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Influence of periodic temperature and concentration on unsteady free convective viscous incompressible flow and heat transfer past a vertical plate in slip-flow regime

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Abstract

The effect of combined heat and mass transfer on unsteady free convective, viscous incompressible flow past a vertical flat plate in slip-flow regime is studied. Assuming variable suction at the plate, approximate solutions are obtained for velocity, skin-friction, temperature, heat transfer and concentration. During the course of discussion, the effects of Gr (Grashof number based on temperature), Gc (modified Grashof number based on concentration difference), A (suction parameter) and ω (frequency parameter) for carbon dioxide ($Sc = 0.94$) in air ($Pr = 0.71$) are presented and discussed graphically.

Keywords: Free convection, unsteady, incompressible fluid, slip-flow, heat and mass transfer.

AMSC(2000): Primary: 76E05, Secondary: 76R10, 80A20.

1 Introduction

In recent years, the requirements of modern technology have stimulated interest in fluid flow, which involve the interaction of several phenomena. The process of heat and mass transfer in free convection flow have attracted the attention of a number of scholars due to their application in many branches of science and engineering, viz. in the early stages of melting adjacent to a heated surface, in chemical engineering processes which are classified as a mass transfer process, in a cooling device. The phenomenon of free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. This can be seen in our everyday life in the atmospheric flow, which is driven by temperature differences. There are many transport processes occurring in nature due to temperature and chemical differences. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries and many engineering applications in which fluid is the working medium. Now, free convective flow past vertical plate has been studied extensively by Ostrach [1], [2] and many others. These studies are confined to steady flows only. Gebhart and Pera [3] studied the natural convection flow from the combined buoyancy effects on thermal and mass diffusion. Also in case of unsteady free convective flows Soundalgekar [4] studied the effects of viscous dissipation on the flow past an infinite vertical porous plate. It was assumed that the plate temperature oscillates in such a way that its amplitude is small. The free convective heat transfer on a vertical semi-infinite plate has been investigated by Berezovsky et al. [5]. Soundalgekar and Wavre [6], [7] studied the unsteady free convection flow past an infinite vertical plate and mass transfer with constant/variable suction. Also the combined heat and mass transfer in mixed convection along vertical and inclined plates has been studied by Chen et al. [8]. Martynenko et al. [9] investigated the laminar free convection from a vertical plate. Also, the free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient is studied by Ramanaiah and Malarvizhi [10]. Das et al. [11] studied the transient free convection flow past an infinite vertical plate with periodic temperature variation, because the free convection is enhanced by superimposing oscillating temperature on the mean plate temperature. Hossain et al. [12] studied the influence of fluctuating surface temperature and concentration on natural convection flow from a vertical flat plate. In many practical applications the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle at the surface has a finite tangential velocity. It "slips" along the surface. The flow regime is called the slip flow regime and this effect can not be neglected. Using this effect Sharma and Chaudhary [13] studied the effect of variable suction on transient free convective viscous incompressible flow past a vertical plate with periodic temperature variations in slip-flow regime. Recently, Sharma and Sharma [14] studied influence of variable suction on unsteady free convective flow from a vertical flat plate and heat transfer in slip-flow regime. Therefore, the object of this paper is to study the effects of combined heat and mass transfer on unsteady free convective, viscous incompressible flow past a vertical flat plate in slip-flow regime, when suction velocity oscillate in time about a constant mean.

2 Formulation of the problem

An unsteady free convective flow of a viscous incompressible fluid past an infinite vertical porous flat plate in slip-flow regime, with periodic temperature and concentration when variable suction velocity distribution, $V^* = -V_0\left(1 + \varepsilon A e^{t\omega^* t^*}\right)$, fluctuating with time is applied. We introduce a coordinate system with wall lying vertically in $x^* - y^*$ plane. The y^*-axis is taken in vertically upward direction along the vertical porous plate and y*-axis is taken normal to the plate. Since the plate is considered infinite in the x*-direction, all physical quantities will be independent of x^*. Under these assumptions, the physical variables are function of y^* and t^* only. Then neglecting viscous dissipation and assuming variation of density in the body force term (Boussinesq's approximation) the problem is governed by the following set of equations

$$\frac{\partial u^*}{\partial t^*} + V^* \frac{\partial u^*}{\partial y^*} = g\beta\,(T^* - T^*_\infty) + g\beta^\circ\,(C^* - C^*_\infty) + \nu\,\frac{\partial^2 u^*}{\partial y^{*2}}, \tag{1}$$

$$\frac{\partial T^*}{\partial t^*} + V^*\,\frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho\, C_p}\frac{\partial^2 T^*}{\partial y^{*2}}, \tag{2}$$

$$\frac{\partial C^*}{\partial t^*} + V^*\,\frac{\partial C^*}{\partial y^*} = D\,\frac{\partial^2 C^*}{\partial y^{*2}}. \tag{3}$$

The boundary conditions of the problem are

$$u^* = L^* \frac{\partial u^*}{\partial y^*}, \quad T^* = T^*_w + \varepsilon\,(T^*_w - T^*_\infty)\,e^{i\omega^* t^*}, \tag{4}$$

$$C^* = C^*_w + \varepsilon\,(C^*_w - C^*_\infty)\,e^{i\omega^* t^*} \quad \text{at} \quad y^* = 0, \tag{5}$$

$$u* \to 0, \quad T^* \to T^*_\infty, \quad C^* \to C^*_\infty, \quad \text{as} \quad y^* \to \infty. \tag{6}$$

We now introduce the following non-dimensional quantities into equations (1) to (4)

$$y = y^* \frac{V^*_0}{\nu}, \quad t = t^* \frac{V_0^{*2}}{4\nu}, \quad u = \frac{u^*}{V^*_0}, \quad \omega = \frac{4\nu\omega^*}{V_0^{*2}},$$

$$\Theta = \frac{T^* - T^*_\infty}{T^*_w - T^*_\infty}, \quad C = \frac{C^* - C^*_\infty}{C^*_w - C^*_\infty},$$

$$Gr = \frac{g\beta\nu\,(T^*_w - T^*_\infty)}{V_0^{*3}}, \quad \text{(Grashof number)},$$

$$Gc = \frac{g\beta^\circ\nu\,(C^*_w - C^*_\infty)}{V_0^{*3}}, \quad \text{(Modified Grashof number)},$$

$$Pr = \mu\frac{C_p}{K} = \frac{\nu\,p\,C_p}{K}, \quad \text{(Prandtl number)},$$

$$Sc = \frac{\nu}{D}, \quad \text{(Schmidt number)},$$

$$h = \frac{V^*_0\,L^*}{\nu}, \quad \text{(rarefaction parameter)}$$

All physical variables are defined in nomenclature. The $(*)$ stands for dimensional quantities. The subscript (∞) denotes the free stream condition. Then equations (1) to (3) reduce to the following non-dimensional form

$$\frac{1}{4}\,\frac{\partial\, u}{\partial\, t} - \left(1 + \varepsilon\,A\,e^{i\,\omega\,t}\right)\,\frac{\partial\, u}{\partial\, y} = Gr\,\Theta + Gc\,C + \frac{\partial^2 u}{\partial y^2}, \tag{7}$$

$$\frac{1}{4}\,\frac{\partial\,\Theta}{\partial\, t} - \left(1 + \varepsilon\,A\,e^{i\,\omega\,t}\right)\,\frac{\partial\,\Theta}{\partial\, y} = \frac{1}{Pr}\,\frac{\partial^2 \Theta}{\partial y^2}, \tag{8}$$

$$\frac{1}{4}\,\frac{\partial\, C}{\partial\, t} - \left(1 + \varepsilon\,A\,e^{i\,\omega\,t}\right)\,\frac{\partial\, C}{\partial\, y} = \frac{1}{Sc}\,\frac{\partial^2 C}{\partial y^2}. \tag{9}$$

The boundary conditions to the problem in the dimensionless form are

$$\begin{array}{lllll} u = h\,\frac{\partial u}{\partial y}, & \Theta = 1 + \varepsilon\, e^{i\,\omega\,t}, & C = 1 + \varepsilon\, e^{i\,\omega\,t}, & \text{at} & y = 0, \\ u \to 0, & \Theta \to 0 & C \to 0, & \text{at} & y \to \infty. \end{array} \tag{10}$$

3 Solution of the problem

Assuming small amplitude oscillations ($\varepsilon << 1$), we can represent the velocity u, temperature Θ and concentration C near the plate as follows

$$\begin{array}{rcl} u\ (y,t) & = & u_0\ (y) + \varepsilon\, u_1\,(y)\ e^{i\,\omega\,t}, \\ \Theta\ (y,t) & = & \Theta_0\ (y) + \varepsilon\,\Theta_1\,(y)\ e^{i\,\omega\,t}, \\ C\ (y,t) & = & C_0\ (y) + \varepsilon\, C_1\,(y)\ e^{i\,\omega\,t}. \end{array} \tag{11}$$

Substituting (9) in (5) to (7), equating the coefficients of harmonic and non harmonic terms, neglecting the coefficients of ε^2, we get

$$\begin{array}{l} \Theta_0'' + Pr\,\Theta_0' = 0, \\ \Theta_1'' + Pr\,\Theta_1' - \frac{i\,\omega\,Pr\,\Theta_1}{4} = -A\,Pr\,\Theta_0', \\ u_0'' + u_0' = -Gr\,\Theta_0 - Gc\,C_0, \\ u_1'' + u_1' - \frac{i\omega\ u_1}{4} = -Gr\,\Theta_1 - Gc\,C_1 - A\,u_0', \\ C_0'' + Sc\,C_0' = 0, \\ C_1'' + Sc\,C_1' - \frac{i\,\omega\,Sc\,C_1}{4} = -A\,Sc\,C_0'. \end{array} \tag{12}$$

The corresponding boundary conditions reduce to

$$\begin{array}{l} u_0 = h\,\frac{\partial u_0}{\partial y}, \quad u_1 = h\,\frac{\partial u_1}{\partial y}, \\ \Theta_0 = 1, \quad \Theta_1 = 1, \quad C_0 = 1, \quad C_1 = 1, \quad \text{at} \quad y = 0, \\ u_0 = 0, \quad u_1 = 0, \quad \Theta_0 = 0, \\ \Theta_1 = 0, \quad C_0 = 0, \quad C_1 = 0, \quad \text{as} \quad y \to \infty, \end{array} \tag{13}$$

where primes denote differentiation with respect to $'y'$. Solving the set of equation (10) under the boundary conditions (11) we get

$$\Theta_0\,(y) = e^{-Pr\,y}, \tag{14}$$

$$C_0\,(y) = e^{-Sc\,y}, \tag{15}$$

$$u_0\,(y) = B_7\,e^{-y} - B_5\,e^{-Pr\,y} - B_6\,e^{-Sc\,y}, \tag{16}$$

$$\Theta_1\,(y) = B_1\,e^{-m_1\,y} + B_2\,e^{-Pr\,y}, \tag{17}$$

$$C_1\,(y) = B_3\,e^{-m_2\,y} + B_4\,e^{-Sc\,y}, \tag{18}$$

$$\begin{array}{rcl} u_1\,(y) & = & B_{13}\,e^{-m_3\,y} - B_8\,e^{-m_1\,y} \\ & & -B_{10}\,e^{-Pr\,y} - B_{11}\,e^{-Sc\,y} - B_{12}\,e^{-y}, \end{array} \tag{19}$$

where

$$m_1 = \frac{Pr + \sqrt{Pr^2 + i\,\omega\,Pr}}{2}, \quad m_2 = \frac{Sc + \sqrt{Sc^2 + i\,\omega\,Sc}}{2}$$
$$m_3 = \frac{1 + \sqrt{1 + i\,\omega}}{2}, \quad B_1 = 1 - \frac{4\,A\,i\,Pr}{\omega}, \quad B_2 = 1 - B_1,$$
$$B_3 = 1 - \frac{4\,A\,i\,Sc}{\omega}, \quad B_4 = 1 - B_3, \quad B_5 = \frac{Gr}{Pr^2 - Pr},$$
$$B_6 = \frac{Gc}{Sc^2 - Sc}, \quad B_7 = \frac{B_5\,(h\,Pr + 1) + B_6\,(h\,Sc + 1)}{h + 1},$$
$$B_8 = \frac{Gr\,B_1}{(Pr - 1)\left(m_1 + \frac{i\,\omega}{4}\right)}, \quad B_9 = \frac{Gc\,B_3}{(Sc - 1)\left(m_2 + \frac{i\,\omega}{4}\right)},$$
$$B_{10} = \frac{Gr\,B_2 + A\,B_5\,Pr}{Pr^2 - Pr - \frac{i\,\omega}{4}}, \quad B_{11} = \frac{Gc\,B_4 + A\,B_6\,Sc}{Sc^2 - Sc - \frac{i\,\omega}{4}},$$
$$B_{12} = \frac{-4\,A\,i\,B_7}{\omega},$$
$$B_{13} = (h\,m_3 + 1)^{-1}\,[B_8\,(h\,m_1 + 1) + B_9\,(h\,m_2 + 1) + B_{10}\,(h\,Pr + 1) + B_{11}\,(h\,Sc + 1) + B_{12}\,(h + 1)]\,.$$

The important characteristics of the problem are the skin-friction and rate of heat transfer at the plate. From the velocity field, we can calculate the skin-friction in main flow direction as:

Skin-friction: The dimensionless shearing stress on the surface of a body, due to a fluid motion, is known as skin-friction and is defined by the Newton's law of viscosity.

$$\tau = \mu\,\frac{\partial\,u^*}{\partial\,y^*}. \tag{20}$$

Substituting equations (14) and (17) into equations (9) we can calculate the shearing stress component in dimensionless form as

$$\tau = \frac{\tau}{\rho\,V_0^{*\,2}} = \frac{\partial\,u}{\partial\,y}\;\Big|_{y=0} \tag{21}$$

In terms of the amplitude and phase, the skin-friction can be written as:

$$\tau = \tau_m + \varepsilon\,|M|\,cos\,(\omega\,t + \Phi)$$

where $\tan\Phi = \frac{M_i}{M_r}$,

$$M = M_r + i\,M_i = -B_{13}\,m_3 + B_8\,m_1 + B_9\,m_2 + B_{10}\,Pr + B_{11}\,Sc + B_{12}$$

and the Mean skin - friction τ_m is given by

$$\tau_m = -B_7 + B_5\,Pr + B_6\,Sc.$$

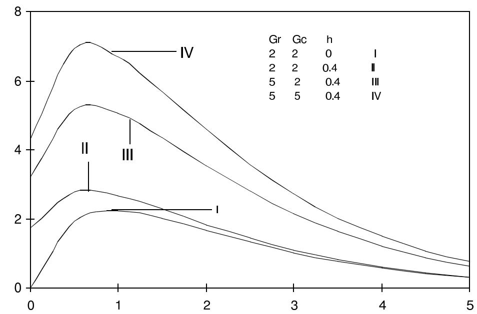

Figure 1: The Velocity profiles of carbon dioxide ($Sc = 0.94$) in air ($Pr = 0.71$)for $\omega = 10$, $\omega t = 0$, $A = 5$ and $\epsilon = 0.2$.

From the temperature field we can calculate the heat transfer at the plate in dimensionless form as:

Heat Transfer: In the dynamics of viscous fluid one is not much interested to know all the details of the velocity and temperature fields but would certainly like to know quantity of heat exchange between the body and the fluid. Since at the boundary the heat exchanged between the fluid and the body is only due to conduction, according to Fourier's law, we have

$$q_w^* = -\kappa \frac{\partial T^*}{\partial y^*} \,|_{y^*=0} \tag{22}$$

where y^* is the direction of the normal to the surface of the body. Substituting equations (12) and (15) into (9), we can calculate the dimensionless coefficient of heat transfer as follows

$$q = \frac{q_w^* \, \nu}{\kappa \, V_0^* \, (T_w^* - T_\infty^*)} = -\frac{\partial \Theta}{\partial y} \,|_{y=0} \tag{23}$$

In terms of the amplitude and phase the rate of heat transfer can be written as:

$$q = Pr + |N| \, cos \, (\omega \, t + \varphi)$$

where

$$N = N_r + iN_i = B_1 \, m_1 + B_2 \, Pr, \quad \text{and} \quad \tan\varphi = \frac{N_i}{N_r}.$$

4 Discussion

The convection flows driven by a combination of diffusion effects are very important in many applications. The foregoing formulations may be analyzed to indicate the nature of the interaction of the various contributions to buoyancy. Here we restricted our discussion to the aiding or favorable case only, for fluids with Prandtl number $Pr = 0.71$ which represent air at $20 \, oC$ at 1 atmosphere. The value of the Schmidt number, Sc is chosen to represent the presence of species Carbon dioxide ($Sc = 0.94$). The values of Gr and Gc are selected arbitrarily. We take $Gr > 0$, which correspond to the cooling of the plate by free convection currents. The velocity profiles in air for carbon dioxide are presented in Fig. (1-3). It is observed from the figures that

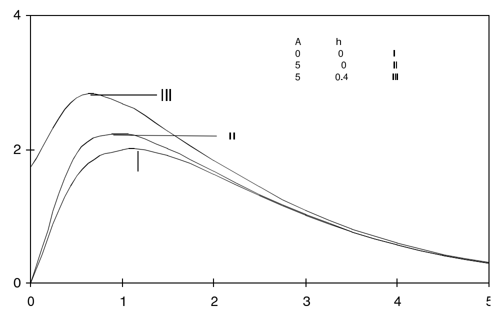

Figure 2: The velocity profiles of carbon dioxide ($Sc = 0.94$) in air ($Pr = 0.71$) for $Gr = 2$, $Gc = 2$, $\omega t = 0$, $\omega = 10$ and $\epsilon = 0.2$.

velocity increases rapidly near the plate and then decreases exponentially far away from the plate. The increase in Gr or Gc leads to an increase in velocity. The values of velocity are greater for $\omega t = 0$ than that of $\omega t = \frac{\pi}{2}$ when $Gr = Gc = 5$, while reverse effect is observed when $Gr = Gc = 2$ or $Gr = 5$, $Gc = 2$. It is also observed from the figures that the velocity increases with the increase of suction parameter A and rarefaction parameter h both. Further we find that when Gc, h and ωt are constant, the velocity of carbon dioxide for air increases due to more cooling of the plate by free convection currents. The mean skin-friction of carbon dioxide for air is shown in Fig.4.

It is evident from this figure that the mean skin-friction decreases with increasing rarefaction parameter. The mean skin-friction increases due to the

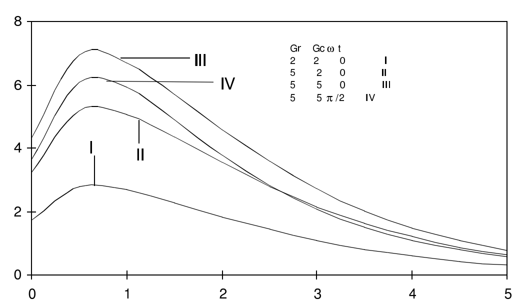

Figure 3: The velocity profiles of carbon dioxide ($Sc = 0.94$) in air ($Pr = 0.71$) for $A = 5$, $h = 0.4$, $\omega = 10$ and $\epsilon = 0.2$.

A	Gr= 2, Gc=2 h=0	Gr= 2, Gc=2 h=0.4	Gr= 5, Gc=2 h=0.4	Gr= 2, Gc=5 h=0.4
0	-0.9525	-1.5939	-1.6190	-1.5683
0.2	-0.7058	-1.1822	-1.1960	-1.1676
0.4	-0.5634	-0.9383	-0.9514	-0.9242
0.6	-0.4708	-0.7770	-0.7919	-0.7607
0.8	-0.4056	-0.6623	-0.6798	-0.6432
1.0	-0.3573	-0.5767	-0.5966	-0.5549

Table 1: The phase of skin-friction ($\tan\Phi$) for carbon dioxide ($Sc = 0.94$), $\omega = 10$ and $Pr = 0.71$.

increase in Gr or Gc both. Hence it may be concluded that the mean skin-friction increases with more cooling of the plate by free convection currents. The amplitude $|M|$ of the skin-friction for CO_2 in air is shown in Fig.5. It is observed from this figure that amplitude increases with increasing A (suction parameter), while reverse effect is observed for h (rarefaction parameter). An increase in Gr or Gc leads to an increase in amplitude of skin-friction in air for CO_2. The numerical values of phase of skin-friction are presented in Table 1. It is observed that the phase of skin-friction increases with increasing A, while reverse effect is observed for h. The phase of skin-friction decreases with the increase of Gr, while the reverse effect is observed for Gc for the same value of rarefaction parameter h.

The temperature profiles are presented in Fig.6. From this figure it is observed that it decreases with increasing the suction parameter while increases with the increase of frequency of fluctuation ω. This figures further shows that the values of temperature are greater in vicinity of the plate and decreases exponentially far away from the plate. The amplitude $|N|$ and phase $\tan\varphi$ of rate of heat transfer are presented in Table 2. This table shows that both increases with increasing frequency ω. The amplitude increases with increasing suction parameter A while, reverse effect is observed for phase of

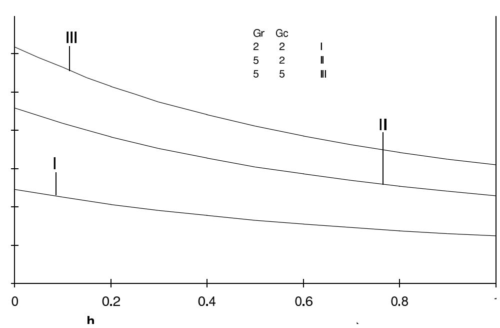

Figure 4: The mean skin-friction of carbon dioxide ($Sc = 0.94$) for (air) $Pr = 0.71$.

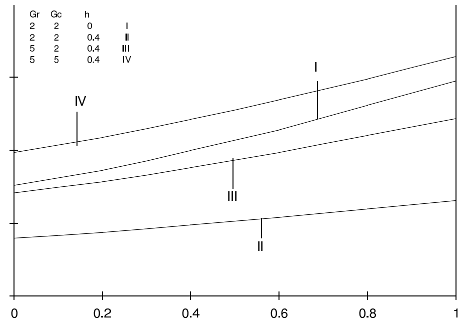

Figure 5: The amplitude of skin-friction of carbon dioxide (Sc = 0.94) in air (Pr=0.71) for $\omega = 10$.

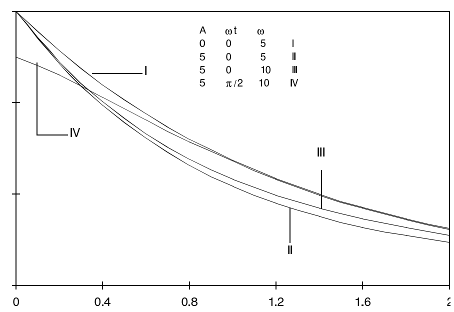

Figure 6: The temperature profiles for (air) $Pr = 0.71$ and $\epsilon = 0.2$.

heat transfer.

The concentration profile of carbon dioxide is presented in Fig.7. The concentration increases with increasing A, the suction parameter. It is also observed that the concentration increases with increasing distance in the vicinity of the plate and thereafter decreases exponentially far away from the plate. It is also found that concentration decreases with the increase of ωt for the same value of suction parameter A.

5 Conclusions

1. The velocity increases with increasing Gr and Gc both. Also, velocity increases with the increase of h and A both.

2. The mean skin-friction increases with increasing either Gr or Gc, while reverse phenomena is observed for h.

3. The amplitude $|M|$ of skin-friction increases with increasing Gr and Gc both.

A	$\omega = 5$ $\lvert N\rvert$	$\omega = 10$ $\lvert N\rvert$	$\omega = 5$ $\tan\varphi$	$\omega = 10$ $\tan\varphi$
0	1.2370	1.6120	0.5800	0.6831
0.2	1.2794	1.6358	0.5083	0.6321
0.4	1.3255	1.6616	0.4449	0.5847
0.6	1.3748	1.6893	0.3886	0.5406
0.8	1.4272	1.7188	0.3380	0.4995
1.0	1.4821	1.7501	0.2925	0.4611

Table 2: The amplitude and phase of rate of heat transfer.

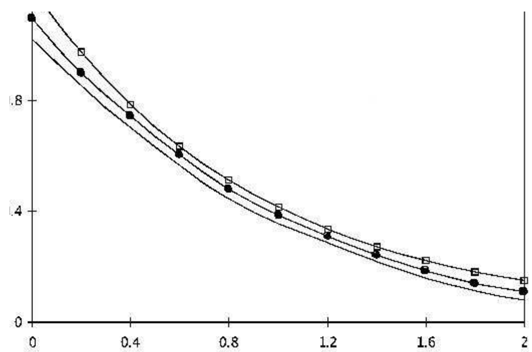

Figure 7: The concentration profile of carbon dioxide ($Sc = 0.94$) for $\omega = 10$ and $\epsilon = 0.2$.

4. The temperature and concentration both are increases near the plate and decreases exponentially far away from the plate.

5. The amplitude $|N|$ of rate of heat transfer increases due to the increase in A and ω both.

6. The phase of rate of heat transfer decreases with increasing A. Also, the rate of heat transfer increases with the increase of for the same value of suction parameter A.

Acknowledgement

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Nomenclature:

ε = amplitude (<< 1),
β = coefficient of thermal expansion,
β° = coefficient of thermal expansion with concentration,
ω = dimensionless frequency,
Θ = dimensionless temperature,
μ = viscosity,
ν = kinematics viscosity,
α = thermal diffusivity,
ω^* = frequency,
κ = thermal conductivity,
ρ = density,
τ = dimensionless shearing stress,
τ^* = shearing stress,
A = suction parameter,
C = dimensionless species concentration of CO_2
C^* = species concentration of CO_2,
C_p = specific heat at constant pressure,
C^*_∞ = species concentration of CO_2 in free stream,
C^*_w = concentration at the wall,
g = gravity,
Gc = modified Grashof number,
Gr = Grashof number,
h = rarefaction parameter,
L^* = constant,
$|M|$ = amplitude of skin-friction,
$|N|$ = amplitude of rate of heat transfer,
q = rate of heat transfer,
Pr = Prandtl number,
q^*_w = heat flux at the wall,
Sc = Schmidt number,
t = dimensionless time,
T^* = temperature,
T^*_∞ = temperature of fluid in free stream,
T^*_w = temperature of wall,
t^* = time,
u = dimensionless velocity component,
u^* = velocity component,
V = suction velocity,
D = molecular diffusivity of the species.

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