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CAPACITATED ARC ROUTING PROBLEM WITH TIME WINDOW IN SOLID WASTE OPERATION

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Vehicle routing problem, also known as node routing problem has been the focus of much research attention. On the contrary, capacitated arc routing problems (CARP) have been comparatively neglected. Both classes are NP-hard and extremely rich in theory and applications. From CARP point of view, a vehicle giving its service whiles it on the route. The capacity of the vehicle get increase along the routes and its service stops when reach capacity. In real life problem, operation of the vehicle is limited to certain time duration and several options occur for a vehicle for routing. In this paper, we introduce an extended problem case model of CARP that is CARP with time window (CARPTW) and its initial heuristic solution which is related to a solid waste operation in Johor Bahru.

3.1 OVERVIEW

In most countries, waste collection problems are frequently considered as environmental and pollution issues thus many approaches have been carried out onto development of policies and improving of waste management at administration level. On the contrary, very few researches have been done to improve the service of waste collection to customers instead. Compared to well-known VRP, CARP has been neglected for a long time, but has a growing interest in two last decades, mainly because its important applications in waste collection, inspection of power lines, and winter gritting.

In term of VRP, some researchers look at waste collection problem as an arc routing model while some other researchers modeled it into various dynamic VRP. The main difference is this; in arc routing model the focus is on the routes, not the nodes. This is because the vehicle/vehicles giving its service when they are on the routes. In other words, in waste collection problem from arc routing point of view, the customers are located along the routes, not at the nodes. The capacity of the vehicle increase when the vehicles are moved along the routes and from one route to another. Some latest expansion of CARP is mixed-CARP (MCARP) by Belenguer et al. [1], Bautista [2] and Mourao et al. [3], periodic CARP (PCARP) by Chu et al. [4] and Lacomme et al. [5], stochastic CARP (SCARP) by Fleury [6] and multiple depot CARP (MDCARP) by Zhengyu [7].

3.2 PROBLEM IDENTIFICATION

3.2.1 CASE STUDY

Our case study and data collection has been carried out onto Syarikat Perniagaan Zawiyah Sdn. Bhd., a waste collection contractor appointed by MBB. Our observation took place in Zone 6 and Zone 7 Taman Setia Indah and Taman Kempas Indah, Johor Bahru. The operation trip of one identical truck starts everyday from workshop depot at Tampoi to the nearest collection zone then to dumpsite area at Seelong (Figure 3.1). Dumpsite will be closed at 5pm everyday due to security reason, so collection of waste must be completed before 4pm, as traveling time to the dumpsite approximately is 45 minutes.

Initially, if vehicle cannot sent the loads to the dumpsite before 5pm due to lateness, it will be kept at the depot overnight before disposal tomorrow morning. By this means, the vehicle does not complete its service in that particular operation day. For next operation day, the vehicle must dispose the loads first before starts its collection at new location zone.

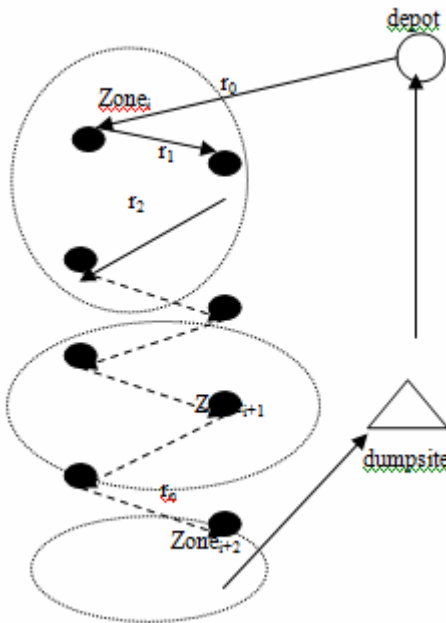


Figure 3.1. Conceptual model of truck operation in solid waste operation.

3.3 FORMULATION OF CARPTW

The CARP formulated by Dror and Langevin [8] is as follows. Given a connected graph $G = (V, E \cup A)$, with V as the set of nodes (vertices), E set of edges ($E \subseteq V \times V$) and A a set of arcs ($A \subseteq V \times V$), the objective of the problem is to find a minimum cost traversal of a given subset of edges and arcs in $R \subseteq E \cup A$. The CARP has an additional traversal cost for each edge and arc with edge (arc) demand $q_{ij} \geq 0$ for each edge (i, j) which must be serviced by one of a fleet of vehicles of capacity W . The problem is to find a number of circuits each of which passes through the depot which satisfies demands at minimal total cost.

We denote c_{ij} as the cost of an edge (arc) $(i, j) \in E(A)$ and x_{ijk} as the number of times edge (arc) $(i, j) \in E \cup A$ is traversed in trip k .

$$y_{ijk} = \begin{cases} 1 & \text{if the edge (arc) } (i, j) \in R \text{ is covered in trip } \\ & k; \\ 0 & \text{otherwise.} \end{cases}$$

M is a large constant greater than or equal to the sum of traversals of edges and arcs in a given $S \subseteq R$, $V[S]$ is the set of nodes incident to the arc set S , k denotes a trip, and K is the maximum number of trips allowed.

The objective function seeks to minimize total cost, and it is given as follows:

$$\text{Minimize } \sum_{(i,j) \in E} \sum_{k=1}^K (fc_{ij} + uc_{ij})x_{ijk},$$

$$\sum_{k=1}^K y_{ijk} = 1, \quad \forall (i, j) \in R.$$

$$x_{ijk} \geq y_{ijk}, \quad \forall (i, j) \in R, \text{ for } k = 1, 2, \dots, K$$

$$\sum_{(i,j) \in R} q_{ij}y_{ijk} \leq W, \text{ for } k = 1, 2, \dots, K$$

$$y_{ijk} \in \{0,1\} \quad \forall (i, j) \in R, k = 1, 2, \dots, K$$

$$x_{ijk} \in Z^+ \quad \forall (i, j) \in E, \quad k = 1, 2, \dots, K$$

The objective function seeks to minimize the total cost where fc denotes the constant fuel cost and uc is cost per unit of

household. Equation (3.1) ensures route discrete continuity. Equation (3.2) state that each edge with positive demand is serviced exactly once. Equation (3.3) guarantees that the traversal circuit k covers the edge $(i, j) \in R$ if it delivers its demand. Vehicle capacity is not violated on account of equation (3.4). Integrality restrictions are given in Equation (3.5) and (3.6).

Expansion of timing element in CARPTW as depicted in Figure 3.2. If T is total operation time then $T \leq A$, where A is a constant maximum service time. This inequation ensures the service times is not exceed the reasonable operation time before traveling to the dumpsite. Then, $T_{s_{rn}} = B$, where B is stochastic traveling time. Moreover, zero demands and service times are defined for this two nodes, that is, $d_0 = d_{r0} = s_0 = s_{r0} = 0$.

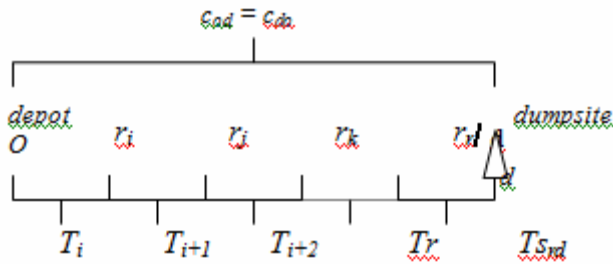


Figure 3.2. Timing element of CARPTW.

CARPTW can be extended to include arc cost $c_{od} = c_{do}$ which determined for traveling cost with loads from depot direct to the dumpsite.

3.4 HEURISTIC METHOD FOR THE INITIAL SOLUTION

3.4.1 NOTATIONS

Given the inherent computational difficulty of the routing problem, a variety of heuristics have been reported e.g. [9], [10] and [11], mostly for the hard time window. We implemented nearest procedure in order to find the first service route after traveling from the depot. Notations of variables are as follows:

- (1) $y_{init} = 0$, number of routes before first cycle starts.
- (2) $q_{init} = 0$, initial capacity for one vehicle before first cycle starts.
- (3) $c_{init} = 0$, initial cost for one vehicle before first cycle starts.
- (4) c_{ij} , cost from point i to point j ,
 E_{i+1} , next successor edge,
 $y = y_{init} + 1$, count of routes after each cycle starts.
- (5) $q_{new} = q_{init} + q_{ij}$, capacity at route ij ,
- (6) $q_{balnew} = q - q_{new}$, balance of capacity after collection at route ij .
- (7) $c_{new} = c_{init} + c_{ij}$, sum of route cost from depot to point i to point j .
- (8) $c = c_{init} + c_{new}$, increase of cost with increase the number of routes.
- (9) $q_{new} := \sum_{q=i}^n i + 1$, sum of capacity from point i to point n ,
assigned to capacity variable.
- (10) $q_{balnew} := q - \sum_{q=i}^n i + 1$, balance capacity from point i to
point n , assigned to balance capacity variable.
- (11) $c_{new} := \sum c_{ij}$, sum of all route cost, assigned to cost variable.

- (12) $q_{init} := q_{new}$, new capacity reassign to initial capacity after each cycle.
- (13) $c_{init} := c_{new}$, new cost reassign to initial cost after each cycle.
- (14) $q_{balnew} \geq q$, decision operator for capacity.

3.4.2 NEAREST PROCEDURE

Nearest procedure build a feasible solution by inserting at every iteration an unrouted customer into a previous continuity serviced routes. This process is performed one route a time.

Step 1: Input V and E . Set depot $O = initial$, $y_{init} = 0$, $q_{init} = 0$, cost $C_{init} = 0$, capacity $q = W$, trip $k = 0$.

Step 2: From O , $k := k + 1$, find the successor customer, V_i , compare and choose nearest j and E_{i+1} . Set $y = y_{init} + 1$, count new weight, $q_{new} = q_{init} + q_i$ and $q_{balnew} = W - q_{new}$. Count new cost, $c_{new} = c_{init} + c_{ij}$.

Step 3: If $q_{balnew} < W$, then check the next successor, V_{i+1} . Assigned $q_{new} := \sum_{q=i}^n i + 1$ and $q_{balnew} := W - \sum_{q=i}^n i + 1$. Assigned $c_{new} := \sum c_{ij}$. $Q_{init} := Q_{new}$ and $C_{init} := C_{new}$. If $Q_{balnew} \geq Q$. Assigned $y_{init} := y$. Terminate all served edges and go to Step 1.

Step 4: Repeat Step 1 until all served $y = V$. Void all served y . Count assigned pre-edges cost.

3.5 CONCLUSION

Despite the version of this problem is NP-hard and no algorithms or procedures are known for this CARPTW model. It seems feasible for small instances and could provide solution for a few routes. But it is too early to provide a computational

evaluation for this model. More robust algorithms must be prepared, appropriate lower bound must be developed, while no other algorithm is available for comparison. All these tasks are in progress.

3.6 REFERENCES

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