Modeling of Temperature Distribution in Orthogonal Cutting with Dual-Zone Contact at Rake Face

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Abstract: In this study, an analytical model is developed in order to calculate the temperature distribution in orthogonal cutting with dual-zone contact at the rake face. The study focuses on heat generation at the primary shear zone and at the rake face. The material behavior at the primary shear zone is represented by Johnson-Cook constitutive equation whereas the contact at the rake face is modeled by sticking and sliding friction zones. This new temperature distribution model allows obtaining the maximum temperature at the rake face and helps determining two dimensional temperature distribution in the chip. The simulation results obtained from the developed model are also compared with experimental results where good agreement is observed.
Keywords: orthogonal cutting, temperature distribution modeling, dual-zone contact.

1. INTRODUCTION

Being one of the most important problems in machining, the heat generation affects selection of cutting parameters strongly. There are three heat generation regions in the cutting process. Depending on the thermal conductivity of the tool and the workpiece, a portion of the heat is transfers to the tool and causes diffusion and thermal stresses which trigger tool wear. Cutting temperatures, especially maximum temperature at tool-chip interface is very important for the tool life. Thus, temperature in machining is an important issue which is the topic of this paper.

First study to calculate temperature analytically was made by [Trigger *et al*., 1951]. In their model, the average tool-chip interface temperature according to primary and secondary deformation zones is calculated. Loewen [Loewen *et al*., 1954] developed analytical model to calculate the temperature on the tool-chip interface considering that the portion of heat flow into the chip is constant along the rake face and tool acts like a quarter-infinite body. Komanduri [Komanduri *et al*., 2000] developed a model using a moving-band heat source for the chip and a stationary square heat source for the tool considering additional boundaries which are the top surface of the chip and the clearance face of the tool. In addition, they took the non-uniform heat intensity caused by the non-uniform heat partition into account. Moufki [Moufki *et al*., 1998]developed a thermomechanical model of the primary shear zone which was combined with a model of the contactat the tool-chip interface. Their assumption for the secondary deformation zone was the Coulomb friction law with a mean friction coefficient depending on the mean temperature.

In this study, an analytical thermal model for two dimensional temperature distribution in chip is developed for orthogonal cutting conditions. Heat generation model in primary deformation zone is taken from [Moufki *et al.*, 1998]. Secondary deformation zone is modelled with dual-zone contact approach which is explained in detail in [Budak *et al.*,2008; Ozlu *et al*., 2009].

2. MODELLING OF THE HEAT GENERATION IN PRIMARY SHEAR ZONE

In this approach, the chip is formed by shearing in a narrow straight band of thickness *h* as shown Figure 1 [Moufki *et al*., 1998; Dudzinski *et al.*, 1997]. It is assumed that the material is deformed only in this band and no deformation occurs before and after the band.

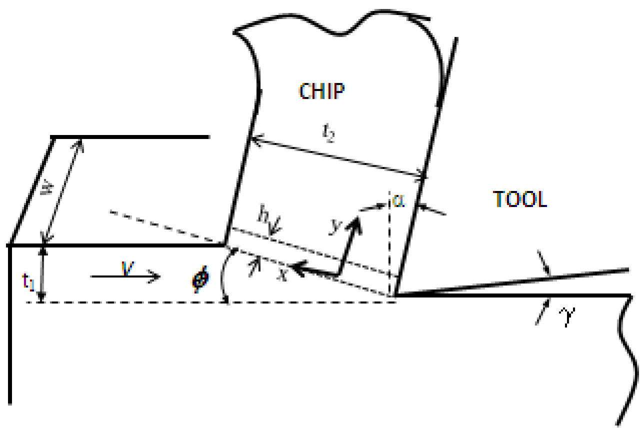


Figure 1; Primary shear zone model.h is thickness of band, ϕ is shear angle.

t_1 is the uncut chip thickness and *w* is the width of cut.The cutting edge of tool is assumed to be perfectly sharp and undeformable. *V* is the cutting speed and α is the rake angle.

It is assumed that the workpiece material is isotropic, rigid and under shear conditions its thermomechanical behaviour is expressed by Johnson-Cook material model.

The model in primary shear zone is restricted to the case of stationary flow. The proposed model is one dimensional and time independent.Johnson-Cook material model can be written as follows:

$$\tau = \frac{1}{\sqrt{3}}\left[A + B\left(\frac{\gamma}{\sqrt{3}}\right)^n\right]\left[1 + ln\left(\frac{\dot{\gamma}}{\dot{\gamma_0}}\right)^C\right]\left[1 - \left(\frac{T_w - T_r}{T_m - T_r}\right)^m\right] \quad (1)$$

where γ is shear strain, $\dot{\gamma}$ is shear strain rate, $\dot{\gamma_0}$ is reference shear strain rate, T_w workpiece temperature, T_r reference temperature, T_m melting temperature, *A, B, n, C* and *m* material constants.

With this model we can calculate the temperature at the exit of the shear band T_1 which is also a boundary condition for the heat equation in the second deformation zone.τ_1 is the shear stress at the exit of the primary shear zone and sticking zone. Shear angle ϕ is calculated iteratively by minimization of the cutting energy.

3. MODELLING OF THE HEAT GENERATION IN SECONDARY SHEAR ZONE

Heat generation in the secondary shear zone occurs due to the friction at the tool-chip interface (Figure 2). In the proposed model, the identification procedure is taken from [Ozlu *et al*., 2009] where the tool-chip contact is divided in to sticking and sliding friction regions.

Plastic deformation in the primary shear zone and the friction at the tool-chip interface affect the temperature distribution in the chip. The stationary temperature distribution is determined by solving the following two dimensional heat equation (eqn. 2). Heat conduction in the flow direction is neglected due to its negligible effect compared to the convection.

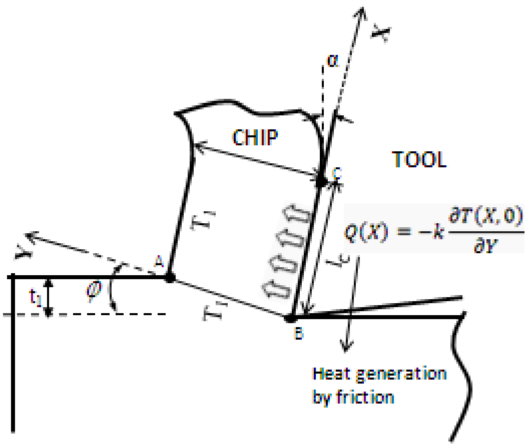


Figure 2; Thermal problem in the tool-chip interface.

$$\alpha \frac{\partial^2 T(x,y)}{\partial y^2} = V_C \frac{\partial T(x,y)}{\partial x} \tag{2}$$

$a = k/\rho c$ is the thermal diffusivity of the workpiece where ρ is the material density, k is the heat conductivity, c is the heat capacity and V_C is the chip velocity.
where the chip velocity is:

$$V_C = V \frac{sin\emptyset}{cos(\emptyset - \alpha)} \tag{3}$$

The term in the right hand side of eqn. (2) is the material derivative of the temperature [Moufki*et al*., 1998].

As can be seen from Figure 2 the temperature is assumed to be equal to T_1 at the enter of the secondary shear zone which is the exit of the primary shear zone and also the temperature at the chip's free surface is assumed as T_1. Therefore boundary conditions for solving eqn. (2) are:

$$T(0,Y) = T_1 \quad x = 0, y \geq 0 \text{(exit of the primary shear zone)} \tag{4.1}$$

$$\lim_{Y\to\infty} T(x,y) = T_1 \quad for\ x \geq 0 \tag{4.2}$$

$$-k\frac{\partial T(x,0)}{\partial y} = Q(x) \quad for\ x \geq 0 \tag{4.3}$$

T_1 is the temperature at the exit of the primary shear zone and *Q(x)* is the heat generated by friction.

Equations (2) and (4) are solved by Laplace transformation similar to the procedure in [Wright *et al*., 1980]. $\bar{T}(s,y)$ is the Laplace transform of the temperature with respect to the *x*.

$$\bar{T}(s,y) = \int_0^\infty T(x,y)e^{-sx}dx \tag{5}$$

$$a\frac{d^2\bar{T}(s,y)}{dy^2} = V_C(s\bar{T}(s,y) - T_1) \tag{6}$$

$$\bar{T}(s,\infty) = \frac{T_1}{s} \tag{7}$$

$$-k\frac{d\bar{T}(s,y)}{dy}\bigg|_{y=0} = f(s) \tag{8}$$

$f(s)$ is the Laplace transform of $Q(x)$ which is the heat generated by friction.

$$f(s) = \int_0^{l_C} Q(x) e^{(-sx)} dx \tag{9}$$

The solution of the equation (5):

$$\bar{T}(s,y) = c_1 e^{y\sqrt{\frac{sV_C}{a}}} + c_2 e^{-y\sqrt{\frac{sV_C}{a}}} + \frac{T_1}{s} \tag{10}$$

From the boundary conditions (6) and (7), c_1 and c_2 are obtained as follows:

$$c_1 = 0,\ c_2 = \frac{f(s)}{k}\sqrt{\frac{a}{sV_C}} \tag{11}$$

$$\bar{T}(s,y) = \frac{f(s)}{k}\sqrt{\frac{a}{sV_C}}\, e^{\left(-y\sqrt{\frac{sV_C}{a}}\right)} + \frac{T_1}{s} \tag{12}$$

Using convolution property of Laplace and inverse Laplace transform, the temperature distribution in the chip is obtained as follows:

$$T(X,Y) = \frac{1}{k}\sqrt{\frac{a}{V_C}} \int_0^X Q(x-u) \frac{1}{\sqrt{\pi u}} e^{\left(-\frac{V_C Y^2}{4au}\right)} du + T_1 \tag{13}$$

Experimental data show that the normal pressure distribution at the tool-chip interface can be modelled as a polynomial variation [Ozlu *et al.*, 2009]:

$$P(x) = P_0 \left(1 - \frac{x}{l_C}\right)^{\xi} \tag{14}$$

l_C: tool-chip contact length.
P_0: pressure on the tool tip.
$\xi\ (\geq 0)$: control parameter for pressure distribution.

Shear stress in the sticking zone on the chip surface (l_p) is equal τ_1 which is material's yield stress. Shear stress in the sliding zone according to Coulomb friction law depends on the sliding friction coefficient (μ) and normal pressure (P) (Figure 3). So that shear stress on the chip surface is:

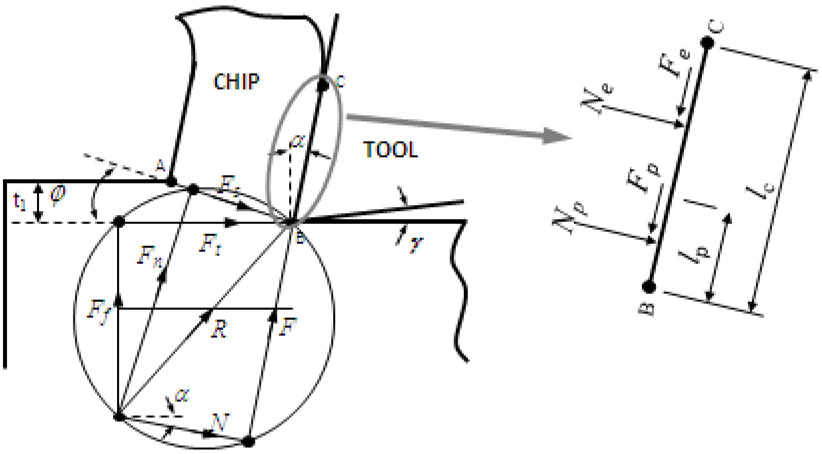


Figure 3; Sticking and sliding zones.

$$\tau = \begin{cases} \tau_1, \; x \leq l_p \\ \mu P, \; l_p \leq x \leq l_c \end{cases} \quad (15)$$

Friction coefficients in the sticking and sliding zones of the tool-chip interface were studied in detail by [Ozlu *et al.*, 2007]. Three major results of their study which are related to the total contact length l_c, sticking zone length l_p and relation between sliding and sticking friction coefficients are given as follows [Ozlu *et al.*, 2009].

$$l_c = t_1 \frac{\xi + 2}{2} \frac{sin(\phi + \lambda - \alpha)}{sin\phi cos\lambda cos\eta} \quad (16)$$

$$l_p = l_c \left(1 - \left(\frac{\tau_1}{P_0 \mu}\right)^{\frac{1}{\xi}}\right) \quad (17)$$

Mechanics of the cutting process, average friction coefficient on the tool-chip surface is obtained from the following equation:

$$\mu_a = tan(\lambda) = \frac{\tau_1}{P_0}\left(1 + \xi\left(1 - \left(\frac{\tau_1}{P_0 \mu}\right)^{1/\xi}\right)\right) \quad (18)$$

For a definite shear angle interval; with a calibrated sliding friction coefficient for a specific material (Table I), the value of μ_a is obtained iteratively from equation (18) [Ozlu *et al.*, 2009].

TableI; Calibrated sliding friction coefficient [Ozlu et al., 2009].

Material	μ_S (V_N (m/min.)
AISI 1050	$0.398 + 6.120 \times 10^{-4} V_N$
AISI 4340	$0.513 + 4.734 \times 10^{-6} V_N^2 - 1.872 \times 10^{-3} V_N$
Ti6Al4V	$0.326 + 1.1 \times 10^{-3} V_N$

Thus, we can express the heat generation on the contact length as follows:

$$Q(x) = \begin{cases} \tau_1 V_c, \; 0 \leq x \leq l_p \\ \mu_s V_c P(x), \; l_p \leq x \leq l_c \\ 0, \; x \geq l_c \end{cases} \quad (19)$$

The shear stress distribution in the sticking and sliding zones is placed in the temperature equation (12) and the following is obtained:

$$T(X,Y) = \frac{1}{k}\sqrt{\frac{a}{V_C}}\left[\int_0^{l_p} \tau_1 V_c \frac{1}{\sqrt{\pi u}} e^{\left(-\frac{V_C Y^2}{4au}\right)} du + \int_{l_p}^{l_c} \mu V_c P_0 \left(1 - \frac{X-u}{l_C}\right)^{\xi} \frac{1}{\sqrt{\pi u}} e^{\left(-\frac{V_C Y^2}{4au}\right)} du\right] + T_1 \quad (20)$$

Since the second term of eqn. (20) cannot be solved analytically, it is obtained numerically by adaptive Simpson quadrature method. In this model, it is assumed that 80% of heat generated by friction pass through into the chip [Wright *et al.*, 1980].

4. RESULTS and DISCUSSION

In this section, experimental data for AISI 1045 steel taken from literature (Table II) [R.W. Ivester *et al.*, 2000] are compared with the results of our model.

Table II;Experimental Results [R.W. Ivester et al., 2000].

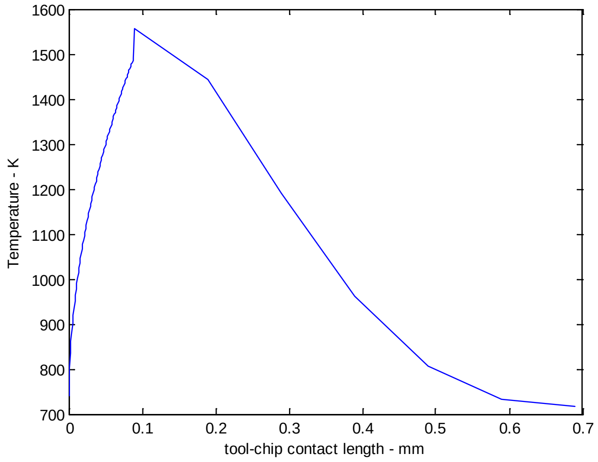
Test	V (m/min)	α (deg)	t_1 (mm)	Exp. max (Tint)
1	200	-7	0.150	1120
2	200	+5	0.150	1250
3	200	-7	0.300	1100
4	200	+5	0.300	1220
5	300	-7	0.150	1310
6	300	+5	0.150	1300
7	300	-7	0.300	1305
8	300	+5	0.300	1300

Calibrated sliding friction coefficient is taken from Table I, and Johnson-Cook material model parameters for the AISI 1045 are given in Table III [Ozlu *et al.*, 2009].

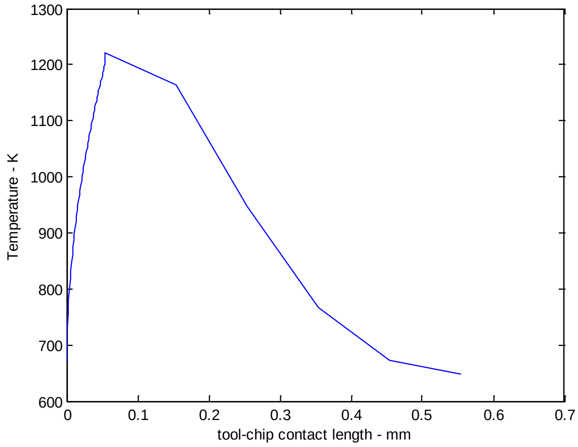
Table III; Calibrated JC parameters.

Malzeme	**A (MPa)**	**B (MPa)**	**n**	**C**	**m**
AISI 1050	880	500	0.234	0.0134	1

Figure 4 shows the temperature distribution calculated by eqn. (20) on the tool-chip contact length for the first two tests.



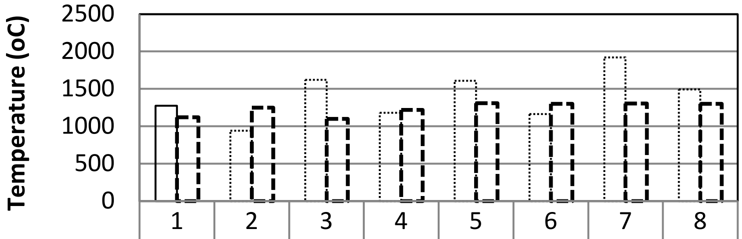
(a)



(b)

Figure 4; Analytical results for the temperature distiribution on the rake contact a)Test 1 and b)Test 2

Comparison of the analytical and experimental results [R.W. Ivester *et al*., 2000] are shown in Figure 5.



	1	2	3	4	5	6	7	8
Analytical	1275	942	1622	1179	1607	1165	1919	1493
Experimental	1120	1250	1100	1220	1310	1300	1305	1300

Figure 5; Comparison of analytical model predictions and experimental results for maximum temperature at the tool-chip contact length.

Results which are obtained from our model are in reasonable agreement with experimental data where the error is between 13-32%.

The maximum temperature value and its' location on the rake face are also investigated by the developed model. For a cutting velocity of 30 m/min and uncut chip thickness of 0.1 mm, the temperature variation with rake angle can be seen in Figure 6. As it can be seen from Figure 6, the maximum temperature decreases with increasing rake angle. As also discussed in [Ozlu et.al, 2009] increasing rake angle decreases normal and shear stresses on the rake face. Moreover from eqn.19 it can be deduced that lower stresses at the rake face yield lower heat generation.

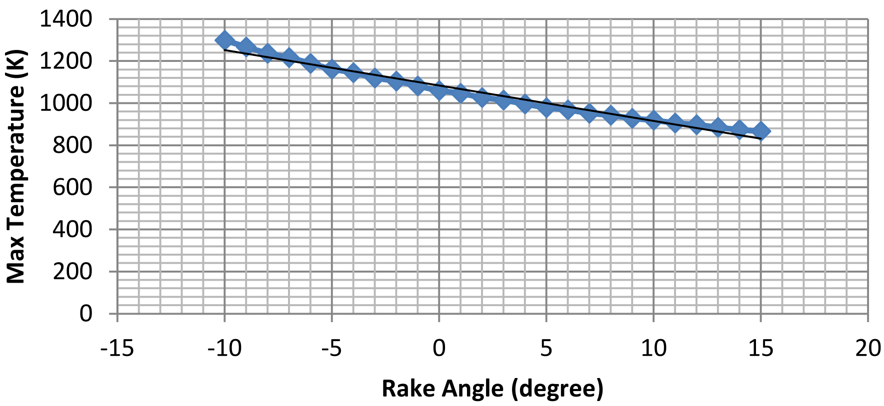


Figure 6; Maximum temperature variation with rake angle.

The location of the maximum temperature also changes with the rake angle. As can be seen in Figure 7, the maximum temperature location, with respect to the tool tip, decreases with the increasing rake angle, meaning that as the rake angle increases the maximum temperature location gets closer to the tool tip. Although this behaviour cannot be explained as explicitly as the previous behaviour, it is known that the sticking contact length / total contact length ratio decreases as the rake angle increases. This mechanism shifts the normal and shear stress distributions closer to the tool tip, where high values of the stresses have more impact. Therefore, again using eqn. 19 the higher heat generation exists at locations closer to the tool tip.

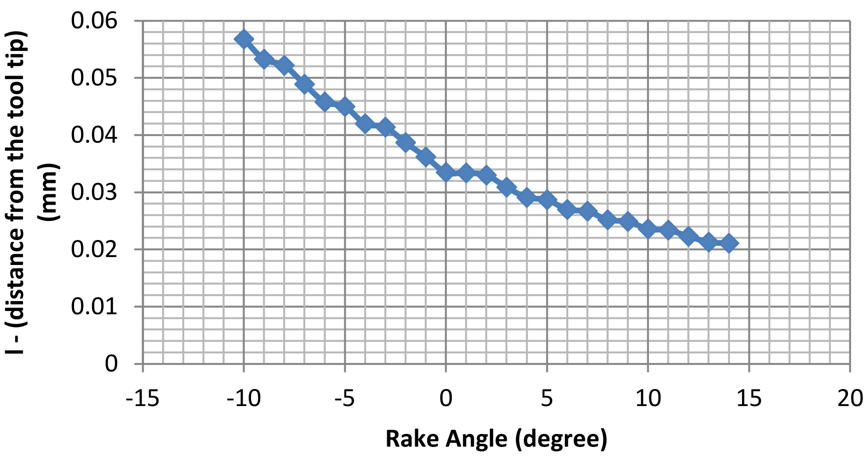


Figure 7; Maximum Temperature Location vs. Rake angle

5. CONCLUSION

In the present study, two dimensional and time independent heat transfer model is developed for the temperature distribution in the chip for orthogonal cutting. Heat effects on the primary and secondary shear zones are considered where the material behaviour in the primary shear zone is expressed using the JC material model. Since the model is analytical the solution times are quite short. The model predictions are compared with the cutting temperature data taken from literature. In general, good correlation is obtained with the experimental data for the maximum temperatures. In addition, several simulations such as effect of rake angle on maximum temperature are also done to show the applications of the model in process analysis.

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