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Manuscript Draft

Manuscript Number: IH-4139R1

Title: Wear prediction of friction material and brake squeal using the finite element method

Article Type: Full-Length Article

Section/Category:

Keywords: Wear; Friction material; Contact analysis; Surface topography; Squeal; the Finite element method

Corresponding Author: Dr. Abd Rahim Abu Bakar, PhD

Corresponding Author's Institution: Universiti Teknologi Malaysia

First Author: Abd Rahim Abu Bakar, PhD

Order of Authors: Abd Rahim Abu Bakar, PhD; Huajiang Ouyang, PhD

Manuscript Region of Origin:

Abstract: This paper presents wear prediction of friction material in a disc brake assembly. A new and unworn pair of brake pads is tested under different durations of brake application to establish wear on their surfaces. One of the wear models available in the literature is adopted and then modified to suit the current work. A detailed 3- dimensional finite element (FE) model of a real disc brake is developed considering the real surface topography of the friction material. Confirmation of the adopted model is made between predicted and measured static contact pressure distribution and surface topography of the friction material. Predicted unstable frequencies and experimental squeal frequencies are shown to be in fairly good agreement.

Wear prediction of friction material and brake squeal using the finite element method

Abd Rahim AbuBakar[1*] and Huajiang Ouyang[2]
[1]Department of Aeronautics and Automotive, Faculty of Mechanical Engineering, Universiti Teknologi Malaysia, 81310 UTM Skudai, Johor, Malaysia
[2]Department of Engineering, the University of Liverpool, Brownlow St, L69 3GH, UK

Abstract

This paper presents wear prediction of friction material in a disc brake assembly. A new and unworn pair of brake pads is tested under different durations of brake application to establish wear on their surfaces. One of the wear models available in the literature is adopted and then modified to suit the current work. A detailed 3-dimensional finite element (FE) model of a real disc brake is developed considering the real surface topography of the friction material. Confirmation of the adopted model is made between predicted and measured static contact pressure distribution and surface topography of the friction material. Predicted unstable frequencies and experimental squeal frequencies are shown to be in fairly good agreement.

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1. Introduction

Wear is a dynamic process which quite often involves progressive dimensional loss from the surface of a solid body due to mechanical interaction between two or more bodies in frictional sliding contact. Wear of engineering components in most cases is regarded as a critical factor influencing the product life and even product performance. Research into wear modelling and prediction has been carried out for over fifty years [1]. To date, there are many wear models proposed for many different

*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159
Tel. No: +60755334572

situations. However, they only work for the particular material pair, contact geometry, operating conditions, and the particular environment and lubricant [2]. Archard [3] was one of the early researchers to develop a linear wear model for metals. In his model, the wear volume per sliding distance was in terms of wear coefficients, which can be interpreted in various ways in literature, for example, in terms of the contact force and material hardness. On the other hand, Rhee [4,5] was the early researcher who proposed a nonlinear wear model for friction material in a disc brake assembly and his seminal work was followed by some other researchers [6-8].

Given various wear models that are available in the literature, it is possible to select an appropriate one and simulate wear numerically. Numeral simulation and prediction of wear were reported in [8-18]. For wear of metals, the authors of [9-15] favoured Archard's wear law [3]. For wear of friction material in brake systems, Bajer *et al.* [17] used a very simple wear formula that is a linear function of the local contact pressure. Barecki and Scieszka [16], on the other hand, used almost the same empirical wear formula of Rhee [4] for their winding gear, post-type brake. AbuBakar *et al.* [18] used a modified Rhee's wear formula [4] and assumed all the constants in the formula as unity. All of the authors mentioned above compared their predicted wear states with experimental data, except in [17, 18].

The effect of wear on squeal generation has been studied experimentally by many researchers [19-21]. However, there is very little investigation by means of numerical methods. The authors of [17,18] recently attempted to relate wear with squeal generation using the finite element method. Bajer *et al.* [17] reported that considering the wear effect, predicted unstable frequencies were close to experimental ones. AbuBakar *et al.* [18] used wear simulation to investigate fugitive nature of disc brake squeal. In this paper, wear prediction and simulation are performed using a new pair of brake pads. The friction material is subjected to three different stages of brake application to establish wear on the pad surface. The wear formula proposed by Rhee [4] is modified for the current investigation. Instead of verifying wear displacement/intensity/volume as has been adopted by the authors of [8-16], this paper attempts to verify wear progress predicted in the simulation using measured static contact pressure distributions from contact tests and measured surface topography of the friction material. Pressure indicating film and an associated pressure analysis system are used to obtain pressure distributions and magnitude. From the comparison, realistic values of those constants required in the modified wear formula are obtained. Then stability analysis through complex eigenvalue analysis is performed to predict unstable frequencies under various wear conditions. The predicted results are then compared with the squeal events observed in the experiments.

2. Wear and contact tests

In this work, a new and unworn pair of brake pads is subjected to three different durations of brake application under a brake-line pressure of 1 MPa and at a rotational speed of 6 rad/s. In the first stage, the brake is applied for 10 minutes. Another 10 minutes is used in the second stage. In the third and final stage, the friction material is run for 60 minutes. At the end of each stage, the disc is stopped and then the stationary disc is subjected to a brake-line pressure of 2.5MP (a contact test). In order to verify predicted results from the FE analysis, Super Low (SL) pressure indicating film, which can accommodate local contact pressure in the range of 0.5 ~ 2.8 MPa, is used. The tested film shown in Fig. 1 can only provide stress marks but cannot reveal the magnitude. To determine the stress levels, a post-process interpretive system

*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159
Tel. No: +60755334572

called Topaq pressure analysis system that can interpret the stress marks is utilised. The system is reported to be accurate to within $\pm 2\%$, which is very accurate in the field of tactile pressure measurement [22].

A disc brake assembly shown in Fig. 2 consists of a pair of a piston (inboard) and a finger (outboard) pads. Once the brake is applied the hydraulic pressure is exerted onto the top of piston and the inside of the piston chamber in the calliper housing. This brings both pads into contact with the sliding disc. Since the pads are much softer than the disc, it wears more rapidly than the disc. Fig. 3 shows measured static contact pressure distributions at the piston and finger pads at the end of all stages of braking. It can be seen from the figures that contact pressure distributions vary as wear progresses in time (note that the blank part with no colour within the circumference of a pad implies no contact of zero pressure). The results also show that the area in contact is also gradually increasing. This may suggest that the surface topography of both pads that initially have a rough surface becomes either smoothened or glazed.

In order to visualise the surface topography, measurements are made in the middle of the circumferential direction of the friction material using a linear gauge LG-1030E and a digital scale indicator (see Fig. 4). Surface topography or height distributions of new friction material at the final stage of braking are illustrated in Fig. 5. It can be seen from Fig. 5a that height distribution of the piston pad after 80 minutes of braking application is in a similar pattern to the initial distribution, except at the trailing edge. For the finger pad as shown in Fig. 5b the height distribution is very similar to the initial distribution but in different magnitude particularly at the trailing edge.

3. Finite element model

A detailed 3-dimensional finite element (FE) model of a Mercedes solid disc brake assembly is developed and illustrated in Fig. 6. The FE model consists of a disc, a piston, a calliper, a carrier, piston and finger pads, two bolts and two guide pins, as shown in Table 1. A rubber seal (attached to the piston) and two rubber washers (attached to the guide pins) in this brake assembly are not included in the FE model. Damping shims are also absent since they have been removed in the squeal experiments. The FE model uses up to 8350 solid elements and approximately 37,100 degrees of freedom (DOFs). Fig. 7 shows a schematic diagram of contact interaction that has been used in the disc brake assembly model. A rigid boundary condition is imposed at the boltholes of the disc and of the carrier bracket, where all six degrees of freedom are rigidly constrained, as those places are stiffly attached to very strong supports in the rig on which the experimental squeal frequency was observed.

Since the contact between the disc and friction material is crucial, a realistic representation of the interface should be made. In this work, actual surfaces at the piston (inboard) and finger (outboard) pads are measured and considered at macroscopic level. A Mitutoyo linear gauge LG-1030E and a digital scale indicator are used to measure and provide readings of the surface respectively, as shown in Fig. 4. The linear gauge is able to measure surface height distribution from 0.01mm up to 12 mm.

Node mapping, as shown in Fig. 4, is required so that surface measurement can be made at desirable positions, which are the FE nodes of the pad surfaces. By doing this, information that is obtained in the measurement can be used to adjust the normal coordinates of the nodes at the contact interfaces. There are about 227 nodes on the

*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159
Tel. No: +60755334572

piston pad interface and 229 nodes on the finger pad interface. Fig. 8 shows the surface topography of the piston and finger pads. The FE model has been validated through three validation stages described in [18] and details of the material data are given in Table 2.

4. Wear simulation

Rhee's wear formula [4] postulates that the material loss ΔW of the friction material at fixed temperature is dependent upon the following parameters:

$$\Delta W = kF^a v^b t^c \tag{1}$$

where k is the wear rate coefficient obtained from experiments, F is the contact normal force, v is the disc speed, t is the sliding time and a, b and c are the set of parameters that are specific to the friction material and the environmental conditions. This original formula however cannot be used in the present investigation. Since mass loss due to wear is directly related to the displacements that occur on the rubbing surface in the normal direction, Rhee's wear formula is then modified as:

$$\Delta h = kP^a (\Omega r)^b t^c \tag{2}$$

where Δh is the wear displacement, P is the normal contact pressure, Ω is the rotational disc speed (rad/s), r is the pad mean radius (m) and a, b and c are all constants which remain to be determined. Since no experimental data on wear rate coefficients have been obtained in this work, this coefficient value is adopted from [23] as $k = 1.78\times10^{-13}$ m^3/Nm because of the same disc material and almost the same brake-line pressure and wear duration. The pad mean radius for current disc brake assembly is r = 0.111 m, the sliding speed is maintained at 6 rad/s and the total sliding time is set to t =4800s (80 minutes). The seemingly short duration of wear tests is due to a numerical consideration. In the wear formula of equation (2), the duration of wear, t, must be specified. The longer the duration of wear, the more the dimensional loss and the greater change of the contact pressure. However, if t is too big, there will be numerical difficulties in an ABAQUS run. It has been found through trial-and-error that t = 200 s gives reasonably good results and good efficiency. Consequently a simulation of 80-minute wear means twenty-four ABAQUS runs. In line with this numerical consideration, wear tests have not lasted for numerous hours as normally done in a proper wear test or a squeal test. In theory, however, numerical simulations of wear may cover an arbitrary length of time.

Firstly, contact analysis of the FE model is performed similarly to the operating conditions of the experiments described in section 2. From the contact analysis, contact pressure can be obtained and hence wear displacements can be calculated. Having obtained the wear displacements for each wear simulation, nodal coordinates at the friction interface model in the axial direction are adjusted. This process continues until it reaches braking duration of 80 minutes. Fig. 9 shows the procedure to predict wear on the friction material interface.

It is worthwhile noting that during this wear calculation all constants in Eq. (2) need to be determined. In the early stage of their investigation, the authors adopted the constant values of [4-7, 18]. However, none of those could produce satisfying contact pressure distributions for the entire braking duration. Having simulated for various values of constants a, b and c, it is found that the wear formula below

$$\Delta h = k_0 \left(\frac{P}{P'}\right)^{0.9} \Omega rt \qquad (3)$$

seems to give a close prediction of contact pressure distributions and surface topography of the friction material to the experimental ones for the entire braking duration. In equation (3), P' is the maximum allowable braking pressure (8MPa for a

*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159
Tel. No: +60755334572

passenger car) and $k_0 = 2.9\times10^{-7}\,\mathrm{m}^3/\mathrm{Nm}$. Fig. 10 illustrates predicted static contact pressure distributions at the piston and finger pads. It is seen that areas in contact increase as braking duration approaches 80 minutes described in Fig. 11. From the figure, the initial contact areas are predicted as about 7.0e-4 m^2 for both pads and then are predicted as much as 2.9e-3 m^2 in the final stage of braking duration. This is an increase by more than four folds.

Comparison is also made on the surface topography or height distribution of the friction material. The height distribution is calculated based on the measured or predicted surface heights minus the lowest height value of the friction material interface. From Fig. 12 it can be seen that predicted height distributions for the piston and finger pads are very close to the measured ones. This work is not intended to verify the wear displacement (Δh). Instead the authors are interested in the changes of surface topography or surface roughness due to wear and its important influence on the squeal generation to be discussed in the following section. Another feature that can be seen from this figure is that the leading edge for both pads is experiencing more wear than the trailing edge. This is because the leading edge is subjected to high pressure due to sliding friction.

The simulated wear progress with time is shown in Fig. 13 for the piston and finger pads. The surface profiles look jagged indicating a rough surface. Later, when the braking duration reaches 80 minutes the surface profiles become smoother and level off. This is reflected by the decreasing arithmetic mean surface roughness R_a (in meter) of the two pads over time. Note that wear causes the axial coordinates of the two pad surfaces to change in an opposite manner as the two surfaces are facing each other in a brake system.

5. Stability analysis

There are typically two big different approaches available to predict squeal noise and they are the transient dynamic analysis and complex eigenvalue analysis. However, complex eigenvalue analysis is much preferred in the brake research community due to its maturity and other advantages over the transient dynamic analysis [24-26]. Nevertheless, both analyses should include frictional contact analysis as the first, integral part of the analysis procedure. For complex eigenvalue analysis, contact pressure distribution is essential to establish asymmetric stiffness matrix that leads to the complex eigenvalues. The positive real parts of the complex eigenvalues are thought to indicate possible squeal noise that occurs in a real disc brake assembly. Therefore, it is important to include realistic surface topography of the friction material, especially when wear is under consideration.

In this paper, complex eigenvalue analysis is conducted using ABAQUS v6.4. The analysis procedure is the same as that reported in [17,18]. In order to perform the complex eigenvalue analysis using ABAQUS, four main steps are required as follows:

- Nonlinear static analysis for applying brake-line pressure
- Nonlinear static analysis to impose rotation of the disc
- Normal mode analysis to extract natural frequencies and modes of undamped system
- Complex eigenvalue analysis that incorporates the effect of contact stiffness and friction coupling

Having completed wear simulations for all stages of brake application, stability analysis is performed using complex eigenvalue analysis. A similar operating condition to that of the experiments is applied. Kinetic friction coefficient is set to $\mu_k = 0.393$ which is determined from the experiments of [27]. From the complex

*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159 9
Tel. No: +60755334572

eigenvalue analysis, it is found that there is an unstable frequency at 4.2 kHz with positive real parts of 39.8 predicted towards the end of braking. Significantly, for the first two stages of braking such an unstable frequency is not predicted. Hence this result is in good agreement with the observation made in the experiments given in Table 3.

6. Conclusions

The experimental results show that the contact area of a new and unworn friction material increases and initial rough surfaces later become smoother or glazed as wear progresses. These are also clear in the simulation results. It is found that the leading edge is prone to more wear than the trailing edge.

The results from simulation show a reasonably good correlation with the experimental results in the static contact pressure distribution and height distributions. Good agreement is also found between the unstable frequency predicted in the stability analysis and the squeal frequency recorded in the experiment. From these results it is suggested that the wear formula modified in this work can be used to predict wear progress and changes in surface topography, based on which disc brake squeal can be captured better in the stability analysis.

Acknowledgements

The first author would like to thank Dr Simon James for running wear tests, Sensor Products LLC for providing Fuji film and analysing images and Universiti Teknologi Malaysia for providing financial support.

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*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159
Tel. No: +60755334572

Figure Captions

Fig. 1. Pressure indicating film before (left) and after (right) testing

Fig. 2. Disc brake assembly

Fig. 3. Measured contact pressure distribution for different braking durations (the top side is the leading edge)

Fig. 4. Facilities for measuring height distribution of the friction material

Fig. 5. Surface topography of the friction material

Fig. 6. FE model of the disc brake assembly

Fig. 7. Contact interaction between disc brake components

Fig. 8. Contact interface model of the friction material: piston pad (left) and finger pad (right)

Fig. 9. FE wear simulation procedure

Fig. 10. Predicted contact pressure distribution for different braking duration (the top side is the leading edge)

Fig. 11.Predicted contact area for different braking duration

Fig. 12. Comparison of height distribution between the FE results and experimental results

Fig. 13. Predicted wear profile for different braking duration

*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159
Tel. No: +60755334572

Table(s)

Table

Table 1 Description of disc brake components

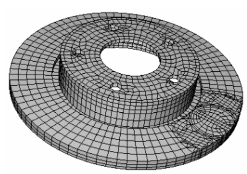
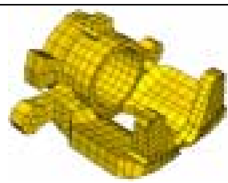
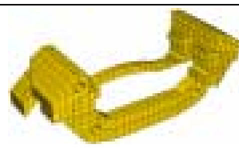
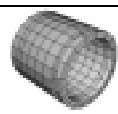
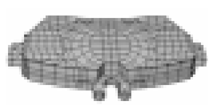
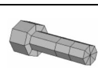
Components		Types of element	No. of elements	No. of nodes
	**Disc**	C3D8 C3D6	3090	4791
	**Calliper**	C3D8 C3D6 C3D4	1418	2242
	**Carrier**	C3D8 C3D6 C3D4	862	1431
	**Piston**	C3D8 C3D6	416	744
	**Back plate**	C3D8 C3D6	2094	2716
	Friction Material	C3D8 C3D6		
	**Guide pin**	C3D8 C3D6	388	336
	**Bolt**	C3D8 C3D6	80	110

Table 2 Material data of disc brake components

	DISC	BACK PLATE	PISTON	CALLIPER	CARRIER	GUIDE PIN	BOLT	FRICTION MATERIAL
Density (kgm^{-3})	7107.6	7850.0	7918.0	7545.0	6997.0	7850.0	9720.0	2798.0
Young's modulus (GPa)	105.3	210.0	210.0	210.0	157.3	700.0	52.0	Orthotropic
Poisson's ratio	0.211	0.3	0.3	0.3	0.3	0.3	0.3	-

Table 3 Observation and prediction of squeal noise

Length of braking application (minutes)	**Experiments Squeal noise**	**FE analysis Unstable frequency**
10	No	No
20	No	No
80	4.0 kHz	4.2 kHz (+39.8)

Figure(s)

Figures

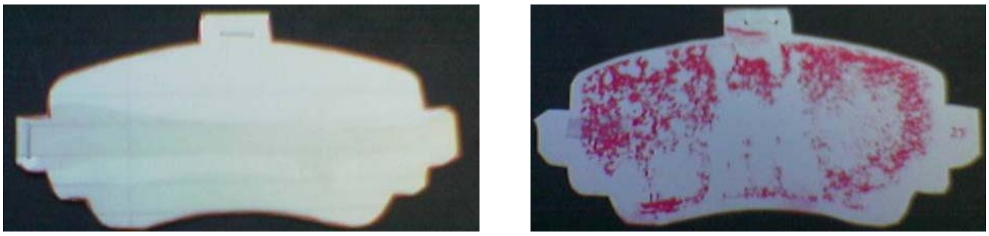
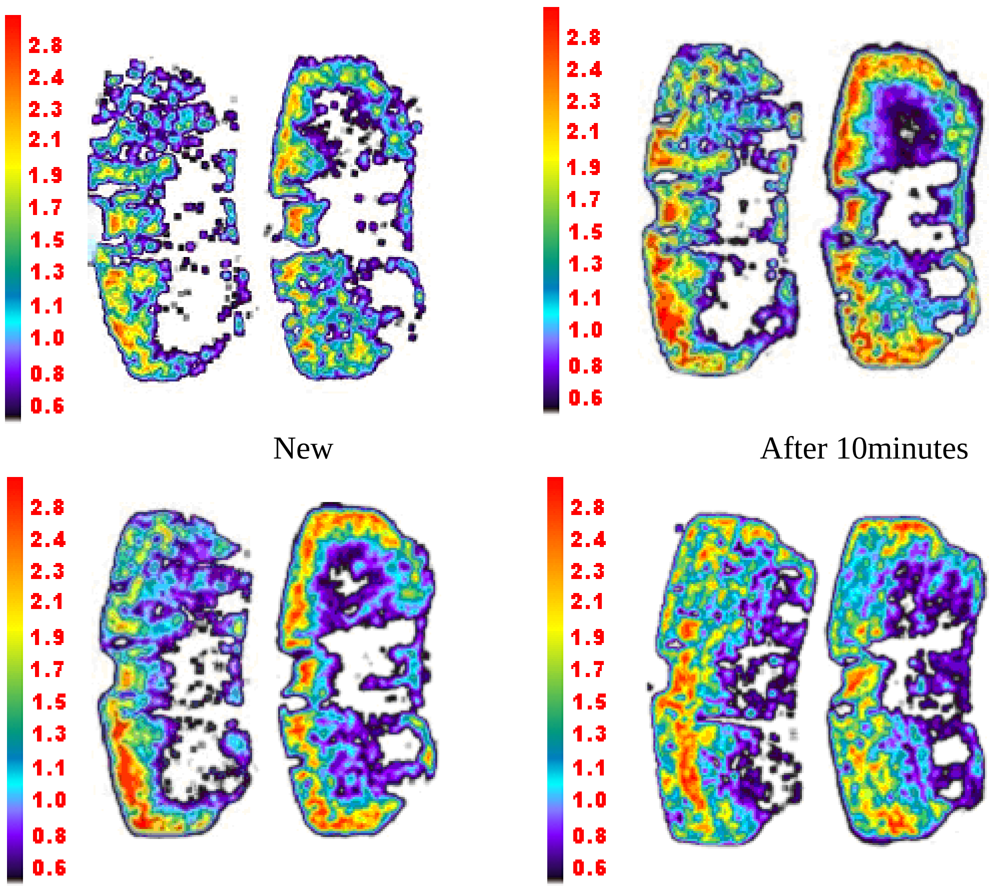


Fig. 1

Fig. 2



*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159 1
Tel. No: +60755334572

After 20 minutes After 80 minutes

Fig. 3

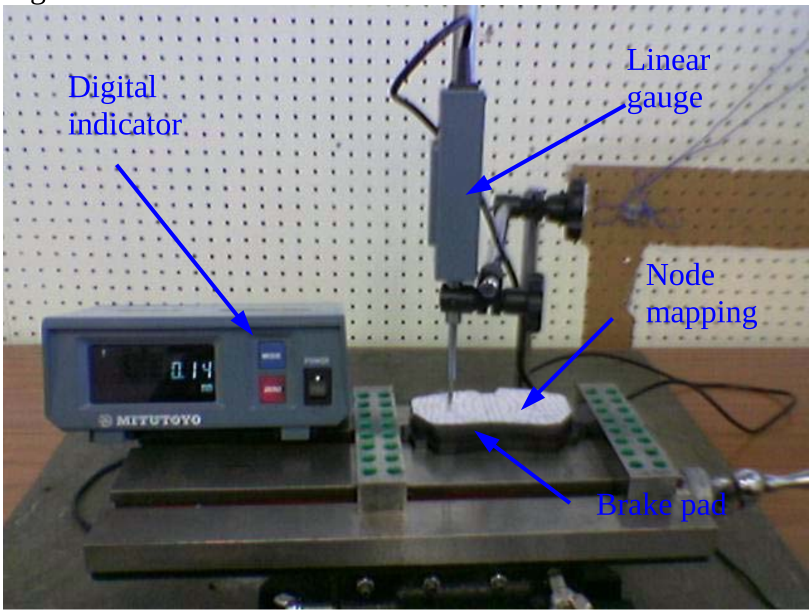
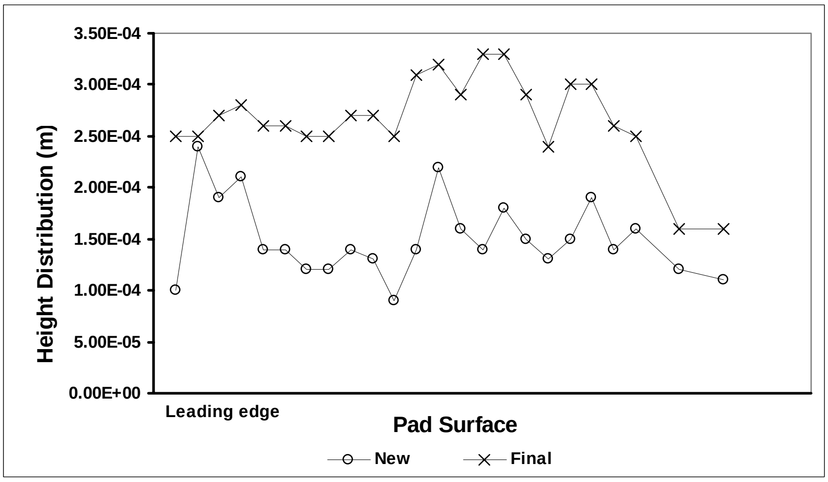
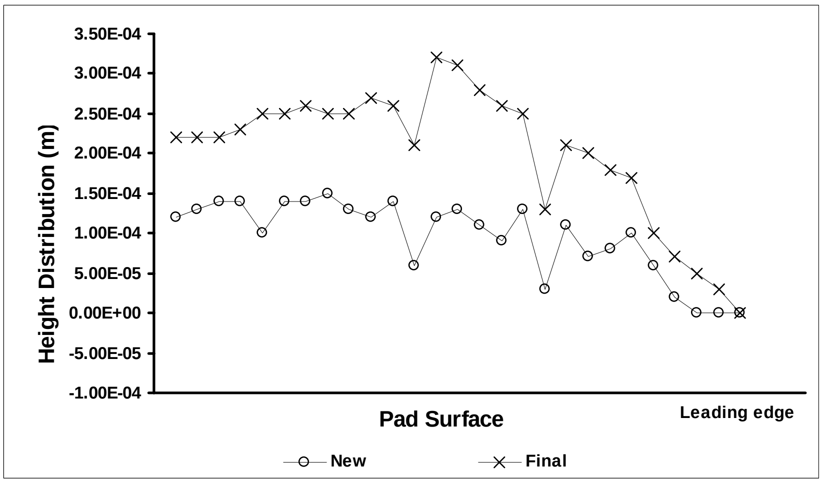


Fig. 4



a) Piston pad



*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159 2
Tel. No: +60755334572

b) Finger pad
Fig. 5

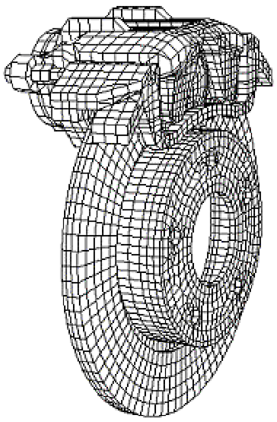


Fig. 6

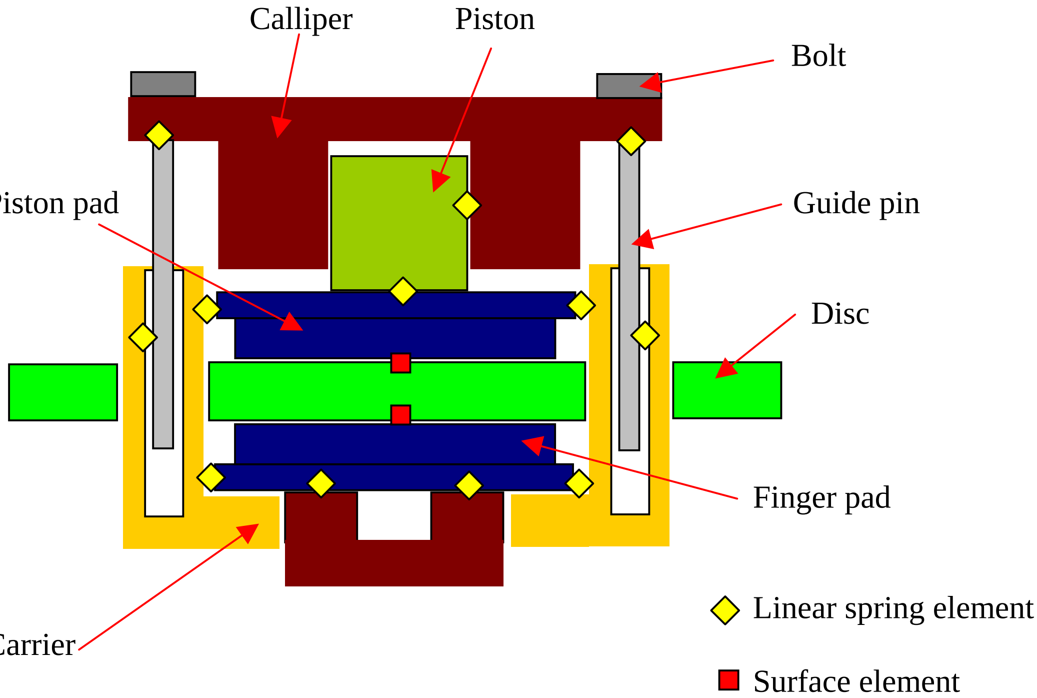
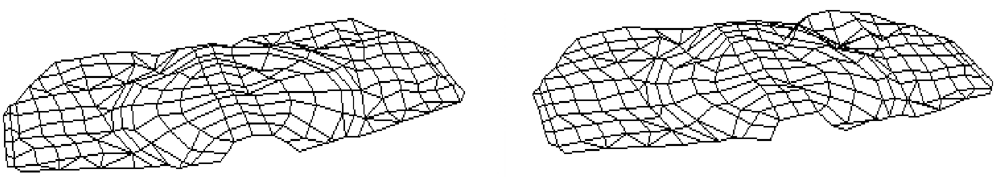


Fig. 7



*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159 3
Tel. No: +60755334572

Fig. 8

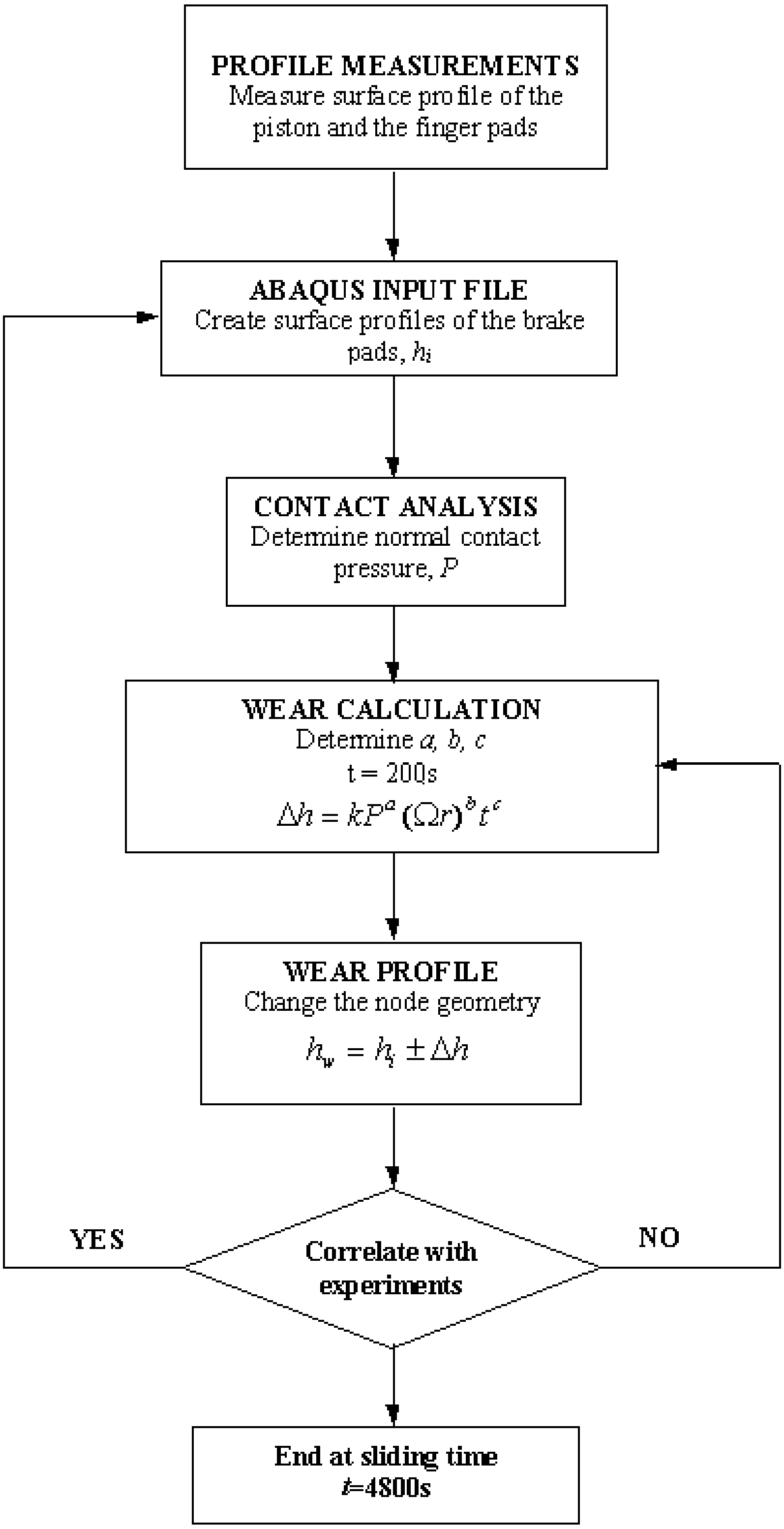
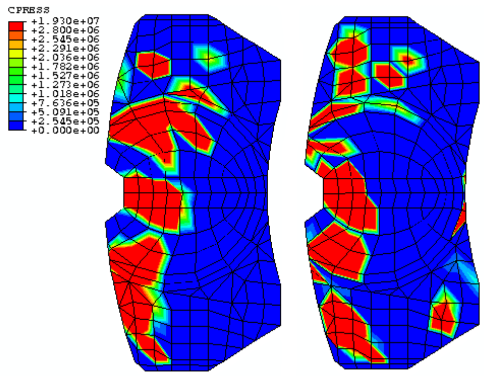
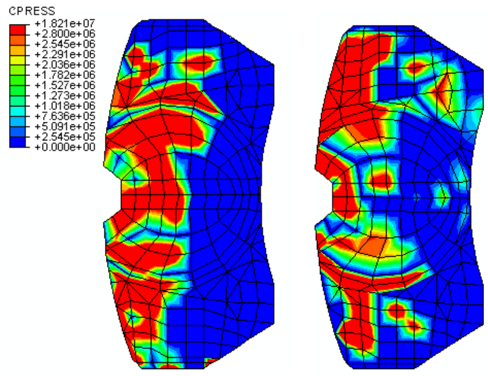


Fig. 9

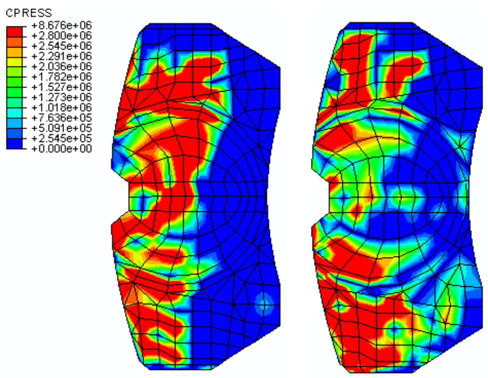
*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159 4
Tel. No: +60755334572



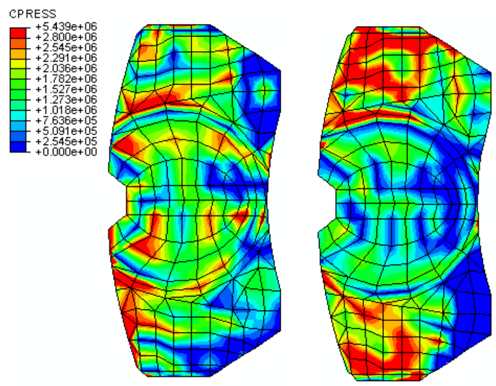
New



After 10 minutes

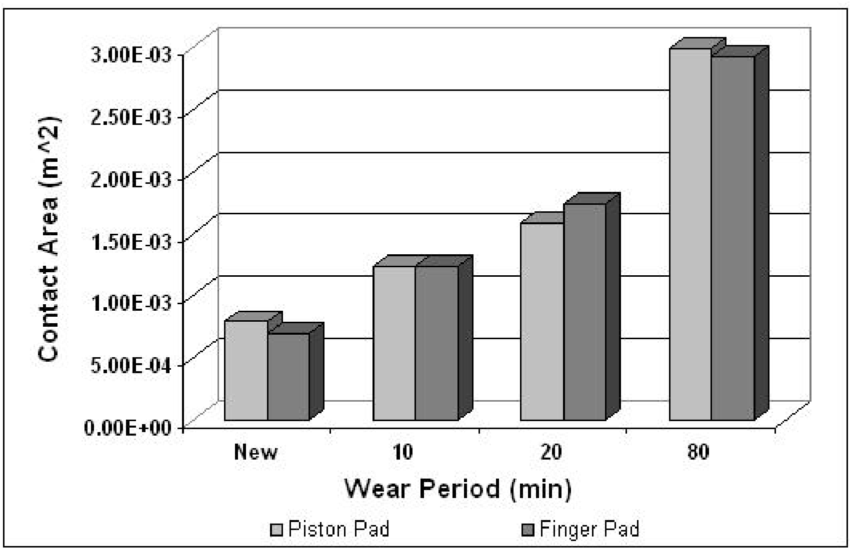


After 20 minutes



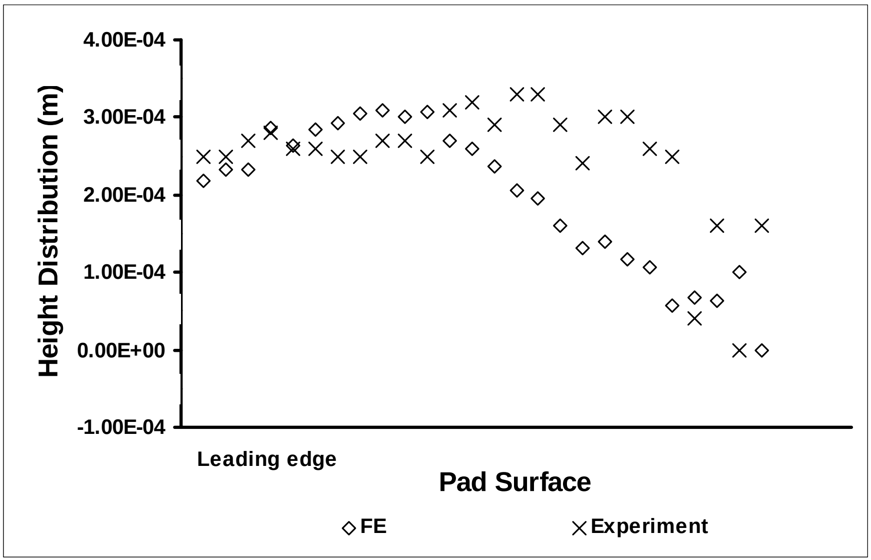
After 80 minutes

Fig. 10

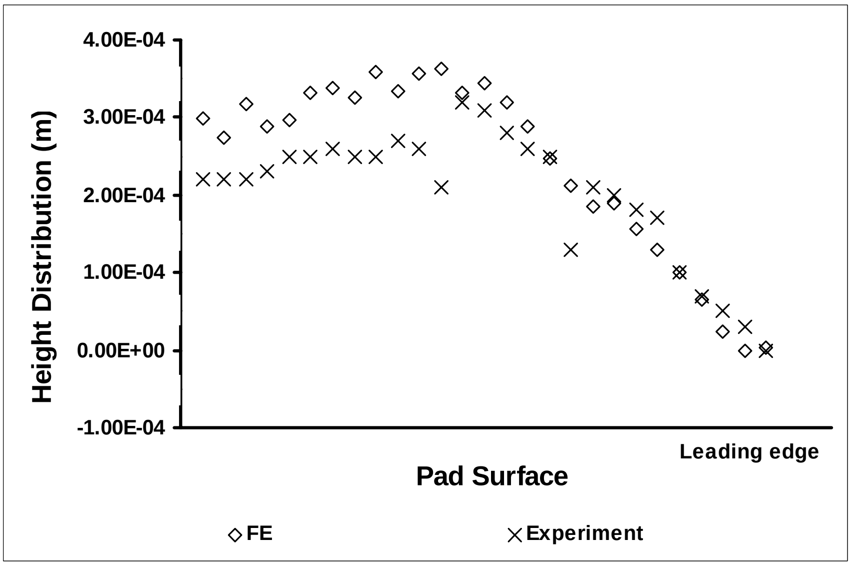


*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159 5
Tel. No: +60755334572

Fig. 11



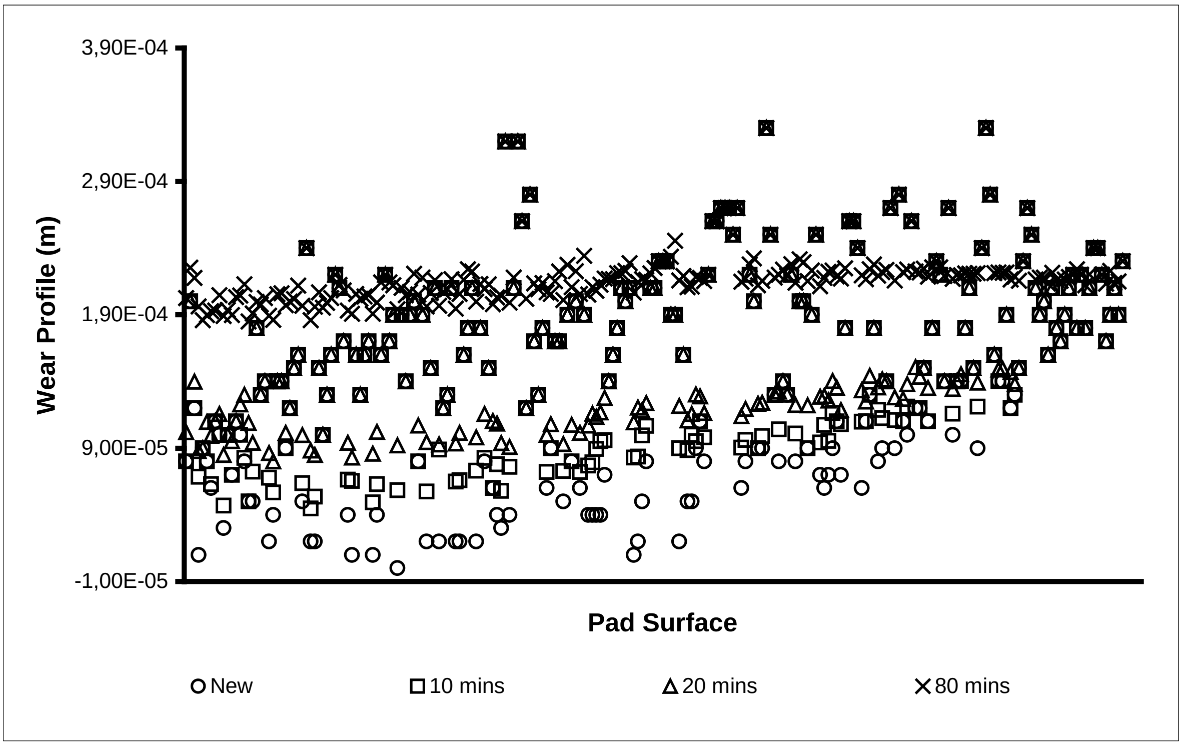
a) Piston pad



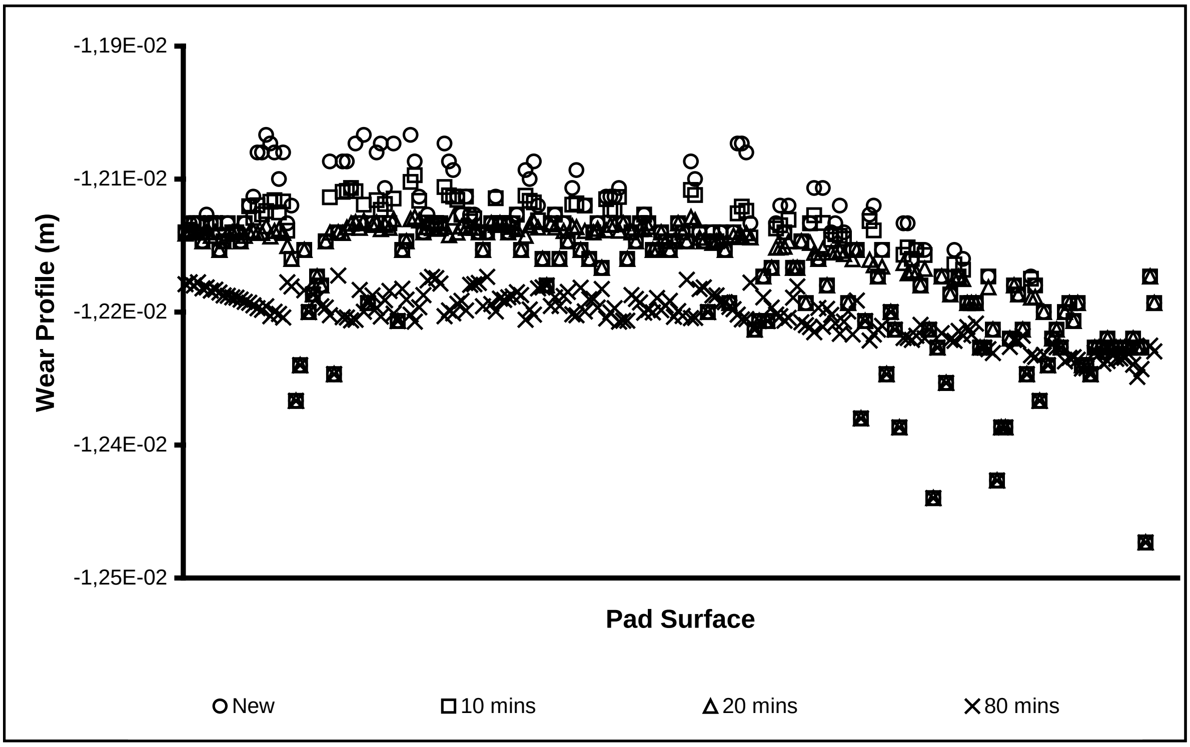
b) Finger pad

Fig. 12

*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159 6
Tel. No: +60755334572



a) Piston pad ($R_a = 6.44\times10^{-5}, 5.60\times10^{-5}, 4.72\times10^{-5}, 1.48\times10^{-5}$ respectively)



b) Finger pad ($R_a = 6.28\times10^{-5}, 5.64\times10^{-5}, 5.04\times10^{-5}, 2.84\times10^{-5}$ respectively)

Fig. 13

*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159 7
Tel. No: +60755334572

*Corresponding author: Abd Rahim Abu Bakar, email: arahim@fkm.utm.my, Fax. No: +6075566159 8
Tel. No: +60755334572

Response to the First Reviewer's Comments.

We are very grateful to the referee for the helpful comments and have made substantial modifications to the paper based on these comments.

1. Abstract & Introduction

Done.

The authors have changed it to "A new and unworn pair of the brake pads". "brand new" means have not been used in a brake before the testing.

2. Wear and contact tests

Done.

Figures 3 and 4 have been swapped accordingly. Figure 3 is enlarged and is now clear.

It is indeed difficult to tell from Figure 5 that the surfaces become smoother as wear progresses. To clarify this important point, values of arithmetic mean roughness Ra are now given for each of the four states of wear progress in Figure 13, as a complement to Figure 5.

3. Finite element model

Done.

Detailed information on the FE models for all brake components is given in a new table (Table 1). It should be clear now that the FE mesh densities of different brake components are normally different. Material data are given in another new table (Table 2).

4. Wear simulation

Done.

The wear model is now re-formulated in terms of a non-dimensional ratio of the applied pressure to the maximum allowable pressure.

We have numerically experimented with continuous wear durations of 100s, 200s, 400s and 600s, and found that 200s is a good compromise between accuracy and efficiency. Simulation of continuous wear over a longer duration leads to divergence in the numerical results.

The exponent values are obtained from a trial-and-error process. The chosen ones result in fairly good agreement between the predicted contact pressure and measured contact pressure overall at the four time instants (no wear at the beginning, at the ends of continuous wear for 10 minutes, continuous wear for another 10 minutes and continuous wear for final 60 minutes), when measurements are taken.

We are applying the wear law to a specific application and admittedly wear and friction themselves are not our areas of research.

5. Stability analysis

The friction coefficient is determined from the experimental results reported in the PhD thesis of Dr Simon James of Liverpool University. If a friction coefficient of 0.4 is used, the unstable frequency will be the same but its real part will be slightly greater.

6. Conclusions

The constants in the wear formula were determined using the measured contact pressure but not measured squeal frequency. In addition, the stiffness values at the interfaces other than the disc and pads interface were validated in the past using modal testing data. Therefore the prediction is genuine --- the measured squeal frequency was not used at any point in the determination of any wear constants or system parameters.

Nonlinearity

We used the linear wear law of a=b=c=1 in a previous investigation reported in Reference 18. In comparison with the nonlinear law of a=0.9, the linear law produced good results for the first braking stage (the first 10 minutes of wear) but deteriorating results afterwards.

Additional modifications

1) We think that we have made a big improvement to the standard of English and the quality of the writing.
2) We have added three more references for the stability analysis and updated Reference 22.
3) Figures 7, 12 and 13 have been made clearer.

We hope the above explanations and modifications are satisfactory. We shall be happy to make further modifications if the referee deems them useful.