money put option on a given currency. As strike prices are different, implied volatilities can also differ. A positive price of a risk reversal means that the call option has higher implied volatility than the put option; this differential is the *risk reversal spread* (hereafter called as risk reversal). An investor who buys a risk reversal believes that the probability of the depreciation of the underlying currency is greater than the probability of appreciation[7] i.e. if the underlying currency pair is the EUR/HUF, the buyer of a RR is betting on HUF depreciation against the euro. That is, the related probability distribution is skewed toward depreciation.

A *strangle* is a strategy consisting of a simultaneous purchase of an out-of-the-money put and an out-of-the-money call option on the underlying currency. The holder of a strangle believes that there will be large exchange rate movements over the life of the options. The higher the probability of extreme exchange rate movements is perceived to be, the higher the price one should be willing to pay for a strangle. In other words, the price of a strangle provides an indication on the degree of kurtosis in the underlying distribution.

For the estimation it is of great importance to know how these option combinations are quoted on the market. For risk reversals and straddles the most commonly quoted combinations consist of options with a delta of 0.25 and 0.75 (measured as deltas of call options). According to option market traders' vocabulary, for simplicity, these combinations are called *25-delta risk reversal* and *25-delta strangle*. The implied volatility of an ATMF straddle is the ATMF implied volatility. The prices of a 25-delta RR and 25-delta strangle can be expressed in terms of volatility *(σ)* related to different deltas (0.25, 0.5 and 0.75):

$$\begin{aligned} &ATMFvol = \sigma(0.5) \\ &RR = \sigma(0.75) - \sigma(0.25) \\ &Strangle = \frac{(\sigma(0.75) + \sigma(0.25))}{2} - \sigma(0.5) \end{aligned} \qquad (3)$$

From this equation system it is possible to express three points of the volatility smile: $\sigma(0.25)$, $\sigma(0.5)$ and $\sigma(0.75)$. The next step is to fit a quadratic function on these three points. The parameters of this quadratic function can be expressed as it follows:

$$\sigma(\Delta) = ATMFvol - 2 * RR(\Delta - 0.5) + 16 * Strangle(\Delta - 0.5)^2 \qquad (4)$$

[6] A more detailed description and the derivation of the presented formulas can be found in Malz (1997a).

[7] This statement is true only when talking about risk-neutral probabilities. Subjective probabilities (i.e. probabilities related to the subjective density) can be different from risk-neutral probabilities; see the discussion of this in Section 6.

Equation (4) represents the volatility smile in function of the delta. However, what we need is the call price in function of the strike price *(c(X))*. This transformation can be done in two steps. First, we wish to obtain the volatility in function of the strike price *(σ(X))*. The only technical difficulty is that the delta (calculated by Black–Scholes formula) is not only a function of strike price, but also depends on the volatility *(Δ (σ,X))*. Consequently, it is not possible to express the volatility in function of the strike price in a closed-form formula, but it can be done by an iteration method.

The next step is to substitute the estimated *σ(X)* function into the Black–Scholes formula, thus we get *c(X)*.[8] Finally, applying equation (2) we can get the estimated implied probability density function. The second-order derivative of the call price with respect to the strike price can be approximated numerically by the following expression:

$$f(X) \approx e^{r\tau} \frac{\left[c\left(X+\Delta X,\tau\right)-c\left(X,\tau\right)\right]-\left[c\left(X,\tau\right)-c\left(X-\Delta X,\tau\right)\right]}{\Delta X^2}\Bigg|_{X=S_T} \tag{5}$$

The use of small enough step size *(ΔX)* allows for reduction of the error arising from the approximation. For practical purposes, the approximation of a continuous probability density function can also be suitable.

[8] Notice, that even if we use the Black–Scholes formula, we do not rely on the assumptions behind the Black–Scholes model. At first, the Black–Scholes formula is only used to create a linkage between option prices and volatility, as by definition the implied volatility for a given delta is the Black–Scholes implied volatility. Second, it is used for the calculation of deltas, but it is only for simplifying the calculation, taking the derivative of option prices with respect to the strike price, almost the same would result.

3 Description of EUR/HUF options data

For the estimation of RNDs we use quotes on EUR/HUF European-style options from OTC markets. The beginning of the sample period is January 2003 and it ends in June 2007, thus the sample covers 4.5 years. The frequency of the available data is weekly, and the maturity of options is 1 month.

Options data are from three different sources. For the period January 2003–December 2004, ATMF implied volatilities and 25-delta risk reversals are from a London-based market maker. These data were collected by contacting this bank directly. The 25-delta strangle quotes for this period are from a different, public source (UBS). For the period January 2005–June 2007, 3 different points of the volatility smile related to 25, 50 and 75 deltas all data are from the same source (Deutsche Bank).

For the first period (2003–2004) we calculated 25, 50 and 75-delta implied volatilities from ATMF volatility, risk reversal and strangle quotes using equation (3). For the second period (2005–2007) 25, 50 and 75-delta implied volatilities were given in the data source. Then, we fitted volatility smiles using equation (4). Altogether, we get 205 distinct volatility smiles.

The use of diverse data sources (for the period 2003–2004) can cause problems if the quotes of different market markers are far from each other. In the case of ATMF volatility and 25-delta risk reversal the discrepancy between different London-based banks' quotes are relatively small (Csávás and Gereben, 2005). However, this problem can be relevant with respect to strangle quotes. We observed that the strangle time series is characterised by stepwise changes: strangle quotes were unchanged on 90% of the days in the sample (see Appendix 1, Chart 7). This can reflect the relatively low liquidity of these option combinations. Nevertheless, this phenomenon is present at both part of the sample, independently from the data source, suggesting that this is rather a characteristic of strangle quotes and not particular to a given market maker. However, we will take this issue into account in the framework of a sensitivity analysis later.

Another matter related to the data is that the Malz method uses extrapolation outside the deltas related to the 3 observed option prices, where the points of the fitted volatility smile can fall far away from market prices. For example, Cincibuch (2002) found that the Malz method significantly underestimates the 'true' volatility smile, while others observed that the extrapolated volatility smile is very close to actual market quotes for deltas between 15 and 85 (Malz, 1997a). Unfortunately, we did not find any regular data source for EUR/HUF option with many points of the volatility smile which could be considered 'real' market quotes. We did, however, receive data from an option pricing software company (Superderivatives), with 13 points of the volatility smile. It was found that the estimated volatility smile fits quite well to these data; there are discrepancies only in the range between 0 and 15 deltas (see Appendix 1, Chart 8).[9] Consequently, it seems that the use of the Malz method affects only the tails of the estimated RNDs, and not the central moments, which are the main focus of our analysis.

For estimation of the RNDs, data on forward exchange rates are also required. These are calculated by using the official exchange rate of the MNB and money market interest rates for the forint and the euro (1-month BUBOR in the forint market and EURIBOR for the euro area).

3.1 THE VOLATILITY SMILE AND THE RND: AN EXAMPLE

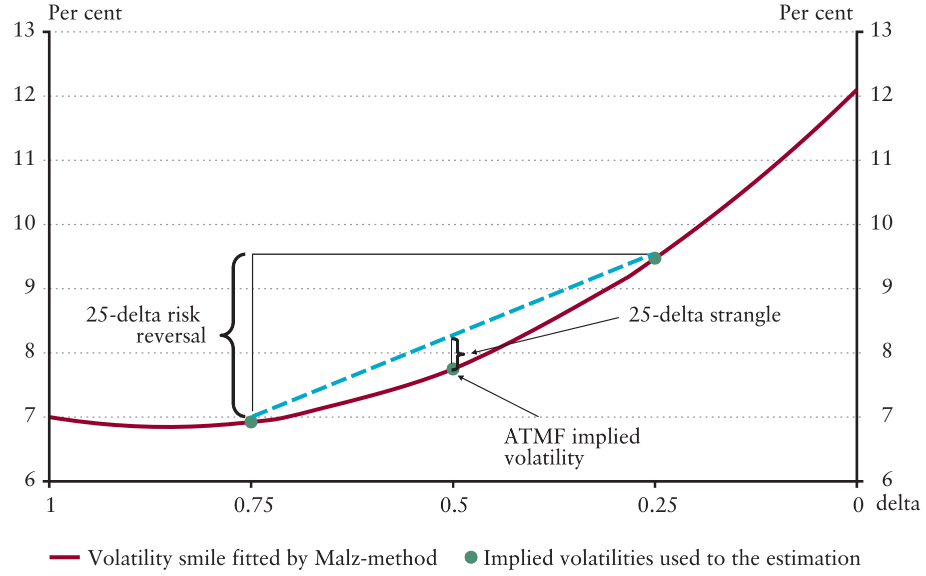
In order to graphically demonstrate the relationship between the input data and the estimated implied RNDs, we have chosen a day when the fitted volatility smile was the closest to the average of the whole sample period. First, let's look at how the prices of option combinations (ATMF straddle, risk reversal and strangle) determine the shape of the volatility smile.

In Chart 1, the ATMF implied volatility is represented by the point related to the 50-delta on the horizontal axis (in the 'middle' of the volatility smile). The positive risk reversal implies that the 25-delta call option has higher volatility than the

[9] Nevertheless, we could not decide if this is evidence for the accuracy of the Malz method or is simply because an extrapolation technique similar to the Malz method is applied in the quoted volatility smile. The reason for the choice not to use this data set for the estimation was the shorter sample period.

Chart 1

Estimated 1-month volatility smile in function of deltas (20 October 2005)



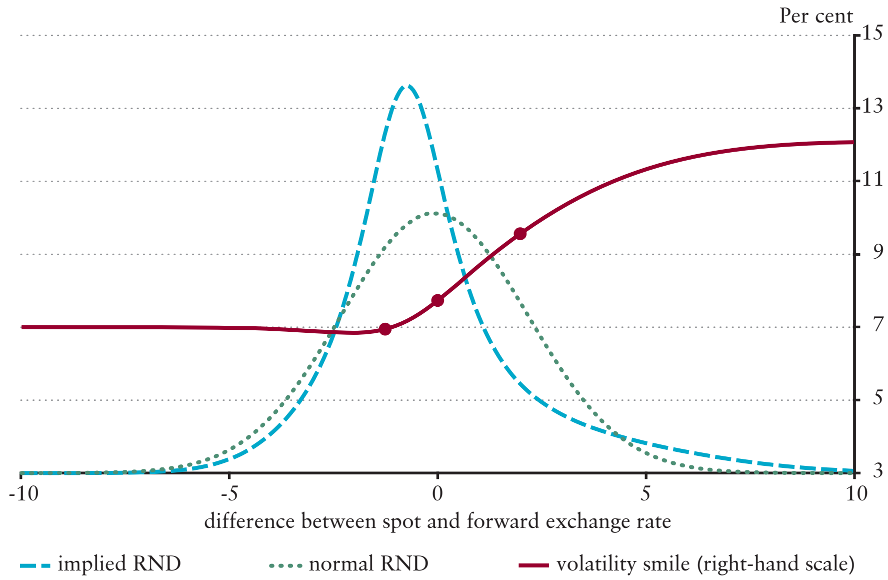
Note: Higher call deltas are related to lower strike prices i.e. where the forint is more appreciated against the euro.

75-delta call, which is reflected in an upward sloping volatility smile between these two points. The positive strangle means that the volatility smile is convex; the average of 25 and 75-delta call option is higher than the ATMF implied volatility (while zero strangle would be consistent with a linear volatility smile).

Based on the fitted, continuous volatility smile, we estimated the related risk-neutral density function using equation (5). In Chart 2, we plotted the estimated density in function of the log difference between the spot exchange rate at maturity and the forward exchange rate. In this way, we can easily compare the estimated density with that assumed by the Black–Scholes model, where the log exchange rate has a normal distribution at maturity.

Chart 2

One-month implied RNDs and the volatility smile (20 October 2005)



Note: Negative values represent appreciation of the forint versus the euro with respect to the forward exchange rate. As we have a histogram instead of a continuous density function, we did not mark the values related to the density functions.

The estimated probability density function and the fitted volatility smile on 20 Oct. 2005 can be seen in Chart 2. As the estimated volatility smile has a quadratic form in function of the delta, if we change from deltas to log exchange rates, the smile will be a transformation of the parabolic curve. For more out-of-the-money options it flattens out unlike the volatility smile in function of the deltas. This is because for more out-of-the-money options the delta is less sensitive to changes to the strike price.

As a consequence of the positive risk reversal, the estimated density function is asymmetric, skewed to the right. This means that the probability of the depreciation of the forint against the euro (exceeding a certain rate) is higher than the appreciation exceeding the same rate, i.e. the right tail is longer. The asymmetry becomes more visible if we compare it with a symmetric density function. We used as a benchmark the implied density function corresponding to the Black–Scholes model, i.e. it was assumed that the log exchange rate follows a normal distribution with a standard deviation equal to the ATMF volatility.

The estimated density has higher kurtosis than the normal one which stems from the fact that the volatility smile is convex. This would imply longer tails at both edges of the distribution, but at the left tail this is not visible since the positive skewness compensates the effect of the higher kurtosis.

4 Comparison of option-based indicators

In this section we present three different types of indicators extracted from option prices which reflect three distinct characteristics of risk-neutral densities: the general level of uncertainty, the asymmetry of the distribution, and the likelihood of extreme movements of the exchange rate. Each characteristic can be described by indicators calculated in three different ways: central moments of the estimated densities; so-called 'shortcut' indicators, which can be calculated from input data and are strongly correlated with the central moments; probability-based indicators (Table 1). The aim of this analysis is threefold. First, we are searching for indicators which can be calculated without the estimation of RNDs, thus apart from saving calculation time, these indicators – or some of them – may be affected less by potential observation errors in the input data. Second, we are seeking indicators which can be expressed in terms of probabilities and are highly correlated with the central moments. An advantage of probability-based indicators with respect to the central moments is that the former are somewhat easier to interpret.[10] Third, we compare the results with those of Lynch and Panigirtzoglou (2003), the only source in the literature where we found a similar comparative analysis.[11] As they analysed RNDs from stock index options and interest rate options, applying a different methodology (spline), our further objective is to check whether we arrive at similar results for currency options data.

The second, third and the fourth central moments of the estimated risk-neutral densities will be used as benchmarks for the comparison (standard deviation, skewness and kurtosis, respectively). These are the most widely used indicators to describe the shape of density functions.

We compare the central moments with shortcut indicators, which can be calculated using 25, 50 and 75-delta implied volatilities. The implied standard deviation will be compared with the ATMF volatility; both indicators are expected to express the uncertainty attached to the distribution. The skewness of the RND can be approximated by the standardised risk reversal, i.e. the 25-delta risk reversal divided by the ATMF implied volatility; this way, both indicators are standardised. With respect to the kurtosis, a straightforward indicator would depend on the strangle. However, we expect that skewness and kurtosis are positively correlated, thus an appropriate shortcut indicator might once again be the standardised risk reversal. The reason behind this assumption is the following. Lynch and Panigirtzoglou (2003) found that the skewness and the kurtosis are closely correlated for stock index options, i.e. higher skewness towards a fall in prices coincides with higher kurtosis. A similarity between stock prices and emerging market exchange rates is that the risk embedded in option prices is generally one sided. As we will see, this is also the case in the EUR/HUF market. Regarding other asset prices, where the skewness changes its sign frequently over the time, the correlation between skewness and kurtosis is supposed to be weak.

If we find that all of the above shortcut indicators show strong co-movement with the corresponding central moments, it means that very similar information content can be extracted from only 1-3 points of the volatility smile, as from the whole density function.

Table 1

Classification of option-based indicators

	General uncertainty	Asymmetry	Likelihood of extreme movements
Central moments of the estimated densities	Standard deviation	Skewness	Kurtosis
Shortcut indicators	ATMF implied volatility	Standardised 25-delta risk reversal	
Probability-based indicators calculated from the estimated densities	Sum of the probabilities of depreciation and appreciation in excess of x%	Difference between the probabilities of depreciation and appreciation in excess of y times the standard deviation	Sum of the probabilities of depreciation and appreciation in excess of z times the standard deviation

[10] For example, a 1% rise in the probability of a more than 5% depreciation of the exchange rate is more telling than i.e. a 1-unit rise in the skewness. However, it should be kept in mind that these are only risk-neutral probabilities.

[11] There are other authors who present probability based indicators, as for example Syrdal (2002), but these were not compared with the central moments.

The third type of indicators show how much the cumulated risk-neutral probability is that the exchange rate will fall in a certain range at maturity. There are many ways to calculate it: based on the level of the exchange rate, or on percentage changes until the maturity; looking at the probability at the tails, or in a certain range in the middle, etc. Thus it is not obvious which kind of probability indicators should correlate strongly with the central moments, but it can be investigated by an empirical analysis.

First, we are seeking for an indicator which measures the sum of the probabilities of depreciation and appreciation in excess of x%. The aim is to find an x parameter which maximises the correlation with the implied standard deviation within the whole sample (it will be called *uncertainty probability indicator)*. A similar indicator for measuring the asymmetry is defined as the difference between the probabilities of depreciation and appreciation in excess of y times the standard deviation (thus it can be considered a standardised indicator, as the skewness). We are looking for a y parameter which maximises the correlation coefficient between the skewness and this indicator (drawing on Lynch and Panigirtzoglou (2003), we will call it the *asymmetry probability indicator,* even though they calculated it using 1 standard deviation). In the case of the probability indicator related to the kurtosis, the only difference from the asymmetry probability indicator is that the sum of the probabilities is calculated, not the difference. Here the parameter is z, which satisfies the strongest co-movement with the kurtosis (we will call it the *extreme movements probability indicator)*. If these probability indicators correlate highly with the central moments, it would imply that from the tails alone it is possible to extract almost the same information as from the whole density function.

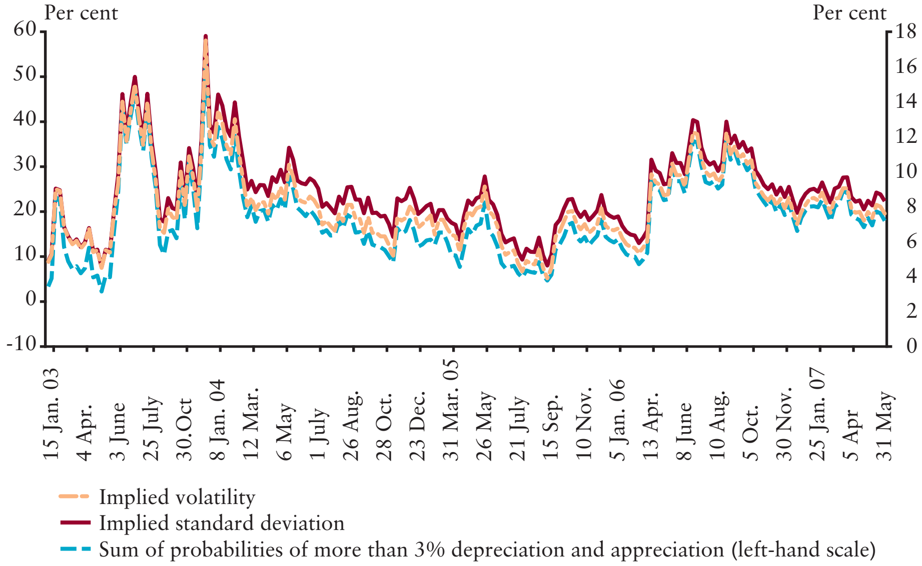
4.1 UNCERTAINTY INDICATORS

The 1-month estimated standard deviation and the ATMF implied volatility were very closely related in the sample period (Chart 3). The correlation between the levels of these two variables is almost perfect, with a correlation coefficient of 0.99 (see the correlation matrix of option implied indicators in Appendix 1, Table 7).

However, the estimated standard deviation is almost always higher than implied volatility, as was also found by Lynch and Panigirtzoglou (2003). The reason behind this is that the volatility smile is not horizontal; if it was, the two indicators would be the same. The implied standard deviation can be considered as a weighted average of the volatility smile. Given that in the entire sample period the strangle was positive, the average of the 75-delta and 25-delta call is higher than the ATMF implied volatility. The same is the true for other points of the volatility smile, thus their average also should be higher than the ATMF implied volatility. The difference between the standard deviation of the estimated distribution and the ATMF implied

Chart 3

One-month estimated standard deviation, ATMF implied volatility and the uncertainty probability indicator



Note: Standard deviation is calculated from the RND related to the log exchange rate, expressed in percentage, and annualised as the ATMF volatility.

volatility fluctuated between 0.5 and 1% which can be considered relatively low. Thus, based on this strong association, the ATMF volatility is a good shortcut indicator of approximating the second central moment of the estimated RNDs.

Within the group of uncertainty probability indicators we found that one which follows the standard deviation very closely is defined this way: the sum of the probabilities of depreciation and appreciation in excess of 3%. The correlation coefficient with the standard deviation was 0.99.[12] It is worth mentioning that 3% depreciation or appreciation in one month's time cannot be considered too high, in the sense that the estimated (not annualised) average standard deviation of the RNDs was about 2.5% in the sample period.

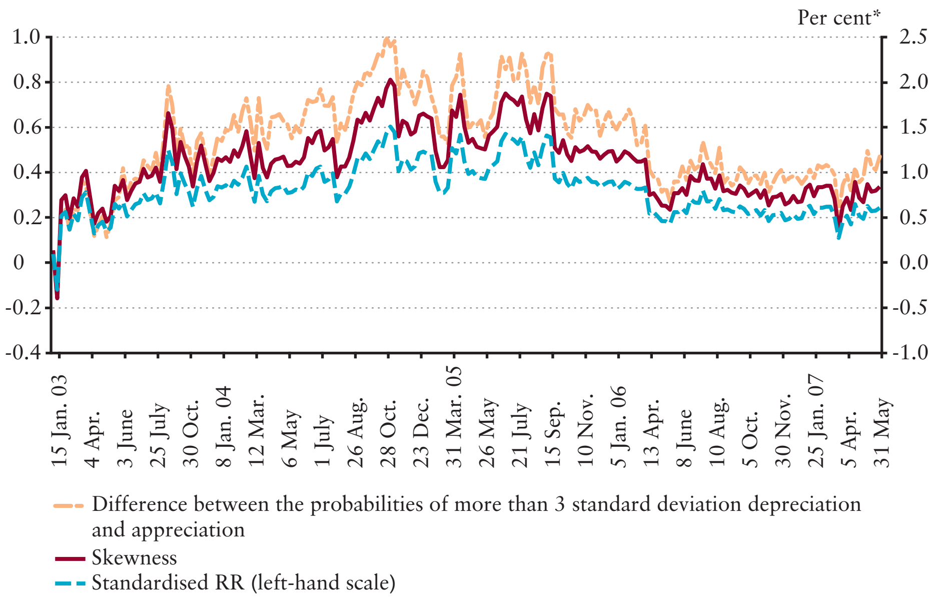
4.2 ASYMMETRY INDICATORS

We compared the skewness of the estimated densities with the 25-delta standardised risk reversal. With respect to the level of these two indicators it can be observed that both indicators reflected a 'negative' skewness only once in the sample period, i.e. a skew toward forint appreciation (Chart 4). On 15 Jan. 2003 the exchange rate of the forint reached the strong edge of the intervention band of the MNB as market participants speculated on further appreciation of the forint. Apart from this, the risk reversal was positive throughout the whole sample implying a distribution skewed to the right, towards depreciation.

We found that standardised risk reversal have almost perfect (0.999) correlation with the estimated skewness. This result is in line with that of Lynch and Panigirtzoglou (2003), as they also found a correlation coefficient higher than 0.99.[13] This confirms that the standardised risk reversal is an appropriate shortcut indicator of the skewness, which is available without the estimation of the RND. A possible reason for the high correlation could be that RNDs were estimated from only 3 points of the volatility smile, thus even if the skewness captures the information from the whole density, it is not possible to extract more information from it than contained by the input data. However, as Lynch and Panigirtzoglou (2003) used many points of the volatility smile, we suppose that the found relationship is not particular to the estimation method of RNDs.

Chart 4

One-month estimated skewness, 25-delta standardised risk reversal and the asymmetry probability indicator



* *Skewness is calculated from the RND related to the log exchange rate. As skewness is the ratio of two numbers expressed in percentages, it is measured in units. The probability asymmetry indicator is expressed in percentage.*

[12] The x parameter which maximised the correlation was 3.4, for simplicity we rounded it to 3.

[13] Despite the strong relationship between the skewness of the RND and the standardised risk reversal, many authors in the literature use the (non-standardised) risk reversal as an asymmetry indicator. (See e.g. the detailed analysis of risk reversals by Dunis and Lequeux, 2001.) One potential reason behind this is that the sign of non-standardised risk reversal and the skewness are generally the same. However, the dynamics of non-standardised risk reversal and the skewness can be very different if ATMF volatility is fluctuating much, as it happened in the Hungarian forint market.

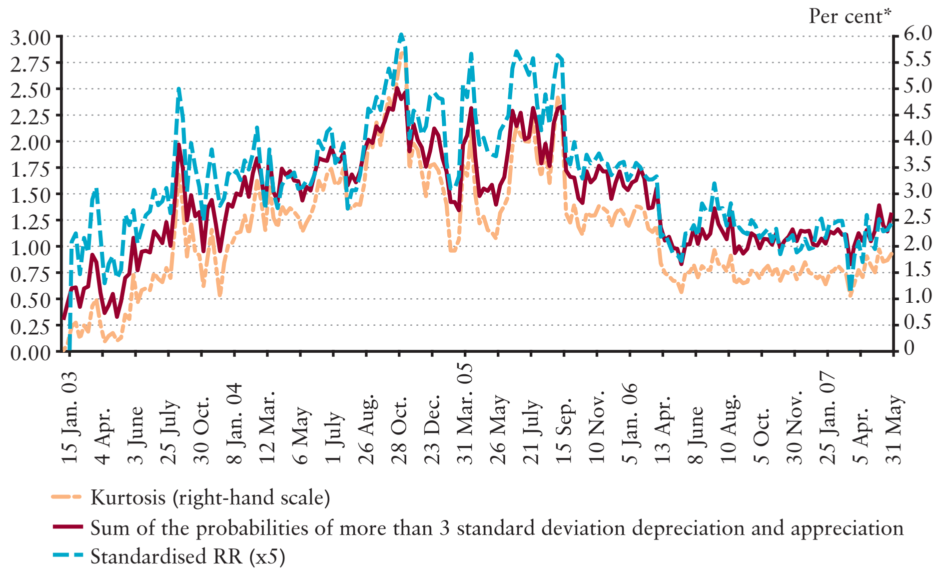
Amongst the asymmetry probability indicators which most closely correlated with the skewness was the following: the difference between the probabilities of depreciation and appreciation by more than 3 times the standard deviation. The correlation coefficient between these two indicators was about 0.96.[14] This result differs from that of Lynch and Panigirtzoglou (2003) in the sense that they found high correlation for the probability indicator based on 1 standard deviation. For us, calculating the asymmetry probability indicator with 1 standard deviation, the correlation was only about 0.7. The difference may stem from the use of different maturity, as the cited authors estimated 3-month RNDs. Moreover, choosing 0.5 times the standard deviation, the correlation coefficient became negative. We can conclude that the definition of the probability-based indicator affects the strength of the correlation with the skewness. However, the strong association between the skewness and the asymmetry probability indicator suggest that this is a general characteristic of RNDs, and does not depend on the underlying asset or the applied method.

4.3 INDICATORS OF EXTREME MOVEMENTS

Prior to the comparison of extreme movement indicators, we calculated the correlation coefficient between the estimated skewness and kurtosis; it was rather high, at 0.94. Based on this relationship, the standardised risk reversal can be a shortcut indicator not only for the skewness, but also for the kurtosis. As expected, the correlation coefficient between the standardised risk reversal and the kurtosis is also high (0.93). The high co-movement between the skewness and the kurtosis can be interpreted in such a manner that a rise in the probability of huge depreciation is associated with a rise in both tails of the RND.

Chart 5

One-month estimated kurtosis, 25-delta standardised risk reversal and the extreme movements probability indicator



* *Kurtosis is calculated from the RND related to log exchange rate, measured in units and deducted 3 (the kurtosis of the normal distribution). The extreme movements probability indicator is expressed in percentage.*

Earlier it was mentioned that a possible explanation for the high correlation between the skewness and the kurtosis is that the skewness is one sided (almost always positive in our sample). This seems to be confirmed by the fact that the strong relation failed on the day when the skewness changed its sign. Another explanation can stem from the data quality: since the variability of strangle quotes is low, the strangle is not able to exert a strong effect on the kurtosis. However, the findings of Lynch and Panigirtzoglou (2003) do not support this hypothesis because they examined more developed underlying markets (S&P 500, FTSE 100), where data problems may be less relevant.

[14] The y parameter which maximised the correlation was 2.6, for simplicity we rounded it to 3. The correlation was measured in the sample excluding the day when the skewness was negative.

Within the group of extreme movements probability indicators, the most closely correlated with the kurtosis was the sum of the probabilities of depreciation and appreciation by more than 3 times the standard deviation (with a correlation coefficient of 0.98). This result again differs slightly from that of Lynch and Panigirtzoglou (2003) as they found high correlation for the probability indicator based on 1 standard deviation. In our case, choosing 1 standard deviation yielded a negative correlation. This also highlights the importance of how to define the probability-based indicators.

The main conclusions from the comparison of different indicators are the followings:

• For measuring the standard deviation of RNDs, an appropriate shortcut indicator is the ATMF implied volatility.

• For measuring the asymmetry of RNDs, the standardised risk reversal can serve as a shortcut indicator.

• In the case of EUR/HUF estimated RNDs, the kurtosis and the skewness are strongly correlated.

• For all three central moments we found probability-based indicators, with which these are highly correlated. The exact definition of these indicators affects the strength of the correlation with the central moments.

5 Sensitivity of estimated central moments to observation errors

In this section we compute the sensitivity of estimated densities to possible observation errors in the 3 'input data' which were used in the estimation. We calculate *ceteris paribus* changes of higher moments of the RNDs in response to changes in ATMF volatility, 25-delta risk reversal and strangle. For the analysis we have chosen the day when the estimated volatility smile was the closest to the average of the sample period (20 October 2005).

Since we compare the sensitivity of central moments with each other, the size of initial changes in input data should be equivalent by some measure. In the case of ATMF volatility and RR, the changes are set to be equal to the half of bid/ask spread, as the spread can be considered a convenient measure of observation errors. The bid/ask spread of ATMF volatility was about 0.5-1% in the whole sample period, while in the case of risk reversal the spread fluctuated between 0.3 and 0.5% (These are indicative spreads, contained by our database). Taking the higher figures, we calculated with ±0.5% change in the ATMF volatility and ±0.25% change in RR. However, we have no information about the bid/ask spread of strangles. In this case we can rely on the fact that strangle quotes moved in a stepwise manner, fluctuating between 0 and 0.7%. This suggests that the observation error may be the highest in this option combination, and we suppose that its true value may fall anywhere between the maximum and minimum. Consequently, we calculated with ±0.35% change in strangle.

Changes in the ATMF volatility have almost one-to-one effect on the estimated standard deviation, while RR has a practically negligible effect on it (see the 1st row of Table 2). *Ceteris paribus* changes to the strangle have a relatively high effect on standard deviation, with a bit higher than a one-to-one effect.

With respect to the skewness, it can be observed that with constant risk reversal, a rise in the ATMF volatility reduces the skewness (2nd row of Table 2). On the other hand, with unchanged volatility, higher RR results in higher skewness. The effect of these two factors is roughly similar in magnitude; there is difference only in the sign. Importantly, it can be observed that the strangle exerts a negligible effect on the skewness, thus the latter is not affected by the possibly huge observation error in the price of this option combination.

Regarding the kurtosis we can make similar remarks, *ceteris paribus* higher ATMF volatility leads to lower kurtosis (3rd row of Table 2). Interestingly, RR has also a significant effect on the kurtosis, but in the other direction. Again, the absolute effect of these two factors is similar. Movements in the strangle cause relatively high changes in the kurtosis, with a positive sign. With respect to the above mentioned effects it can also be observed that central moments are affected the most sensitively by the input data to which these are related also intuitively (standard deviation vs. ATMF volatility; skewness vs. RR; kurtosis vs. strangle; see the diagonal of the Table).

In order to assess the importance of the sensitivity of the estimated indicators we analyse the aggregate changes of central moments to changes in the input data, i.e. we take into account that each of these can suffer from observation errors. We

Table 2

Sensitivity of the implied density function of 20 October 2005

	Original value	Effect of changes to input data			Aggregate effect of changes to input data
		ATMF (±0.5%)	RR(±0.25%)	Strangle (±0.35%)	
Standard deviation	8.44	+0.49/-0.49	+0.02/-0.01	+0.48/-0.46	+0.99/-0.95
Skewness	1.13	-0.07/0.08	+0.10/-0.11	+0.01/-0.04	+0.19/-0.21
Kurtosis	2.22	-0.18/+0.25	+0.24/-0.18	+0.90/-0.85	+1.39/-1.20

Note: On the left hand side of the cells there are responses to rises in ATMF volatility, RR and strangle; on the right hand side there are responses to reductions in these.

compare the changes in each central moment with their original value and also with their standard deviation in the whole sample (see descriptive statistics in the Appendix, Table 8.)

Changes to all of the 3 input data cause about 1 percentage point change in the level of the standard deviation. In relative terms, compared with its original value (8.44%), it means more than a 10% change. Even though this can be considered a bit high, we will see that this is the lowest relative change in comparison with the other central moments. On the other hand, this 1% is relatively low compared to the sample standard deviation of the implied standard deviation (about 2.4%).

The aggregate sensitivity of the skewness is about 0.2 unit, which in relative terms implies an almost 20% change. However, this is equal to about the half of the standard deviation of the skewness (0.38), roughly similar to what was seen in the case of the implied standard deviation.

Lastly, the sensitivity of the kurtosis can be considered extremely high by both measures. The approximately 1.2-1.4 unit change means more than 50% in relative terms, and it is in the same magnitude as the sample standard deviation of the kurtosis (1.2). This confirms that the kurtosis is the most sensitive to possible observation errors, affected mostly by changes to the strangle.

The main conclusions from the sensitivity analysis are the followings:

- The sensitivity of the estimated standard deviation and skewness to observation errors in input data can be considered low, albeit not negligible.

- The estimated kurtosis is the most sensitive central moment to possible observation errors.

6 Forecasting ability of RNDs

From a central bank's point of view, the estimation of currency option implied RNDs is important because these are related to market expectations about the future exchange rate. However, it is widely stated in the literature that RNDs cannot be interpreted straightforwardly as the 'true' expectations of market participants on the future probability distribution *(subjective density function)* (see e.g. Merton, 1971). The equivalence between the two densities would hold only if market participants were risk-neutral. The aim of this section is to analyse whether risk-neutral densities coincide with subjective densities.

Our null hypothesis – in economic terms – is that RNDs are identical with the subjective market expectations on the density of the exchange rate. Provided that investors are rational, subjective densities should correspond, over the average of a long time period, to the distribution of the future realization of the exchange rate. (These latter are sometimes also called physical densities, from which the realizations are drawn; for simplicity, we will denote it also as the *historical density* of the exchange rate.) Thus, subjective densities are expected to have forecasting power with respect to the historical density. Consequently, if risk-neutral and subjective densities are identical, risk-neutral densities are also expected to have accurate forecasting ability. In econometric terms, this latter statement is what will be tested, i.e. RNDs having forecasting power with respect to the realised exchange rate (more details are presented in the next subsection).

Many authors found that risk-neutral densities do not provide accurate forecasts (see, among others, Christoffersen and Mazzotta, 2004 or Castrén, 2005). We expect the same for the Hungarian forint. In the presence of a non-zero and time varying risk premium, the mean of the estimated RNDs can be different from the mean of the subjective density (this difference is the risk premium). If this is so, then the whole density function would have low forecasting power. However, Rubinstein (1994) stated that under certain conditions, even if there is a difference in the mean of risk-neutral and subjective densities, the shape of the densities are broadly the same. We test this hypothesis by analysing whether the higher moments of the RNDs are responsible for the poor forecasting ability.

In this section, first, we give a description of the methods of testing the forecasting ability of density functions. In the remaining part we present the results of the tests and provide some interpretations. Finally, we perform a robustness check.

6.1 METHODS FOR TESTING DENSITY FORECASTS

For testing the accuracy of density forecasts we have chosen a recent technique proposed by Berkowitz (2001). It is based on the analysis of the so-called *probability transform variable,* which is defined below:

$$z_{t,h} \equiv \int_{-\infty}^{S_{t+h}} f_{t,h}(u)\,du \tag{6}$$

where h is the forecast horizon (1 month), t is the time of forecasting, S_{t+h} is the realised value of the exchange rate, and $f_{t,h}(.)$ is the RND used to forecast. As shown by Diebold et al. (1998), amongst others, if the historical and the $f_{t,h}(.)$ densities coincide, the probability transform variable is an identically and independently distributed variable (iid) with uniform distribution on the interval (0,1). The equivalence of the two densities can be interpreted as the RNDs having accurate forecasting ability.

Then, the probability integral transform variable can be transformed using the inverse of the standard normal cumulative density function:

$$y_{t,h} = N^{-1}(z_{t,h}) \tag{7}$$

This new variable is called *normal transform variable,* for simplicity, we will refer to it in following as the $y_{t,h}$ *variable*. Under the null hypothesis of the equality of the historical density and the density used to forecast, $y_{t,h}$ should follow an *iid standard normal distribution.*

We will use a normality test for $y_{t,h}$ which is based on some moment conditions.[15] We have chosen a GMM (generalised method of moments) estimation method proposed by Christoffersen and Mazzotta (2004). The motivation behind this choice was that in the only paper where we found tests on EUR/HUF density forecast, the same method was applied (Castrén, 2005).[16] Accordingly, this gives us the opportunity to compare his results with ours.

The idea of testing the null by GMM is simple: the standard normality of the variable in question can be tested by the estimation of the first four moments of its distribution. Christoffersen and Mazzotta (2004) estimated by GMM the parameters of the following equation system:

Method 1

$$\begin{aligned} y_{t,h} - c(1) &= 0 + \varepsilon_{t,h}^{(1)} \\ y_{t,h}^2 - c(2)^2 &= 0 + \varepsilon_{t,h}^{(2)} \\ y_{t,h}^3 - c(3) &= 0 + \varepsilon_{t,h}^{(3)} \\ y_{t,h}^4 - c(4) &= 0 + \varepsilon_{t,h}^{(4)} \end{aligned} \tag{8}$$

where equations represent the first four moment conditions, and *c(i)* denotes the i-th central moment of $y_{t,h}$. Under the null that $y_{t,h}$ is a standard normal variable, the following should hold: *c(1)=0; c(2)=1; c(3)=0; c(4)=3*. Notice that here we estimate 4 parameters using 4 conditions (and instruments will be not used), thus this is an exactly-identified system. Consequently, the parameter estimates are the same as estimating the 4 equations separately by OLS, as a specific case of GMM. The advantage of estimating in a system is that the joint significance of the parameters also can be tested.

The above method assumes at the estimation of the i-th moment that lower moments are equal to that of the standard normal density, which is not necessarily true. Consequently, only the joint significance of the parameters can be interpreted with respect to the forecasting ability. However, we also wish to know which central moments are responsible for the possible rejection of the forecasting ability. To correct for this, we also estimate the following equation system:

Method 2

$$\begin{aligned} y_{t,h} - c(1) &= 0 + \varepsilon_{t,h}^{(5)} \\ \left(y_{t,h} - c(1)\right)^2 - c(2)^2 &= 0 + \varepsilon_{t,h}^{(6)} \\ \left(y_{t,h} - c(1)\right)^3 / c(2)^3 - c(3) &= 0 + \varepsilon_{t,h}^{(7)} \\ \left(y_{t,h} - c(1)\right)^4 / c(2)^4 - c(4) &= 0 + \varepsilon_{t,h}^{(8)} \end{aligned} \tag{9}$$

where equations again represent the first four moment conditions, and *c(i)* again denotes the i-th central moment of $y_{t,h}$. The null hypothesis is the same: *c(1)=0; c(2)=1; c(3)=0; c(4)=3*. The estimated values of *c(2), c(3)* and *c(4)* can be different from that of estimated by Method 1.

At both methods, the weighting matrix of the GMM will be estimated as the inverse of the HAC covariance matrix, in order to address the potential serial correlation in $y_{t,h}$.

6.2 RESULTS OF DENSITY FORECASTING

In this subsection, we first present the results of the forecasting ability of the RNDs estimated from option prices, tested by Methods 1 and 2. We present possible interpretations of the estimated parameters. The result will also be compared with similar tests in the empirical literature.

[15] The independence, in general, is not tested in practice, because if the null hypothesis about the central moments can be rejected, it is enough to reject the forecasting power.

[16] Other motivations are the well-known advantages of the GMM estimator, i.e. it also gives a robust estimator when the error term is non-normal (unlike the maximum likelihood), the serial correlation of the error term can also be taken into account, and parameter uncertainty also can be handled well (Bontemps and Meddahi, 2005).

Table 3

Results of the option-implied density forecasts (Sample: June 2003–June 2007)

	Method 1 (equation system 8)	Method 2 (equation system 9)
Mean (c(1))	-0.49 (0.632)	-0.49 (0.632)
Standard deviation (c(2))	0.836 (0.001)	0.898 (0.016)
Skewness (c(3))	-0.385 (0.078)	-0.283 (0.022)
Kurtosis (c(4))	1.183 (0.000)	2.140 (0.000)
Wald test	407.104 (0.000)	126.226 (0.000)

Estimation method: GMM – Time series (HAC). Kernel: Bartlett; Bandwidth: Fixed (3); No prewhitening. Instruments were not used. In parenthesis, p-values are denoted. Null hypothesis: $c(1)=0$; $c(2)=1$; $c(3)=0$; $c(4)=3$.

All tests are based on the $y_{t,h}$ variable defined in equations (6) and (7). The density functions whose forecasting ability will be tested are the 1-month risk-neutral density functions estimated as shown in Section 3 (hereafter called as *option-implied densities)*. Using the realised values of the EUR/HUF exchange rate in one month's time we calculated the probability transform variable defined by equation (6), then transformed it by equation (7). That is, we simply calculated the value of the cumulative density function related to the realised exchange rate. The sample period is June 2003–June 2007, the number of observations is 187, with weekly observations.[17]

The results of the forecasting ability of option-implied RNDs tested by estimation Methods 1 and 2 are reported in Table 3. First, it can be observed that although the two estimation methods are different, the results are broadly the same. The estimated mean is the same, as it is simply the sample average of $y_{t,h}$. The higher moments differ slightly. For simplicity's sake, we will address the results only from Method 2 in the following.

The mean of $y_{t,h}$ is not significantly different from zero. However, all higher moments of $y_{t,h}$ differ from that of a standard normal variable: the standard deviation is lower than 1, the skewness is negative and the kurtosis is lower than 3. Obviously, this is reflected in the Wald statistic as well: it can be rejected that the $y_{t,h}$ variable would follow a standard normal variable. Consequently, we can reject that the option-implied RNDs would provide accurate forecasts for the realised distribution of the EUR/HUF exchange rate.

In the following, we provide some interpretations related to the estimated values of the central moments of $y_{t,h}$. The zero mean implies that on average, the exchange rate was not different from the mean of the option implied densities, i.e. the forward exchange rate. In our sample there was a slight appreciation trend in the forint exchange rate (in the sense that at the end of the sample the EUR/HUF exchange rate was lower than at the beginning of the sample), while the forward rate was more depreciated than the spot exchange rate (see Chart 9 in Appendix 1). This can explain the slightly negative (but not significant) mean of $y_{t,h}$.

The reason for the lower-than-one standard deviation of $y_{t,h}$ is as it follows. The 1-month-ahead spot exchange rate, on average, fell more frequently in the 'middle' of the option-implied RNDs than would have been consistent with the standard deviation of the RNDs, and only very rarely at the tails. In other words, the standard deviation of option-implied RNDs was higher than that of historical density in the examined period. The explanation for the negative skewness of $y_{t,h}$ is similar. The exchange rate fell more frequently to the left tail than would have been required by the skewness of the option-implied densities. It reflects that the skewness of the historical densities is lower that that of the RNDs, i.e. RNDs are more asymmetric. Similar statements can be made about the kurtosis: the fourth central moment of the RNDs is higher that that of the historical density.

The main question of our analyses is the interpretation of risk-neutral densities in situations where the subjective densities are what we are really interested in. It is widely stated in the literature that the evidence of rejecting the forecasting power of risk-

[17] It should be mentioned that the sample here is shorter than in Section 3. The reason for excluding the period January 2003-June 2003 was that we will rely on the rationality assumption at the interpretation, while in this period two enormous shocks affected the exchange rate. With respect to these two events it would be less plausible to assume that the exchange rate in one month's time was equal to the expected exchange rate.

neutral densities suggests that risk-neutral and subjective densities differ (Bliss and Panigirtzoglou, 2004). Rejecting at least one of the necessary conditions for $y_{t,h}$ having an iid standard normal distribution, we can conclude that risk-neutral and subjective densities are different. This conclusion is based on the rationality assumption, which implies that the subjective and the historical densities coincide over the average of a long time period.

We can interpret the results under the rationality assumption since our sample is rather long (about 4 years). Based on the estimated parameters of $y_{t,h}$, the results can be summarised as follows: option-implied RNDs have the same mean as the subjective densities, but higher standard deviation, skewness and kurtosis (higher skewness in the sense that more skewed toward forint depreciation).[18]

A possible interpretation of the difference between risk-neutral and subjective standard deviation is that the risk-neutral standard deviation contains a volatility risk premium with respect to the standard deviation expected by market participants (see some arguments about why the risk-neutral volatility can contain risk premium in Breuer, 2003). This could be related to the activity of market makers in the option market. It is reported for developed currencies that currency option market maker banks have short option positions, i.e. they generally write options instead of buying them (Csávás and Gereben, 2005). Although for EUR/HUF options markets we have no data on this, it is also reasonable that market makers are in short position since customers have to place collateral when they sell options to market makers, while customers just have to pay the option premium when they buy the options. Accordingly, for a market maker the rise in the volatility is what can cause losses if they cannot hedge the risk perfectly, thus they require risk premium for being at the short side of the options.

It is also worth mentioning that given the estimated value of the standard deviation of $y_{t,h}$ (0.9), the supposed difference between the risk-neutral standard deviation and the standard deviation of the subjective densities cannot be considered too high. This can be illustrated by a comparison between the average standard deviation of option-implied RNDs in the sample and the historical standard deviation estimated from 1-month exchange rate changes. The implied standard deviation was about 8.8%, while the historical standard deviation was 7.8% (both in annualised measures). In economic terms this difference is relatively low; however, it is high enough to reject the equivalence of risk-neutral and historical densities.

In the case of the skewness, similar interpretation can be provided as for the standard deviation. According to anecdotal information, when there is substantial trade in risk reversal option combinations, the customers in option markets generally 'buy' risk reversal, i.e. they bet on the rise of the skewness. Demand for this option combination can raise the level of risk-neutral skewness with respect to the skewness expected by the market.

Remember that the mean of $y_{t,h}$ and was not significantly different from zero. However, it does not necessarily mean that investors are risk-neutral in the sense that the expected exchange rate is always equal to the forward rate. It is also possible that the risk premium is changing its sign over the time in the 1-month horizon.

Finally, we compare our results with the findings of the empirical literature where similar methods were used and/or where the subject of the study was the EUR/HUF option market.

Christoffersen and Mazzotta (2004) and Castrén (2005) performed density forecast evaluation tests based on the estimation Method 1 (for RNDs estimated from developed and East European currency options, respectively). They found that for most currency pairs the standard deviation of $y_{t,h}$ was significantly lower than 1. Thus for these currencies there is also evidence that option-implied RNDs have higher standard deviation than the historical density. Interestingly, the estimated standard deviation of $y_{t,h}$ was lower than in our case, ranging between 0.4 and 0.8 (when it was significantly lower than 1). They also found that for many currency pairs, the standard deviation of $y_{t,h}$ was different from zero despite having a zero mean.

Castrén (2005) tested the forecasting ability of 1-month EUR/HUF option-implied densities as well. With his tests it was not possible to reject the hypothesis that option-implied densities have the same moments individually as the historical density. This stands in clear contrast to our results. Moreover, he found that the HUF was the only East European currency where it

[18] In the case of the standard deviation, for example, the interpretation of the rationality assumption is the following. The standard deviation of RNDs is higher than that of the historical density. Supposing the equality of risk-neutral and subjective densities, it is against the rationality, because it suggests that market participants do not diminish the standard deviation of their subjective densities with the time going, with which they would produce better forecasts. Thus, assuming rationality, the standard deviation of risk-neutral and subjective densities should be different.

Table 4

Results of the option-implied density forecasts (Sample: January 2003–December 2003)

	Method 1 (equation system 8)
Mean (c(1))	0.081 (0.775)
Standard deviation (c(2))	1.179 (0.502)
Skewness (c(3))	2.489 (0.321)
Kurtosis (c(4))	0.112 (0.118)
Wald test	26.391 (0.000)

Estimation method: GMM – Time series (HAC). Kernel: Bartlett; Bandwidth: Fixed (3); No prewhitening. Instruments were not used. In parenthesis, p-values are denoted. Null hypothesis: c(1)=0; c(2)=1; c(3)=0; c(4)=3.

was not possible to reject the forecasting power of RNDs. In order to make comparable our results with that of Castrén (2005), we performed the same test (based on Method 1) for a very similar sample period. He analysed the period between Sept. 2002 and Dec. 2003. As we have options data only from 2003, we have chosen the period Jan. 2003–Dec. 2003. The mentioned author used daily frequency, we have only weekly data; the data source is also different, while the estimation method for RNDs is the same: the Malz method.

We found that for this shorter period none of the central moments of $y_{t,h}$ were significantly different from that of the standard normal variable, as it was reported by Castrén (2005). However, there the joint significance of the parameters was not reported, while we found that the four parameters are jointly significant thus the forecasting ability can be rejected. Based on this, we suggest that our results are different mostly because of having the possibility of using a much larger sample.

Our results also can be compared with those obtained by Gereben and Pintér (2005). They examined the forecasting ability of EUR/HUF implied volatility with respect to the realised volatility. For 1-month horizon they found that the implied volatility overestimated the historical volatility, as in our case the implied standard deviation was higher than the historical standard deviation. Even though the applied method was different and the sample period was also shorter, our results are in line with the aforementioned authors.

To summarise our findings, the forecasting ability of option-implied risk-neutral density functions can be rejected. Our results suggest that risk-neutral standard deviation, skewness and kurtosis are significantly different from that of the subjective densities (supposing that the rationality assumption holds). All in all, these examinations emphasise that using the risk-neutral densities instead of subjective densities can be misleading because in general, the shape of the two densities is different. We will study in the next section whether, apart from these findings, changes in risk-neutral central moments are or are not connected to changes in the central moments of subjective densities.

6.3 ROBUSTNESS CHECK

In this subsection, we show how robust the result of option-implied RNDs' forecasting power is to some modifications in the specification of Method 2.

High positive values of $y_{t,h}$ related to huge depreciations of the forint can be considered as outliers. Excluding the 5 highest outliers the value of the standard deviation and the kurtosis became lower, while the skewness lost its significance.

In order to correct for the autocorrelation of $y_{t,h}$, HAC standard errors were used in the original specification. Another way to get rid of the serial correlation is the use of non-overlapping observation. We split the whole sample in sub-samples in a way that only each 4th observation was taken into account. The main conclusion did not change, only the significance of some of the parameters became lower.

We performed the test on sub-samples defined in another way. The whole sample was divided into 4 equally-sized parts, each of them covering about one year. The standard deviation and the kurtosis remained almost the same, only loosing the significance in some of the sub-samples. However, in two of the sub-samples the skewness became positive. Thus, the estimated skewness is rather sensitive to the examined period.

Since the parameter estimates of GMM can be sensitive to the choice of the weighting matrix, we run the tests with two alternative specifications. The prewhitening of the central moments prior to the estimation yielded roughly to the same results. The main difference was that the skewness of $y_{t,h}$ lost its significance, while other parameters remained roughly unchanged. The weighting matrix was also estimated by the use of instruments. We chose the 4-period lagged of $y_{t,h}$, because of the overlapping nature of the data. The results changed only with respect to the skewness, which again lost its significance.

So far, we tested the null hypothesis that $y_{t,h}$ follows a standard normal distribution only by the shown GMM-based methods. In order to see how a different method performs, we also tested this hypothesis by a simple linear regression method, proposed by Berkowitz (2001), where the mean and the standard deviation of $y_{t,h}$ are estimated by maximum likelihood method (see in the Appendix 2 in more details). The result was again that with these tests it is possible to reject the null that the option-implied densities would provide accurate forecasts for the historical density.

Based on the above test we can conclude that the results are rather robust with respect to the standard deviation and the kurtosis, while the interpretation given to the skewness should be treated with caution.

7 The relationship between RNDs' central moments and the risk premium

The aim of this section is to test indirectly whether *changes in the higher moments* (standard deviation and skewness) of RNDs coincide with those of the subjective densities. We consider this analysis important because the above results indicate that higher moments of RNDs are different in levels from those of subjective densities. We will test whether changes in higher moments of RNDs are able to explain *changes in the risk premium.*

If investors are risk-averse, they need to be compensated for investing in risky assets. Thus the risk premium is affected by their subjective expectations, which can be described by the higher central moments of the subjective densities. Not only the standard deviation, but also the skewness can affect the risk premium.[19] Thus the null hypothesis in *economic terms* is the following: risk-neutral central moments are good proxies for the moments of subjective densities in explaining the development of the risk premium. In *econometric terms,* it will be tested if the estimated risk-neutral standard deviation and skewness have a significant effect on the estimated risk premium during the sample period (see more details below).

In this section, we first describe the methods of testing the relationship between the RNDs' central moments and the risk premium. In the last subsection we present the results of the tests and provide some interpretations.

7.1 METHODS FOR TESTING THE RELATIONSHIP BETWEEN THE RNDS' CENTRAL MOMENTS AND THE RISK PREMIUM

Based on the empirical literature (Malz, 1997b; Gereben, 2002), the following equation can be tested in order to study whether the higher moments of the RNDs are able to explain the time series dynamic of the risk premium:

$$F_t^{t+h} - E_t\left(S_{t+h}\right) = \alpha + \beta \cdot STDEV_t + \gamma \cdot SKEW_t + u_t \tag{10}$$

where h is the forecast horizon (1 month), t is the time of forecasting, F_t is the forward, S_{t+h} is the spot exchange rate in 1-month time and $E_t(S_{t+h})$ refers to the expectations at time t on the spot exchange rate in 1-month time. $STDEV_t$ and $SKEW_t$ refer to the 1-month estimated risk-neutral standard deviation and skewness coefficient.[20]

On the left hand side, there is the risk premium, which is by definition the difference between the forward and the expected exchange rate. E.g. a positive risk premium implies that foreign investors expect a more appreciated forint in one month's time with respect to the forward rate. As market expectations, and also the risk premium are unobservable variables, they should be estimated. We will use two different estimates: a survey-based risk premium and the ex-post premium (see more methods for estimating EUR/HUF exchange rate risk premium in Rezessy, 2007).

First, the risk premium will be estimated supposing that the expected exchange rate is equal to the realised exchange rate one month later (hereafter called *ex-post risk premium*):

$$RP\left(ex\_post\right) = F_t^{t+h} - S_{t+h} \tag{11}$$

The reason for this choice is the following. Under the rationality assumption, the expected exchange rate can be expressed as the sum of the future spot exchange rate and the forecast error, the latter being white noise:

$$E_t\left(S_{t+h}\right) = S_{t+h} + v_t \tag{12}$$

[19] The theoretical considerations behind this relationship can be based on a CAPM framework, see e.g. Campbell (2000) on the derivation of a version of the three moment CAPM.

[20] In contrast to the cited authors, we do not use the kurtosis of the risk-neutral densities within the right-hand-side variables, because the skewness and the kurtosis are almost perfectly correlated (see Appendix 1, Table 7).

Replacing equation (12) into the equation (11) we get:

$$RP(ex\_post) = F_t^{t+h} - E_t(S_{t+h}) + v_t \tag{13}$$

Thus, the ex-post risk premium estimates the risk premium with noise, but it can be considered as an unbiased estimate if the rationality holds.

Replacing equation (13) into (10) and rearranging it, we get an equation which can be tested directly:

$$RP(ex\_post) = \alpha_1 + \beta_1 \cdot VOL_t + \gamma_1 \cdot STDEV_t + (u_t + v_t) \tag{14}$$

If investors are rational, the forecast error (v_t) should be independent from the information available at time t, i.e. uncorrelated with the right-hand-side variables. Thus estimating the risk premium this way, the estimated β and γ parameters from equation (14) are unbiased estimates for the parameters in equation (10), supposing that the rationality assumption holds.

Second, differently from Malz (1997b) and Gereben (2002), we will also use a survey-based risk premium. From the Reuters-poll we can obtain a direct estimate of $E_t(S_{t+h})$, and consequently, for the risk premium (we will call it the *ex-ante risk premium*):

$$RP(ex\_ante) = F_t^{t+h} - E_t(S_{t+h})^{REUTERS\_POLL} \tag{15}$$

Replacing the left-hand-side variable of (11) with the ex-ante risk premium, we again get a testable relationship:

$$RP(ex\_ante) = \alpha_2 + \beta_2 \cdot STDEV_t + \gamma_2 \cdot SKEW_t + w_t \tag{16}$$

Here, we do not have information about how the estimated ex-ante and the 'true' risk premium is connected, and therefore we marked the error term of (16) differently than in (10).

We expect the β parameter to be positive by both estimations. Risk-averse investors require risk premium for the risk they assume when investing in the underlying asset, which is expected to be proportional to the standard deviation of the subjective density. The parameter γ is also expected to be positive, as with unchanged standard deviation, foreign investors require higher premium for the higher expected probability of huge depreciations. (Notice that we defined the skewness to be positive for densities skewed towards forint depreciation.)

In the regressions (14) and (16) we use risk-neutral indicators instead of the central moments of subjective densities, which are expected to have explanatory power. Risk-neutral moments should not be in a causal relationship with the risk premium. If market participants were risk-neutral, the estimated ex-post risk premium would be equal to the forecast error, which could not be explained by risk-neutral moments, assuming rationality. If we arrive at significant parameter estimates with the expected signs, a reasonable conclusion is that risk-neutral standard deviation and skewness are good proxies for the central moments of the subjective densities (proxies in the sense that their levels are highly correlated). As the parameters are able to capture the effect of changes in the explanatory variables, the significance of parameters suggests that risk-neutral and subjective indicators, in general, move in the same direction.

7.2 TEST RESULTS OF THE EXPLANATORY POWER OF RNDS' CENTRAL MOMENTS

In this subsection we present the regression results for the explanatory power of the second and the third central moments of the RNDs regarding the estimated risk premium (based on equations 14 and 16). First, we describe exactly how these two time series are estimated. We provide interpretation of the results as to whether changes in risk-neutral central moments can be used as proxies for subjective central moments. Finally, we compare the results of the two regressions.

First, we regressed the ex-post, 1-month risk premium on the risk-neutral standard deviation and the skewness (as in equation 14). The *ex-post risk premium* was estimated as the difference between the 1-month forward exchange rate and the 1-month-

Table 5

Regression results of the ex-post risk premium (based on Equation 14)

Dependent Variable: 1 month ex-post risk premium

Method: Least Squares

Sample: 01.2003 - 06.2007

Included observations: 50

Newey-West HAC Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-58.897	17.929	-3.285	0.002
β_1 (STDEV)	4.794	1.446	3.316	0.002
γ_1 (SKEW)	20.557	7.609	2.701	0.001
R-squared	0.241		F-statistic	7.473
Adjusted R-squared	0.209		Prob(F-statistic)	0.002

Note: Ex-post risk premium measured in annualised percentages, log difference between the forward and the future spot exchange rate. Standard deviation (STDEV) is also annualised, expressed in percentages. Skewness (SKEW) measured in units, not annualised.

ahead realization of the spot exchange rate. Standard deviation and skewness were estimated from the option-implied densities, as illustrated in Charts 3 and 4, respectively. The sample is the same as described in Section 3 (2003–mid-2007). The frequency is one month, in order to avoid overlapping observations. Because of the possible autocorrelation in the error term, Newey-West standard errors were used. As neither the dependent, nor the independent variables contain unit root according to the standard tests, a linear least squares regression was run on the level of the variables.

According to the results, the standard deviation and the skewness have highly significant effect on the ex-post risk premium (Table 5). Both of them have the expected, positive sign. As mentioned earlier, only the central moments of the subjective densities are expected to have explanatory power, not the risk-neutral ones, as only risk averse investors are expected to require risk premium, risk-neutrals do not. Thus, these results suggest that risk-neutral standard deviation and skewness can be used as proxies for the subjective ones. Since the parameters are able to capture the effect of changes to the explanatory variables, the results indicate that there may be a strong (even though unobservable) relationship between the changes in risk-neutral and subjective central moments. A rise in the risk-neutral standard deviation or in the skewness can be interpreted as an increase in the subjective central moments. At least, as the results are valid for the average of the long sample period, this could be the case most of the time, even though it cannot be ruled out that on occasion, from one week to another, the opposite could happen. Moreover, as the functional form between risk-neutral and subjective indicators is unknown, we cannot say that the extent of changes would be equal (e.g. 1% rise in the risk-neutral standard deviation would be equivalent to 1% rise in the standard deviation of the subjective density). But the direction of the changes may coincide.

Notice that the above interpretation is not in contradiction with the results of Section 6. Even though the level of the central moments of the subjective and risk-neutral densities are presumably different, changes to the risk-neutral indicators can coincide with changes to subjective indicators.

The R-squared of the regression was about 0.25; this explanatory power can be considered relatively high, as similar regressions are tested in the literature on much larges samples (Gereben, 2002), with similar or lower R-squared. On the other hand, much higher R-squared could not be expected as the ex-post risk premium also contains the forecast error, which causes higher standard errors. Moreover, we are concentrated on the question whether the two independent variables are significant; if the question were to explain the time series development of the risk premium, adding other variables possibly could raise the explanatory power of the model. The results were also broadly unchanged when we run the same regression on two, equally sized sub-samples, with positive and significant parameter values.

It is also worth mentioning the potential problems arising from multicollinearity. If the correlation between the 2 independent variables were positive, it would be possible that one of them partly captures the effect of the other, thus causing the significance of both explanatory variables. However, the correlation was negative between the skewness and the standard deviation (with a coefficient of about -0.33), suggesting that the significance of the parameters is not due to purely technical

Table 6

Regression results of the ex-ante risk premium (based on Equation 16)

Dependent Variable: 1 month ex-ante risk premium

Method: Least Squares

Sample: 01.2003 - 06.2007

Included observations: 50

Newey-West HAC Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-36.158	13.283	-2.722	0.009
β_2 (STDEV)	3.193	1.183	2.700	0.010
γ_1 (SKEW)	11.736	4.024	2.916	0.006
AR(1)	0.730	0.074	9.813	0.000
R-squared	0.581		F-statistic	21.270
Adjusted R-squared	0.554		Prob(F-statistic)	0.000

Note: Ex-ante risk premium measured in annualised percentages, log difference between the forward and the expected future spot exchange rate. Standard deviation (STDEV) is also annualised, expressed in percentages. Skewness (SKEW) measured in units, not annualised.

reasons. On the other hand, because of the multicollinearity the parameters cannot be interpreted as *ceteris paribus* changes. However, this is not a problem for us since we relied only on the sign of the parameters in the above interpretation.

We have performed a similar exercise as above, but now for the *ex-ante risk premium* (as in equation 16). The risk premium was estimated based on the Hungarian Reuters survey, as the difference between the 1-month forward exchange rate and the average expected spot exchange rate in one month's time.[21] Standard deviation and skewness were the same as defined above. The frequency of the time series is the same, as the Reuters survey is available once a month. The sample period is the same as in the first regression (2003–mid-2007). As the time series of the ex-ante risk premium proved to be highly autocorrelated, an autoregressive term was also included in the regression. The remaining autocorrelation in the error term was handled by using Newey-West standard errors. The ex-ante risk premium do not contain unit root according to the standard tests, thus again a linear least squares regression was run on the levels of the variables.

Results show that the parameters of the standard deviation and the skewness are significant and positive (Table 6). Thus, we can reiterate the interpretation given above: risk-neutral indicators can be used as proxies for the subjective ones.

Here, the AR term is responsible for the relatively high R-squared. Omitting this, the R-squared diminished to about 0.25; the sign of the parameters remained unchanged, while the significance of the coefficient of the skewness became lower. This explanatory power also can be considered satisfactory given the relatively small sample.

Finally, comparing the regressions on the ex-post and the ex-ante risk premium, we can make two important notes. First, it can be observed that the value of the parameters of the independent variables is rather different. This is reasonable, because not only the risk premia are estimated in different ways, but the specification of the regressions is also somewhat different. However, the ratio of the two parameters, β and γ is very similar. From the regression based on equation (14) this ratio is about 0.23, while in the regression related to equation (16) it is 0.27. This observation suggests the robustness of the results.

Second, we compared the time series of the estimated ex-post and ex-ante risk premium. The correlation coefficient between these two time series was practically zero in the examined period (-0.03). Since both estimates depend on the same forward exchange rate, this phenomenon can be considered surprising. The difference between the two estimates is that for the ex-post risk premium the future spot exchange rate was subtracted from the forward rate, while in the case of the ex-ante risk premium the expected exchange rate was subtracted. Comparing the survey-based expected 1-month change in the exchange

[21] In this survey the respondents give forecasts for the spot exchange for the end of next month. We estimated the expected spot exchange rate in one month's time by exponential interpolation between the spot exchange rate and the survey forecast.

rate with the subsequent realization, we found again a non-significant correlation between these two (-0.11). It suggests that an average respondent in the survey is not able to forecast well the exchange rate in one month's time, which is reasonable assuming that the FX market is efficient. Another explanation for the low association between the two estimates of the risk premia is that the ex-post risk premium also contains the forecast error, and the variance of this can be much higher than that of the unobservable risk premium. All in all, the low correlation between the two estimated risk premia confirms the robustness of the results, as despite it, we arrived at rather similar results.

The main conclusions of this subsection are the following:

- Risk-neutral standard deviation and skewness are able to explain a significant part of the variability in the estimated risk premium.

- The direction of the changes to risk-neutral central moments (standard deviation and skewness) presumably coincides with changes to central moments of subjective densities, on the average of the period in review.

8 Conclusions

In this paper we estimated 1-month risk-neutral density functions from EUR/HUF option prices. First, we compared the time series of risk-neutral central moments with other option-based indicators. The key results from the comparison can be summarised as follows. The ATMF implied volatility can be considered as an appropriate 'shortcut' indicator for measuring the standard deviation of RNDs, i.e. it is highly correlated with the standard deviation and there is no use for estimating densities to calculate it. For measuring the skewness of the risk-neutral densities, the standardised 25-delta risk reversal (risk reversal divided by the ATMF volatility) proved to be a good shortcut indicator. For the third central moment, the kurtosis, it is also possible to find a shortcut indicator. Since in our case the kurtosis was extremely strongly correlated with the skewness, the standardised 25-delta risk reversal can be considered as a shortcut indicator for the kurtosis as well.

For each of the central moments, we calculated indicators from the RNDs which are expressed in probabilities and have strong co-movement with the standard deviation, skewness and kurtosis, respectively. These indicators are calculated as the sum or the difference of probabilities at the tails of the densities. These indicators can be used as alternative measures for the central moments. It was found that it is not irrelevant how these probability-based indicators are defined; the exact definition of these indicators affects the strength of the correlation with the central moments.

We also calculated to what extent the risk-neutral central moments change in response to changes in the input data arising from potential observation errors. This analysis showed that the sensitivity of the standard deviation and the skewness can be considered low in relative terms, even though not negligible. On the other hand, the estimated kurtosis proved to be extremely sensitive. Thus the information content of this indicator should be treated with caution.

According to several tests, we rejected the hypothesis that risk-neutral density functions would provide good forecasting power. The standard deviation, the skewness and the kurtosis of risk-neutral densities are also responsible for the poor forecasting ability. Based on these tests, our interpretation is that risk-neutral densities are significantly different from the central moments of subjective densities. The explanation for this may be that the risk-neutral central moments contain a risk premium. Thus, unfortunately, the risk-neutral densities (and their central moments) cannot be used as if these were equal to the expectations of market participants on the future probability distribution.

Based on two different estimates of the risk premium, we found that the risk-neutral standard deviation and the skewness are able to explain a significant part of the variability in the risk premium. These results suggest that the direction of the changes to risk-neutral central moments coincides with changes to central moments of subjective densities, i.e. these indicators, in general, move in the same direction.

All in all, we can conclude that the estimated risk-neutral standard deviation and skewness can be considered as good proxy indicators of market expectations regarding the future distribution of the Hungarian forint. However, instead of its level, the changes to these are recommended to use for analysing purposes.

Appendix 1: Charts and Tables

Chart 6

Payoff diagrams of standard option combinations in function of the exchange rate at maturity

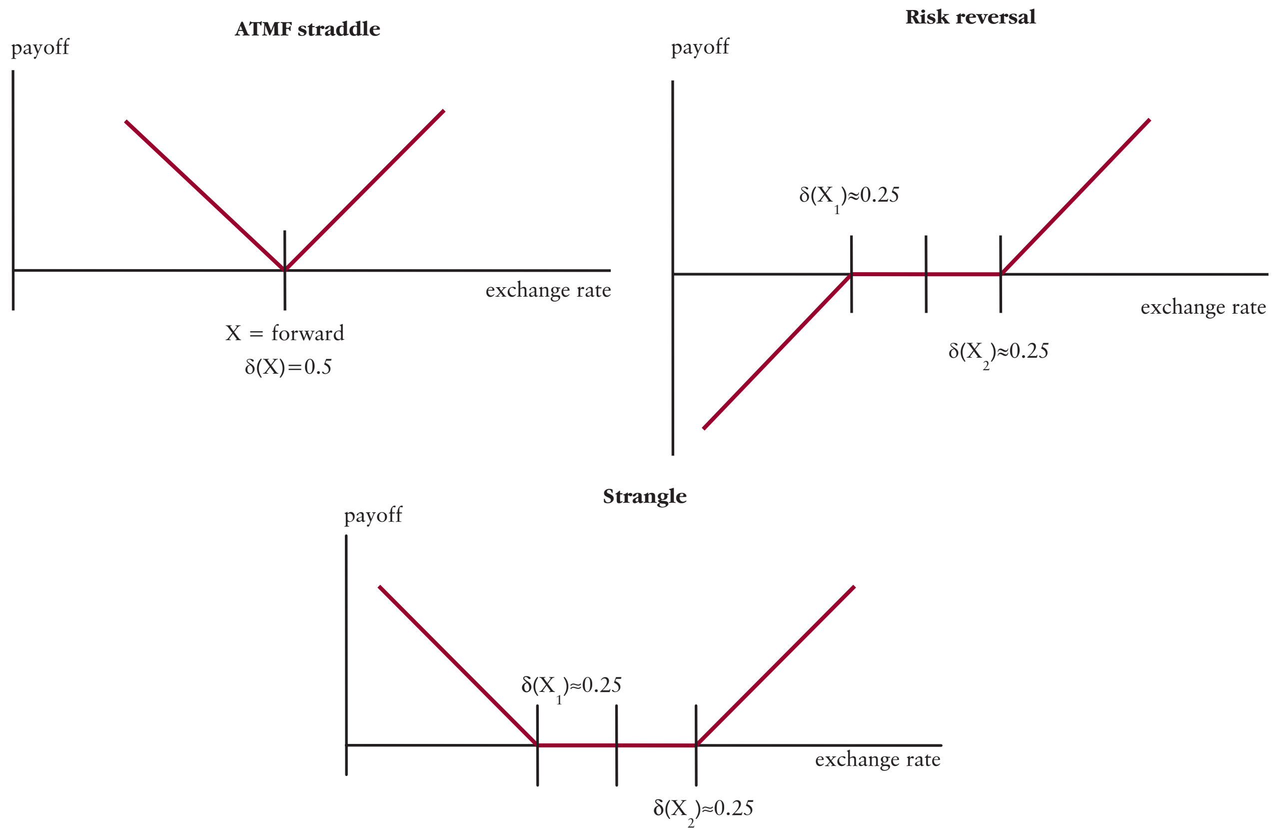


Chart 7

Input data used to the estimation

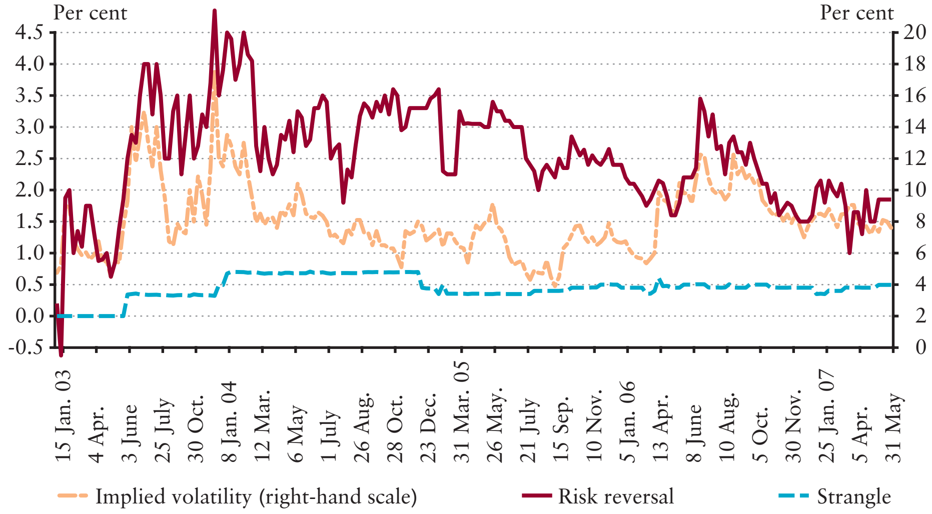
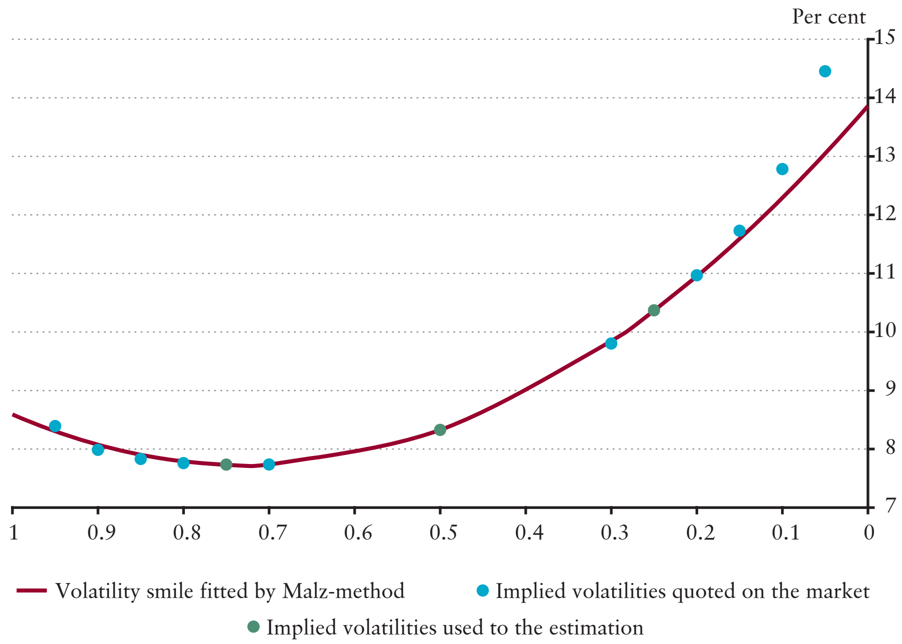


Chart 8

One-month volatility smile from 13 cross section data (2003–2004)



Note: Data source for the points of the volatility smile is Superderivatives. On the chart we plotted the points of the volatility smile as the average for the period 2003–2004, calculated from weekly data. Green points denote the 3 points on which the smile was fitted by the Malz-method.

Chart 9

The forward exchange rate and the future spot exchange rate

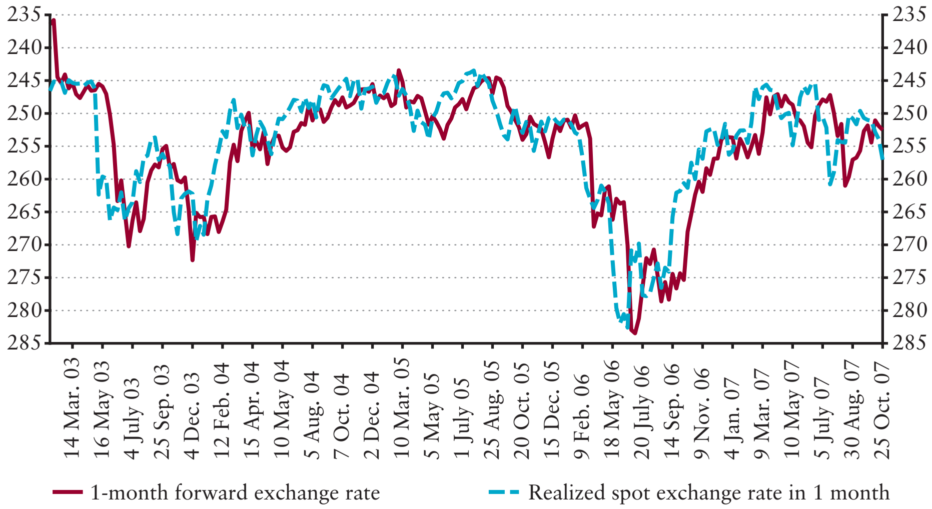
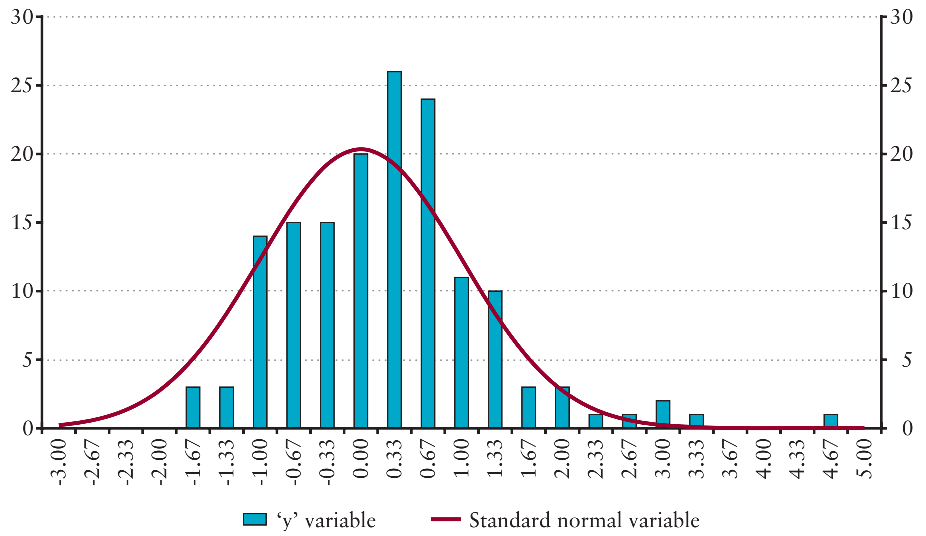


Chart 10

The histogram of $y_{t,h}$ related to the option-implied densities



Note: Positive values are related to cases when the forint depreciated more than it would have been implied by the forward exchange rate.

Table 7

Correlation matrix of option based indicators (Sample: January 2003-June 2007)

	ATMF implied volatility	Standard deviation	Uncertainty probability indicator (x=3%)	Standardised risk reversal	Skewness	Probability asymmetry indicator (y=3 std)	Kurtosis	Extreme movements probability indicator (z=3 std)
ATMF implied volatility	**1.000**	**0.992**	**0.992**	-0.408	-0.403	-0.377	-0.413	-0.373
Standard deviation	**0.992**	**1.000**	**0.990**	-0.336	-0.326	-0.281	-0.313	-0.272
Uncertainty probability indicator (x=3%)	**0.992**	**0.990**	**1.000**	-0.402	-0.393	-0.351	-0.376	-0.341
Standardised risk reversal	-0.408	-0.336	-0.402	**1.000**	**0.999**	**0.950**	**0.930**	**0.927**
Skewness	-0.403	-0.326	-0.393	**0.999**	**1.000**	**0.962**	**0.943**	**0.942**
Probability asymmetry indicator (y=3 std)	-0.377	-0.281	-0.351	**0.950**	**0.962**	**1.000**	**0.974**	**0.996**
Kurtosis	-0.413	-0.313	-0.376	**0.930**	**0.943**	**0.974**	**1.000**	**0.979**
Extreme movements probability indicator (z=3 std)	-0.373	-0.272	-0.341	**0.927**	**0.942**	**0.996**	**0.979**	**1.000**

Table 8

Descriptive statistics of option-implied indicators (Sample: January 2003–June 2007)

	Mean	Max	Min	Standard deviation
ATMF implied volatility	7.831	17.500	3.900	2.337
Standard deviation	8.527	17.761	4.541	2.372
Uncertainty probability indicator (x=3%)	17.565	53.576	2.247	9.504
Risk reversal	2.678	4.850	-0.625	0.845
Standardised risk reversal	0.355	0.603	-0.119	0.114
Skewness	1.197	2.028	-0.394	0.382
Probability asymmetry indicator (y=3 std)	1.475	2.503	-0.388	0.530
Strangle	0.444	0.706	0.000	0.209
Kurtosis	2.493	5.712	0.020	1.182
Extreme movements probability indicator (z=3 std)	1.519	2.506	0.300	0.501

Appendix 2: Testing forecasting power by Berkowitz LR statistics

Berkowitz (2001) suggested testing the following autoregressive model for the $y_{t,h}$ variable defined in equation (7):

$$y_{t,h} - \mu = \rho\left(y_{t-1,h} - \mu\right) + \varepsilon_{t,h} \tag{17}$$

The null hypothesis is that $y_{t,h}$ follows an iid N(0,1): $\mu=0$, $\rho=0$ and $\sigma=Var(\varepsilon)=1$. The null is tested by the following likelihood ratio test (see Berkowitz, 2001 for the exact likelihood function):

$$LR_3 = -2\left[L(0,1,0) - L\left(\hat{\mu}, \hat{\sigma}^2, \hat{\rho}\right)\right] \sim \chi^2(3) \tag{18}$$

Berkowitz also tested the independence assumption of $y_{t,h}$ variable *($\rho=0$)* by the following likelihood ratio:

$$LR_1 = -2\left[L\left(\mu, \sigma^2, 0\right) - L\left(\hat{\mu}, \hat{\sigma}^2, \hat{\rho}\right)\right] \sim \chi^2(1) \tag{19}$$

The 3 possible outcomes of these tests can be interpreted this way:

- Reject the null of (18) by LR_3 and not reject the null of (19) by LR_1: estimated densities do not produce accurate forecasts of the historical density.

- Reject the null of (18) by LR_3 and reject the null of (19) by LR_1: not possible to conclude if there is lack of forecasting power or we reject it only because of serial correlation arising from overlapping data.

- Not reject the null of (18) by LR_3 and not reject the null of (19) by LR_1: estimated densities provide accurate forecasts of the historical density.

These tests were applied for the $y_{t,h}$ variables calculated from the option-implied densities. For the whole sample period it was found that we can reject both null at any reasonable significance level (see Table 9). In accordance with the above, we cannot state anything firm about forecasting ability. However, it can be supposed that it stems from the high serial correlation because of overlapping data, as the first-order autocorrelation coefficient was about 0.75.

To improve the test, we divided the whole sample into 3 parts in order to minimise the period of overlapping forecasts. We took into account only every third observation and we ran the test again for these shorter time series. As a result, we found that the null of (18) was rejected by LR_3 and the null of (19) was not rejected by LR_1, in each of the sub-samples. These results suggest that the estimated densities do not provide accurate forecasts of the historical density.

Table 9

Berkowitz LR statistics for the Malz-method based densities

	LR(1)	LR(3)
Full sample	66.395 (0.000)	76.512 (0.000)
1. sub-sample	1.355 (0.244)	9.038 (0.029)
2. sub-sample	1.947 (0.163)	9.526 (0.023)
3. sub-sample	1.869 (0.172)	6.980 (0.073)

Note: P-values are in parenthesis. 1. sub-sample: observations no. 1, 4, 7, … 2. sub-sample: observations no. 2, 5, 8, … 3. sub-sample: observations no. 3, 6, 9, …

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