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On The "Group Non-bossiness" Property

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Abstract

We extend the concept of non-bossiness to groups of agents and say that a mechanism is group non-bossy if no group of agents can change the assignment of someone else while theirs being unaff ected by misreporting their preferences. First, we show that they are not equivalent properties. We, then, prove that group strategy-proofness is sufficient for group non-bossiness. While this result implies that the top trading cycles mechanism is group non-bossy, it also provides a characterization of the market structures in which the deferred acceptance algorithm is group non-bossy.

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1 Introduction

In a single-unit assignment problem, a collectively owned set of indivisible objects are to be assigned to a number of agents where each agent demands only one object, and no monetary transfer is allowed. Student placement in public schools and university housing allocation are two important examples of such assignment problems. Each agent has a strict preference relation over the objects; and a mechanism specifies an assignment of the objects to the agents for each preference profile.

Satterthwaite and Sonnenschein (1981) introduce the notion of *non-bossiness*, which is important in many assignment problems. A mechanism is non-bossy if no agent can change the assignment of someone else without being affected by misreporting his preference. Besides its theoretically appealing definition, it is also desirable due to normative and strategic manipulation concerns.[1]

Even though the concept of non-bossiness has received much attention in the literature,[2] it only considers "bossy" individuals, in other words, it does not capture the situations where no individual is bossy but a group of agents. In the current study, we extend the concept of non-bossiness to groups of agents and introduce the *group non-bossiness* notion. We say that a mechanism is *group non-bossy* if no group of agents (including individual ones) can change the assignment of someone else while theirs being unaffected by collectively misreporting their preferences. First, we show that group non-bossiness is not equivalent to non-bossiness in the sense that the latter does not imply the former. We, then, demonstrate that group strategy-proofness is sufficient for a mechanism to be group non-bossy. As immediate implications of this result, we obtain a characterization of the market structures in which the well-known *agent-proposing deferred acceptance algorithm* (Gale and Shapley (1962); hereafter, DA) is group non-bossy; and another well-known one, *top trading cycles mechanism* (hereafter, TTC), turns out to be group non-bossy.

The current study is important for at least two reasons. First, the concept of group non-bossiness is theoretically interesting per se, as, in economics, it has been always desired to understand the effects of groups of agents' actions on the outcome in various settings. Secondly, it gives a sufficient condition for group non-bossiness in terms of the well understood group strategy-proofness property, which immediately enables us to have a say in the group non-bossiness of some well-known mechanisms.

2 Model & Results

In a single-unit assignment problem, there are sets of agents N and objects O along with the null object, denoted by $\emptyset$. Each agent i has a *preference relation* R_i, which is a strict, complete, and transitive binary relation over $O \cup \{\emptyset\}$. We write aP_ib if aR_ib and $a \neq b$.

[1]The reader could refer to Kojima (2010) for a detailed argument on this point.

[2]Kojima (2010) shows that stability and non-bossiness are incompatible, and Matsubae (2010) extends this result to a weaker version of non-bossiness, called "Non-damaging bossiness". Papai (2000) proves that the combination of strategy-proofness and non-bossiness is equivalent to group strategy-proofness; and Ergin (2002) characterizes the market structures in which the student-proposing deferred acceptance algorithm is non-bossy. In the multi-unit assignment setting, Papai (2001) introduces the concept of total nonbossiness, which is more stringent than non-bossiness, and characterizes the serial dictatorship rule in terms of it.

Each object $a \in O \cup \{\emptyset\}$ has a *quota* q_a, and the null object $\emptyset$ is not scarce, i.e., $|q_\emptyset| = |N|$. An *allocation* is an assignment of objects to agents such that each agent receives exactly one object in $O \cup \{\emptyset\}$,[3] and each object is assigned to as many agents as at most its quota.

Let $R = (R_i)_{i \in N}$ be the preference profile of agents; and, for $S \subset N$, we write $R_S = (R_i)_{i \in S}$ for the preference profile of the agents in S. Let $\mathcal{A}$ and $\mathcal{R}$ denote the sets of allocations and preference relations, respectively. A *mechanism* is a function $\psi : \mathcal{R}^{|N|} \to \mathcal{A}$.

Definition 1 (Satterthwaite and Sonnenschein (1981))**.** *A mechanism ψ is non-bossy if, for all $i \in N$, $\psi_i(R'_i, R_{-i}) = \psi_i(R)$ implies $\psi(R'_i, R_{-i}) = \psi(R)$ for all $R'_i \in \mathcal{R}$ and $R \in \mathcal{R}^{|N|}$.*

In words, a mechanism is non-bossy if no individual agent can ever change the assignment of someone else without being affected by misreporting his preference. It only eliminates bossy individuals, yet, there might be instances where no agent is bossy but a group of agents. The following notion is a direct generalization of non-bossiness incorporating bossy groups as well.

Definition 2. *A mechanism ψ is group non-bossy if, for all $S \subset N$, $\psi_i(R'_S, R_{-S}) = \psi_i(R)$ for any $i \in S$ implies $\psi(R'_S, R_{-S}) = \psi(R)$ for all $R'_S \in \mathcal{R}^{|S|}$ and $R \in \mathcal{R}^{|N|}$.*

The first question that should be answered is that whether group non-bossiness is equivalent to non-bossiness. It is obvious that the former implies the latter, yet, the following theorem shows that the converse is not true, which means that they are not equivalent.

Theorem 1. *Group non-bossiness is not equivalent to non-bossiness.*

Proof. For the proof, we need show that there exists a mechanism which is non-bossy but not group non-bossy.

Let $N = \{i, j, k\}$ and $O = \{a, b, c\}$, with $q_a = q_b = q_c = 1$. We write $top(R_i)$ for the best choice of agent i with respect to preference R_i. Let define a mechanism ψ as follows:

$$\psi(R) \ = (\psi_i(R), \psi_j(R), \psi_k(R)) = \begin{cases} (a, b, c) & \text{if } top(R_i) = top(R_j) \in \{a, c, \emptyset\} \\ (a, b, \emptyset) & \text{if } top(R_i) = top(R_j) = b \\ (\emptyset, \emptyset, \emptyset) & \text{otherwise} \end{cases}$$

It is easy to verify that ψ is non-bossy. Yet, it is not group non-bossy. To see this, consider two preference profiles R and R' where $top(R_i) = top(R_j) = a$, and $top(R'_i) = top(R'_j) = b$. Then, $\psi(R) = (a, b, c)$ and $\psi(R') = (a, b, \emptyset)$, showing that ψ is not group non-bossy.

□

Definition 3. *A mechanism ψ is group strategy-proof if there exist no $S \subseteq N$, $R'_S \in \mathcal{R}^{|S|}$, and $R \in \mathcal{R}^{|N|}$ such that $\psi_i(R'_S, R_{-S}) R_i \psi_i(R)$ for all $i \in S$, and for some $j \in S$, $\psi_j(R'_S, R_{-S}) P_j \psi_j(R)$.*

[3] All the arguments carry over to the multi-unit assignment case where each agent might receive more than one object. I will discuss this point in detail in Remark 1.

Theorem 2. *If a mechanism is group strategy-proof, then it is group non-bossy.*

Proof. Let ψ be a group strategy-proof mechanism, and assume that it is not group non-bossy. Then, this implies that there exist $S \subset N$, $R'_S \in \mathcal{R}^{|S|}$, and $R \in \mathcal{R}^{|N|}$ such that $\psi_i(R'_S, R_{-S}) = \psi_i(R)$ for all $i \in S$, but $\psi(R'_S, R_{-S}) \neq \psi(R)$. Let $A = \{i \in N \setminus S : \psi_i(R'_S, R_{-S}) P_i \psi_i(R)\}$.

If $A \neq \emptyset$, let $i \in A$. Then, however, the group of agents $S \cup \{i\}$ would manipulate ψ by reporting false preference profile $R'_{S\cup\{i\}} = (R'_S, R_i)$, which contradicts ψ being group strategy-proof.[4]

On the other hand, if $A = \emptyset$, then this implies that there exists an agent $j \in N \setminus S$ such that $\psi_j(R) P_j \psi_j(R'_S, R_{-S})$ (since $\psi(R) \neq \psi(R'_S, R_{-S})$). In this case, however, the grand coalition N would manipulate ψ at the problem instance where the true preference profile is $R' = (R'_S, R_{-S})$ by reporting false preference profile R,[5] which contradicts the group strategy-proofness of ψ. □

Yet, group strategy-proofness is not necessary for a mechanism to be group non-bossy as the following example shows.

Example 1. Let $N = \{i, j, k\}$ and $O = \{a, b, c\}$, with $q_a = q_b = q_c = 1$. Consider the following mechanism ψ:

$$\psi(R) = (\psi_i(R), \psi_j(R), \psi_k(R)) = \begin{cases} (a, b, c) & \text{if } top(R_i) = top(R_j) \\ (\emptyset, \emptyset, c) & \text{otherwise} \end{cases}$$

It is easy to verify that ψ is group non-bossy. However, it is not group strategy-proof, as agents i and j would benefit from misreporting whenever (i) their top choices are not the same, and (ii) each of them prefers any object in O to the null object.

One advantage of Theorem 2 is that it gives a sufficient condition in terms of group strategy-proofness, which is well-studied in the literature. This makes us able to say something about the group non-bossiness of two well-known mechanisms: TTC and DA, which are widely used for priority-based assignment problems.[6] It is known that TTC is group strategy-proof, hence, it is group non-bossy by Theorem 2.

Corollary 1. *TTC is group non-bossy.*

Ergin (2002) demonstrates that DA is separately group strategy-proof and non-bossy if and only if the priority structure of objects is acyclic.[7] Hence, this along with Theorem 2 enable us to characterize the market structures in which DA is group non-bossy.

Corollary 2. *DA is group non-bossy if and only if the priority structure of objects is acyclic.*

Remark 1. It is obvious that all the arguments except Corollaries 1&2 carry over to the multi-unit assignment case as well, where each agent might receive more than one object. For DA, on the other hand, Kojima (2011) shows that it is not even strategy-proof under acyclic

[4] Agent i would be strictly better off while all the other agents in S would not be affected by the misreporting.

[5] No agent in S would be affected by the misreporting. Whereas, all the agents in $N \setminus S$ would be at least weakly better off while at least agent j would be strictly better off by the misreporting.

[6] In such problems, objects have priority orders over agents.

[7] In Ergin (2002), schools are the objects, and students are the agents.

priority structures in multi-unit assignment problems. He recovers the strategy-proofness of DA under the more stringent priority structures. Yet, it is not known whether it is also the case for group strategy-proofness. On the other hand, for TTC, to my best knowledge, there is no study investigating the group strategy-proofness of TTC in the multi-unit assignment environment. Hence, as of now, the current paper does not have a say in the group non-bossiness of TTC and DA in the multi-unit assignment setting.

Remark 2. It is easy to see that *serial dictatorship*, which is another well-known assignment mechanism where there is a fixed ordering of agents according to the which they choose objects, is group non-bossy even in the multi-unit assignment case.

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References

Abdulkadiroğlu, A. and T. Sönmez (2003): "School Choice: A Mechanism Design Approach," *American Economic Review*, 93(3), 729–747.

Ergin, H. I. (2002): "Efficient Resource Allocation on the Basis of Priorities," *Econometrica*, 88, 485–494.

Gale, D. and L. S. Shapley (1962): "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 69, 9–15.

Kojima, F. (2010): "Impossibility of stable and nonbossy matching mechanisms," *Economics Letters*, 107(1), 69–70.

——— (2011): "Efficient Resource Allocation under Multi-unit Demand," *mimeo*.

Matsubae, T. (2010): "Impossibility of Stable and Non-damaging bossy Matching Mechanism," *Economics Bulletin*, 30(3), 2092–2096.

Papai, S. (2000): "Strategyproof Assignment by Hierarchical Exchange," *Econometrica*, 68(6), 1403–1433.

——— (2001): "Strategyproof and Nonbossy Multiple Assignments," *Journal of Public Economic Theory*, 3(3), 257–271.

Roth, A. E. and M. O. Sotomayor (1990): *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*, Econometric Society Monographs, Cambridge Univ. Press, Cambridge.

Satterthwaite, M. A. and H. Sonnenschein (1981): "Strategy-Proof Allocation Mechanisms at Differentiable Points," *Review of Economic Studies*, 48(4), 587–597.