OPTIMAL INCOME TAXATION UNDER LABOR INTERDEPENDENCE

by Abdullah Selim Öztek

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APPROVED BY

Assist. Prof. Dr. Hakkı Yazıcı ....
(Thesis Supervisor)

Prof. Dr. İsmail Sağlam ....

Assist. Prof. Dr. Remzi Kaygusuz ....

Assist. Prof. Dr. Şerif Aziz Şimşir ....

Assoc. Prof. Dr. Özgür Kıbrıs ....

DATE OF APPROVAL: ....

to my parents ...

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Abdullah Selim Öztek
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Supervisor: Hakkı Yazıcı

Abstract

Keywords: Optimal non-linear taxation, redistribution, positional goods.

In this thesis, I consider optimal redistributive income taxation under a Mirrleesian framework while adding utility interdependence over labor choice and analyze whether the optimal tax schedule is regressive or progressive. In this environment, I show that optimal marginal income taxation could be progressive depending on the parameters of the model. There are two separate forces that are at work in determining the optimal tax schedule. First, due to the informational problems, there is a usual Mirrleesian force that works towards the regressivity of taxes. Second effect is a novel force that arises from labor externality and has a progressive effect on the income tax. This effect could be called as Pigouvian tax. Labor externality requires subsidies for agents which are asymmetric according to productivities. Because of this asymmetry, there should be higher subsidies for low types which has a progressive effect on the optimal tax schedule. Pigouvian and Mirrleesian effects are in a multiplicative form in the tax function, therefore the tax schedule is identified by the effect which is more powerful. I also show that, when we consider the labor interdependence, zero tax at the top of the skill distribution result is no longer valid. Additionally, I show that even under full information the market is not efficient and there is a need for progressive income taxes, as there is a need to correct the labor externality. Moreover, the numerical examples of the paper show the progressive effect of labor externality on the tax schedule. This additional concern about labor externality makes the income taxation schedule more consistent with the current tax policies.

İŞGÜCÜNÜN KARŞILIKLI BAĞIMLI OLMASI DURUMUNDA
OPTİMAL GELİR VERGİLENDİRMESİ

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Özet

Anahtar Kelimeler: Doğrusal olmayan optimal vergilendirme, yeniden dağıtım, konumsal mallar.

Bu tezde, bireylerin fayda fonksiyonlarının, her bireyin bağımsız olarak yaptığı işgücü arzı tercihi kanalıyla birbirine bağımlı olabileceği, diğer bir ifadeyle emek dışsallığı Mirrlees modeline eklenerek yeniden dağıtımcı gelir vergilendirme problemi ele alınmış ve optimal vergi tarifesinin azalan oranlı mı yoksa artan oranlı mı olması gerektiği incelenmiştir. Bu yeni ortamda, optimal marjinal gelir vergisi tarifesinin model parametrelerine bağlı olarak artan oranlı olabileceği ortaya konmuştur. Optimal vergi tarifesini belirleyen birbirinden ayrılabilir iki farklı dinamik bulunmaktadır. Bunlardan birincisi, asimetrik bilgi problemi nedeniyle, vergilerin azalan oranlı olmasına sebep olan, olağan Mirrlees etkisidir. İkincisi ise emek dışsallığından doğan yeni bir etkendir ve gelir vergisi üzerinde artan oranlı etki yapmaktadır. Bu kuvvet, Pigou vergisi olarak adlandırılabilir. Emek dışsallığı, kişilerin üretkenliklerine göre asimetrik olarak sübvansiyonlar gerektirmektedir. Bu asimetriden ötürü, düşük üretkenliğe sahip kişiler daha fazla sübvanse edilmelidir ve bu durum vergi tarifesi üzerinde artan oranlı bir etki yaratmaktadır. Pigou ve Mirrlees etkileri, vergi fonksiyonunda çarpım şeklinde bulunurlar, bundan ötürü vergi tarifesi daha güçlü olan etki tarafından belirlenir. Ayrıca bu tezde, emek dışsallığını göz önünde bulundurduğumuzda, üretkenlik dağılımının en üst tabakasında bulunanlara sıfır vergi oranı uygulanmalı sonucunun artık geçerli olmadığı ortaya konmuştur. Buna ek olarak, tam bilgi seviyesinde bile piyasanın etkin olmadığını ve emek dışsallığının etkilerini düzeltmek için artan oranlı vergiye ihtiyaç duyulduğu tespit edilmiştir. Tezde verilen sayısal örnekler emek dışsallığının vergi tarifesi üzerindeki artan oranlı etkisini göstermektedir. Emek dışsallığı fikrinin modele dâhil edilmesi, teorik vergi tarifesini, güncel vergi politikalarıyla daha tutarlı hale getirmektedir.

TABLE OF CONTENTS

1 Introduction

The modern optimal income taxation literature started by the seminal work of Mirrlees (1971). The idea behind this study was the trade-off between efficiency and distributional concerns. Mirrlees indicates that a high marginal tax rate implemented to the high productive worker will distort the labor decision and leads to a disincentive for working. By integrating this incentive consideration into the existing optimal income taxation literature, Mirrlees had changed both the context and the direction of the debate.

According to Mirrlees (1971), when the efficiency loss is considered, optimal income taxation may follow a regressive fashion as the low income earners should pay higher taxes than the high income earners. After Mirrlees, a huge literature has developed. Sadka (1976) and Seade (1977) showed that the optimal income tax implemented at the top of the income (ability) distribution should be zero when there is a finite maximum to the skill distribution.[1]The intuition here is simple; the disincentive effect of the high taxes on productive individuals causes a crucial efficiency loss which leads to a considerable decrease in total welfare. By using log-linear skill distribution, Atkinson (1973), Tuomala (1983,1990) showed that the optimal income tax is regressive as Mirrlees said. Furthermore according to Sadka (1976) even there is no disincentive problem, the progressivity in the tax policy is not necessarily desirable.

There is an important conflict between the theory of optimal income taxation and the current tax policies as there is almost no country in the world which has a regressive income tax schedule. It was an unexpected result to have a regressive income tax schedule in theory, as Mirrlees confess.[2] Conversely, Diamond (1998) has shown that if the skill distribution is unbounded,

[1]Diamond, P. (1998): "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates," American Economic Review, 88(1), 83–95.

[2]I must confess that I had expected the rigorous analysis of income-taxation in the utilitarian manner to provide an argument for high taxes. It has not done so. [Sir James A. Mirrlees, "An Exploration in the Theory of Optimum Income Taxation".]

optimal tax schedule could be progressive in the upper tail of the income distribution. Saez (2001) mentioned that progressivity of the tax schedule could be possible when the link between optimal tax formulas and elasticities of earnings are considered. However these results are very sensitive to the skill distribution assumptions. While Mirrlees and Tuomala are using Log-normal skill distribution, Diamond and Saez use Pareto distribution for the upper tail of the skill distribution, and this result does not hold when the distribution assumption is changed.

In this study, I consider optimal redistributive income taxation under a Mirrleesian framework while adding utility interdependence over labor choice and analyze whether the optimal tax schedule is regressive or progressive. Previous studies on optimal redistributive income taxation consider the consumption externalities but ignore the labor interdependency of agents[3]. In my setup, agents' labor decisions are affecting each other and generating externalities which lead to a utility interdependence among agents. The main concern of the Mirrlees (1971) is information asymmetry that is agents are heterogenous and productivity is private information of an individual. I add labor externality to the Mirrlees setup and show that this additional concern about labor interdependence entails government to intervene in order to correct these externality effects. Although the widely accepted model states that individuals derive disutility from their own work, $V(l)$, people also care about their position in the society. Instead of a form of disutility $V(l)$, I use the combination of their own and other people's labor choice $V(l, L)$, where L denotes the average working hour in the society. Specifically, if disutility depends on the average work hour, the increase in an agent's working hour has a positive externality on other agents because it lowers the disutility of others.

In this environment, I show that optimal marginal income taxation could be progressive depending on the parameters of the model. There are two sepa-

[3] Kanbur and Tuomala (2010), Oswald (1983), Samano (2009), Tuomala (1990).

rate forces that are at work in determining the optimal tax schedule. First, due to the informational problems, there is a usual Mirrleesian force that works towards the regressivity of taxes. Second effect is a novel force that arises from labor externality and has a progressive effect on the income tax. This effect could be called as Pigouvian tax. Labor externality requires subsidies for agents which are asymmetric according to productivities. Because of this asymmetry, there should be higher subsidies for low types which has a progressive effect on the optimal tax schedule. Pigouvian and Mirrleesian effects are in a multiplicative form in the tax function, therefore the tax schedule is identified by the effect which is more powerful. In general it is not obvious which force dominates the other. I provide conditions on parameters under which optimal income tax schedule is progressive. These results show that if we believe in the existence of labor interdependence, the progressivity of actual tax systems can be rationalized. This is the main contribution of this study.

I also show that, when we consider the labor interdependence, zero tax at the top of the skill distribution result is no longer valid. Additionally, I show that even under full information the market is not efficient and there is a need for progressive income taxes, as there is a need to correct the labor externality. The reason of this certain progressivity is the asymmetry of the externalities generated by agents. Moreover, the numerical examples of the paper show the change in tax schedule when there is labor externality, and these examples also show that the societies which have a less dispersed skill distribution may have a more progressive income tax schedule. This result is consistent with the current tax policies when we look at the marginal tax data of the US and Europe.

There are several studies that test the positional concerns of individuals over the labor interdependency between agents. It is a historical fact, which was mentioned by many economists like Thorstein Veblen, John Stuart Mill, and Arthur Pigou, that people are affecting each other's labor decisions. People are also considering other people's leisure or work hours as well as their income and consumption. Veblen (1899) mentioned the importance of relative position in society, under the concept of "conspicuous consumption and conspicuous leisure".[4] Taxing of positional goods was pointed out by John Stuart Mill 150 years ago.[5] Arthur Pigou (1920) said that "men do not desire to be rich, but richer than other men." Pingle and Mitchell (2002), who find evidence of leisure positionality in a questionnaire-based study, mention that most income is derived from allocating time toward labor and away from leisure; any observed positional concern for income is potentially confounded with a positional concern for leisure.

There are several micro level studies in labor economics and social psychology have shown that each individual labor decision is dependent to the other agent's labor supply decisions. In economics, it is usually assumed that agents are interacting only over the market. However, other social sciences pay attention to direct interactions between the agents. Blomquist (1993) mentions that if there are direct interactions, then the results and predictions are really biased because of omitting these interactions. Grodner and Kniesner, (2006) and (2009) showed that labor interdependency has a significant effect on the agent's labor supply decision. Aronsson, Blomquist, and Sacklén (1999) test the hypothesis that individual's choices of hours of work are influenced by the

[4] "...the utility of both (conspicuous leisure and conspicuous consumption) alike for the purposes of reputability lies in the waste that is common to both. In the one case it is a waste of time and effort, in the other it is a waste of goods. Both are methods of demonstrating the possession of wealth, and the two are conventionally accepted as equivalents." The Theory of the Leisure Class, 1899.

[5] ...a great portion of the expenses of the higher and middle classes in most countries, and the greatest in this, is not incurred for the sake of the pleasure afforded by the things on which the money is spent, but from regard to opinion, and an idea that certain expenses are expected from them, as an appendage of station; and I cannot but think that expenditure of this sort is a most desirable subject of taxation. If taxation discourages it some good is done, and if not, no harm; for, in so far as taxes are levied on things which are desired and possessed from motives of this description, nobody is the worse for them. "Principles of Political Economy" MILL, 1848. (This is quoted in Carlsson et al. 2007.)

average hours of work in a social reference group. Their results support the hypothesis of interdependent behavior and neglecting the interdependence can lead to serious underestimates of the labor supply effects of income taxes. As an empirical example Weinberg et al. (2004) find that an extra hour worked by the social reference group of an individual can increase the individuals total working hours by about 0.6 hours in United States. And there are several other studies that verifies the labor interdependency among agents[6].

Surveys could be used in order to test concerns of people about other individual's working hours, by asking them hypothetical questions regarding their choices among alternative states. Pingle and Mitchell (2002) quoted the survey study of Solnick and Hemenway (1998) and mentioned that some of the responses Solnick and Hemenway identified as resulting from positional concerns for income could have resulted because of positional concerns for leisure.

There are several studies which discuss the optimal tax schedule with consumption externalities, however my study investigates the effect of labor externality on the optimal marginal income tax schedule. Samano (2009) investigates the consumption externalities under the Mirrleesian setup, and mentioned the progressivity effect of the consumption externality over tax schedules. Oswald (1983), Tuomala (1990) and Kanbur and Tuomala (2010) look at the tax schedule when agents value their consumptions relative to the average consumption. Kanbur and Tuomala (2010) find support for greater progressivity in the tax structure as relative consumption concern increases. According to Oswald (1983) if there is utility interdependence over consumption then zero marginal tax at the top of the skill (income) distribution result does not

[6]Other studies about labor interdependency are; Baskaya, Y., and Kilinc, M. (2010), Woittiez and Kapteyn (1998), Becker and Murphy (2000) Glaeser, Sacerdote, and Scheinkman (2003), Elster (1989), Fryer and Payne (1986), Jackson and Warr (1987), Feather (1990), Platt and Kreitman (1990) and Platt, Micciolo and Tansella (1992).

hold. Tuomala (1990) shows that optimal income taxes are progressive when there is utility interdependence.

As far as I know, the optimal tax schedule with labor externalities under a Mirrleesian framework has never been analyzed. This is the first study that shows labor interdependence has a progressive effect on the optimal income taxes. It is a common fact that current tax systems around the world are progressive. This study shows that by changing some underlying assumptions of the economic model, the current tax systems of countries could be rationalized.

The rest of the paper proceeds as follows: section 2 presents the model, section 3 presents the full information benchmark, section 4 presents the two-type model under private information, section 5 presents the N-type model under private information, section 6 and 7 investigate a specific form of utility in two-type and N-type cases respectively, section 8 shows the numerical examples, and section 9 concludes.

2 Model

In section 3 and 4, I study the labor externality effect on the tax policy in a two-type model because it is easier to illustrate the results and the intuition behind. In section 5 and 6, I analyze the general N-type model and show that the results are general. As in Mirrlees (1971) agents are heterogeneous about their privately known productivity levels. An agent with a productivity level θ has a separable utility function in the form of consumption and labor;

$$U(c, y, L, \theta) = u(c) - v(\frac{y}{\theta}, L)$$

where c is consumption, y is agent's income, and $L = \sum_{i=1}^{n} \pi_i \frac{y_i}{\theta_i}$ is the average working hour of the society. Production function is $y = \theta l$, therefore labor is $l = \frac{y}{\theta}$, as number of work hours.

Assumptions of the model are as follows:

i) Preferences satisfy the usual assumptions that; $u'(c) > 0$, $u''(c) < 0$ and $v_1(\frac{y}{\theta}, L) > 0$, $v_{11}(\frac{y}{\theta}, L) > 0$.

ii) There are two additional assumptions that;

1-) $v_2(\frac{y}{\theta}, L) < 0$ which means disutility decreases when L increases.

2-) $v_{21}(\frac{y}{\theta}, L) > 0$ which means the agent who works more is getting a higher disutility decrease from the increased L.

While agents are deriving utility from their consumption, as usual, working is a source of disutility. In this setup they also derive utility from the increase in the average working hour, because agent's disutility is decreasing while average working hour of the society is increasing.

3 Full Information Benchmark and Inefficiency of the Laissez-Faire Market

In this section, I analyze the full information case as a benchmark. After finding the allocation which the social planner offers to the agents, the optimal marginal income taxes under full information will be identified. Also the planner's allocation is compared with the Laissez-Faire market equilibrium in order to have an idea on whether the market is efficient or not.

3.1 Social Planning Problem Under Full Information

The aim of the social planner is to maximize the overall welfare of the society while evaluating all agents equally by giving them the same weight. π_l and π_h are the proportions of the low and high productive agents in the society and they are normalized to 1, which means the summation of proportions is equal to 1. When planner has full information, which means knowing each agent's productivity level, the social planner's problem is as follows:

$$\max_{c_l, c_h, y_l, y_h} \pi_l \left[u(c_l) - v\left(\frac{y_l}{\theta_l}, L\right)\right] + \pi_h \left[u(c_h) - v\left(\frac{y_h}{\theta h}, L\right)\right]$$

subject to

$$\pi_h c_h + \pi_l c_l \leq \pi_h y_h + \pi_l y_l$$

$$L = \pi_h \frac{y_h}{\theta_h} + \pi_l \frac{y_l}{\theta_l}$$

Letting λ be the multiplier on the resource (feasibility) constraint, FOC are as follows:

$(c_l): \quad \pi_l u'(c_l) - \lambda \pi_l = 0$

$(c_h): \quad \pi_h u'(c_h) - \lambda \pi_h = 0$

$$u'(c_l) = u'(c_h)$$

Therefore, under full information, consumption levels of the agents with high and low productivities are equal.

$$c_h^f = c_l^f$$

$$(y_l): \qquad \pi_l\left[-v_1(\tfrac{y_l}{\theta_l}, L)\tfrac{1}{\theta_l} - v_2(\tfrac{y_l}{\theta_l}, L)\tfrac{\pi_l}{\theta_l}\right] + \pi_h\left[-v_2(\tfrac{y_h}{\theta_h}, L)\tfrac{\pi_l}{\theta_l}\right] + \lambda\pi_l = 0$$
$$(y_h): \qquad \pi_h\left[-v_1(\tfrac{y_h}{\theta_h}, L)\tfrac{1}{\theta_h} - v_2(\tfrac{y_h}{\theta_h}, L)\tfrac{\pi_h}{\theta_h}\right] + \pi_l\left[-v_2(\tfrac{y_l}{\theta_l}, L)\tfrac{\pi_h}{\theta_h}\right] + \lambda\pi_h = 0$$

By using the FOC, the optimality conditions for both types can be characterized.

Proposition 1 *The conditions that characterize the social planner's problem are;*

$u'(c_h) = v_1(\frac{y_h}{\theta_h}, L)\frac{1}{\theta_h} + v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h}{\theta_h} + v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_l}{\theta_h}$
$u'(c_l) = v_1(\frac{y_l}{\theta_l}, L)\frac{1}{\theta_l} + v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_l}{\theta_l} + v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h}{\theta_l}$

From High type agent's FOC (c_h) and (y_h),

$$u'(c_h) = v_1(\frac{y_h}{\theta_h}, L)\frac{1}{\theta_h} + v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h}{\theta_h} + v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_l}{\theta_h}$$

Left hand side of the equation is the marginal benefit of one additional unit of consumption. $v_1(\frac{y_h}{\theta_h}, L)\frac{1}{\theta_h}$ term in the right hand side is the marginal cost of working for one more unit of consumption. Because of the increased average working hour, $v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h}{\theta_h}$ term is the marginal benefit for high type while $v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_l}{\theta_h}$ term is the marginal benefit for low type. Second and third terms are negative values, because disutility is decreasing while average work hour is increasing. So it can be concluded as:

$$v_1(\frac{y_h}{\theta_h}, L)\frac{1}{\theta_h} > u'(c_h)$$

Because working generates a positive externality which gives utility to each agent, the cost of working for one more unit of consumption is not directly equal to the marginal benefit of consuming the additional good. As it is stated in the proposition the positive externality effect must be subtracted from the marginal cost of working in order to get the marginal benefit of additional consumption.

From FOC's of Low type agent (c_l) and (y_l);

$$u'(c_l) = v_1(\frac{y_l}{\theta_l}, L)\frac{1}{\theta_l} + v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_l}{\theta_l} + v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h}{\theta_l}$$

Terms are counterparts of high type terms for low the type agent, the interpretation is same and again the second and third terms of the right hand side are negative. So similarly it can be said that:

$$v_1(\frac{y_l}{\theta_l}, L)\frac{1}{\theta_l} > u'(c_l)$$

3.2 Laissez-Faire Market

In order to compare with the social planner allocation, in this part, the paper examines the Laissez-Faire market solution. In the market, agents know that they derive utility from the average working hour, but they are not aware of the fact that they can affect the average working hour. Agents are maximizing their utility subject to their resource constraint and their problem is as follows:

Agent's Problem;

$\max_{c,y} \; u(c) - v\left(\frac{y}{\theta}, L\right)$

subject to

$c \leq y - \tau(y)$

$L = \pi_h \frac{y_h}{\theta_h} + \pi_l \frac{y_l}{\theta_l}$.

Letting λ be the multiplier on the resource (feasibility) constraint, FOC are as follows:

$(c):\quad u'(c) - \lambda = 0$

$(y):\quad -v_1(\frac{y}{\theta}, L)\frac{1}{\theta} + \lambda - \lambda\tau'(y) = 0$

which gives;

$$u'(c)(1 - \tau'(y)) = v_1(\frac{y}{\theta}, L)\frac{1}{\theta}$$

where $u'(c)$ is the marginal benefit of one more unit of consumption, and right hand side is the marginal cost of working in order to consume one more unit. If there were no taxes, agents would equalize their costs and benefits.

$$u'(c) = v_1(\frac{y}{\theta}, L)\frac{1}{\theta}$$

Theorem 1 *Laissez-faire market allocation is inefficient.*

Proof. The condition that characterize the Laissez-faire market is $u'(c) = v_1(\frac{y}{\theta}, L)\frac{1}{\theta}$. However this condition gives an inefficient allocation, because agents do not know that they cause a positive externality while they are working. In fact when we consider this effect, as the social planner does, the efficient allocation conditions are those in *Proposition 1* and from those conditions it is shown that $v_1(\frac{y}{\theta}, L)\frac{1}{\theta} > u'(c)$ for both type. ■

Social planner knows the distortionary effect of labor externality, that is $v_1(\frac{y}{\theta}, L)\frac{1}{\theta} > u'(c)$. In order to compensate the agents welfare, planner should subsidize the agents. From these conditions one can say that there is no zero marginal income tax under full information case. And both marginal taxes should be negative, $\tau'(y_h) < 0$ and $\tau'(y_l) < 0$.

3.3 Optimal Income Taxes Under Full Information

Marginal tax is an important public policy instrument which mainly cares about the distributional concerns. To obtain an efficient redistributive income tax, social planner should take the market behavior of the agents into account. In order to get the efficient tax schedule, social planner uses the market conditions of the taxes and determines the appropriate taxes for each agent. The following part examines the optimal non-linear income taxes of each agent under full information.

Proposition 2 *Under full information (First-Best) with labor externalities,*

i) Marginal Taxes are:

$$(1-\tau'(y_h)) = 1 - \frac{1}{\lambda\theta_h}\left[v_2(\tfrac{y_h}{\theta_h}, L)\pi_h + v_2(\tfrac{y_l}{\theta_l}, L)\pi_l\right]$$

$$(1-\tau'(y_l)) = 1 - \frac{1}{\lambda\theta_l}\left[v_2(\tfrac{y_h}{\theta_h}, L)\pi_h + v_2(\tfrac{y_l}{\theta_l}, L)\pi_l\right]$$

ii) Tax schedule is progressive:

$$\tau'(y_h) > \tau'(y_l).$$

Proof. From market solution it is known that marginal tax condition is;

$$(1-\tau'(y)) = \frac{v_1(\frac{y}{\theta}, L)\frac{1}{\theta}}{u'(c)}$$

From FOC; $u'(c_l) = u'(c_h)$

Marginal Tax for High-type;

From $(y_h) : v_1(\frac{y_h}{\theta_h}, L)\frac{1}{\theta_h}\pi_h = \lambda\pi_h - v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h\pi_h}{\theta_h} - v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_h\pi_l}{\theta_h}$

From $(c_h) : u'(c_h)\pi_h = \lambda\pi_h$

Dividing both side gives;

$$(1-\tau'(y_h)) = 1 - \frac{v_2(\frac{y_h}{\theta_h}, L)\pi_h}{\lambda\theta_h} - \frac{v_2(\frac{y_l}{\theta_l}, L)\pi_l}{\lambda\theta_h}$$

$$(1-\tau'(y_h)) = 1 - \frac{1}{\lambda\theta_h}\left[v_2(\tfrac{y_h}{\theta_h}, L)\pi_h + v_2(\tfrac{y_l}{\theta_l}, L)\pi_l\right]$$

Marginal Tax for Low-type;

From $(y_l) : v_1(\frac{y_l}{\theta_l}, L)\frac{1}{\theta_l}\pi_l = \lambda\pi_l - v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h\pi_l}{\theta_l} - v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_l\pi_l}{\theta_l}$

From $(c_l) : u'(c_l)\pi_l = \lambda\pi_l$

Dividing both side gives;

$(1-\tau'(y_l)) = 1 - \frac{v_2(\frac{y_h}{\theta_h},L)\pi_h}{\lambda\theta_l} - \frac{v_2(\frac{y_l}{\theta_l},L)\pi_l}{\lambda\theta_l}$

$(1-\tau'(y_l)) = 1 - \frac{1}{\lambda\theta_l}\left[v_2(\frac{y_h}{\theta_h},L)\pi_h + v_2(\frac{y_l}{\theta_l},L)\pi_l\right]$

Disutility is decreasing while L is increasing, so both $v_2(\frac{y_h}{\theta_h},L)$ and $v_2(\frac{y_l}{\theta_l},L)$ are negative terms. Since $\theta_h > \theta_l$ one can conclude that;

$(1-\tau'(y_l)) > (1-\tau'(y_h))$ which means marginal tax is progressive;

$$\tau'(y_h) > \tau'(y_l)$$

■

The reason of this progressivity is the asymmetry of the externalities. Agents are choosing their consumption and labor levels, and in order to increase his income, a low type worker has to work more than a high type worker. This means the positive externality that a low type generates when producing one unit of output is higher than the externality that a more productive agent generates. Therefore social planner should tax the agent's income while considering this externality asymmetry. Taxes will be negative (subsidy) and the low type worker will get more subsidy than the high type worker. This additional concern about labor interdependency is eliminating the result of zero marginal tax under full information.

4 Social Planning Problem Under Private Information

When we consider the information asymmetry, which arises when the productivity of agents is a private information of individuals and cannot be observed by the social planner, the equally weighted Social Planning Problem becomes as follows:

$$\max_{c_l, c_h, y_l, y_h} \pi_l \left[u(c_l) - v\left(\frac{y_l}{\theta_l}, L\right)\right] + \pi_h \left[u(c_h) - v\left(\frac{y_h}{\theta h}, L\right)\right]$$

subject to

$$\pi_h c_h + \pi_l c_l \leq \pi_h y_h + \pi_l y_l \qquad (\lambda)$$

$$u(c_h) - v\left(\frac{y_h}{\theta h}, L\right) \geq u(c_l) - v\left(\frac{y_l}{\theta_h}, L\right) \qquad (\mu)$$

$$L = \pi_h \frac{y_h}{\theta_h} + \pi_l \frac{y_l}{\theta_l}$$

Letting λ and μ be the multipliers on the feasibility and incentive compatibility constraints respectively, FOC are as follows:

$$(c_l): \quad \pi_l u'(c_l) - \lambda \pi_l - \mu u'(c_l) = 0$$
$$(c_h): \quad \pi_h u'(c_h) - \lambda \pi_h + \mu u'(c_h) = 0$$

From (c_l) and (c_h),
$\frac{u'(c_l)}{u'(c_h)} = \frac{\pi_l(\pi_h+\mu)}{\pi_h(\pi_l-\mu)}$

$$u'(c_l) > u'(c_h).$$

Under private information, marginal utility derived by the low type agent from an additional consumption is greater than the marginal utility derived by the high type. By the concavity of utility function one can conclude that under private information, high ability worker consumes more than the low productive one.

$$c_h^p > c_l^p.$$

(y_l) : $\pi_l \left[-v_1(\frac{y_l}{\theta_l}, L)\frac{1}{\theta_l} - v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_l}{\theta_l} \right] + \pi_h \left[-v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_l}{\theta_l} \right] + \lambda\pi_l + \mu \left[-v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_l}{\theta_l} + v_1(\frac{y_l}{\theta_h}, L)\frac{1}{\theta_h} + v_2(\frac{y_l}{\theta_h}, L)\frac{\pi_l}{\theta_l} \right] = 0$

(y_h) : $\pi_h \left[-v_1(\frac{y_h}{\theta_h}, L)\frac{1}{\theta_h} - v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h}{\theta_h} \right] + \pi_l \left[-v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_h}{\theta_h} \right] + \lambda\pi_h + \mu \left[-v_1(\frac{y_h}{\theta_h}, L)\frac{1}{\theta_h} - v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h}{\theta_h} + v_2(\frac{y_l}{\theta_h}, L)\frac{\pi_h}{\theta_h} \right] = 0$

From the FOC of agents, social planner's optimality conditions can be derived.

Proposition 3 *The conditions that characterize the social planner's problem are;*

$u'(c_h) = v_1(\frac{y_h}{\theta_h}, L)\frac{1}{\theta_h} + v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h}{\theta_h} + \frac{\pi_l \left[v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_h}{\theta_h} \right] - \mu \left[v_2(\frac{y_l}{\theta_h}, L)\frac{\pi_h}{\theta_h} \right]}{\pi_h + \mu}$

$u'(c_l) = \frac{\pi_l}{(\pi_l - \mu)} \left[v_1(\frac{y_l}{\theta_l}, L)\frac{1}{\theta_l} \right] + \frac{\pi_l}{(\pi_l - \mu)} \left[v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_l}{\theta_l} \right] + \frac{(\pi_h + \mu)}{(\pi_l - \mu)} \left[v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_l}{\theta_l} \right] + \frac{\mu}{(\pi_l - \mu)} \left[-v_1(\frac{y_l}{\theta_h}, L)\frac{1}{\theta_h} - v_2(\frac{y_l}{\theta_h}, L)\frac{\pi_l}{\theta_l} \right].$

By integrating (c_h) into (y_h),

$$u'(c_h) = v_1(\frac{y_h}{\theta_h}, L)\frac{1}{\theta_h} + v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h}{\theta_h} + \frac{\pi_l \left[v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_h}{\theta_h} \right] - \mu \left[v_2(\frac{y_l}{\theta_h}, L)\frac{\pi_h}{\theta_h} \right]}{\pi_h + \mu}$$

$v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h}{\theta_h}$ term in the right hand side is the marginal benefit that high type gets from the labor externality and it is negative. And because $\pi_l > \mu$ and by the cross derivative of $v_{21} > 0$ it can be said that $v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_h}{\theta_h} > v_2(\frac{y_l}{\theta_h}, L)\frac{\pi_h}{\theta_h}$ in absolute value, because the agent who is working more than the others, will get a higher marginal benefit from the increase in the average number of working hour. Therefore the third term in the right hand side will also be negative. So;

$$u'(c_h) < v_1(\frac{y_h}{\theta_h}, L)\frac{1}{\theta_h}$$

Labor externalities distort the condition for the market. In order to correct this externality effect, there must be a tax for the high type, which eliminates well-known result: implementing zero marginal tax at the top of the ability distribution.

From (c_l) and (y_l);

$$u'(c_l)\left[\pi_l - \mu\right] = \pi_l \left[v_1(\tfrac{y_l}{\theta_l}, L)\tfrac{1}{\theta_l}\right] + \pi_l \left[v_2(\tfrac{y_l}{\theta_l}, L)\tfrac{\pi_l}{\theta_l}\right] + (\pi_h + \mu)\left[v_2(\tfrac{y_h}{\theta_h}, L)\tfrac{\pi_l}{\theta_l}\right] + \mu\left[-v_1(\tfrac{y_l}{\theta_h}, L)\tfrac{1}{\theta_h} - v_2(\tfrac{y_l}{\theta_h}, L)\tfrac{\pi_l}{\theta_l}\right]$$

Under private information, marginal cost of working could be higher or lower than the marginal benefit of consumption for low type agent. This condition is identified by the two opposite effects which are disincentive and externality effects. If the labor externality effect is higher than the disincentive effect, low productive agent gets a subsidy as the high productive agent. The size of the subsidies will identify the tax schedule.

Now I will investigate the optimal marginal tax schedule under private information.

4.1 Optimal Marginal Income Taxes Under Private Information

When information asymmetry is added to the model, the role of marginal taxes are more crucial, because a higher tax above the optimal level will have a disincentive effect on the high productive agent and would cause considerable efficiency losses.

Proposition 4 *Marginal Taxes under private information with labor externalities are;*

$(1-\tau'(y_h)) = 1 + \frac{\Sigma}{\lambda\theta_h}$ *and* $(1-\tau'(y_l)) = (1 + \frac{\Sigma}{\lambda\theta_l})\Psi$

where $\Sigma = -\left[v_2(\frac{y_h}{\theta_h}, L)\left[\pi_h + \mu\right] + v_2(\frac{y_l}{\theta_l}, L)\pi_l - v_2(\frac{y_l}{\theta_h}, L)\mu\right]$ *and* $\Psi = \left[\frac{\pi_l - \mu}{\pi_l - \phi\mu}\right]$.

Proof. From market solution it is known that marginal tax condition is;

$(1-\tau'(y)) = \frac{v_1(\frac{y}{\theta}, L)\frac{1}{\theta}}{u'(c)}$.

Marginal Tax for High-type;

From (y_h) : $v_1(\frac{y_h}{\theta_h}, L)\frac{1}{\theta_h}\left[\pi_h + \mu\right] = \lambda\pi_h - v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_h(\pi_h+\mu)}{\theta_h} - v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_h\pi_l}{\theta_h} + v_2(\frac{y_l}{\theta_h}, L)\frac{\mu\pi_h}{\theta_h}$.

From (c_h) : $u'(c_h)\left[\pi_h + \mu\right] = \lambda\pi_h$.

Dividing both side gives:

$(1-\tau'(y_h)) = 1 - v_2(\frac{y_h}{\theta_h}, L)\frac{(\pi_h+\mu)}{\lambda\theta_h} - v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_l}{\lambda\theta_h} + v_2(\frac{y_l}{\theta_h}, L)\frac{\mu}{\lambda\theta_h}$

Marginal Tax for Low-type;

From (y_l) : $v_1(\frac{y_l}{\theta_l}, L)\frac{1}{\theta_l}\left[\pi_l - \mu\frac{v_1(\frac{y_l}{\theta_h}, L)\frac{1}{\theta_h}}{v_1(\frac{y_l}{\theta_l}, L)\frac{1}{\theta_l}}\right] = \lambda\pi_l - v_2(\frac{y_h}{\theta_h}, L)\frac{\pi_l(\pi_h+\mu)}{\theta_l} - v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_l\pi_l}{\theta_l} + v_2(\frac{y_l}{\theta_h}, L)\frac{\mu\pi_l}{\theta_l}$.

From (c_l) : $u'(c_l)\left[\pi_l - \mu\right] = \lambda\pi_l$.

Dividing both side gives:

$(1-\tau'(y_l))\left[\frac{\pi_l - \mu\phi}{\pi_l - \mu}\right] = 1 - v_2(\frac{y_h}{\theta_h}, L)\frac{(\pi_h+\mu)}{\lambda\theta_l} - v_2(\frac{y_l}{\theta_l}, L)\frac{\pi_l}{\lambda\theta_l} + v_2(\frac{y_l}{\theta_h}, L)\frac{\mu}{\lambda\theta_l}$

where ϕ is $0 < \frac{v_1(\frac{y_l}{\theta_h}, L)\theta_l}{v_1(\frac{y_l}{\theta_l}, L)\theta_h} < 1$ because convexity of $v(.)$ implies $v_1(\frac{y_l}{\theta_l}, L) > v_1(\frac{y_l}{\theta_h}, L)$.

Then the marginal taxes become:

$(1-\tau'(y_h)) = 1 - \frac{1}{\lambda\theta_h}\left[v_2(\frac{y_h}{\theta_h}, L)\left[\pi_h + \mu\right] + v_2(\frac{y_l}{\theta_l}, L)\pi_l - v_2(\frac{y_l}{\theta_h}, L)\mu\right]$

$(1-\tau'(y_l)) = \left\{1 - \frac{1}{\lambda\theta_l}\left[v_2(\frac{y_h}{\theta_h}, L)\left[\pi_h + \mu\right] + v_2(\frac{y_l}{\theta_l}, L)\pi_l - v_2(\frac{y_l}{\theta_h}, L)\mu\right]\right\}\left[\frac{\pi_l - \mu}{\pi_l - \phi\mu}\right]$

Since $\phi < 1$ then $\Psi = \left[\frac{\pi_l - \mu}{\pi_l - \phi\mu}\right] < 1$.

Cross derivative of $v_{21} > 0$, therefore $v_2(\frac{y_l}{\theta_h}, L) < v_2(\frac{y_l}{\theta_l}, L)$ and $v_2(\frac{y_l}{\theta_h}, L) < v_2(\frac{y_h}{\theta_h}, L)$ in absolute value. Then the summation in the brackets is negative.

Let $\Sigma = -\left[v_2(\frac{y_h}{\theta_h}, L)\left[\pi_h + \mu\right] + v_2(\frac{y_l}{\theta_l}, L)\pi_l - v_2(\frac{y_l}{\theta_h}, L)\mu\right]$ we have $\Sigma > 0$

The marginal income taxes for both types are as follows:

$(1 - \tau'(y_h)) = 1 + \frac{\Sigma}{\lambda\theta_h}$ *and* $(1 - \tau'(y_l)) = (1 + \frac{\Sigma}{\lambda\theta_l})\Psi$ ■

Since $\theta_h > \theta_l$, it is obvious that $1+\frac{\Sigma}{\lambda\theta_h} < 1+\frac{\Sigma}{\lambda\theta_l}$. There fore the progressivity of tax schedule is identified by the multiplication of $(1 + \frac{\Sigma}{\lambda\theta_l})\Psi$.

When the tax schedule under labor interdependence is compared with the Mirrlees taxes, it is seen that the disincentive and the externality effects can be separated. Following remark shows the tax functions of Mirrlees and labor externality cases.

Remark 1 *Labor externality is seen in a multiplicative fashion over the standard Mirrlees information problem. For the $U(c)-V(\frac{y}{\theta})$ form of utility Mirrlees setup taxes are;*

$(1 - \tau'(y_h)) = 1$ *and* $(1 - \tau'(y_l)) = \Psi$ *where* $\Psi = \frac{\pi_l - \mu}{\pi_l - \phi\mu} < 1$.

And when the Labor externality added to the model, the utility form becomes $U(c) - V(\frac{y}{\theta}, L)$ and taxes are as follows;

$(1 - \tau'(y_h)) = 1 + \frac{\Sigma}{\lambda\theta_h}$ *and* $(1 - \tau'(y_l)) = (1 + \frac{\Sigma}{\lambda\theta_l})\Psi$.

Optimal marginal tax functions have two components that are Mirrleesian and Pigouvian taxes. The term $1+\frac{\Sigma}{\lambda\theta_h}$ in high type tax is the term that comes from labor externality. But in low type tax, Ψ comes from private information and it is a regressive force for taxation. On the other hand $1+\frac{\Sigma}{\lambda\theta_l}$ term comes from the externality effect and it is a progressive force for marginal tax. The multiplication of these two opposite forces identifies the tax schedule as regressive or progressive. These two tax effects can be distinguished as Mirrleesian tax and Pigouvian tax. The force that makes the tax schedule regressive is Mirrleesian tax and the tax that arise from externality could be called as

Pigouvian tax. In some cases where the externality effect dominates the informational problem effect, low productive agent gets more subsidy than the high productive one which forms a progressive marginal income tax schedule. If $\Psi > \frac{1+\frac{\Sigma}{\lambda\theta_h}}{(1+\frac{\Sigma}{\lambda\theta_l})}$ marginal taxes are progressive, otherwise they have a regressive form.

In general it is not obvious which component dominates, however I found a condition on model parameters under which optimal tax schedule is always progressive.

Proposition 5 *Optimal marginal income tax schedule is progressive while* $\pi_l \to 1$

Proof. We know that $\Psi = 1$, is sufficient condition for progressivity of taxes, from the tax function defined by the equation:

$(1-\tau'(y_h)) = 1 + \frac{\Sigma}{\lambda\theta_h}$ *and* $(1-\tau'(y_l)) = (1+\frac{\Sigma}{\lambda\theta_l})\Psi$, because $\theta_h > \theta_l$.

Therefore, if we show that $\lim_{\pi_l \to 1} \Psi = 1$, then we can conclude that the tax schedule will be progressive when $\pi_l \to 1$.

First, from the FOC's of the SPP, we have $\lambda = \frac{u'(c_l)u'(c_h)}{\pi_h u'(c_l)+\pi_l u'(c_h)}$.

Then the limit of this term is given by:

$\lim_{\pi_l \to 1} \lambda = \lim_{\pi_l \to 1} \frac{u'(c_l)u'(c_h)}{\pi_h u'(c_l)+\pi_l u'(c_h)}$

$= \lim_{\pi_l \to 1} \frac{u'(c_l)u'(c_h)}{(1-\pi_l)u'(c_l)+\pi_l u'(c_h)} = \lim_{\pi_l \to 1} \frac{u'(c_l)u'(c_h)}{\pi_l u'(c_h)} = u'(c_l)$.

From FOC's we also know, $\mu = \pi_l(1 - \frac{\lambda}{u'(c_l)})$ which results

$\lim_{\pi_l \to 1} \mu = \lim_{\pi_l \to 1} \pi_l(1 - \frac{\lambda}{u'(c_l)})$ $=\lim_{\pi_l \to 1} 1 - \frac{\lim_{\pi_l \to 1} \lambda}{u'(c_l)} = 0$.

Finally, Ψ is given by, $\Psi = \frac{\pi_l - \mu}{\pi_l - \phi\mu}$. Hence,

$\lim_{\pi_l \to 1} \Psi = \lim_{\pi_l \to 1} \frac{\pi_l - \mu}{\pi_l - \phi\mu} = \lim_{\pi_l \to 1} \frac{\pi_l - \lim_{\pi_l \to 1} \mu}{\pi_l - \phi \lim_{\pi_l \to 1} \mu} = \lim_{\pi_l \to 1} \frac{\pi_l - 0}{\pi_l - \phi.0}$ $=1$.

So, while low ability proportion, π_l, is increasing and reaches to 1, the income tax schedule is going to have a progressive fashion for certain. ■

Here is the intuition for this result. The only source of regressivity is the Mirrleesian component. As π_l goes to 1, incentive compatibility constraint multiplier μ goes to 0. Therefore Mirrleesian effect becomes less important. So there exist a π_l high enough above which Pigouvian effect always dominates Mirrleesian effect which means the optimal tax schedule is progressive.

Equations for marginal taxes contains endogenous variables. Therefore it is impossible to give a precise condition over the model parameters that makes the income schedule progressive.

5 N-Type Case With General Utility Form

Before testing the model in a specific utility from, the paper investigates the N-type problem in order to show the impossibility of having a precise condition that makes the tax schedule progressive. It will be better to try the N-type problem in a general utility form like $u(c) - v(\frac{y}{\theta}) + L$. Because in the form $u(c) - v(\frac{y}{\theta}, L)$ it is not possible to interpret the results easily. Then the social planner's problem will be in the following form:

$$\max_{c_i, y_i} \left[\sum_{i=1}^{N} \pi_i (u(c_i) - v\left(\frac{y_i}{\theta_i}\right) + L) \right]$$

subject to

$$\sum_{i=1}^{n} \pi_i c_i \leq \sum_{i=1}^{n} \pi_i y_i \qquad (\lambda)$$

$$u(c_i) - v\left(\frac{y_i}{\theta_i}\right) \geq u(c_{i-1}) - v\left(\frac{y_{i-1}}{\theta_i}\right) \qquad (\mu_i)$$

$$L = \sum_{i=1}^{n} \pi_i \frac{y_i}{\theta_i} \ and \ \ \mu_1 = 0$$

Letting λ and μ_i be the multipliers on the feasibility and incentive compatibility constraints respectively, FOC are as follows:

$$(c_i): \quad \pi_i u'(c_i) - \lambda \pi_i + \mu_i u'(c_i) - \mu_{i+1} u'(c_i) = 0$$
$$(c_N): \quad \pi_N u'(c_N) - \lambda \pi_N + \mu_N u'(c_N) = 0$$

$$(y_i): \quad \pi_i \left[-v'(\tfrac{y_i}{\theta_i}) \tfrac{1}{\theta_i} \right] + \tfrac{\pi_i}{\theta_i} + \lambda \pi_i - \mu_i v'(\tfrac{y_i}{\theta_i}) \tfrac{1}{\theta_i} + \mu_{i+1} v'(\tfrac{y_i}{\theta_{i+1}}) \tfrac{1}{\theta_{i+1}} = 0$$
$$(y_N): \quad \pi_N \left[-v'(\tfrac{y_N}{\theta_N}) \tfrac{1}{\theta_N} \right] + \tfrac{\pi_N}{\theta_N} + \lambda \pi_N - \mu_N v'(\tfrac{y_N}{\theta_N}) \tfrac{1}{\theta_N} = 0$$

From these conditions, one can get the optimal tax schedule.

Proposition 6 *Marginal Taxes under private information with labor externalities are;*

$(1-\tau'(y_N)) = 1 + \frac{1}{\lambda\theta_N}$

$(1-\tau'(y_i)) = (1 + \frac{1}{\lambda\theta_i})\Psi_i$

where $\Psi_i = \left[\frac{\pi_i\omega_i+\mu_i-\mu_{i+1}}{\pi_i\omega_i+\mu_i-\phi_i\mu_{i+1}}\right]$ *and* $\lambda = \left[\sum_{j=1}^{n}\frac{\pi_j}{u'(c_j)}\right]^{-1}$

Proof. *Marginal Tax for type-N:*

From the market; $(1-\tau'(y)) = \frac{v'(\frac{y}{\theta})\frac{1}{\theta}}{u'(c)}$

From $(y_N): \quad v'(\frac{y_N}{\theta_N})\frac{1}{\theta_N}(\pi_N + \mu_N) = \lambda\pi_N + \frac{\pi_N}{\theta_N}$

From $(c_N): \quad u'(c_N)(\pi_N + \mu_N) = \lambda\pi_N$

Dividing both side gives;

$(1-\tau'(y_N)) = 1 + \frac{1}{\lambda\theta_N}$

Marginal Tax for type-i:

From $(y_i): \quad v'(\frac{y_i}{\theta_i})\frac{1}{\theta_i}\left[\pi_i + \mu_i - \mu_{i+1}\frac{v'(\frac{y_i}{\theta_{i+1}})\frac{1}{\theta_{i+1}}}{v'(\frac{y_i}{\theta_i})\frac{1}{\theta_i}}\right] = \lambda\pi_i + \frac{\pi_i}{\theta_i}$

From $(c_i): \quad u'(c_i)(\pi_i + \mu_i - \mu_{i+1}) = \lambda\pi_i$

Dividing both side gives;

$(1-\tau'(y_i)) = (1 + \frac{1}{\lambda\theta_i})\left[\frac{\pi_i+\mu_i-\mu_{i+1}}{\pi_i+\mu_i-\phi_i\mu_{i+1}}\right]$ *where* $\phi_i = \frac{v'(\frac{y_i}{\theta_{i+1}})\frac{1}{\theta_{i+1}}}{v'(\frac{y_i}{\theta_i})\frac{1}{\theta_i}} < 1$

$(1-\tau'(y_i)) = (1 + \frac{1}{\lambda\theta_i})\Psi_i$ *where* $\Psi_i = \left[\frac{\pi_i\omega_i+\mu_i-\mu_{i+1}}{\pi_i\omega_i+\mu_i-\phi_i\mu_{i+1}}\right] < 1$ ■

As in two-type model the tax functions has two separable effects that are Mirrleesian and Pigouvian taxes, and the tax schedule will be identified by these two effects. For a progressive tax schedule the following condition must be satisfied.

Proposition 7 *Optimal marginal tax schedule is progressive if and only if*

$$\frac{\lambda\frac{\pi_i}{u'(c_i)}}{(1-\phi_i)[\pi_i+\Omega]+\phi_i\lambda\frac{\pi_i}{u'(c_i)}} > \frac{\lambda\frac{\pi_{i+1}}{u'(c_{i+1})}}{(1-\phi_{i+1})\left[\pi_i+\pi_{i+1}+\Omega-\lambda\frac{\pi_i}{u'(c_i)}\right]+\phi_{i+1}\lambda\frac{\pi_{i+1}}{u'(c_{i+1})}}\frac{\theta_i(\lambda\theta_{i+1}+1)}{\theta_{i+1}(\lambda\theta_i+1)}$$

$$where\ \Omega = \sum_{j=1}^{i-1} \pi_j - \left[\sum_{j=1}^{n} \frac{\pi_j}{u'(c_j)}\right]^{-1} \sum_{j=1}^{i-1} \frac{\pi_j}{u'(c_j)} \quad and \quad \lambda = \left[\sum_{j=1}^{n} \frac{\pi_j}{u'(c_j)}\right]^{-1} and$$

$$\phi_i = \frac{v'(\frac{y_i}{\theta_{i+1}})\frac{1}{\theta_{i+1}}}{v'(\frac{y_i}{\theta_i})\frac{1}{\theta_i}}$$

Proof. When we look at ith and i+1th agents in the society, their tax functions are as follows;

$$(1 - \tau'(y_i)) = (1 + \frac{1}{\lambda\theta_i})\Psi_i \qquad and \qquad (1 - \tau'(y_{i+1})) = (1 + \frac{1}{\lambda\theta_{i+1}})\Psi_{i+1}$$

where $\Psi_i = \left[\frac{\pi_i\omega_i + \mu_i - \mu_{i+1}}{\pi_i\omega_i + \mu_i - \phi_i\mu_{i+1}}\right]$ and $\Psi_{i+1} = \frac{\pi_{i+1} + \mu_{i+1} - \mu_{i+2}}{\pi_{i+1} + \mu_{i+1} - \phi_{i+1}\mu_{i+2}}$ and $\lambda = \left[\sum_{j=1}^{n} \frac{\pi_j}{u'(c_j)}\right]^{-1}$

If $(1 + \frac{1}{\lambda\theta_i})\Psi_i > (1 + \frac{1}{\lambda\theta_{i+1}})\Psi_{i+1}$ then the optimal taxation scheme will be progressive.

For; $\qquad (1 + \frac{1}{\lambda\theta_i})\frac{\pi_i + \mu_i - \mu_{i+1}}{\pi_i + \mu_i - \phi_i\mu_{i+1}} > (1 + \frac{1}{\lambda\theta_{i+1}})\frac{\pi_{i+1} + \mu_{i+1} - \mu_{i+2}}{\pi_{i+1} + \mu_{i+1} - \phi_{i+1}\mu_{i+2}}$

where $\phi_i = \frac{v'(\frac{y_i}{\theta_{i+1}})\frac{1}{\theta_{i+1}}}{v'(\frac{y_i}{\theta_i})\frac{1}{\theta_i}}$ and $\mu_i = \sum_{j=1}^{i-1} \pi_j - \lambda \sum_{j=1}^{i-1} \frac{\pi_j}{u'(c_j)}$

After plugging and manipulating the terms gives the condition:

$$\frac{\lambda\frac{\pi_i}{u'(c_i)}}{(1-\phi_i)[\pi_i + \Omega] + \phi_i\lambda\frac{\pi_i}{u'(c_i)}} > \frac{\lambda\frac{\pi_{i+1}}{u'(c_{i+1})}}{(1-\phi_{i+1})\left[\pi_i + \pi_{i+1} + \Omega - \lambda\frac{\pi_i}{u'(c_i)}\right] + \phi_{i+1}\lambda\frac{\pi_{i+1}}{u'(c_{i+1})}} \frac{\theta_i(\lambda\theta_{i+1} + 1)}{\theta_{i+1}(\lambda\theta_i + 1)}$$

where $\Omega = \sum_{j=1}^{i-1} \pi_j - \left[\sum_{j=1}^{n} \frac{\pi_j}{u'(c_j)}\right]^{-1} \sum_{j=1}^{i-1} \frac{\pi_j}{u'(c_j)}$ and $\lambda = \left[\sum_{j=1}^{n} \frac{\pi_j}{u'(c_j)}\right]^{-1}$ and

$\phi_i = \frac{v'(\frac{y_i}{\theta_{i+1}})\frac{1}{\theta_{i+1}}}{v'(\frac{y_i}{\theta_i})\frac{1}{\theta_i}}$ ■

This condition contains some endogenous variables, so it is impossible to give a precise condition over exogenous parameters that make the tax progressive in the N-type case. Next part of the paper assumes a specific form of utility to interpret the results.

6 Private Information Case With Linear Utilities

Because Ψ is determined by the endogenous variables in the general form of utility, this part assumes a specific form of utility to interpret the results. Let us assume the following form of utility and disutility functions which Diamond (1998) uses:

$u(c) = c$ and $v\left(\frac{y}{\theta}, L\right) = \left(\frac{y}{\theta}\right)^{\alpha} - L$

First, I will analyze the 2-type linear utility case, after that I will generalize it to the N-type case.

6.1 Social Planner Problem With Linear Utilities

Social planner problem solves the following maximization problem: In this kind of a utility form, without a weighting parameter incentive compatibility constraint is not binding, and the problem is becoming meaningless. Therefore social planner has to give weight to the low type agent more than the high type, and the problem becomes:

$$\max_{c_l, c_h, y_l, y_h} \varphi \pi_l \left[c_l - (\left(\frac{y_l}{\theta_l}\right)^{\alpha} - L) \right] + \pi_h \left[c_h - (\left(\frac{y_h}{\theta_h}\right)^{\alpha} - L) \right]$$

subject to

$$\pi_h c_h + \pi_l c_l \leq \pi_h y_h + \pi_l y_l \qquad (\lambda)$$

$$c_h - \left(\left(\frac{y_h}{\theta_h}\right)^{\alpha} - L\right) \geq c_l - \left(\left(\frac{y_l}{\theta_h}\right)^{\alpha} - L\right) \qquad (\mu)$$

$$L = \pi_h \frac{y_h}{\theta_h} + \pi_l \frac{y_l}{\theta_l} \text{ and } \varphi > 1$$

Letting λ and μ be the multipliers on the feasibility and incentive compatibility constraints respectively, FOC are as follows:

$$(c_l): \qquad \varphi\pi_l - \lambda\pi_l - \mu = 0$$
$$(c_h): \qquad \pi_h - \lambda\pi_h + \mu = 0$$

$$(y_l): \qquad \varphi\pi_l\left[-\alpha\frac{y_l^{\alpha-1}}{\theta_l^{\alpha}} + \frac{\pi_l}{\theta_l}\right] + \pi_h\left[\frac{\pi_l}{\theta_l}\right] + \lambda\pi_l + \mu\left[\alpha\frac{y_l^{\alpha-1}}{\theta_h^{\alpha}}\right] = 0$$
$$(y_h): \qquad \pi_h\left[-\alpha\frac{y_h^{\alpha-1}}{\theta_h^{\alpha}} + \frac{\pi_h}{\theta_h}\right] + \varphi\pi_l\left[\frac{\pi_h}{\theta_h}\right] + \lambda\pi_h + \mu\left[-\alpha\frac{y_h^{\alpha-1}}{\theta_h^{\alpha}}\right] = 0$$

From the FOC of agents, social planner optimality conditions are derived in the following proposition.

Proposition 8 *The conditions that characterize the social planner's problem are;*

$$\alpha\frac{y_h^{\alpha-1}}{\theta_h^{\alpha}} = 1 + \frac{1}{\theta_h}$$
$$\alpha\frac{y_l^{\alpha-1}}{\theta_l^{\alpha}} = (1 + \frac{1}{\theta_l})\frac{\varphi\pi_l+\pi_h}{\varphi-\frac{\theta_l^{\alpha}}{\theta_h^{\alpha}}(\varphi-\varphi\pi_l-\pi_h)}.$$

From the FOC of high type agent (y_h) and (c_h);

$$\pi_h\left[-\alpha\frac{y_h^{\alpha-1}}{\theta_h^{\alpha}} + \frac{\pi_h}{\theta_h}\right] + \varphi\pi_l\left[\frac{\pi_h}{\theta_h}\right] + \lambda\pi_h + (\lambda\pi_h - \pi_h)\left[-\alpha\frac{y_h^{\alpha-1}}{\theta_h^{\alpha}}\right] = 0$$

Manipulating the equation gives:

$$\frac{\varphi\pi_l+\pi_h}{\theta_h} + \lambda = \lambda(\alpha\frac{y_h^{\alpha-1}}{\theta_h^{\alpha}}) \qquad where \qquad \lambda = \varphi\pi_l + \pi_h$$

$$\alpha\frac{y_h^{\alpha-1}}{\theta_h^{\alpha}} = 1 + \frac{1}{\theta_h} > 1 = u'(c_h)$$

In the social planner's efficient allocation conditions, because of the labor externalities, marginal cost of consumption is not equal to the marginal benefit of consumption for high type. So, planner should impose a tax on agent's income which leads to a tax at the top of the ability distribution.

(y_l) and (c_l) implies:

$\varphi\pi_l\left[-\alpha\frac{y_l^{\alpha-1}}{\theta_l^\alpha}+\frac{\pi_l}{\theta_l}\right]+\pi_h\left[\frac{\pi_l}{\theta_l}\right]+\varphi\pi_l-\mu+\mu\left[\alpha\frac{y_l^{\alpha-1}}{\theta_h^\alpha}\right]=0$
Manipulating the equation gives:
$\alpha\frac{y_l^{\alpha-1}}{\theta_l^\alpha}\left[\varphi\pi_l-\mu\frac{\theta_l^\alpha}{\theta_h^\alpha}\right]=\frac{\varphi\pi_l\pi_l}{\theta_l}+\frac{\pi_h\pi_l}{\theta_l}+\varphi\pi_l-\mu$ where $\mu=\varphi\pi_l-\lambda\pi_l$
$=\frac{\varphi\pi_l\pi_l}{\theta_l}+\frac{\pi_h\pi_l}{\theta_l}+\varphi\pi_l-\varphi\pi_l+\lambda\pi_l$
$=\lambda\pi_l(1+\frac{1}{\theta_l})$ where $\lambda=\varphi\pi_l+\pi_h$
$\alpha\frac{y_l^{\alpha-1}}{\theta_l^\alpha}=(1+\frac{1}{\theta_l})\frac{\varphi\pi_l+\pi_h}{\varphi-\frac{\theta_l^\alpha}{\theta_h^\alpha}(\varphi-\varphi\pi_l-\pi_h)}$
The term $\frac{\varphi\pi_l+\pi_h}{\varphi-\frac{\theta_l^\alpha}{\theta_h^\alpha}(\varphi-\varphi\pi_l-\pi_h)}=1$ if only if $\pi_l=1$ otherwise $\frac{\varphi\pi_l+\pi_h}{\varphi-\frac{\theta_l^\alpha}{\theta_h^\alpha}(\varphi-\varphi\pi_l-\pi_h)}<1$

The condition for low productive agent is not as clear as the condition of the agent with high productivity. The informational problem and the labor externalities will identify the condition for low type.

6.2 Optimal Marginal Income Taxes With Linear Utilities

In order to identify taxes, the optimal tax condition must be obtained from the market. The condition is derived from the agent's problem. The agent's problem as follows:

$max\ \ c-(\frac{y}{\theta})^\alpha+L$
subject to
$c\leq y-\tau(y)$
$L=\pi_h\frac{y_h}{\theta_h}+\pi_l\frac{y_l}{\theta_l}$

Letting λ is the feasibility constraint multiplier, FOC are as follows:
$(c):\qquad 1-\lambda=0$
$(y):\qquad -\alpha\frac{y^{\alpha-1}}{\theta^\alpha}+\lambda-\lambda\tau'(y)=0$

$$(1-\tau'(y))=\alpha\frac{y^{\alpha-1}}{\theta^\alpha}$$

Where $\alpha\frac{y^{\alpha-1}}{\theta^{\alpha}}$ marginal cost of one more unit of consumption. When tax is zero, agents would equalize the marginal benefit and cost of additional one more unit of consumption.

$MB = u'(c) = 1 = \alpha\frac{y^{\alpha-1}}{\theta^{\alpha}} = MC$

The next proposition shows the optimal taxes for both agents.

Proposition 9 *If the utilities are linear, under private information with labor externalities, marginal taxes are:*

$$(1-\tau'(y_h)) = 1 + \frac{1}{\theta_h}$$
$$(1-\tau'(y_l)) = (1+\frac{1}{\theta_l})\Psi \qquad where\ \Psi = \frac{\theta_h^{\alpha}(\pi_h+\varphi\pi_l)}{\varphi\theta_h^{\alpha}-\varphi\theta_l^{\alpha}+\theta_l^{\alpha}\pi_h+\varphi\theta_l^{\alpha}\pi_l}$$

Proof. Marginal Tax for High-type;

From $(y_h):\quad \alpha\frac{y_h^{\alpha-1}}{\theta_h^{\alpha}}(\pi_h+\mu) = \lambda\pi_h + \frac{\pi_h\pi_h}{\theta_h} + \frac{\varphi\pi_l\pi_h}{\theta_h}$

From $(c_h):\quad \pi_h + \mu = \lambda\pi_h$

Dividing both side gives;

$(1-\tau'(y_h)) = 1 + \frac{1}{\theta_h}$

Marginal Tax for Low-type;

$$\text{From } (y_l):\quad \alpha\frac{y_l^{\alpha-1}}{\theta_l^{\alpha}}\left[\varphi\pi_l - \mu\frac{\alpha\frac{y_l^{\alpha-1}}{\theta_h^{\alpha}}}{\alpha\frac{y_l^{\alpha-1}}{\theta_l^{\alpha}}}\right] = \lambda\pi_l + \frac{\pi_h\pi_l}{\theta_l} + \frac{\varphi\pi_l\pi_l}{\theta_l}$$

$$\alpha\frac{y_l^{\alpha-1}}{\theta_l^{\alpha}}\left[\varphi\pi_l - \mu\frac{\theta_l^{\alpha}}{\theta_h^{\alpha}}\right] = \lambda\pi_l + \frac{\lambda\pi_l}{\theta_l}$$

From $(c_l):\quad \varphi\pi_l - \mu = \lambda\pi_l$

Dividing both side gives;

$(1-\tau'(y_l)) = (1+\frac{1}{\theta_l})\left[\frac{\varphi\pi_l-\mu}{\varphi\pi_l-\mu\phi}\right] \qquad where\ \phi = \frac{\theta_l^{\alpha}}{\theta_h^{\alpha}} < 1$

$(1-\tau'(y_l)) = (1+\frac{1}{\theta_l})\Psi \qquad where\ \Psi = \left[\frac{\varphi\pi_l-\mu}{\varphi\pi_l-\mu\phi}\right] < 1$

$\lambda = \varphi\pi_l + \pi_h$ and $\mu = \varphi\pi_l - \pi_l(\varphi\pi_l + \pi_h)$ then;

$\Psi = \frac{\theta_h^{\alpha}(\pi_h+\varphi\pi_l)}{\varphi\theta_h^{\alpha}-\varphi\theta_l^{\alpha}+\theta_l^{\alpha}\pi_h+\varphi\theta_l^{\alpha}\pi_l}$ ■

In the tax function of high type, the term $1+\frac{1}{\theta_h}$ is the labor externality effect. For high type the information problem effect is 1. On the other hand for low type agent while labor externality effect term is $1+\frac{1}{\theta_l}$, the disincentive effect is Ψ. Therefore the optimal tax schedule will be identified by these two multiplicative effects. Following proposition gives the condition that makes the optimal tax schedule progressive.

Proposition 10 *Optimal tax schedule is progressive if and only if* $\frac{\theta_h^{\alpha+1}(\pi_h+\varphi\pi_l)}{\theta_h^{\alpha}\theta_l\varphi-\theta_l^{\alpha+1}(\varphi-\pi_h-\varphi\pi_l)} > \frac{\theta_h+1}{\theta_l+1}$.

Proof. For a progressive tax schedule it must be the case that;

$(1-\tau'(y_l)) > (1-\tau'(y_h))$.

$(1+\frac{1}{\theta_l})\Psi > 1+\frac{1}{\theta_h}$

plugging $\Psi = \frac{\theta_h^{\alpha}(\pi_h+\varphi\pi_l)}{\varphi\theta_h^{\alpha}-\varphi\theta_l^{\alpha}+\theta_l^{\alpha}\pi_h+\varphi\theta_l^{\alpha}\pi_l}$ and rearranging the terms gives the condition. ■

In this proposition, the condition that makes the tax schedule progressive consists of the exogenous parameters. So with the appropriate parameters tax schedule have to be progressive. This result shows that if we add the labor interdependency over to the standard Mirrleesian setup, under this condition there could be a progressive income tax schedule.

The proposition below, gives the First-Best case with the weighting parameter being 1, which makes incentive compatibility constraint multiplier 0.

Proposition 11 *Problem becomes the First-Best case when $\varphi = 1$, and optimal marginal tax schedule is progressive.*

Proof. $\mu = \varphi\pi_l - \pi_l(\varphi\pi_l+\pi_h) = \pi_l - \pi_l(\pi_l+\pi_h) = 0$

Incentive compatibility constraint is not binding. In the First-Best;

$(1-\tau'(y_h)) = 1+\frac{1}{\theta_h}$

$(1-\tau'(y_l)) = (1+\frac{1}{\theta_l})\left[\frac{\varphi\pi_l-\mu}{\varphi\pi_l-\mu\phi}\right]$

$\varphi = 1$ and $\mu = 0$ then $(1-\tau'(y_l)) = (1+\frac{1}{\theta_l})$

Since $\theta_h > \theta_l$, $1 + \frac{1}{\theta_l} > 1 + \frac{1}{\theta_h}$. This means there is progressive marginal income taxation.

$$\tau'(y_h) > \tau'(y_l)$$

■

7 N-type Case With Linear Utilities

In this section, the economy is populated by N agents and the utility function is linear. Social planner's problem is as follows:

$$\max_{c_i,y_i} \left[\sum_{i=1}^{N} \omega_i \pi_i (c_i - \left(\frac{y_i}{\theta_i}\right)^{\alpha} + L) \right]$$

subject to

$$\sum_{i=1}^{n} \pi_i c_i \leq \sum_{i=1}^{n} \pi_i y_i \qquad (\lambda)$$

$$c_i - \left(\frac{y_i}{\theta_i}\right)^{\alpha} + L \geq c_{i-1} - \left(\frac{y_{i-1}}{\theta_i}\right)^{\alpha} + L \qquad (\mu_i)$$

$$L = \sum_{i=1}^{n} \pi_i \frac{y_i}{\theta_i} \text{ and } \theta_i < \theta_{i+1}$$

Letting λ and μ_i be the multipliers on the feasibility and incentive compatibility constraints multiplier respectively, FOC are as follows:

(c_i) : $\omega_i \pi_i - \lambda \pi_i + \mu_i - \mu_{i+1} = 0$

(c_N) : $\omega_N \pi_N - \lambda \pi_N + \mu_N = 0$

(y_i) : $\omega_i \pi_i \left[-\alpha \frac{y_i^{\alpha-1}}{\theta_i^{\alpha}} + \frac{\pi_i}{\theta_i} \right] + \lambda \pi_i - \mu_i \alpha \frac{y_i^{\alpha-1}}{\theta_i^{\alpha}} + \mu_{i+1} \alpha \frac{y_i^{\alpha-1}}{\theta_{i+1}^{\alpha}} = 0$

(y_N) : $\omega_N \pi_N \left[-\alpha \frac{y_N^{\alpha-1}}{\theta_N^{\alpha}} + \frac{\pi_N}{\theta_N} \right] + \lambda \pi_N - \mu_N \alpha \frac{y_N^{\alpha-1}}{\theta_N^{\alpha}} = 0$

Following proposition shows the optimal tax schedule in N-type case.

Proposition 12 *If the utilities are linear, under private information with labor externalities, marginal taxes are:*

$(1 - \tau'(y_N)) = 1 + \frac{\omega_N \pi_N}{\lambda \theta_N}$

$(1 - \tau'(y_i)) = (1 + \frac{\pi_i \omega_i}{\lambda \theta_i}) \Psi_i$ *where* $\Psi_i = \left[\frac{\pi_i \omega_i + \mu_i - \mu_{i+1}}{\pi_i \omega_i + \mu_i - \phi_i \mu_{i+1}} \right]$

Proof. *Marginal Tax for type-N:*

From $(y_N): \quad \alpha\frac{y_N^{\alpha-1}}{\theta_N^{\alpha}}(\omega_N\pi_N+\mu_N)=\lambda\pi_N+\frac{\omega_N\pi_N\pi_N}{\theta_N}$

From $(c_N): \quad \omega_N\pi_N+\mu_N=\lambda\pi_N$

Dividing both side gives;

$(1-\tau'(y_N))=1+\frac{\omega_N\pi_N}{\lambda\theta_N}$

Marginal Tax for type-i:

From $(y_i): \quad \alpha\frac{y_i^{\alpha-1}}{\theta_i^{\alpha}}\left[\omega_i\pi_i+\mu_i-\mu_{i+1}\frac{\alpha\frac{y_i^{\alpha-1}}{\theta_{i+1}^{\alpha}}}{\alpha\frac{y_i^{\alpha-1}}{\theta_i^{\alpha}}}\right]=\lambda\pi_i+\frac{\omega_i\pi_i\pi_i}{\theta_i}$

From $(c_i): \quad \omega_i\pi_i+\mu_i-\mu_{i+1}=\lambda\pi_i$

Dividing both side gives;

$(1-\tau'(y_i))=(1+\frac{\pi_i\omega_i}{\lambda\theta_i})\left[\frac{\pi_i\omega_i+\mu_i-\mu_{i+1}}{\pi_i\omega_i+\mu_i-\phi_i\mu_{i+1}}\right] \qquad where\ \phi_i=\frac{\theta_i^{\alpha}}{\theta_{i+1}^{\alpha}}<1$

$(1-\tau'(y_i))=(1+\frac{\pi_i\omega_i}{\lambda\theta_i})\Psi_i \qquad where\ \Psi_i=\left[\frac{\pi_i\omega_i+\mu_i-\mu_{i+1}}{\pi_i\omega_i+\mu_i-\phi_i\mu_{i+1}}\right]<1 \quad \blacksquare$

The condition that makes the optimal tax schedule progressive is in following proposition.

Proposition 13 *Optimal marginal tax schedule is progressive if and only if*

$$\frac{\pi_i\Upsilon}{(1-\frac{\theta_i^{\alpha}}{\theta_{i+1}^{\alpha}})[\pi_i\omega_i+\Omega]+\frac{\theta_i^{\alpha}}{\theta_{i+1}^{\alpha}}[\pi_i\Upsilon]}>\frac{\pi_{i+1}\Upsilon}{(1-\frac{\theta_{i+1}^{\alpha}}{\theta_{i+2}^{\alpha}})[\omega_i\pi_i+\pi_{i+1}\omega_{i+1}+\Omega-\pi_i\Upsilon]+\frac{\theta_{i+1}^{\alpha}}{\theta_{i+2}^{\alpha}}[\pi_{i+1}\Upsilon]}\frac{\theta_i(\Upsilon\theta_{i+1}+\pi_{i+1}\omega_{i+1})}{\theta_{i+1}(\Upsilon\theta_i+\pi_i\omega_i)}$$

$$where\ \Omega=\sum_{j=1}^{i-1}\omega_j\pi_j-(\sum_{j=1}^{n}\omega_j\pi_j)\sum_{j=1}^{i-1}\pi_j \qquad and \qquad \Upsilon=\sum_{j=1}^{n}\omega_j\pi_j.$$

Proof. When we look two ith and i+1th agents in the society their tax functions are;

$(1-\tau'(y_i))=(1+\frac{\pi_i\omega_i}{\lambda\theta_i})\Psi_i \qquad and \qquad (1-\tau'(y_{i+1}))=(1+\frac{\pi_{i+1}\omega_{i+1}}{\lambda\theta_{i+1}})\Psi_{i+1}$

If $(1+\frac{\pi_i\omega_i}{\lambda\theta_i})\Psi_i>(1+\frac{\pi_{i+1}\omega_{i+1}}{\lambda\theta_{i+1}})\Psi_{i+1}$ then the optimal taxation scheme will be progressive.

For;

$(1+\frac{\pi_i\omega_i}{\lambda\theta_i})\frac{\pi_i\omega_i+\mu_i-\mu_{i+1}}{\pi_i\omega_i+\mu_i-\phi_i\mu_{i+1}}>(1+\frac{\pi_{i+1}\omega_{i+1}}{\lambda\theta_{i+1}})\frac{\omega_{i+1}\pi_{i+1}+\mu_{i+1}-\mu_{i+2}}{\omega_{i+1}\pi_{i+1}+\mu_{i+1}-\phi_{i+1}\mu_{i+2}}$

$$\lambda = \sum_{j=1}^{n} \omega_j \pi_j \quad \text{and} \quad \phi_i = \frac{\theta_i^{\alpha}}{\theta_{i+1}^{\alpha}} \quad \text{and} \quad \mu_i = \sum_{j=1}^{i-1} \omega_j \pi_j - (\sum_{j=1}^{n} \omega_j \pi_j) \sum_{j=1}^{i-1} \pi_j$$

Plugging these terms and rearranging them gives the condition;

$$\frac{\pi_i \Upsilon}{(1-\frac{\theta_i^{\alpha}}{\theta_{i+1}^{\alpha}})[\pi_i \omega_i + \Omega] + \frac{\theta_i^{\alpha}}{\theta_{i+1}^{\alpha}}[\pi_i \Upsilon]} > \frac{\pi_{i+1} \Upsilon}{(1-\frac{\theta_{i+1}^{\alpha}}{\theta_{i+2}^{\alpha}})[\omega_i \pi_i + \pi_{i+1}\omega_{i+1} + \Omega - \pi_i \Upsilon] + \frac{\theta_{i+1}^{\alpha}}{\theta_{i+2}^{\alpha}}[\pi_{i+1} \Upsilon]} \frac{\theta_i(\Upsilon \theta_{i+1} + \pi_{i+1}\omega_{i+1})}{\theta_{i+1}(\Upsilon \theta_i + \pi_i \omega_i)}$$

■

In an N-type economy, if the parameters are satisfying the condition stated in the proposition, the optimal income tax schedule should be in a progressive form. As it was shown before in the paper, in a general utility form the condition that makes the tax schedule has some endogenous parameters which makes it impossible to have an analytical solution. However in a specific form of utility such as linear utility, it is possible to have a analytical solution over the exogenous parameters.

8 Numeric Examples

In this part of the paper, I give numeric examples with log utility in order to see the change in tax schedules when labor interdependence is considered. Because Mirrlees (1971) uses log-utility as an example, in order to be consistent and be able to see the effect of labor externality, the paper uses the same form of utility function.

8.1 Log-Utility N-type

First of all, I analyze the N-type model with log-utility which Mirrlees (1971) uses. The social planner problem that maximizes the total welfare is as follows.

$$\max_{c_i, y_i} \left[\sum_{i=1}^{n} \pi_i (\log c_i + \log(1 - \frac{y_i}{\theta_i}) + L) \right]$$

subject to

$$\sum_{i=1}^{n} \pi_i c_i \leq \sum_{i=1}^{n} \pi_i y_i \qquad (\lambda)$$

$$\log c_i + \log(1 - \frac{y_i}{\theta_i}) \geq \log c_{i-1} + \log(1 - \frac{y_{i-1}}{\theta_i}) \qquad (\mu)$$

$$L = \sum_{i=1}^{n} \frac{y_i}{n\theta_i} \text{ and } \theta_i < \theta_{i+1}$$

In order to compare with the Mirrlees case, first, I give the Mirrlees results. The parameters are as follows: There are 100 agents, first agent's productivity is 2, it increases by 0.01 and reaches 3 for most productive agent. The tax schedule is as in the Figure 1. As seen in the graph, marginal taxes are positive, decrease in upper tail and show a regressive fashion. With the same parameters, the marginal tax schedule under interdependency is shown in Figure 2. In this case, marginal taxes are negative and increasing. In other words, subsidies are decreasing while agent's productivity is increasing. But these results are sensitive to number of agents and the productivity difference between agents. In Figure 3, population and first agent productivity is the same but productivity increment is 0.04. In this case, marginal tax is going to be regressive in the upper tail. This consideration could be an explanation of why European countries like Germany, France, Belgium has a more progressive marginal income tax schedule than the US. The studies that are using the International Adult Literacy Survey shows that the skills are more unequally distributed in the US than in the EU. The productivity differences in Europe are less than the US. And in Figure 4, first agent productivity and productivity increment is the same with Figure 2, but this time population increases to 200. Again in this case, marginal tax decreases in upper tail.

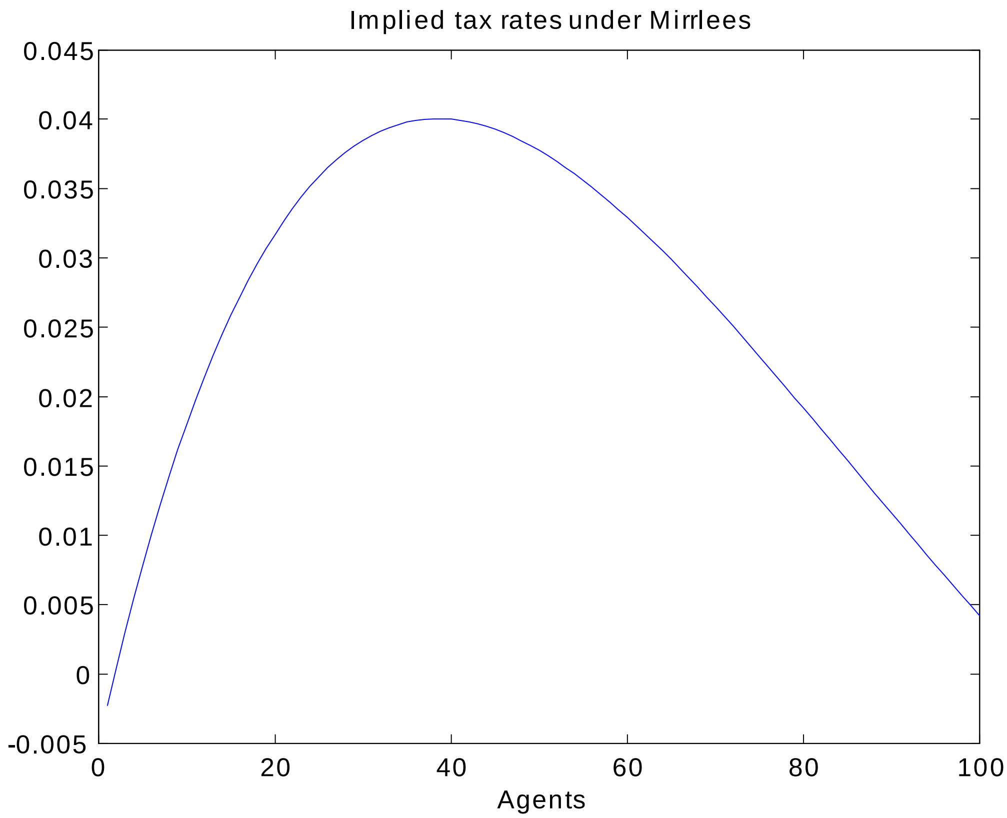

Figure 1

$N = 100$

$\theta \in \{2, 2.01, 2.02...3\}$

$\pi = 1/100$

This graph shows the standard Mirrlees optimal income taxation result where the marginal tax is regressive in the upper tail of the skill distribution. The tax is generally positive and after it increases up to the middle of the skill distribution, it starts to decrease while productivity increases. A higher skill dispersion makes the tax schedule even more regressive.

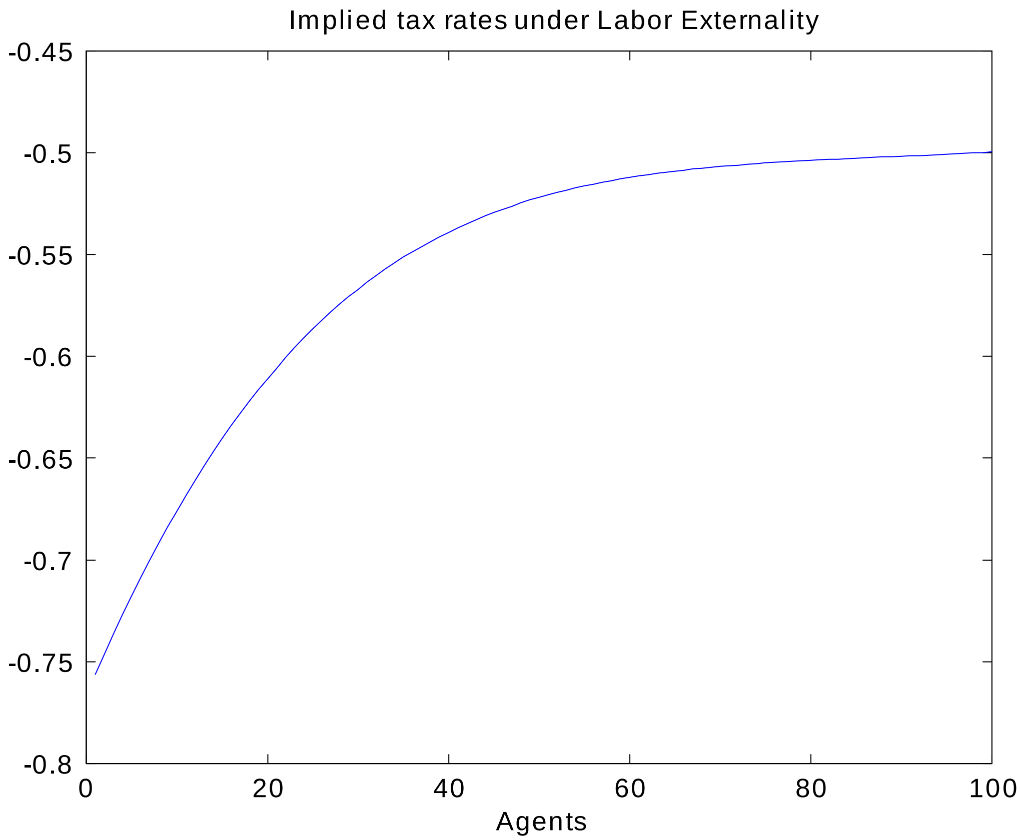

Figure 2

$N = 100$

$\theta \in \{2, 2.01, 2.02...3\}$

$\pi = 1/100$

As it is seen in Figure 2 that with exactly the same parameters, when the labor externality added to the model, the optimal tax schedule seen in a progressive fashion. When the taxes are negative, it means that low type agents get more subsidies than the high ones. As it is mentioned in the text, a low type agent has to work more than the high type in order to increase his income at same amount. Therefore the asymmetry of externalities generated by agents is the reason of tax progressivity. But, as one would expect that the shape of the optimal income tax schedule is sensitive to the distribution of skills within the population. As shown in Figure 3, while the ability distribution getting dispersed, the optimal tax schedule is going to have a regressive fashion in the upper tail.

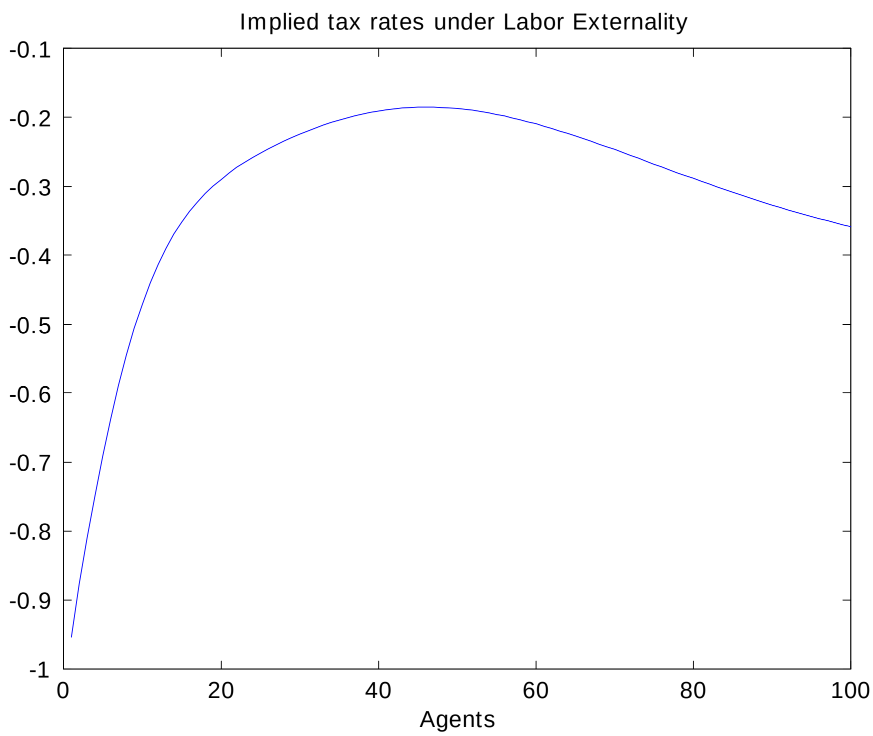

Figure 3

$N = 100$

$\theta \in \{2, 2.04, 2.08...6\}$

$\pi = 1/100$

This change in tax policy shows that we can expect an even more regressive income schedule in the societies that have a more dispersed skill distribution. From the Tax Database of OECD, the European countries like Germany, France, Netherlands, Belgium has a more progressive marginal tax schedule than the US.[7] And it is also known that according to the studies that are mostly using the International Adult Literacy Survey shows that the skills are more unequally distributed in the US than in the EU. The productivity differences in Europe are less than the US.[8]

Following graph shows the effect of population increase while the skill increment is same.

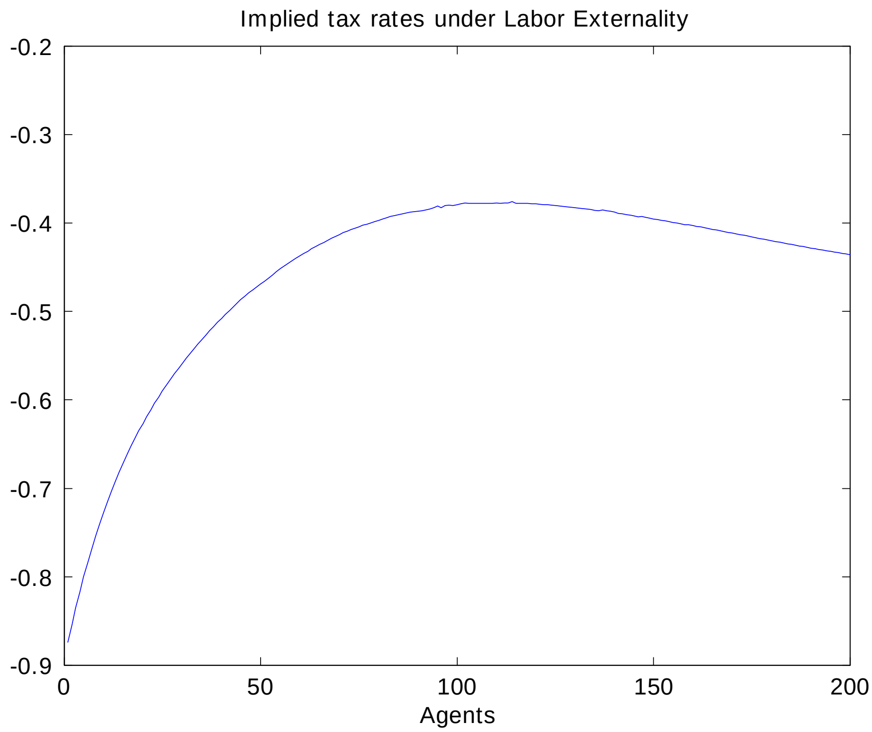

Figure 4

[7]OECD Tax Database, or the marginal tax calculation in Mankiw et al. (2009).

[8]There are several studies that uses this fact and try to explain some other economic isues. Devroye and Freeman (2000) "Does Inequality in Skills Explain Inequality of Earnings Across Countries?" and Bombardini, Giovanni Gallipoli and Pupato (2011), "Skill Dispersion and Trade Flows".

$N = 200$
$\theta \in \{2, 2.01, 2.02...4\}$
$\pi = 1/200$

This figure shows that when the ability difference is increasing between the first and the last agent, the income tax schedule is going to have a regressive fashion.

8.2 Log-Utility 2-type

The social planner's maximization problem is as follows.

$$\max_{c_l,c_h,y_l,y_h} \pi_l \left[\log c_l + \log\left(1 - \frac{y_l}{\theta_l}\right) + L\right] + \pi_h \left[\log c_h + \log\left(1 - \frac{y_h}{\theta_h}\right) + L\right]$$

subject to

$$\pi_h c_h + \pi_l c_l \leq \pi_h y_h + \pi_l y_l \qquad (\lambda)$$

$$\log c_h + \log\left(1 - \frac{y_h}{\theta_h}\right) + L \geq \log c_l + \log\left(1 - \frac{y_l}{\theta_h}\right) + L \qquad (\mu)$$

$$L = \pi_h \frac{y_h}{\theta_h} + \pi_l \frac{y_l}{\theta_l}$$

First, I will give the results of the two type model; while θ_l is constant, θ_h is increasing. In this case $\frac{\theta_h}{\theta_l}$ matters. In order to compare with Mirrlees result, first I will give the result of Mirrlees two type case in Figure 5 when the parameters are as follows: Proportions of each agent are 0.5 and θ_l is 3 and θ_h starts from 3.1 and iterates 100 times and finally reaches 13. The tax schedule is as in Figure 5.

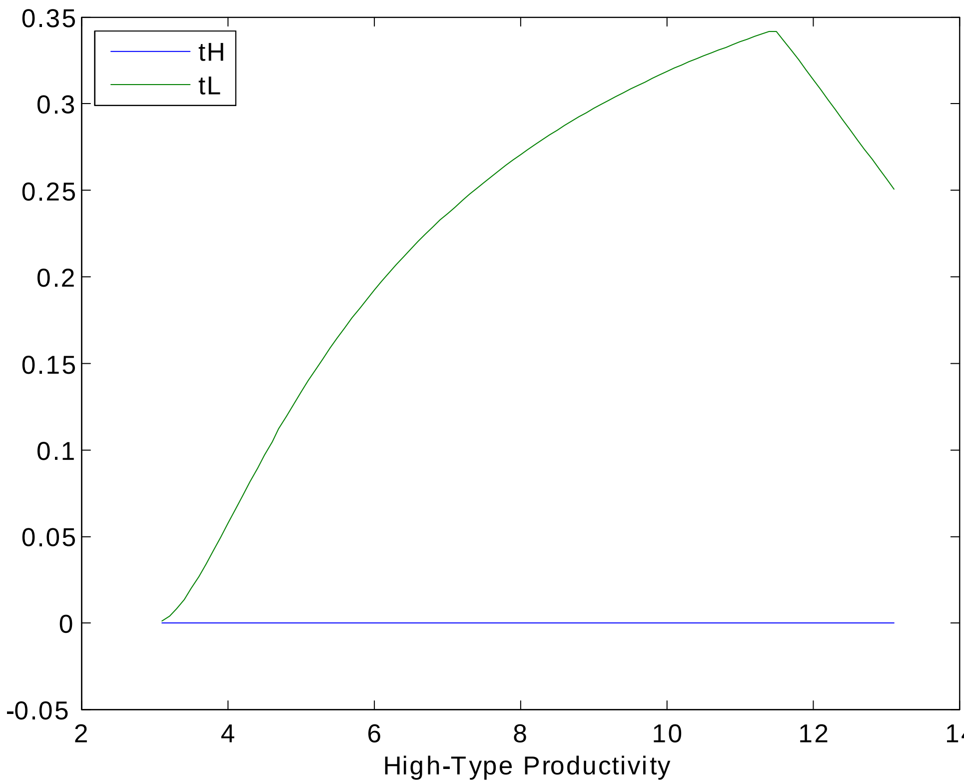

Figure 5

$\theta_l = 3$

$\theta_h \in \{3.1; 3.2; 3.3; ...13\}$

$\pi = 0.5$ for both type

This result reflecting the analytical solution of Mirrlees problem in a two type model. The tax of high productive worker is zero, but there is positive tax on low type and it is increasing while the ability difference is increasing.

With the same parameters, Figure 6 shows the case when labor externality is added to the model.

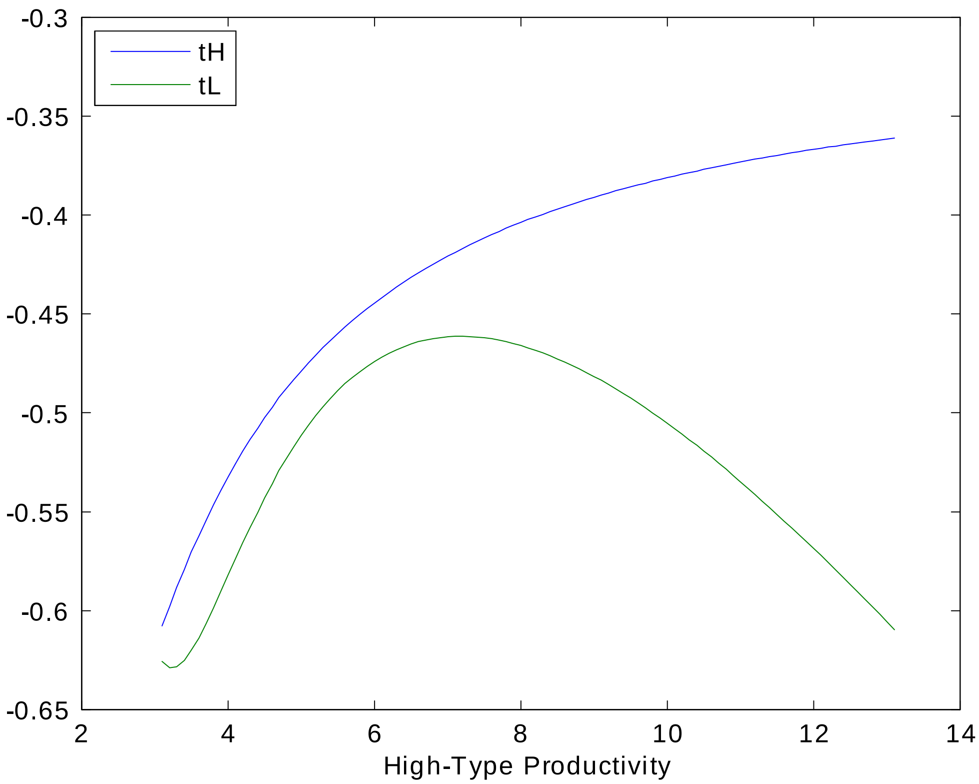

Figure 6

$\theta_l = 3$

$\theta_h \in \{3.1; 3.2; 3.3; ...13\}$

$\pi = 0.5$ for both type

When labor externality is added, tax schedules turn to be negative values and low type agent gets more subsidy which means a progressive marginal tax. In order to see the effect of proportion of low type agents, I give two examples while other parameters are the same. While in Figure 7, low type proportion is 0.8, in Figure 8 low type proportion is 0.2.

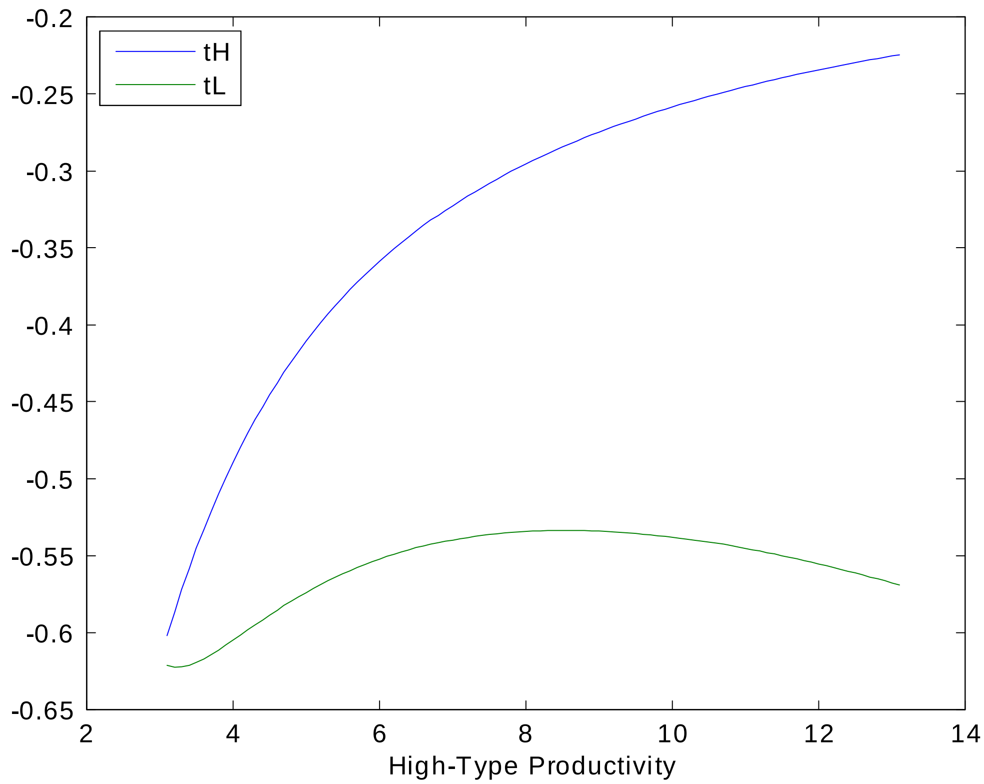

Figure 7

$\theta_l = 3$

$\theta_h \in \{3.1; 3.2; 3.3; ...13\}$

$\pi_l = 0.8$

$\pi_h = 0.2$

This figure shows that an increase in the proportion of low type agents causes a stronger progressive taxation.

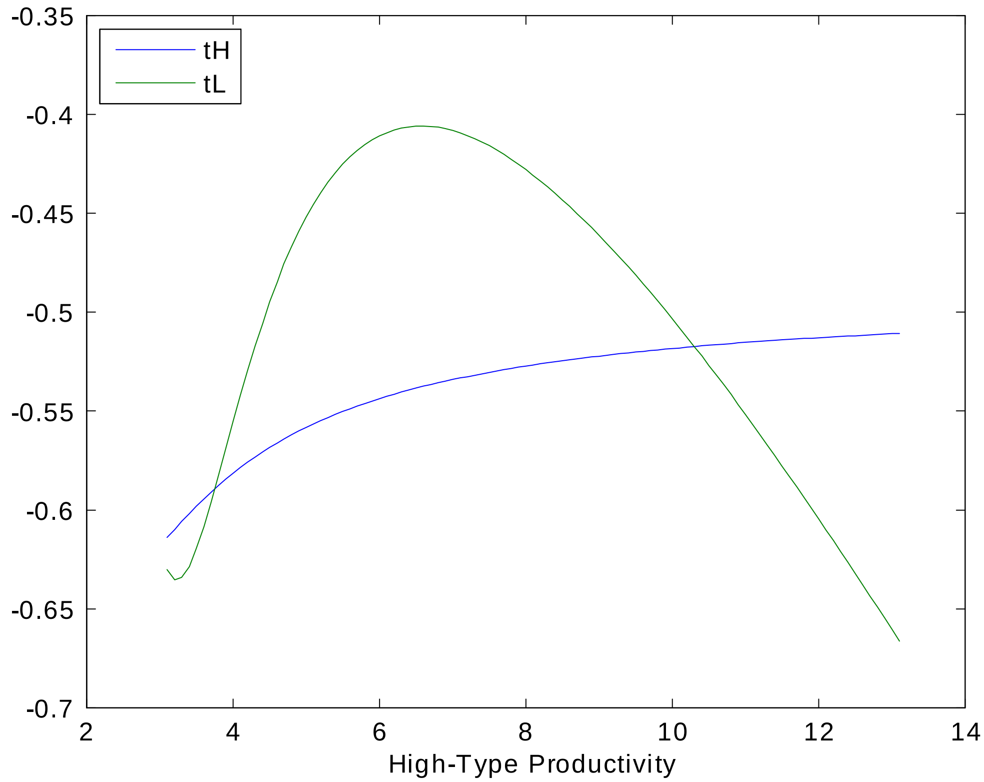

Figure 8

$\theta_l = 3$

$\theta_h \in \{3.1; 3.2; 3.3; ...13\}$

$\pi_l = 0.2$

$\pi_h = 0.8$

In Figure 8 the proportion of low type agent is 0.2. When the low type agent's proportion is too small, the tax policy could be regressive. However while the productivity difference increases, the tax schedule turns to a progressive fashion.

Following part shows an example of two-type model, using log utility. And the analysis shows the effects of change in low type proportion and ability difference at the same time. Proportion of low type starts from 0.1 and increases by 0.09, iterates 10 times and becomes 1. θ_l is 1 and θ_h is starting from 1 and increases by 0.2 and at the end reaches to 3. Three graphs are showing high type tax, low type tax and the tax difference respectively. As it can be seen on the graphs, high type marginal tax is increasing while proportion of low type and productivity of high type are increasing. Low type tax is increasing with the increase in the productivity of the high type. It shows a slightly decreasing pattern while the proportion of low type is increasing. The last graph shows the tax difference; if it is above zero, that means marginal tax is progressive. We can see that after a certain level of π_l optimal taxation is always progressive.

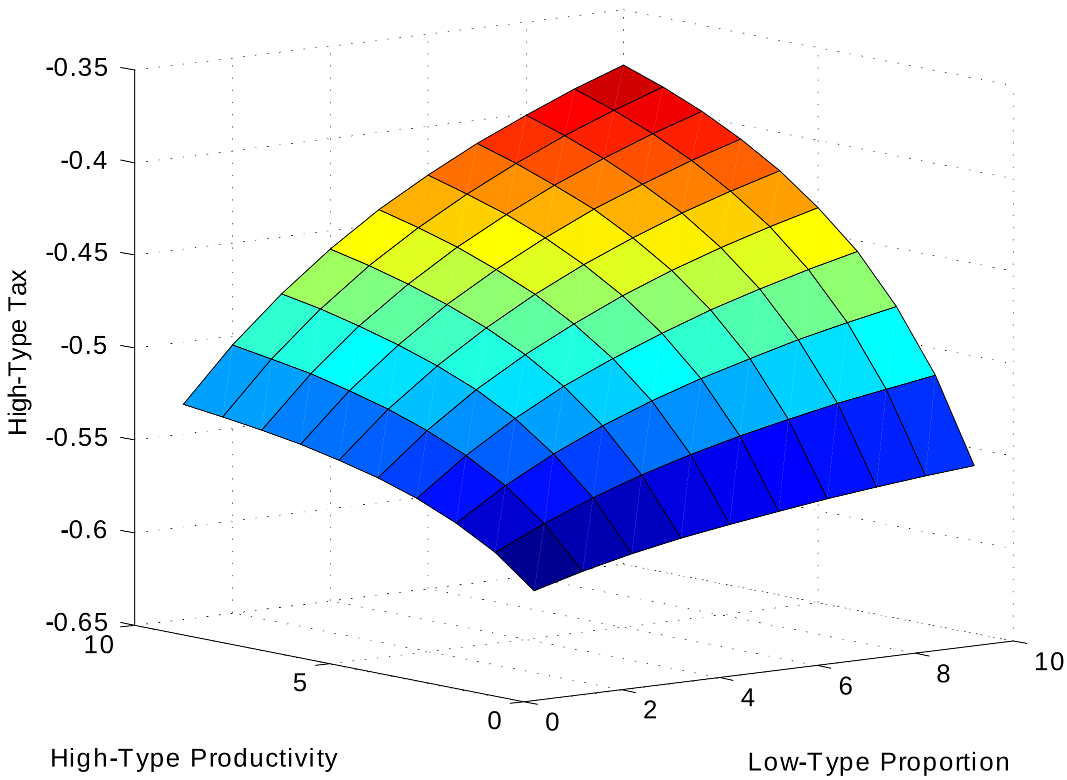

Figure 9

$\theta_l = 1$

$\theta_h \in \{1.2; 1.4; 1.6; ...3\}$

$\pi_l \in \{0.1; 0.19; 0.28; ...1\}$

$\pi_h \in \{0.9; 0.81; 0.72; ...0\}$

Figure 9 shows that high type tax is identified by both low type proportion and the productivity change of high type. These two changing parameters have a joint effect on the tax of highly productive agent.

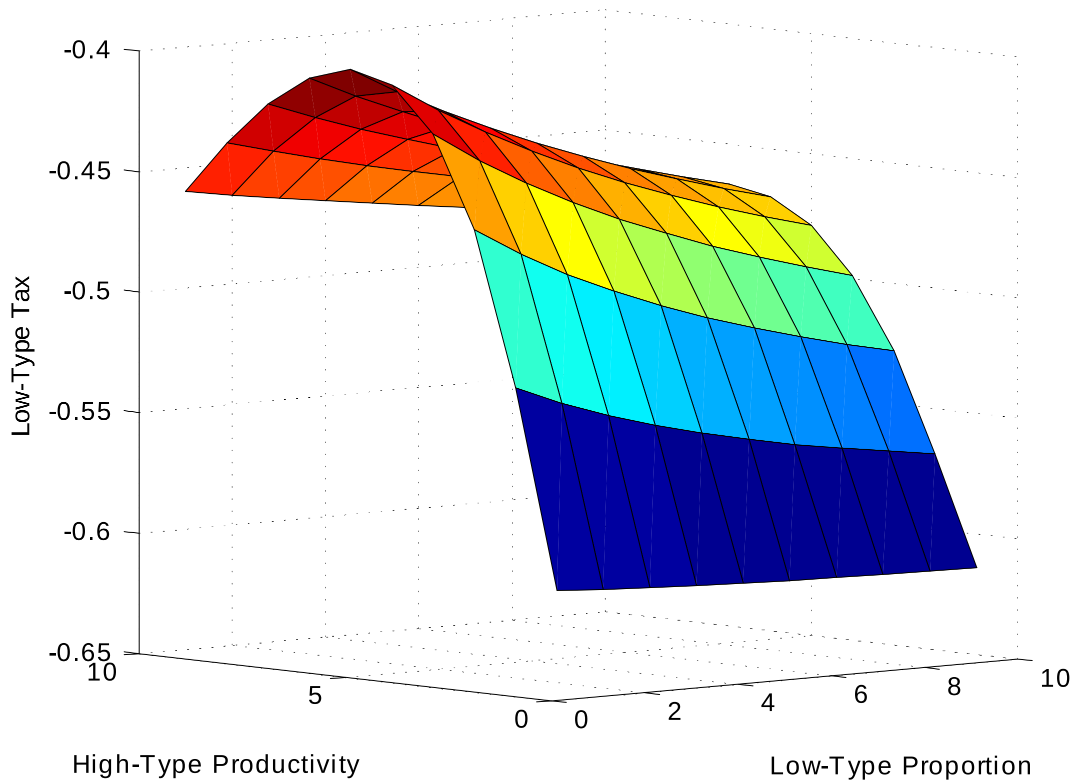

Figure 10

$\theta_l = 1$

$\theta_h \in \{1.2; 1.4; 1.6; ...3\}$

$\pi_l \in \{0.1; 0.19; 0.28; ...1\}$

$\pi_h \in \{0.9; 0.81; 0.72; ...0\}$

Figure 10 shows that low type tax is generally identified by the productivity difference. Low type proportion has less influence on the tax difference.

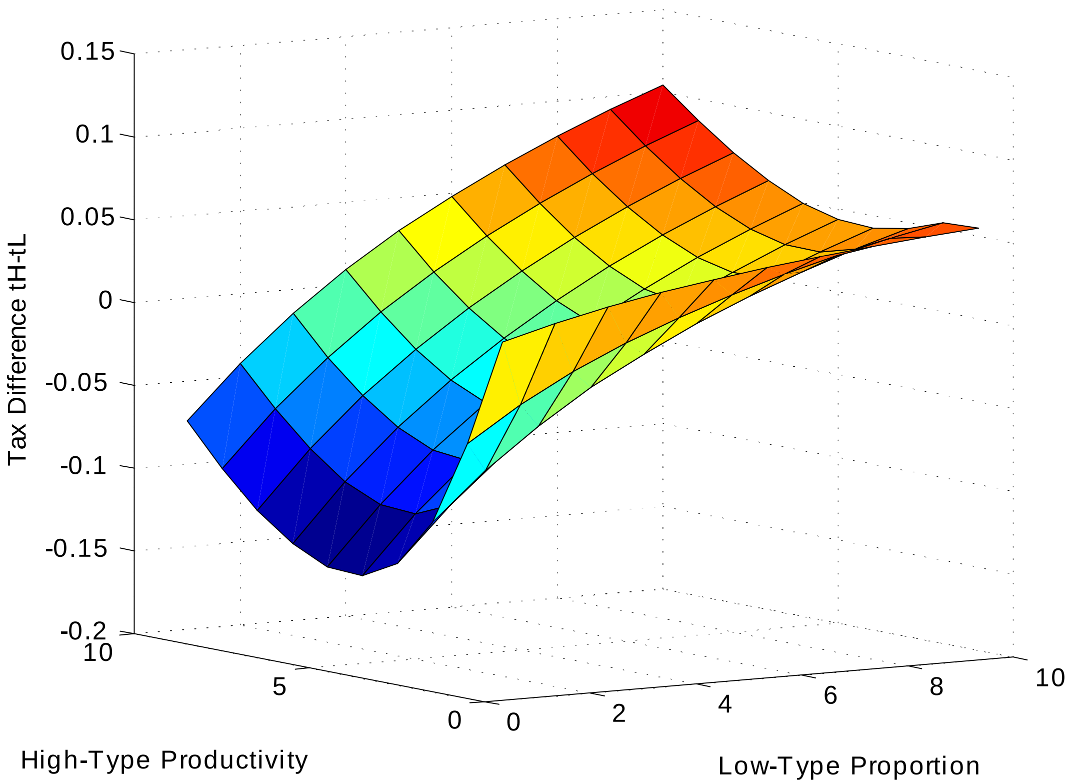

Figure 11

$\theta_l = 1$

$\theta_h \in \{1.2; 1.4; 1.6; ...3\}$

$\pi_l \in \{0.1; 0.19; 0.28; ...1\}$

$\pi_h \in \{0.9; 0.81; 0.72; ...0\}$

This figure illustrates the same thing that proposition 5 says in the model that there is a critical π_l value that, after this proportion, the income tax schedule will be progressive.

9 Conclusion

In this study, I have presented a model which contains the utility interdependence over labor choice among the agents. The results showed that labor externality has a progressive effect on optimal tax schedules. Even in the full information case, because of the labor externality, there will be a negative taxes for all agents, and taxes are increasing with the increase in productivity. This additional concern is eliminating the result that there should be a zero tax at the top of the skill distribution. The model in the paper is implementing a tax even to the highest productive agent.

In private information case, the tax is consisting of two separate parts. One is due to the informational problem that Mirrlees stated. And the second is due to the labor externality effect. While labor externality has a progressive force on marginal tax, information asymmetry has a regressive force. As it is shown in the paper, with the appropriate parameters externality effect dominates the informational problem effect and tax schedule becomes in a progressive form. Also the numerical examples are reflecting the tax schedule changes when labor externality is added to the standard Mirrlees optimal taxation problem. This additional concern about labor externality makes the income taxation schedule more consistent with the current tax policies. With these corrective concerns that makes the model's environment even closer to the real life, the progressivity of actual tax systems can be rationalized.

I understood from this study that corrective concerns about the model are make the model even closer to the real life. I analyzed the labor effect solely. I believe that adding the consumption and labor externalities or any other considerations at the same time may better explain why the tax policies are in the current state.

Furthermore, while it is easier to see the intuition in an N-type model, in the literature there are several studies working with continuum agents. Therefore it could be further work to study labor interdependence in taxation problem with a continuum agent model.

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