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Resource Dedication Problem in a Multi-Project Environment*

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Abstract There can be different approaches to the management of resources within the context of multi-project scheduling problems. In general, approaches to multi-project scheduling problems consider the resources as a pool shared by all projects. On the other hand, when projects are distributed geographically or sharing resources between projects is not preferred, then this resource sharing policy may not be feasible. In such cases, the resources must be dedicated to individual projects throughout the project durations. This multi-project problem environment is defined here as the Resource Dedication Problem (RDP). RDP is defined as the optimal dedication of resource capacities to different projects within the overall limits of the resources and with the objective of minimizing a predetermined objective function. The projects involved are multi-mode resource constrained project scheduling problems (MRCPSP) with finish to start zero time lag and non-preemptive activities and limited renewable and nonrenewable resources. Here, the characterization of RDP, its mathematical formulation and two different solution methodologies are presented. The first solution approach is a genetic algorithm employing a new improvement move called combinatorial auction for RDP, which is based on preferences of projects for resources. Two different methods for calculating the projects' preferences based on linear and Lagrangian relaxation are proposed. The second solution approach is a Lagrangian relaxation based heuristic employing subgradient optimization. Numerical studies demonstrate that the proposed approaches are powerful methods for solving this problem.

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1 Introduction

Multi-project management is a major way of doing business both in manufacturing and services and, being a large-scale complex problem, it constitutes an important research area. The available approaches to this problem in literature generally address a multi-project management environment where the individual projects can share the available resources from a common pool. Such an approach allows the representation of the problem using a large project network formed by combining the networks of the individual projects, and hence, the employment of the available single-project scheduling solution methods. This kind of approach relies on the basic assumption that all the renewable resource capacity is available for all projects. However, this assumption can be invalid in certain cases where projects are distributed geographically, or sharing resources are not preferred, or some characteristics of the projects do not allow resource sharing. In such situations, the multi-project environment becomes completely different.

Multi-project environments, where resource sharing is not allowed, require the dedication of resources to individual projects in such a way that the projects can be scheduled to optimize the overall multi-project environment objective. To the best of our knowledge, the multi-project scheduling problem for such project scheduling environments has not been investigated or referred to in the project scheduling literature, and will be designated here as the Resource Dedication Problem (RDP). Both the mathematical model and the solution approaches proposed for this problem are original contributions of the present paper.

The further characterization of the multi-project environment considered in this paper is as follows: All projects are assumed to be ready to start initially. Uncertainties are not considered. The projects involve finish to start zero time lag and non-preemptive activities. There are both renewable and non-renewable resources with limited capacities. Thus, a static and deterministic multi-project multi-mode resource constrained project scheduling problem (MMRCPSP) is addressed. The minimization of the total weighted tardiness of the projects is the objective criterion under consideration.

The paper continues in the following section with a brief literature review on resource-constrained single and multi-project scheduling. In section 3, RPD is defined and a mathematical formulation is presented. In section 4, two different solution approaches for RDP are proposed: one is a genetic algorithm (GA) application based on the resource preference concept and the other is a Lagrangian Relaxation based heuristic. Numerical studies for the proposed methods are reported in section 5. Finally, conclusions and directions for further research are stated in section 6.

2 Literature Review

The Multi-mode Resource Constrained Project Scheduling Problem (MRCPSP), which is a generalized version of the Resource Constrained Project Scheduling Problem (RCPS), can be considered as the base problem for the specific multi-project scheduling problem that is addressed in this paper. Therefore, first a brief survey on MRCPSP, and then some of the basic work on multi-project scheduling will be presented in this section.

2.1 Single Project Scheduling

The general case of the single project scheduling problem of interest here is known as the MRCPSP, where the activities can be executed by choosing one of the available resource usage modes that are manifested as pre-determined recipes of resource usages and the respective activity durations. The resources in the environment can be renewable, nonrenewable or doubly constrained. MRCPSP is a well studied problem in project scheduling literature and various authors contributed to the solution of the problem with exact solution approaches (see e.g., [25]; [23]), and rule based or meta heuristic approaches (see e.g., [2]; [19]; [24]; [3]; [6]; [8]; [1]; [16]). Surveys on MRCPSP can be found in [20]; [7]; [4] and [12]. While there are several mathematical models suggested for the MRCPSP (e.g., [17] and [28]) the mathematical formulation given in [25] is employed throughout this paper. This model is presented below:

Sets:

- J set of activities, $j = 1 \dots N$
- T set of time periods, $t = 1 \dots T$
- P set of all precedence relationships
- M_j set of modes for activity j , $m = 1 \dots M_j$
- K set of renewable resources, $k = 1 \dots K$
- I set of nonrenewable resources, $i = 1 \dots I$

Parameters:

- E_j Earliest finish time of activity j
- L_j Latest finish time of activity j
- R_{kt} Capacity of renewable resource k , at time period t
- r_{jkm} Renewable resource k usage of activity j , operating on mode m
- W_i Capacity of nonrenewable resource i
- w_{jim} Nonrenewable resource i usage of activity j , operating on mode m
- d_{jm} Duration of activity j operating on mode m

Decision Variable:

$$x_{jmt} = \begin{cases} 1 & \text{if activity } j \text{ is finished operating on mode } m \text{ at time period } t \\ 0 & \text{otherwise} \end{cases}$$

Mathematical Model PS

$$\min. z = \sum_{m=1}^{M_N} \sum_{t=E_N}^{L_N} tx_{Nmt} \quad (1)$$

Subject to

$$\sum_{m=1}^{M_j} \sum_{t=E_j}^{L_j} x_{jmt} = 1 \quad \forall j \in N \quad (2)$$

$$\sum_{m=1}^{M_b} \sum_{t=E_b}^{L_b} (t - d_{bm})x_{bmt} \geq \sum_{m=1}^{M_a} \sum_{t=E_a}^{L_a} tx_{amt} \quad \forall (a, b) \in P \quad (3)$$

$$\sum_{j=1}^N \sum_{m=1}^{M_j} \sum_{q=t}^{t+d_{jm}-1} r_{jkm}x_{jmq} \leq R_{kt} \quad \forall k \in K \text{ and } \forall t \in T \quad (4)$$

$$\sum_{j=1}^N \sum_{m=1}^{M_j} \sum_{t=E_j}^{L_j} w_{jim}x_{jmt} \leq W_i \quad \forall i \in I \quad (5)$$

$$x_{jmt} \in \{0, 1\} \quad \forall j \in J, \forall m \in M_j \text{ and } \forall t \in T \quad (6)$$

The objective (1) is defined as minimizing the completion time of the project schedule. The resulting schedule in turn minimizes all regular objectives for the given scheduling problem. Constraint set (2) ensures that an activity is completed exactly once. Constraint set (3) ensures the precedence relations among activities. The precedence relations imposed here are of “finish to start with no time lag” type. Constraint sets (4) and (5) enforce renewable and nonrenewable resource capacities, respectively.

2.2 Multi-Project Scheduling

The multi-project scheduling problem consists of several projects where the individual projects have the characteristics defined in the previous section. The modeling approaches in the literature for multi-project scheduling allow resource sharing between the projects, the resource dedication policy, on the other hand, has not been considered to the best of our knowledge.

One of the first studies on multi-project scheduling is a general integer programming formulation proposed in [21]. The proposed formulation covers most of the aspects of multi-project scheduling like renewable resource constraints, multiple modes and preemption. The general setting given in the paper can be easily converted to a multi-project environment with multiple modes and renewable resource constraints.

Kurtulus and Narula [13] analyze the tardiness cost performance of scheduling rules for multi-project problem with different weights for projects. Note that in this study, for all activities only single mode is considered. The authors propose various parameters for the characterization of the project networks like Maximum Load Factor and Average Utilization Factor, which are commonly used to characterize project networks. Apart from these summary measures related with the characteristics of activities, the penalties for projects are defined with different functions, which are related with total work content and

critical path. The proposed scheduling rules behave differently for different ranges of the defined project network characterizing parameters.

Tsubakitani and Decro [26] apply the scheduling rules for multi-project problem proposed in [13] by creating an applicable framework for the housing industry. First of all, the characteristics for the different projects of housing industry are determined by the measures proposed in [13], which in turn determine the specific scheduling rules used for the different settings. The proposed frame includes an update scheme for the ongoing projects which enables rescheduling.

Lawrance and Morton [14] propose resource pricing based priority heuristic rules for multi-project scheduling. The aim is to minimize the total weighted tardiness cost of projects, where the weight of a project is defined as the relative importance of a project. The authors define different approaches for resource price estimation and develop heuristics based on those price estimations.

Sperenza and Vercellis [22] propose a two stage approach for multi-project scheduling where all the projects can have precedence relations among themselves. The first stage of the approach aims to define projects as “activities” with multiple modes, where each mode is determined by solving a mathematical model for the project with a given budget that yields a finish time for the project. The authors assume that a set of possible budget limitations for a project can be estimated which in turn constitutes different modes for a project. The network formed in this manner is expressed as an RCPSP where the objective is the maximization of the net present value. The solution to this problem gives the required start and finish times of the projects, and the total nonrenewable and renewable resource capacities that a project can use in a given period. In the second stage of the framework, this information is used for detailed scheduling of individual projects for makespan minimization, and the activity mode assignments of each project within the given amount of renewable and nonrenewable resources are obtained.

Another hierarchical approach for multi-project scheduling is proposed in [9]. The problem environment consists of multiple projects with both nonrenewable and renewable resource constraints. The detailed scheduling of projects is based on relative resource indices for projects. The overall scheduling procedure is an iterative one. It starts with an initial early and late finish times for projects and updates these values at each iteration. The procedure iterates till a desired value is reached for the total weighted tardiness over all projects.

Yang and Sum [27] approach the multi-project scheduling with various considerations such as due date assignment, resource allocation, project release and activity scheduling. The authors see multi-project problems as a dual level management problem in a dynamic environment where new projects arrive during the execution of the current set of projects. The problems in the first level are related with individual project scheduling tasks, which are activity selection and resource allocation. The upper management decisions are related with release dates of projects and due date assignments. For these problems the authors propose priority rules. For due date assignment, workload related and critical path length related priority rules are defined. The release of projects is handled by limiting the number of active projects.

Lova et. al [15] study multi-project scheduling with different objectives at different levels. The authors' approach to the problem consists of two levels and two different objective types associated with each level. First, the problem is solved with the objective of minimizing mean project duration or maximum project duration; namely, time related objectives. A forward-backward heuristic is applied to achieve these objectives. With a starting feasible solution, activities are left and right shifted according to different priority rules for different objectives. When no further improvement can be achieved with forward-backward heuristics, the objectives that are resource related are considered and heuristics are applied to minimize idle resources and to achieve resource leveling.

Goncalves [5] use a GA approach to multi-project scheduling problems where only renewable resources are considered. The basic approach of the authors is combining all activities of the projects into a big project network and solving this combined project network using GA. The chromosome structure is different from the generally used structures in the project scheduling literature. The genes corresponding to activities are priority coefficients for the activities and are used in schedule generation procedures. In addition to these genes, there are different genes for delay times of activities and start times for individual projects for schedule generation. The performance measure of an individual in the GA is a composition of tardiness, earliness and flow time deviation of projects from the makespan calculated for the unconstrained case using CPM.

Mittal and Kanda [18] propose a two-phase heuristic for multi-project scheduling. Activities do not have alternative modes and only renewable resources are present. The objective is minimizing the makespan and deviation from critical path duration. The rationale behind the two-phase heuristic is the distinction between project and activity selection. Basically, project selection is done first (phase one) and an activity is selected from the selected project (phase two) with defined priority rules for both project and activity selection. The priority rules for activity selection are the common priority rules used in the literature like minimum duration or minimum slack. The priority rules used for project selection are critical path length, remaining critical path length, total work content and total work remaining.

3 Resource Dedication Problem

RDP is the dedication of a set of limited resources to a set of projects in a multi-project environment in such a way that individual project schedules would result in an optimal solution for a specified objective. Here, the objective is the minimization of the total weighted tardiness over all projects. Weighted tardiness is one of the more preferred objective functions both in literature as well as in practice. Once the resources are dedicated to individual projects they are not anymore allowed to be shared with other projects and the problem becomes solving an MRCPSP for each project with the dedicated resource amounts. The general problem environment is depicted in Figure 1 below.

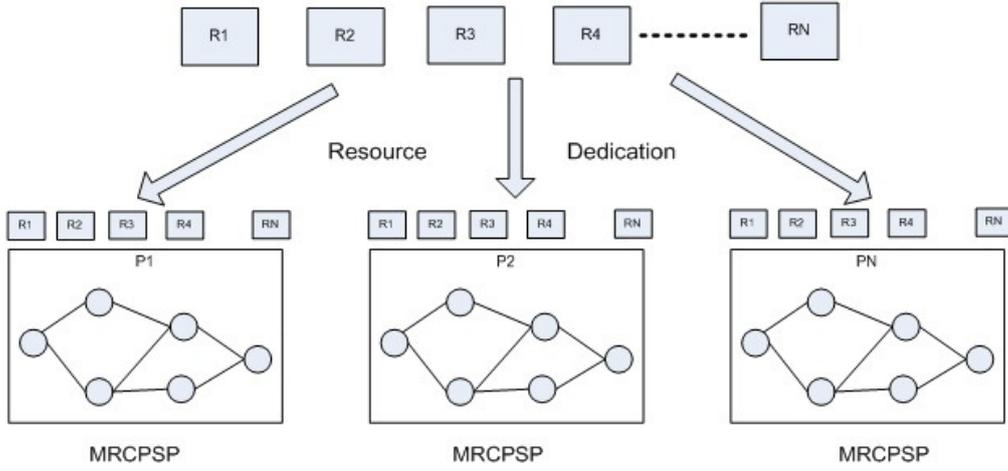


Fig. 1 General problems in multi-project environment

The proposed formulation for resource dedication problem is given below;

Additional Sets:

- V set of projects, $v = 1 \dots V$
- J_v set of activities of project v , $j = 1 \dots J_v$
- P_v set of all precedence relationships of project v
- M_{vj} set of modes for activity j of project v , $m = 1 \dots M_{vj}$

Additional Parameters:

- E_{vj} Earliest finish time of activity j of product v
- L_{vj} Latest finish time of activity j of project v
- d_{vjm} Duration of activity j of project v , operating on mode m
- $r_{vjk m}$ Renewable resource k usage of activity j of product v , operating on mode m
- $w_{vij m}$ Nonrenewable resource i usage of activity j of project v , operating on mode m
- C_v Relative weight of project v
- DD_v Due date of project v
- R_k Capacity of renewable resource k
- W_i Capacity of nonrenewable resource i

Decision Variables:

$$x_{vjmt} = \begin{cases} 1 & \text{if activity } j \text{ of project } v \text{ is finished operating on mode } m \text{ at time period } t \\ 0 & \text{otherwise} \end{cases}$$

- BR_{vk} : Amount of renewable resource k dedicated to project v
- BW_{vi} : Amount of nonrenewable resource i dedicated to project v
- TC_v : Weighted tardiness of project v

Mathematical Model RDP

$$\min. z = \sum_{v=1}^V TC_v \quad (7)$$

Subject to

$$\sum_{m=1}^{M_{vj}} \sum_{t=E_{vj}}^{L_{vj}} x_{vjmt} = 1 \quad \text{for } \forall j \in N_v \text{ and } \forall v \in V \quad (8)$$

$$\sum_{m=1}^{M_{vb}} \sum_{t=E_{vb}}^{L_{vb}} (t - d_{vbm}) x_{vbm} \geq \sum_{m=1}^{M_{va}} \sum_{t=E_{va}}^{L_{va}} t x_{vam} \quad \forall (a, b) \in P_v \text{ and } \forall v \in V \quad (9)$$

$$\sum_{j=1}^{N_v} \sum_{m=1}^{M_{vj}} \sum_{q=t}^{t+d_{vjm}-1} r_{vjkm} x_{vjmq} \leq BR_{vk} \quad \forall k \in K, \forall t \in T \text{ and } \forall v \in V \quad (10)$$

$$\sum_{j=1}^{N_v} \sum_{m=1}^{M_{vj}} \sum_{t=E_{vj}}^{L_{vj}} w_{vjim} x_{vjmt} \leq BW_{vi} \quad \forall i \in I \text{ and } \forall v \in V \quad (11)$$

$$\sum_{v=1}^V BR_{vk} \leq R_k \quad \forall k \in K \quad (12)$$

$$\sum_{v=1}^V BW_{vi} \leq W_i \quad \forall i \in I \quad (13)$$

$$TC_v \geq C_v \left(t \sum_{m=1}^{M_{vN}} x_{vNmt} - DD_v \right) \quad \forall t = E_{vN} \dots L_{vN} \text{ and } \forall v \in V \quad (14)$$

$$BR_{vk} \in Z^+ \quad \forall v \in V \text{ and } \forall k \in K \quad (15)$$

$$BW_{vi} \in Z^+ \quad \forall v \in V \text{ and } \forall i \in I \quad (16)$$

$$TC_v \in Z^+ \quad \forall v \in V \quad (17)$$

$$x_{vjmt} \in \{0, 1\} \quad \forall v \in V, \forall j \in J_v, \forall t \in T \text{ and } \forall m \in M_{vj} \quad (18)$$

Objective function (7) minimizes total weighted tardiness cost over all projects. Constraint set (8) ensures that all activities are scheduled once and only once for all projects. Constraint set (9) implies predecessor relationships for all activities of all projects. Constraint set (10) sets the maximum level of renewable resource needed for each project. Constraint set (11) determines necessary nonrenewable resources for each project. Constraint sets (12) and (13) limit the total dedicated renewable and non-renewable resources according to the given general resource capacities, respectively. Constraint set (14) calculates the weighted tardiness cost for project v .

4 Solution Methodologies

The given mathematical model conceptually contains two different problems: (i) Dedication of the resource capacities to projects and (ii) scheduling of the individual activities of the corresponding project. Scheduling of activities with multi-modes under nonrenewable and renewable resource constraints as a generalization of resource constrained project scheduling problem is NP hard [10] and the given multi-project problem not only encapsulates a multiple number of this problem but also the decision for resource dedications for the projects. In preliminary test runs it has been observed that exact solution approaches using the given mathematical *model RDP* employing *ILOG CPLEX 11.2* could not find any feasible solutions for a considerable number of the test problems consisting of 6 projects with 22 or 32 activities in each project and 4 alternative modes for each activity under different resource capacities. Therefore, heuristic solution approaches will be employed here for the solution of *Model RDP* above. A GA with a new local improvement heuristic called combinatorial auction (CA) for RDP is introduced here. Furthermore, a Lagrangian relaxation based heuristic is presented.

4.1 A Genetic Algorithm Based on Combinatorial Auction for Resource Dedication Problem

In this study, GA is used as an intelligent search infrastructure supported by the new local improvement heuristic CA for RDP. Below detailed information about CA for RDP is given.

4.1.1 Combinatorial Auction for Resource Dedication Problem

CA for RDP is a local improvement heuristic based on the preferences of the projects for the resources. The preference of a project for a resource can be defined as the value of the resource for that project according to the current state of the resources of that project. Hence, the preference of a project for a resource can be taken as an indication for a possible improvement that would result in the objective when a unit of that resource is obtained by some means by that project. If one can calculate the preferences of the projects for the resources for a given resource dedication state for the projects, then the current solution can be moved to a more preferable solution with respect to the preferences. If the preference calculation reflects the importance of the resources for the projects correctly, then this move will improve the objective. In this paper, two different preference calculation approaches are used: (i) *Linear relaxation* (LR) based and (ii) *Lagrangian relaxation* (LA) based.

Linear Relaxation Based Preference Calculation

If the linear relaxation of the single project scheduling model [(1)-(6)] [25] is solved for each project with some set levels of dedicated resources, the solution can be used to determine the preferences of the projects for the resources. LR of the given formulation (*SP-LR*) results in two basic information: (i) Dual variable values (or shadow prices) and (ii) allowable upper and lower bounds for the right hand sides (*RHS*) of

the constraints. According to the duality theory, dual variable values can be interpreted as the amount of improvement in the objective function, if the RHS of the constraint is increased by one unit. On the other hand, allowable upper and lower bounds represent the limits of the *RHS* values where the optimal basis will not change. There are two basic groups of constraints in the project scheduling model. First group is the sets of technology constraints (activity assignment (2) and precedence constraints (3)), and the other group consists of the sets of resource capacity constraints (4-5). Resource capacity constraints are not generally binding in the solution of *SP-LR*, this in turn results with distinct allowable upper and lower bounds from the *RHS* values of resource capacity constraints. If the resource constraints are increased to their allowable upper bounds, the new basis should give, at worst, as good a feasible solution as the previous basis, since the capacity is increased for the resource. The allowable upper bounds for the RHS of the resource constraints can be used to determine the sensitivity of the projects for resources and can be used as preferences for these resources. In other words, the resource levels that cause a change in the optimal basis in the *LR* are interpreted as a direction for the resource levels that can improve the objective of the original problem. The preference for a resource by a project is defined as the closeness of the allowable upper bound to the current *RHS* value as follows:

For a renewable resource k

$$a_{kv} = \frac{1}{\max_t \{AUB_{ktv} - DR_{kv}\}} \quad (19)$$

$$p_{kv} = \frac{a_{kv}}{\sum_v a_{kv}} \quad (20)$$

For a nonrenewable resource i

$$a_{iv} = \frac{1}{AUB_{iv} - DW_{iv}} \quad (21)$$

$$p_{iv} = \frac{a_{iv}}{\sum_v a_{iv}} \quad (22)$$

where a_{kv} and a_{iv} are the closeness of the allowable upper bounds to the dedicated renewable and nonrenewable resource levels, respectively; AUB_{ktv} is the allowable upper bound value for renewable resource k resulting from the usage constraint for time period t (constraint set 4); AUB_{iv} is the allowable upper bound value for nonrenewable resource i resulting from the usage constraint for the project (constraint set 5); DR_{kv} and DW_{iv} are the dedicated renewable and nonrenewable resources for project v , respectively; p_{kv} and p_{iv} are the preferences of project v for renewable resource k and nonrenewable resource i , respectively.

Lagrangian Relaxation Based Preference Calculation:

Another preference calculation approach can be defined by employing Lagrangian relaxation. Consider the Lagrangian relaxation of the modified MRCPSPP formulation, given below, where both nonrenewable and renewable resource constraints are relaxed:

Mathematical Model LA-SP

$$\begin{aligned} \min. z_{LA-SP} = & TC + \sum_{k=1}^K \sum_{t=1}^T \lambda_{kt} \left\{ \sum_{j=1}^N \sum_{m=1}^{M_j} \sum_{q=t}^{t+d_{jm}-1} r_{jkm} x_{jmq} - DR_k \right\}^+ \\ & + \sum_i^I \mu_i \left\{ \sum_{j=1}^{N_v} \sum_{m=1}^{M_j} \sum_{t=E_j}^{L_j} w_{jim} x_{jmt} - DW_i \right\}^+ \end{aligned} \quad (23)$$

Subject to

$$(2), (3), (6)$$

$$TC \geq C(t \sum_{m=1}^{M_N} x_{Nmt} - DD) \quad \forall t = E_N \dots L_N \quad (24)$$

$$TC = TC_{min} \quad (25)$$

where DR_k and DW_i are dedicated renewable and nonrenewable resources respectively, TC_{min} is the minimum reachable weighted tardiness for the project, which can be easily calculated from the project network employing CPM without considering any resource constraints.

This formulation is exactly the Lagrangian relaxation model of MRCPSP except constraint (25), which sets the weighted tardiness equal to the calculated minimum tardiness for the project. In addition to this, in the calculation of the objective function *LA-SP*, penalty values are added if they are nonnegative. In other words, only the renewable and nonrenewable resource usages exceeding the given capacities DR_k and DW_i are included in the summations in (23).

First, note that this Lagrangian relaxation formulation is not for solving the MRCPSP. Rather, it is used to calculate the preferences of the projects for the resources. The rationale of these modifications is to define a methodology to accurately calculate preferences. Since the total weighted tardiness cost part of the objective is set to the minimum reachable value, the formulation will accept the necessary resource infeasibilities to reach that minimum possible weighted tardiness value. Thus, when the subgradient optimization approach is applied to this formulation, after a number of iterations, the values of λ_{kt} and μ_i can be used as an estimation for the value of the corresponding resource to the project, or in other words, as the preference of the project for that resource to reach the minimum possible weighted tardiness. In addition to this, penalizing only the resource over-utilizations prevents the abuse of the Lagrangian objective function by under-utilization of the resources. Note that, especially for the renewable resource case, where only the constraint for the time period with the maximum usage will be active, the abuse will be excessive.

For each resource, when a series of subgradient optimization iteration is executed, the preference values for each project v can be calculated as follows:

$$p_{kv} = \max_t \{\lambda_{vkt}\} \quad (26) \text{ preference of project } v \text{ for a renewable resource } k,$$

$$p_{iv} = \mu_{vi} \quad (27) \text{ preference of project } v \text{ for a nonrenewable resource } i.$$

Moving to a More Preferable Solution:

After calculating the preference of projects for the resources, the next issue is moving to a more preferable solution using these preferences. This is carried out by calculating the slack resources (resources that are not used for the current solution of the projects) and distributing these slack resources among the projects according to the projects' preferences. The amount of a resource that a project can give up can easily be determined from the solution of the individual project scheduling problem. The smallest slack for each renewable resource observed over the project duration and the slacks of the nonrenewable resource capacity constraints will be the unnecessary amount of the corresponding resources for a project. Preference and slack values can be used to form a continuous knapsack problem to distribute the calculated slack resources for maximizing the total gain for all projects. The knapsack model for renewable and nonrenewable resources is given below.

For nonrenewable resources

$$\max. z = \sum_i \sum_v p_{iv} y_{iv} \quad (28)$$

Subject to

$$\sum_v y_{iv} \leq b_i \quad \forall i \in I \quad (29)$$

$$y_{iv} \in R^+ \quad (30)$$

For renewable resources

$$\max. z = \sum_k \sum_v p_{kv} y_{kv} \quad (31)$$

Subject to

$$\sum_v y_{kv} \leq b_k \quad \forall k \in K \quad (32)$$

$$y_{kv} \in R^+ \quad (33)$$

where p_{iv} (p_{kv}) is the preference of project v for resource i (k), b_i (b_k) is the amount of resource i (k) available for dedication to different projects, and y_{iv} (y_{kv}) is the nonnegative continuous decision variable for the amount of spare resource i (k) dedicated to project v . This formulation tries to maximize the total preference value gained by dedicating the spare resources to projects.

To summarize CA for RDP a numerical example in Figure 2 is given below.

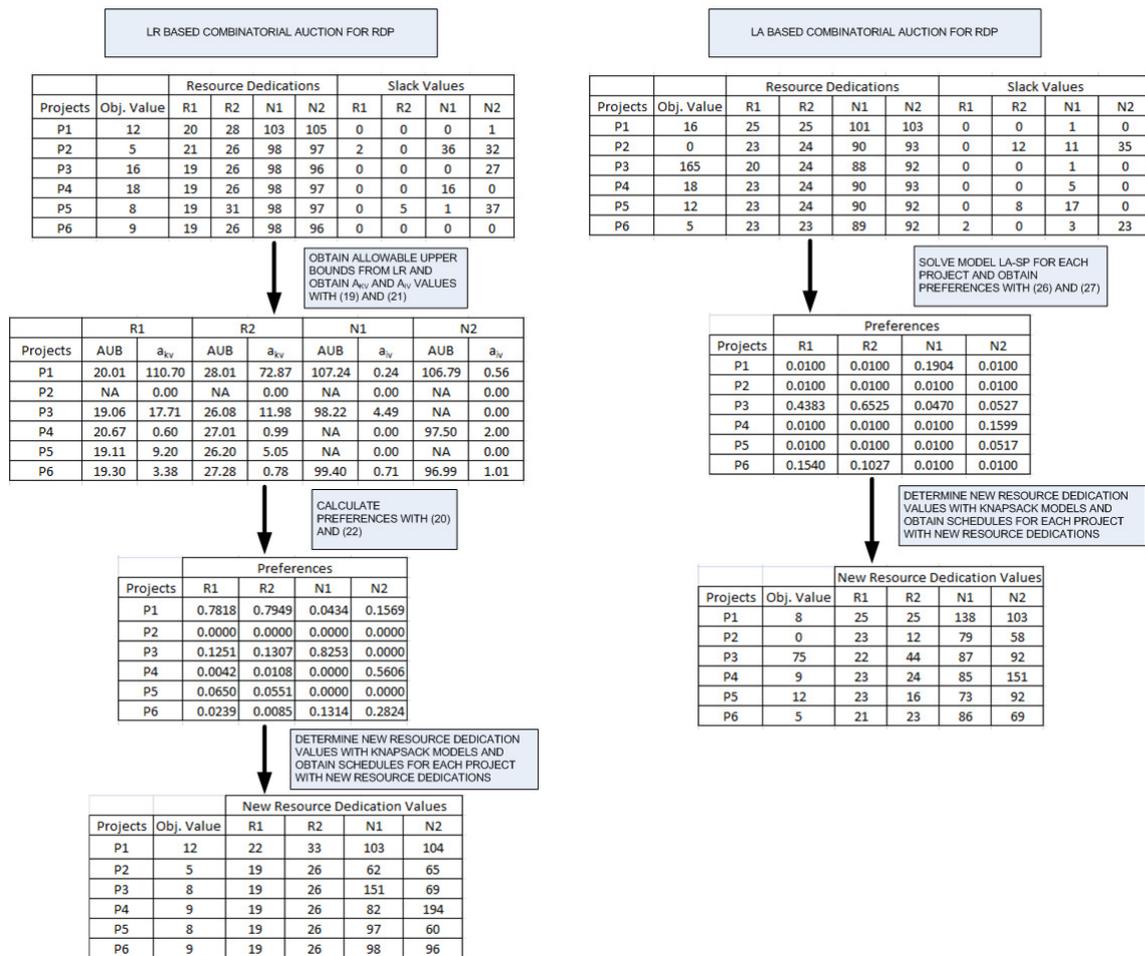


Fig. 2 General procedure for CA for RDP

4.1.2 The Basics of Genetic Algorithm Applied

An individual in the proposed GA is defined so as to reflect a multi-project environment with dedicated resource values. Only resource feasible individuals are present in the population. In Figure 3 the general view of an individual is given. Note that a chromosome represents a resource dedication instance with the determined resource allocations to the projects. This representation can also be interpreted as a single string that represents resource dedication values for projects.

Resources	Projects		
	P1	P2	P3
R1	15	12	20
R2	8	14	21
R3	10	21	11
R4	15	12	13

Fig. 3 Individual representation

There are three different ways to generate individuals for the initial population. The first two methods employ the no-delay resource requirements of the projects. No-delay resource requirement of a project is calculated by solving the resource unconstrained project scheduling problem and calculating resource requirement from the resulting schedule. In the first method, resource dedications are determined proportional to the no-delay resource requirements of the projects with respect to the given general resource capacities. One individual is generated in this manner. In the second method, for each project, the no-delay resource requirement is satisfied from the general resource capacities, and resource dedications of the remaining projects are determined proportional to the no-delay resource requirements of the corresponding projects. And finally, resource dedications are randomly generated for each project. Based on the preliminary test runs the population size is selected as 10.

Here, three basic procedures are employed to create new individuals in the proposed GA; namely crossover, mutation and CA for RDP. Generally speaking, a crossover operator selects two different individuals from the population in a predetermined way and exchanges sub-strings from the selected individuals to generate two new offsprings. With the crossover operators it is intended to allow inheritance of the successful parts of individuals to the children generated. For the individual representation presented, a successful part of an individual can be the resource dedication values of a project (a column under a project in the individual representation) or dedicated resource values for a resource (a row for a resource in the individual representation). The crossover operators are defined to apply the aforementioned ideas. There are three different types of crossover operations defined for the proposed GA. The first crossover operation takes two projects from different individuals and changes sub-strings of resource dedications for selected projects in different individuals. The second crossover operation similarly selects a resource and exchanges the resource dedications for all projects of the parents. The last crossover operation selects a sub-string where the cut off points do not have to be project or resource changing positions of the chromosome, and then exchanges these sub-strings on the parents. The general idea behind these crossover operations is the generation of individuals inheriting good parts of their parents. On the other hand, mutation operators try to establish the diversity in the population by changing an individual. There are three different mutation operations employed here. The first mutation operation selects two projects and a resource, and then resource dedications of these two selected projects are exchanged. In the second mutation operation one project and two resources are selected, and then the resource dedications of selected project for these two resources are exchanged. The last mutation operation selects two different resource dedications of two projects and exchanges them.

The individuals generated from GA operators can have resource infeasibility caused by corresponding resource dedication values. These infeasibilities (if any exists) are corrected by decreasing resource dedications of the projects according to the general resource capacities. These repair mechanisms require at worst (number of projects x number of resources) or (number of projects) operations for different crossover operators and as many as (number of projects) operations for mutation operators and hence consume negligible amount of CPU time. In GA, CA for RDP is used on randomly selected individuals.

Based on preliminary test runs, the probabilities for each crossover and mutation type operators as well as CA for RDP is set to 0.1.

A pseudo-code for the execution of GA is given below.

Initialization: Generate initial population as described above

Run: Apply the steps 1-4 until the allowed execution time limit is reached

Step 1 (Crossover): For each crossover operator check crossover probability for each individual in the current population. If crossover operator is issued to be applied, select a mate individual randomly and apply crossover operator. Check resource dedication infeasibility, if any infeasibility exists, restore feasibility.

Step 2 (Mutation): For each mutation operator, check mutation probability for each individual in the current population. If mutation operator is issued to be applied, execute mutation operator on the individual. Check resource dedication infeasibility, if any infeasibility exists, restore feasibility.

Step 3 (Combinatorial Auction for RDP): Check CA for RDP probability for each individual. If CA for RDP is issued to be applied, execute CA on RDP for the individual.

Step 4 (Passing to the Next Generation): Select the best 10 individuals as the next generation, delete the remaining individuals.

4.2 Lagrangian Relaxation Based Heuristic for Resource Dedication Problem

The proposed solution approach employs a Lagrangian relaxation of RDP and the subgradient optimization methodology to search for the Lagrangian multipliers. In the following sections, Lagrangian relaxation formulation and the details of the subgradient optimization approach are given.

4.2.1 Lagrangian Relaxation of Resource Dedication Problem

The *Model RDP* has two basic groups of constraints. The first group is related with the scheduling of individual projects, namely the activity assignment constraint sets (8), precedence constraint sets (9) and weighted tardiness calculation (14) for each project. The other group of constraints is related with the resource dedication. Constraint sets (10) and (11) limit the renewable and nonrenewable resource usage with the corresponding dedicated resource limits, respectively. And finally constraint sets (12) and (13) limit the total resource dedication to projects with the available overall resource capacities.

Relaxing constraint sets (10) and (11) leads to the following Lagrangian relaxation designated as *LA-RDP*:

Mathematical Model LA-RDP

$$\begin{aligned} \min. z_{LA-RDP} = & \sum_{v=1}^V TC_v + \sum_{v=1}^V \sum_{k=1}^K \sum_{t=1}^T \lambda_{vkt} \left\{ \sum_{j=1}^{N_v} \sum_{m=1}^{M_{vj}} \sum_{q=t}^{t+d_{vjm}-1} r_{vjkm} x_{vjmq} - BR_{vk} \right\} \\ & + \sum_{v=1}^V \sum_{i=1}^I \mu_{vi} \left\{ \sum_{j=1}^{N_v} \sum_{m=1}^{M_{vj}} \sum_{t=E_{vj}}^{L_{vj}} w_{vjim} x_{vjmt} - BW_{vi} \right\} \end{aligned} \quad (34)$$

Subject to

$$(8),(9),(12),(13),(14),(15),(16), (17), (18)$$

Note that the objective function of *LA-RDP* has two main parts, one in terms of the activity finish time decision variables and the other in terms of the resource dedication decision variables. If this separation is taken as the basis, the constraint sets can also be decomposed. Constraint sets (8), (9) and (14) contain activity finish decision variables; whereas constraint sets (12) and (13) contain renewable and nonrenewable resource dedication decision variables, respectively. Thus, model *LA-RDP* decomposes into three subproblems *SP1*, *SP2* and *SP3* related with activity finish, renewable resource dedication and nonrenewable resource dedication decision variables, respectively.

SP1

$$\begin{aligned} \min. z_{SP1} = & \sum_{v=1}^V TC_v + \sum_{v=1}^V \sum_{k=1}^K \sum_{t=1}^T \lambda_{vkt} \left\{ \sum_{j=1}^{N_v} \sum_{m=1}^{M_{vj}} \sum_{q=t}^{t+d_{vjm}-1} r_{vjkm} x_{vjmq} \right\} \\ & + \sum_{v=1}^V \sum_{i=1}^I \mu_{vi} \left\{ \sum_{j=1}^{N_v} \sum_{m=1}^{M_{vj}} \sum_{t=E_{vj}}^{L_{vj}} w_{vjim} x_{vjmt} \right\} \end{aligned} \quad (35)$$

Subject to

$$(8), (9), (14), (18)$$

SP1 is basically scheduling the individual projects without any resource constraints and can be further decomposed with respect to individual projects. The resulting schedules will be precedence feasible but can be resource infeasible for each project according to the results of resource subproblems *SP2* and *SP3*.

SP2

$$\min. z_{SP2} = \sum_{v=1}^V \sum_{k=1}^K \sum_{t=1}^T \lambda_{vkt} (-BR_{vk}) \quad (36)$$

Subject to

$$BR_{vk} \leq ur_{vk} \quad \forall k \in K \text{ and } \forall v \in V \quad (37)$$

$$(12), (15)$$

SP3

$$\min. z_{SP3} = \sum_{v=1}^V \sum_i^I \mu_{vi}(-BW_{vi}) \quad (38)$$

Subject to

$$BW_{vi} \leq uw_{vi} \quad \forall i \in I \text{ and } \forall v \in V \quad (39)$$

$$(13), (16)$$

where in *SP2* and *SP3*, ur_{vk} and uw_{vi} are resource usage values for renewable and nonrenewable resources, respectively when CPM is applied to individual project networks. Thus, they are no-delay resource requirements for individual projects.

SP2 and *SP3* are for resource dedication decision variables. They are further decomposable in terms of individual renewable and nonrenewable resources, respectively. The additional constraints (37) and (39) limit the dedicated renewable and nonrenewable resources, respectively. Note that if these constraints are not added to *SP2* and *SP3*, all the general resource capacity would be dedicated to the project, which has the largest λ_{vkt} summed over t and similarly to the one with the largest μ_{vi} . Thus, these constraints prevent the abuse of the general resource capacities by the corresponding project. The resulting resource dedications for the projects will be feasible for the general resource limits (R_k, W_i) .

The combination of optimal solutions of the subproblems *SP1*, *SP2*, and *SP3* is a relaxation for RDP, since the additional constraints (37) and (39) limit the resource dedication decision variables with their possible maximum value (no-delay resource requirements). Note that this relaxation also gives a tighter lower bound than the original relaxation of the problem with the additional constraints (37) and (39) to *SP2* and *SP3*, respectively, since these additions prevent the abuse of the renewable and nonrenewable resource dedication decision variables.

4.2.2 Subgradient Optimization for Lagrangian Relaxation of Resource Dedication Problem

Subgradient optimization approach is used to obtain a solution to RDP. There are three basic steps in an iteration of subgradient optimization: (i) obtaining a lower bound (LB) with a given set of Lagrangian multipliers, (ii) obtaining an upper bound (UB) and (iii) updating the Lagrangian coefficients. On solving the three subproblems, an LB for the RDP can be obtained. *SP1* can be solved using exact solution approaches. The resource subproblems are continuous knapsack problems, which are also easy to solve.

An efficient UB is calculated using the results of the Lagrangian relaxation. The solutions of resource subproblems could lead to a resource dedication, which is infeasible for some projects. Thus, resource dedication results cannot be used directly for an UB calculation. In order to overcome this deficiency for UB calculation, resource dedication for each project is calculated as follows:

$$BR_{vk}^{UB} = R_k \frac{BR_{vk}}{\sum_{v=1}^V BR_{vk}} \quad (40) \quad \text{for each renewable resource } k, \text{ for each project } v$$

$$BW_{vi}^{UB} = W_i \frac{BW_{vi}}{\sum_{v=1}^V BR_{vi}} \quad (41) \quad \text{for each nonrenewable resource } i, \text{ for each project } v$$

This resource dedication calculation approach normalizes the resource dedication over the resource dedication values of the Lagrangian relaxation problem. After setting BR_{vk}^{UB} and BW_{vi}^{UB} values, RDP reduces to solving individual MRCPSP for each project. The UB calculation for RDP is carried out by solving individual MRCPSP for each project with the given BR_{vk}^{UB} and BW_{vi}^{UB} values and Lagrangian coefficients are calculated accordingly.

5 Experimental Results

To test the solution approaches proposed here for RDP a series of test problems are used. Test problems are grouped according to the network complexity (NC) and the maximum utilization factor (MUF) [13]. NC is calculated as the total number of arcs divided by total number of nodes in the project network. For the multi-project scheduling problems presented in this study, for a determined network complexity value for the multi-project problem, the NC values of all the projects in the problem are set to this value (i.e. if the multi-project problem has a NC value of 1.4 then all the individual projects in the multi-project problem has a NC value of 1.4).

MUF is the ratio of the resource requirement of no-delay schedule of the project to available resource, i.e., resource capacity. Thus, if MUF is less than or equal to one, the project can be scheduled without any delays. Since the resources cannot be shared, combining the projects and calculating the MUF values are not suitable for the proposed multi-project problem environment. In order to take into account resource dedication concept, the no-delay resource requirement of the multi-project problem is calculated as the sum of no-delay resource requirements of the individual projects. With this approach, if MUF value is less than or equal to one, the multi-project problem can have a no-delay schedule for the dedicated resources case. Similarly when MUF value is increased, resources become tight for the multi-project problem.

Multi-project problems are created with activity-on-node representation combining 6 different projects from j20 and j30 sets in PSPLIB (<http://129.187.106.231/psplib/>) [11]. Two different levels of NC (1.4 and 1.8) and three different levels of MUF (1.2, 1.4, and 1.5) are selected and a full factorial design with 10 problems in each combination is created. In order to compare different methods, for each combination a total of 10 base problems are used. For example, the problems in combination (NC 1.4 - MUF 1.5) and (NC 1.8 - MUF 1.5) have the same network structure, but the projects in the first combination has some of its arcs deleted to achieve a NC value of 1.4. Similarly, (NC 1.8 - MUF 1.4) and (NC 1.8 - MUF 1.5) have exactly the same network structure (same number of arcs, nodes and same modes for activities)

but the general resource capacities differ. With this approach, when an optimal solution is found for a problem, it can be used as a lower bound for the other cases with larger NC and/or MUF values. In other words, the weighted tardiness values for the combination (NC 1.4 - MUF 1.2) can be used as a lower bound for the other test cases.

To have a positive weighted tardiness value for projects in the multi-project problem environments, the following approach is used. The due date of the project with the highest weight is set as the makespan calculated for the unconstrained case using CPM (no-delay due date). As the weight decreases, projects are assigned tighter due dates than their no-delay due date. As a result, the minimum possible total weighted tardiness becomes a fixed value for the multi-project problem as shown in Table 1. This value can be considered as a lower bound for the multi-project problem. The test problems can be downloaded from the link “<http://www.bufaim.boun.edu.tr/flexset.zip>”.

Table 1 Possible least weighted tardiness values for individual projects

Project	Due date	Weight	Possible Least Weighted Tardiness
Project1	Nodelay due date	6	0
Project2	Nodelay due date - 1	5	5
Project3	Nodelay due date - 2	4	8
Project4	Nodelay due date - 3	3	9
Project5	Nodelay due date - 4	2	8
Project6	Nodelay due date - 5	1	5
Possible Least Total Weighted Tardiness			35

Solution approaches are coded with *Microsoft Visual Studio 2010 C#*. For all the problems that are solved with an exact solution approach, *ILOG CPLEX Component Library* is used employing *CPLEX 11.2*. For the exact solution approach of RDP, a total of 4096 Mb working memory is allocated and the hard drive is used when this allocation is exceeded. Test runs are carried out on an *Intel Xeon X 5492, 3.40 Ghz* processor.

Results for multi-project problems consisting of projects with 22 and 32 activities are presented in Table 2 and Table 3, respectively, where GA-LA and GA-LinR refer to GA with CA based on Lagrangian relaxation and CA based on linear relaxation, respectively. SO column refers to the Subgradient Optimization based solution procedure. Exact column is for the exact solutions. The NC-MUF column shows the corresponding network complexity and maximum utilization factors used for the problem groups. As it is mentioned earlier, there are 10 problem instances in a problem group. The AWT column reports the average weighted tardiness for a problem group, whereas the values in the ART column show the average execution time of the solution approaches in minutes for a problem group. If a solution approach cannot reach a feasible solution for an instance within the execution time limit, then AWT value is set to NA (not available). All of the solution approaches have a execution time limit of 120 minutes. OS column shows the number of instances for which optimal solution is found in a problem group whereas

NS column shows the number of instances where no solution could be found in a problem group within the execution time limit. When the sum of OS and NS is less than 10, it shows that the exact solution approach could only find an incumbent solution (feasible but not proven optimal) in the given execution time limit for the remaining problem instances.

Table 2 Results for the problem groups consisting of 10 problem instances each and containing 6 projects with 22 activities

NC-MUF	GA-LA				GA-LinR				SO				Exact			
	AWT	ART	OS	NS	AWT	ART	OS	NS	AWT	ART	OS	NS	AWT	ART	OS	NS
1.4-1.2	35	6.19	10	0	35	6.19	10	0	35	4.65	10	0	35	1.39	10	0
1.4-1.4	42.6	94.4	3	0	46.6	100.6	3	0	56.6	111.76	2	0	NA	101.17	4	6
1.4-1.5	52.9	117.29	3	0	66.9	120	1	0	86.7	120	0	0	NA	120	0	10
1.8-1.2	35	3.38	10	0	35	4.42	10	0	35	16.02	10	0	35	1.44	10	0
1.8-1.4	41.4	98.99	5	0	45.6	100.07	4	0	64.6	120	0	0	NA	95.38	4	5
1.8-1.5	50.6	120	2	0	67.9	120	0	0	101.7	120	0	0	NA	112.1	1	9

Table 3 Results for the problem groups consisting of 10 problem instances each and containing 6 projects with 32 activities

NC-MUF	GA-LA				GA-LinR				SO				Exact			
	AWT	ART	OS	NS	AWT	ART	OS	NS	AWT	ART	OS	NS	AWT	ART	OS	NS
1.4-1.2	35	14.81	10	0	35	18.22	10	0	35	5.98	10	0	35	12.03	10	0
1.4-1.4	56	74.07	5	0	155	103.11	3	0	131.9	120	0	0	NA	90.11	3	5
1.4-1.5	181	110.08	1	0	269	109.24	1	0	314.5	120	0	0	NA	102.23	2	8
1.8-1.2	35	8.73	10	0	35	17.59	10	0	38.6	90.05	2	0	35	12.07	10	0
1.8-1.4	82.2	88.45	5	0	131	112.77	1	0	132	123	0	0	NA	70.26	5	4
1.8-1.5	148	110.15	1	0	247	110.06	1	0	302.9	120	0	0	NA	104.82	2	7

The results for RDP are examined for solution quality and solution times and compared using *paired t-test* with 0.05 level of significance. First of all, when MUF values are closer to 1, all solution approaches give optimal values for all or most of the problems in a problem group. But as MUF values increase, exact solution approach begins to fail finding solutions in the given solution runtime limit and falls behind the other approaches. Both GA approaches give results above the given lower bound. Thus, problem groups with MUF values closer to 1 can be thought as relatively easy problems whereas problem groups with higher MUF values can be thought of as relatively hard problems, which is in fact an expected result.

SO approach falls behind GA approaches even though it can find feasible solution for all instances for all problem groups. Both of the GA approaches GA-LA and GA-LinR give overall good results for all problem groups. GA-LA is significantly better than the other solution approaches according to the

solution quality aspect. Especially when the resources are tight, Lagrangian relaxation based preference calculation is significantly better than linear relaxation based preference calculation. The statistical test results also show that network complexity is not a significant factor for solution quality and only have an additional initialization load for solution time.

The run times are compared for the cases where all solution approaches reach the optimum objective values, because otherwise the procedures are stopped at the run time limit. When these values are compared, no statistically significant difference is observed among the solution approaches. But note that solution times are reasonable for relatively easy problems for all approaches.

6 Summary and Further Research Topics

In this paper a new approach for multi-project scheduling environments is presented where resources cannot be shared among projects and must be dedicated to individual projects. The case of dedicated resources and the corresponding scheduling problem is defined as the Resource Dedication Problem. A general mathematical model for RDP is given. Different solution approaches based on GA and Lagrangian relaxation based heuristics are proposed for RDP. The crucial part of the proposed GA is the new improvement heuristic CA for RDP. CA for RDP is based on the preference definition of projects for resources, which has two different approaches proposed for preference calculation: (i) Linear relaxation based and (ii) Lagrangian relaxation based. The other solution approach for RDP is based on the Lagrangian relaxation formulation of RDP. The resulting Lagrangian relaxation formulation is separated with respect to project scheduling and resource dedication decision variables and corresponding subproblems are solved to obtain a lower bound. To update the Lagrangian coefficients subgradient optimization approach is used.

Different test problems with different characteristics are used to test and compare solution approaches. The proposed solution approaches give satisfactory results for test problems according to the solution times and solution quality. Exact solution approach with the given formulation for RDP fails to give results within the given time limit when problems are relatively hard to solve. It is seen that CA for RDP gives overall best results. In addition to this, subgradient optimization and GA employing linear relaxation based preference calculation gives overall good results in reasonable solution times.

For future research, two different extensions for RDP can be examined. The first case would be the introduction to the problem of a budget for resource investment, which would be used to determine the overall resource capacities. In other words, the problem will be moved to a higher level of decision where the amounts of general resource capacities become a decision variable constrained among others by the given resource investment budget. The other case for future research would be the investigation of RDP in a dynamic multi-project environment.

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